Reheating in α-attractors

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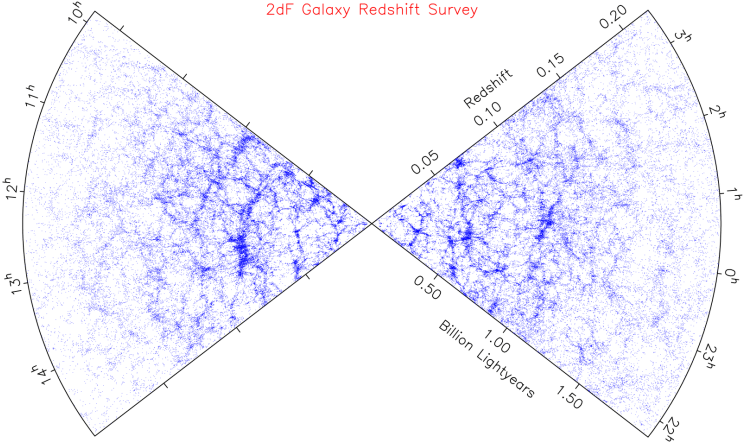
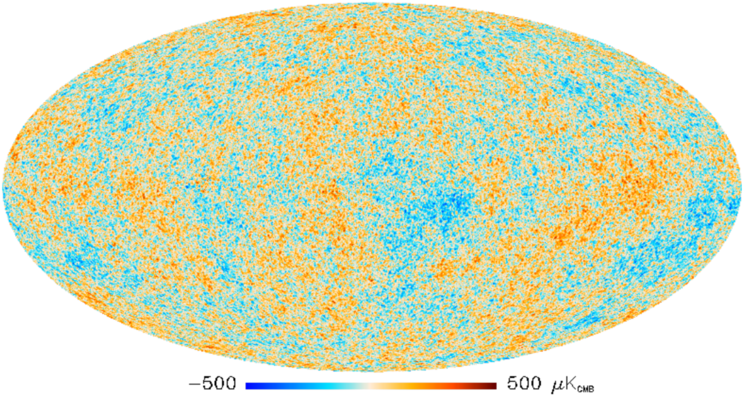
Large Scale Structure



Visualization of data from 2dF galaxy catalog.[1]

1. Colless, M. *et al. Mon. Not. Roy. Astron. Soc.* **328,** 1039. arXiv: astro-ph/0106498 [astro-ph] (2001).

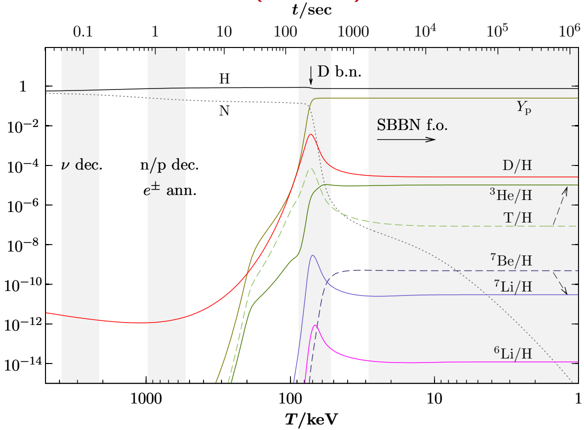
Cosmic Microwave Background (CMB) Radiation



CMB map reconstructed from Planck data.[2]

2. Ade, P. A. R. *et al. Astron. Astrophys.* **571,** A1. arXiv: 1303.5062 [astro-ph.CO] (2014).

Big Bang Nucleosynthesis (BBN)



Production of light elements in BBN.[3]

3. Pospelov, M. & Pradler, J. *Ann. Rev. Nucl. Part. Sci.* **60,** 539–568. arXiv: 1011.1054 [hep-ph] (2010).

Conclusions from observational data

- The current Universe is homogeneous at scales larger than $100\mathrm{Mpc}$.
- The Universe was (nearly) isotropic during recombination. Relative fluctuations of the CMB radiation are of the order of:

$$\frac{\Delta T}{T} \sim 10^{-5}.$$

- The energy density of the Universe during BBN was dominated by radiation (particles moving with relativistic speeds).

