

Systematic uncertainties from the implementation of higher twists in QCD analyses

Sanghwa Park
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CTEQ-JLab Collaboration

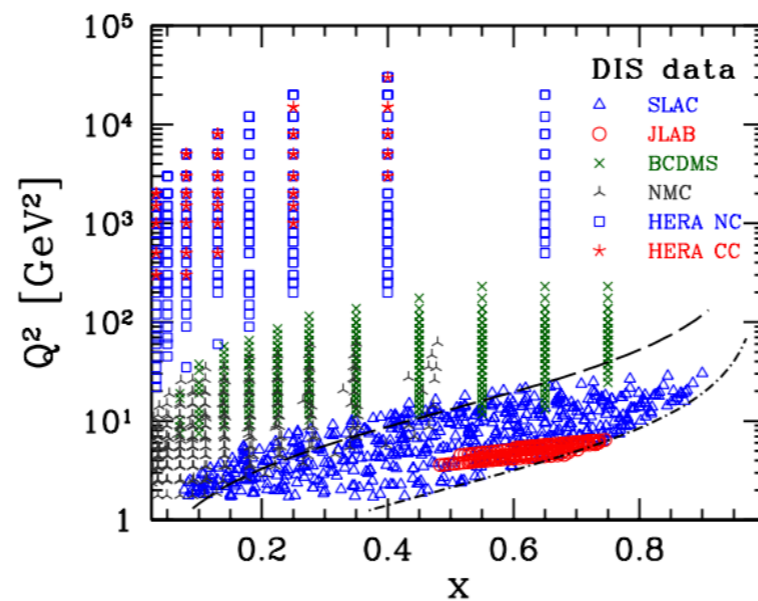
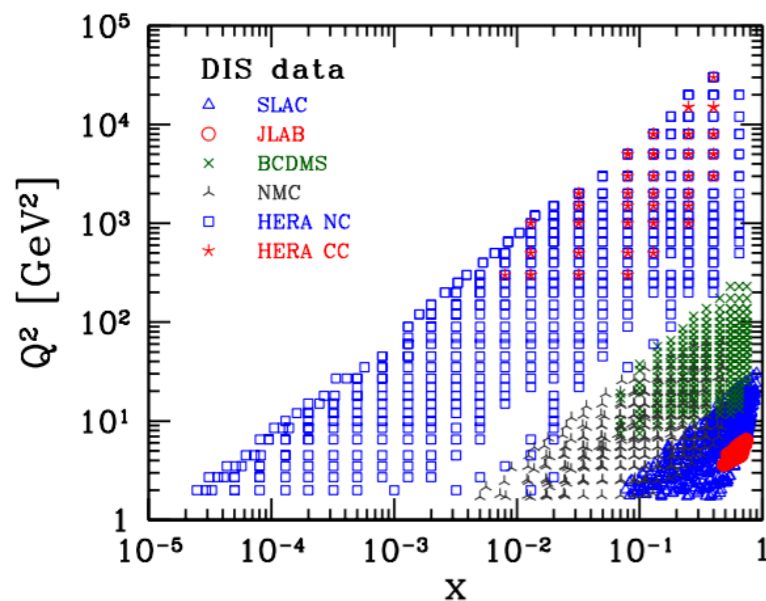
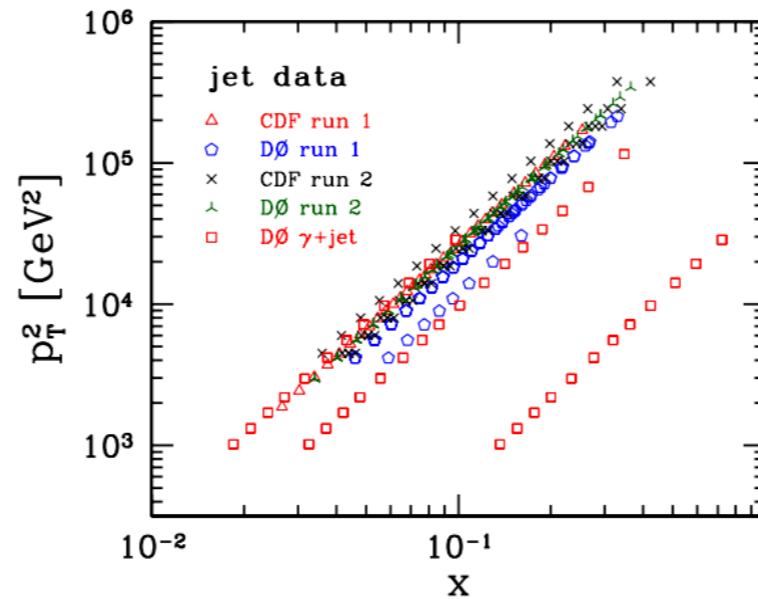
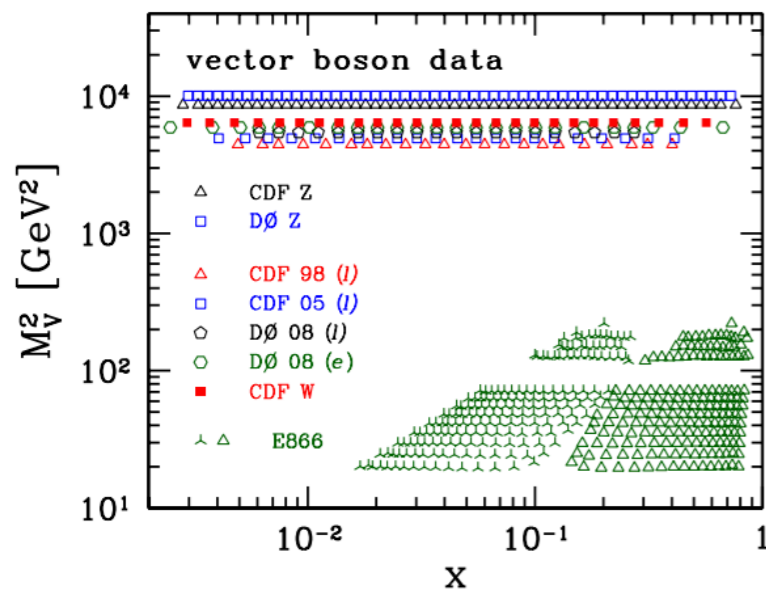
A. Accardi, M. Cerutti, I. Fernando, X. Jing, S. Li, J.F. Owens,
C.E. Keppel, W. Melnitchouk, P. Monahan, S. Park

Outline

- PDFs in large- x region, in particular dealing with Deuterium target data
- Higher twist and off-shell corrections (and their interplay)
- Discuss bias introduced from the choice of implementation methods of higher-twist correction
- How to reduce the bias and systematic uncertainty?
- Resulting systematic uncertainty needs to be considered. How to properly take into account the systematic uncertainty?

PDFs in Large-x

Understand the behaviour of PDFs in the large-x region



Main focus: d
—
 u

u-quark

DIS on proton target
Drell—Yan data

...

d-quark

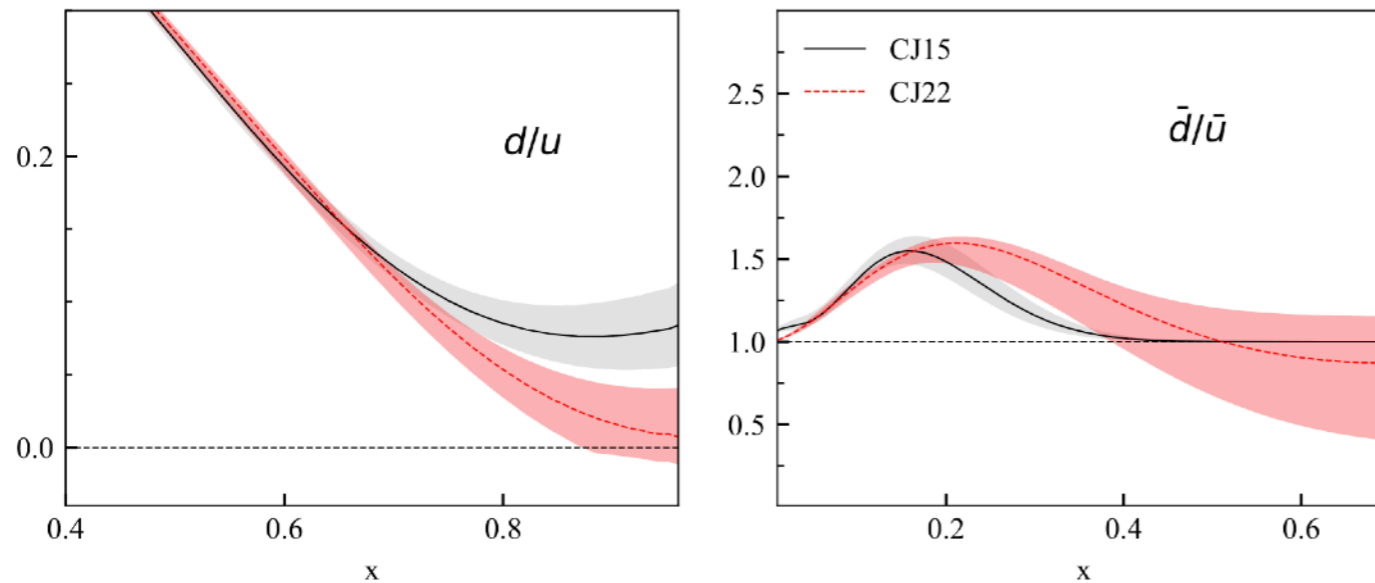
W-boson asymmetry
DIS on Deuterium targets
Proton-Tagged DIS (BONuS)

...

