Fast NNLO Implementation of the approximate ACOT scheme for DIS

CTEQ Fall Meeting – Christopher Newport University

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nCTEQ
nuclear parton distribution functions

soon to be published, preliminary results: 2307.08269 (DIS23 Proceedings)

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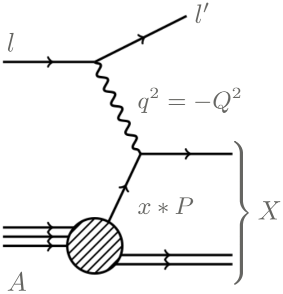
Introduction to heavy quark schemes in DIS

Factorization in DIS structure functions

$$F_\lambda(x,Q^2) = \sum_k C_k^\lambda \otimes f_k = \sum_k \int_x^1 \frac{\mathrm{d}\xi}{\xi} C_k^\lambda\left(\frac{\chi}{\xi}, \frac{Q}{\mu}, \frac{M_i}{\mu}, \alpha_s(\mu)\right) f_k(\xi,\mu)$$



Introduction to heavy quark schemes in DIS

Factorization in DIS structure functions

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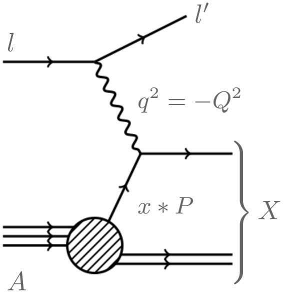

- Wilson coefficients have a **complicated** α_s expansion
  - these are the hard scattering amplitudes
- **heavy quark mass effects** sizable at $Q \sim M_H$
- extremely **important for global PDF** fits
  - bulk of experimental data from DIS
  - need fast theory predictions
- older implementations **too slow to be applicable**

DIS mass schemes

Zero Mass Variable Flavor Number Scheme (ZM-VFNS)

- consider only quarks below threshold: $M_H < Q$
- neglect all mass terms part of the Wilson coefficients
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Fixed Flavor Number Scheme (FFNS)

- treat all quarks as massless except for the heaviest M_H
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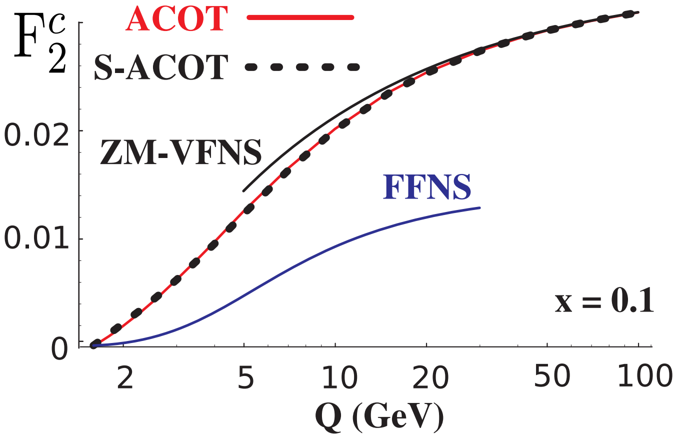
Fixed Flavor Number Scheme (FFNS)

- treat all quarks as massless except for the heaviest M_H
- this mass appears explicitly in the Wilson coefficients

- **good results for $\mathbf{Q \sim M_H}$ unreliable as Q becomes large**

General Mass Variable Flavor Number Schemes (GM-VFNS)

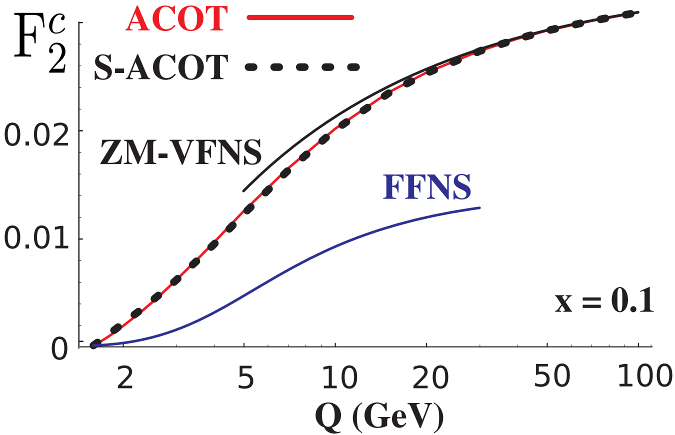
- 'interpolating' between FFNS and ZM-VFNS