Cosmological inflation

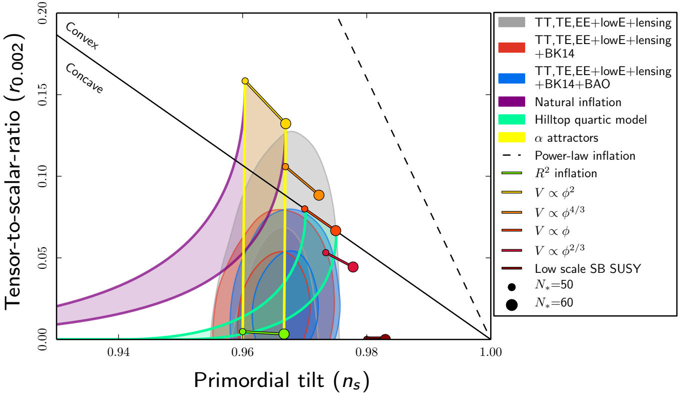
Cosmological inflation allows for simultaneous solution for many problems in cosmology:

- horizon problem
- flatness problem
- magnetic monopoles problem

Moreover, it provides a very natural explanation of CMB inhomogeneities.

CMB fluctuations measurement

Predictions of some multi-field inflationary models (especially α-attractor ones[4]) fits very well to current measurements of CMB radiation inhomogeneities.



Constraints on inflationary models from Planck2018[5].

4. Carrasco, J. J. M. *et al.* *Phys. Rev. D* **92,** 063519. arXiv: `1506.00936 [hep-th]` (2015).
5. Akrami, Y. *et al.* arXiv: `1807.06211 [astro-ph.CO]` (2018).

α-attractor T-model

α-attractor models of inflation origin from supergravity.
T-model of inflation is characterized by the superpotential:

$$W_H = \sqrt{\alpha}\mu S \left(\frac{T-1}{T+1} \right)^n,$$

and by the Kähler potential:

$$K_H = -\frac{3\alpha}{2} \log \left(\frac{(T-\bar{T})^2}{4T\bar{T}} \right) + S\bar{S}.$$

Two-fields α-attractor T-model

The scalar sector can be expressed in terms of two real scalar fields ϕ and χ. The scalar Lagrangian takes particularly simple form:

$$\mathcal{L} = -\frac{1}{2}\left(\partial_\mu\chi\partial^\mu\chi + e^{2b(\chi)}\partial_\mu\phi\partial^\mu\phi\right) - V(\phi,\chi), \qquad b(\chi) := \log(\cosh(\beta\chi)),$$

with the potential:

$$V(\phi,\chi) = M^4\left(\frac{\cosh(\beta\phi)\cosh(\beta\chi) - 1}{\cosh(\beta\phi)\cosh(\beta\chi) + 1}\right)^n \left(\cosh(\beta\chi)\right)^{2/\beta^2}, \qquad M^4 := \alpha\mu^2,$$

and

$$\beta := \sqrt{\frac{2}{3\alpha}}.$$

One-field α-attractor T-model

In the literature one-field simplification is considered with the potential of the form:

$$V(\phi, 0) = M^4 \tanh^{2n}\left(\frac{\beta|\phi|}{2}\right).$$

In order to find inflationary trajectory (at least the part along $\chi = 0$ direction) one need to solve set of coupled differential equations:

$$H^2 = \frac{1}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi, 0)\right], \qquad \ddot{\phi} + 3H\dot{\phi} + V_\phi(\phi, 0) = 0.$$

Perturbations in α-attractors

Linear perturbations are described in terms of gauge invariant Mukhanov-Sasaki variables:

$$Q_\phi := \delta\phi + \frac{\dot{\phi}}{H}\Psi, \qquad Q_\chi := \delta\chi + \frac{\dot{\chi}}{H}\Psi,$$

which obey the following equations of motion

$$\ddot{Q}_\varphi + 3H\dot{Q}_\varphi + \left(\frac{k^2}{a^2} + m_\varphi^2\right)Q_\phi = 0, \quad \text{for } \varphi = \phi, \chi,$$

where the effective masses m_φ^2 of perturbations are:

$$m_\phi^2 = V_{\phi\phi}(\phi,\chi), \qquad m_\chi^2 = V_{\chi\chi}(\phi,\chi) + \frac{1}{2}\dot{\phi}^2\mathbb{R} \quad \text{for } \mathbb{R} = -2\beta^2.$$