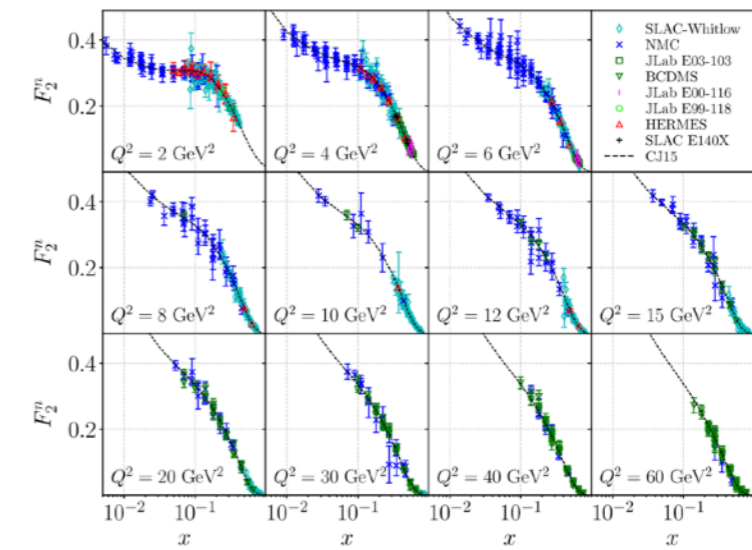
We have to deal with Deuterium target at large-x

CJ Analysis

Accardi, Jing, Owens, Park, PRD 107 (2023) 11, 113005



S.Li et al, PRD 109, 074036 (2024)



CJ global data set:

- W asymmetry data
- 1000+ data points on **Deuterium target**
- high- x and low- Q^2
- $W^2 > 3 \text{ GeV}^2, Q^2 > 1.69 \text{ GeV}^2$

Nuclear corrections

TMC

Higher Twists

The choice of their implementation may be a source of systematic error

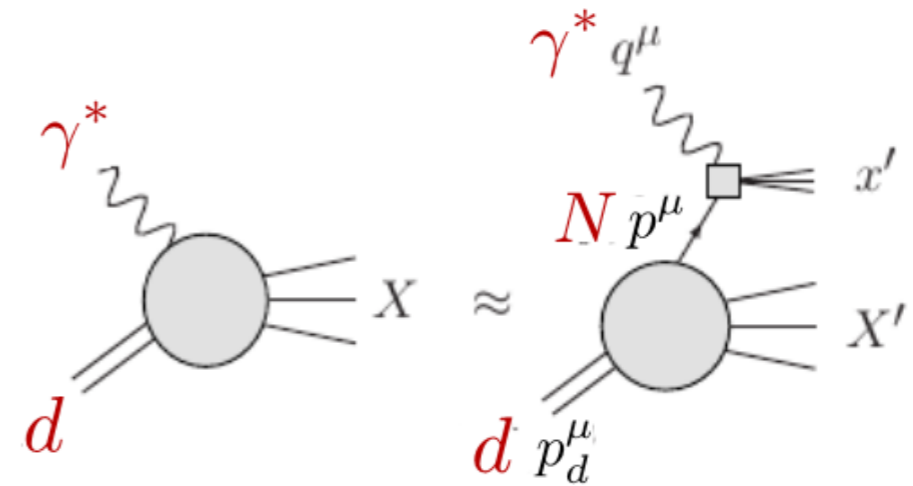
Deuterium: nuclear smearing

Nuclear impulse approximation

Melnitchouk, Schreiber, Thomas, PRD 49 (1994)

Kulagin, Piller, Weise, PRC 50 (1994)

Kulagin and Petti, NPA 765 (2006)



$$F_{2,D}(x_D, Q^2) = \int_{y_{Dmin}}^{y_{Dmax}} dy_D dp_T^2 f_{N/D}(y_D, p_T^2; \gamma) F_{2,N}\left(\frac{x_D}{y_D}, Q^2, p^2\right)$$

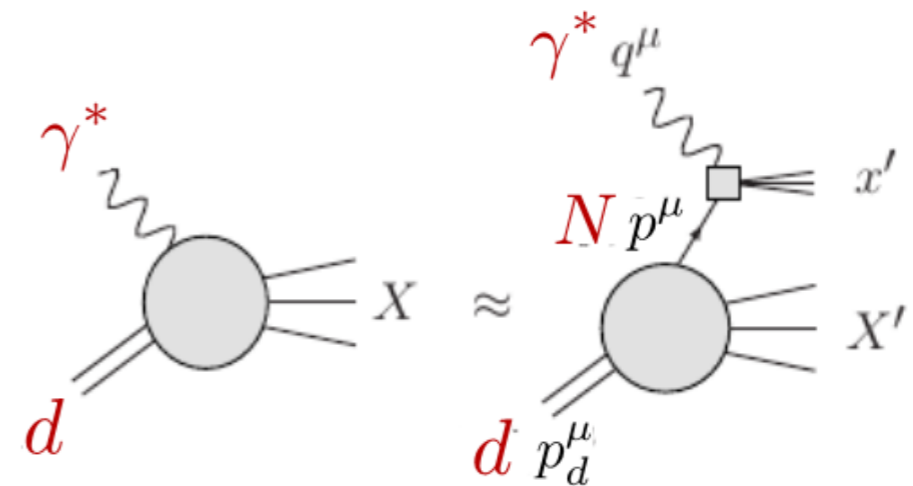
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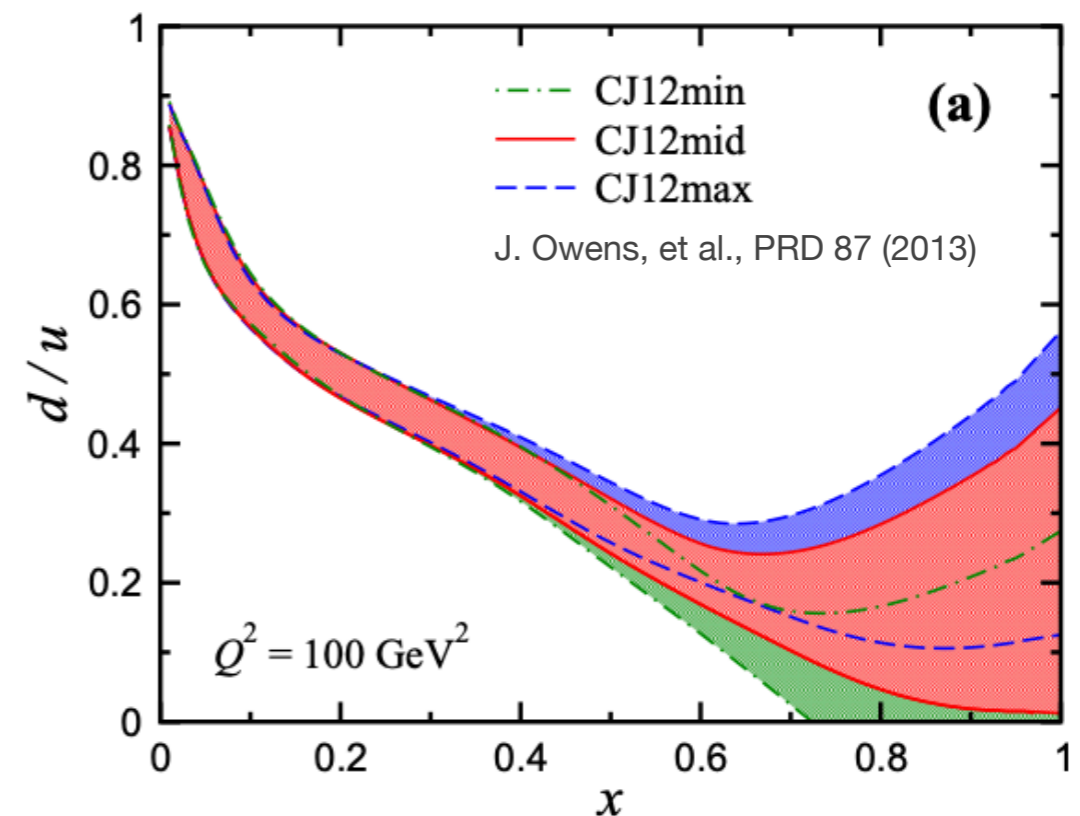
Smearing function:

Weak Binding approximation (WBA)

