

Speed of Sound In Dense Matter



Udita Shukla
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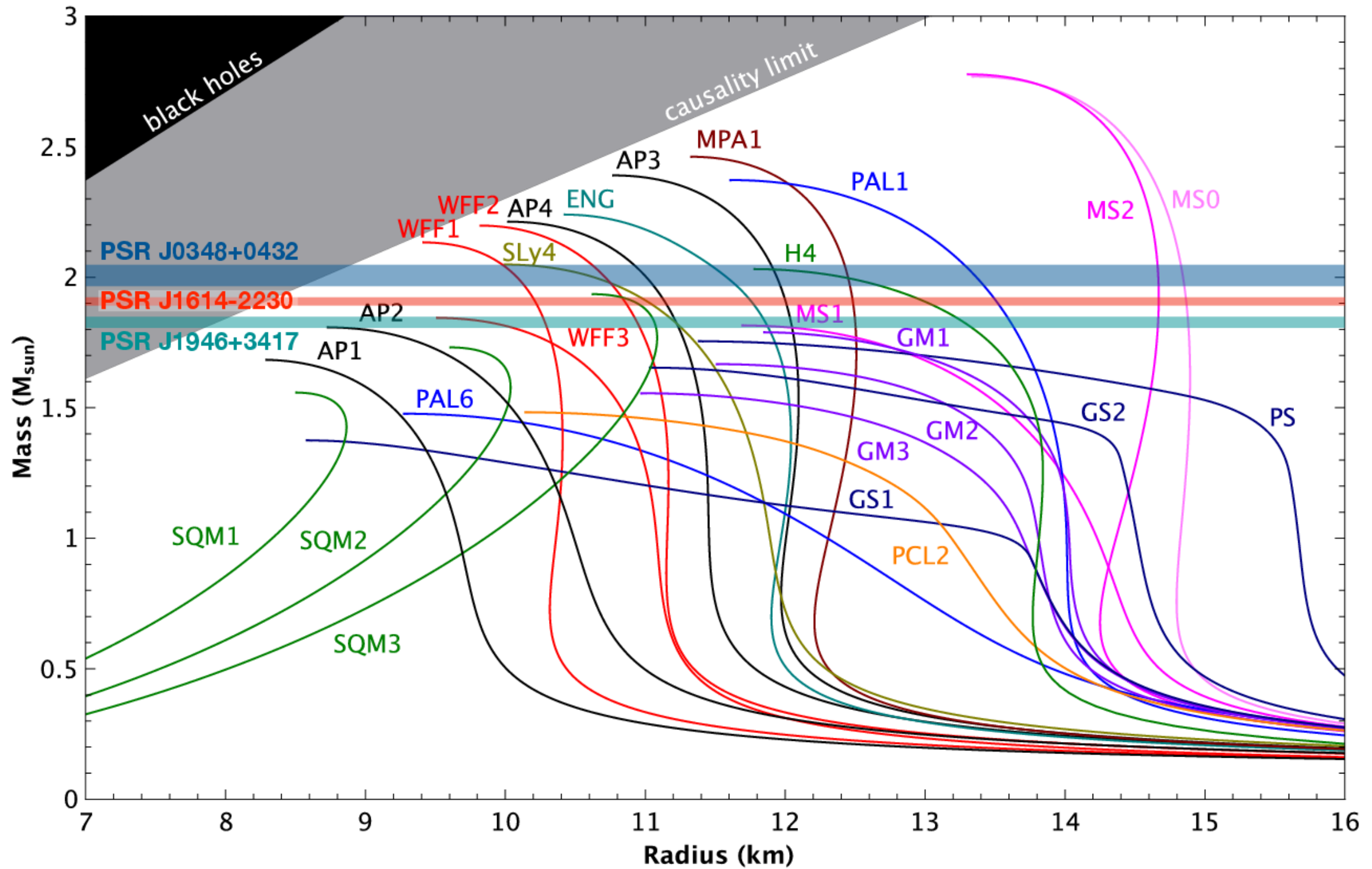
(In collaboration with Pok Man Lo, Péter Kovács, Gyozo Kovács)

V4HEP#3: THEORY AND EXPERIMENT IN HIGH ENERGY PHYSICS

01.10.2024 - 04.10.2024
PRAGUE, CZECH REPUBLIC

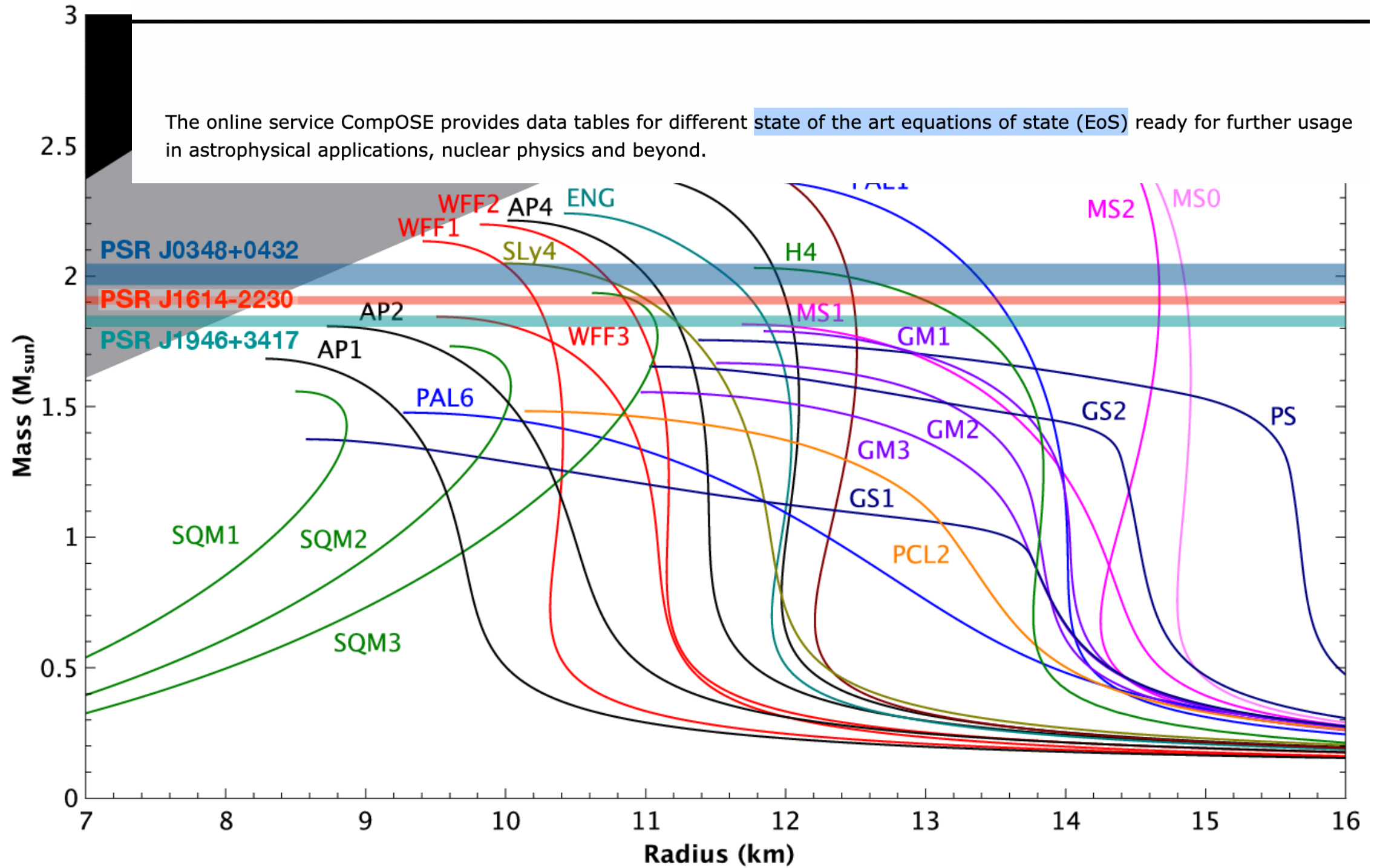


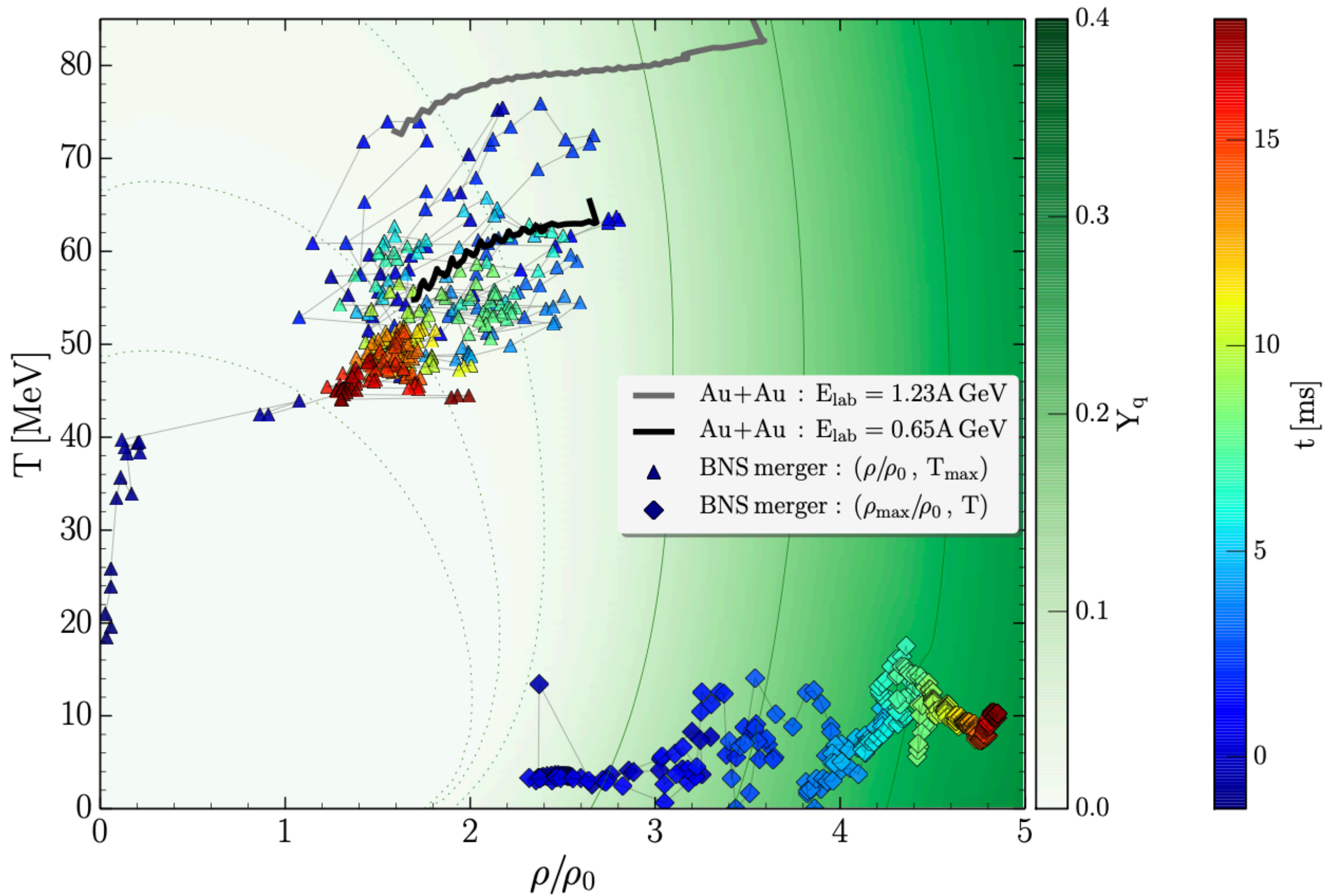
MASS : Astrophysical Constraint for high μ , low T

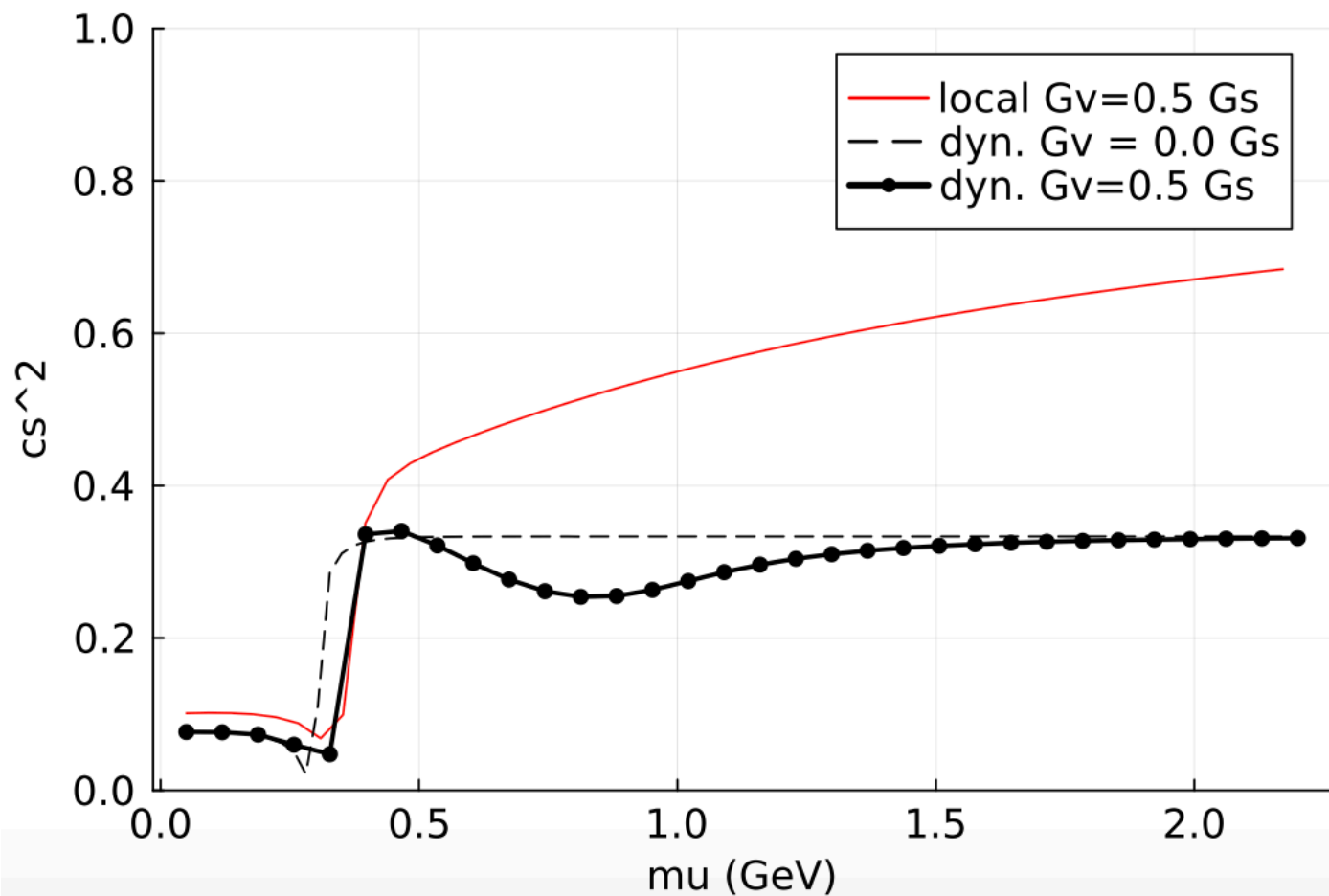
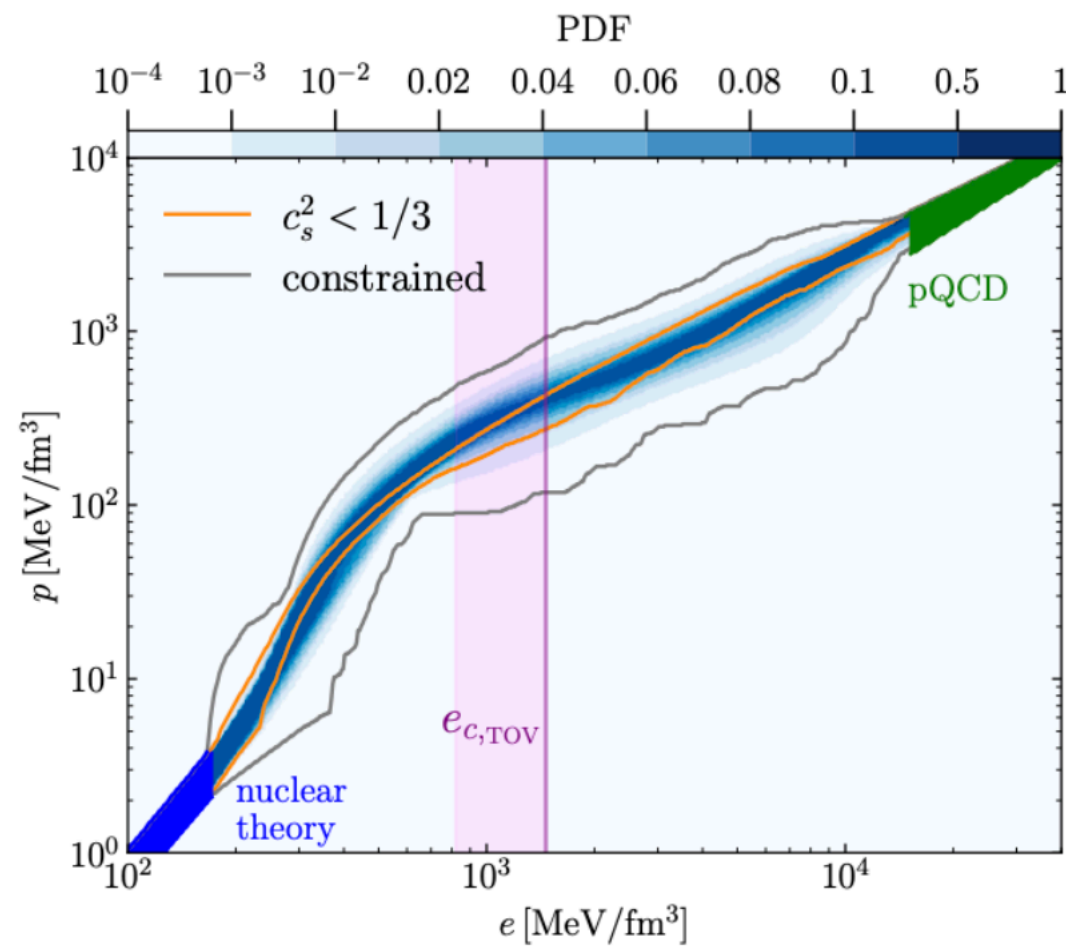
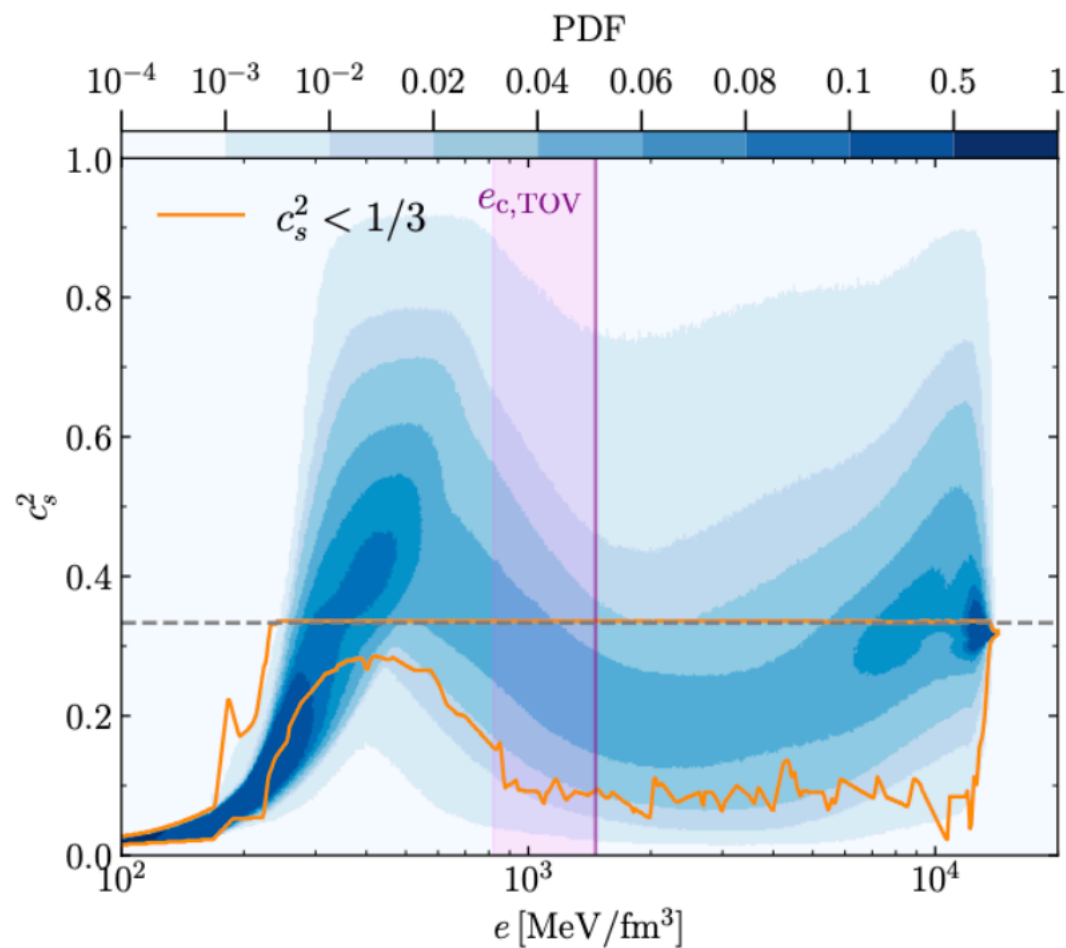




The online service CompOSE provides data tables for different state of the art equations of state (EoS) ready for further usage in astrophysical applications, nuclear physics and beyond.







NJL-like theory fails

*Dynamical
chiral Quark Model*

Table of Contents

- Model(s) for Confinement
 - Results-
 - Speed of Sound
 - QCD Phase Diagram
 - Polyakov Loop

QED IN COULOMB GAUGE

$$\mathcal{H}_{\text{Coulomb}} = \bar{\psi} (-i\vec{\gamma} \cdot \nabla + m) \psi - g\bar{\psi}\vec{\gamma}\psi \cdot \vec{A}_{\perp} + \frac{1}{2}\vec{\Pi}_{\perp}^2 + \frac{1}{2}\vec{B}^2$$

$$+ \frac{1}{2}g^2\rho \frac{-1}{\nabla^2} \rho$$

where $\rho = \bar{\psi}\gamma^0\psi$

$$\nabla \cdot \vec{A} = 0 \rightarrow \vec{A} = \vec{A}_{\perp}$$

(A^0, \vec{A})

only 2 DoFs

A_0 is **NOT** dynamical

Trade it w Gauss Law!