Perturbation around oscillating inflaton

After inflation inflaton oscillates around minimum of the potential loosing its energy in favor of other degrees of freedom. Assuming that this process is slow one may approximate evolution of (non)harmonic oscillations with slowly decaying amplitude.
The equations of motion for Mukhanov-Sasaki variables can be written in Fourier space as first order ordinary differential equations:

$$\begin{pmatrix} \dot{Q}_{\varphi,k} \\ \dot{\Pi}_{\varphi,k} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\left(\frac{k^2}{a^2} + m_\varphi^2\right) & -3H \end{pmatrix} \begin{pmatrix} Q_{\varphi,k} \\ \Pi_{\varphi,k} \end{pmatrix} =: \Lambda_\varphi(t) \begin{pmatrix} Q_{\varphi,k} \\ \Pi_{\varphi,k} \end{pmatrix},$$

with $\Lambda_\varphi(t)$ being (nearly) periodic matrices.

Floquet Theorem

Let

$$\dot{x}(t) = U(t)x(t) \tag{1}$$

be a linear first order differential equation, where:

- $x(t)$ is a column vector of dim N,
- $U(t)$ is a periodic $N \times N$ matrix valued function with period T.

The fundamental solution $O(t, t_0)$ of eq. (1) can be expressed as:

$$O(t, t_0) = P(t, t_0) \exp\left((t - t_0)V\right)$$

where:

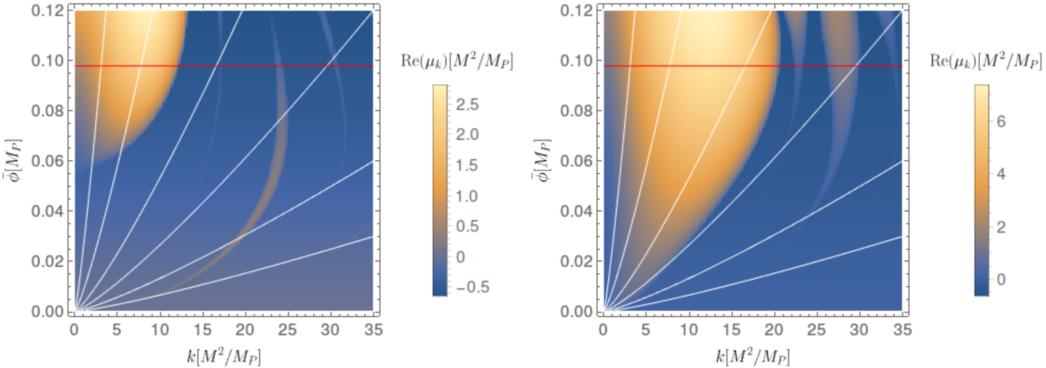
- $P(t, t_0)$ is a periodic matrix valued function with period T,
- V is constant $N \times N$ matrix satisfying $O(t_0 + T, t_0) = \exp(TV)$.

Amplification of linear perturbations

In virtue of Floquet theorem one deduces that linear perturbations can be approximated by:

$$Q\varphi, k(t) = \sum_{\psi=\phi,\chi} Q^{\psi}_{\varphi,k}(t, t_0) \exp\left(\mu^{\psi}_{\varphi,k}(t - t_0)\right)$$

during few oscillations of inflaton field.
Large positive real parts of Floquet exponents $\mu^{\psi}_{\varphi,k}$ indicate amplification of perturbations, thus instability.