(Non) Relativistic approach

Nucleon wave function

Nuclear binding and Fermi motion effects



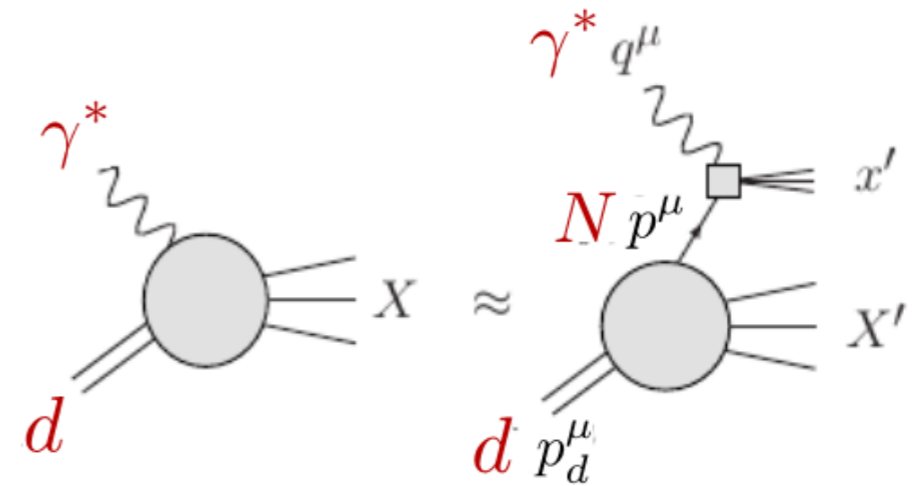
Deuterium: off-shell corrections

Nuclear impulse approximation

Melnitchouk, Schreiber, Thomas, PRD 49 (1994)

Kulagin, Piller, Weise, PRC 50 (1994)

Kulagin and Petti, NPA 765 (2006)



$$F_{2,D}(x_D, Q^2) = \int_{y_{Dmin}}^{y_{Dmax}} dy_D dp_T^2 f_{N/D}(y_D, p_T^2; \gamma) F_{2,N}\left(\frac{x_D}{y_D}, Q^2, p^2\right)$$

Off-shell expansion (in nucleon virtuality p^2)

$$q_N(x, Q^2, p^2) = q_N^{\text{free}}(x, Q^2) \left[1 + \frac{p^2 - M^2}{M^2} \delta f(x) \right]$$

Kulagin, Piller, Weise, PRC 50 (1994)

Kulagin, Melnitchouk, et al., PRC 52 (1995)

Kulagin and Petti, NPA 765 (2006)

$$F_{2N}(x, Q^2, p^2) = F_{2N}^{\text{free}}(x, Q^2) \left[1 + \frac{p^2 - M^2}{M^2} \delta F(x) \right]$$



Free nucleon pdfs/SFs

$$p^2 = m_N^2$$



Off-shell function

(To be fitted)



Structure function

of a bound, off-shell nucleon

Off-shell function: parameterization

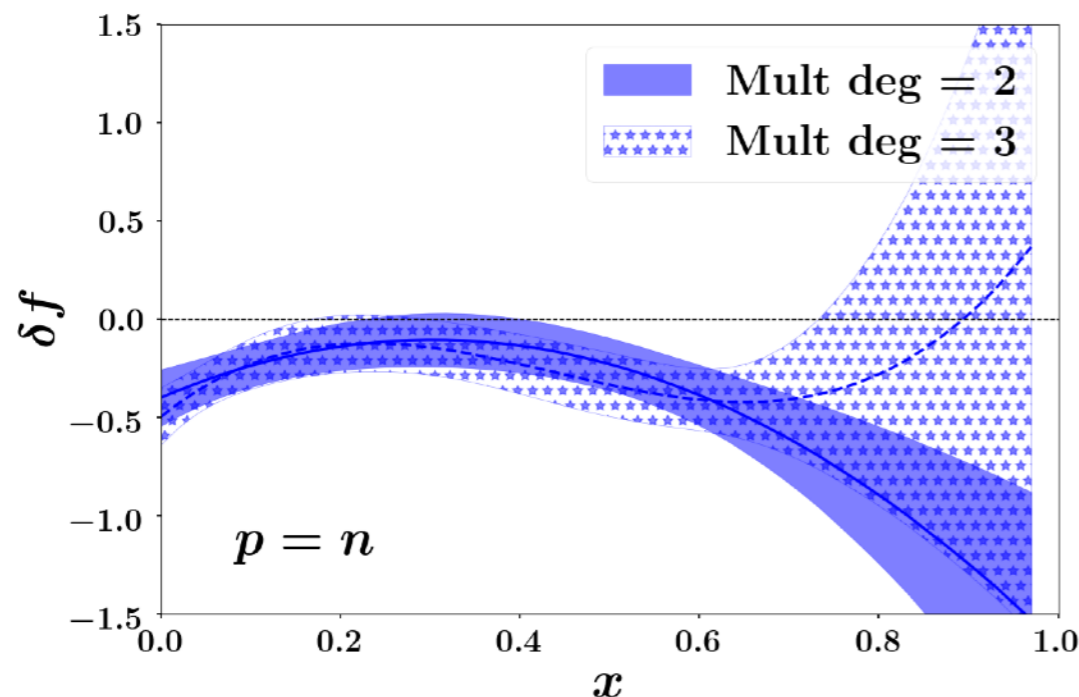
Off-shell corrections

$$q_N(x, Q^2, p^2) = q_N^{\text{free}}(x, Q^2) \left[1 + \frac{p^2 - M^2}{M^2} \delta f(x) \right]$$

○ KP-like model $\delta f^N = C(x - x_0)(x - x_1)(1 + x_0 - x)$ Kulagin and Petti, NPA 765 (2006)

+ valence sum rule $\int_0^1 dx \delta f^N(x) [q(x) - \bar{q}(x)] = 0$ Accardi, et al., PRD 93 (2016)
Accardi, et al., PRD 107 (2023)

○ Polynomial model $\delta f(x) = \sum_n a_{off}^{(n)} x^n$ Alekhin, Kulagin, Petti, PRD 96 (2017)
Alekhin, Kulagin, Petti, PRD 105 (2022)
Alekhin, Kulagin, Petti, PRD 107 (2023)



Constrain power of CJ dataset
only up to $x = 0.6$

Higher-Twist function

Higher Twist correction

Multiplicative (CJ fits)

Additive

$$F_2(x, Q^2) = F_2^{LT}(x, Q^2) \left(1 + \frac{C(x)}{Q^2} \right)$$

$$F_2 = F_2^{LT}(x, Q^2) + \frac{H(x)}{Q^2}$$

$$C(x) = a_{ht}^{(0)} x^{a_{ht}^{(1)}} (1 + a_{ht}^{(2)} x)$$

$$H(x) = a_{ht}^{(0)} x^{a_{ht}^{(1)}} (1 - x)^{a_{ht}^{(2)}} (1 + a_{ht}^{(3)} x)$$

they are related

$$\begin{aligned} F_2^{LT}(x, Q^2) \left(1 + \frac{C(x)}{Q^2} \right) &= F_2^{LT}(x, Q^2) + F_2^{LT}(x, Q^2) \frac{C(x)}{Q^2} \\ &= F_2^{LT}(x, Q^2) + \frac{\tilde{H}(x, Q^2)}{Q^2} \end{aligned}$$

Impact of HT on n/p ratio

Are experimental observables independent of the choice of the HT?

$$\frac{F_{2,n}}{F_{2,p}} = \frac{n}{p} \xrightarrow{x \rightarrow 1} \frac{4d + u}{4u + d} \simeq \frac{1}{4} \quad \text{(extrapolation region)}$$