T. Stavreva et al., arXiv: 1203.0282

General Mass Variable Flavor Number Schemes (GM-VFNS)

- 'interpolating' between FFNS and ZM-VFNS
- several choices can be made, resulting in different schemes:
  - **ACOT**: minimal extension of the $\overline{\text{MS}}$ renormalisation scheme
  - **FONLL**: interpolating between schemes with a damping function
  - **TR-method**: requiring smooth transition at $Q = M_H$



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Overview: Heavy Quark schemes in DIS

- need for heavy quark contributions
- General Mass Variable Flavor Number Schemes

The approximate SACOT-χ scheme at NNLO

- neutral current (Stavreva *et al*, arXiv:1203.0282)
- charged current

Numerical implementation

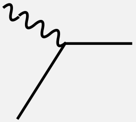
- APFEL++
- results

The aSACOT-χ scheme

Stavreva *et al*, arXiv:1203.0282

aSACOT-χ: approximate simplified ACOT-χ

approximate: $\text{ACOT}\left[\mathcal{O}(\alpha_s^0) + \mathcal{O}(\alpha_s^1)\right] + \text{ZM-VFNS}\left[\mathcal{O}(\alpha_s^2)\Big|_\chi\right]$



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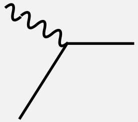
simplified: zero mass coefficients for

- incoming heavy quarks
- internal on-shell cuts on a heavy quark line



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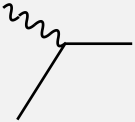
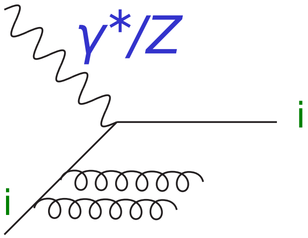


χ-prescription: set lower integration bound: $\chi(n) = x\left[1 + \left(\frac{nM_H}{Q}\right)^2\right]$

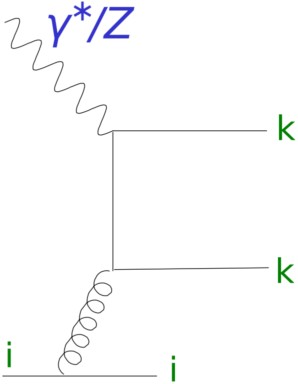
aSACOT-χ: Dissection into flavor contributions at NNLO

$$\sum_{i=0}^{n_f} \boldsymbol{F}_{\boldsymbol{\lambda}}^{\boldsymbol{i}} \Rightarrow$$



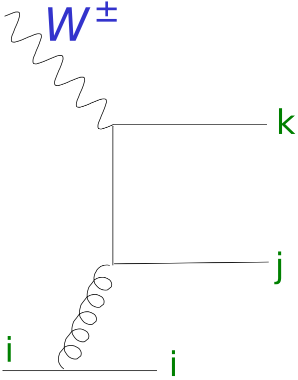
aSACOT-χ: Dissection into flavor contributions at NNLO

$$\sum_{i=0}^{n_f} \sum_{k=1}^{n_f} F_\lambda^{i\;\;\;,k} \Rightarrow$$



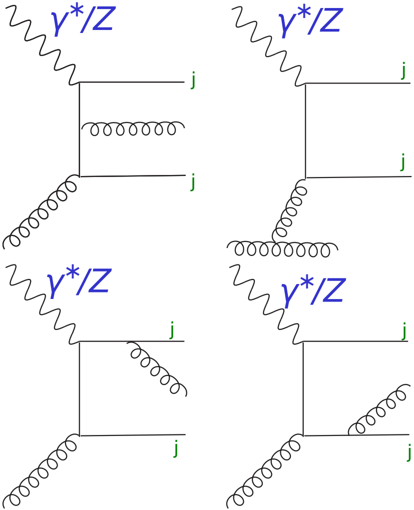
aSACOT-χ: Dissection into flavor contributions at NNLO (NEW)

$$\sum_{i=0}^{n_f} \sum_{j=1}^{n_f} \sum_{k=1}^{n_f} F_{\lambda}^{i,j,k} \Rightarrow$$