Potential

$$\frac{-1}{\nabla^2} \rightarrow \frac{1}{4\pi r}$$

$$\begin{aligned} -\nabla^2 A^0 &= \rho \\ \Rightarrow A^0 &= -\frac{1}{\nabla^2} \rho \end{aligned}$$

QCD IN COULOMB GAUGE

$$\mathcal{H} = -i\bar{\psi}\vec{\gamma} \cdot \nabla\psi + m\bar{\psi}\psi + \frac{1}{2}(\vec{E}^2 + \vec{B}^2) - g\bar{\psi}\vec{\gamma}T^a \cdot \vec{A}^a$$

$$+ \frac{1}{2} \rho \left[\frac{g}{\nabla \cdot D} (-\nabla^2) \frac{g}{\nabla \cdot D} \right] \rho$$

$$\rho^a = \bar{\psi}\gamma^0 T^a \psi + f^{abc} A_i^b E_c^i$$

$$\vec{D}^{ab} = \delta^{ab} \vec{\nabla} + ig T_{ab}^c \vec{A}^c$$

both quarks and gluons
 are color charged &
 confined

Potential

$$V_{ab}(x, y; \vec{A}_\perp) = \langle x, a | \left[\frac{g}{\nabla \cdot D} (-\nabla^2) \frac{g}{\nabla \cdot D} \right] | y, b \rangle$$

QCD IN COULOMB GAUGE

$$\mathcal{H} = -i\bar{\psi}\vec{\gamma} \cdot \nabla\psi + m\bar{\psi}\psi + \frac{1}{2}(\vec{E}^2 + \vec{B}^2) - g\bar{\psi}\vec{\gamma}T^a \cdot \vec{A}^a$$

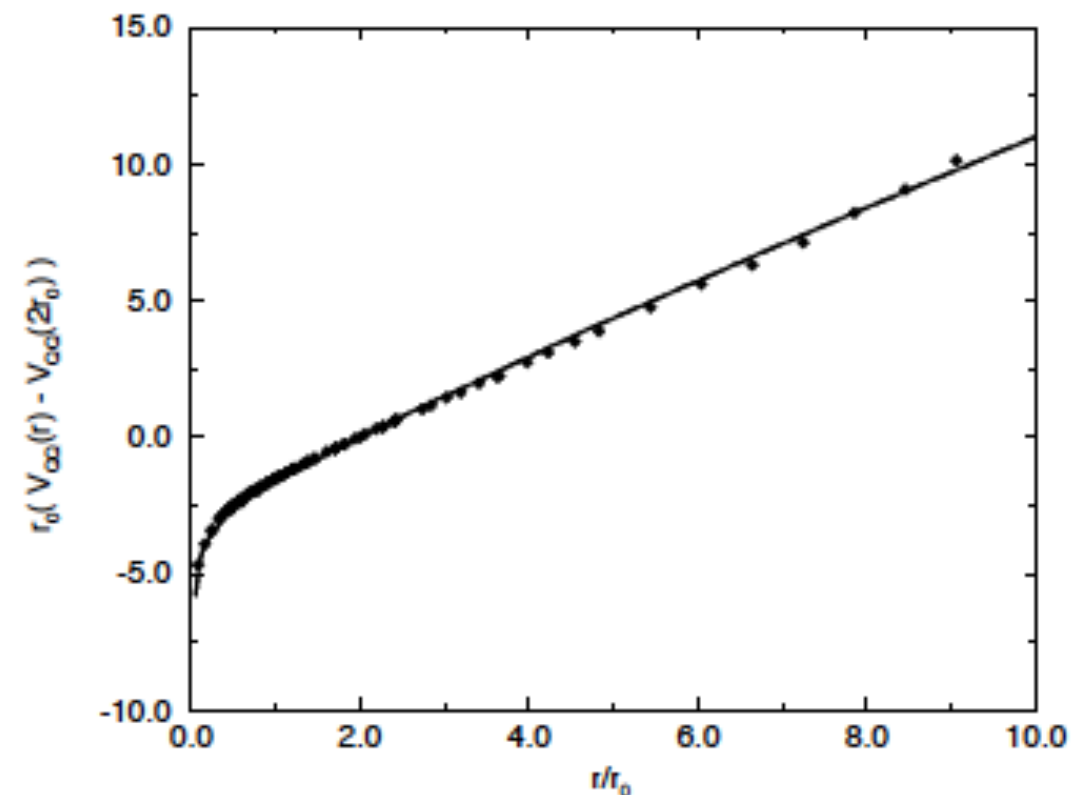
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$$\vec{D}^{ab} = \delta^{ab}\vec{\nabla} + igT_{ab}^c \vec{A}^c$$

Confining Potential



How confinement works?

“confinement” via thermal suppression

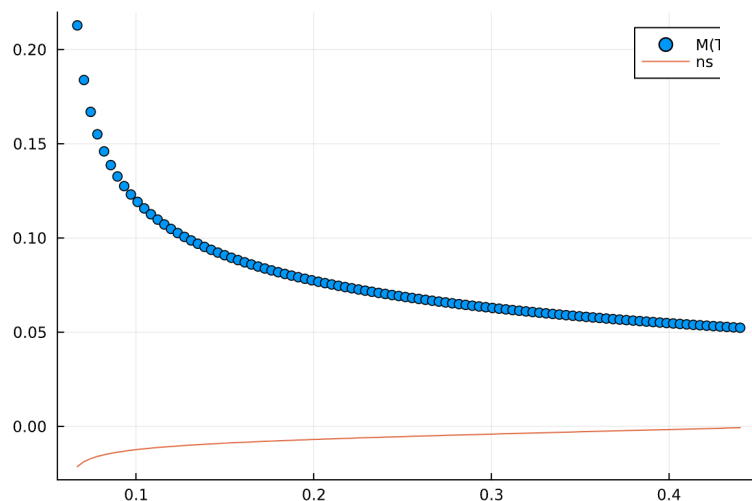
$$n_Q = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{e^{\beta(\sqrt{p^2 + M_Q^2} - \mu_Q)} + 1}$$

“confinement” via thermal suppression

$$n_Q = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{e^{\beta(\sqrt{p^2 + M_Q^2} - \mu_Q)} + 1}$$

string-flip model

$$M_Q \propto 1/n^{\frac{1}{3}} \rightarrow \infty$$



PHYSICAL REVIEW D

VOLUME 34, NUMBER 11

1 DECEMBER 1986

Pauli quenching effects in a simple string model of quark/nuclear matter

G. Röpke and D. Blaschke

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H. Schulz

Central Institute for Nuclear Research, Rossendorf, 8051 Dresden, German Democratic Republic

and The Niels Bohr Institute, 2100 Copenhagen, Denmark

(Received 16 December 1985)

*G. Roepke, D. Blaschke and H. Schulz
PRD 34 11 (1986)*

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PHYSICAL REVIEW D

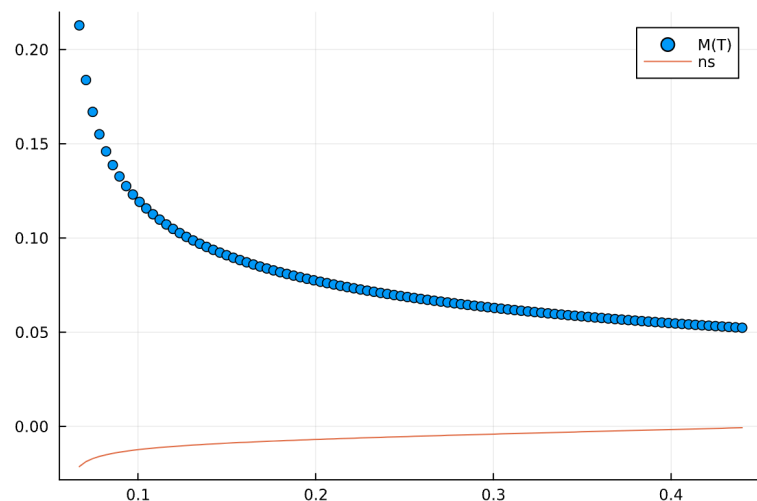
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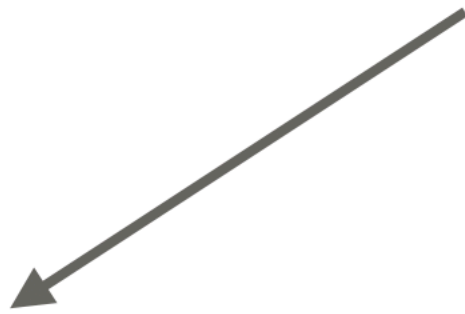
G.

BUT...

not really confinement...
mess up chiral physics in vac.
mess up hadron spectrum

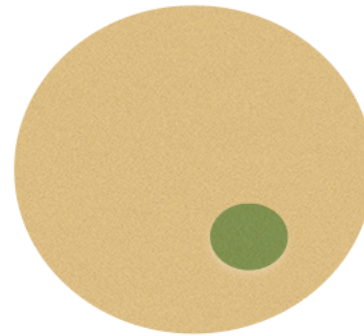
“confinement” via thermal suppression

$$n_Q = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{e^{\beta(\sqrt{p^2 + M_Q^2} - \mu_Q)} + 1}$$



Bag Model

$$\int \frac{d^3 p}{(2\pi)^3} \rightarrow \sum_{p_n}$$



$$p_{\min.} = \pi/L$$

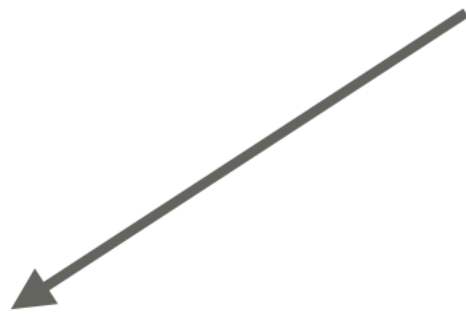
Better model of
confinement

motivates an IR scale

$$\Lambda_{\text{IR}} \approx 0.2 \text{ GeV}$$

“confinement” via thermal suppression

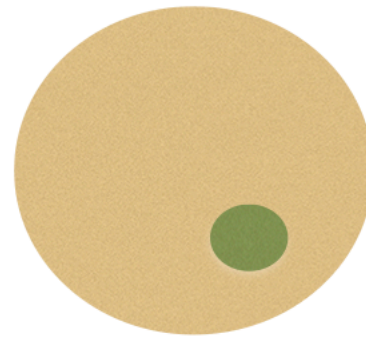
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Bag Model

$$\int \frac{d^3 p}{(2\pi)^3} \rightarrow \sum_{p_n}$$

Proton Wavefunction



$$p_{\min.} = \pi/L$$

Better model of confinement

motivates an IR scale

$$\Lambda_{\text{IR}} \approx 0.2 \text{ GeV}$$

also recently

CONFINEMENT MECHANISM IN COUOMB GAUGE QCD

“confinement” via thermal suppression

$$n_Q = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{e^{\beta(\sqrt{p^2 + M_Q^2} - \mu_Q)} + 1}$$

A-conf

$$E(p) \rightarrow \tilde{E}(p) = A(p) \sqrt{p^2 + M(p)^2}$$

$A(p), M(p)$ satisfy a coupled
Dyson eqns.

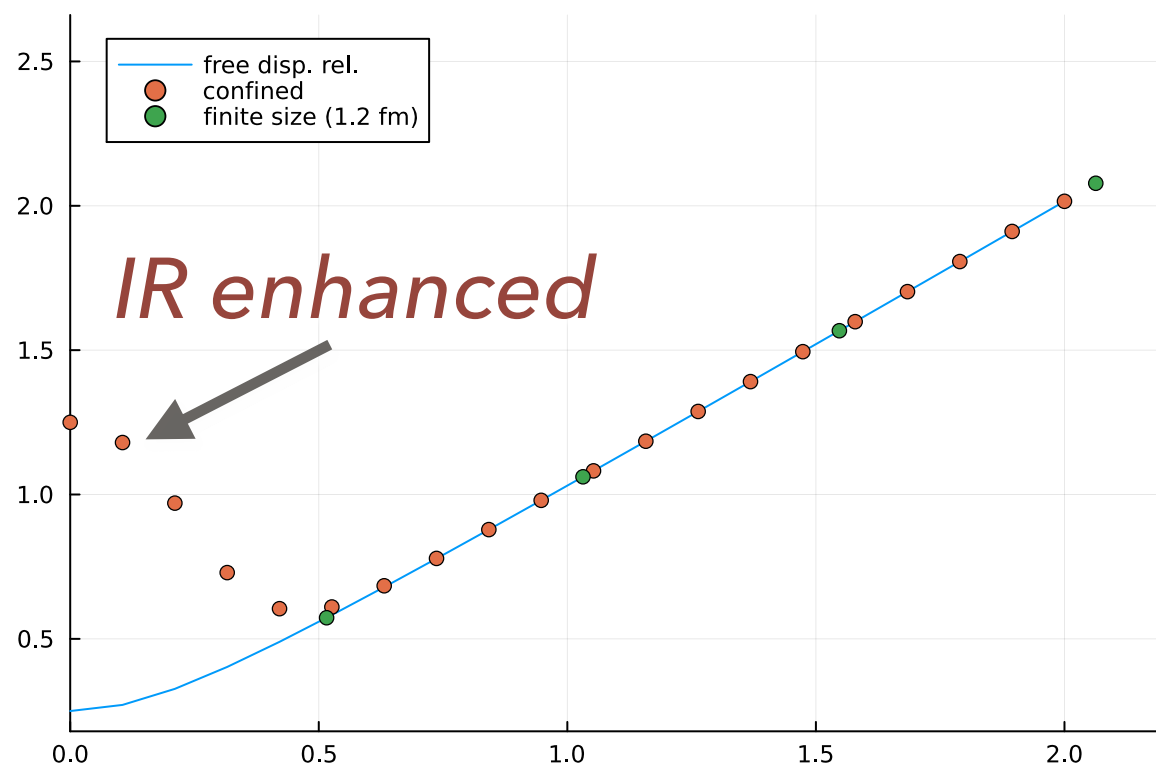
infrared enhanced!

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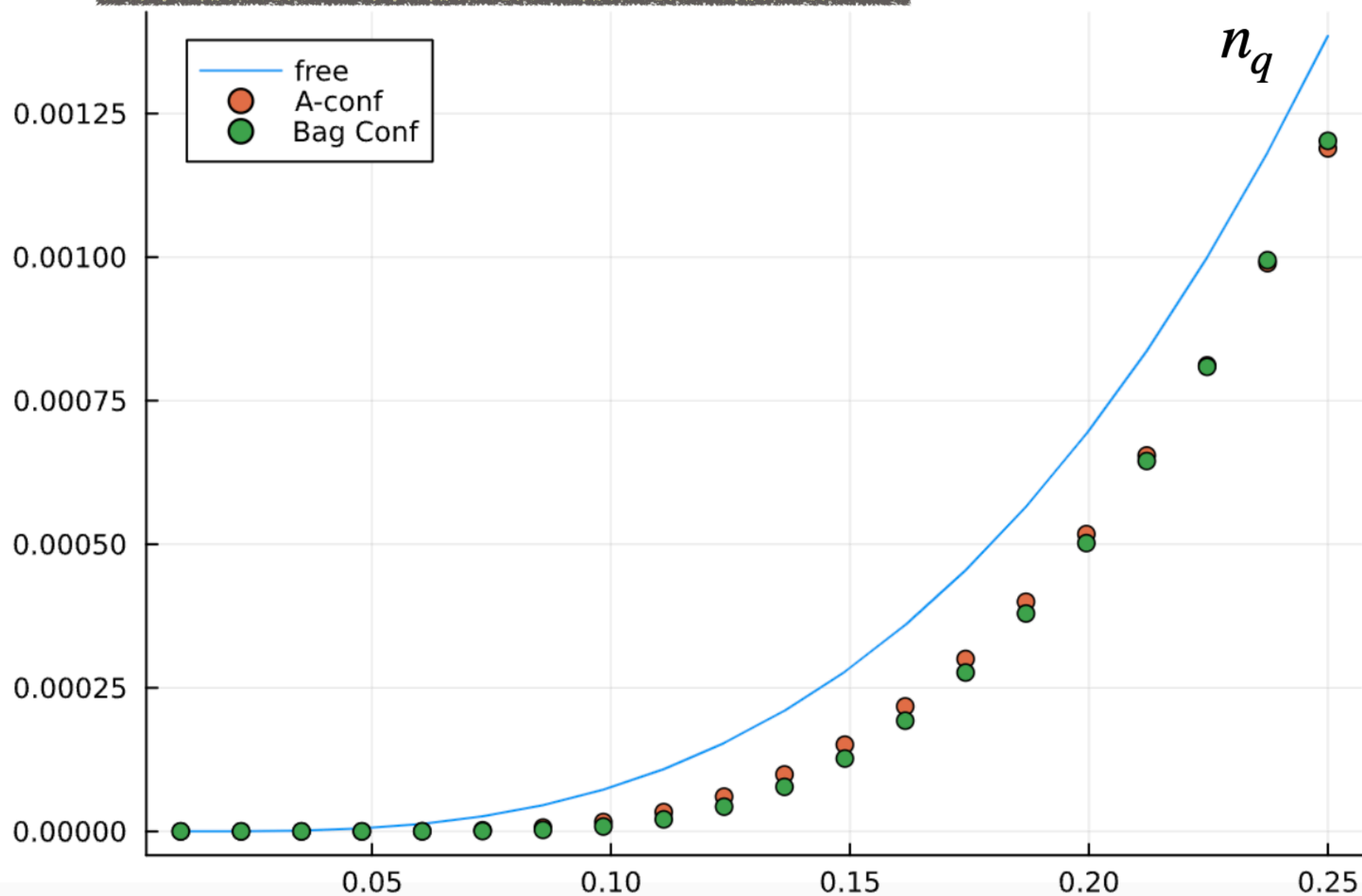
$A(p), M(p)$ satisfy a coupled Dyson eqns.

infrared enhanced!

“C

thermal suppression

effects on thermal obs.
are similar!



—
L
 $\overline{p)^2}$

coupled
n eqns.

ced!

P. M. Lo
(2010)

Confinement of Quarks

$$S^{-1}(p) = A_0(p) p^0 \gamma^0 - A(p) \vec{p} \cdot \vec{\gamma} - B(p)$$

$$\Sigma(p) \approx C_F \int \frac{d^4 q}{(2\pi)^4} V(\vec{p} - \vec{q}) i \gamma^0 S(q) \gamma^0$$

$$V_{ab}(x, y; \vec{A}_\perp) = \langle x, a | \left[\frac{g}{\nabla \cdot D} (-\nabla^2) \frac{g}{\nabla \cdot D} \right] | y, b \rangle$$

$A(p), B(p)$ are IR div! But

$$M(p) = \frac{B(p)}{A(p)} \quad \text{is finite!}$$

VS string-flip model:
M -> infinity
(too large sigma mass)

$$\langle \bar{\psi} \psi \rangle = N_c \int \frac{d^3 q}{(2\pi)^3} \frac{-4 B(q)}{2 \sqrt{A(q)^2 q^2 + B(q)^2}}$$

QUARK SDE

- Momentum-dependence
- Confining potential

$$\mu'(p) = \mu + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{1}{2} (n(\tilde{E}) - \bar{n}(\tilde{E}))$$

$$B(p) = m + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{B(q)}{2\tilde{E}(q)} (1 - n(\tilde{E}) - \bar{n}(\tilde{E}))$$

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$$\tilde{E}(p) = \sqrt{A(p)^2 p^2 + B(p)^2}$$

$$n(\tilde{E}) = \frac{1}{e^{\beta(\tilde{E}(q) - \mu'(q))} + 1}$$

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Conf. via:
 A -> Infinity
 thermal weights -> 0;
 non-sense??
 Quark Suppression

QUARK SDE

- Momentum-dependence
- Confining potential

$$\mu'(p) = \mu + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{1}{2} (n(\tilde{E}) - \bar{n}(\tilde{E}))$$

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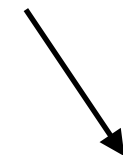
Conf. via:
 A -> Infinity
 thermal weights -> 0;
 non-sense!

Alkofer et al. A->1 in thermal

**ASYMPTOTIC FREEDOM
WITH
SEPARABLE
APPROXIMATION**

Mean fields

$$\mu' = \mu - 2G_V \int \frac{d^3 q}{(2\pi)^3} (n_F - \bar{n}_F)$$


$$\propto \mu'^3$$

versus

$$\mu' \propto \mu^{\frac{1}{3}} \longrightarrow c_S^2 \rightarrow 1$$

C-gauge: Separable approximation

$$\mu'(p) = \mu + \int \frac{d^3 q}{(2\pi)^3} \frac{1}{2} V(\vec{p} - \vec{q}) (n_F - \bar{n}_F)$$

If $V \rightarrow 0$ as $p \rightarrow \text{Inf}$: $\mu' \rightarrow \mu \longrightarrow c_s^2 \rightarrow \frac{1}{3}$

e.g. $V(p, q) \approx V_0 e^{-p^2/\Lambda^2} e^{-q^2/\Lambda^2}$ (SEPARABLE APPROX.)

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- UV SCALE
- p-dep.

e.g.

$V(p, q) \approx V_0 e^{-p^2/\Lambda^2} e^{-q^2/\Lambda^2}$ (SEPARABLE APPROX.)

Dynamical Interaction

- $M(p)$ saturates asymptotically to current quark mass
- $\mu(p)$ saturates asymptotically to bare μ

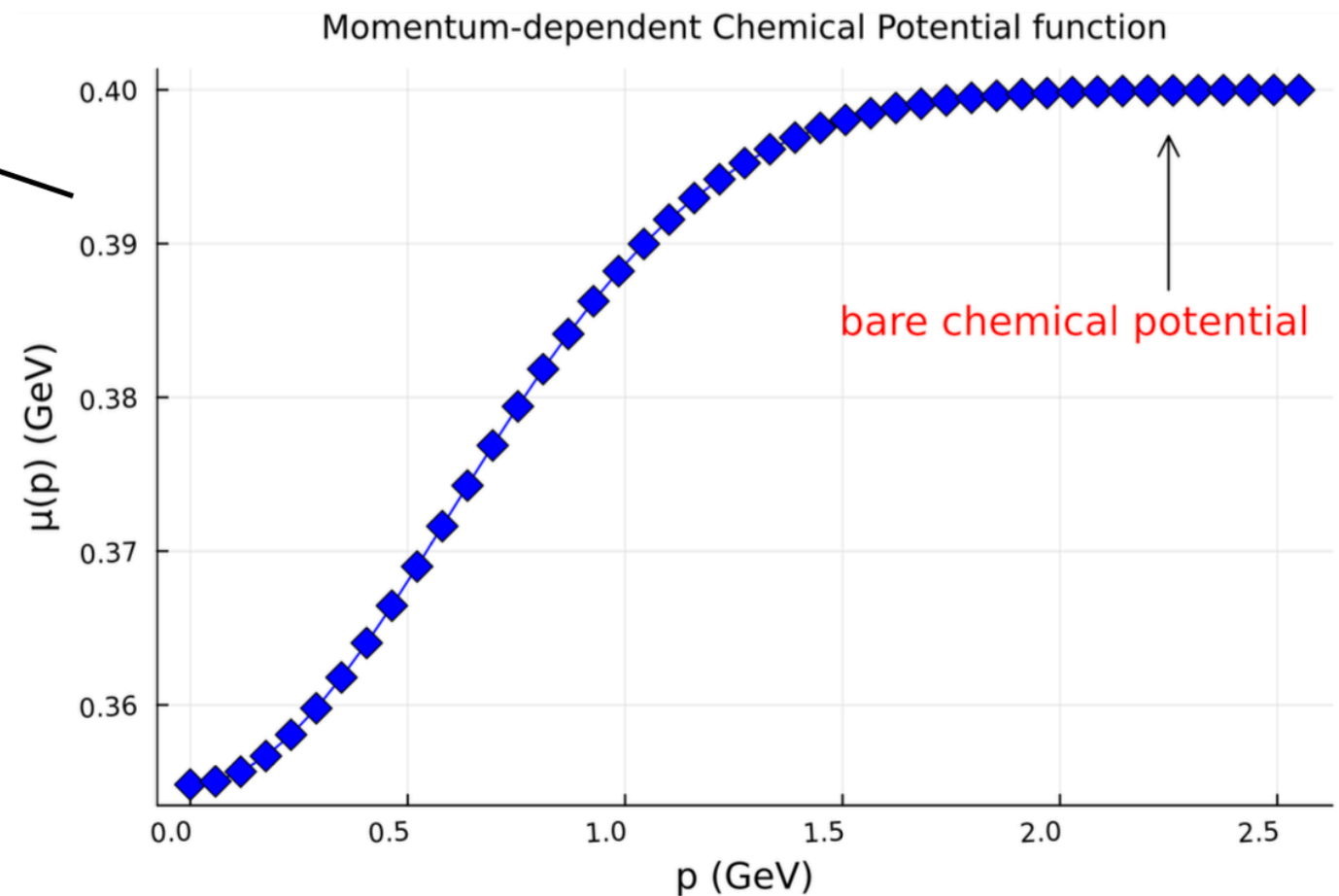
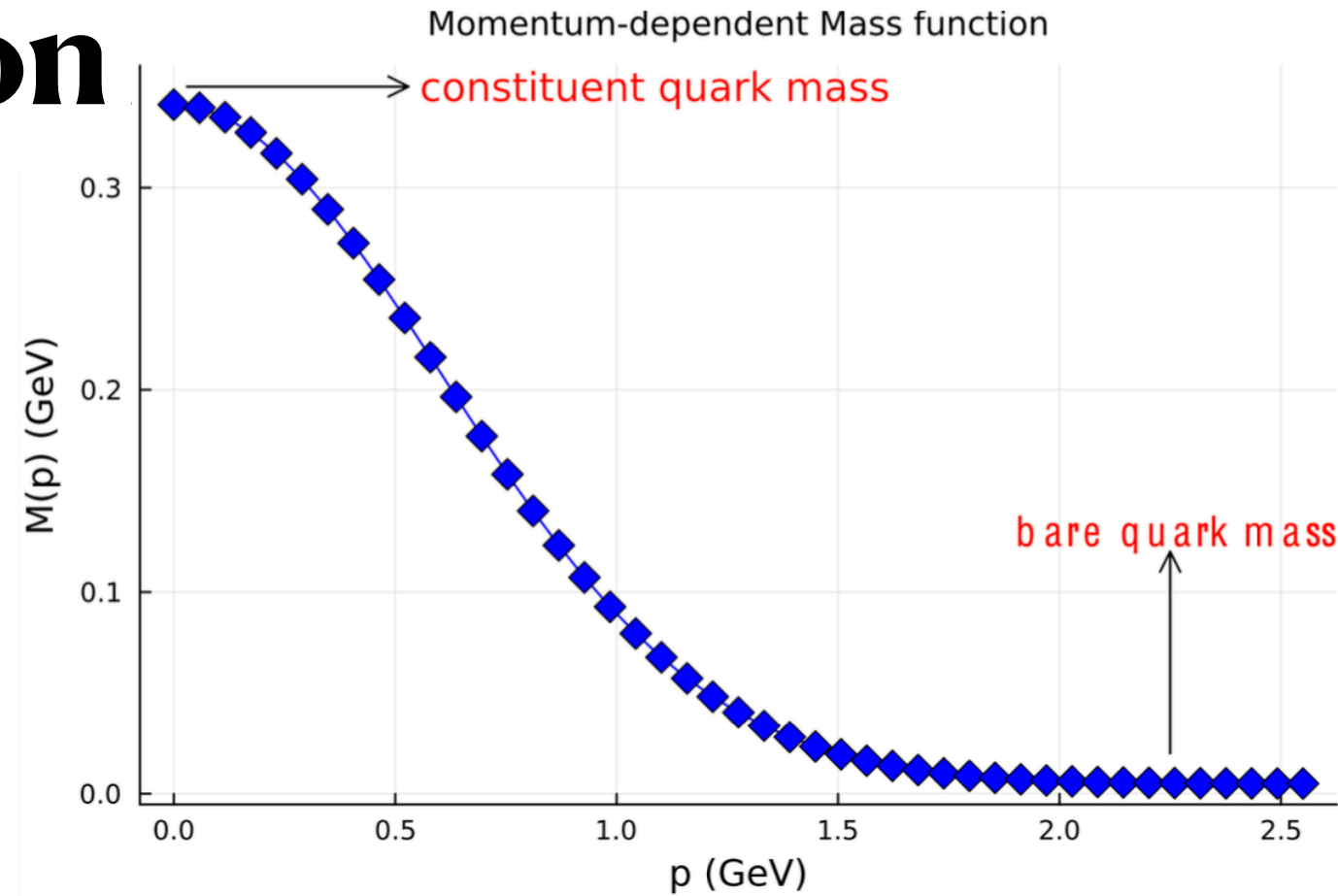
saturation dictated by UV scale

—> interaction becomes weak

—> asymptotic freedom !