Case 1: isospin-symmetric HT

Mult HT

$$C_p(x) = C_n(x) = C(x)$$

$$\frac{(4d + u)(1 + C/Q^2)}{(4u + d)(1 + C/Q^2)} \simeq \frac{1}{4}$$

No effect of HT

Add HT

$$H_p(x) = H_n(x) = H(x)$$

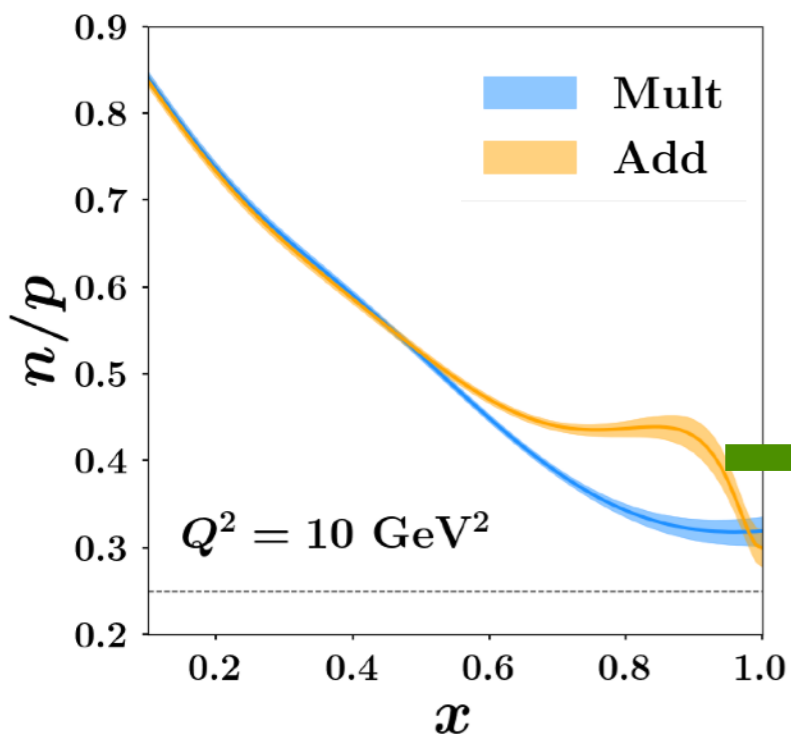
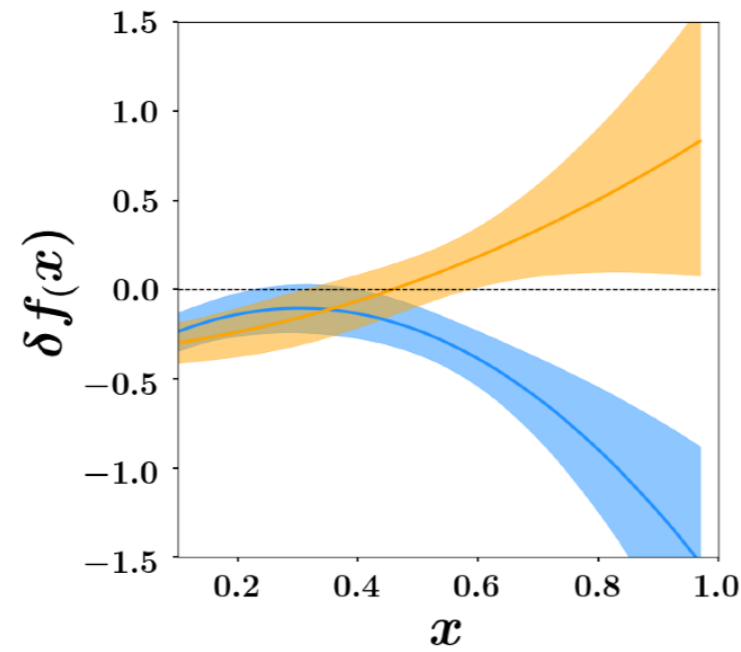
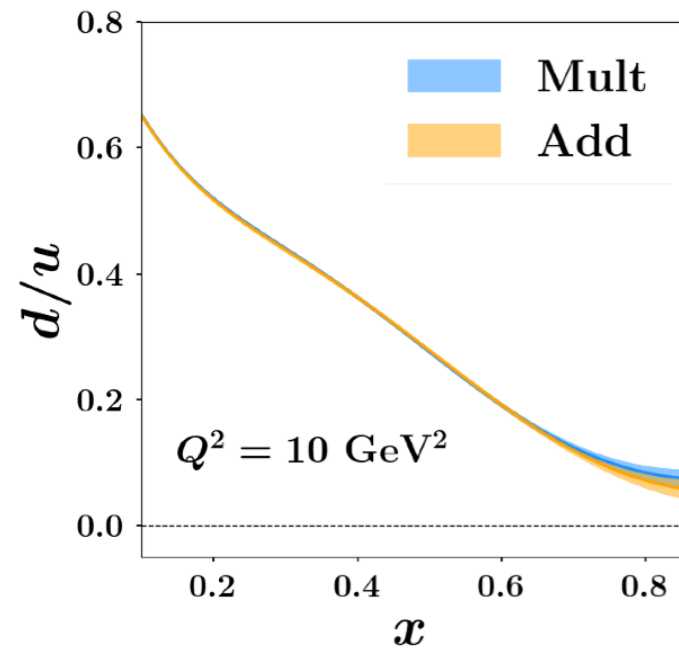
$$\frac{4d + u + H/Q^2}{4u + d + H/Q^2} \simeq \frac{1}{4} + 27 \frac{H}{16uQ^2}$$

Strong effect of HT

Bias identified!!

