Domain Walls & Gravitational Waves

Alexander Vikman

02.10.2024

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This talk is based on

- Beyond freeze-in: dark matter via inverse phase transition and gravitational wave signal
  e-Print: 2104.13722, PRD

- Gravitational shine of dark domain walls
  e-Print: 2112.12608, JCAP

- NANOGrav spectral index γ=3 from melting domain walls
  e-Print: 2307.04582, PRD

- Revisiting evolution of domain walls and their gravitational radiation with CosmoLattice
  e-Print: 2406.17053, JCAP

Eugeny Babichev (IJCLab, Orsay)
Ivan Dankovsky (Moscow State U. and INR)
Dmitry Gorbunov (INR and MIPT, Moscow)
Sabir Ramazanov (ITMP, Moscow State U.)
Rome Samanta (INFN & SSM, Naples)
Alexander Vikman (CEICO, FZU Prague)

Main Message

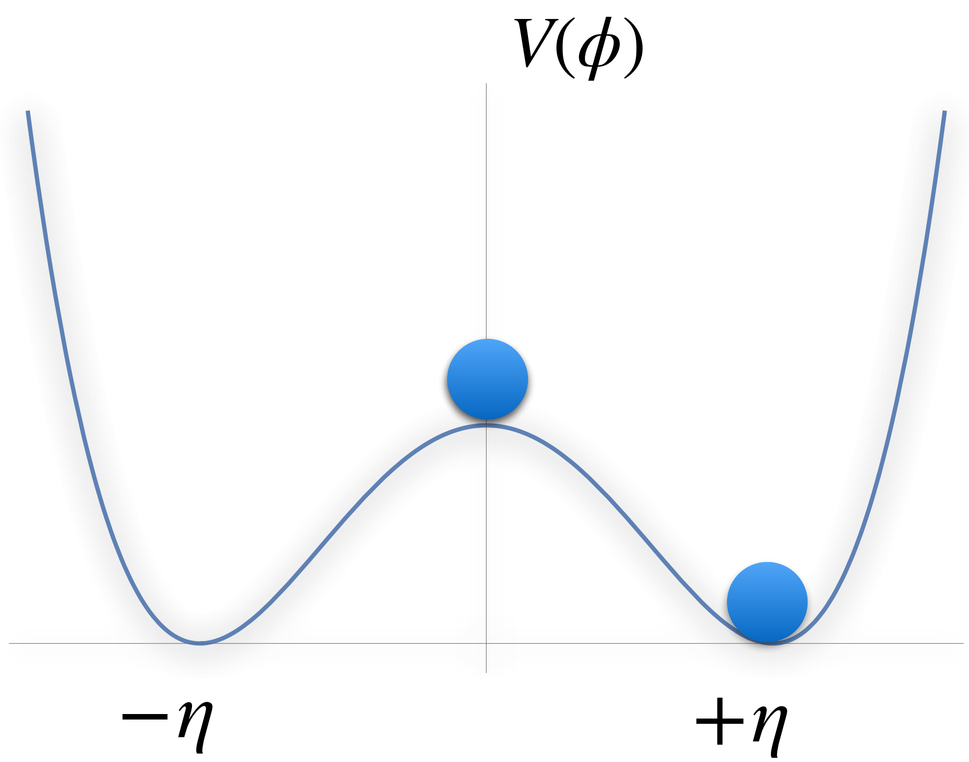
- Ultralight DM can be created for minuscular couplings and still produce observable GW

- NANOGrav GW can be from Melting Domain Walls of DM

- Scaling Regime in DW evolution seems to be only a local attractor, details depend on initial conditions

Spontaneous Breaking of Discrete Symmetry

Z_2 -symmetric scalar field



$$V = \frac{\lambda}{4}\left(\phi^2 - \eta^2\right)^2$$

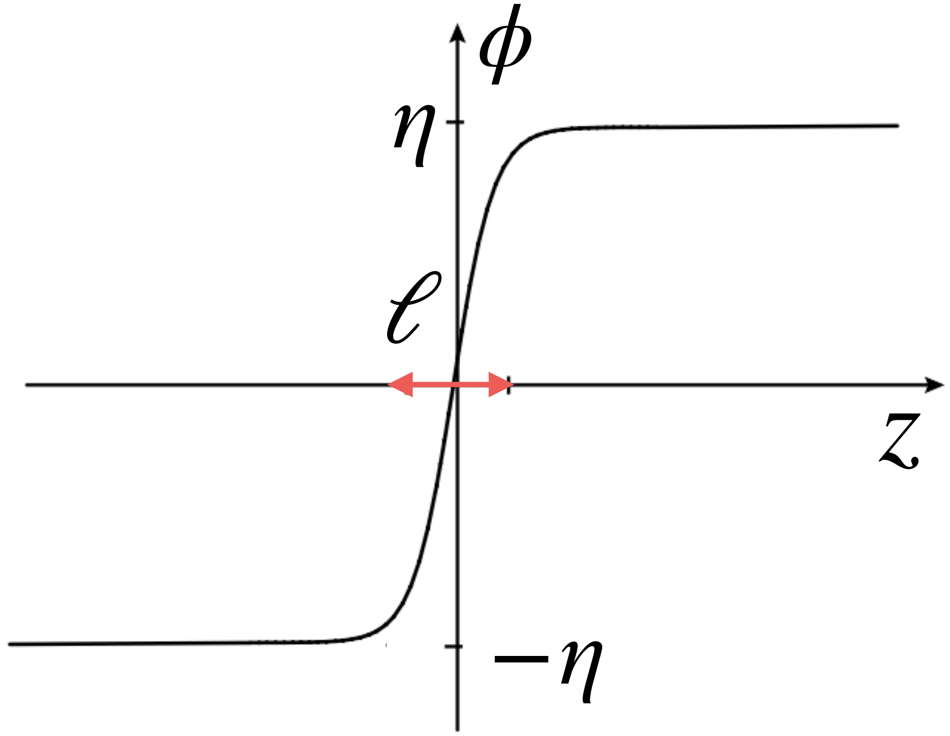
$$\varepsilon = \frac{1}{2}\left(\nabla\phi\right)^2 + V$$

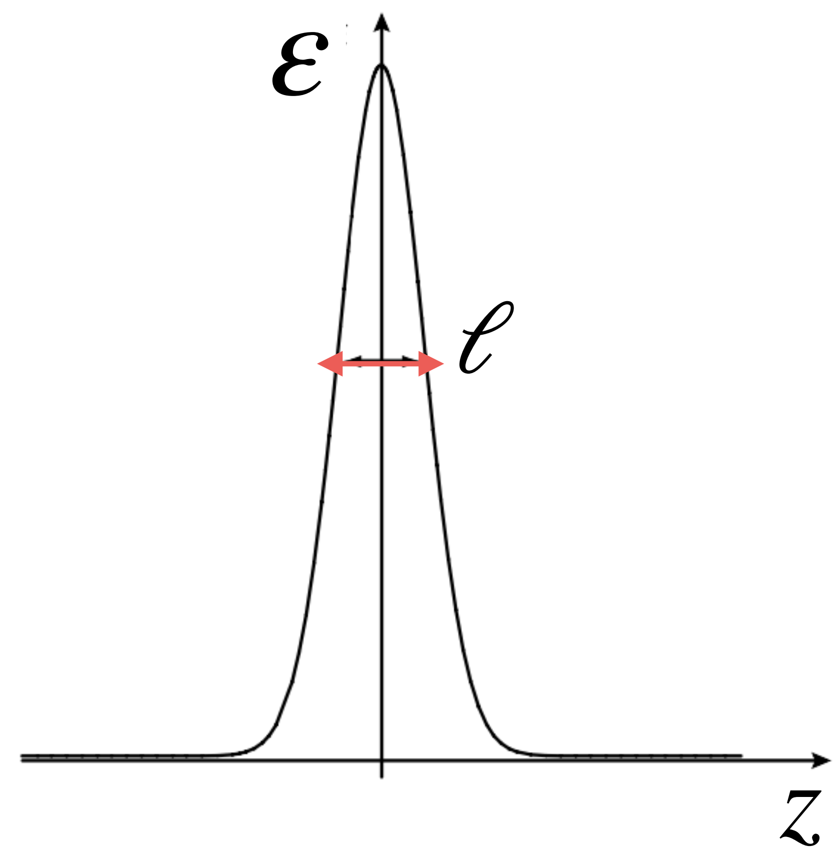
Minimising $\quad \sigma \simeq \varepsilon\ell \simeq \left(\frac{\eta}{\ell}\right)^2 \ell + \lambda\eta^4\ell$

$$\ell \simeq \lambda^{-1/2}\eta^{-1} \qquad \sigma \simeq \lambda^{1/2}\eta^3$$

Kink

$$\phi(z) = \eta \tanh\left(z/\ell\right)$$





$$\ell = (\lambda/2)^{-1/2}\,\eta^{-1}$$

Energy-Momentum and Surface Tension

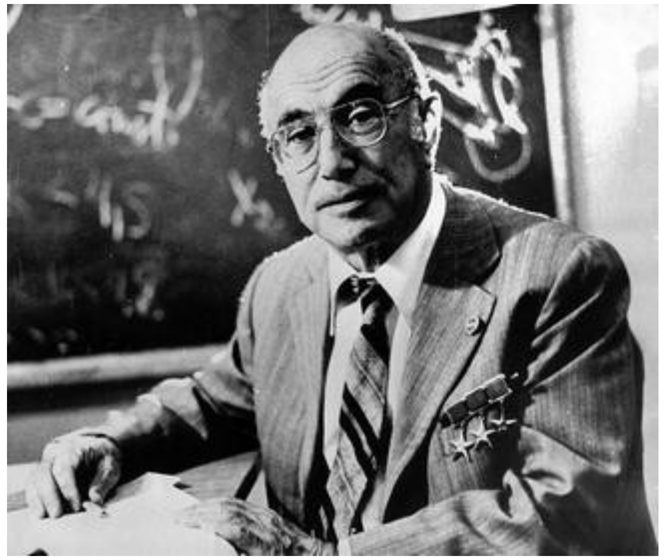
$$T^{\mu}_{\nu} = \partial^{\mu}\phi\,\partial_{\nu}\phi - \delta^{\mu}_{\nu}\left(\frac{1}{2}\left(\partial\phi\right)^{2} - V\left(\phi\right)\right)$$

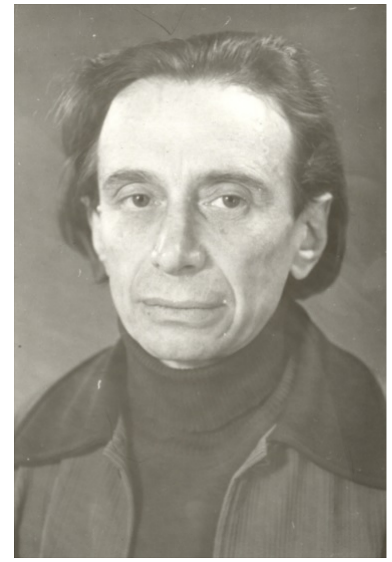
$$T^{\mu}_{\nu} = \frac{\lambda}{2}\eta^{4}\cosh^{-4}\left(\frac{z}{\ell}\right) diag\left(1,1,1,0\right)$$

$$\sigma_{wall} = \int dz T^{0}_{0} = \frac{2\sqrt{2\lambda}}{3}\eta^{3}(T)$$

Note that Great Wall

Even a Greater One!





