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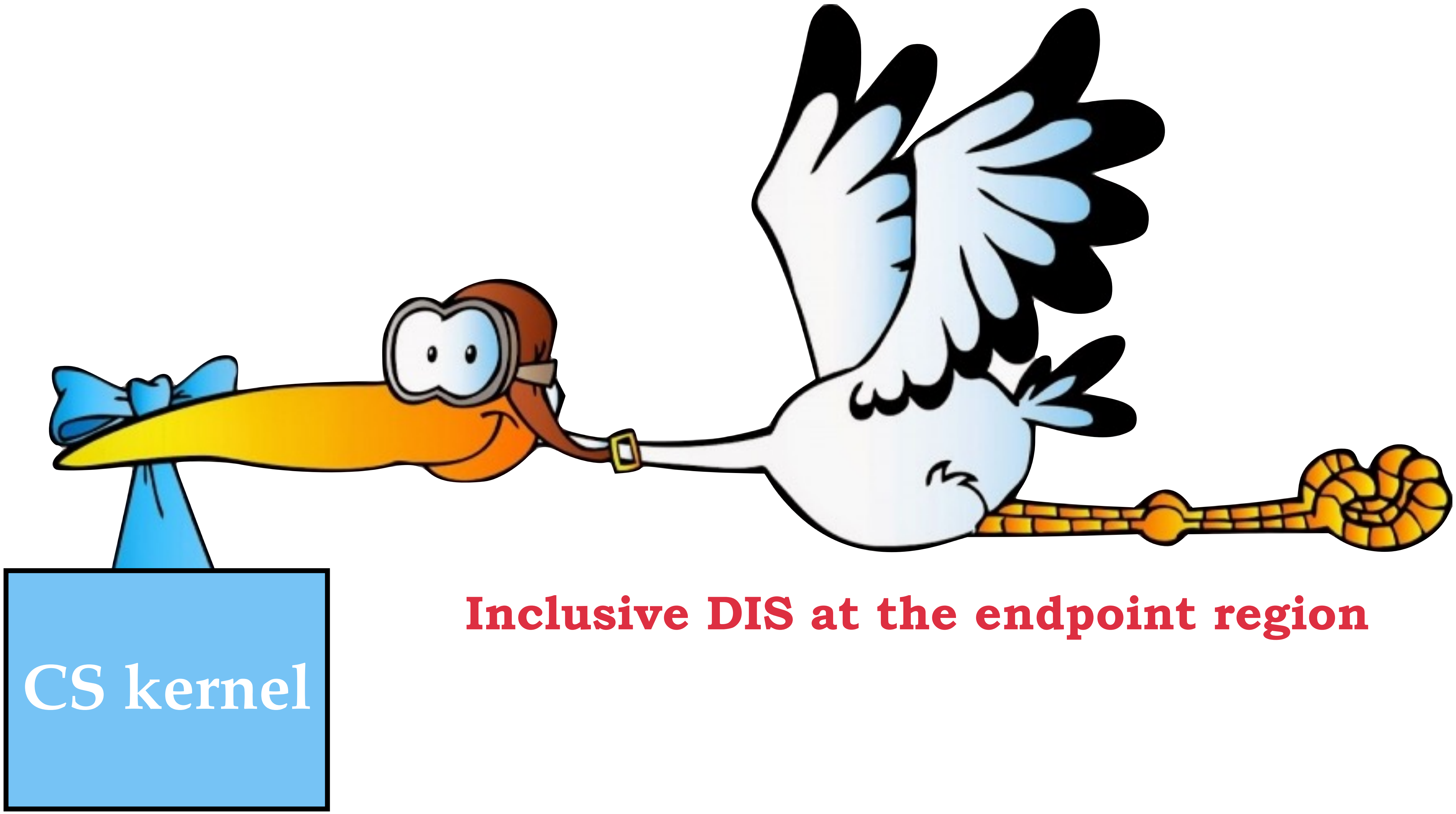
Collins—Soper kernel from collinear factorization