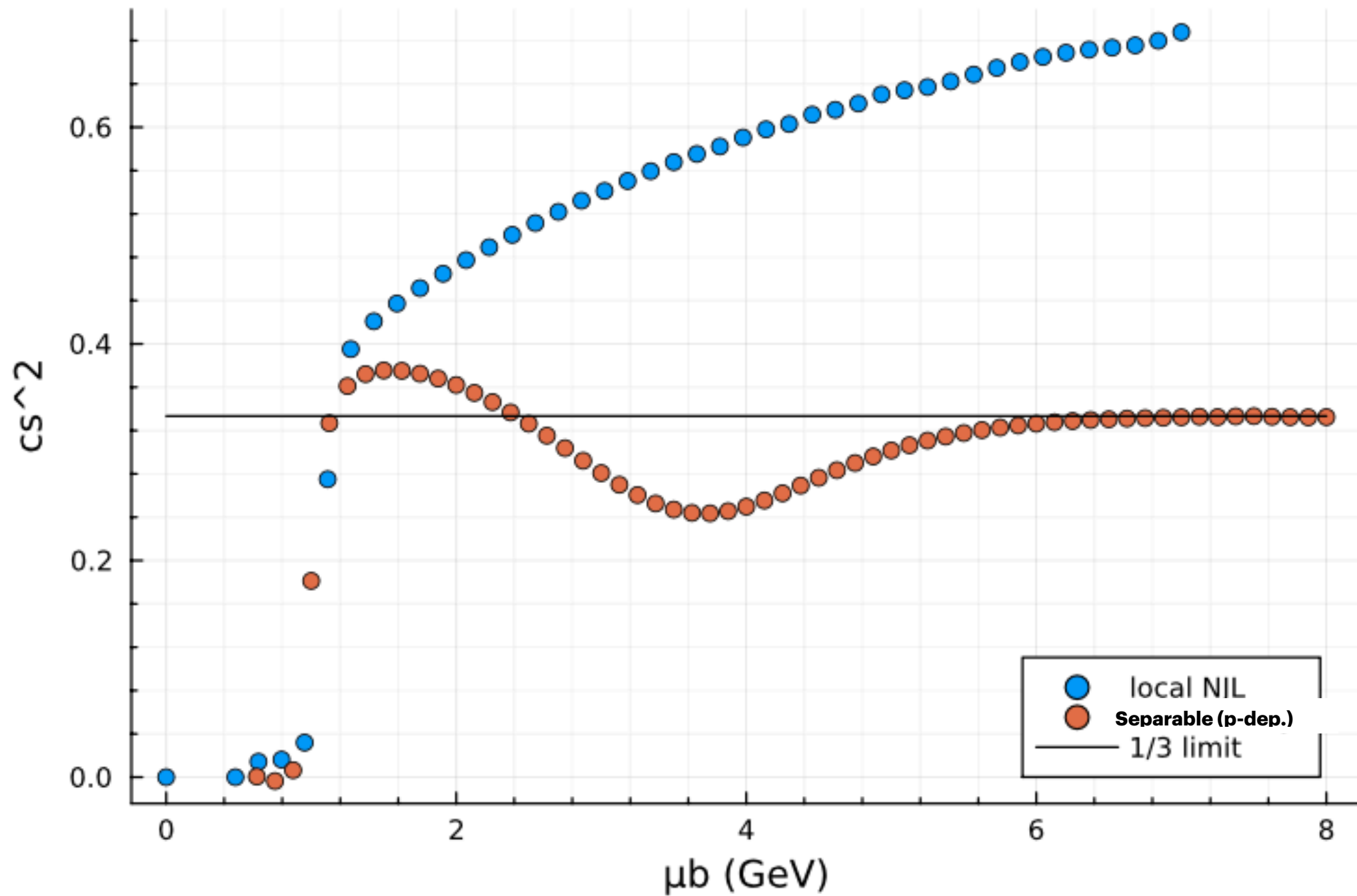
- Change in dispersion relation -

$$E(p) = \sqrt{p^2 + M(p)^2}$$

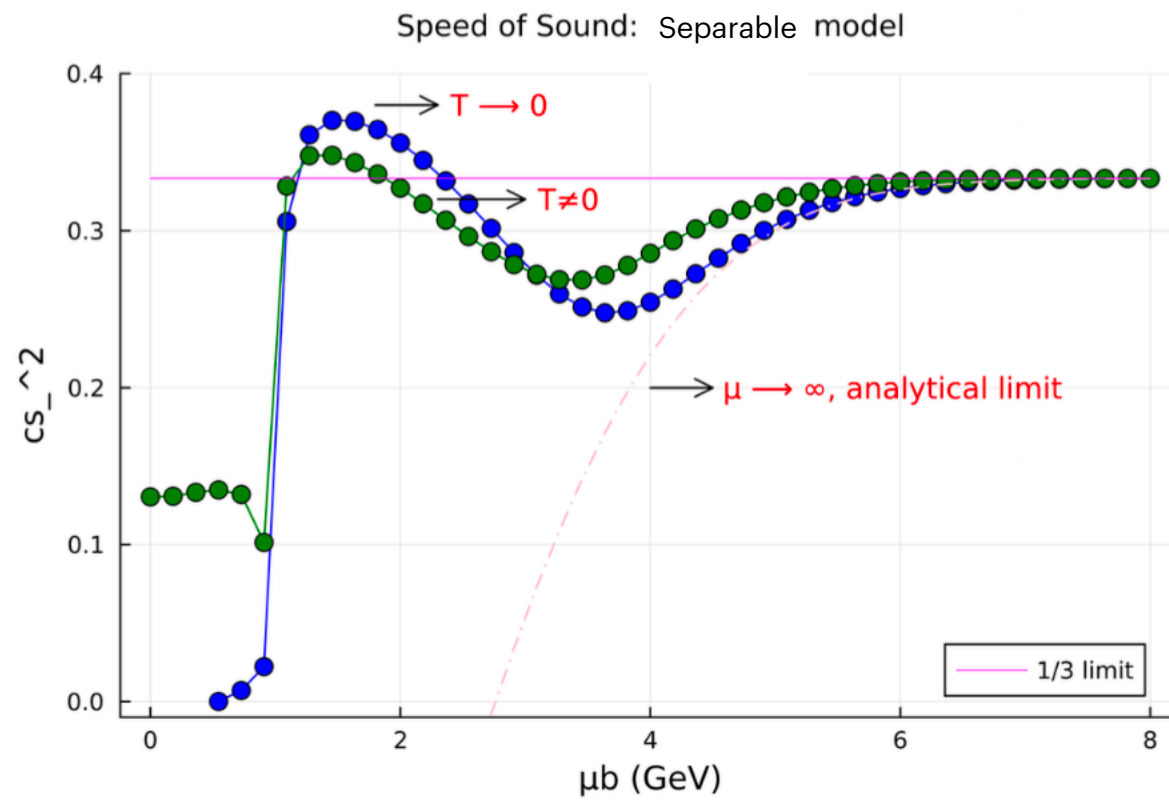


SEPARABLE (NO CONFINEMENT)

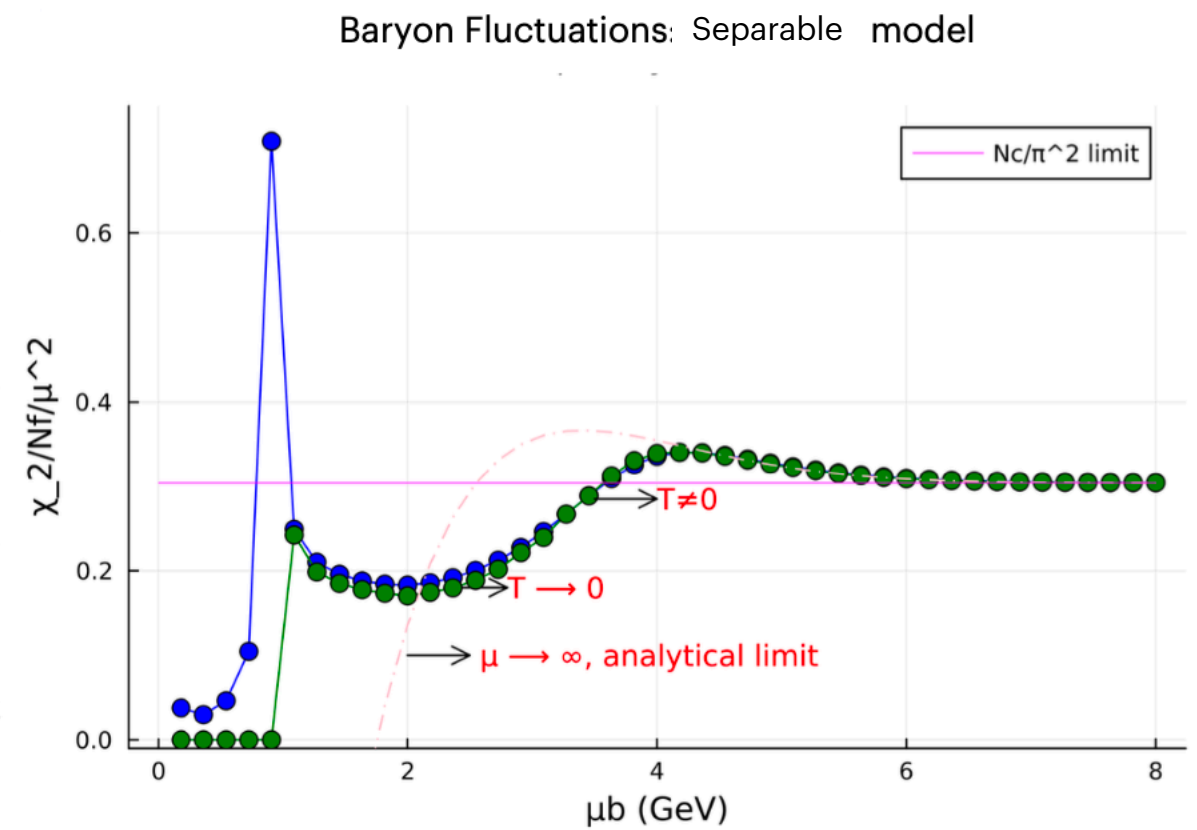
(squared) Speed of Sound



SPEED OF SOUND V/S BARYON FLUCTUATIONS



$$cs^2_{\mu \rightarrow \infty} \approx \frac{1}{3} \left[1 - \frac{\omega_\infty}{\mu} \left(1 + \frac{2\mu^2}{UV^2} \right) e^{-\frac{\mu^2}{UV^2}} \right]$$



$$\chi^2_{\mu \rightarrow \infty} \approx \frac{N_c N_f}{\pi^2} \mu^2 \left[1 - \frac{2\omega_\infty}{\mu} \left(1 - \frac{\mu^2}{UV^2} \right) e^{-\frac{\mu^2}{UV^2}} \right]$$

**BARYON FLUCTUATIONS
IN
FINITE TEMPERATURE**

BARYON FLUCTUATIONS

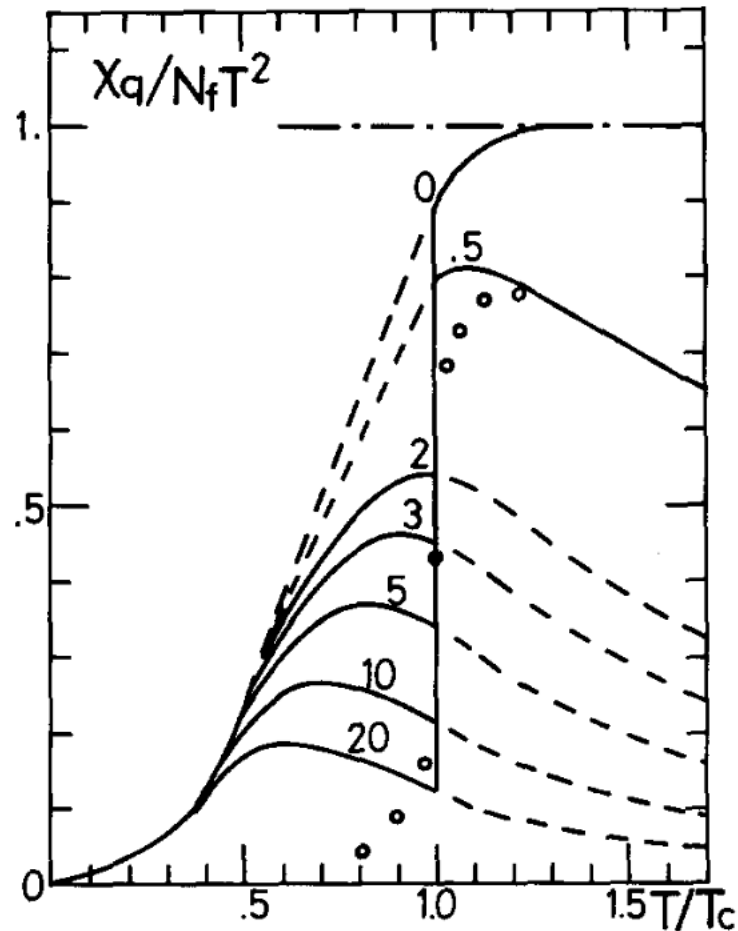


Fig. 1. The temperature dependence of the quark-number susceptibility χ_q in the unit of $N_f T^2$ with some of the vector coupling $g_v A^2$: $g_v A^2 = 0, 0.5, 2, 3, 5, 10, 20$, which are indicated with the numbers attached to the respective curves. The dash-dotted line shows the free massless case. The small circles are the lattice result on an $8^3 \times 4$ lattice with the quark mass $m/T = 0.2$ [7] compiled in ref. [9].

Physics Letters B 271 (1991) 395–402
North-Holland

PHYSICS LETTERS B

$$\chi_2 = \frac{\partial n_v}{\partial \mu}$$

Quark-number susceptibility and fluctuations in the vector channel at high temperatures \star

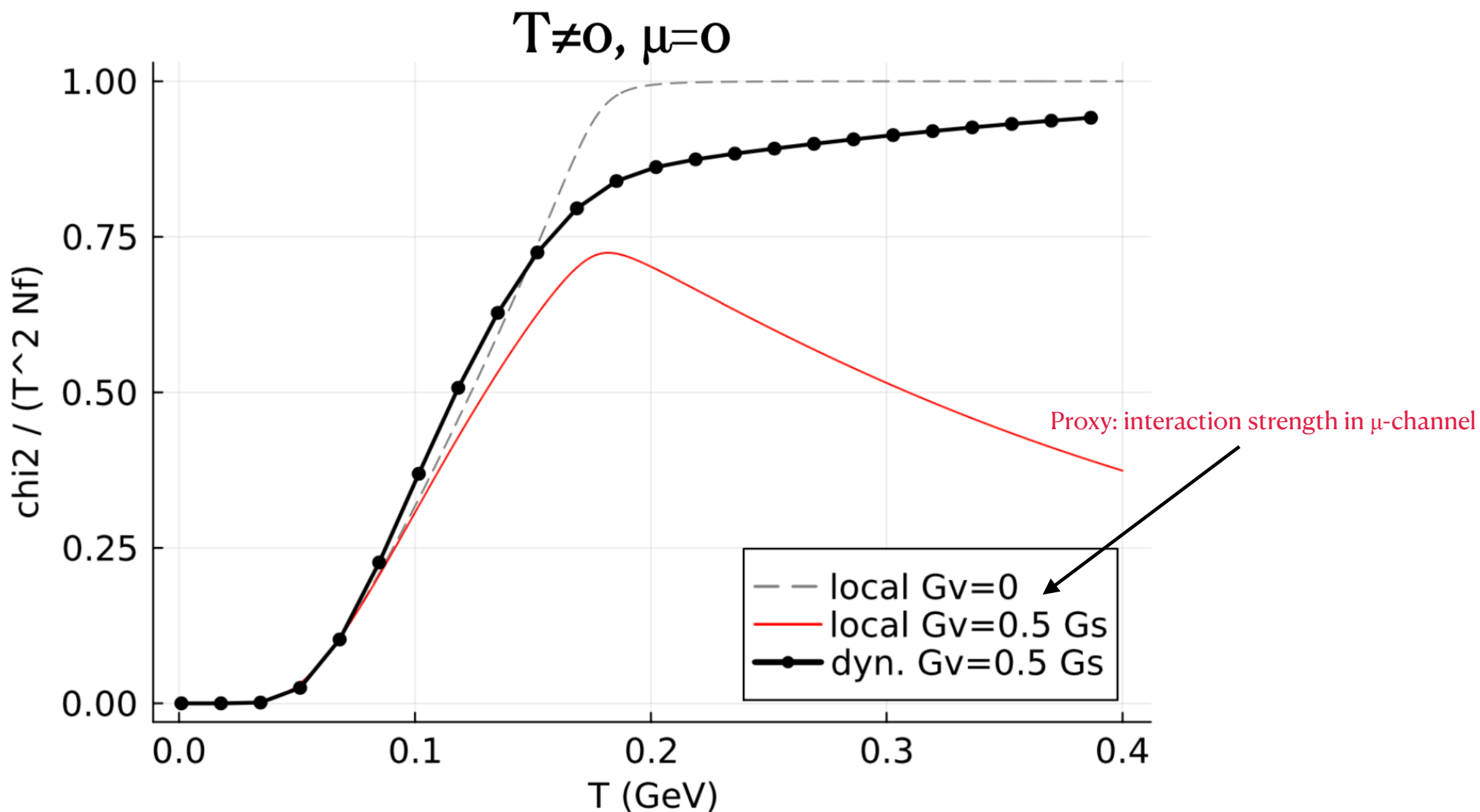
Teiji Kunihiro

Faculty of Science and Technology, Ryukoku University, Seta, Otsu-city 520-21, Japan

Received 9 July 1991; revised manuscript received 10 September 1991

The quark-number susceptibility χ_q is examined as an observable which may help to reveal the physical picture of the high-temperature phase of QCD. It is emphasized that χ_q is intimately related with the fluctuations in the vector channel of the system. It is shown that the results of the recent lattice simulations of χ_q can be understood in terms of a possible change of the interactions between quark and anti-quarks in the vector channel, and imply that the fluctuations in the vector channel is greatly suppressed in the high-temperature phase in contrast with those in the scalar and pseudo-scalar ones.