Cosmological consequences of a spontaneous breakdown of a discrete symmetry

Ya. B. Zel'dovich, I. Yu. Kobzarev, and L. B. Okun'

Institute for Applied Mathematics, USSR Academy of Sciences

(Submitted January 31, 1974)

Zh. Eksp. Teor. Fiz. **67**, 3–11 (July 1974)

In theories involving spontaneous symmetry breakdown one may expect a domain structure of the vacuum. Such a structure does not exist near a cosmolgical singularity, when the temperature is above the Curie point, but this structure must appear later during the cosmological expansion and cooling down. We discuss the properties of the domain interfaces and of the space with domains in the large, the law of cosmological expansion in the presence of domains, and the influence of domains of the homogeneity of the Universe at a late stage.

Large CMB fluctuations

photon temperature due to redshift $T \propto \frac{1}{a}$

$$\frac{\delta T}{T} \propto \frac{\delta a}{a} \simeq \Phi$$

Poisson Equation $\Delta\Phi \sim G\,\sigma_{wall}\,\delta(z)$

$$\Phi \sim G\,\sigma_{wall} z$$

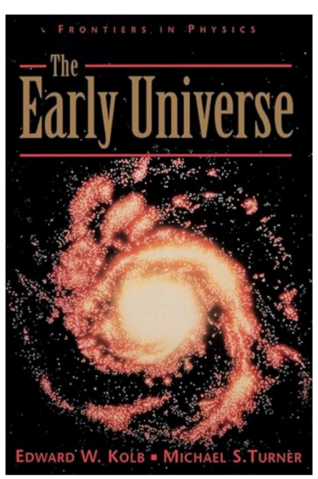
$$\frac{\delta T}{T} \simeq G\,\sigma_{wall} H_0^{-1} \simeq 10^{10}\ \lambda^{1/2}\left(\frac{\eta}{100\,GeV}\right)^3$$

Even Larger Mass

Mass inside the horizon H^{-1}

$$M_{wall} \sim \sigma_{wall}/H^2$$

$$\simeq 4 \times 10^{65}\ \lambda^{1/2} \left(\frac{\eta}{100\,GeV} \right)^3 \text{ grams}$$



"Apparently, domain walls are cosmological bad news…"

PHYSICAL REVIEW D VOLUME 9, NUMBER 12 15 JUNE 1974

Gauge and global symmetries at high temperature*

Steven Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 19 February 1974)

It is shown how finite-temperature effects in a renormalizable quantum field theory can restore a symmetry which is broken at zero temperature. In general, for both gauge symmetries and ordinary symmetries, such effects occur only through a temperature-dependent change in the effective bare mass of the scalar bosons. The change in the boson bare mass is calculated for general field theories, and the results are used to derive the critical temperatures for a few special cases, including gauge and nongauge theories. In one case, it is found that a symmetry which is unbroken at low temperature can be broken by raising the temperature above a critical value. An appendix presents a general operator formalism for dealing with higher-order effects, and it is observed that the one-loop diagrams of field theory simply represent the contribution of zero-point energies to the free energy density. The cosmological implications of this work are briefly discussed.

PHYSICAL REVIEW D VOLUME 23, NUMBER 4 15 FEBRUARY 1981

Gravitational field of vacuum domain walls and strings

Alexander Vilenkin
Department of Physics, Tufts University, Medford, Massachusetts 02155
(Received 10 October 1980)

The gravitational properties of vacuum domain walls and strings are studied in the linear approximation of general relativity. These properties are shown to be very different from those of regular massive planes and rods. It is argued that the domain walls are gravitationally unstable and collapse at a certain time $\sim t_c$ after their creation. If the vacuum walls ever existed, they must have disappeared at $t < t_c$.

Z_2 -symmetric DM scalar field χ coupled to ϕ - a multiplet of N *thermal* degrees of freedom

portal coupling

$$V = \frac{1}{2}\left(M^2 - g^2\phi^\dagger\phi\right)\cdot\chi^2 + \frac{\lambda}{4}\chi^4 + \frac{\lambda_\phi}{4}\left(\phi^\dagger\phi\right)^2$$

tachyonic thermal mass

$$\mu^2 = g^2\langle\phi^\dagger\phi\rangle \simeq \frac{Ng^2T^2}{12}$$

increasing during preheating, then red-shifting

potential bounded from below

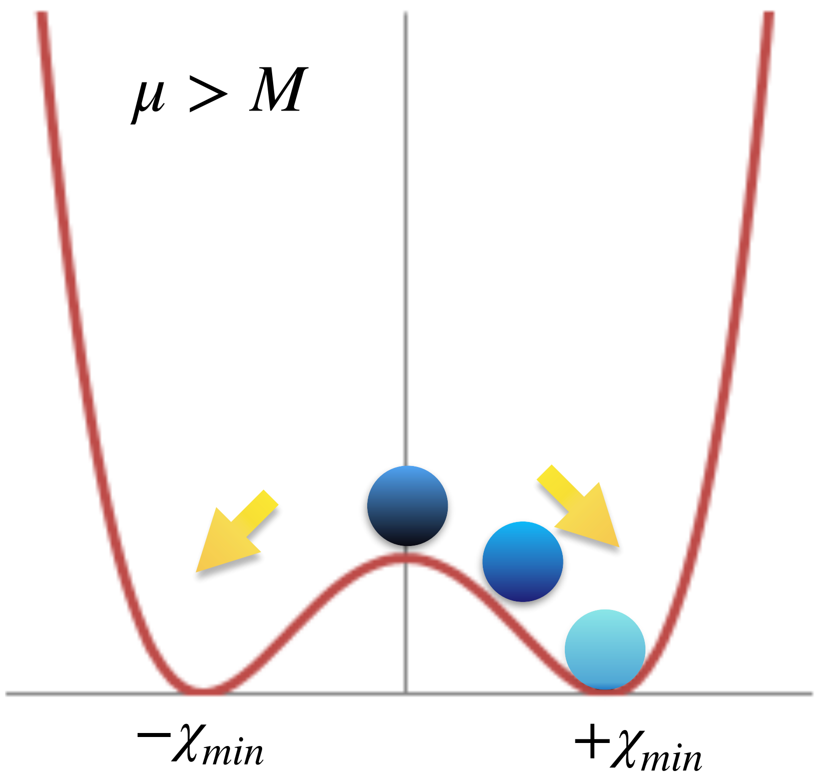
$$\beta = \frac{\lambda}{g^4} \geq \frac{1}{\lambda_\phi} \geq 1$$

potential bounded from below

weak coupling

Direct Phase Transition

Early universe spontaneously Broken Phase



Domain Walls!

Avoid too much friction to start rolling

$$\mu \gtrsim H$$

$$\sqrt{\frac{N}{12}} g T_i \simeq \sqrt{\frac{\pi^2 g_*}{90}} \frac{T_i^2}{M_{pl}}$$

$$T_i \simeq g \, M_{Pl} \sqrt{\frac{N}{g_*(T_i)}} \times \frac{1}{\sqrt{B}}$$

Correction taking into account time to get to the minimum

$$V_{eff} \simeq \frac{\lambda \cdot \left(\chi^2 - \eta^2\right)^2}{4} \qquad \eta(T) \simeq g\sqrt{\frac{N}{12\lambda}}\, T$$

Tension/energy per unit surface $\sigma_{wall} = \frac{2\sqrt{2\lambda}}{3}\eta^3(T)$ melting away as $\propto T^3$!

In the **scaling regime (Kibble 1976):** one domain wall per Hubble volume:

$$M_{wall} \sim \sigma_{wall}/H^2$$

$$\rho_{wall} \sim M_{wall}H^3 \sim \sigma_{wall}\, H \propto T^5$$

Usual Constant tension DW
$\rho_{wall} \propto T^2$

Let us first study **usual constant tension** DW and associated GW

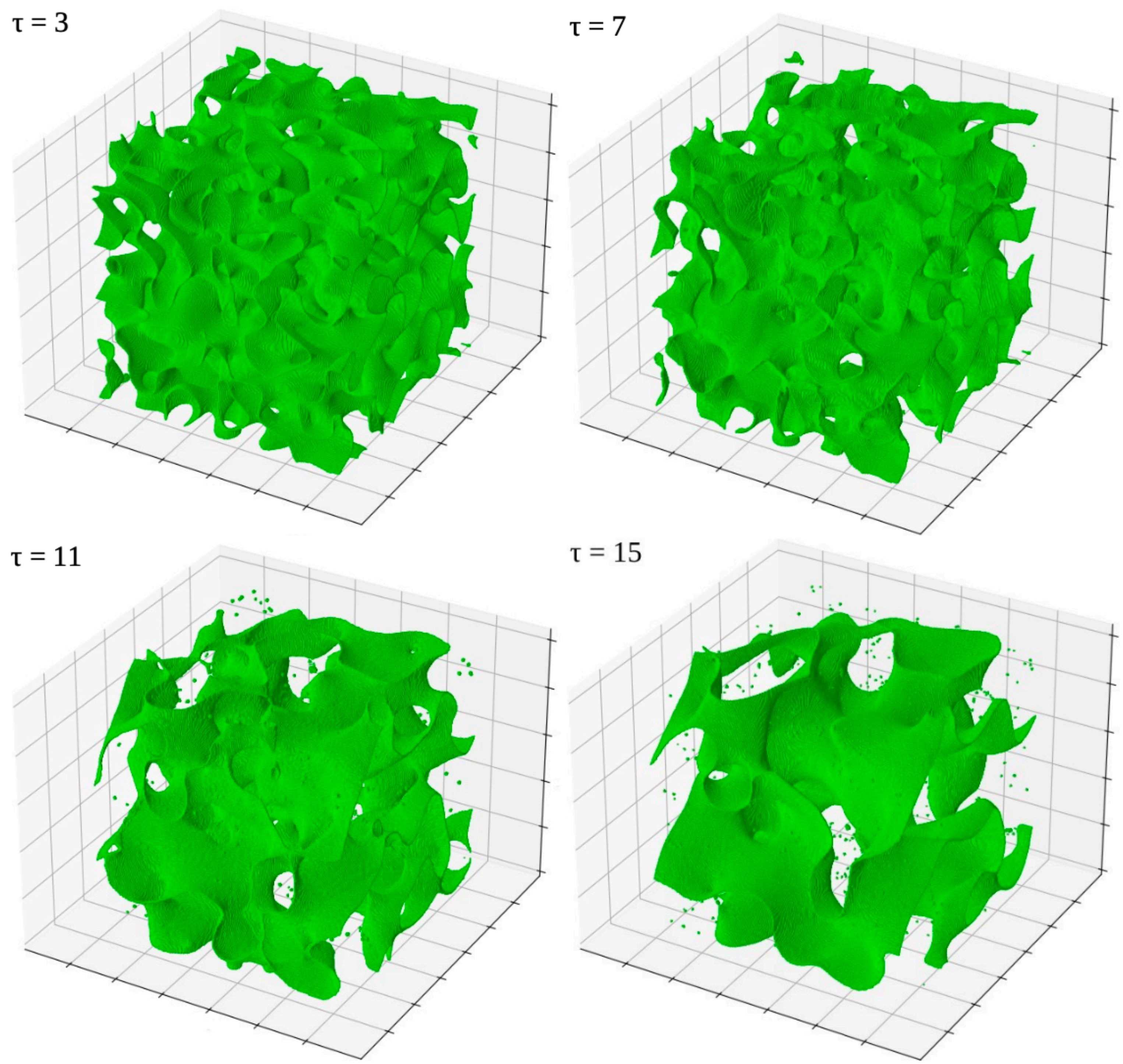


Figure 2: Snapshots of domain wall evolution in the case of vacuum initial conditions at different conformal times τ in units of $\frac{1}{\sqrt{\lambda}\eta}$. Simulations have been performed starting from vacuum initial conditions on a lattice with the grid number $N = 512$. The visible dot-like structures are small size domain walls.