aSACOT-χ: NNLO gluon contribution

Stavreva *et al*, arXiv:1203.0282



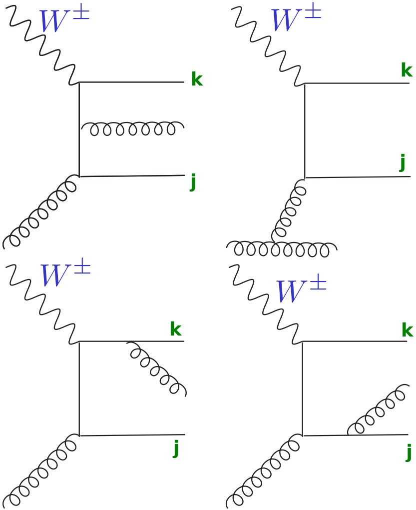
Neutral current

- filter out the individual quark flavor contributions
- apply $\chi(n)$-prescription to coefficient
  - $\chi(n) = x\left[1 + \left(\frac{nM_j}{Q}\right)^2\right] \qquad n = 0, 1, 2, 3$
- repeat for all flavors

aSACOT-χ: NNLO gluon contribution

Charged current (NEW)

- filter out the individual quark flavor contributions
  - take care of the flavor change at the $W^{\pm}$ vertex
- apply $\chi(n)$-prescription to coefficient
  - $\chi(n) = x\left[1 + \left(\frac{nM_H}{Q}\right)^2\right] \qquad n = 0, 1, 2, 3$
  - use heaviest quark mass: $M_H = \max(M_j, M_k)$
- repeat for all flavors



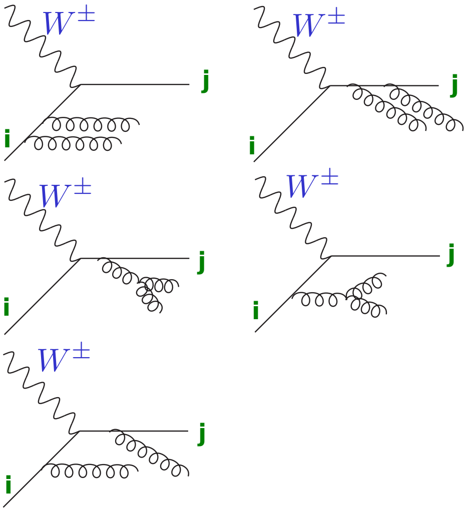
aSACOT-χ: NNLO non-singlet contribution (NEW)

Non-singlet combination

$$q^{\mathrm{NS}} = \sum_{i=1}^{n_f} \left(\hat{e}_i^2 - \langle \hat{e}^2 \rangle\right) (q_i + \bar{q}_i) \quad \text{with} \quad \langle \hat{e}^2 \rangle = \frac{1}{n_f} \sum_{i=1}^{n_f} \hat{e}_i^2$$

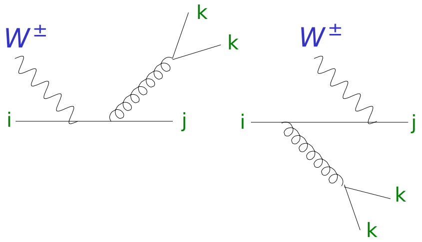
"whenever the incoming parton couples to the EW-boson"

- filter out the individual quark flavor contributions
  - take care of the flavor change at the $W^\pm$ vertex
- apply $\chi(n)$-prescription to coefficient
  - $\chi(n) = x \left[1 + \left(\frac{n M_H}{Q}\right)^2\right] \qquad n = 0, 1, 2, 3$
  - use heaviest quark mass: $M_H = \max(M_i, M_j)$
- repeat for all flavors



aSACOT-χ: NNLO non-singlet contribution (NEW)

- filter out the individual quark flavor contributions
  - take care of the flavor change at the $W^{\pm}$ vertex
    - **note that we have three flavors now**
- apply $\chi(n)$-prescription to coefficient
  - $\chi(n) = x\left[1 + \left(\frac{n M_H}{Q}\right)^2\right] \qquad n = 0, 1, 2, 3$
  - use heaviest quark mass: $M_H = \max(M_i, M_j, M_k)$
- repeat for all flavors
  - both incoming and outgoing flavors



aSACOT-χ: NNLO purely singlet contribution (NEW)

Purely singlet contribution

$$F_{a,q_i} \simeq C^{\mathbf{PS}}_{a,q} \otimes \langle \hat{e}^2 \rangle (q_i + \bar{q}_i)$$

"the EW-boson does not couple to the incoming quark"