Matteo Cerutti

A. Accardi, MC, C. Costa, A. Signori, A. Simonelli

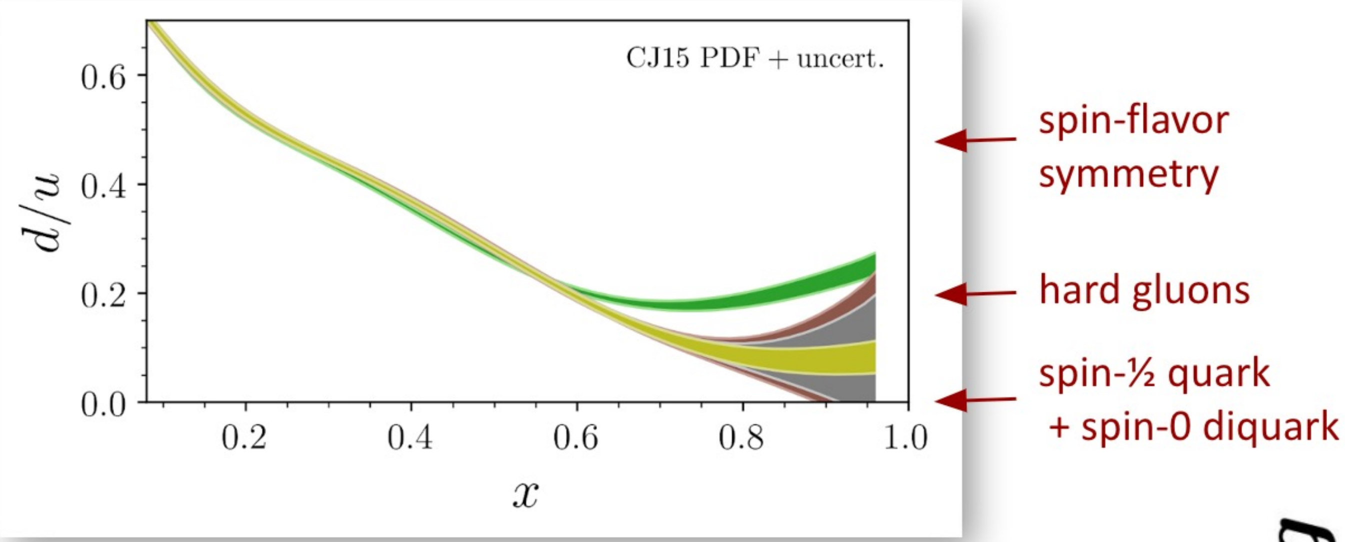


Inclusive DIS at the endpoint region

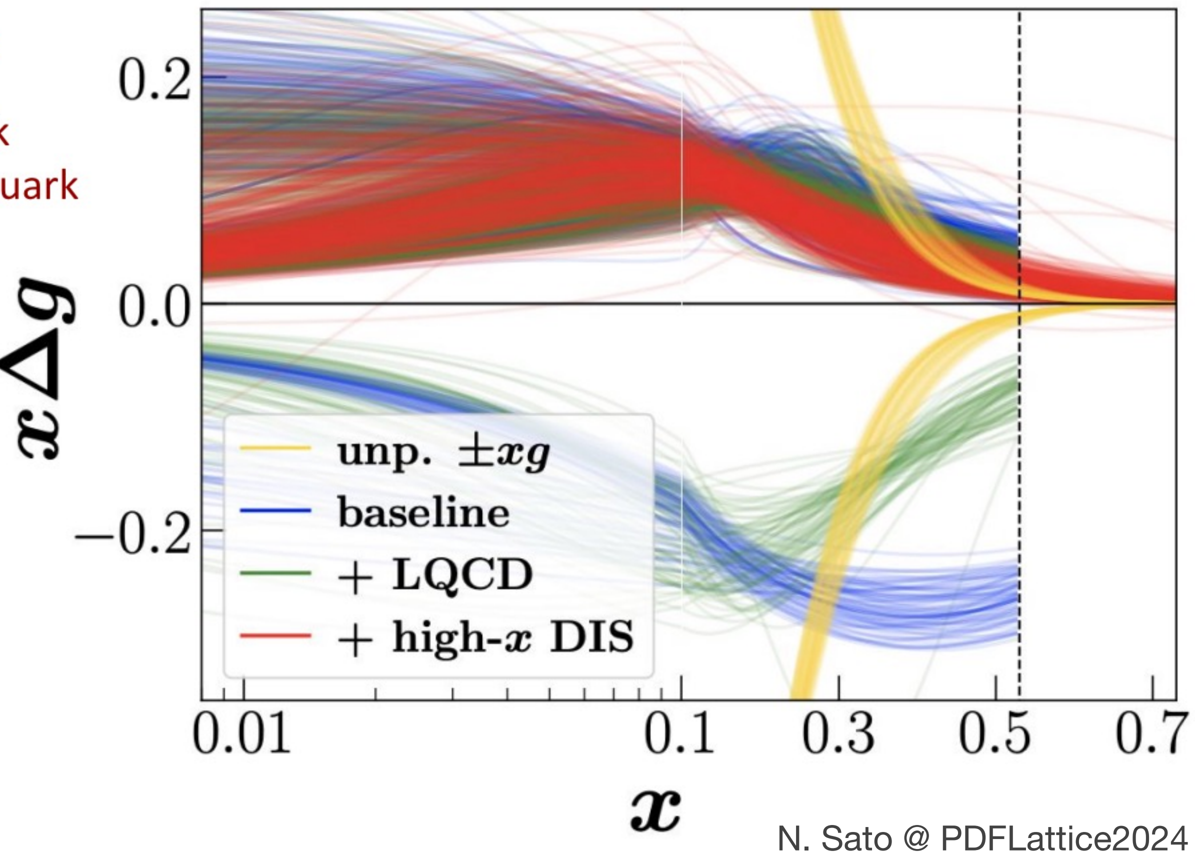
Motivations

Understand the behaviour of PDFs in the large-x region

- d/u ratio as a tool for investigating confinement
- Theoretical constraints (positivity bounds)



Li, Accardi et al, PRD 109 (2024)

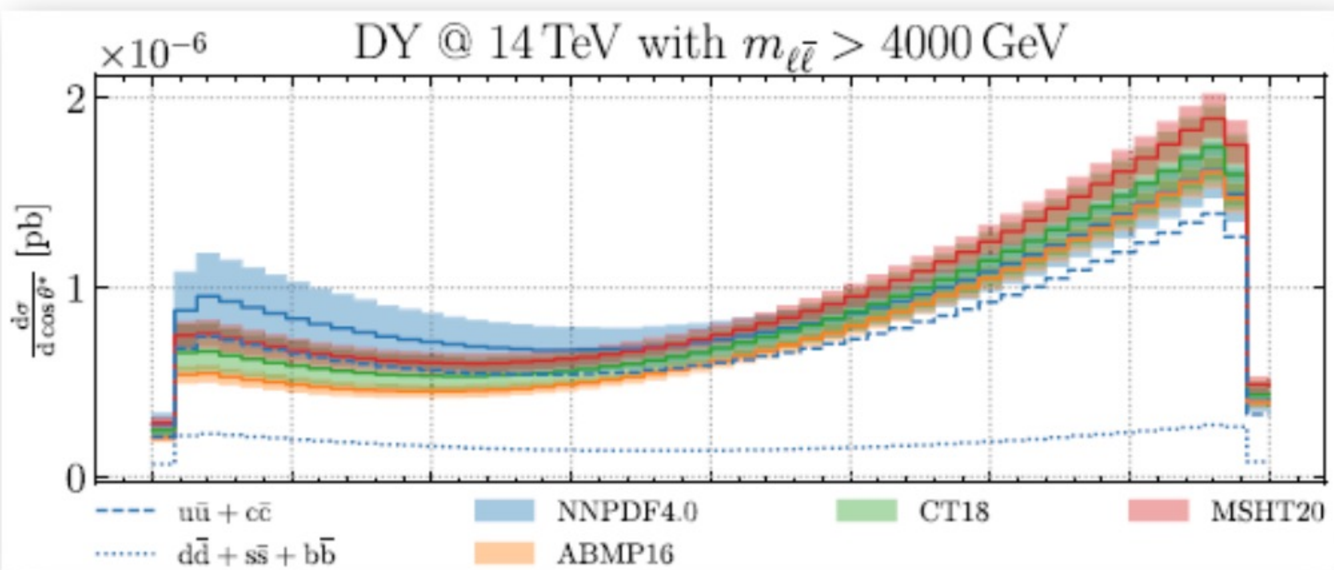


N. Sato @ PDFLattice2024

Motivations

Understand the behaviour of PDFs in the large-x region

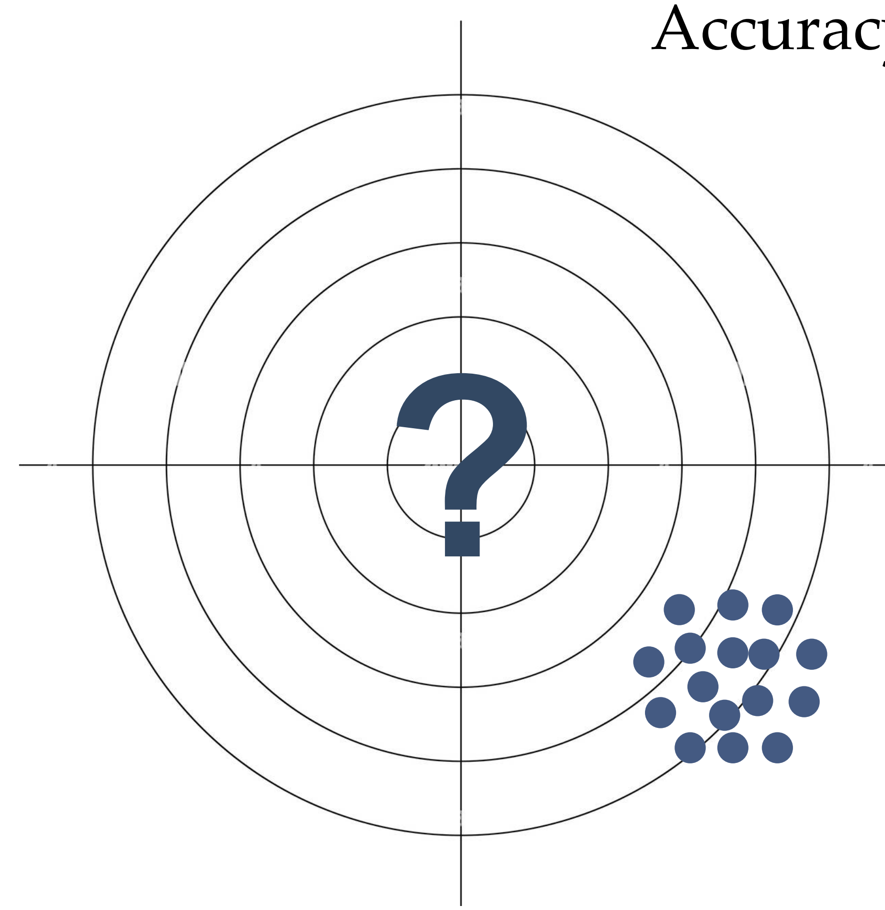
- Beyond-Standard-Model searches
- Forward facilities (LHC)



Ball et al, EPJC 82 (2022)