Figueroa, Florio, Torrenti, Valkenburg

CosmoLattice

"vacuum" initial conditions

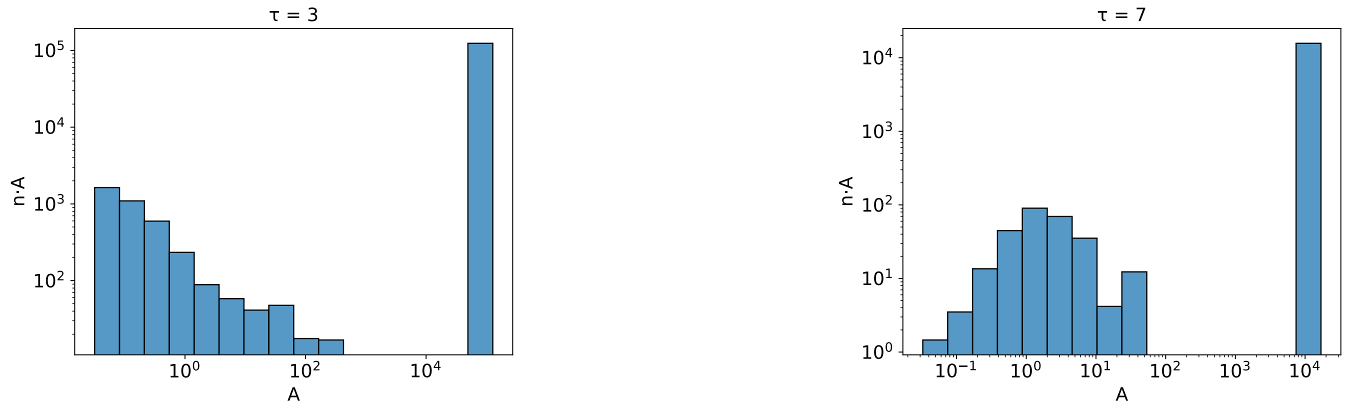
$$\left\langle \chi(\mathbf{k}) \chi^*(\mathbf{k}') \right\rangle = \frac{1}{2k} \delta(\mathbf{k} - \mathbf{k}')$$

$$\left\langle \dot{\chi}(\mathbf{k}) \dot{\chi}^*(\mathbf{k}') \right\rangle = \frac{k}{2} \delta(\mathbf{k} - \mathbf{k}')$$

"thermal" initial conditions

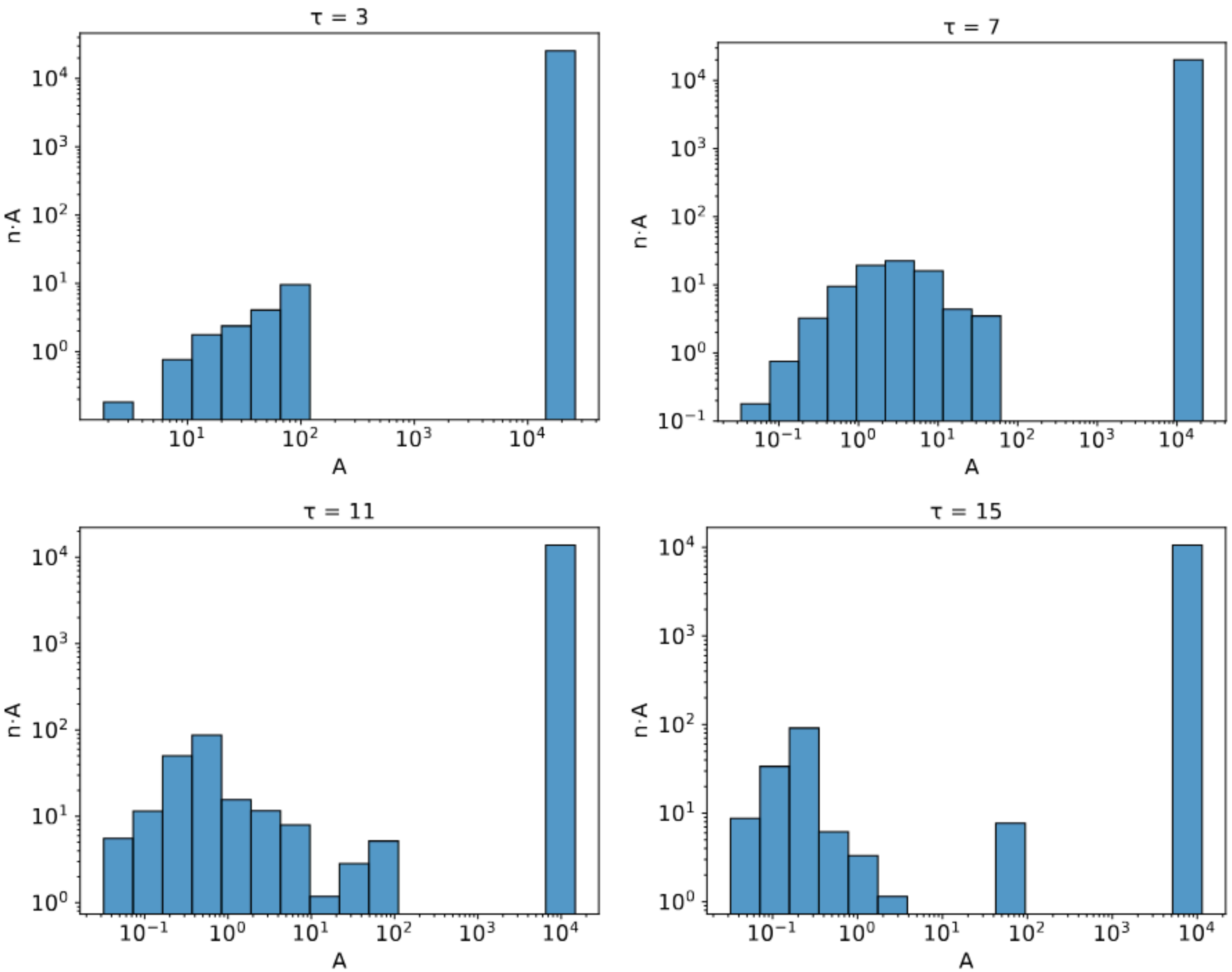
$$\times \frac{2}{e^{k/T} - 1}$$

Dankovsky, Babichev, Gorbunov, Ramazanov, Vikman (2024)



Figueroa, Florio, Torrenti, Valkenburg

CosmoLattice



Dankovsky, Babichev, Gorbunov, Ramazanov, Vikman (2024)

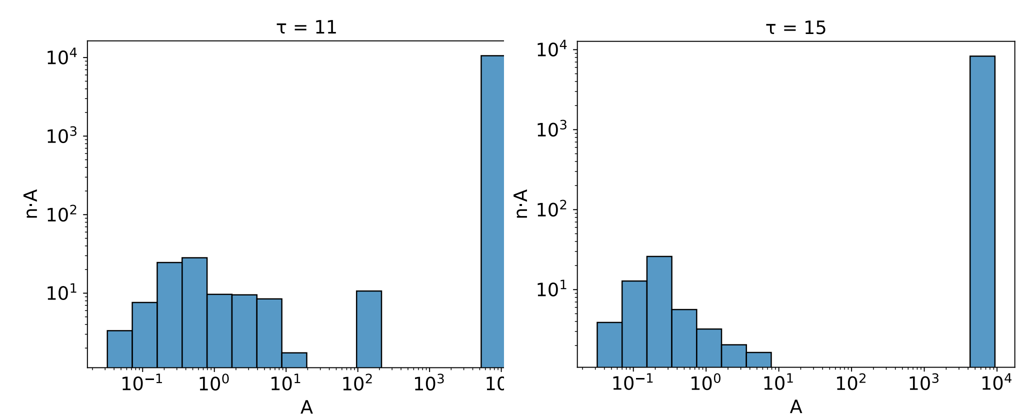
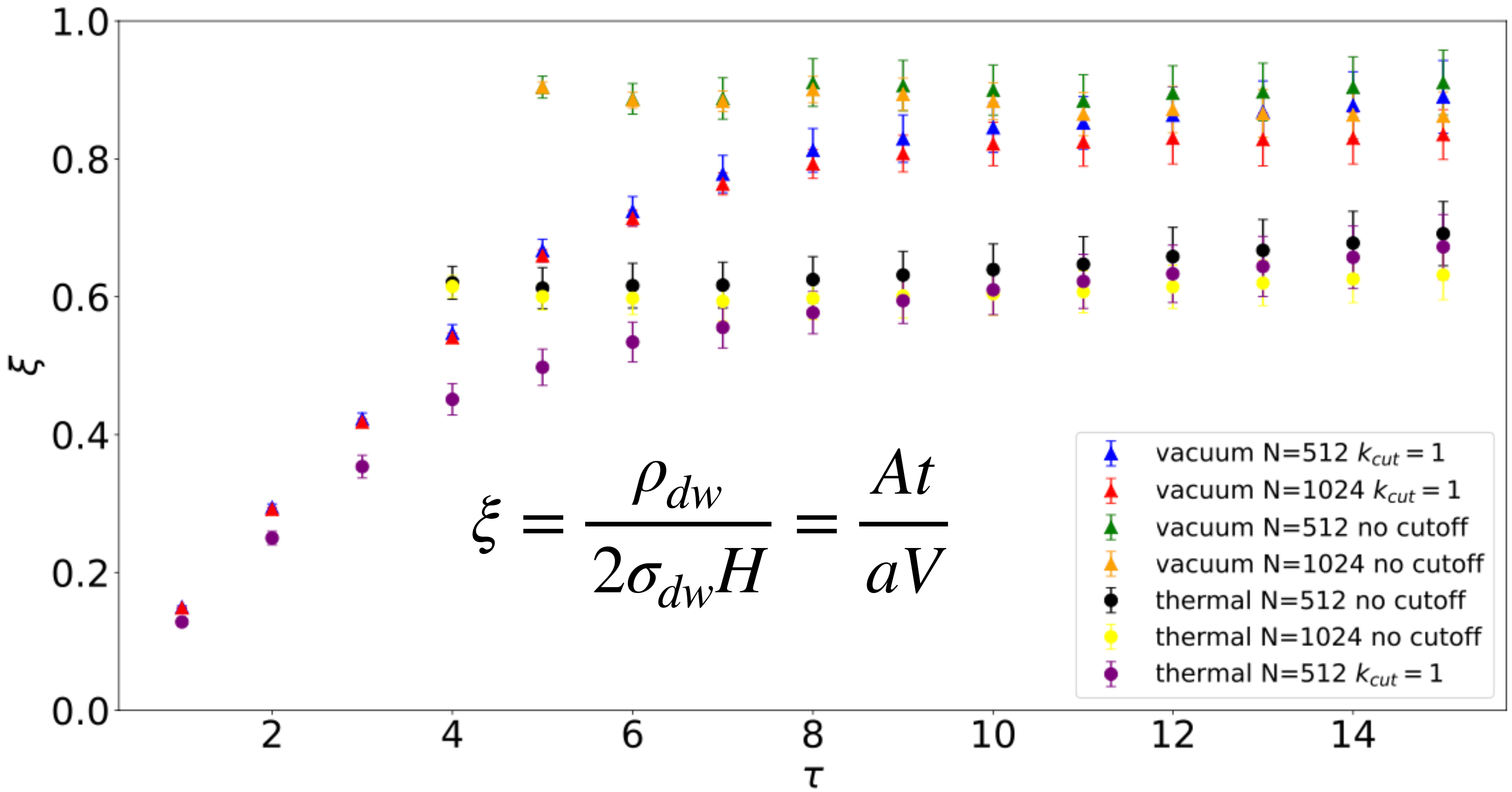


Figure 5: Histograms showing the distribution of the quantity $n \cdot A$ versus A, where A is the comoving domain wall area in units of $\frac{1}{\lambda\eta^2}$, and n is the number of domain walls with the area A. Distributions are considered at different conformal times τ in units of $\frac{1}{\sqrt{\lambda}\eta}$. Simulations have been performed starting from vacuum initial conditions on a lattice with the grid number $N = 512$.