BARYON FLUCTUATIONS: SEPARABLE MODEL



local

$$\chi_2 = \frac{dn_v}{d\mu} \propto \frac{T^2}{1 + CT^2}$$

dynamical

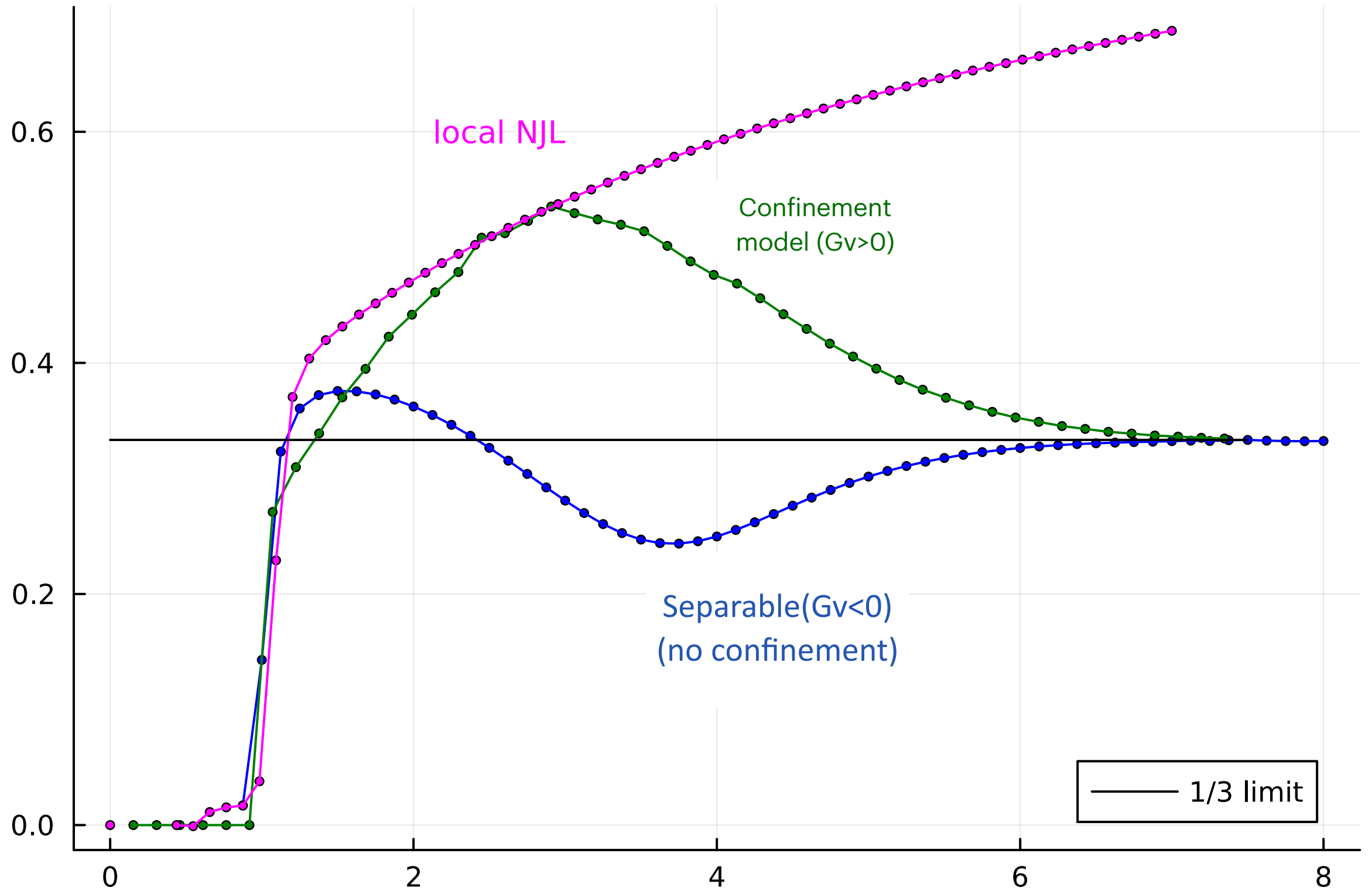
$$\chi_2 = \frac{dn_v}{d\mu} \propto T^2$$

CONFINEMENT

&

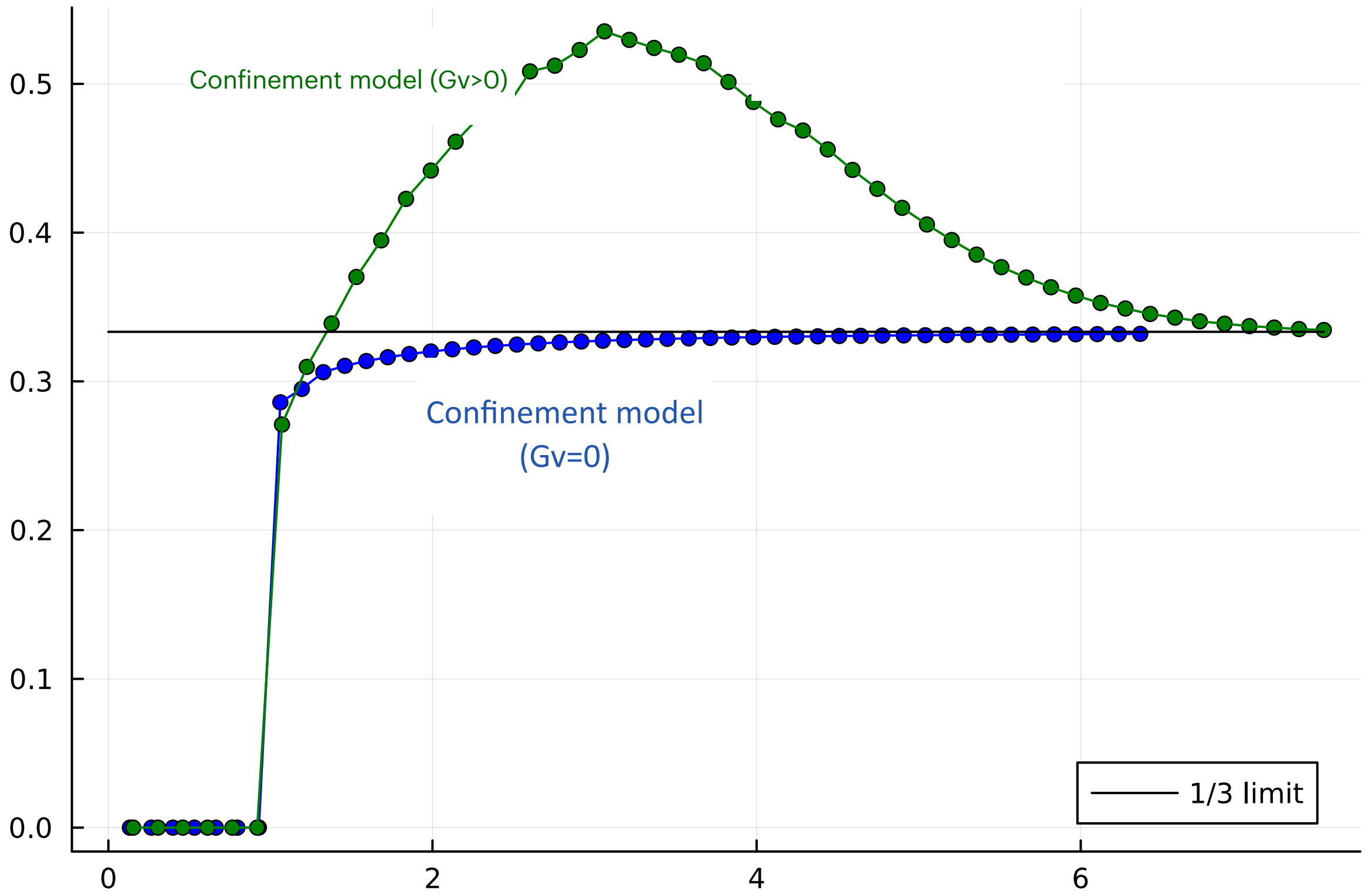
ASYMPTOTIC FREEDOM

CONFINEMENT MODEL SPEED OF SOUND $T \rightarrow 0$



preliminary result

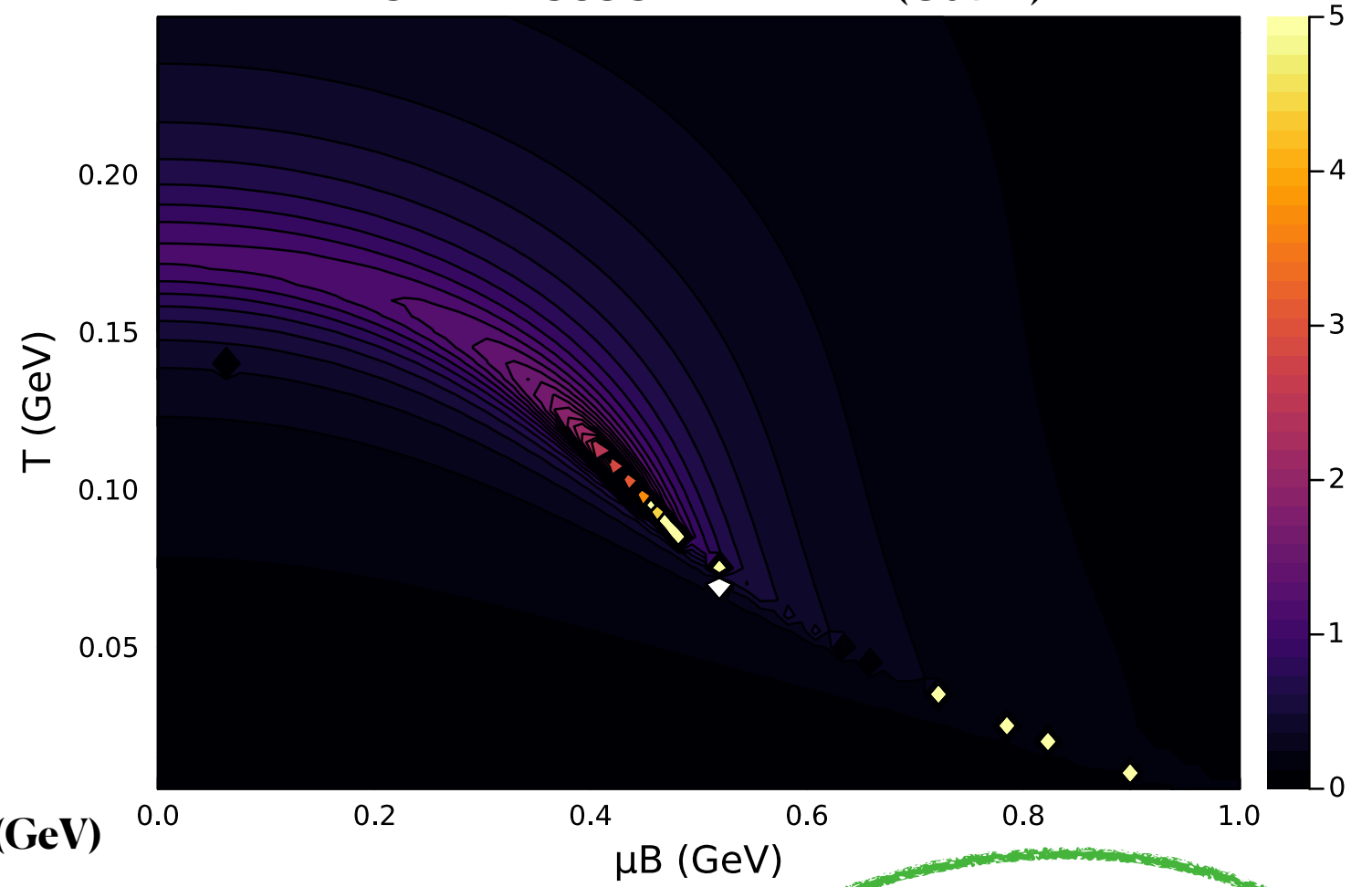
CONFINEMENT MODEL FOR VARYING INTERACTION STRENGTH SPEED OF SOUND $T \rightarrow 0$



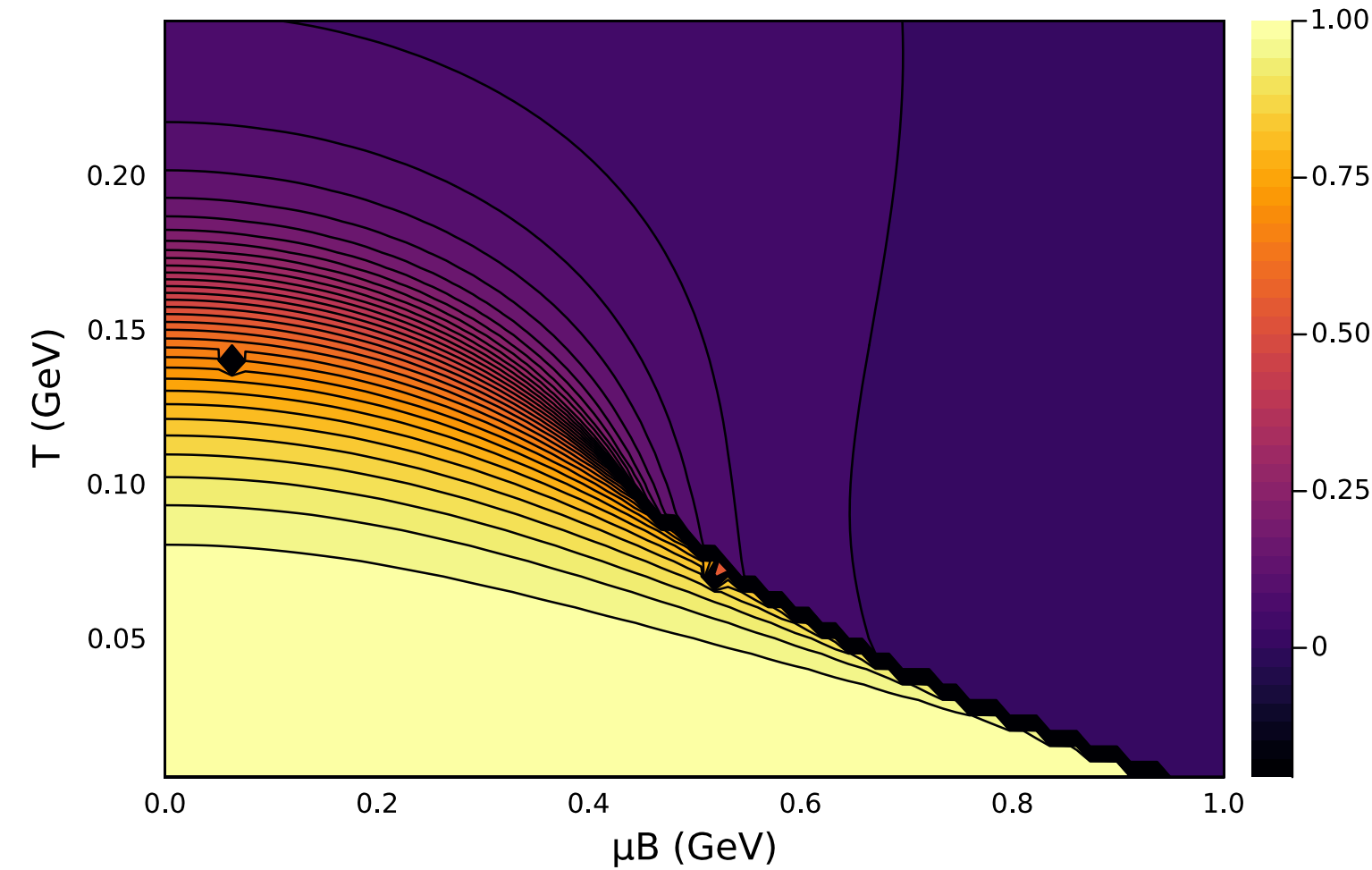
preliminary result

QCD PHASE DIAGRAM

CHIRAL SUSCEPTIBILITY (GeV²)



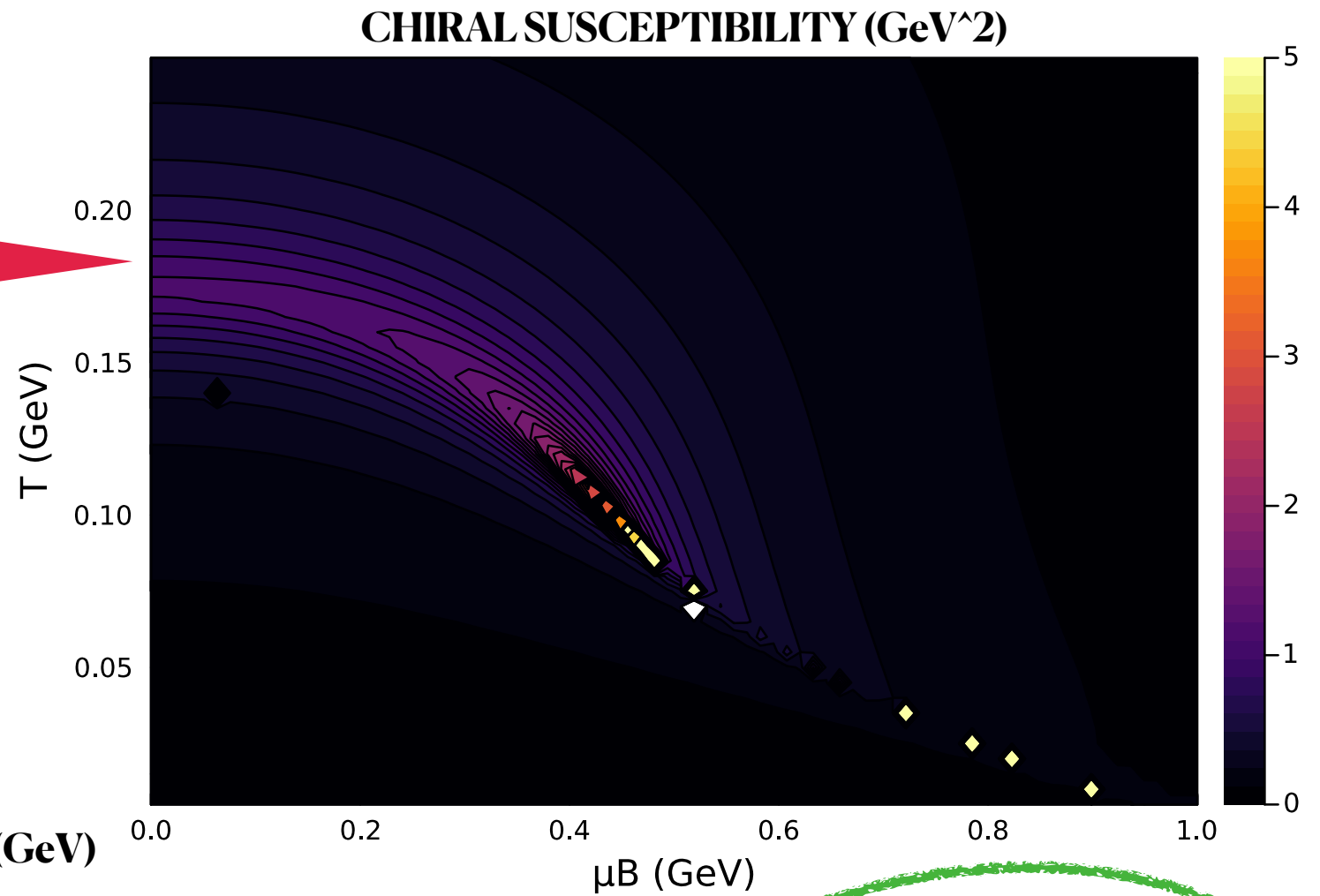
(NORMALISED) EFFECTIVE QUARK MASS (GeV)
M_{vac} = 330 MeV



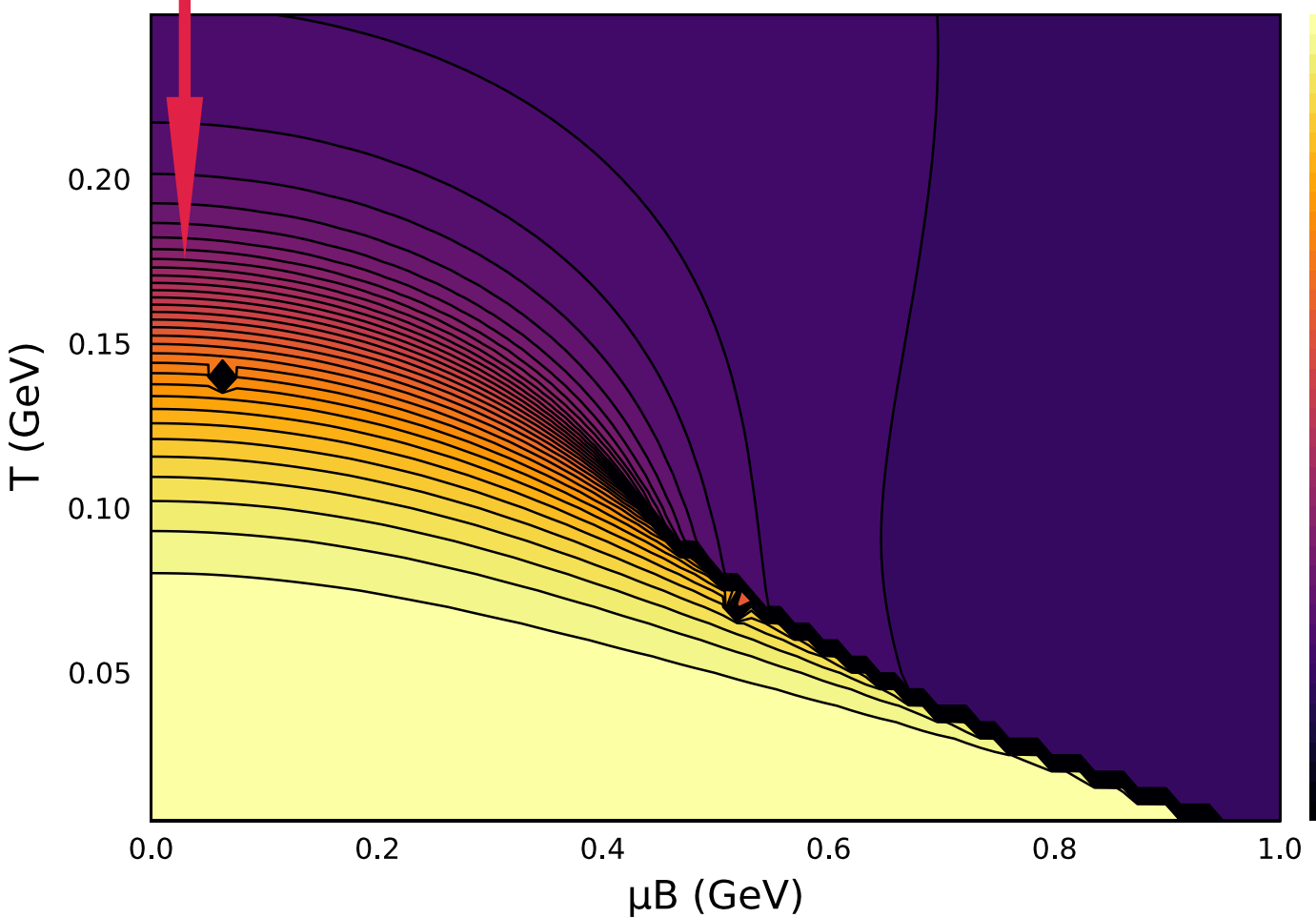
$$\chi_{sus} = \frac{d \langle \bar{q}q \rangle}{dm_{current}}$$