Precision ✗

Accuracy ✓



Motivations

Understand the behaviour of PDFs in the large-x region

- High-precision data from Jefferson Lab

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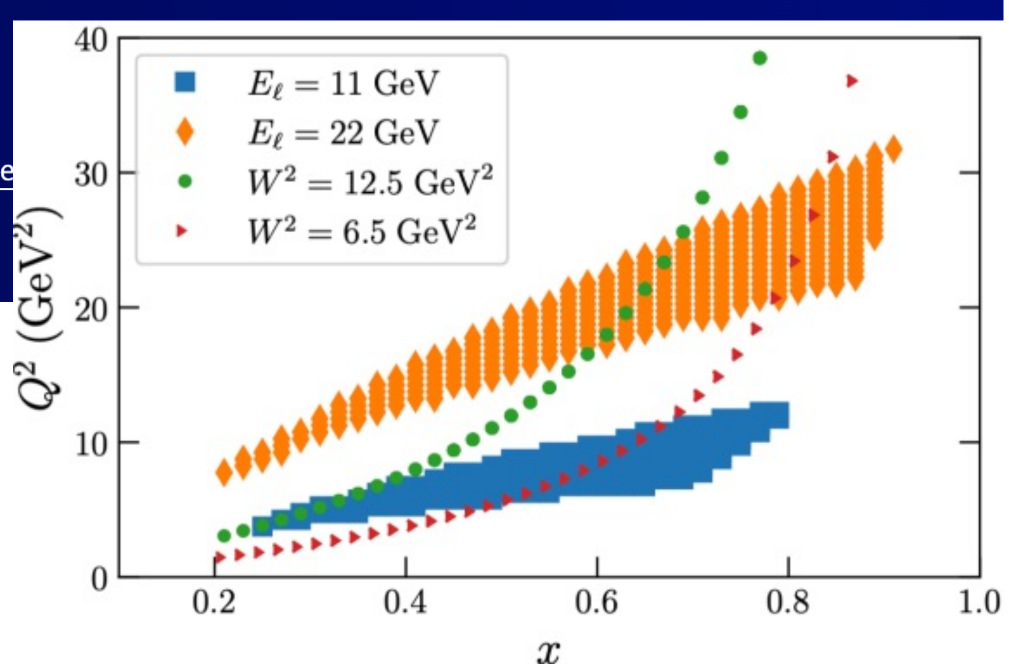
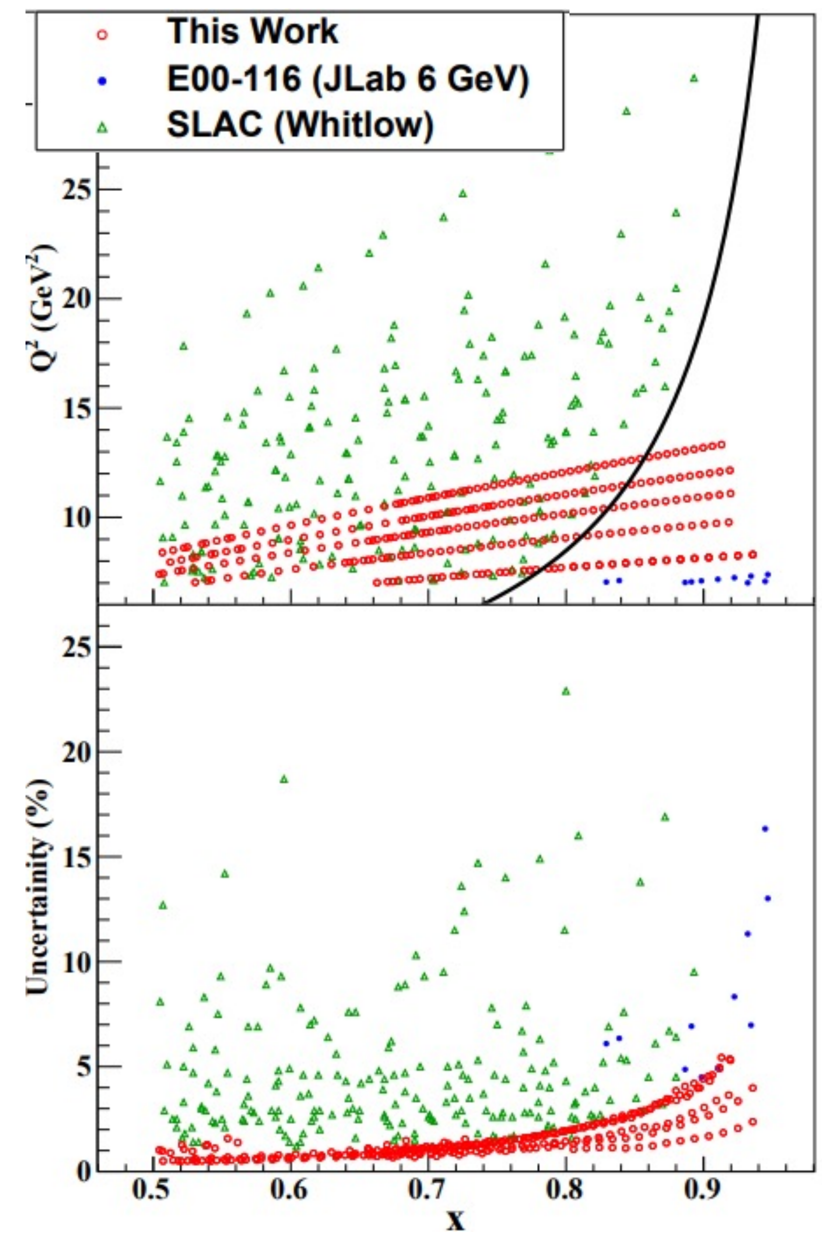
Review | Published: 04 September 2024

Volume 60, article number 173, (2024) Cite





Hall C (11 GeV), 2409.15236

Past Literature (resummation)

QCD

Summation of Large Corrections to Short Distance Cross-Sections *Sterman (1986)*

Resummation of the QCD Perturbative Series *Catani, Trentadue (1989)*

SCET

Factorization and Momentum-Space Resummation in DIS *Becher, Neubert, Pecjak (2007)*

Rapidity Divergences and DIS in the Endpoint Region *Fleming, Labun (2012)*

Proper factorization in high-energy scattering near the endpoint *Chay, Kim (2013)*

Is there a final answer?

Role of Soft Function

Soft Function

QCD

Sterman (1986)

$$F(x,Q^2) = |H_{\mathrm{DI}}(Q^2)|^2 \int_x^1 (\mathrm{d}y/y)\,\phi(y,Q^2) \int_0^{y-x} (\mathrm{d}w/[1-w])\,V(wQ) \times J[Q^2(y-x-w)/2x, Q] + O(1-x)^0. \quad (3.13)$$

and in SCET?

SCET

$$F_2^{\mathrm{ns}}(x,Q^2) = \sum_q e_q^2\,|C_V(Q^2,\mu)|^2\,Q^2 \int_x^1 d\xi\; J\Big(Q^2\,\frac{\xi - x}{x}, \mu\Big)\,\phi_q^{\mathrm{ns}}(\xi,\mu)$$

Issues with rapidity divergences

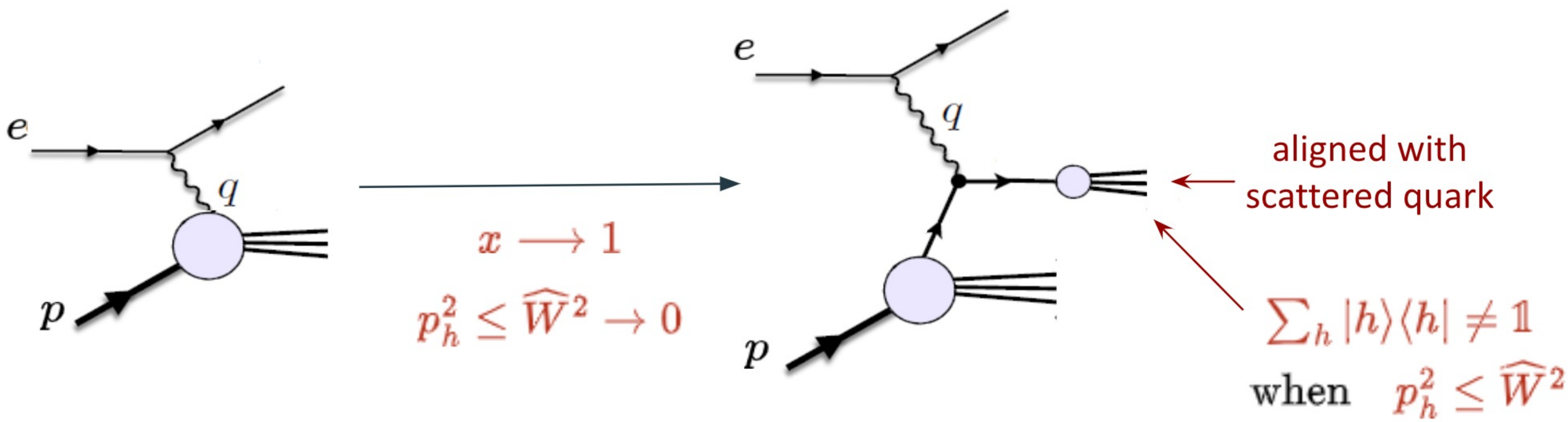
"[the rapidity anomalous dimensions] reveal sensitivity to IR scales which *may signal a breakdown of factorization*"