Scaling Parameter (constant tension DW)



Figueroa, Florio, Torrenti, Valkenburg

CosmoLattice

Figure 4: The area parameter ξ inferred in Eq. (44) is obtained from numerical simulations performed on lattices with the grid numbers $N = 512$ and $N = 1024$ starting from vacuum and thermal initial conditions with and without cutoffs at high momenta. Conformal time τ and conformal momentum k are in units of $\frac{1}{\sqrt{\lambda}\eta}$ and $\sqrt{\lambda}\eta$, respectively. The parameter ξ taking a constant value reflects that the domain wall network enters the scaling regime. Expectation values and error bars are obtained from 10 simulations run with different base seed values.

Dankovsky, Babichev, Gorbunov, Ramazanov, Vikman (2024)

Cf. Hiramatsu, Kawasaki, Saikawa (2013);
Ferreira, Gasparotto, Hiramatsu, Obata, Pujolas (2023);
Kitajima, Lee, Takahashi, Yin (2023)

Gravitational Waves

Einstein's formula

$$P \sim \dddot{Q}_{ij}^2 / M_{Pl}^2$$

works well for domain wall network!!!

On the estimation of gravitational wave spectrum from cosmic domain walls

Takashi Hiramatsu (Kyoto U., Yukawa Inst., Kyoto), Masahiro Kawasaki (Tokyo U., ICRR and Tokyo U., IPMU), Ken'ichi Saikawa (Tokyo Inst. Tech.) (Sep 19, 2013)

Published in: *JCAP* 02 (2014) 031 • e-Print: 1309.5001 [astro-ph.CO]

Quadrupole Moment

$$|Q_{ij}| \sim M_{wall}/H^2$$

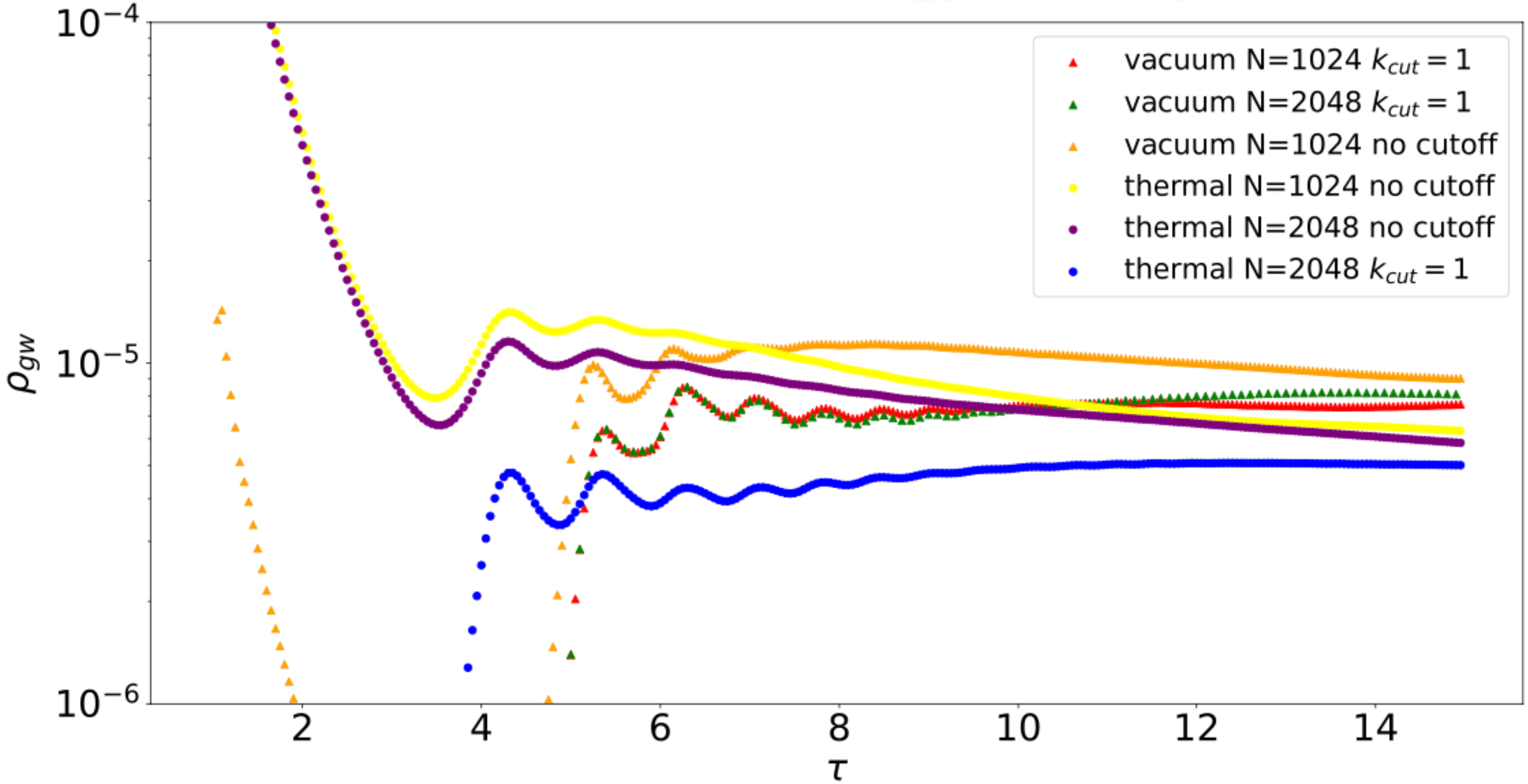
$$M_{wall} \sim \sigma_{wall}/H^2$$

$$\rho_{gw} \sim P \cdot t \cdot H^3 \sim \frac{\sigma_{wall}^2}{M_{Pl}^2} \propto T^6$$

If scaling regime attained almost instantaneously, the **peak frequency** is H_i!

$$f = H_i$$

GW energy density



Figueroa, Florio, Torrenti, Valkenburg

CosmoLattice

Figure 6: The energy density of GWs in units of $\lambda\eta^4$ emitted by the domain wall network is obtained from numerical simulations on lattices with the grid numbers $N = 1024$ and $N = 2048$ starting from vacuum and thermal initial conditions with and without cutoffs. Conformal time τ is in units of $\frac{1}{\sqrt{\lambda}\eta}$. The expectation value η is set at $\eta = 6 \cdot 10^{16}$ GeV. Rescaling to arbitrary η is achieved by multiplying the energy density ρ_{gw} by $(\eta/6 \cdot 10^{16}\ \text{GeV})^2$.

Dankovsky, Babichev, Gorbunov, Ramazanov, Vikman (2024)

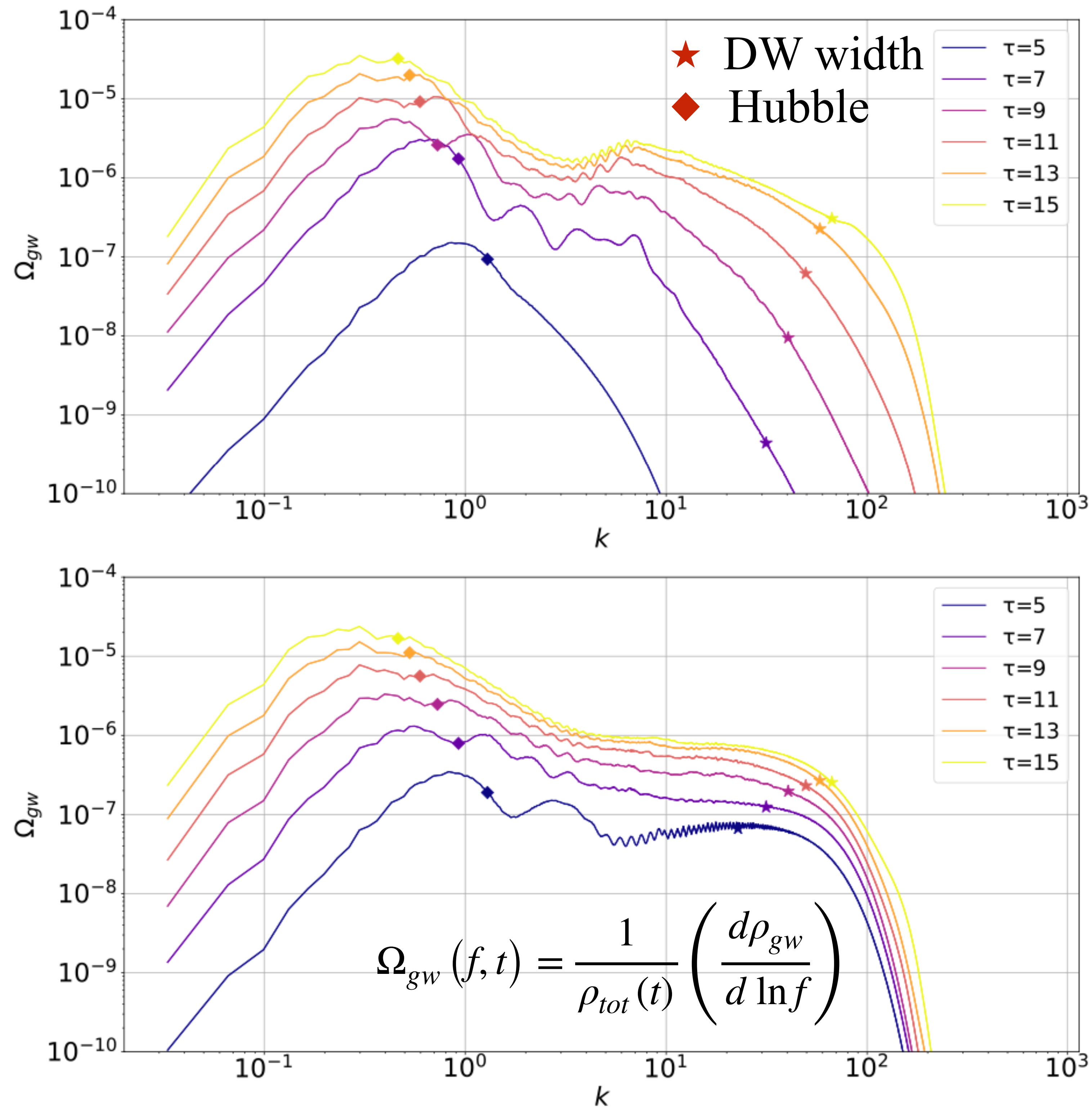


Figure 1: Spectrum of GWs emitted by the domain wall network at radiation domination starting with vacuum (top panel) and thermal (bottom panel) initial conditions defined in Eqs. (36) and (37), respectively. Conformal momenta and conformal times are in units of $\sqrt{\lambda}\eta$ and $\frac{1}{\sqrt{\lambda}\eta}$, respectively. The sharp upper cutoff at $k_{cut} = 1$ is applied in the case of vacuum initial conditions. The expectation value η is set at $\eta = 6 \cdot 10^{16}$ GeV. Rescaling to arbitrary η is achieved by multiplying the spectra by $(\eta/6 \cdot 10^{16}\ \text{GeV})^4$. Simulations have been performed on a lattice with a grid number $N = 2048$. The positions of diamonds correspond to the comoving Hubble scale $k = 2\pi Ha$ at the time associated with the corresponding curves, while stars show the inverse domain wall width $1/\delta_w$, i.e., $k = 2\pi a/\delta_w$.