Results in the CJ fitting framework

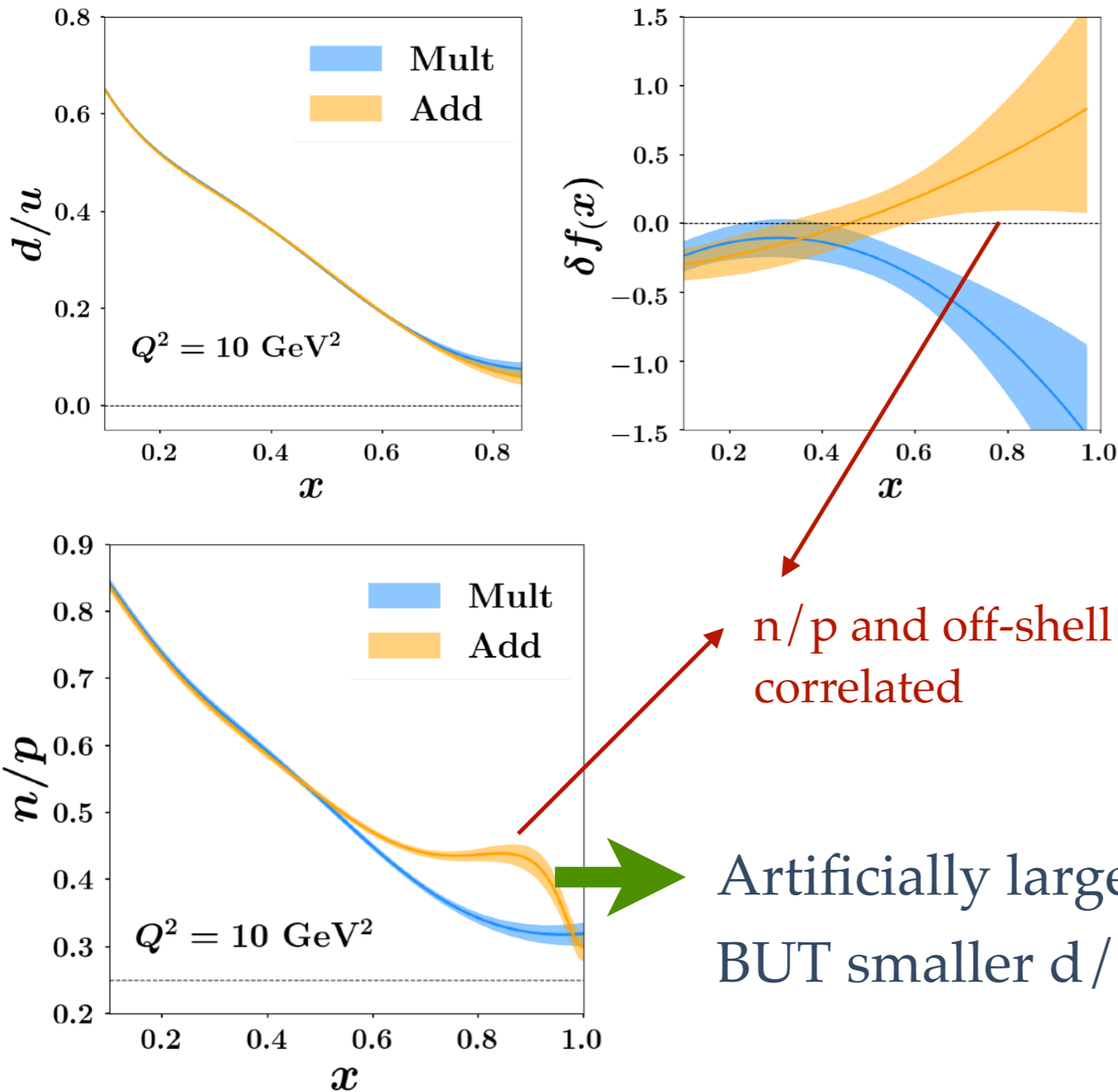
Case 1: isospin-symmetric HT



Artificially large n/p
BUT smaller d/u than Mult

Results in the CJ fitting framework

Case 1: isospin-symmetric HT

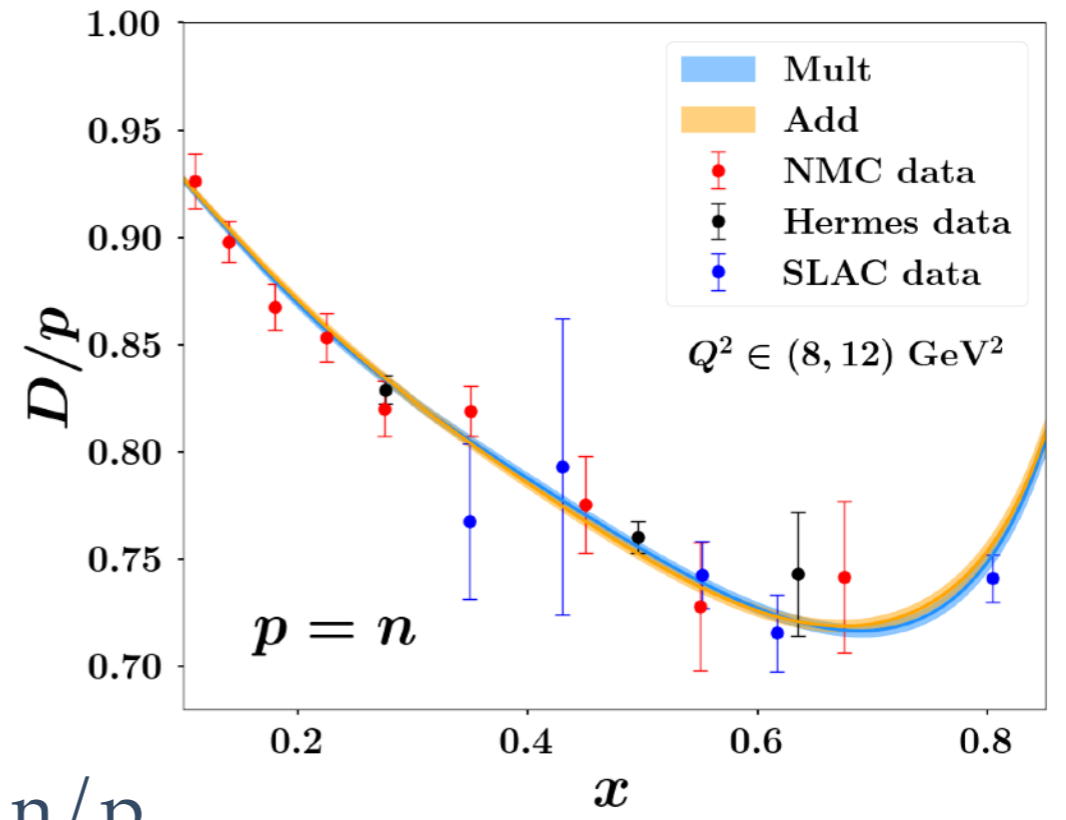


n/p and off-shell correlated

Artificially large n/p
BUT smaller d/u than Mult

Bias identified

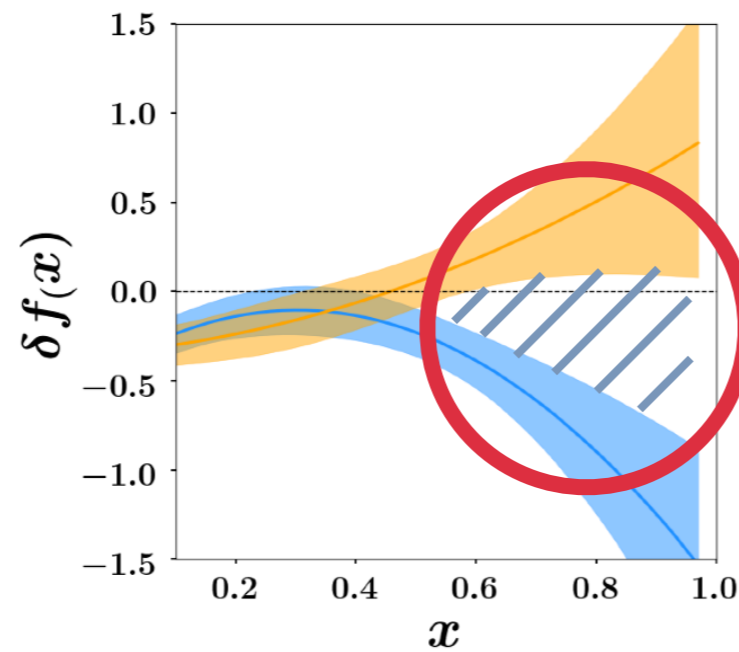
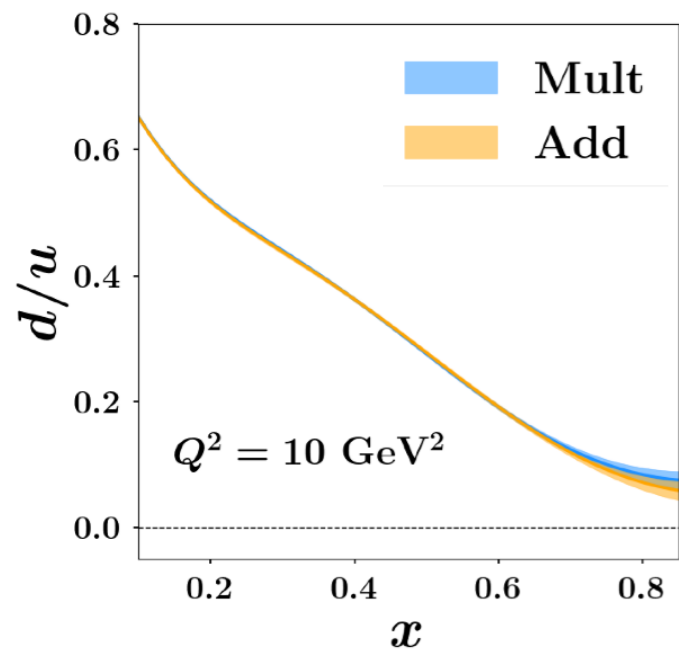
Off-shell compensates n/p



$p = n$

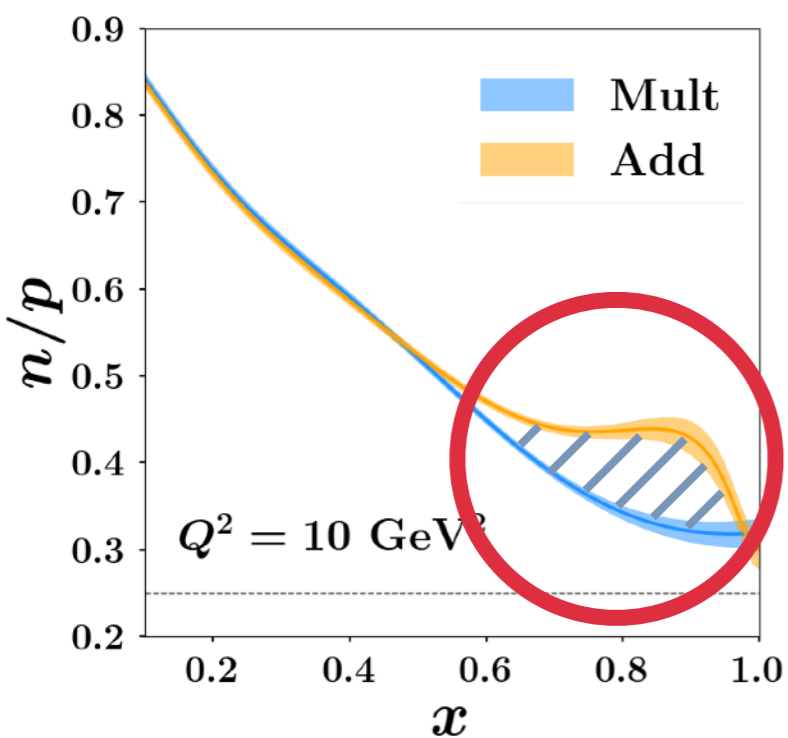
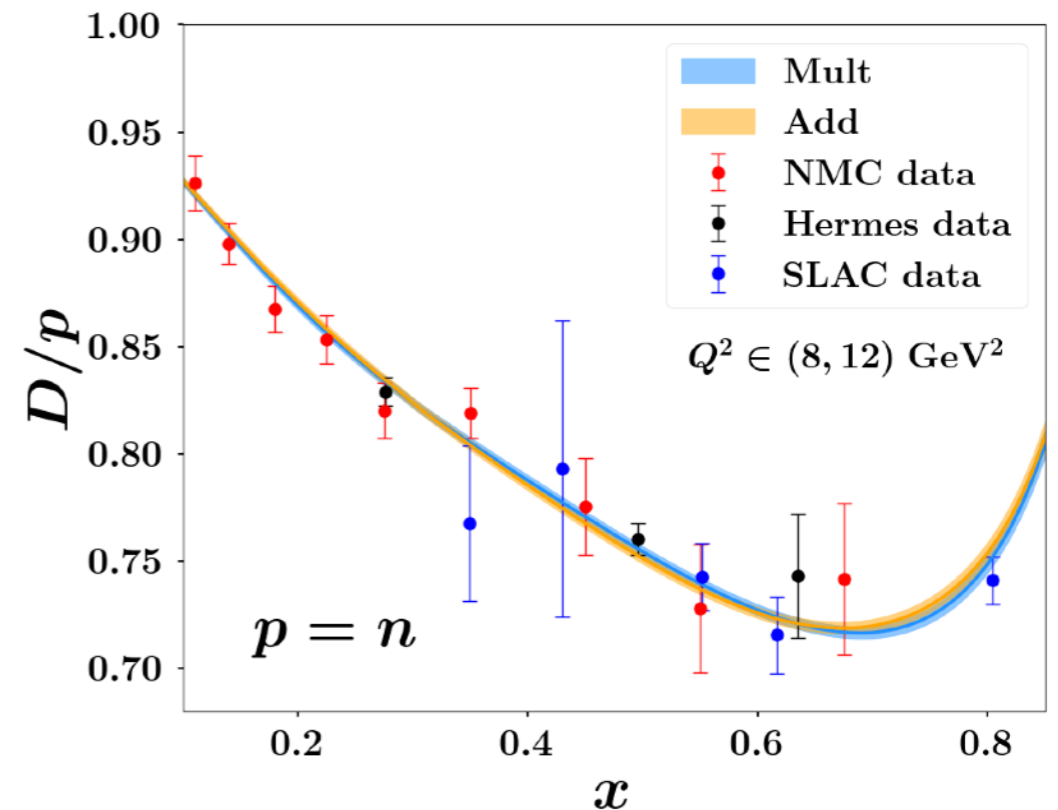
Results in the CJ fitting framework

Case 1: isospin-symmetric HT



Bias identified

Off-shell compensates n/p



**Systematic “implementation” uncertainty
in a region of extrapolation**

Impact of HT on n/p ratio

Are experimental observables independent of the choice of the HT?

$$\frac{n}{p} \xrightarrow{x \rightarrow 1} \frac{1}{4} \quad \text{LT} \quad \text{Mult HT} \quad C_p(x) = C_n(x) = C(x)$$