Fleming, Labun (2012)

We need a careful treatment of rapidity divergences

Kinematics and Dynamics at large x

Kinematic limit on invariant mass $$W^2 = \frac{1 - x_B}{x_B} Q^2 + M^2$$



The process has 2 natural scales: W^2 **and** Q^2

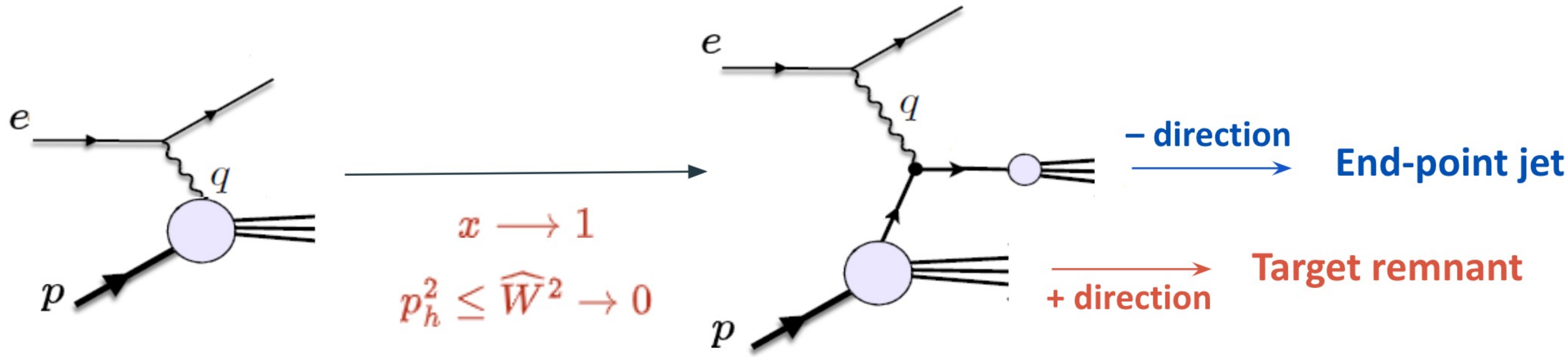
The final state becomes more and more jet-like as x increases

the completeness relation cannot be used anymore

the spread of transverse momentum is limited

Kinematics and Dynamics at large x

Kinematic limit on invariant mass $$W^2 = \frac{1-x_B}{x_B}Q^2 + M^2$$



Peculiar case: neither inclusive nor semi-inclusive

The process has 2 dominant light-cone (opposite) directions

same as SIDIS in TMD factorization

off-lightcone factorization

Off-lightcone collinear factorization

Why do we want to go off the light cone?

Pros

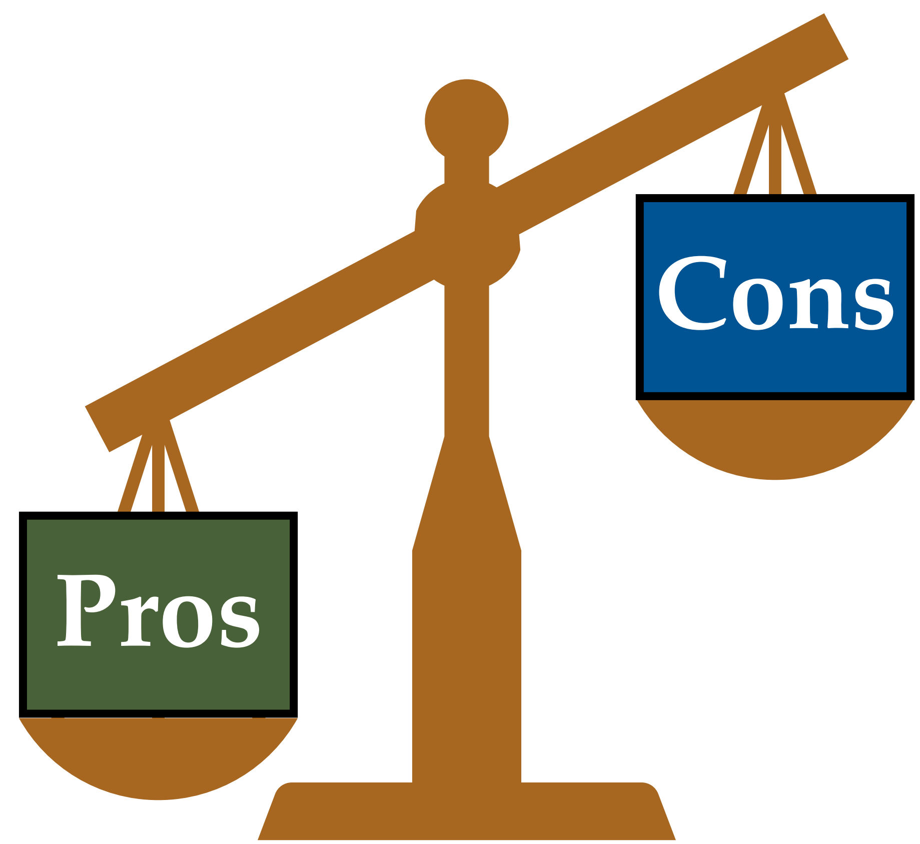
1. **Gauge-invariance** is preserved
2. **Soft exponentiation** is preserved
3. Explicit **tracking of the rapidity effects** that may break factorization

Their cancellation

- *is not guaranteed a priori*
- *often happens at the cross section level*

Cons

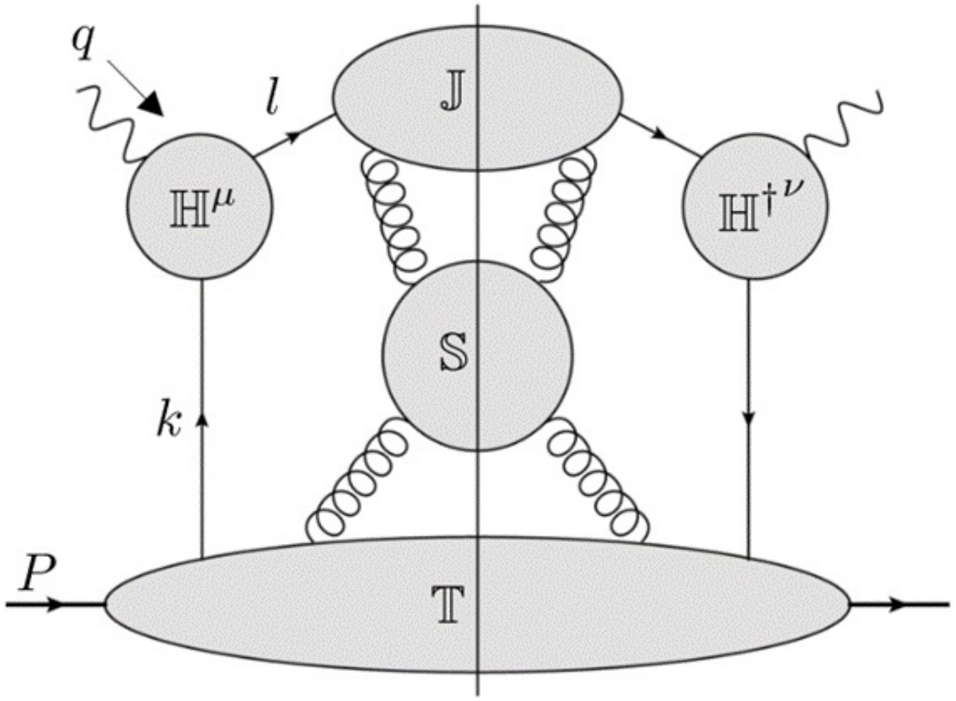
1. The calculation of the diagrams appears to be more complicated