GW from constant tension DW

Figueroa, Florio, Torrenti, Valkenburg

CosmoLattice

Dankovsky, Babichev, Gorbunov, Ramazanov, Vikman (2024)

cf.

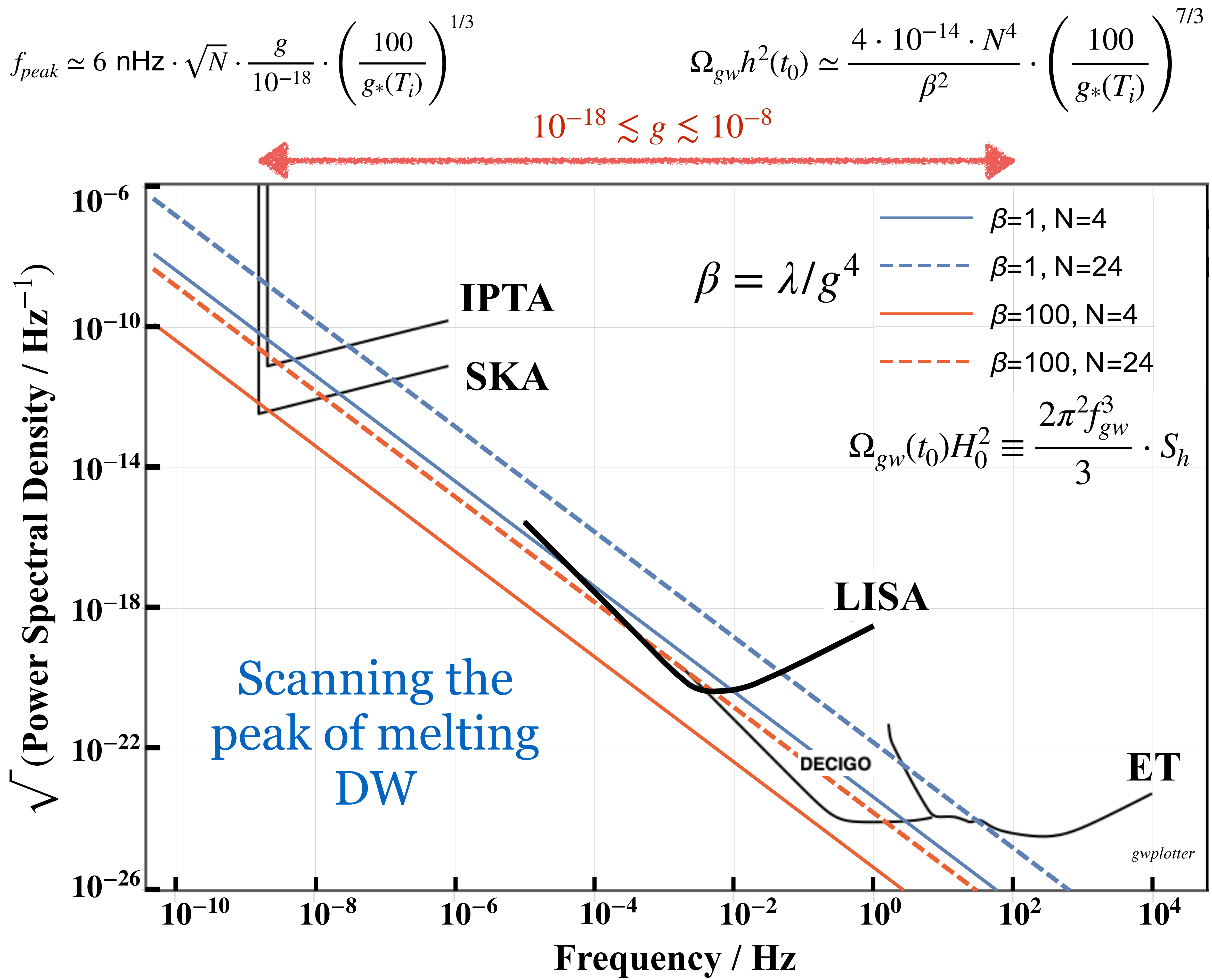
Hiramatsu, Kawasaki, Saikawa (2013)

Yang Li, Ligong Bian, Yongtao Jia; Yang Li, Ligong Bian, Rong-Gen Cai, Jing Shu (2023)

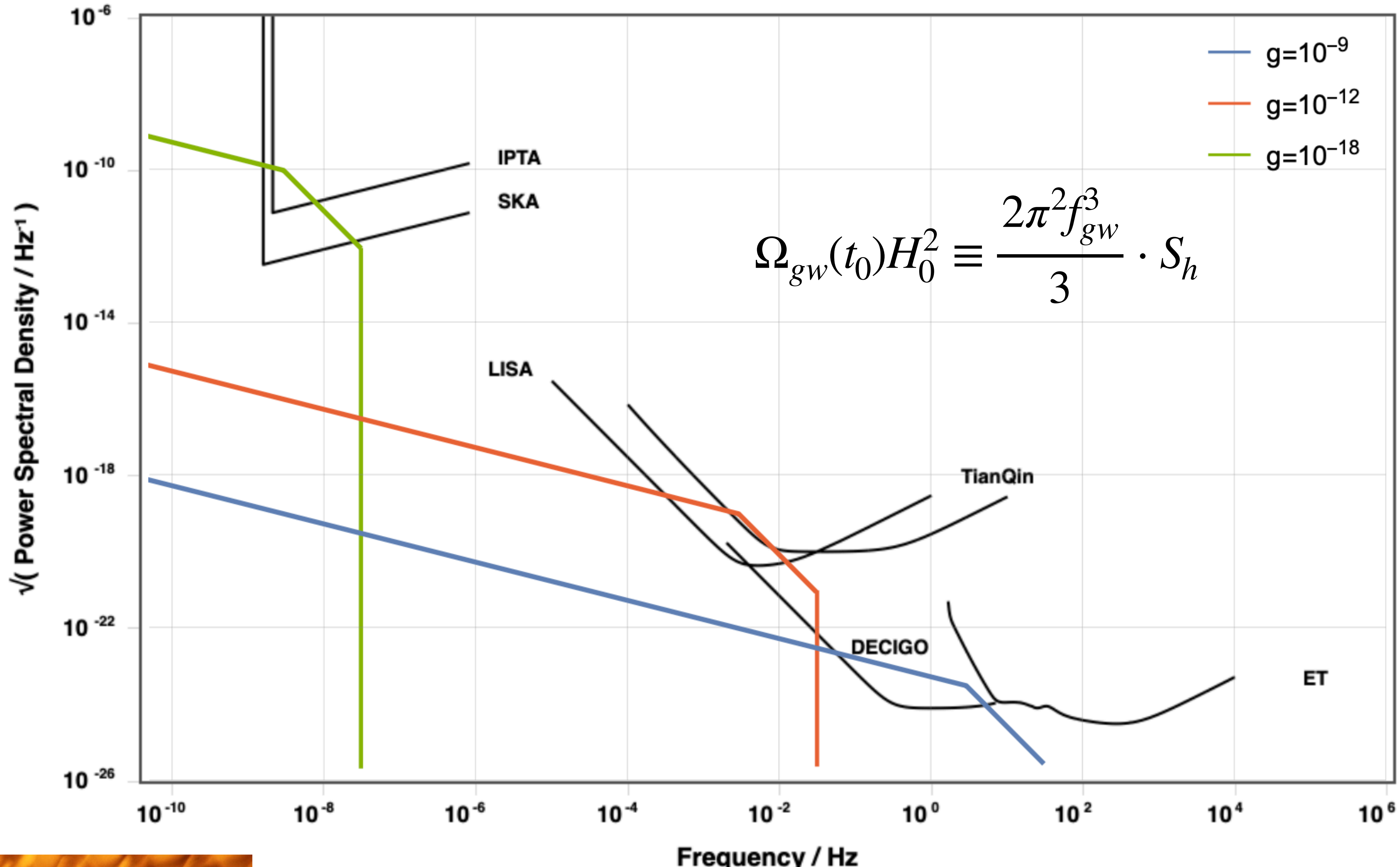
$$f_{peak} \simeq 0.7\, H_i \qquad f^0_{peak} \propto T_i$$

$$\Omega^{peak}_{gw} \sim \frac{\sigma^2_{dw}}{H^2_i} \quad \propto \left(f^0_{peak}\right)^2$$

Melting Domain Walls



$f_{peak} \simeq 6\ \mathrm{nHz} \cdot \sqrt{N} \cdot \frac{g}{10^{-18}} \cdot \left(\frac{100}{g_*(T_i)}\right)^{1/3}$
$\Omega_{gw} h^2(t_0) \simeq \frac{4 \cdot 10^{-14} \cdot N^4}{\beta^2} \cdot \left(\frac{100}{g_*(T_i)}\right)^{7/3}$
$10^{-18} \lesssim g \lesssim 10^{-8}$
$\beta = \lambda / g^4$
β=1, N=4
β=1, N=24
β=100, N=4
β=100, N=24
$\Omega_{gw}(t_0) H_0^2 \equiv \frac{2\pi^2 f_{gw}^3}{3} \cdot S_h$
IPTA
SKA
LISA
DECIGO
ET
Scanning the peak of melting DW
gwplotter
$\sqrt{\text{(Power Spectral Density / Hz}^{-1})}$
Frequency / Hz
10^{-6}
10^{-10}
10^{-14}
10^{-18}
10^{-22}
10^{-26}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^{0}
10^{2}
10^{4}



$$\Omega_{gw}(IR) \sim f^{2} \qquad \Omega_{gw}(UV) \sim f^{-1\,,1.28\,,1.53}$$

Cutoff $\ell = (\lambda/2)^{-1/2}\,\eta^{-1}$

Usual Domain Walls $\Omega_{gw}(IR) \sim f^{3}$

More on f^2 in IR

Dimensional analysis
supported by simulation
for constant tension

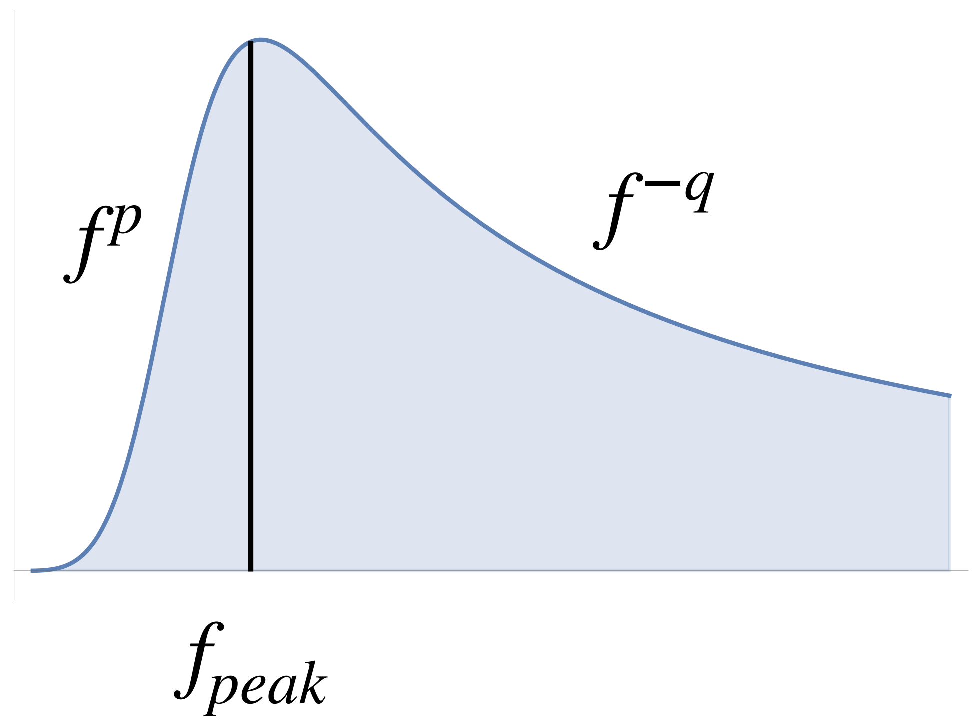
$$\Omega_{gw}\left(t_{now}\right)_{peak} \simeq A\left(\frac{f_{peak}}{F_{max}}\right)^2$$