preliminary result

Fixing vacuum naturally
 leads to a $T_c \sim 155$ MeV
 \rightarrow Alkofer



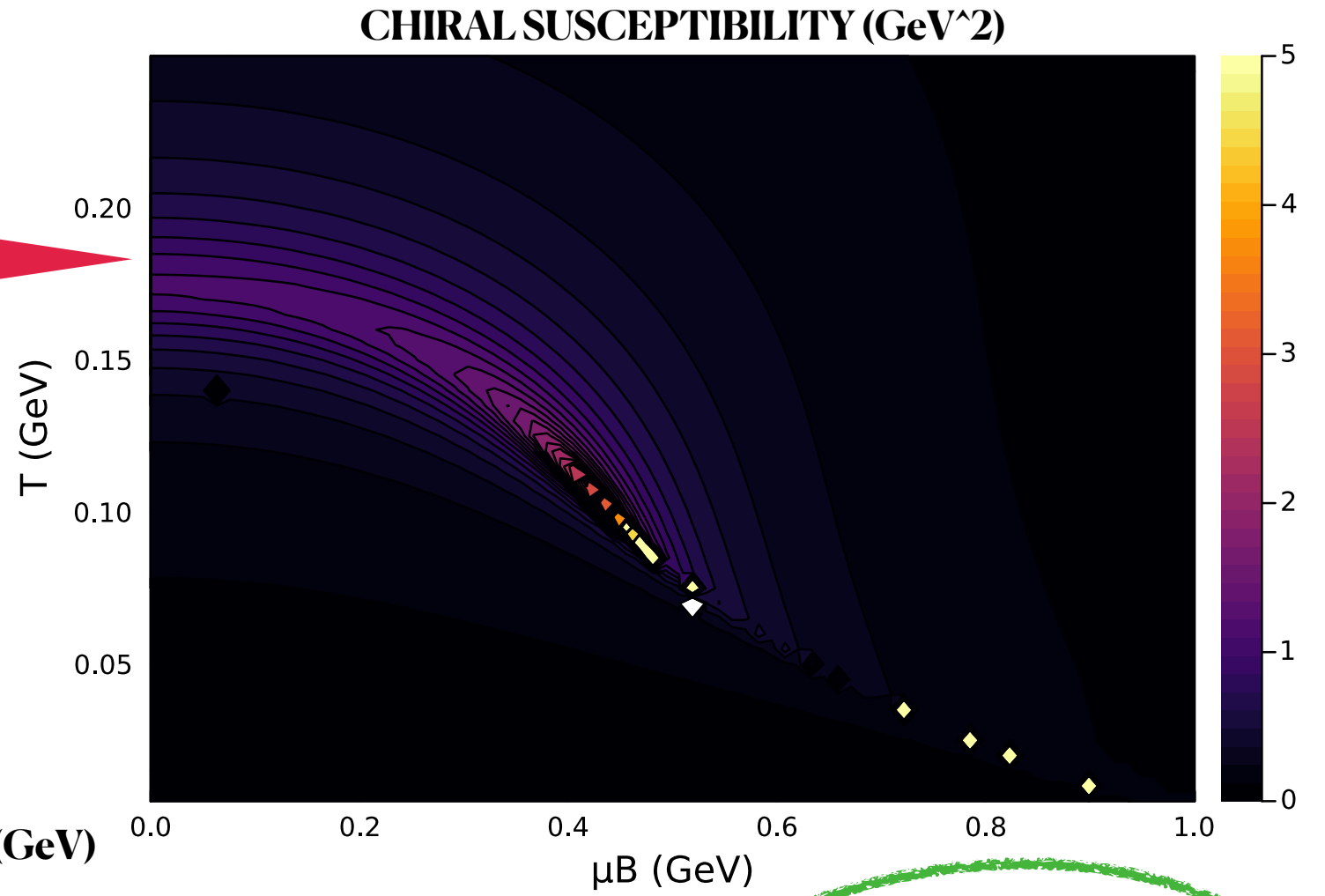
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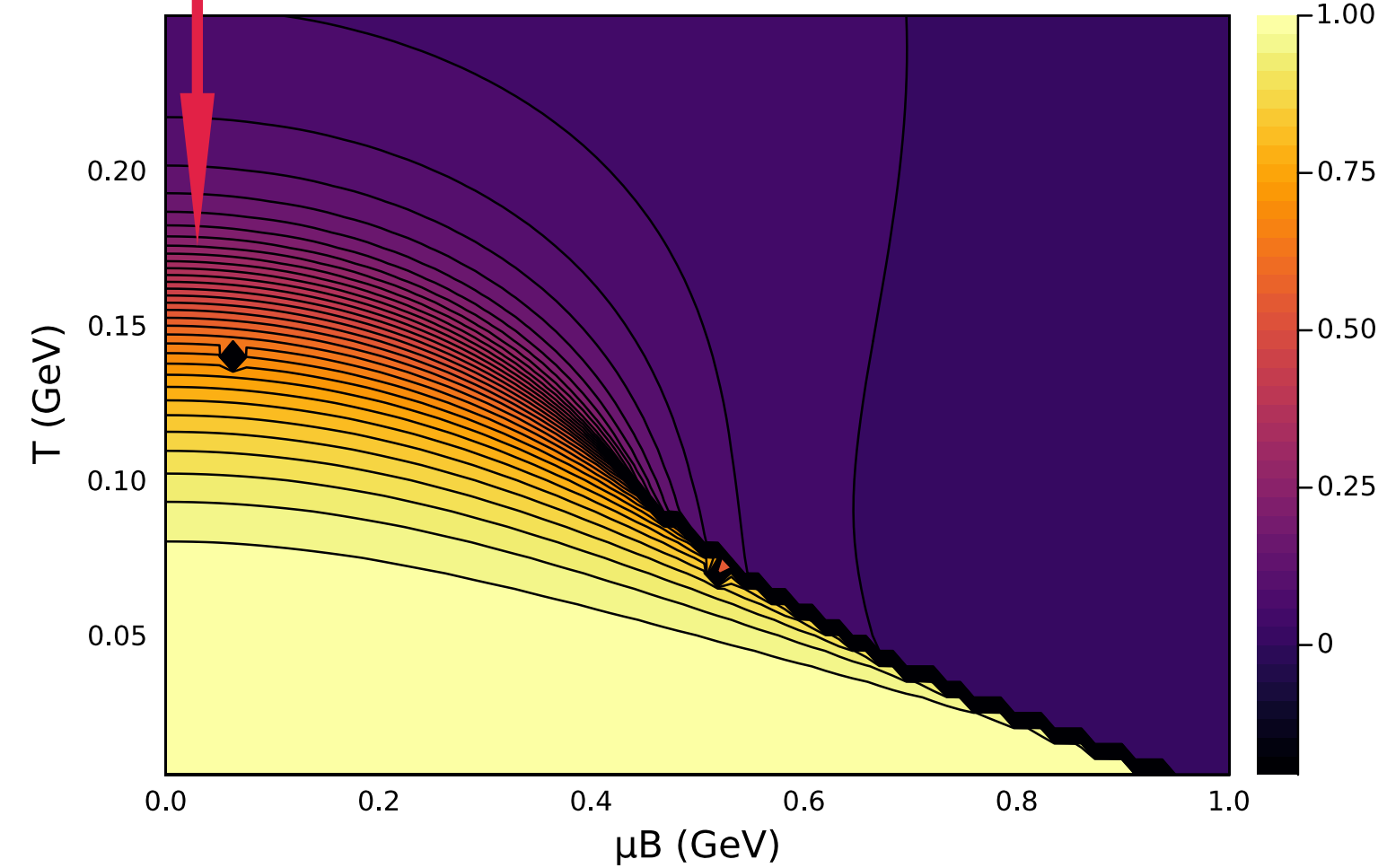
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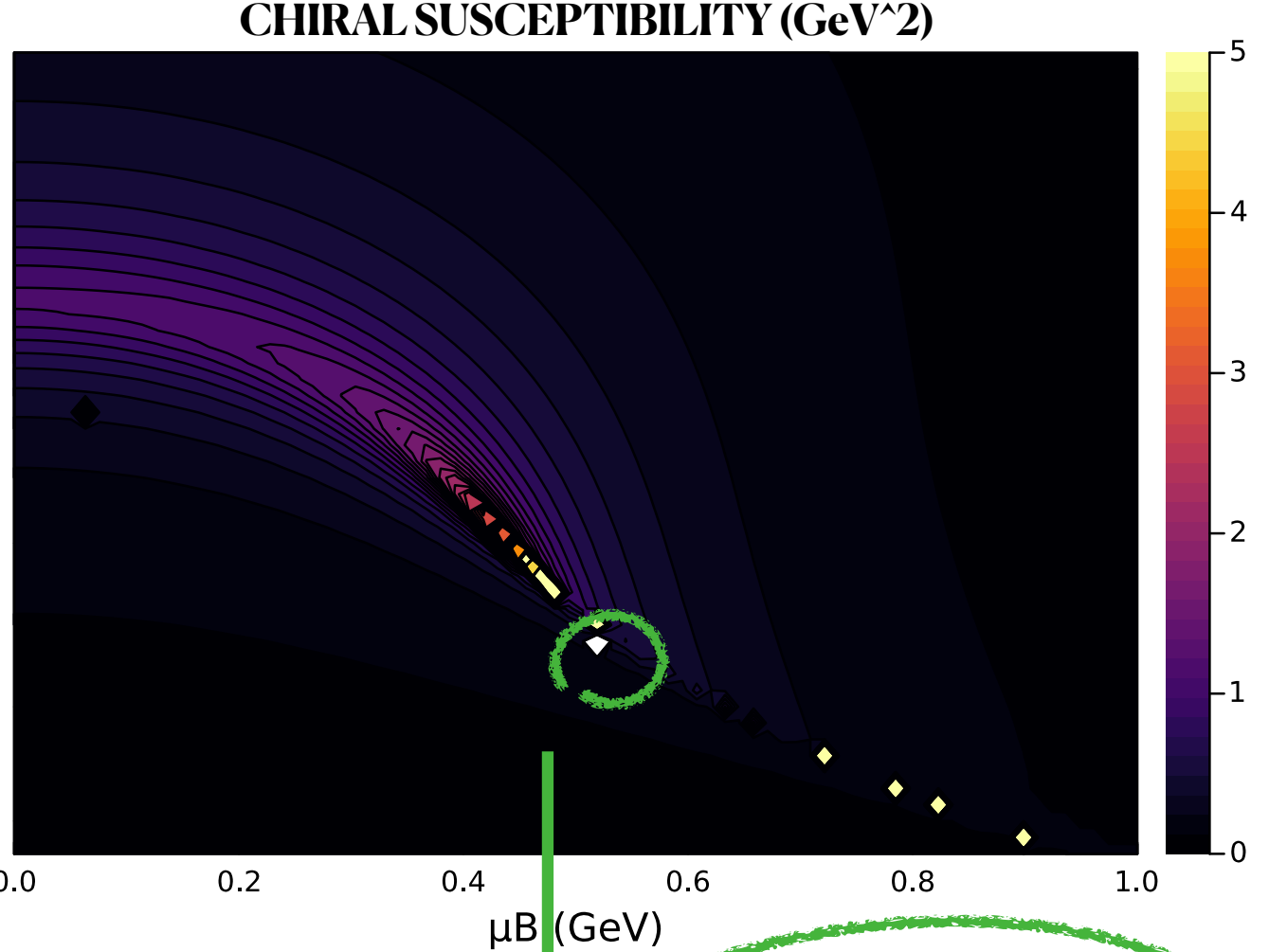
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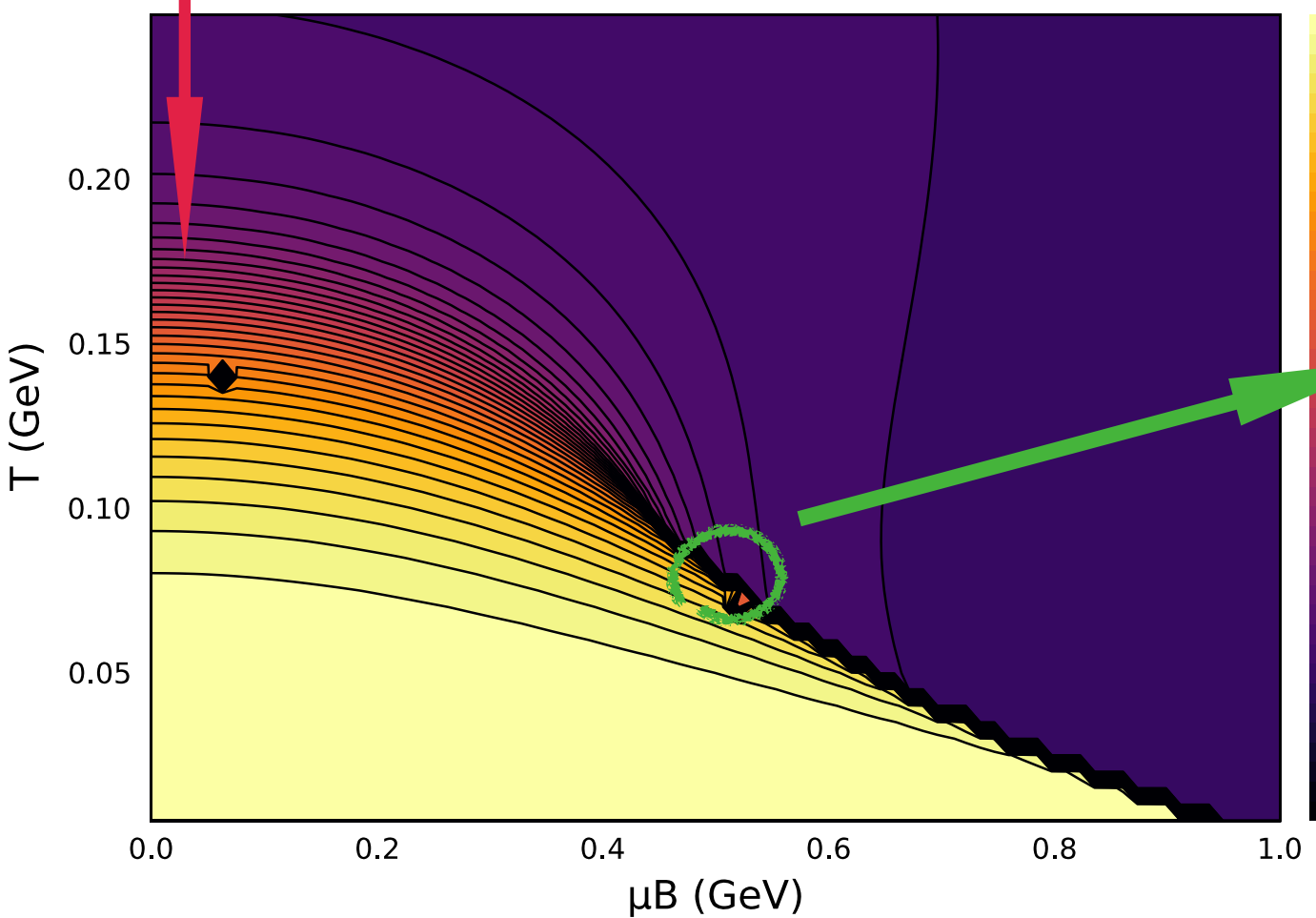
Fixing vacuum naturally
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 \rightarrow Alkofer approximation



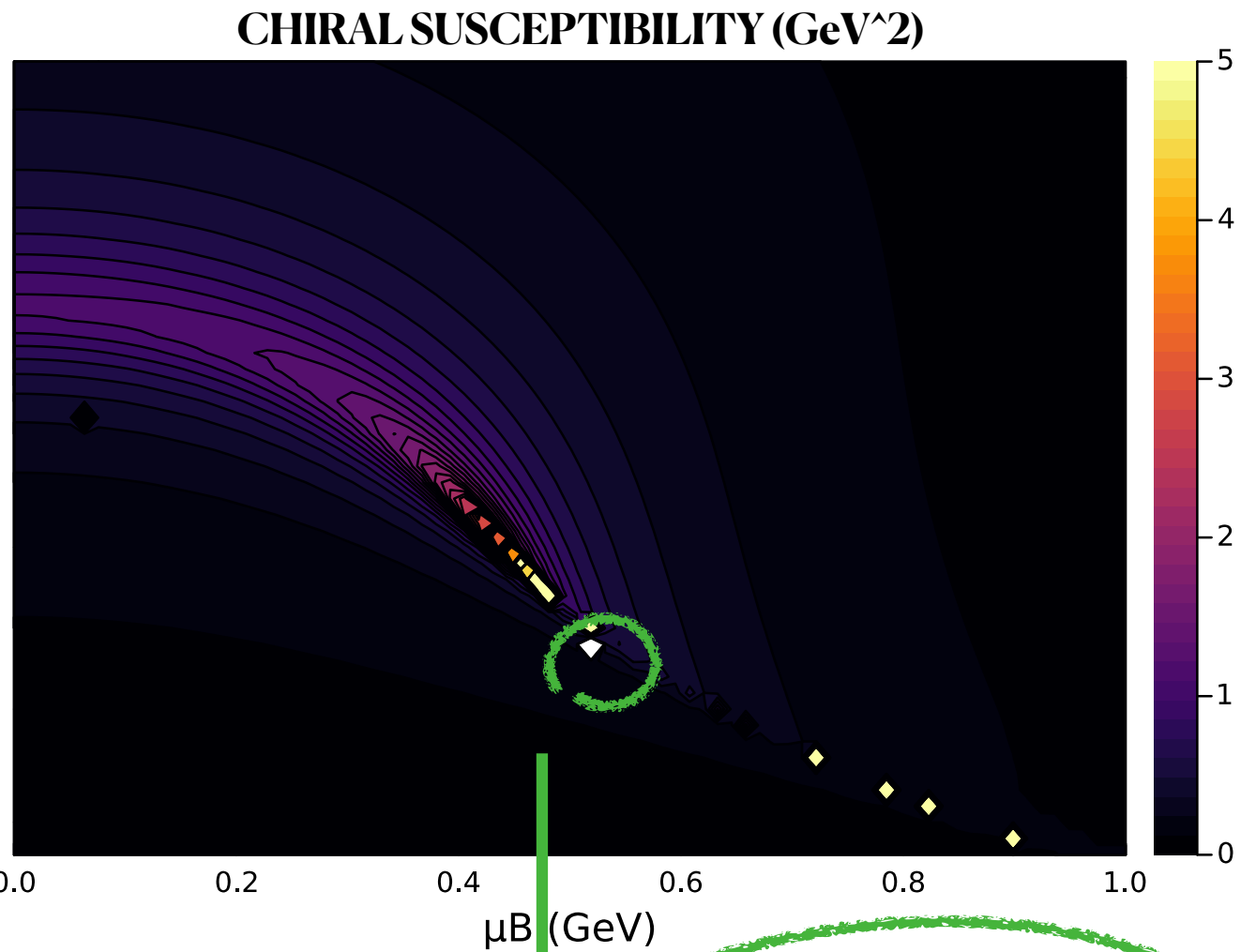
$$\chi_{sus} = \frac{d \langle \bar{q}q \rangle}{dm_{current}}$$

$T_{cep} \sim 0.7$ GeV for $Gv \neq 0$ (< 1)

(NORMALISED) EFFECTIVE QUARK MASS (GeV)
 $M_{vac} = 330$ MeV



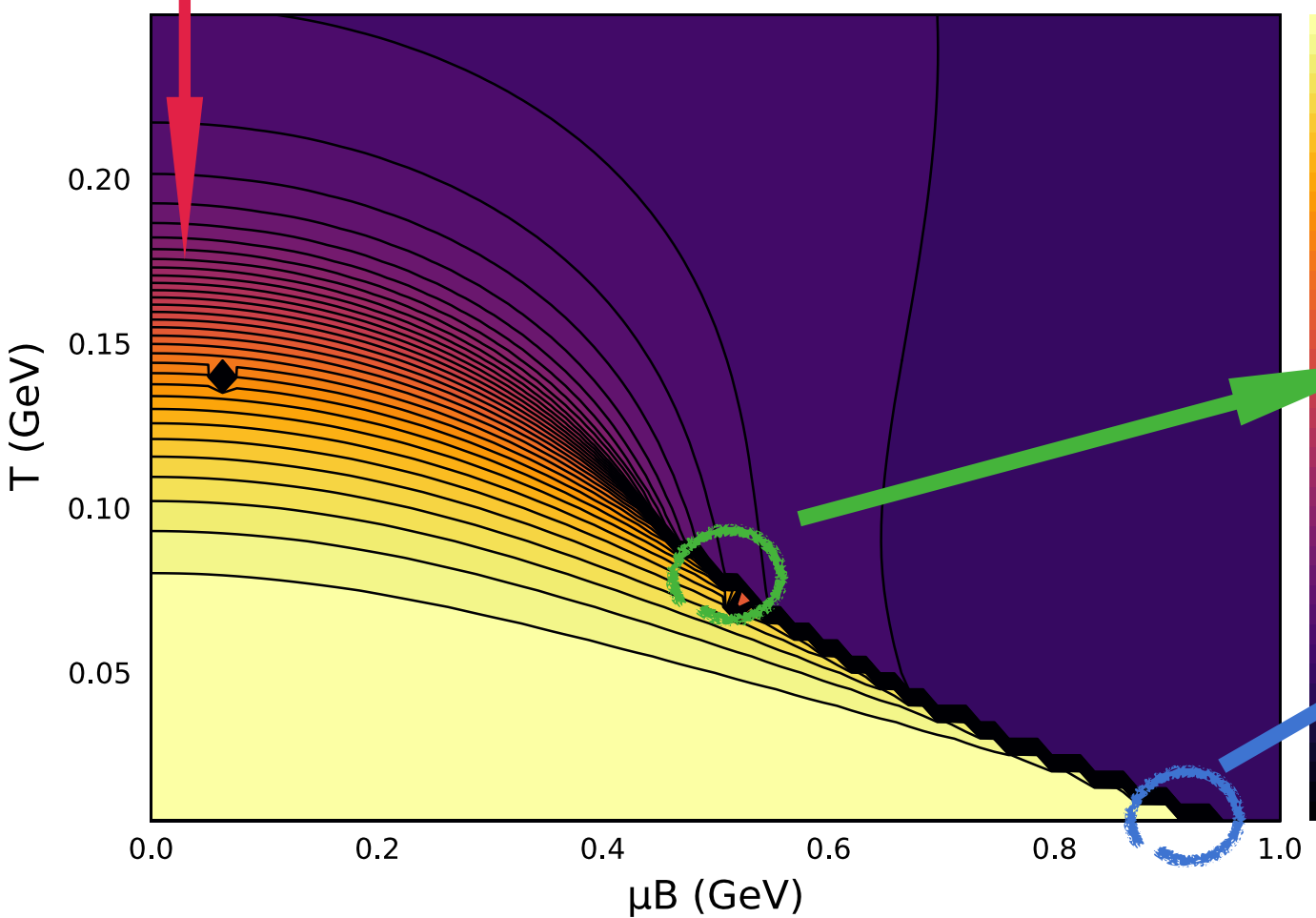
Fixing vacuum naturally
 leads to a $T_c \sim 155$ MeV
 \rightarrow Alkofer approximation



$$\chi_{sus} = \frac{d \langle \bar{q}q \rangle}{dm_{current}}$$

$T_{cep} \sim 0.7$ GeV for **$Gv \neq 0$ (<1)**
 $\mu B_{critical} \sim 0.9$ GeV (low)
 \rightarrow Confinement model

(NORMALISED) EFFECTIVE QUARK MASS (GeV)
 $M_{vac} = 330$ MeV

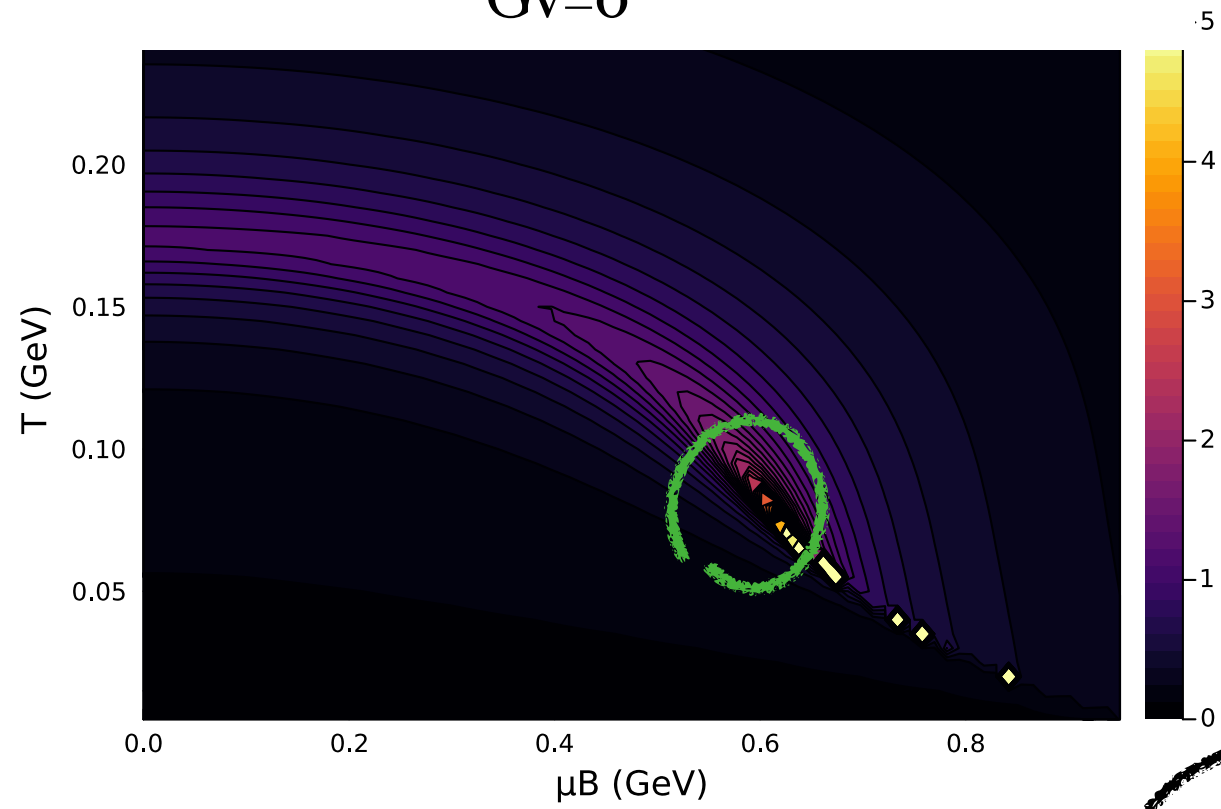


preliminary result

EXISTENCE & MOVEMENT OF
CEP & $\mu_{critical}$
(CONFINEMENT MODEL)