Case 2: isospin-breaking HT

Add HT
 $H_p(x) \neq H_n(x)$

$$\frac{u + H_n/Q^2}{4u + H_p/Q^2} \approx \frac{1}{4} + 9 \frac{4H_n - H_p}{16uQ^2}$$

$\nearrow H_p(x) = H_n(x)$
 $\rightarrow H_p(x) = 2H_n(x)$

$$\frac{1}{4} + 27 \frac{H}{16uQ^2}$$

$$\frac{1}{4} + 9 \frac{H}{16uQ^2}$$

n/p ratio is smaller

Mult HT
 $C_p(x) \neq C_n(x)$

$$\frac{u + \tilde{H}_n/Q^2}{4u + \tilde{H}_p/Q^2}$$

same as Add

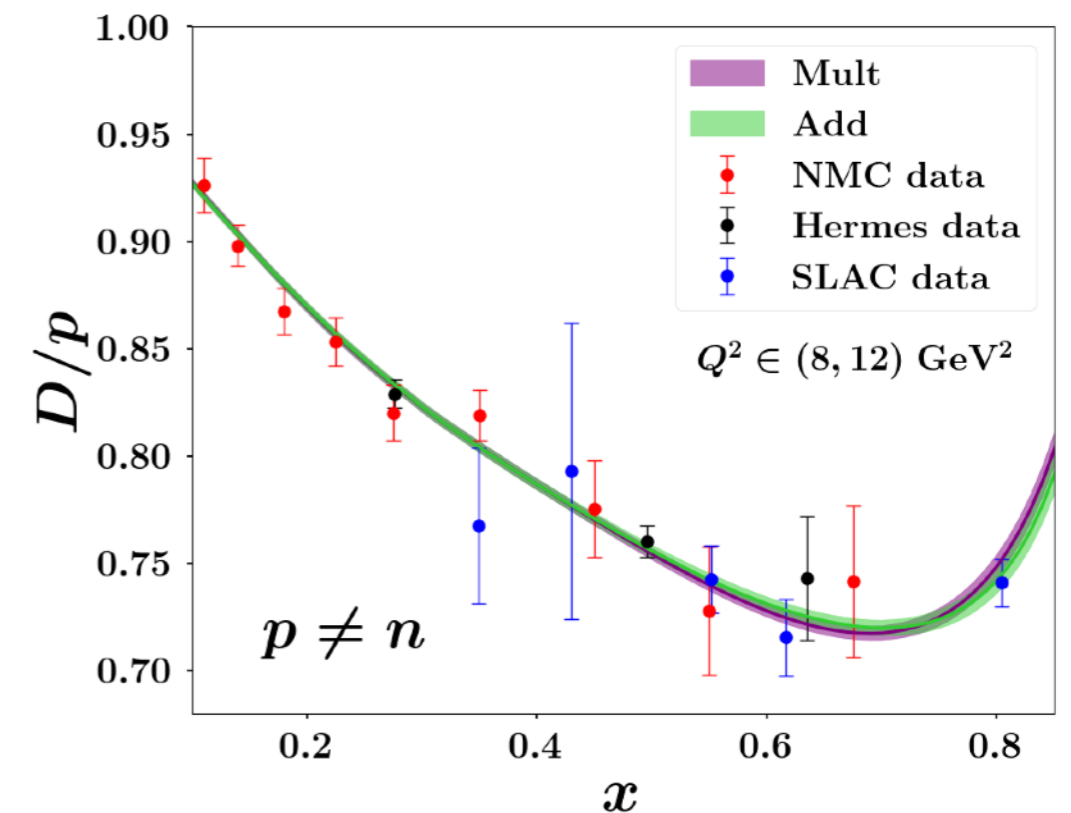
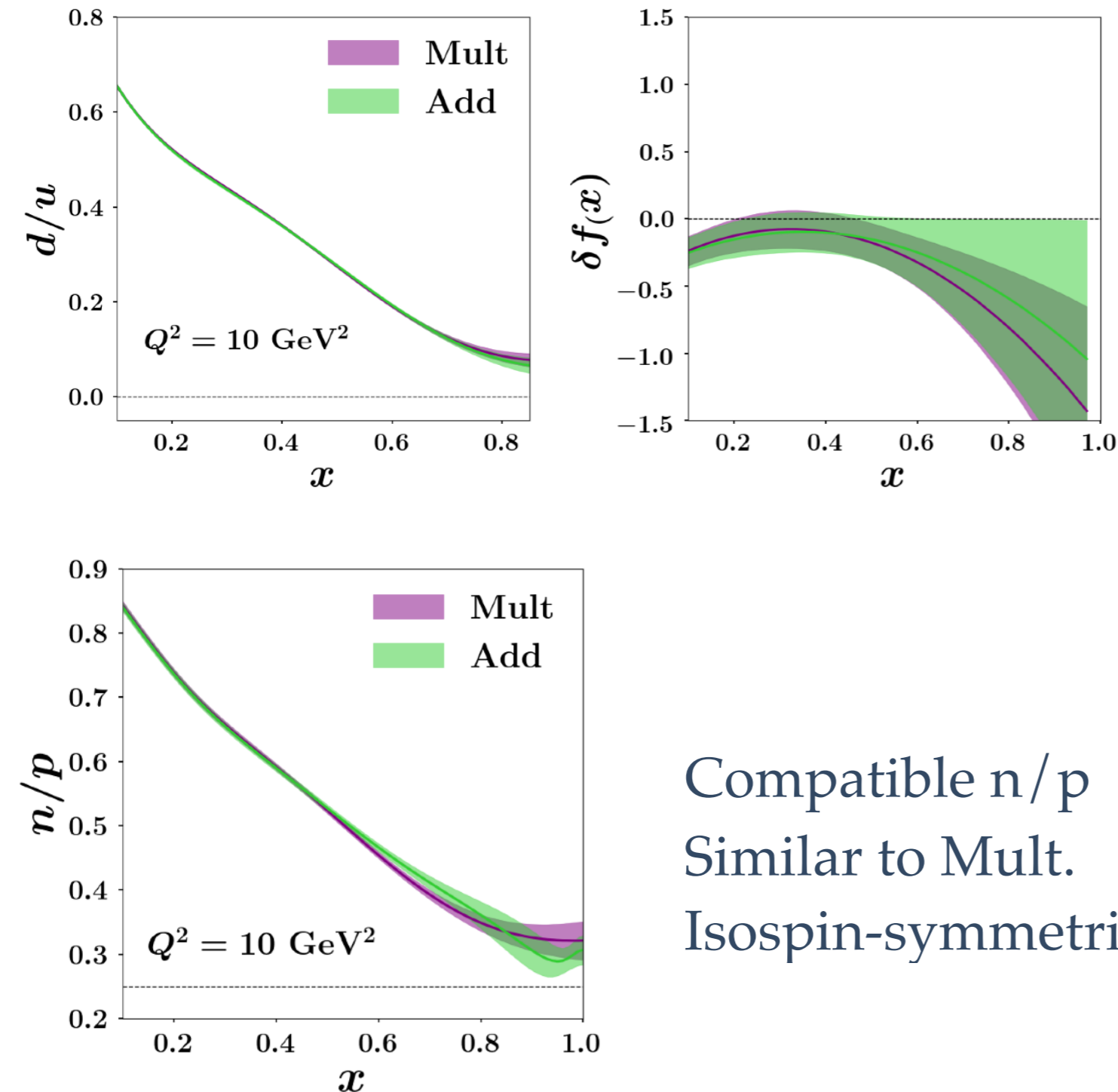
Bias removed!

Results in the CJ fitting framework

Case 2: isospin-breaking HT

Bias removed

No need of compensation by off-shell
Theory expectations confirmed!



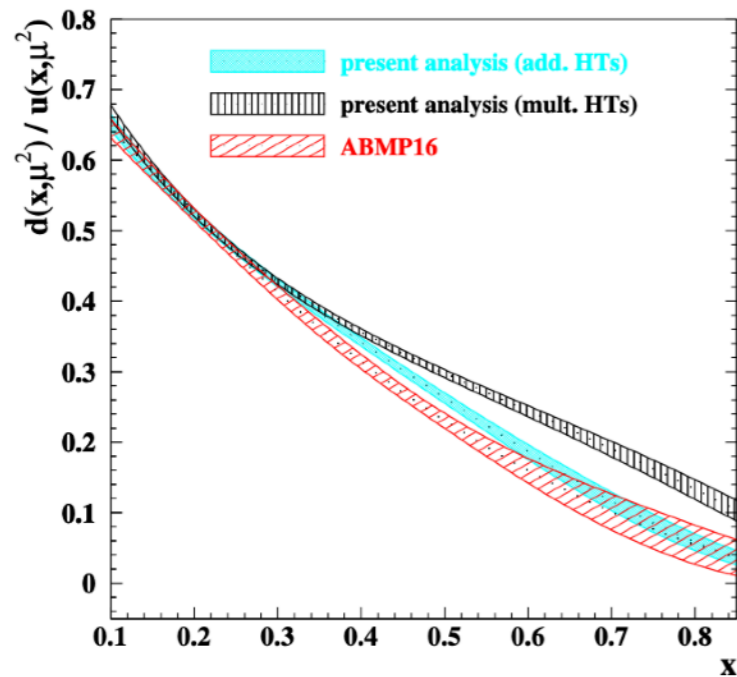
Compatible n/p
Similar to Mult.
Isospin-symmetric HT case