energy is additive

Σ over t_{em} = Σ over f_{peak}

$$\delta\Omega_{gw}(f) = 2A\left(\frac{f_{peak}}{F_{max}^2}\right)\delta f_{peak}\left(\frac{f}{f_{peak}}\right)^p \frac{2}{1+\left(f/f_{peak}\right)^{p+q}}$$

for $f_{min} \ll f \ll F_{max}$



$$\Omega_{gw}(f) = \int_{f_{min}}^{F_{max}} \delta\Omega_{gw}(f) \propto \left(\frac{f}{F_{max}}\right)^2\left[1-\mathcal{O}\left(\frac{f}{F_{max}}\right)^n - \mathcal{O}\left(\frac{f_{min}}{f}\right)^m\right]$$

e.g. J0437–4715 has a period of 0.005757451936712637 s with an error of 1.7×10^{-17} s

15 years of observations of 68 millisecond pulsars

The New York Times

The Cosmos Is Thrumming With Gravitational Waves, Astronomers Find

Radio telescopes around the world picked up a telltale hum reverberating across the cosmos, most likely from supermassive black holes merging in the early universe.

June 28, 2023

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The Very Large Array on the Plains of San Agustin, N.M., one of three radio telescopes that worked with a global consortium to detect the timing of pulsars. NRAO/AUI/NSF

The Washington Post

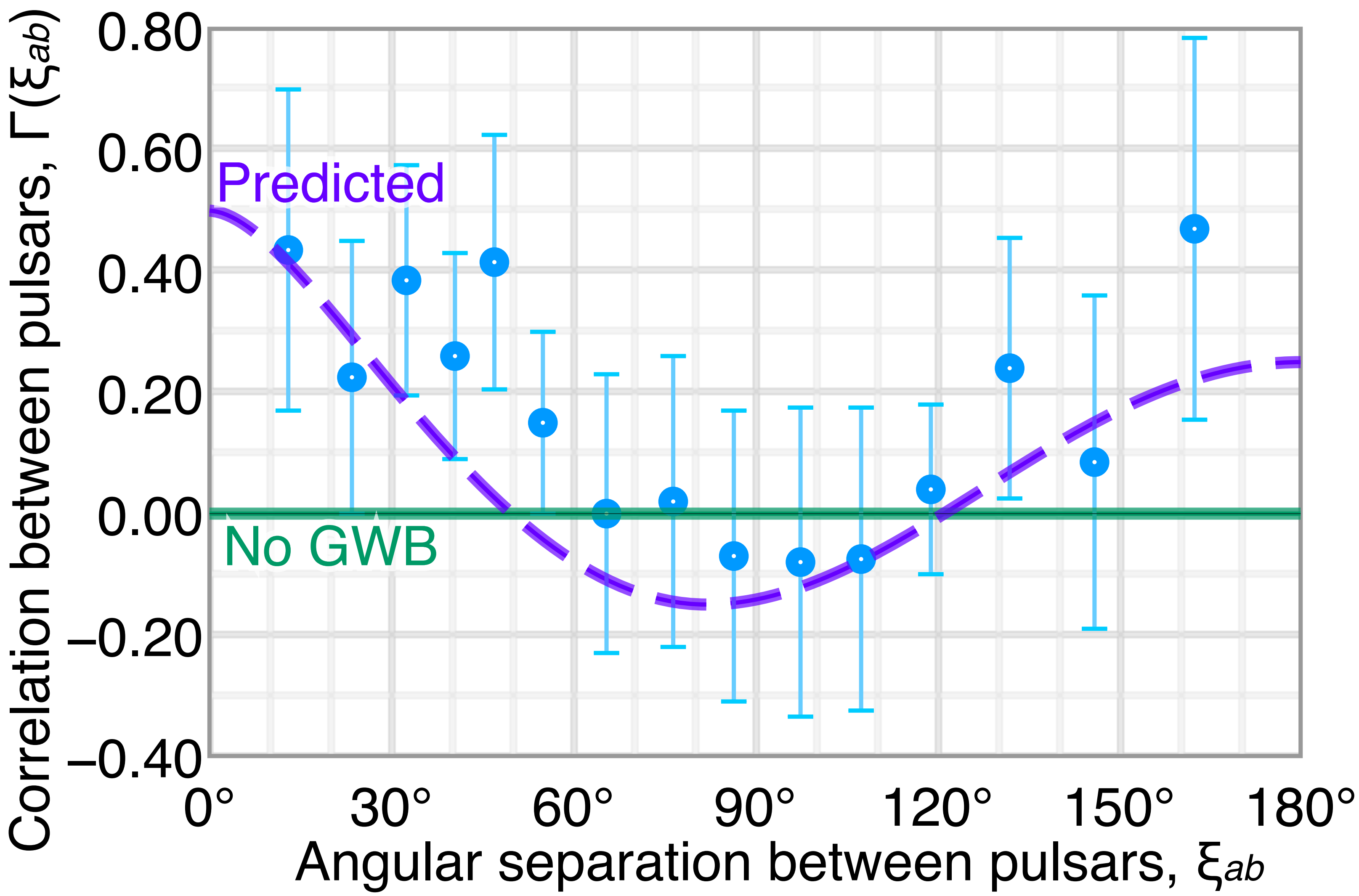
In a major discovery, scientists say space-time churns like a choppy sea

The mind-bending finding suggests that everything around us is constantly being roiled by low-frequency gravitational waves

By Joel Achenbach and Victoria Jaggard
June 28, 2023 at 8:00 p.m. EDT

The Green Bank Observatory in Green Bank, W.Va., was among the observatories used to track pulsars as a way of detecting low-frequency gravitational waves. (Michael S. Williamson/The Washington Post)