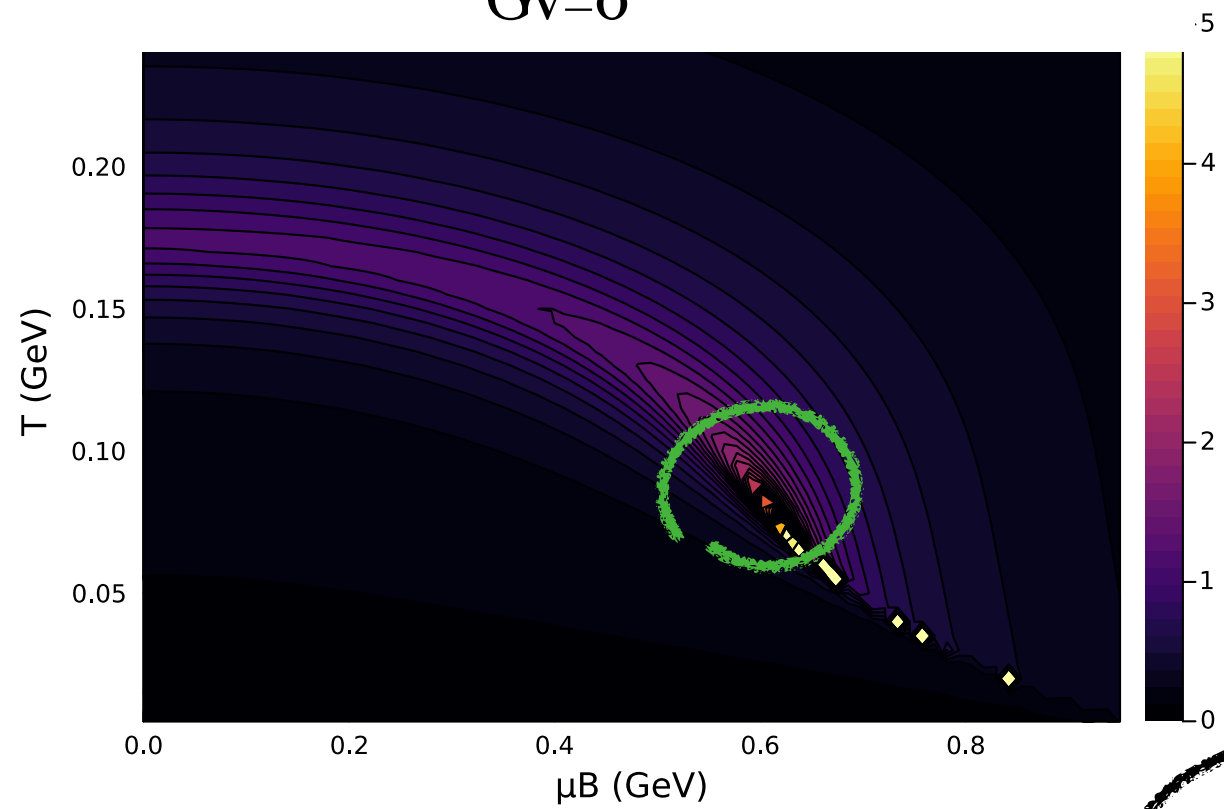
**CHIRAL
SUSCEPTIBILITY**

$G_V=0$

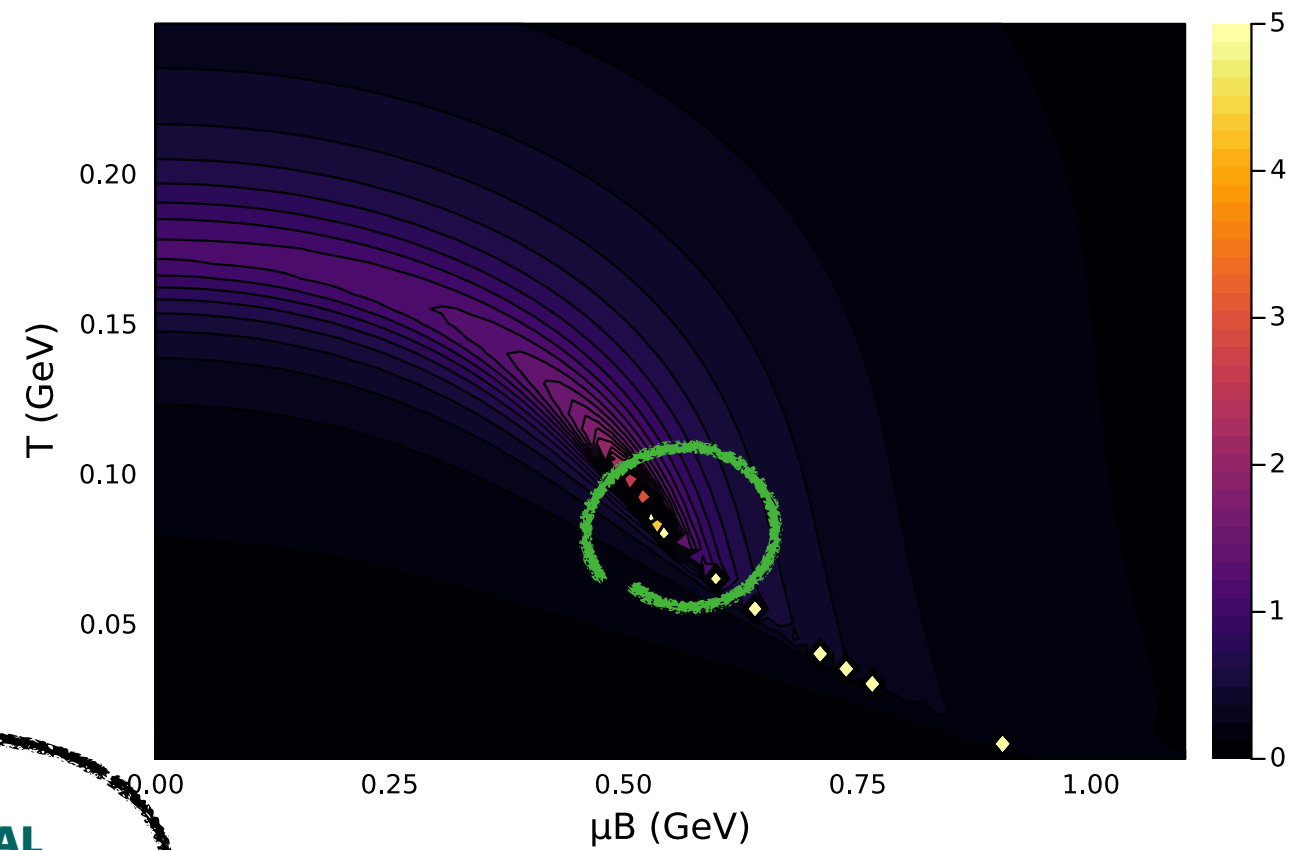


**CHIRAL
SUSCEPTIBILITY**

$G_V=0$

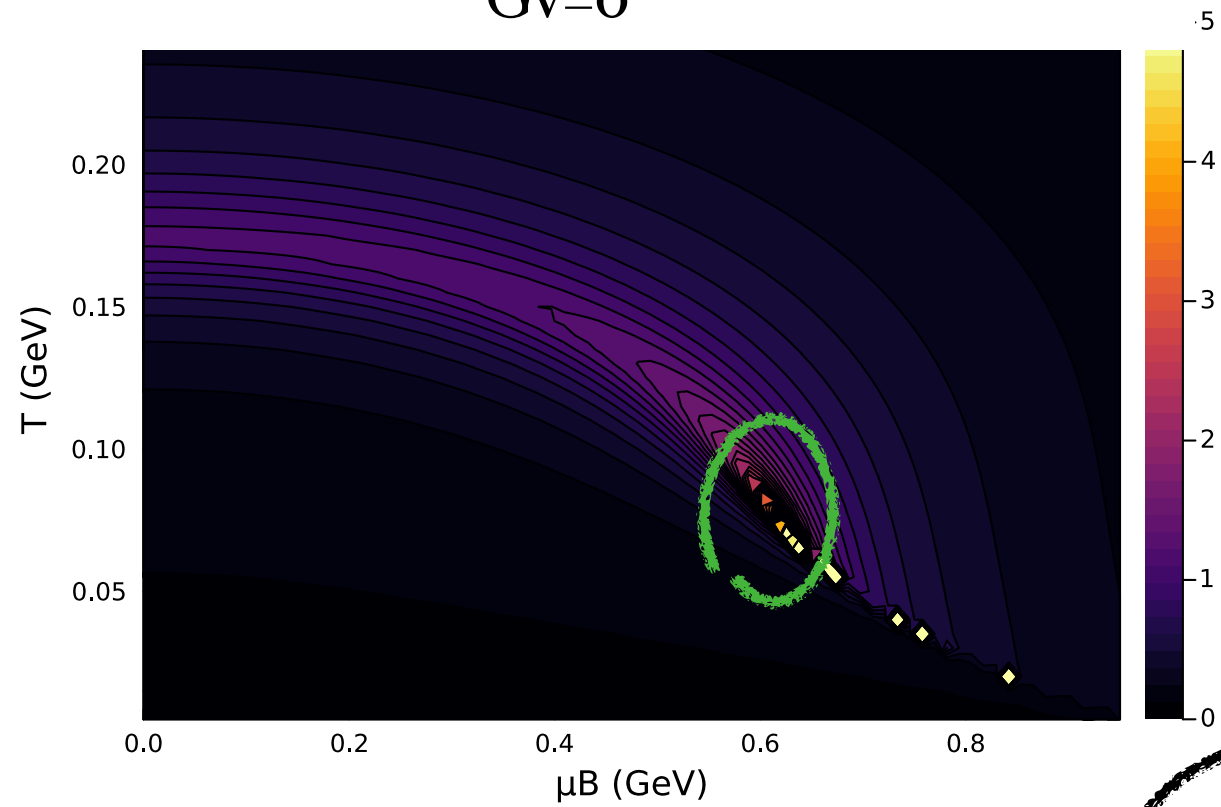


$G_V=0.2$

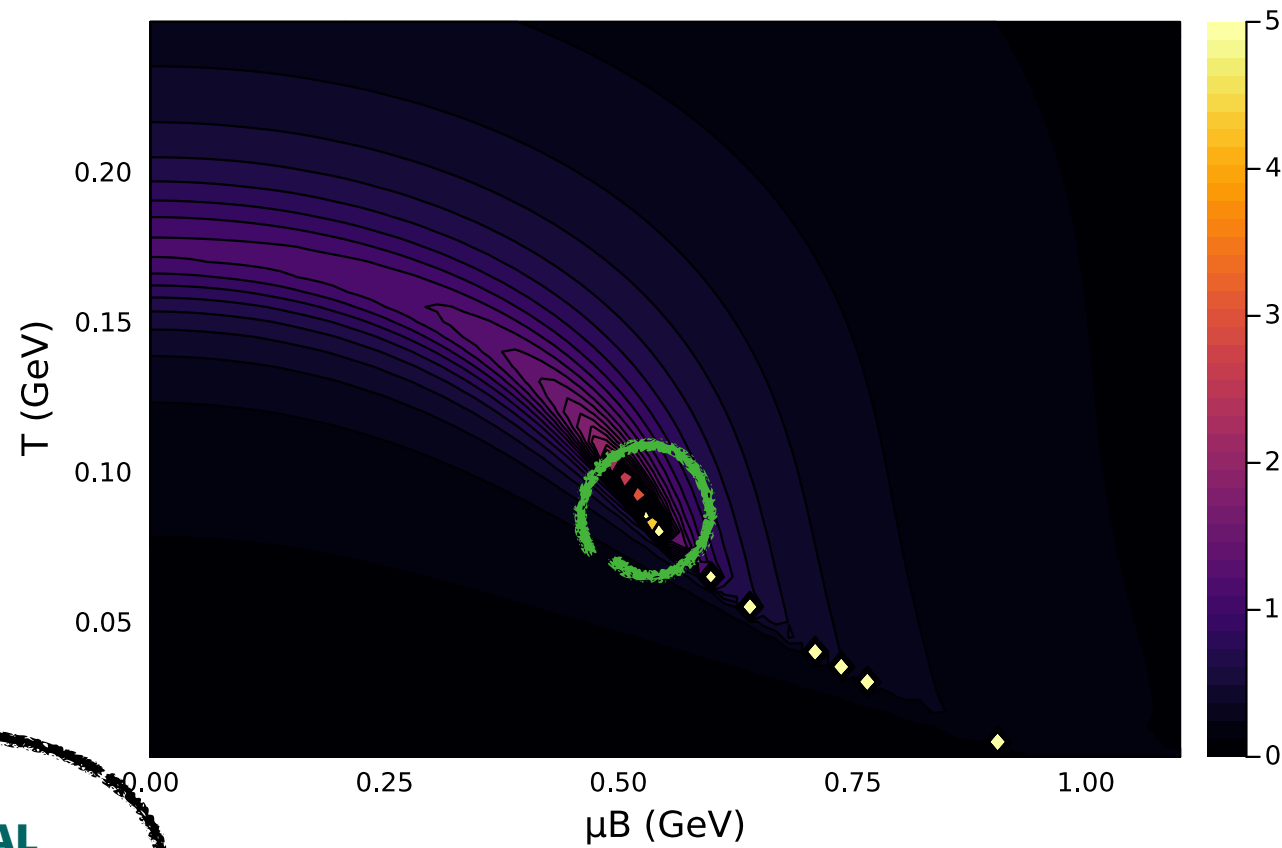


**CHIRAL
SUSCEPTIBILITY**

$G_V=0$

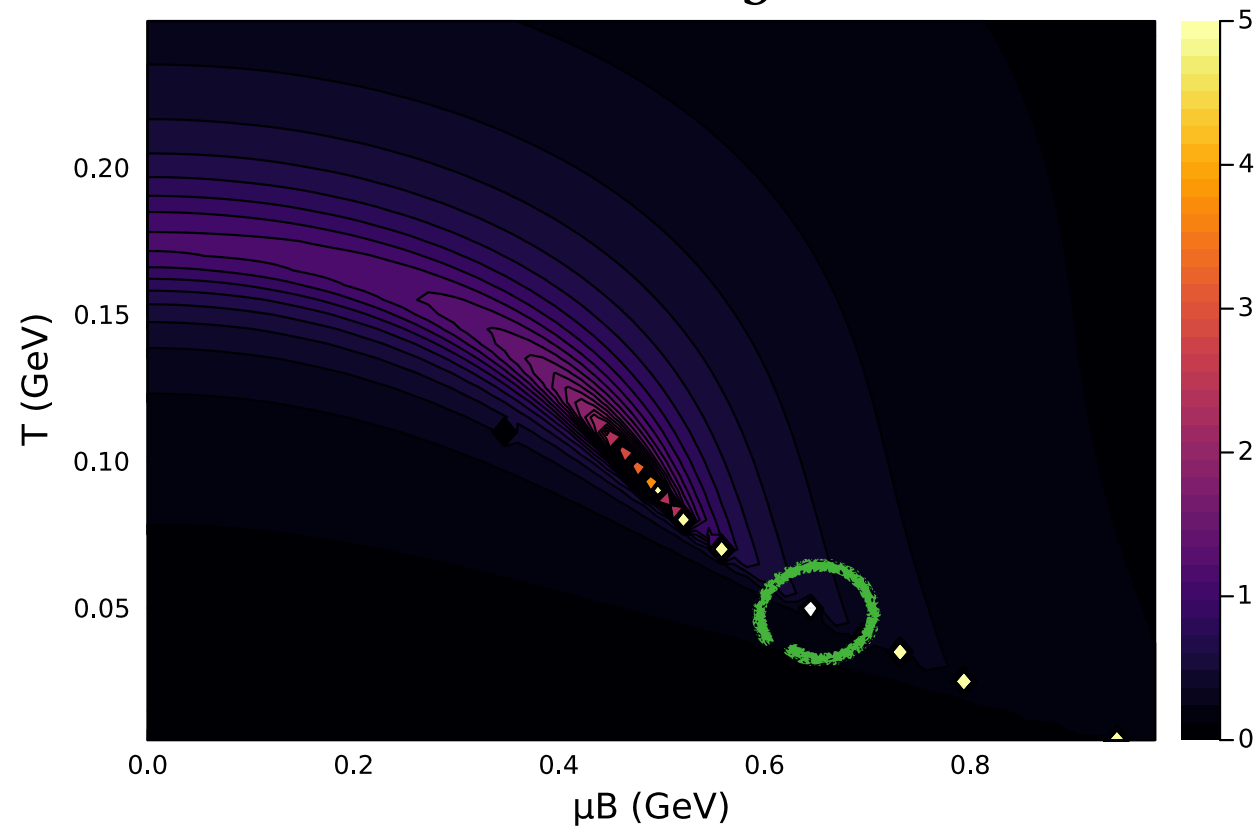


$G_V=0.2$

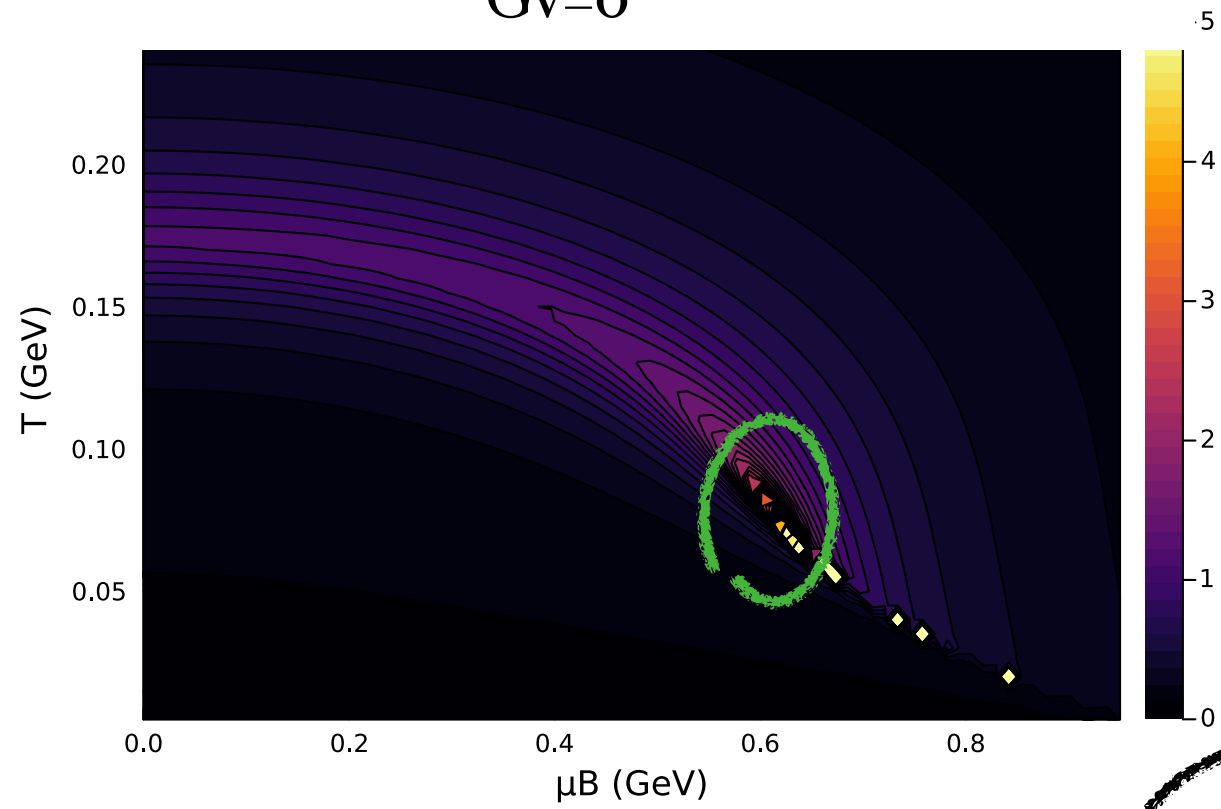


**CHIRAL
SUSCEPTIBILITY**

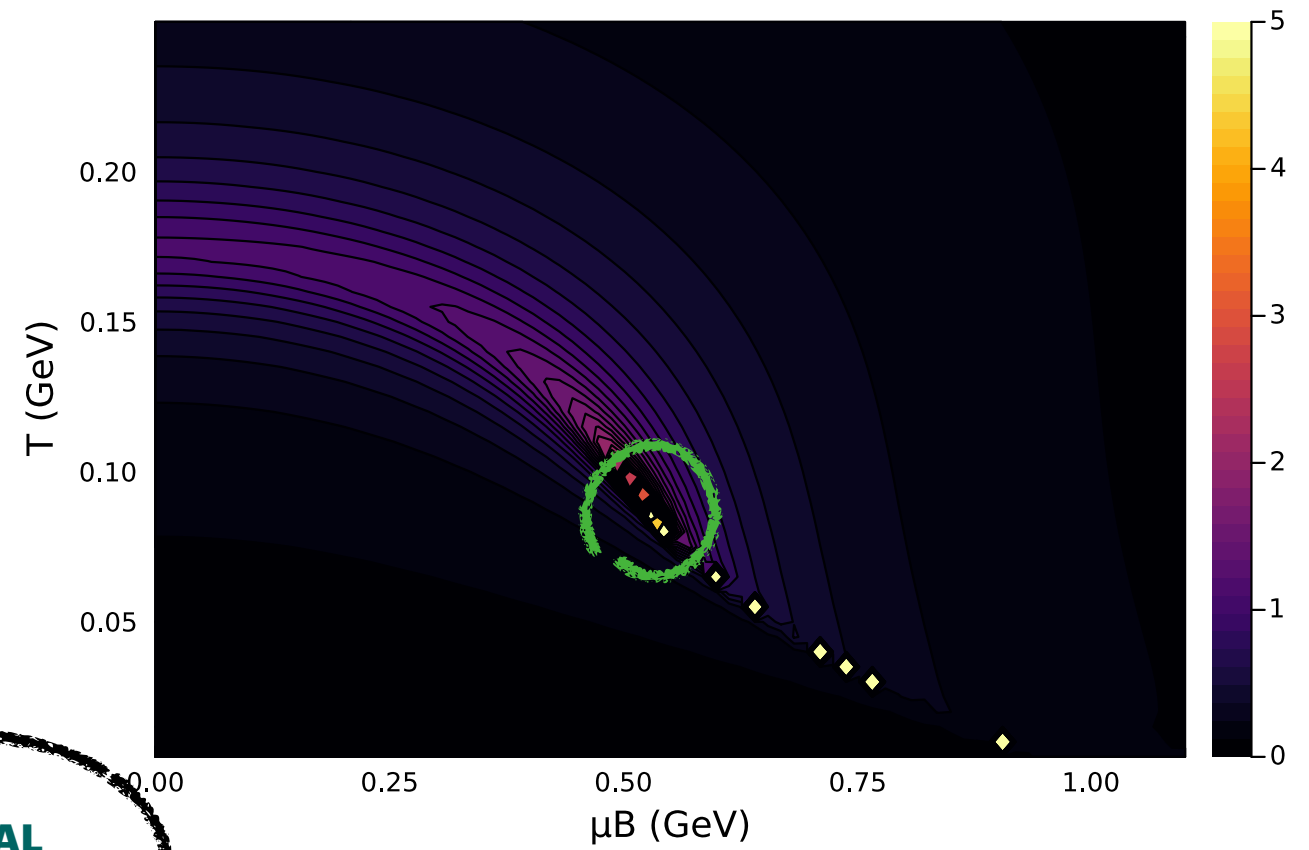
$G_V=0.3$



$G_V=0$

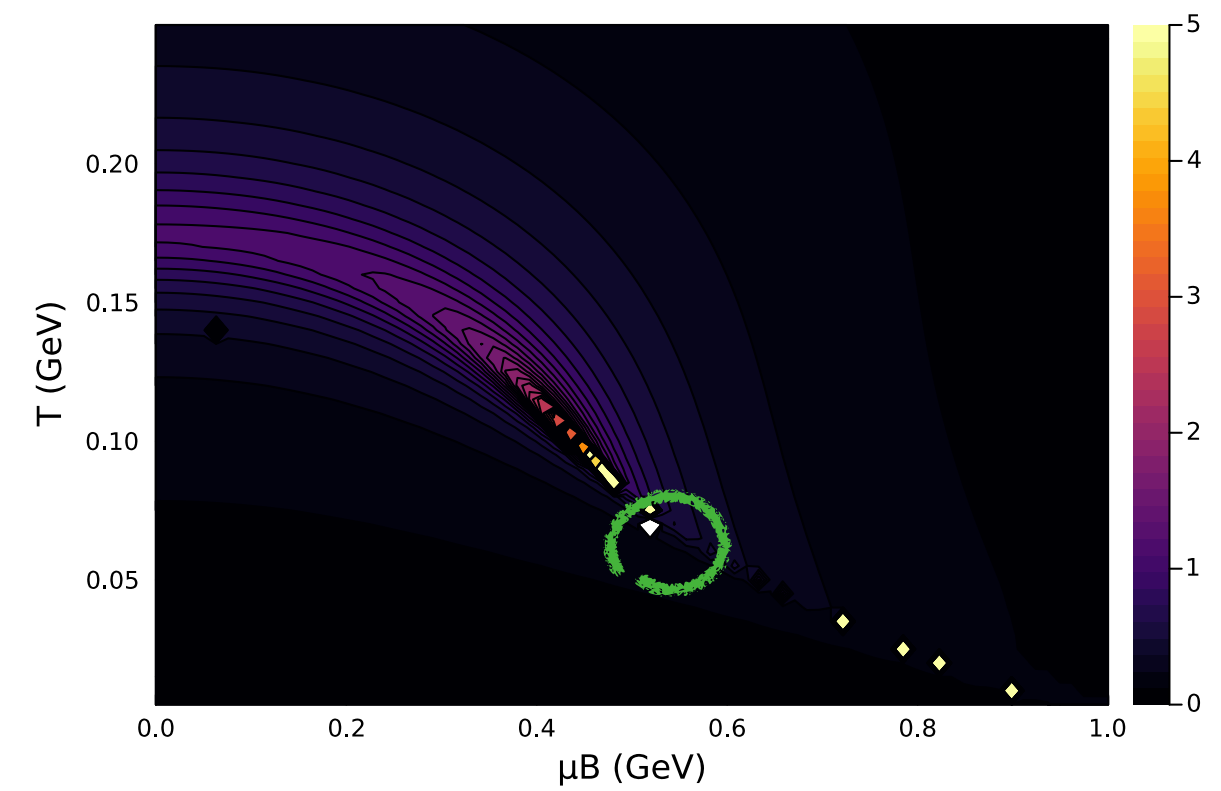


$G_V=0.2$

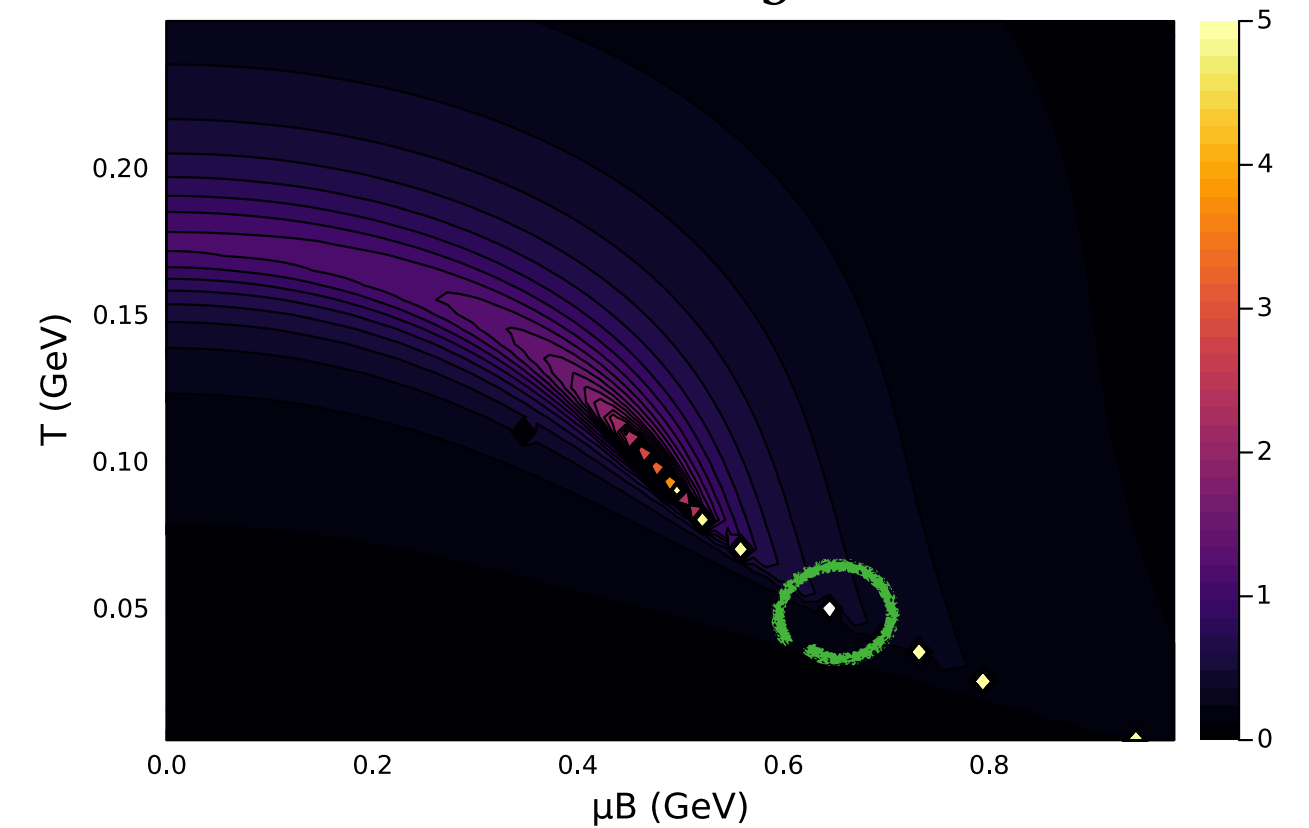


**CHIRAL
SUSCEPTIBILITY**

$G_V=0.4$



$G_V=0.3$



preliminary result

POLYAKOV LOOP

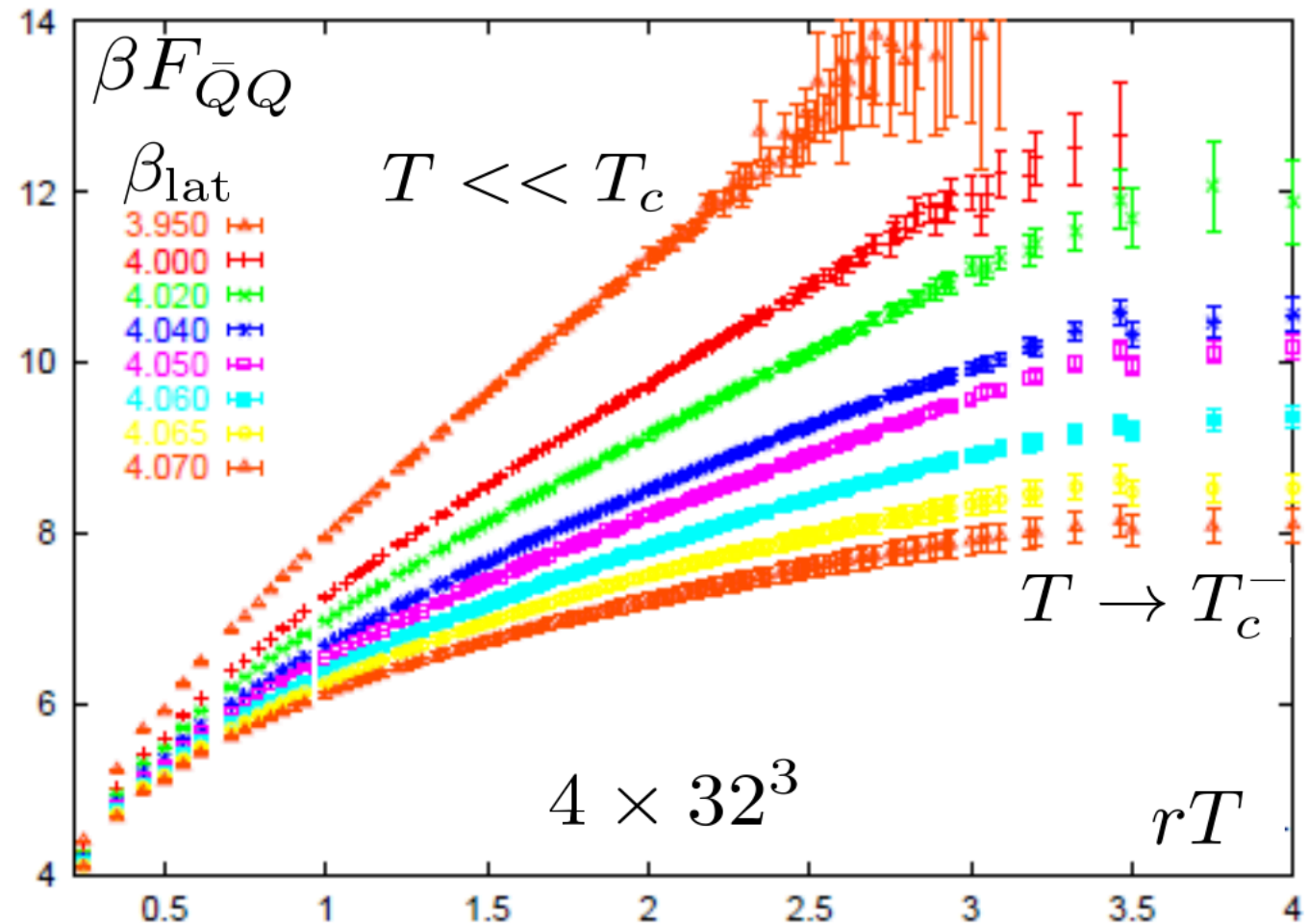
HEAVY QUARK FREE ENERGY

$$|\langle L \rangle|^2 = e^{-\beta F_{Q\bar{Q}}[r \rightarrow \infty, T]}$$

Kaczmarek *et. al.*

$T < T_c$
 $\langle L \rangle = 0$
 confined

$T > T_c$
 $\langle L \rangle \neq 0$
deconfined



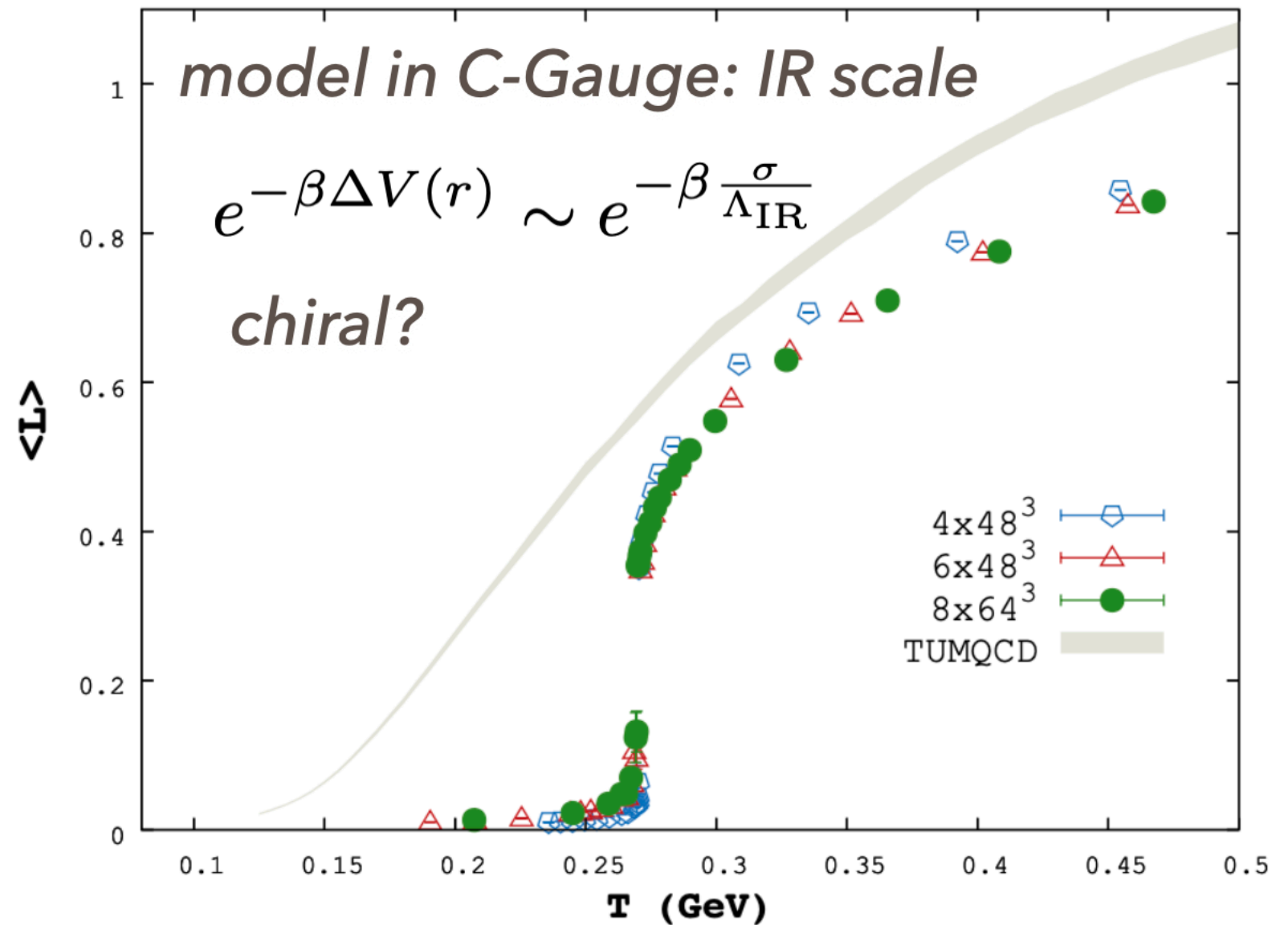
HEAVY QUARK FREE ENERGY

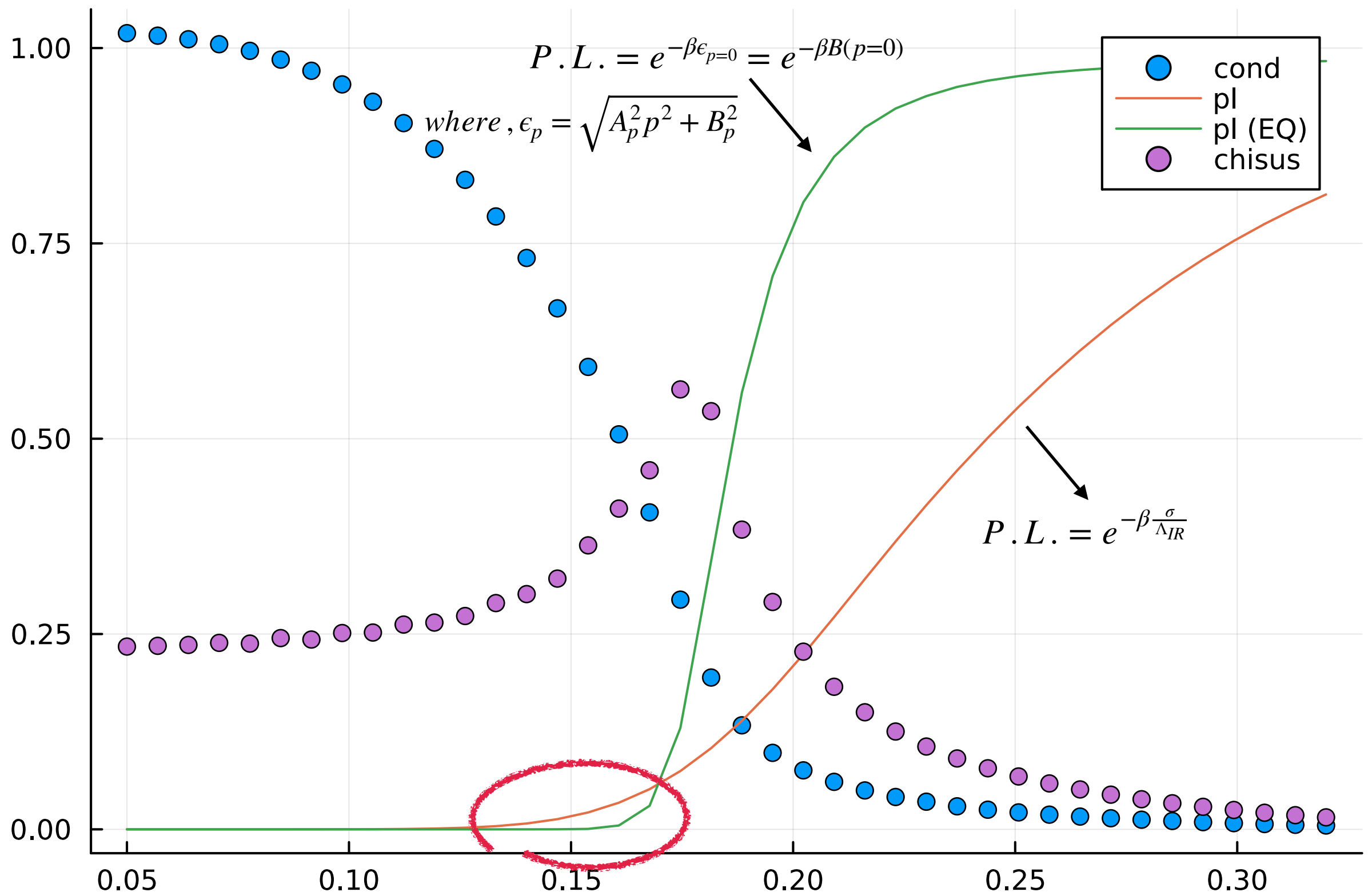
IR-regulated potential

$$|\langle L \rangle|^2 = e^{-\beta F_{Q\bar{Q}}[r \rightarrow \infty, T]} \quad V(r) - V(0) = \frac{-\sigma}{\Lambda_{IR}} [e^{-r\Lambda_{IR}} - 1]$$

$T < T_c$
 $\langle L \rangle = 0$
 confined

$T > T_c$
 $\langle L \rangle \neq 0$
deconfined





preliminary result

SUMMARY & CONCLUSIONS

- Infrared (confinement) & UV scales serve very different purposes (within the regime of current model).