Comparison to other extractions

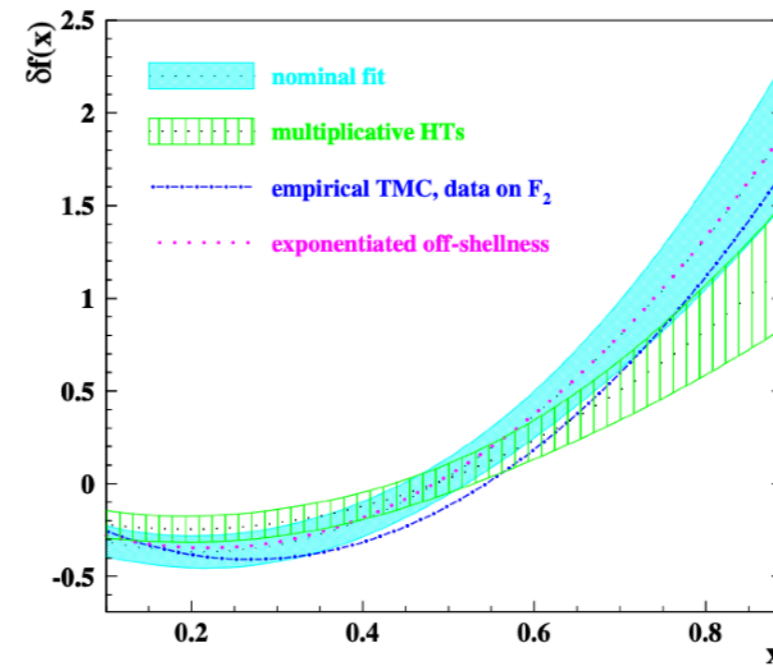
AKP

Alekhin, Kulagin, Petti, PRD 105 (2022)

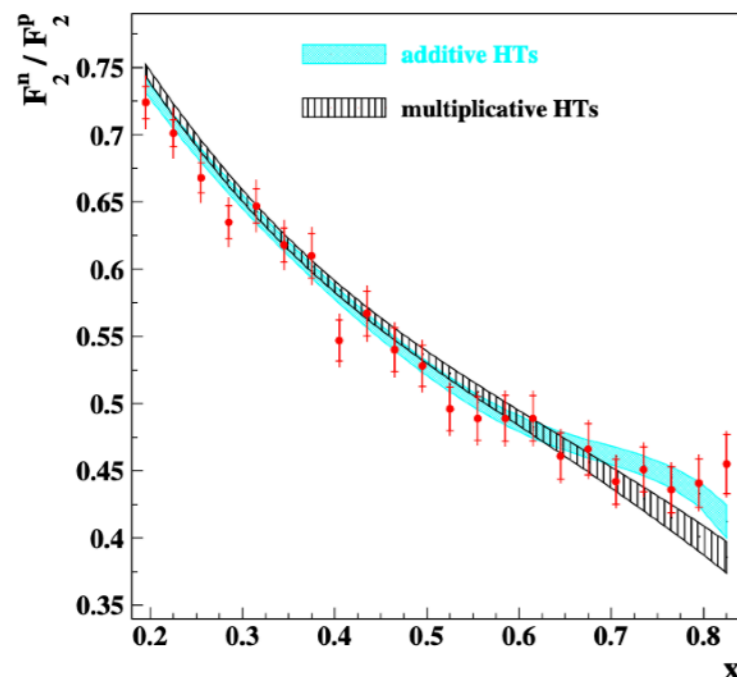
Alekhin, Kulagin, Petti, PRD 107 (2023)



Assume $p=n$ for HT



compatible $\delta f(x)$

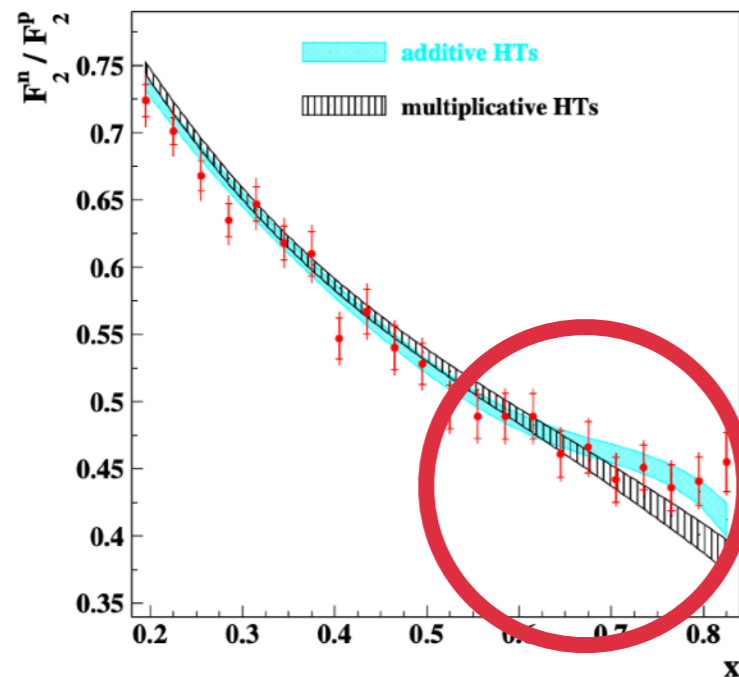
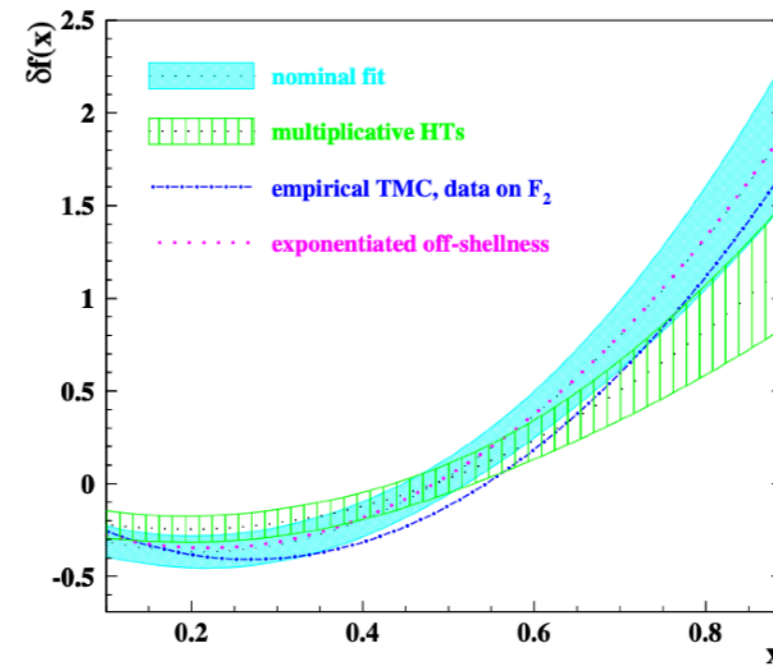
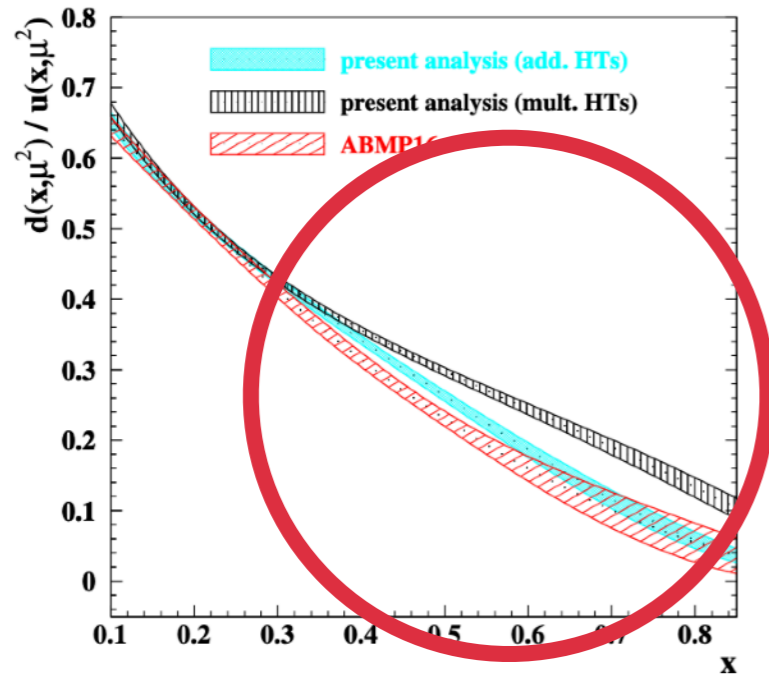


Artificially large n/p
BUT smaller d/u than Mult

Comparison to other extractions

AKP

Alekhin, Kulagin, Petti, PRD 105 (2022)
Alekhin, Kulagin, Petti, PRD 107 (2023)