Floquet exponents for the inflaton (left panel) and the spectator (right panel) perturbations with $n = 3/2$ and $\alpha = 10^{-3}$.[6]

6. Krajewski, T. *et al. Eur. Phys. J. C* **79,** 654. arXiv: `1801.01786 [astro-ph.CO]` (2019).

Hypernatural α-attractor T-model

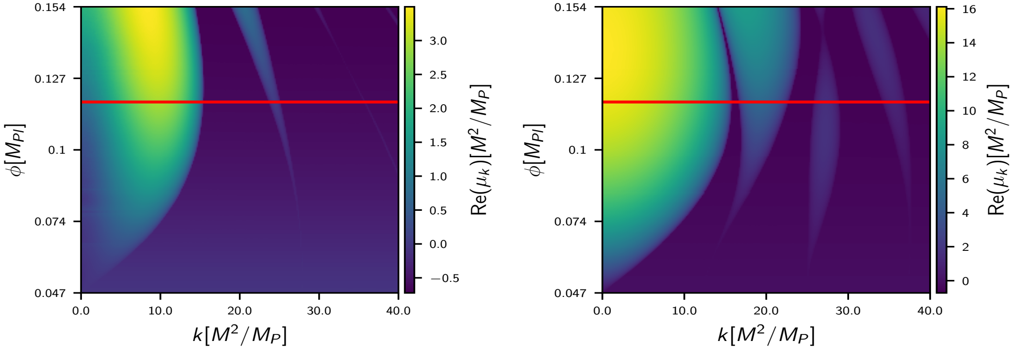
The somewhat different parametrization $Z = e^{i\theta} \tanh \frac{\varphi}{2}$ scalar Lagrangian takes the following form:[7]

$$\mathcal{L} = -\frac{1}{2}\left(\partial_\mu\varphi\partial^\mu\varphi - \frac{3\alpha}{4}\sinh^2\left(\beta\varphi\right)\partial_\mu\theta\partial^\mu\theta\right) - V(\varphi,\theta),$$

with the potential:

$$V(\varphi,\theta) = M^4\left[\left(1 - c^{-2}\tanh^2\frac{\varphi}{\sqrt{6\alpha}}\right) + 4A\cos^2\frac{n\theta}{2}\tanh^{n+2}\frac{\varphi}{\sqrt{6\alpha}}\right].$$

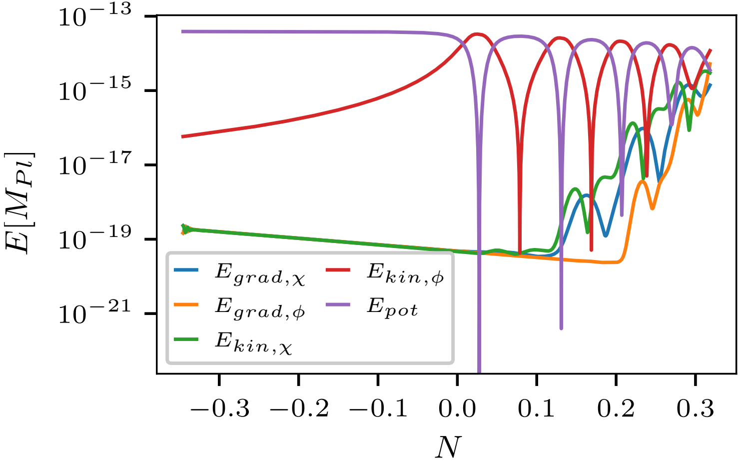
7. Linde, A. *et al.* *JCAP* **07,** 035. arXiv: 1803.09911 [hep-th] (2018).



Floquet exponents for the inflaton (left panel) and the spectator (right panel) perturbations with $c = 0.75$, $A = 0.1$, $n = 3/2$ and $\alpha = 10^{-3}$.[8]

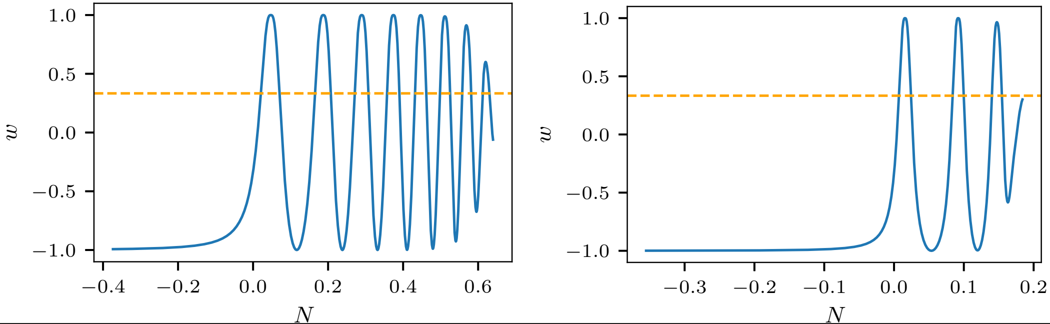
8. Kulejewski, M. Bachelor's Thesis (Faculty of Physics, University of Warsaw, 2022).