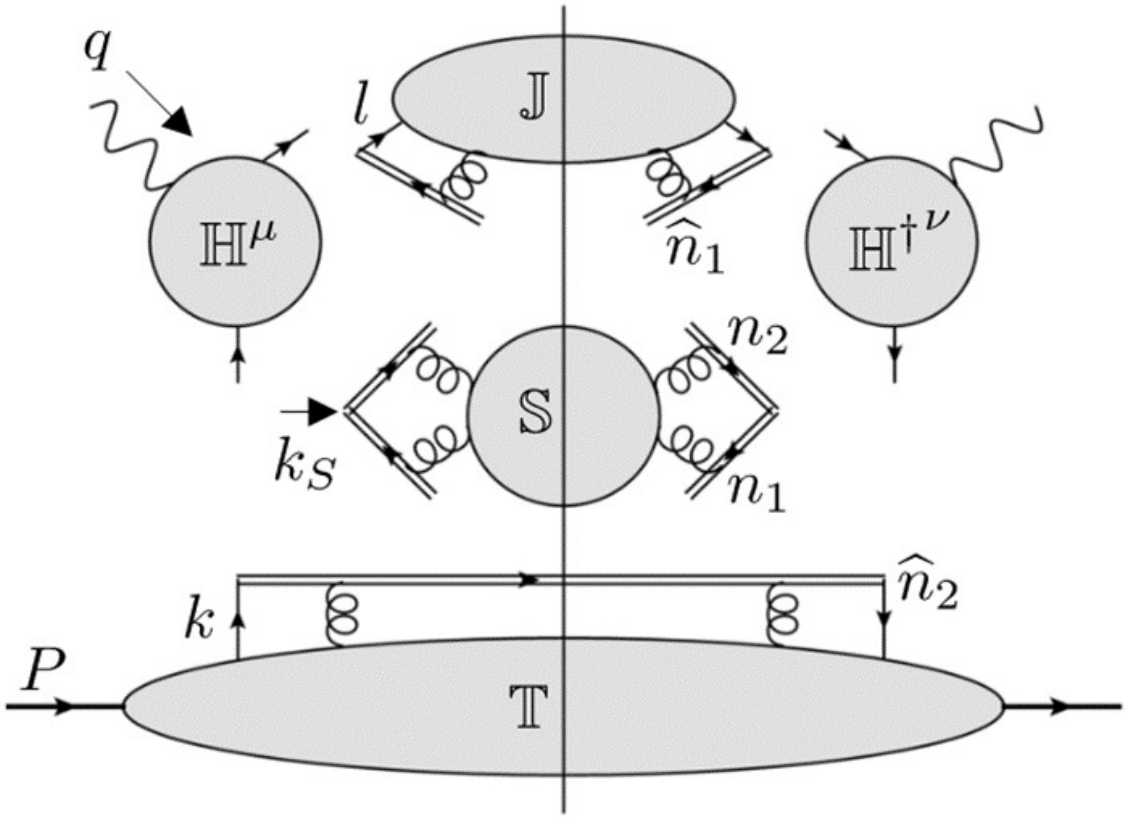


Off-lightcone collinear factorization

Wilson lines are tilted off the light cone

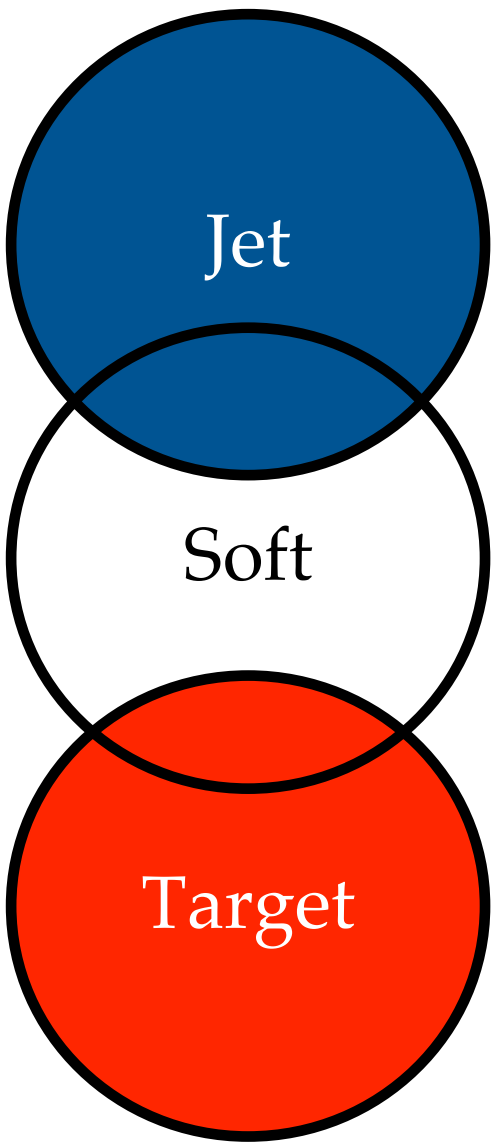
$$n = (1, 0, \vec{0}_T) \quad \mapsto \quad n_1 = (1, -e^{-2y_1}, \vec{0}_T),$$
$$\overline{n} = (0, 1, \vec{0}_T) \quad \mapsto \quad n_2 = (e^{2y_2}, 1, \vec{0}_T).$$