 - Asymptotic freedom \rightarrow p-dep. & UV scale in μ -channel \rightarrow cs^2 and χ_2 receive **essential, quantifiable** contribution towards their respective conformal limits (**even without confinement**: separable model).
 - **Unrealistic** $Gv=0$ case (no p-dep. in μ -channel) : **only scale is μ** \rightarrow leads to conformal limit by accident.
- Towards a first attempt to determine phase diagram of Coulomb gauge QCD
 - Fixing vacuum naturally leads to a $T_c \sim 155$ MeV \rightarrow confinement model.
 - **CEP moves up as interaction strength in μ -channel (Gv) increases** \rightarrow confinement model.
- Polyakov loop can be computed as an observable \rightarrow **doesn't perform well as a deconfinement order parameter.**
- **TO DO**: include confinement in dense matter (ring *Eur. Phys. J. A* 58 (2022) 9, 172), include diquark.

Thank you for your attention.

Back-up slides

CHIRAL CONDENSATE

The constituent mass is found from the gap equation:

$$M(p) = m_q + C(p)\bar{\sigma} \quad \text{with the mean field} \quad \bar{\sigma} = \langle \sigma \rangle$$

By the principle of least action we can get the mean field:

$$\bar{\sigma} = 8N_c G \int \frac{d^4 p}{(2\pi)^4} C(p) \frac{M(p)}{p^2 + M^2(p)}$$

How to get the condensate:

$$\langle \bar{\psi} \psi \rangle = -4N_f N_c \int \frac{d^4 p}{(2\pi)^4} \left[\frac{M(p)}{p^2 + M^2(p)} - \frac{m_q}{p^2 + m_q^2} \right]$$

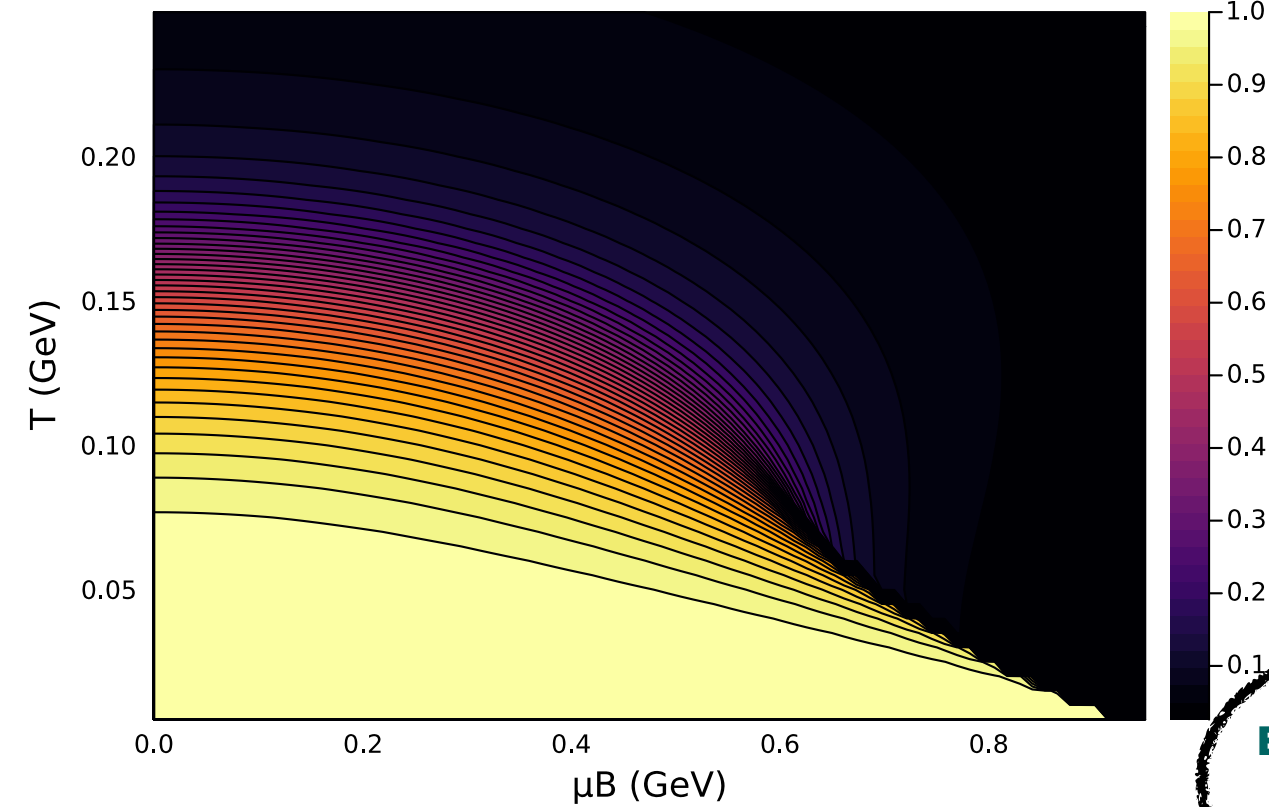
Differences with the local version? $M = -G \langle \bar{\psi} \psi \rangle = \bar{\sigma}$