Energy transfer

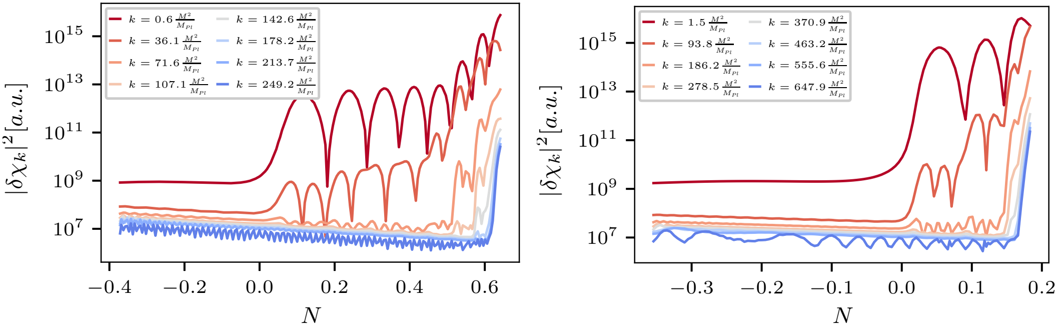


Barotropic parameter

$$w = \frac{p}{\rho} = \frac{\langle \frac{1}{2}\left(\mathrm{e}^{2b(\chi)}\dot{\phi}^2 + \dot{\chi}^2\right) - \frac{1}{6a^2}\left(\mathrm{e}^{2b(\chi)}(\nabla\phi)^2 + (\nabla\chi)^2\right) - V(\phi,\chi)\rangle}{\langle \frac{1}{2}\left(\mathrm{e}^{2b(\chi)}\dot{\phi}^2 + \dot{\chi}^2\right) + \frac{1}{2a^2}\left(\mathrm{e}^{2b(\chi)}(\nabla\phi)^2 + (\nabla\chi)^2\right) + V(\phi,\chi)\rangle},$$



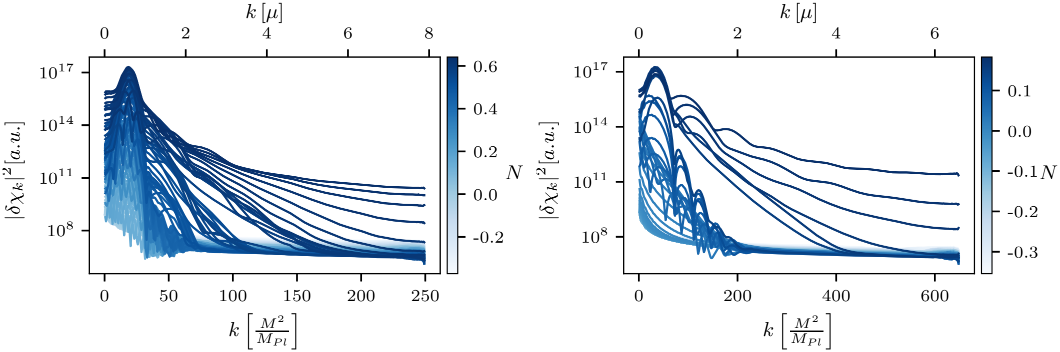
Evolution of spectator field perturbations



The time evolution of spectator field perturbations for different number of wavenumbers k for $n = 1.5, \alpha = 10^{-3}$ (left panel), $n = 1.5, \alpha = 10^{-4}$ (right panel).[9]