Hellings–Downs curve



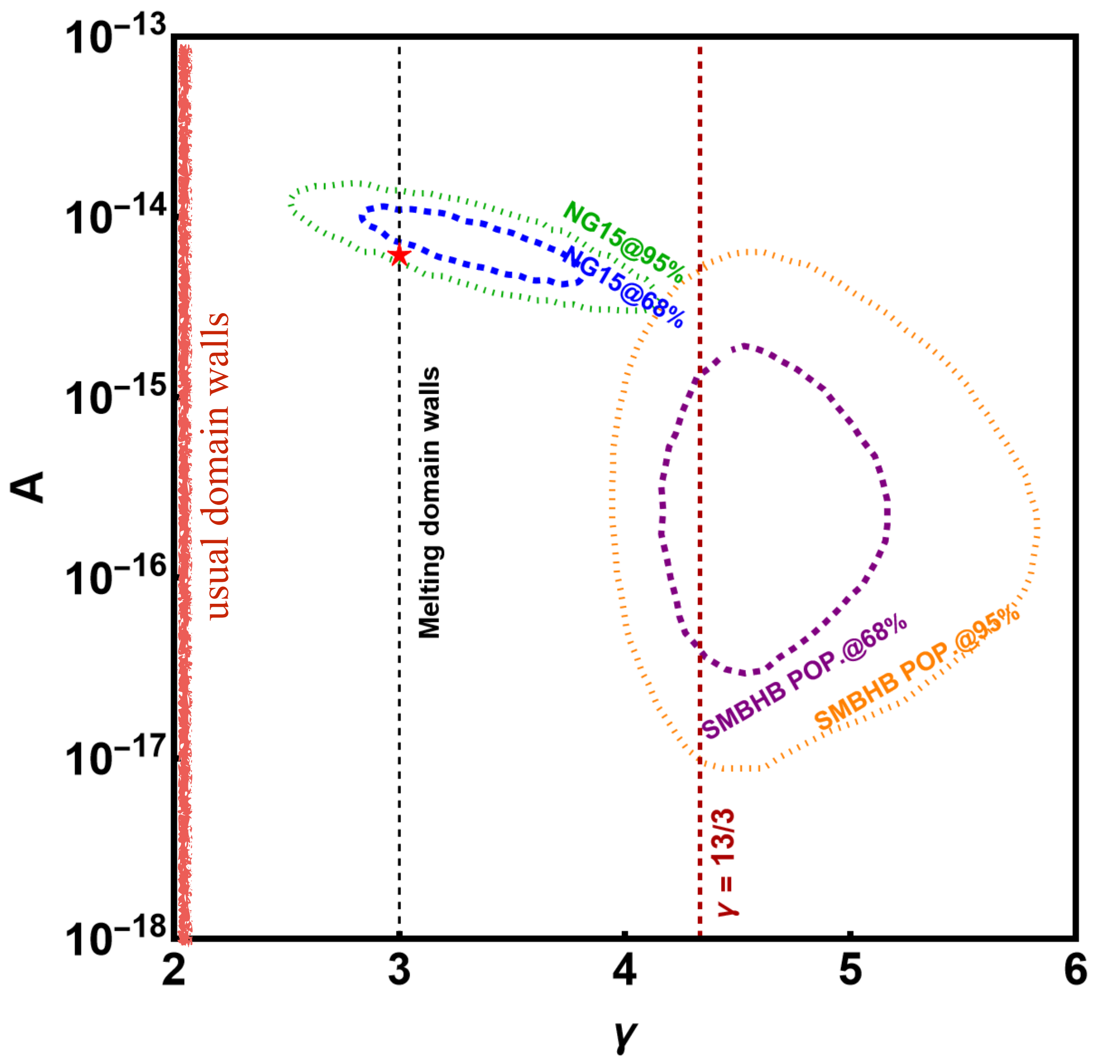
NANOGrav 15-year data set

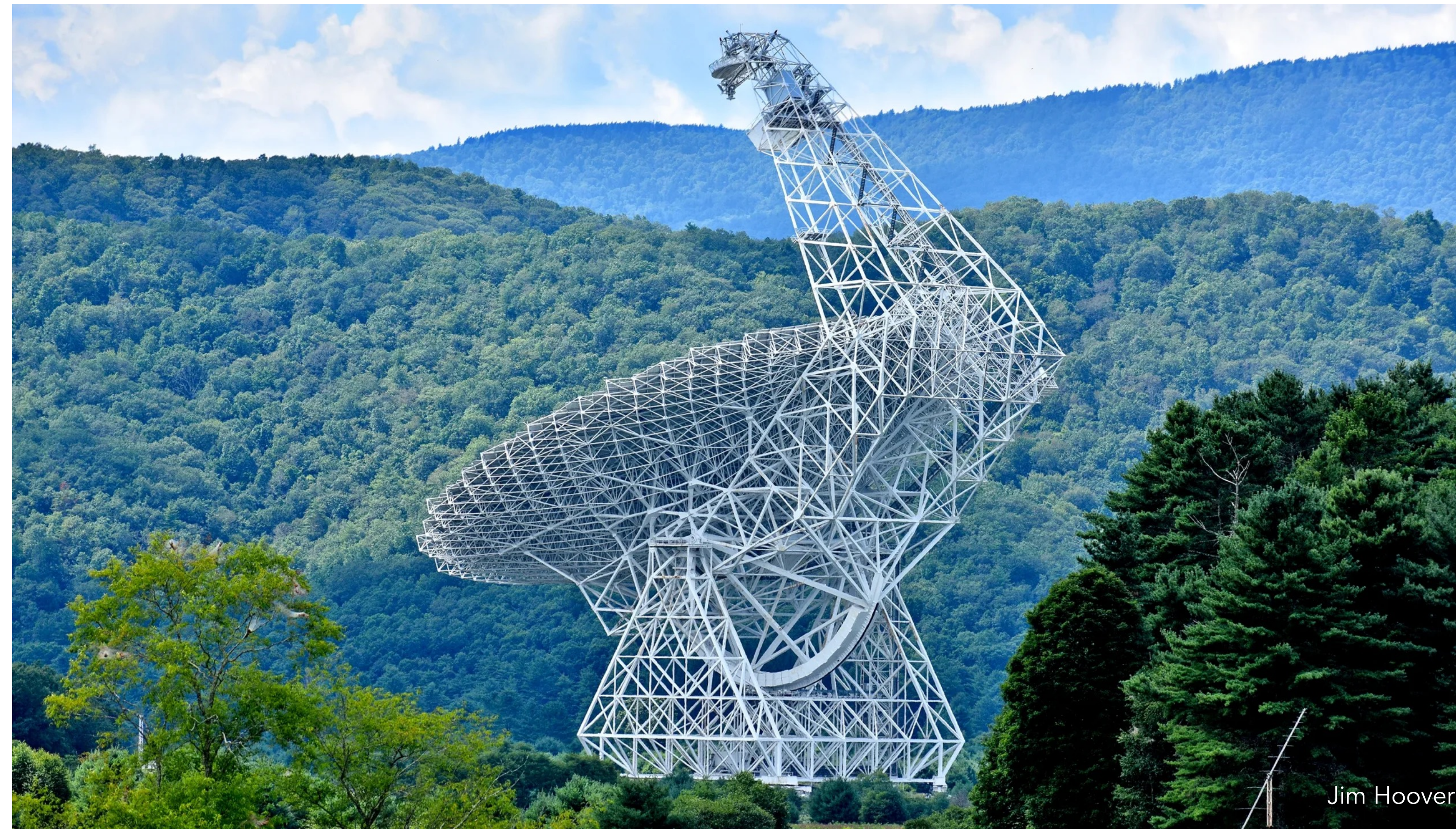
Perfect for NANOGgrav

$$\Omega_{\rm GW}(f) = \Omega_{yr}\left(f/f_{\rm yr}\right)^{5-\gamma},$$

$$f_{yr} = 32\,\mathrm{nHz}$$

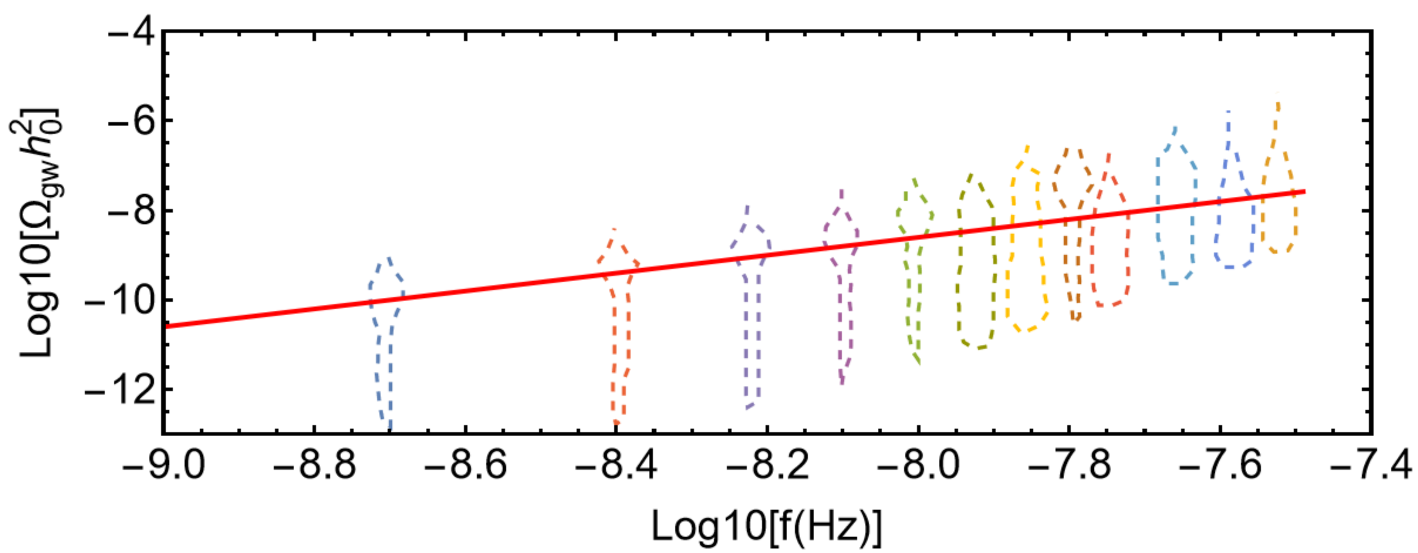
$$\Omega_{yr} = \frac{2\pi^2}{3H_0^2}A^2 f_{yr}^2$$





The 100-meter Green Bank Telescope, the world's largest fully steerable telescope and a core instrument for pulsar timing array experiment.

parameters $g = 10^{-18}$, $\beta = \lambda/g^4 = 1$, $N = 24$, $g_* = 75$

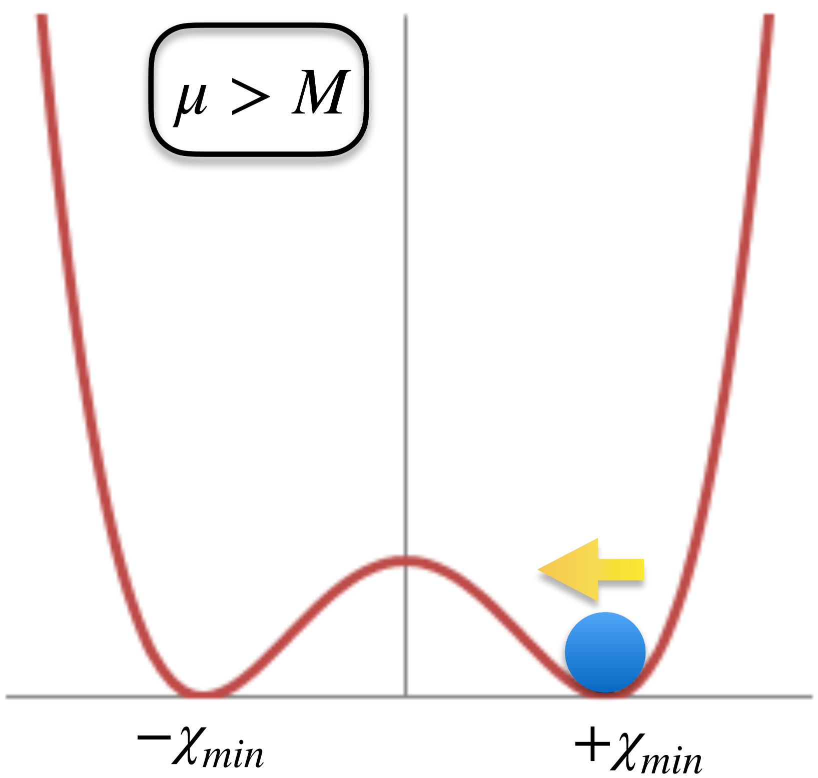


Inverse Phase Transition At Meltdown

$$\mathscr{L} = \frac{(\partial\chi)^2}{2} - \frac{\left(M^2 - \mu^2(t,\mathbf{x})\right)\cdot\chi^2}{2} - \frac{\lambda\chi^4}{4}$$

Early Universe

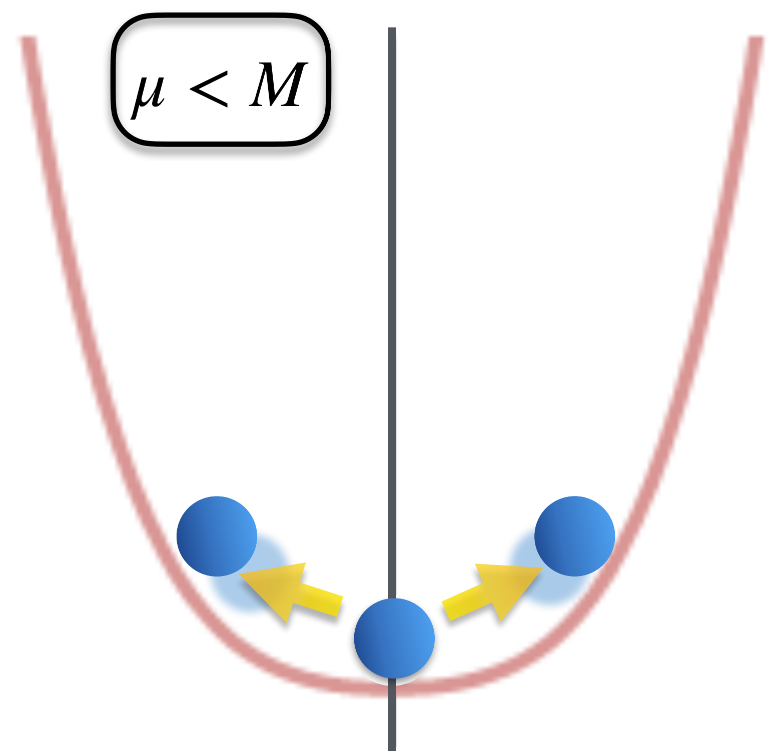
spontaneously Broken Phase with VEV slowly moving