Off-lightcone collinear factorization

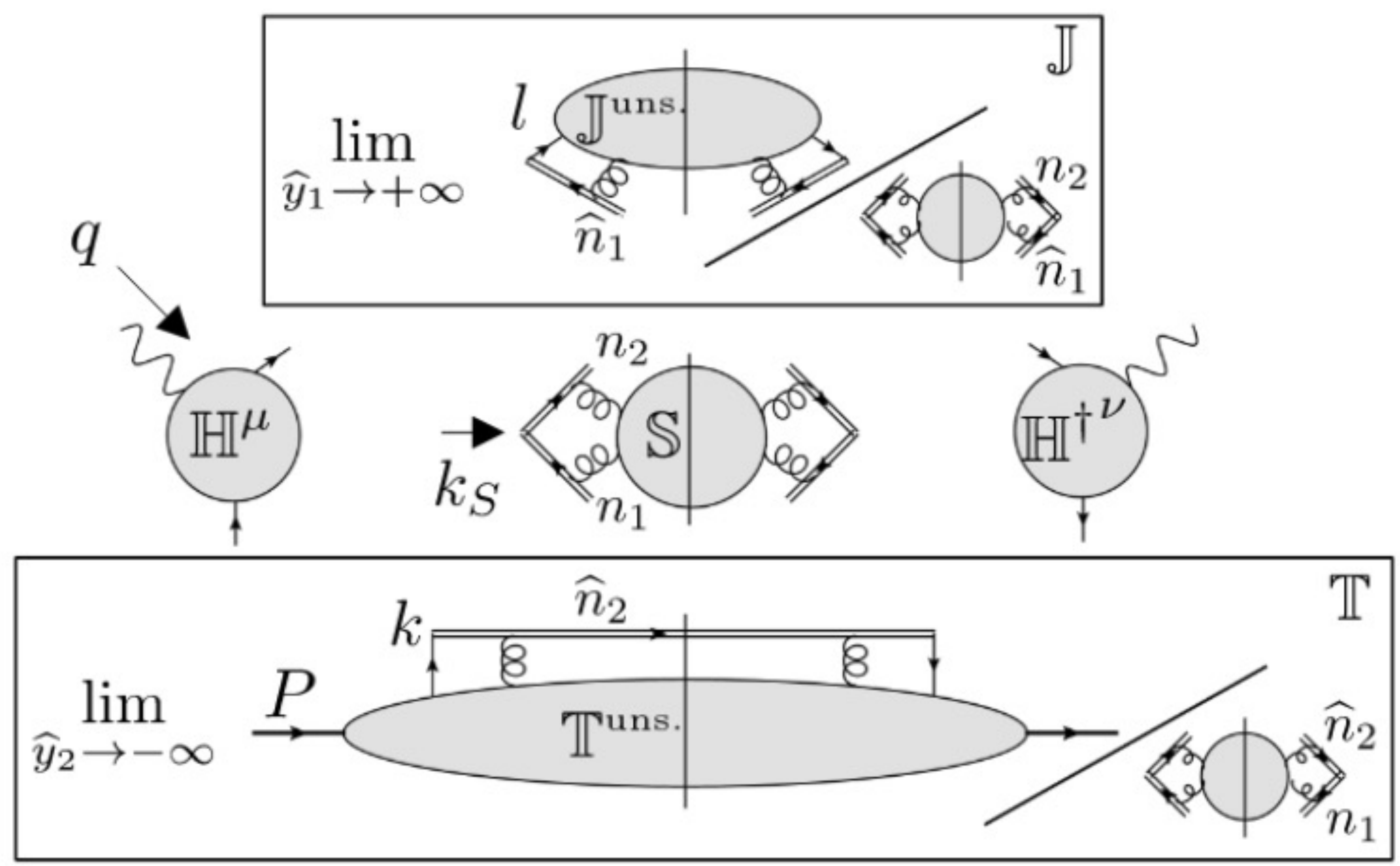
Subtracting the regions of overlap



soft-collinear

soft-collinear

off-lightcone jet function



off-lightcone target function

Off-lightcone collinear factorization

Factorization theorem (off-lightcone)

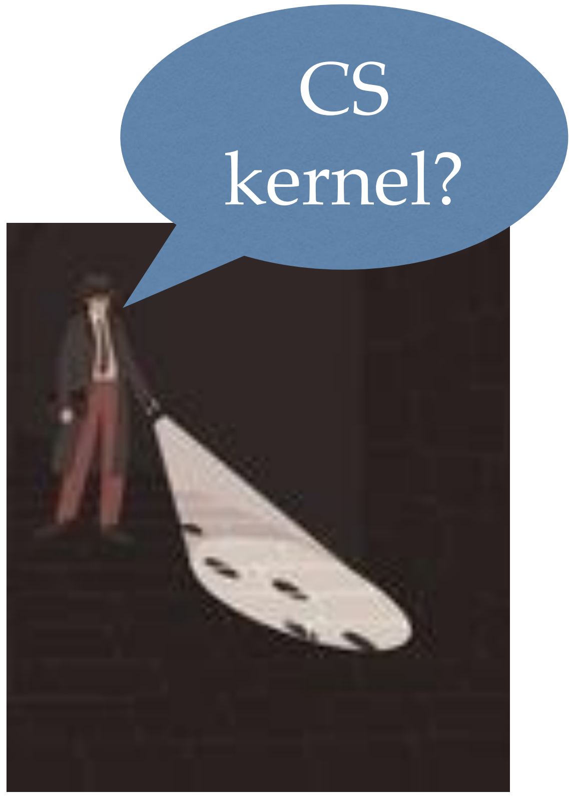
$$x\,W^{\mu\nu} = N_C\,\mathbf{u}^{\mu\nu}\,H(Q^2)\sum_i \int_x^1 \frac{d\xi}{\xi}\int_0^{\xi-x} d\rho \times \phi_i^{\mathrm{thr}}(\xi, y_1)\,\boxed{S\left(\frac{\rho}{\xi}; y_1, y_2\right)}\,\mathcal{J}_i^{\mathrm{thr}}\left(\frac{\xi-x-\rho}{x}; y_2\right)$$

y_1: large and positive $\quad$ y_2: large and negative

The soft function appears naturally

It bridges the rapidity gaps of the target and the jet

Same result as Sterman (1986)



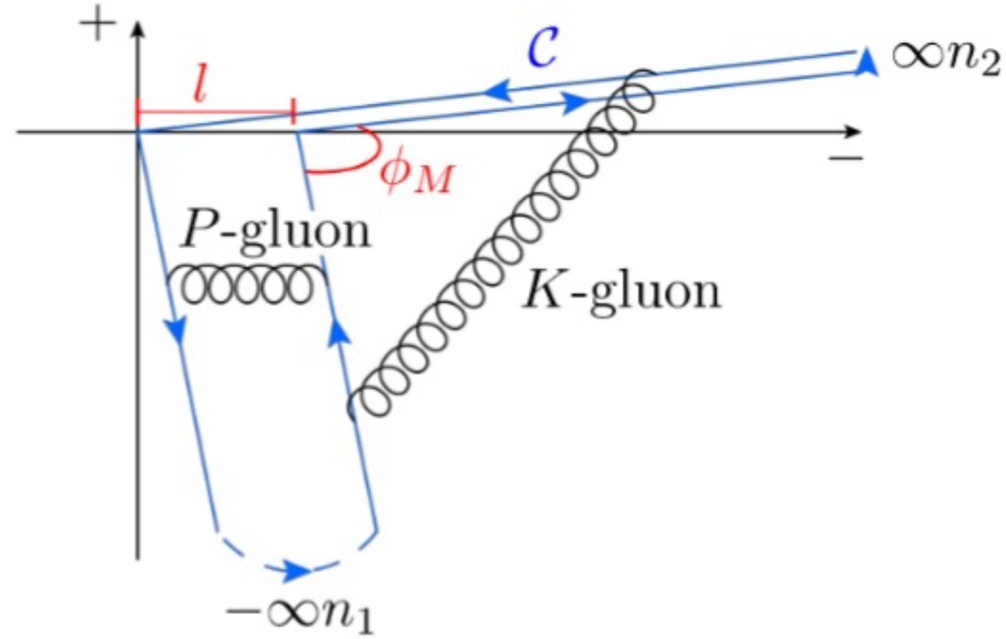
Off-lightcone collinear factorization

Off-lightcone collinear factorization

Soft function in Mellin space

$x \to 1 \Leftrightarrow N$ asymptote

K-P decomposition



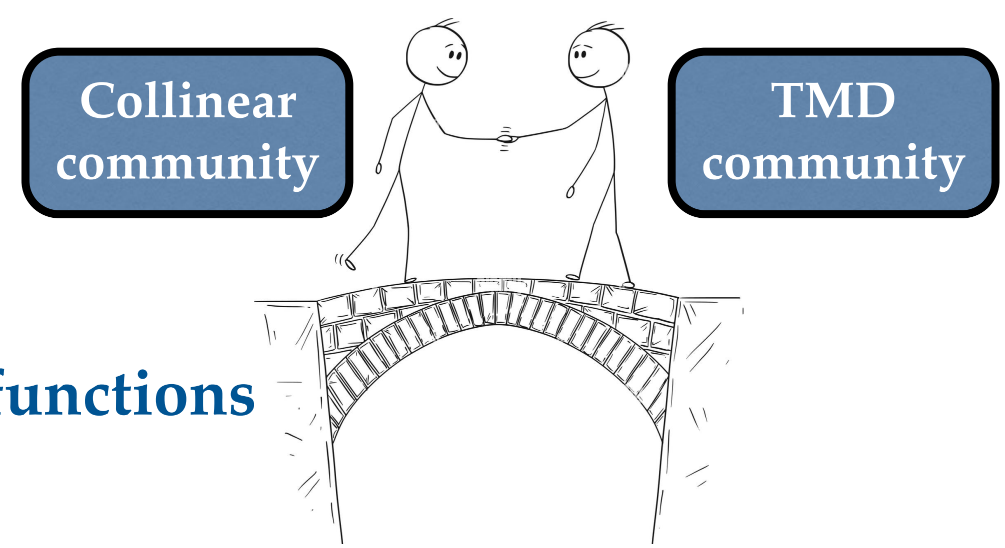
The soft function is the vacuum expectation value of this Wilson loop

$$S_{\rm DIS}(N,\mu,y_1,y_2) = e^{\int_{y_2}^{y_1+i\frac{\pi}{2}} dy\, K(a_S(\mu),L_N+y) + \frac{1}{2}\left[P\left(a_S(\mu),L_N+y_1+i\frac{\pi}{2}\right)+P(a_S(\mu),L_N+y_2)\right] + \mathcal{O}(e^{-2y_1},e^{2y_2})}$$