9. Krajewski, T. & Turzyński, K. *JCAP* **10,** 005. arXiv: 2204.12909 [astro-ph.CO] (2022).

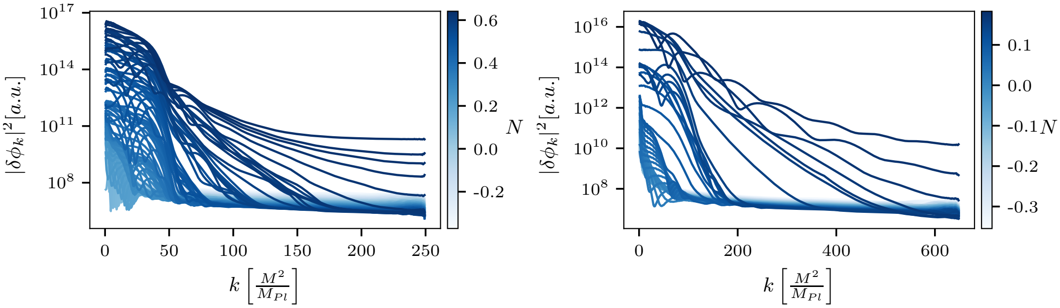
Evolution of spectator field perturbations



The time evolution of spectrum of spectator field perturbations for $n = 1.5$, $\alpha = 10^{-3}$ (left panel), $n = 1.5$, $\alpha = 10^{-4}$ (right panel).[9]

9. Krajewski, T. & Turzyński, K. *JCAP* **10,** 005. arXiv: `2204.12909 [astro-ph.CO]` (2022).

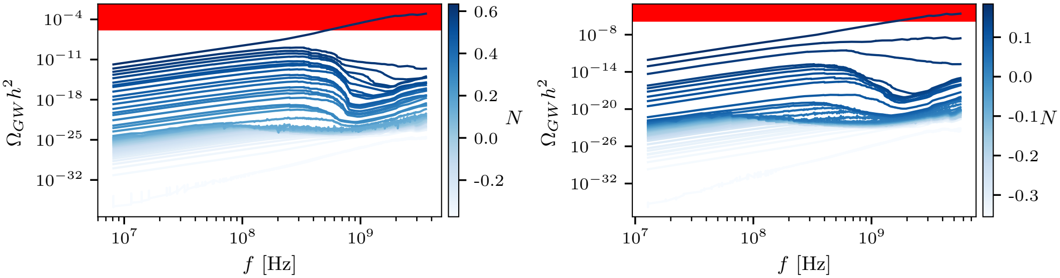
Evolution of inflaton field perturbations



The time evolution of spectrum of inflaton field perturbations for $n = 1.5$, $\alpha = 10^{-3}$ (left panel), $n = 1.5$, $\alpha = 10^{-4}$ (right panel).[9]

9. Krajewski, T. & Turzyński, K. *JCAP* **10,** 005. arXiv: 2204.12909 [astro-ph.CO] (2022).

Spectrum of gravitational waves



Evolution of the spectrum of gravitational waves as a function of the number of e-folds N from the end of inflation for $n = 1.5$, $\alpha = 10^{-3}$ (left panel), $n = 1.5$, $\alpha = 10^{-4}$ (right panel).[9]

9. Krajewski, T. & Turzyński, K. *JCAP* **10,** 005. arXiv: 2204.12909 [astro-ph.CO] (2022).

Summary

1. Geometrical destabilization may take place in multi-field inflationary models with negative curvature of the field space.
2. Initial stages of preheating can be studied using Floquet analysis, however it breaks down when the produced fluctuations backreact.
3. Lattice simulations proofed that short wavelength fluctuations of the inflaton field are produced by non-linear interactions from spectator ones.
4. In α-attractors production of inflaton fluctuations is accompanied by intensive emission of gravitational waves.

Thank you for your attention.

Field space metric

Field space in α-attractor T-model has non-trivial structure:

$$\mathcal{G} = \begin{pmatrix} 1 & 0 \\ 0 & e^{2b(\chi)} \end{pmatrix},$$

with negative curvature:

$$\mathbb{R} = -2\beta^2.$$

The so called geometrical destabilization is possible with two scenarios:

- during inflation leading to perturbation of inflationary trajectory or premature end of inflation,
- around the end of inflation leading to fast (p)reheating.

Multi-field models of inflation

Geometrical destabilization may take place when action for scalar fields contain non-canonical kinetic terms.