- filter out the individual quark flavor contributions
  - take care of the flavor change at the $W^{\pm}$ vertex
    - **note that we have three flavors now**
- apply $\chi(n)$-prescription to coefficient
  - $\chi(n) = x \left[1 + \left(\frac{n M_H}{Q} \right)^2 \right] \qquad n = 0, 1, 2, 3$
  - use heaviest quark mass: $M_H = \max(M_i, M_j, M_k)$
- repeat for all flavors
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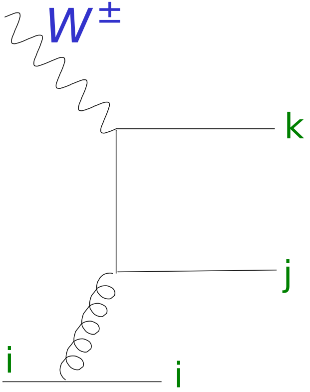

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APFEL++ – A PDF evolution library in C++

Bertone, arXiv:1708.00911

- main author: **V. Bertone**

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$$F_\lambda(x,Q^2) = \sum_k \int_\chi^1 \frac{\mathrm{d}\xi}{\xi} C_k^\lambda\left(\frac{\chi}{\xi},\frac{Q}{\mu},\frac{M_i}{\mu},\alpha_s(\mu)\right) f_k(\xi,\mu)$$

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Precompute observables

$$F_\lambda(x, Q^2) = \sum_k \int_\chi^1 \frac{\mathrm{d}\xi}{\xi} C_k^\lambda \left(\frac{\chi}{\xi}, \frac{Q}{\mu}, \frac{M_i}{\mu}, \alpha_s(\mu) \right) f_k(\xi, \mu)$$

Replace with interpolating functions: $\Uparrow$

$$\sum_\alpha^{N_\xi} w_\alpha(\xi) f_k(\xi_\alpha, \mu)$$

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$\Rightarrow F_\lambda(x,Q^2)$ as a function of the PDFs without the need for repeating the integration!

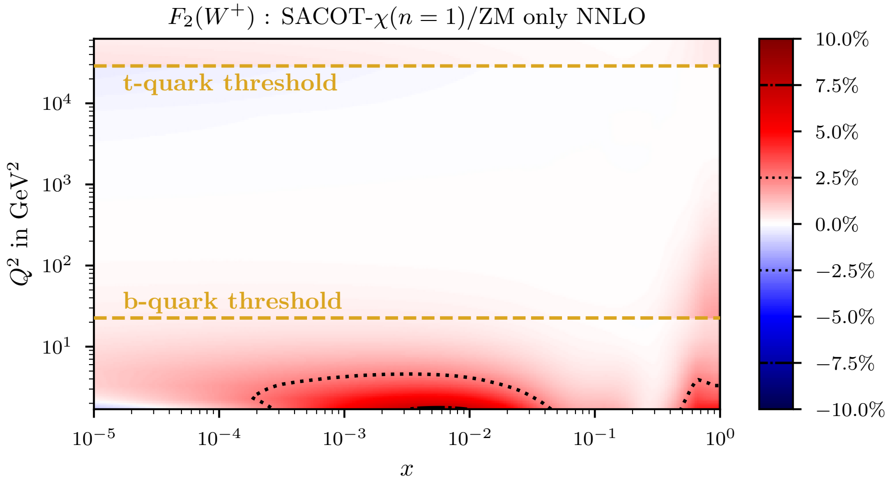
Available schemes in APFEL++

scheme	$\mathcal{O}(\alpha_s)$		NC: F_2	NC: F_3	NC: F_L		CC: F_2	CC: F_3	CC: F_L
ZM	N2LO		✓	✓	✓		✓	✓	✓
FONLL-C	N2LO		✓	✗	✓		✗	✗	✗
ACOT	NLO		✗	✗	✗		✗	✗	✗
sACOT-χ	NLO		✗	✗	✗		✗	✗	✗
approx. sACOT-χ	N2LO		✗	✗	✗		✗	✗	✗