CS Kernel

$$L_N = \log\left(\frac{\mu}{\sqrt{2}P^+} N e^{\gamma_E}\right)$$

log of the displacement l



K and P are the truly universal functions

Off-lightcone collinear factorization

Factorization theorem (1)

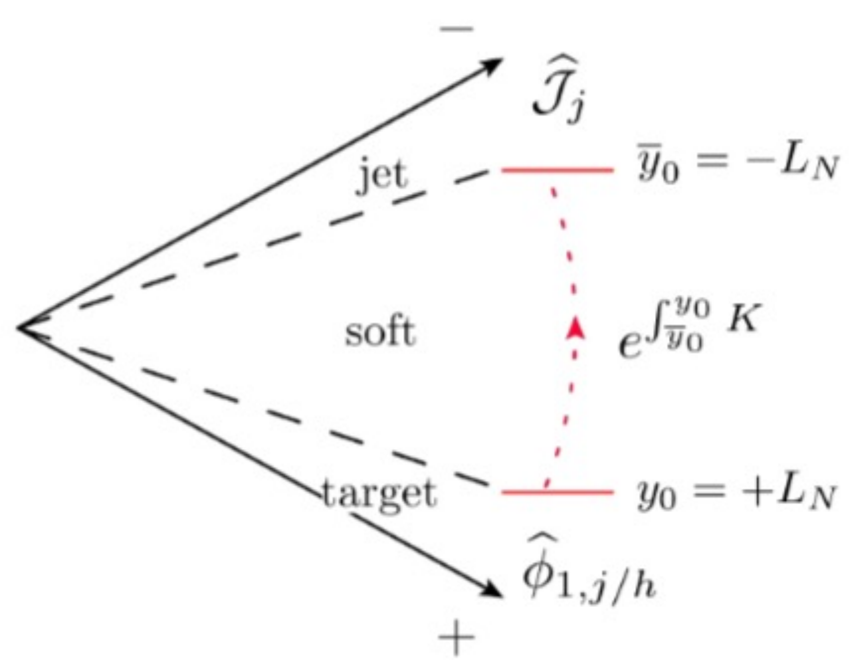
$$W^{\mu\nu} = N_C\, \mathbf{u}^{\mu\nu}\, H(\mu, Q) \sum_j \int_0^1 dx\, x^{N-1} \times \widehat{\phi}_j^{\text{thr}}(N, \mu, y_0)\, e^{\int_{\bar{y}_0}^{y_0} dy\, K(a_S(\mu), L_N + y)}\, \widehat{\mathcal{J}}_j^{\text{thr}}(N, \mu, \bar{y}_0)$$

CS kernel EXPLICIT in inclusive DIS cross section

Off-lightcone effects (P functions) cancel at the cross section level

$y_0, \bar{y}_0$ arbitrary rapidity scales

natural symmetric choice $y_0 = +L_N$, $\bar{y}_0 = -L_N$



Clear and transparent rapidity separation

Off-lightcone collinear factorization

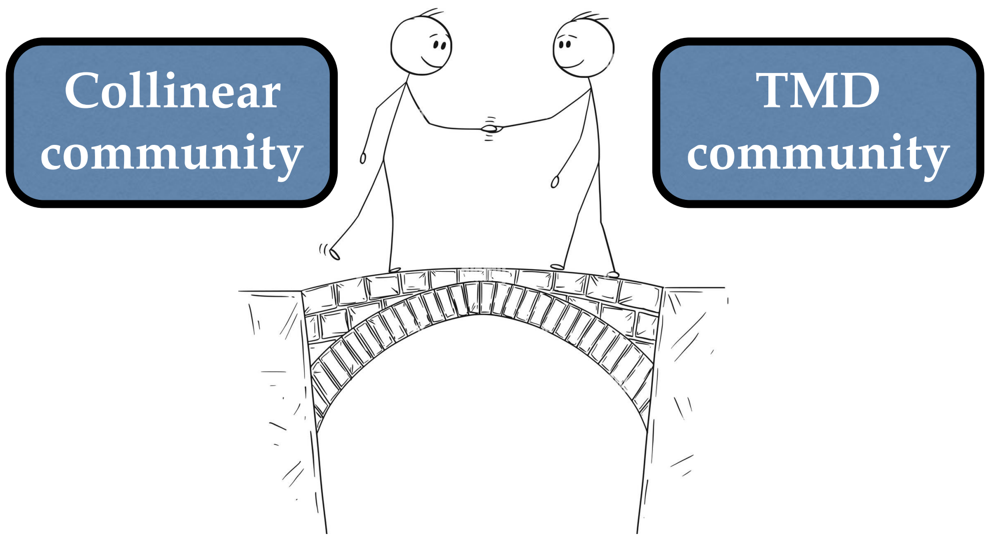
Off-lightcone effect still present at the operator level

$\Rightarrow$ Complicated evolution equations

It is possible to cancel P functions at the operator level

Square-root definitions

$$\widehat{\mathcal{J}}^{\rm sqrt}(y_n) = \lim_{\substack{\hat{y}_1 \to +\infty \\ \hat{y}_2 \to -\infty}} \widehat{\mathcal{J}}^{\rm uns}(\hat{y}_1) \sqrt{\frac{\widehat{S}(y_n, \hat{y}_2)}{\widehat{S}(\hat{y}_1, \hat{y}_2)\widehat{S}(\hat{y}_1, y_n)}}$$



Very simple evolution equations

$$\frac{\partial}{\partial y_n} \log \widehat{\mathcal{J}}^{\rm sqrt}(N, \mu, y_n) = +K(a_S(\mu), L_N + y_n),$$
$$\frac{\partial}{\partial y_n} \log \widehat{\phi}^{\rm sqrt}(N, \mu, y_n) = -K(a_S(\mu), L_N + y_n).$$

$$\widehat{\phi}^{\rm thr}(N, \mu, y_0) e^{\int_{\bar{y}_0}^{y_0} dy\, K(a_S, L_N + y)} \widehat{\mathcal{J}}^{\rm thr}(N, \mu, \bar{y}_0) = \widehat{\phi}^{\rm sqrt}(N, \mu, y_n) \widehat{\mathcal{J}}^{\rm sqrt}(N, \mu, y_n)$$