Let us concentrate on non-linear sigma models with action given by:

$$S = \int d^4x\sqrt{-g}\left[M_{Pl}^{-2}R - \frac{1}{2}G_{IJ}\left(\phi^K\right)\partial_\mu\phi^I\partial^\mu\phi^J - V\left(\phi^K\right)\right].$$

Non-canonical kinetic terms can be introduced directly into an inflationary model (as in the case of supergravity) or can come from quantum corrections in effective theory approach.

$$\mathcal{L}_{\mathrm{eff}}\left(\phi^I\right) = \mathcal{L}_\ell\left(\phi^I\right) + \sum_i c_i\frac{\mathcal{O}_i\left(\phi^I, \partial\phi^I, \ldots\right)}{\Lambda^{\delta_i - 4}}$$

Geometry of field space

- Non-canonical kinetic terms can be interpreted as a manifestation of non-trivial geometry of the field space.

- Field space has a Riemman geometric structure with metric given by G_{IJ}.

- If the Riemann tensor associated to the field-space metric $\mathcal{R}^{I}{}_{KLJ}$ is non-trivial the field space is curved.

Inflationary trajectory

The inflationary trajectory is solution of following set of equations:

$$3H^2 M_{Pl}^2 = \frac{1}{2}\dot{\sigma}^2 + V\,,$$
$$\dot{H} M_P^2 = -\frac{1}{2}\dot{\sigma}^2\,,$$
$$\mathcal{D}_t\dot{\phi}^I + 3H\dot{\phi}^I + G^{IJ}V_{,J} = 0\,.$$

where $\frac{1}{2}\dot{\sigma}^2 \equiv \frac{1}{2}G_{IJ}\dot{\phi}^I\dot{\phi}^J$ is the kinetic energy of the fields, $\mathcal{D}_t A^I \equiv \dot{A}^I + \Gamma^I_{JK}\dot{\phi}^J A^K$ is the covariant derivative in the field space and $H := \dot{a}/a$ is the Hubble parameter with a being the scale factor of the FRW metric.

Linear perturbations

The behavior of linear fluctuations around inflationary trajectory is described by the second-order action:

$$S_{(2)} = \int dt\, d^3x\, a^3 \left(G_{IJ}\mathcal{D}_t Q^I \mathcal{D}_t Q^J - \frac{1}{a^2} G_{IJ}\partial_i Q^I \partial^i Q^J - M_{IJ} Q^I Q^J \right),$$

where $Q^I := \delta\phi^I + \frac{\dot{\phi}^I}{H}\Psi$'s are so-called Mukhanov-Sasaki variables and M_{IJ} is a mass matrix:

$$M^I{}_J = V^{,I}{}_{;J} - \frac{1}{a^3 M_{Pl}^2}\mathcal{D}_t \left(\frac{a^3}{H}\dot{\phi}^I \dot{\phi}_J \right) - \mathcal{R}^I{}_{KLJ}\dot{\phi}^K \dot{\phi}^L.$$

Equations of motion read:

$$\mathcal{D}_t\mathcal{D}_t Q^I + 3H\mathcal{D}_t Q^I + \frac{k^2}{a^2} Q^I + M^I{}_J Q^J = 0.$$

Effective mass

We can rewrite EOMs in the adiabatic-entropic basis (e^I_σ, e^I_s) where $e^I_\sigma := \dot{\phi}^I/\dot{\sigma}$ is tangent to inflationary trajectory and e^I_s is orthonormal to e^I_σ.
The EOM for superhorizon modes of the entropic fluctuations simplifies to

$$\ddot{Q}_s + 3H\dot{Q}_s + m^2_{s(\mathrm{eff})} Q_s = 0\,,$$

with the effective entropic mass

$$\frac{m^2_{s(\mathrm{eff})}}{H^2} = \frac{V_{;ss}}{H^2} + 3\eta_\perp^2 + \epsilon\,\mathbb{R} M_{Pl}^2,$$

where $\eta_\perp \equiv -\frac{V_{,s}}{H\dot{\sigma}}$, $\mathbb{R}$ is the field-space Ricci scalar and ϵ is the slow-roll parameter.