Late Universe

DW melt down and disappear

then oscillations around restored symmetric vacuum



Tachyonic mass $\mu(t)$ slowly decreases / redshifts due to cosmological expansion

for Hubble parameter

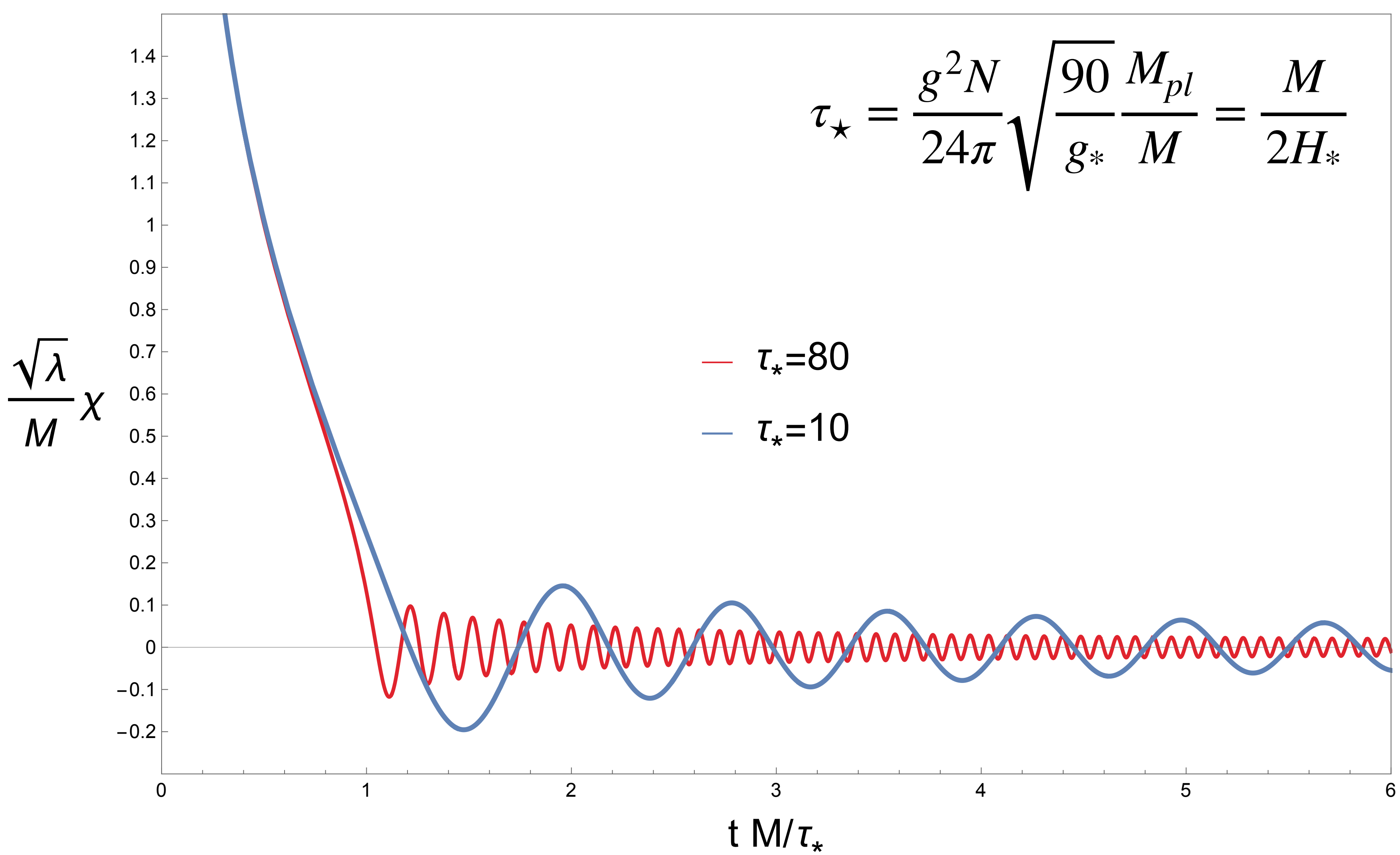
$H < M$

scalar field traces vacuum $\chi_{min}(t) = \sqrt{\dfrac{\mu^2(t) - M^2}{\lambda}}$ as long as $\left|\dfrac{\dot{M}_{eff}}{M_{eff}^2}\right| \ll 1$

Dynamics only depends on one single free dimensionless parameter

$$\ddot{\chi} + 3H\dot{\chi} + \left(M^2 - \frac{g^2 N T^2}{12}\right)\chi + \lambda\chi^3 = 0 \quad \text{with} \quad H = \frac{1}{2t} = \sqrt{\frac{\pi^2 g_*}{90}}\frac{T^2}{M_{pl}}$$

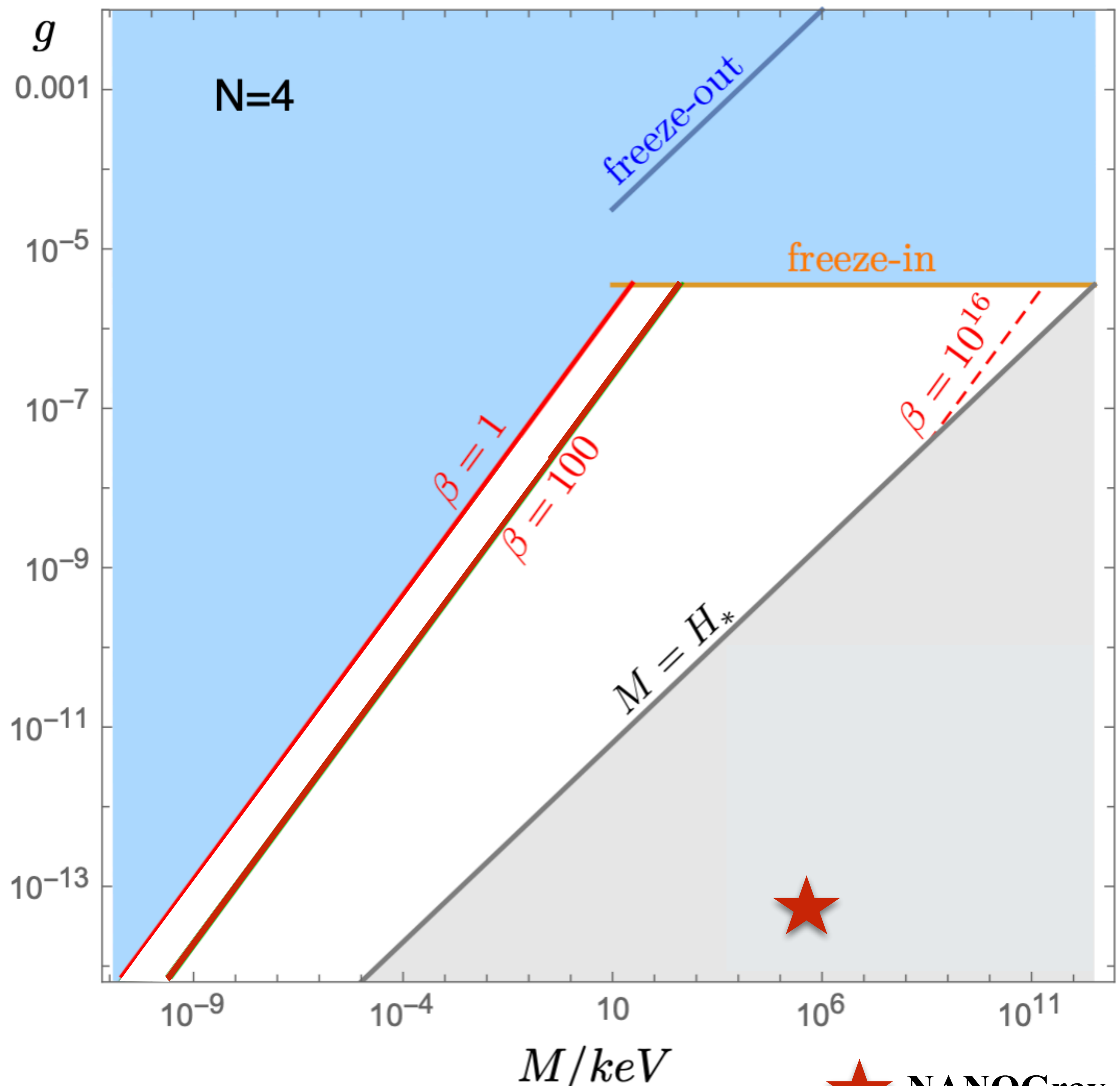
$$\frac{1}{\tau_\star^2}\left(\bar{\chi}'' + \frac{3}{2}\frac{\bar{\chi}'}{\tau}\right) + \left(1 - \frac{1}{\tau}\right)\bar{\chi} + \bar{\chi}^3 = 0$$



$$\tau_\star = \frac{g^2 N}{24\pi}\sqrt{\frac{90}{g_*}}\frac{M_{pl}}{M} = \frac{M}{2H_*}$$

Allowed Parameter Space

$$M \simeq \; 10^{-13}\,\text{eV} \cdot \frac{\beta^{3/5}}{\sqrt{N}} \cdot \left(\frac{g_*(T_*)}{100}\right)^{2/5} \cdot \left(\frac{g}{10^{-18}}\right)^{7/5}$$

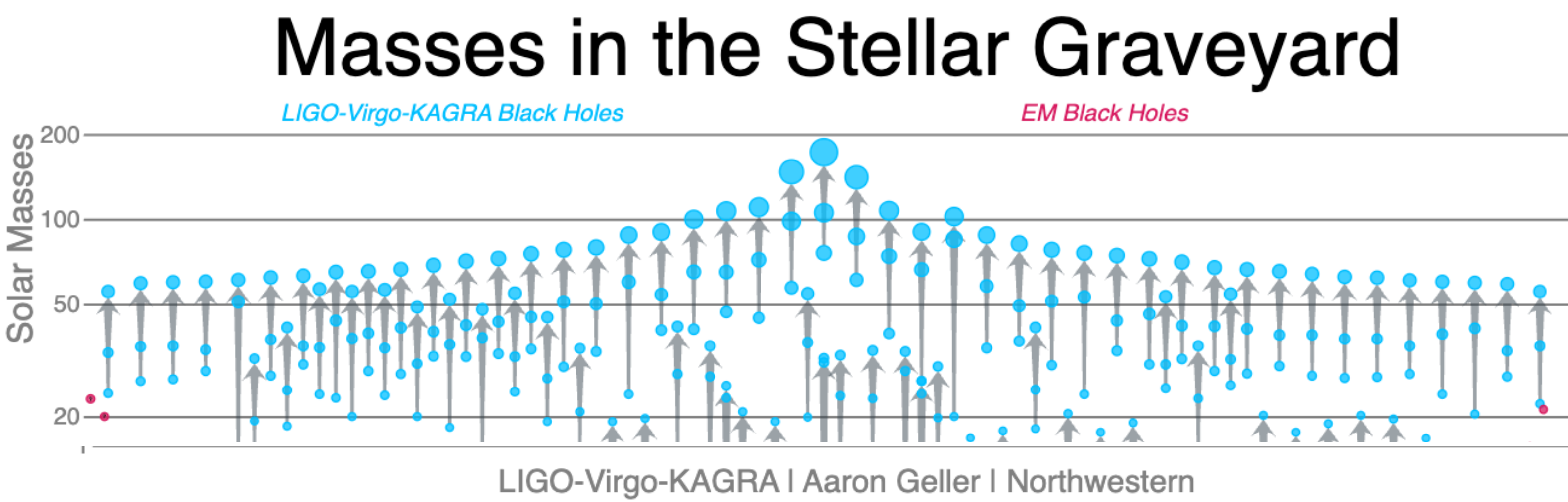


★ **NANOGrav**

A bridge between NANOGrav and LIGO!

$$M_\chi \simeq 10^{-12}\,\text{eV}\cdot B^{9/20}\cdot\left(\frac{g_*(T_{sym})}{100}\right)^{1/5}\cdot\left(\frac{g_*(T_i)}{100}\right)^{1/20}\cdot\left(\frac{m_\phi}{10\ \text{MeV}}\right)^{1/2}\times\left(\frac{f_{peak}}{30\ \text{nHz}}\right)^{6/5}\cdot\left(\frac{10^{-8}}{\Omega_{gw,peak}h_0^2}\right)^{3/20}$$

Superradiance for $M_{BH} \simeq 10^2 M_\odot$ → Just on on edge of LIGO!




A highly promising path to the origins
of DM and NANOGrav signal
Thanks a lot for attention!