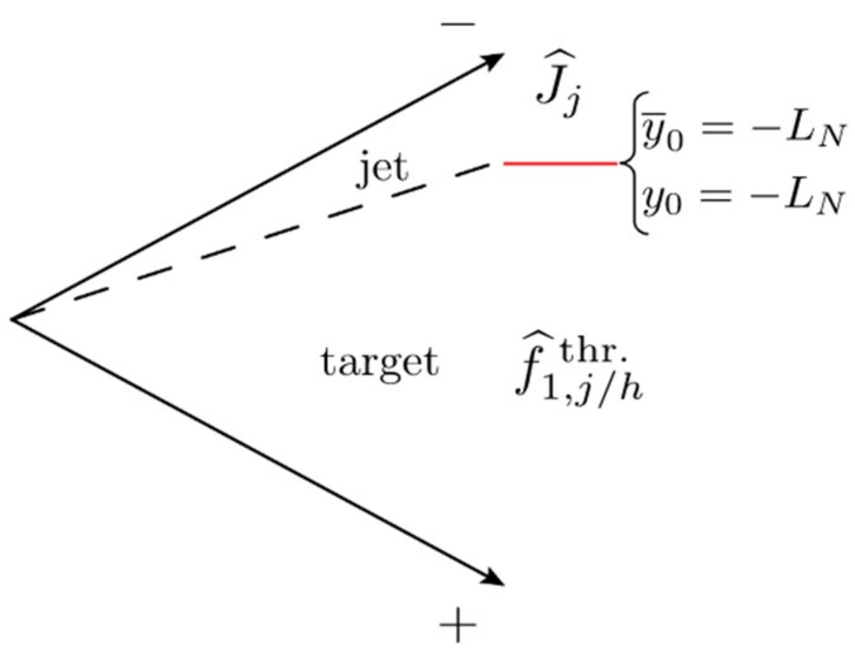
Off-lightcone collinear factorization

Factorization theorem (2)

$y_0, \bar{y}_0$ **arbitrary rapidity scales**

lucky choice

$$y_0 = -L_N$$
$$\bar{y}_0 = -L_N$$



Soft physics in the target operator

Matching to the (standard) light-cone operators

$$\widehat{\mathcal{J}}_i^{\text{sqrt}}(N,\mu,-L_N) = C(a_S(\mu),L_N)\,\widehat{J}_j(N;\mu),$$
$$\widehat{\phi}_j^{\text{sqrt}}(N,\mu,-L_N) = \frac{\widehat{f}_j(N,\mu)}{C(a_S(\mu),L_N)}.$$

$$W^{\mu\nu} = N_C\,\mathbf{u}^{\mu\nu}\,H(\mu,Q)\sum_j \int_0^1 dx\, x^{N-1}\ \widehat{f}_j(N,\mu)\,\widehat{J}_j(N;\mu),$$

Same result as SCET — but all factorization issues are now resolved
CS kernel IMPLICIT in inclusive DIS cross section

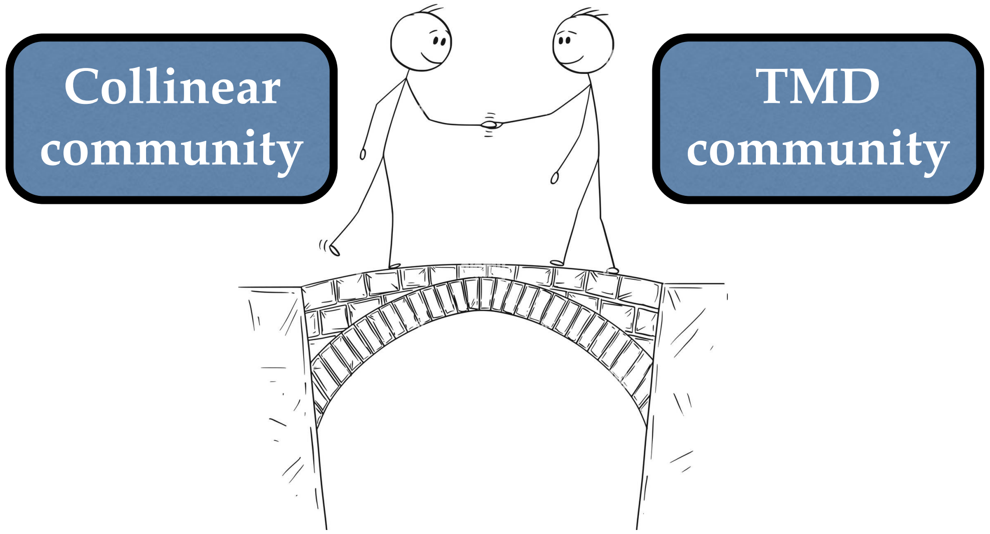
TAKE-HOME message

Factorization theorem (1)

$$W^{\mu\nu} = N_C\, \mathbf{u}^{\mu\nu}\, H(\mu,Q) \sum_j \int_0^1 dx x^{N-1} \times \hat{\phi}_j^{\rm thr}(N,\mu,y_0) e^{\int_{\bar{y}_0}^{y_0} dy\, K(a_S(\mu), L_N+y)}\, \hat{\mathcal{J}}_j^{\rm thr}(N,\mu,\bar{y}_0)$$

Factorization theorem (2)

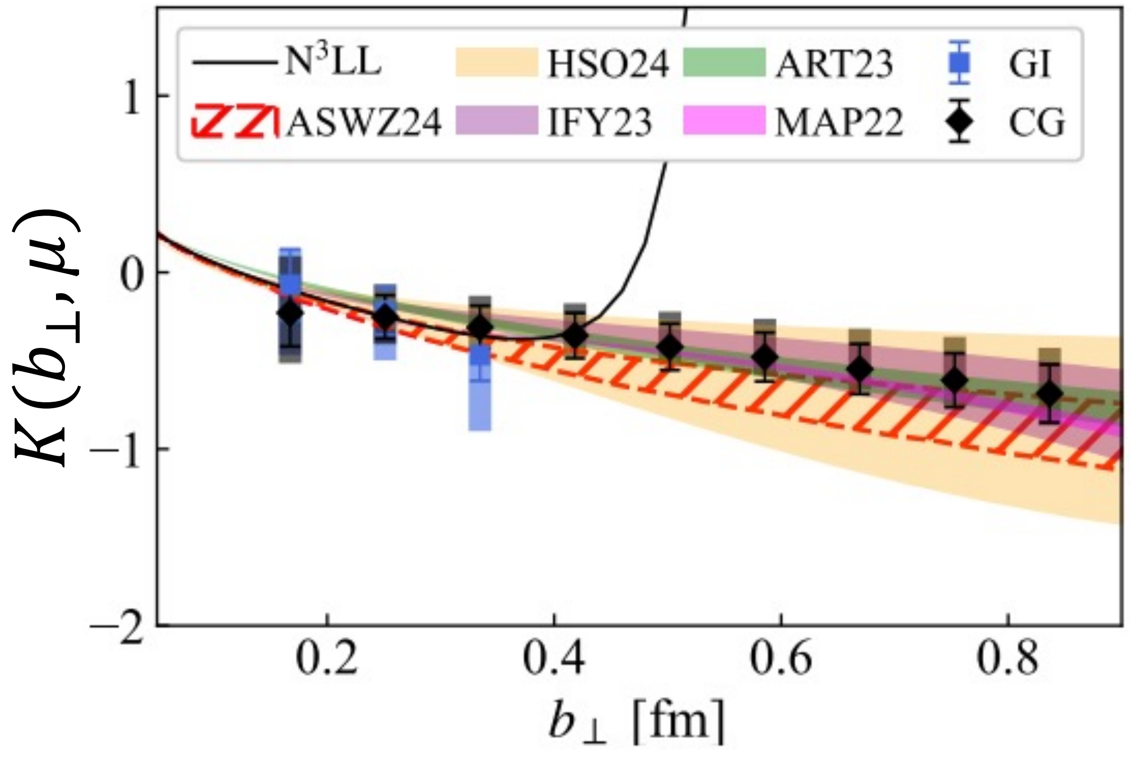
$$W^{\mu\nu} = N_C\, \mathbf{u}^{\mu\nu}\, H(\mu,Q) \sum_j \int_0^1 dx x^{N-1}\; \hat{f}_j(N,\mu)\, \hat{\mathcal{J}}_j(N;\mu),$$



Rapidity evolution of target/jet operator

$$\frac{\partial}{\partial y_n} \log \hat{\mathcal{J}}^{\,\rm sqrt}(N,\mu,y_n) = +K(a_S(\mu), L_N + y_n),$$
$$\frac{\partial}{\partial y_n} \log \hat{\phi}^{\,\rm sqrt}(N,\mu,y_n) = -K(a_S(\mu), L_N + y_n).$$

New way for accessing the CS kernel a typical TMD object



Bollweg, et al., PLB 852 (2024)