$$\Lambda_{IR} = 0.1 \text{ GeV}$$

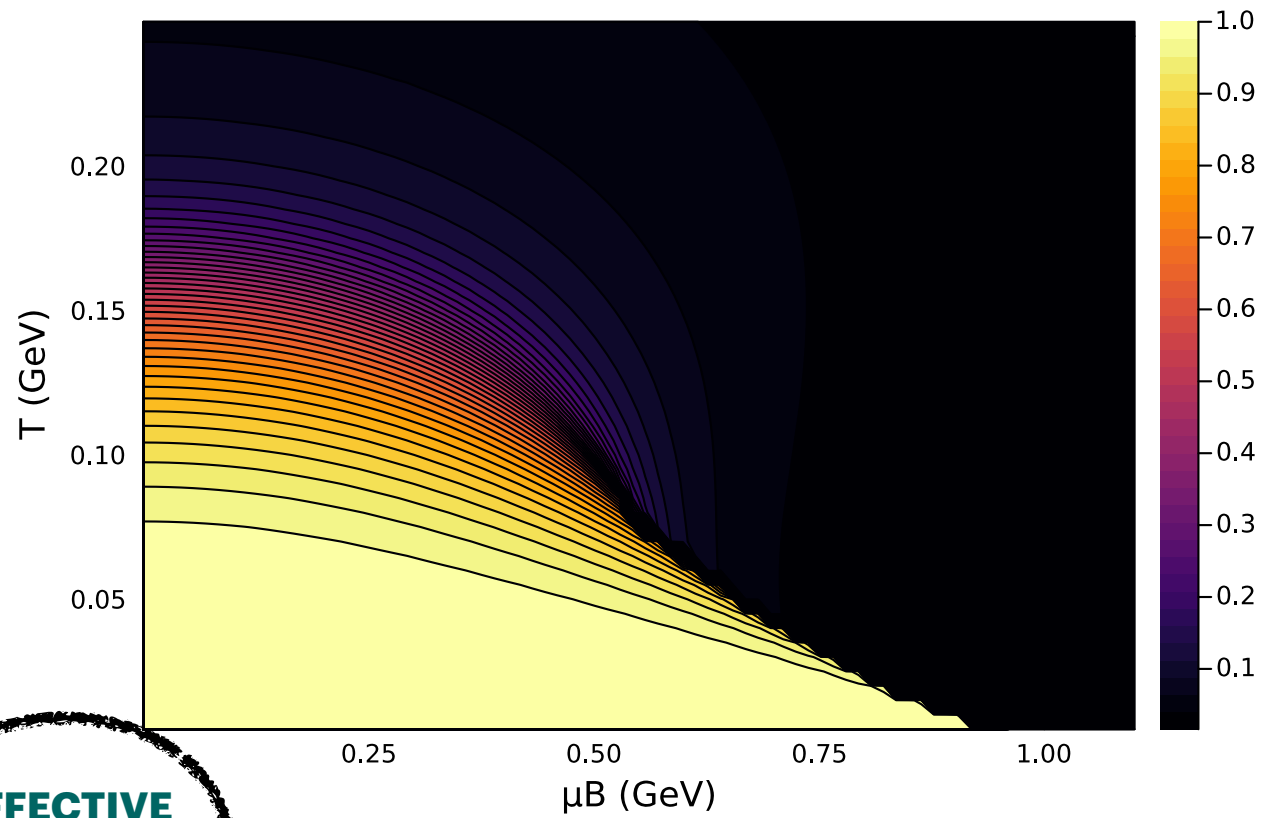
$$\sigma = 0.18 * 15$$

$$C_f = \frac{N_c^2 - 1}{2N_c}$$

$Gv=0$

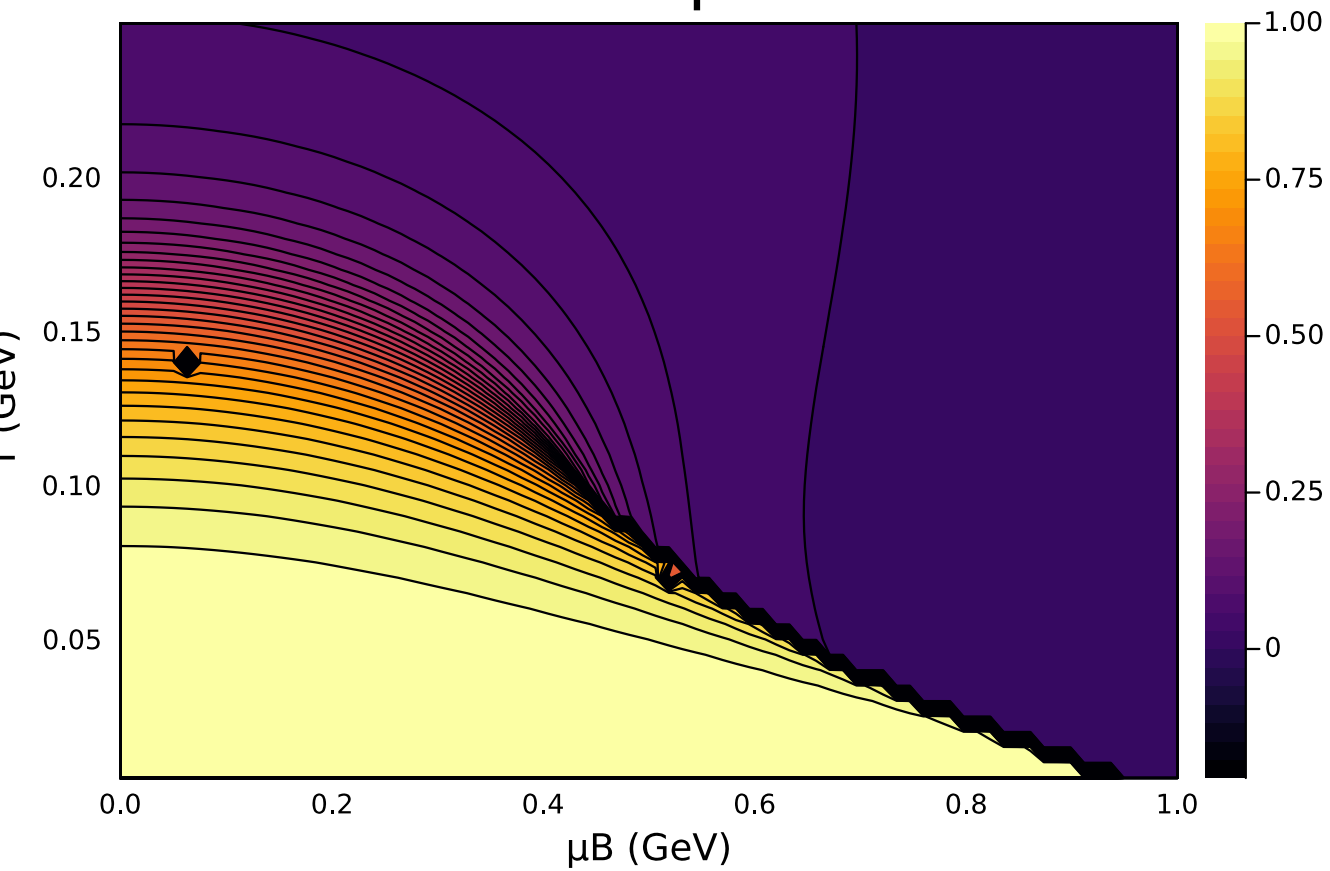


$Gv=0.2$

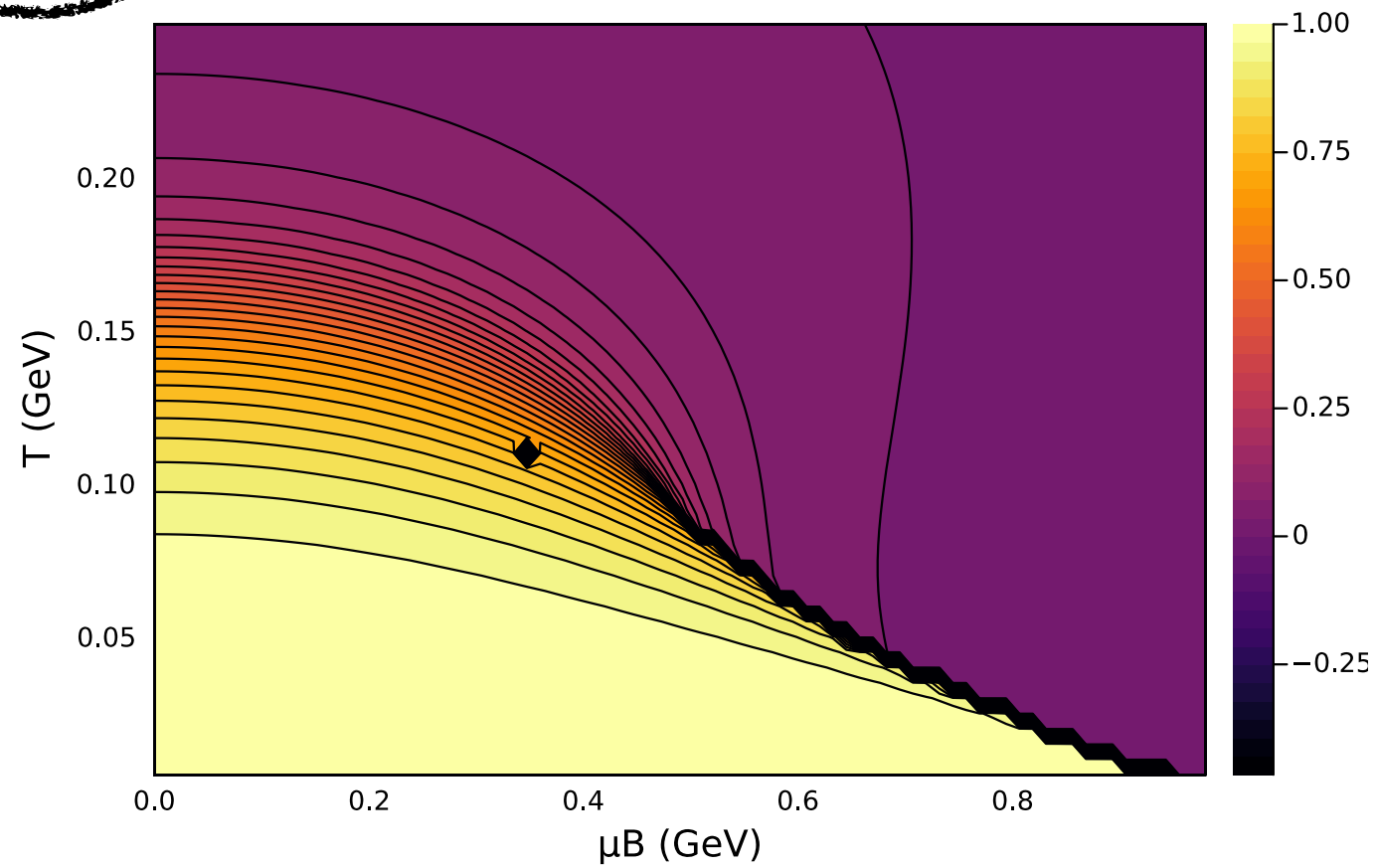


**EFFECTIVE
QUARK
MASS**

$Gv=0.4$



$Gv=0.3$



preliminary result

SPEED OF SOUND: Astrophysical Constraint for high μ , low T

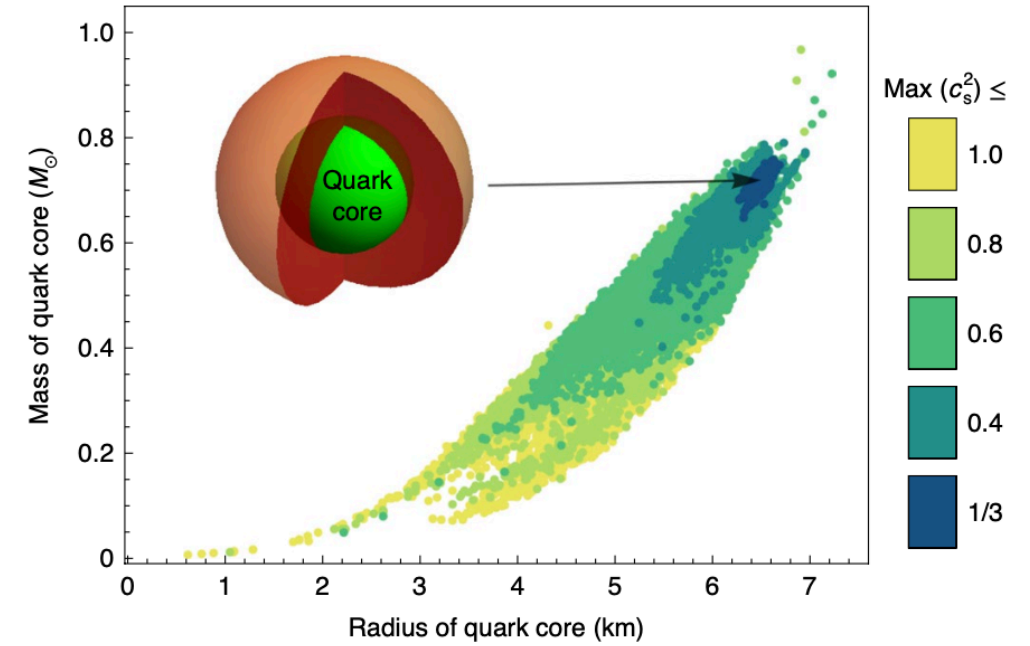
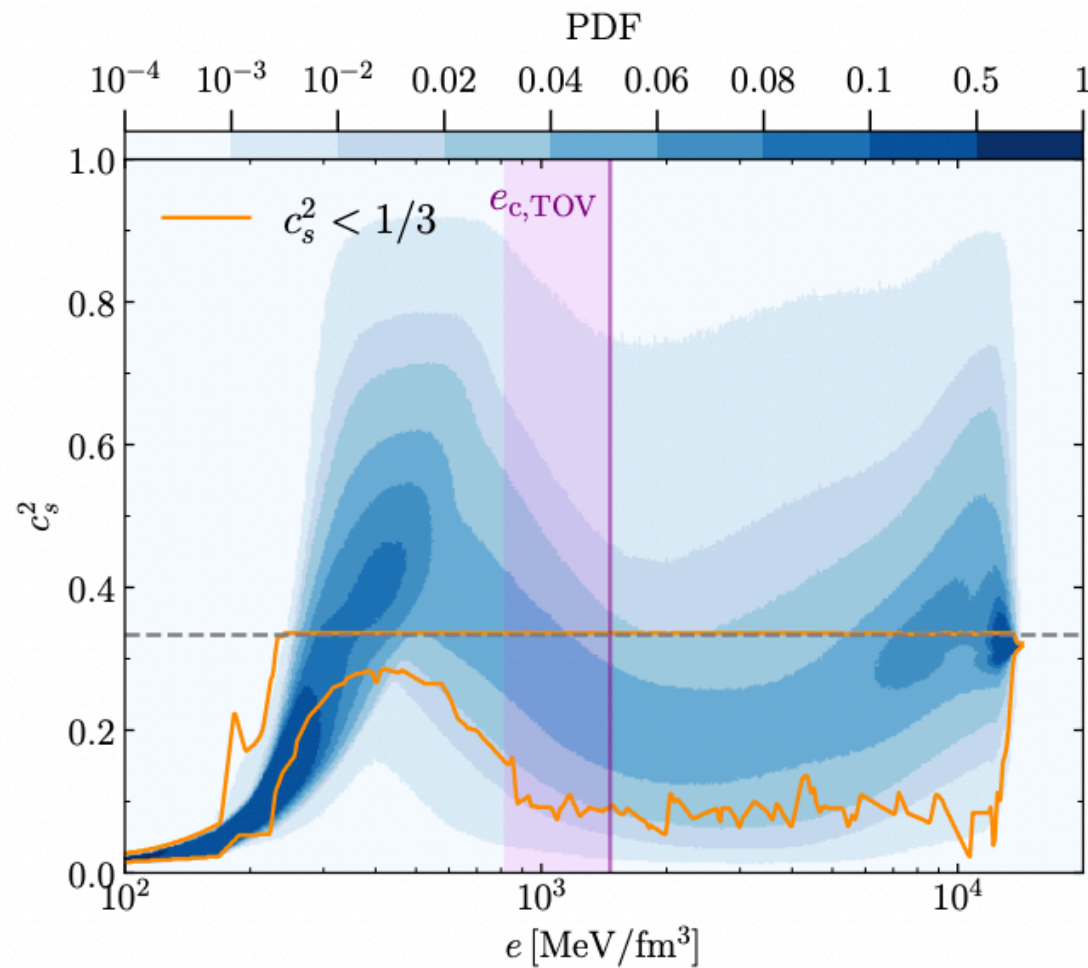


Fig. 3 | The size of the quark core. Predictions for the radius and the mass of the quark cores in maximally massive NSs are displayed. The maximal value that the speed of sound squared c_s^2 reaches in each individual EoS is indicated by the colour coding of the corresponding point. Points corresponding to lower c_s^2 values are drawn on top of those corresponding to higher ones. The NS in the inset visualizes a 12 km, $2M_\odot$ star with a 6.5 km quark core, built with a subconformal ($c_s^2 < 1/3$) EoS.

astronomical observations. In this way, we find that EOSs with sub-conformal sound speeds, i.e., with $c_s^2 < 1/3$ within the stars, are possible in principle but very unlikely in practice, being only 0.03% of our sample. Hence,

with mass $M \approx 2M_\odot$, the presence of quark matter is found to be linked to the behaviour of the speed of sound c_s in strongly interacting matter. If the conformal bound $c_s^2 \leq 1/3$ is not strongly violated, massive neutron stars are predicted to have sizable quark-matter cores.

SPEED OF SOUND: Astrophysical Constraint for high μ , low T

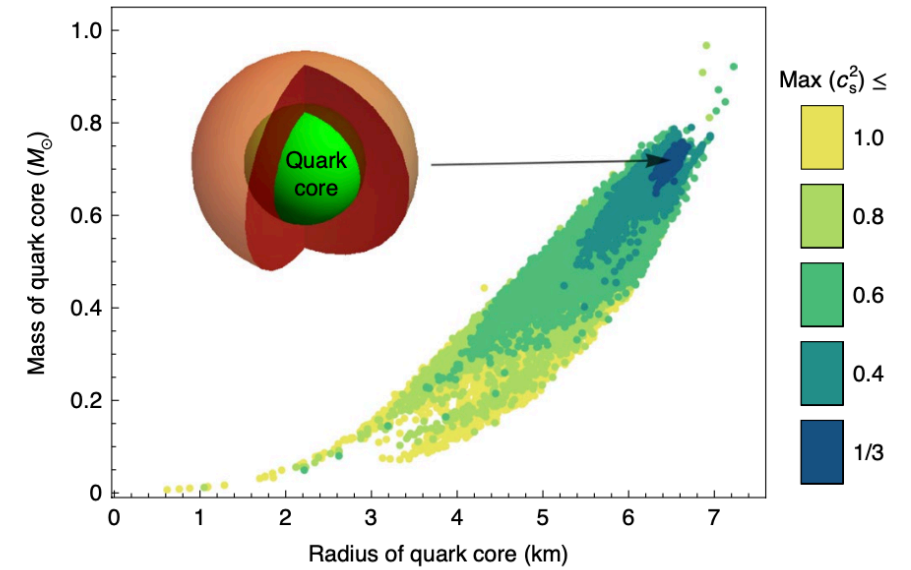
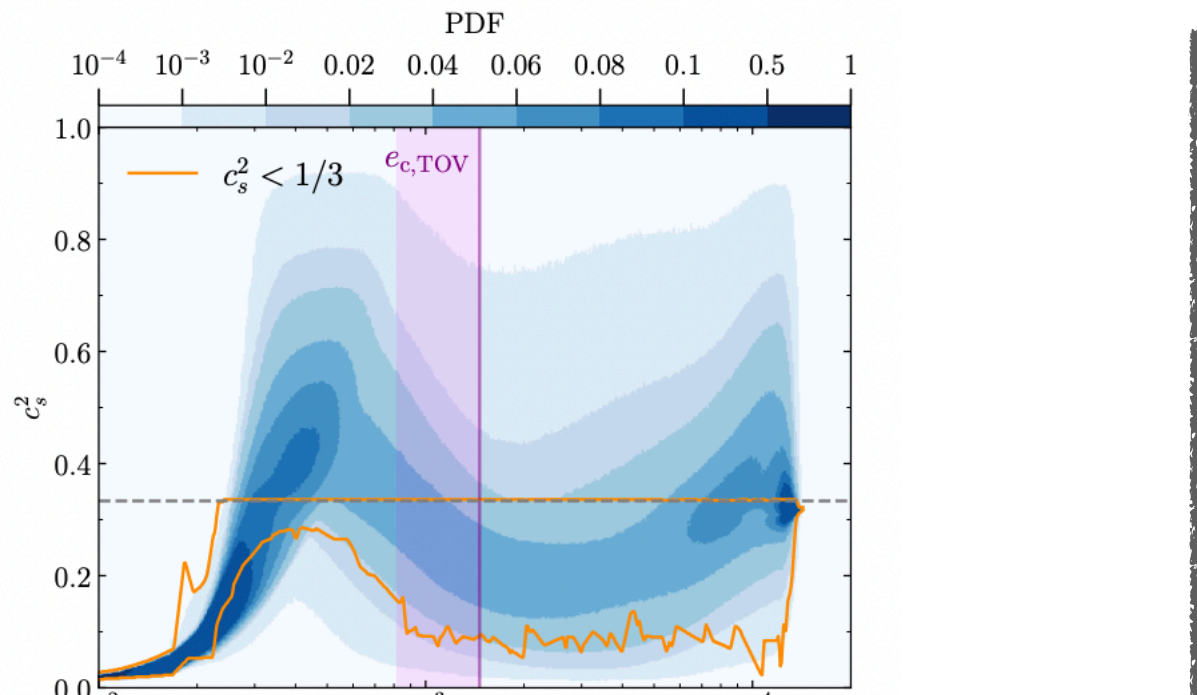
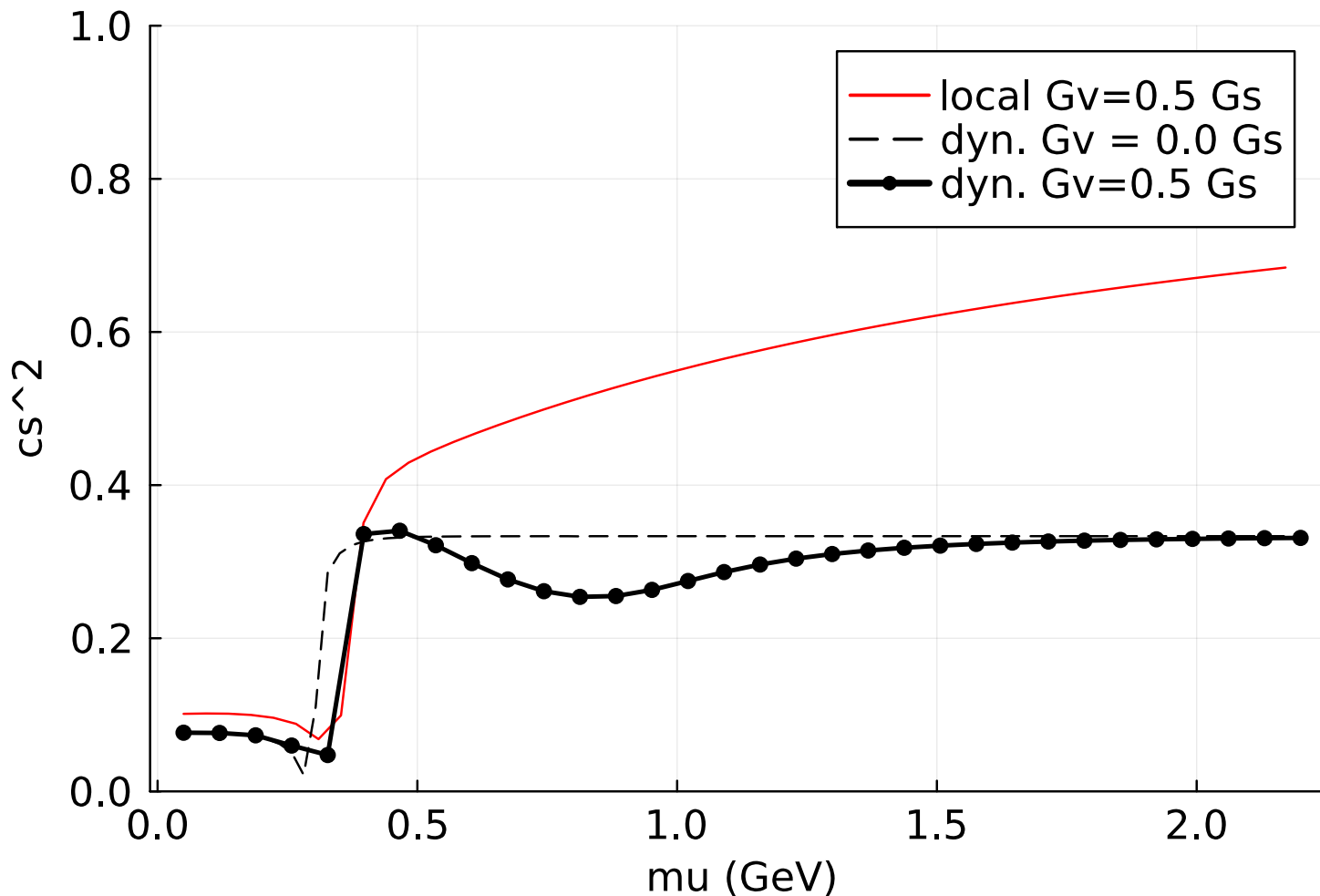


Fig. 3 | The size of the quark core. Predictions for the radius and the mass



→ *NJL-like theory fails*

→ *Dynamical
chiral Quark Model*

Local v/s Dynamical

$T \rightarrow 0$

Local Interaction

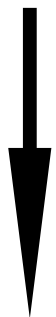
$$n_v = N_c N_f \frac{1}{\pi^2} \int_0^\infty dq q^2 \Theta(\mu^* - q)$$

$$= N_c N_f \frac{1}{\pi^2} \int_0^{\mu^*} dq q^2$$

$$= N_c N_f \frac{1}{3\pi^2} \mu^*(\mu)^3$$

$$n_v = n_v(\mu^*(\mu))$$

$$\mu^* = \mu - G_v n_v$$



Trivial Fermi surface

Dynamical Interaction

$$n_v = N_c N_f \frac{1}{\pi^2} \int_0^\infty dq q^2 \Theta(\mu_q^* - q)$$

$$= N_c N_f \frac{1}{\pi^2} \int_0^{\mu_{pf(\mu)}^*} dq q^2$$

$$n_v = n_v[\mu_{pf(\mu)}^*]$$

$$\mu^*(p) = \mu - \gamma(p) G_v n_v$$



NATURAL DENSITY-DEPENDENCE

Non-trivial Interacting Fermi surface !