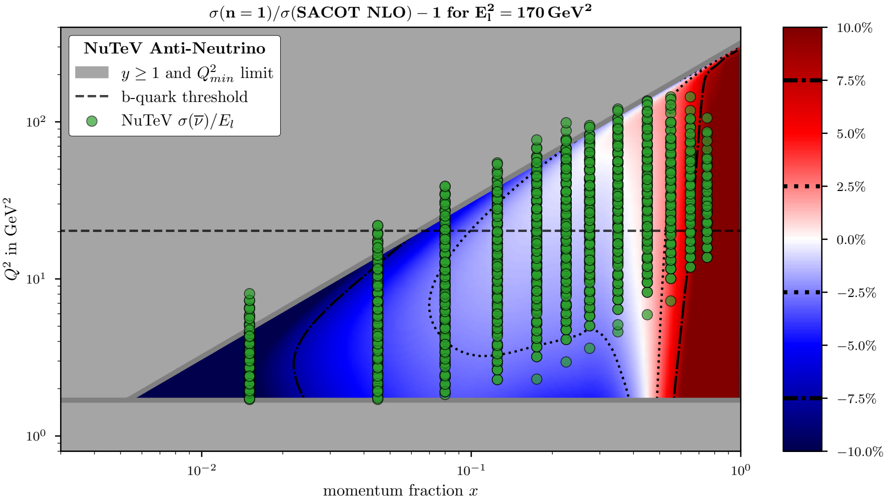
Available schemes in APFEL++ (NEW)

scheme	$\mathcal{O}(\alpha_s)$		NC: F_2	NC: F_3	NC: F_L		CC: F_2	CC: F_3	CC: F_L
ZM	N2LO		✓	✓	✓		✓	✓	✓
FONLL-C	N2LO		✓	✗	✓		✗	✗	✗
ACOT	NLO		✓	✓	✓		✗	✗	✗
sACOT-χ	NLO		✓	✓	✓		✓	✓	✓
approx. sACOT-χ	N2LO		✓	✓	✓		✓	✓	✓

Results: Mass effects at NNLO for F_2



NuTeV predictions: NNLO over NLO comparison



Conclusion

Extension of SACOT-χ to charged current

- using ZM-coefficients at NNLO
- dominant mass effects taken into account

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- fast implementation in `APFEL++`
  - suitable for PDF extractions
- publicly available!

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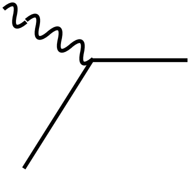
Thank you for your intention!

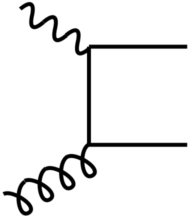
backup

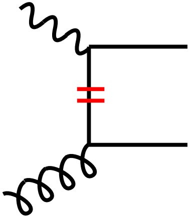
ACOT at NLO

Aivazis *et al*, arXiv:9312318 and arXiv:9312319

LO quark-boson scattering + NLO gluon-boson scattering − subtraction term







ACOT at NLO

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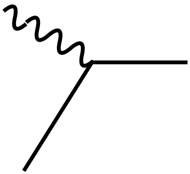
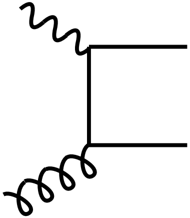
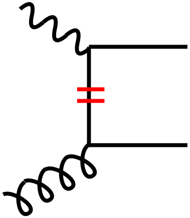
LO quark-boson scattering $+$ NLO gluon-boson scattering $-$ subtraction term





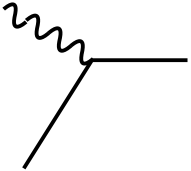


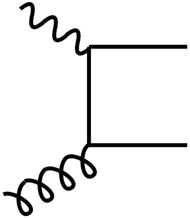
- $Q \gtrsim M_H$: removed by subtraction
- $Q \gtrsim M_H$: dominant contribution
- $Q \gtrsim M_H$: approx. equal to quark-boson scattering

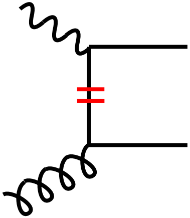
ACOT at NLO

Aivazis *et al*, arXiv:9312318 and arXiv:9312319

LO quark-boson scattering + NLO gluon-boson scattering − subtraction term







LO quark-boson scattering
- $Q \gtrsim M_H$: removed by subtraction
- $Q \gg M_H$: dominant contribution

NLO gluon-boson scattering
- $Q \gtrsim M_H$: dominant contribution
- $Q \gg M_H$: removed by subtraction

subtraction term
- $Q \gtrsim M_H$: approx. equal to quark-boson scattering
- $Q \gg M_H$: approx. equal to gluon-boson scattering