**Systematic
“implementation” uncertainty**

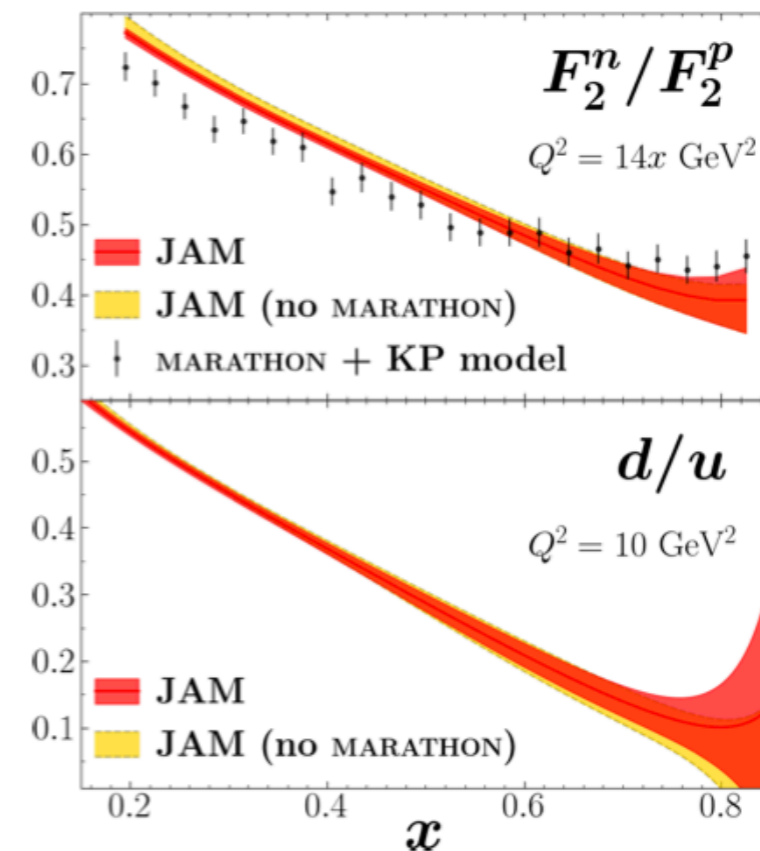
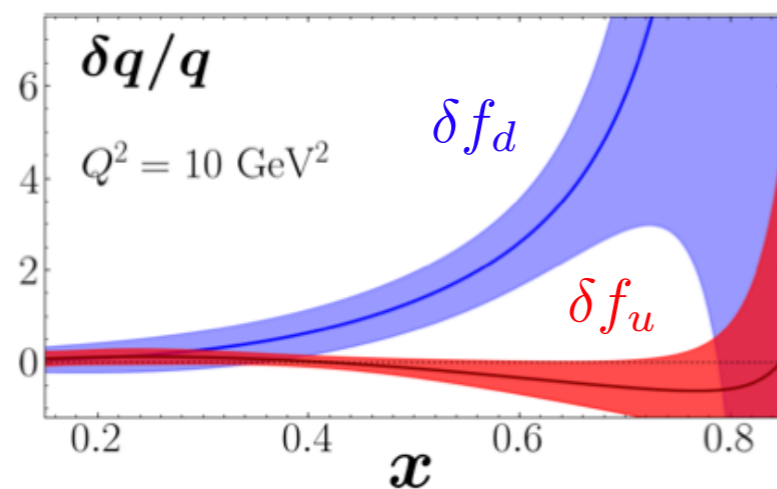
appears in both $\frac{n}{p}$ and $\frac{d}{u}$

JAM results

JAM Fit including $A=3$ data δf_u δf_d

JAM Collaboration, PRL 127 (2021)

Mult HT (p=n) as default choice

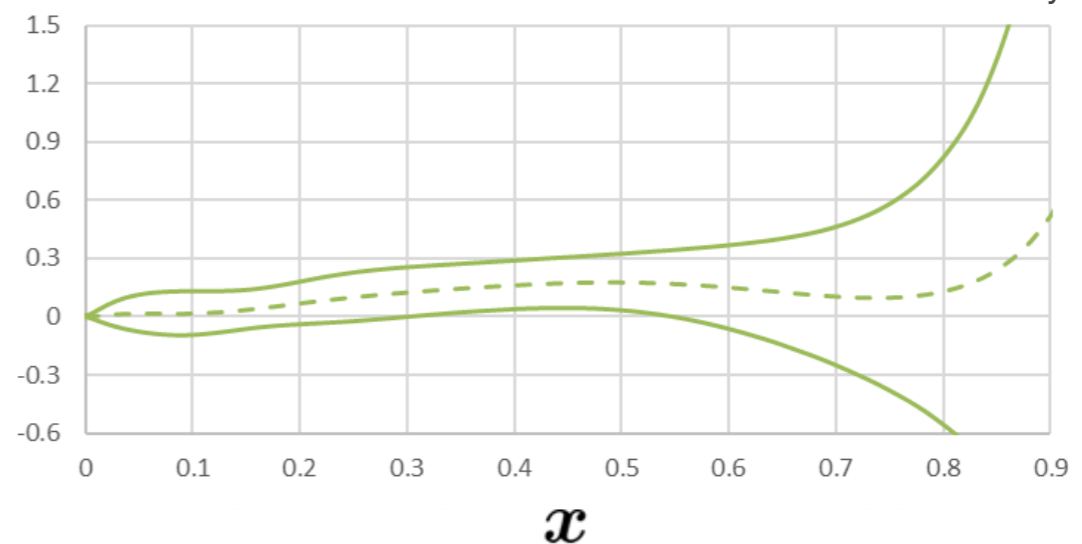


$$\delta f(x)|_{\text{CJ-like}} = \frac{u\delta f_u + d\delta f_d}{u + d}$$

Compatible with CJ results

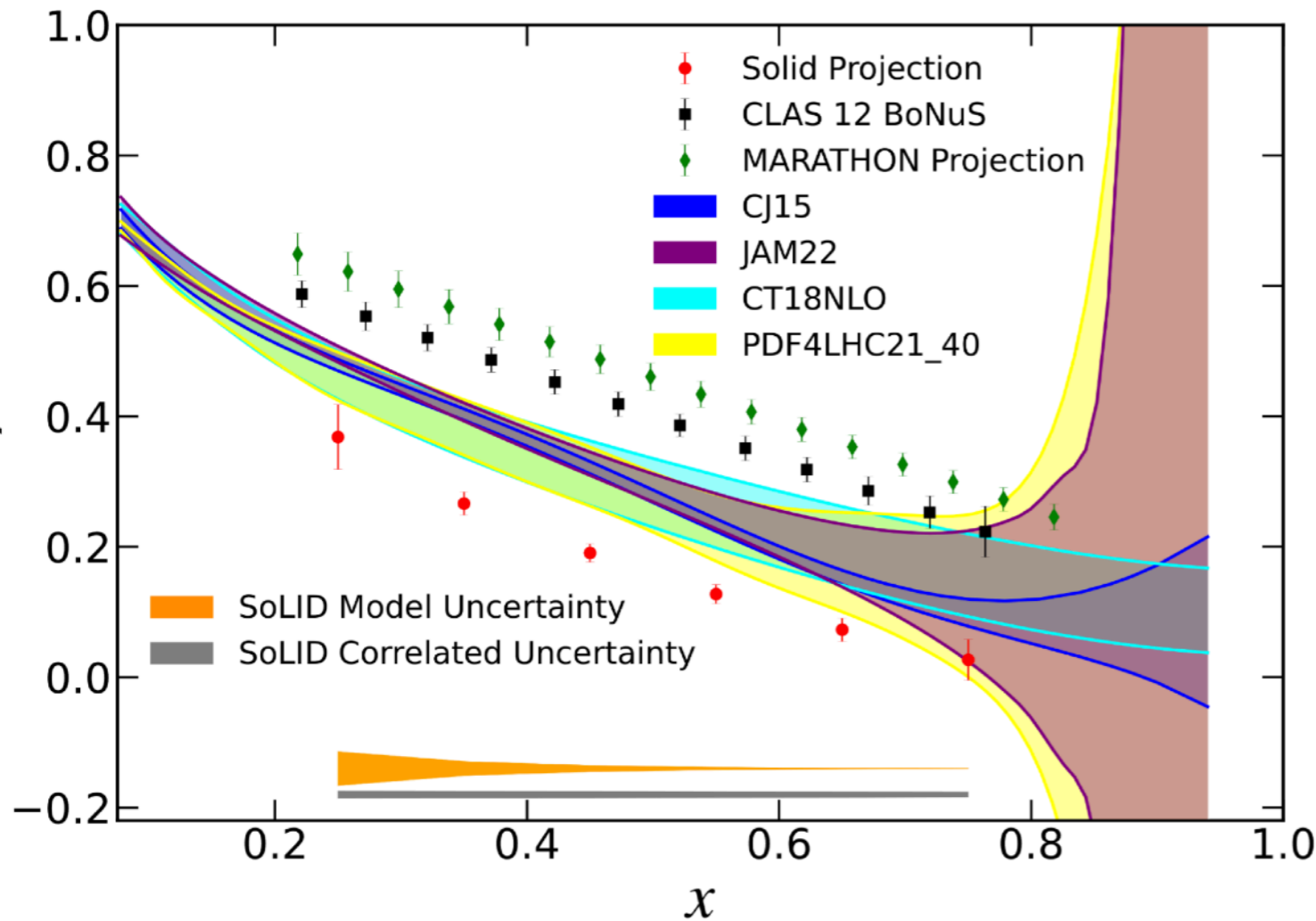
Isoscalar offshell function (JAM)

Courtesy of C. Cocuzza



Constraints on d/u from JLab 12GeV

New experimental data in the large- x region are needed to understand the correct interconnection of d/u , n/p ratios and off-shell corrections



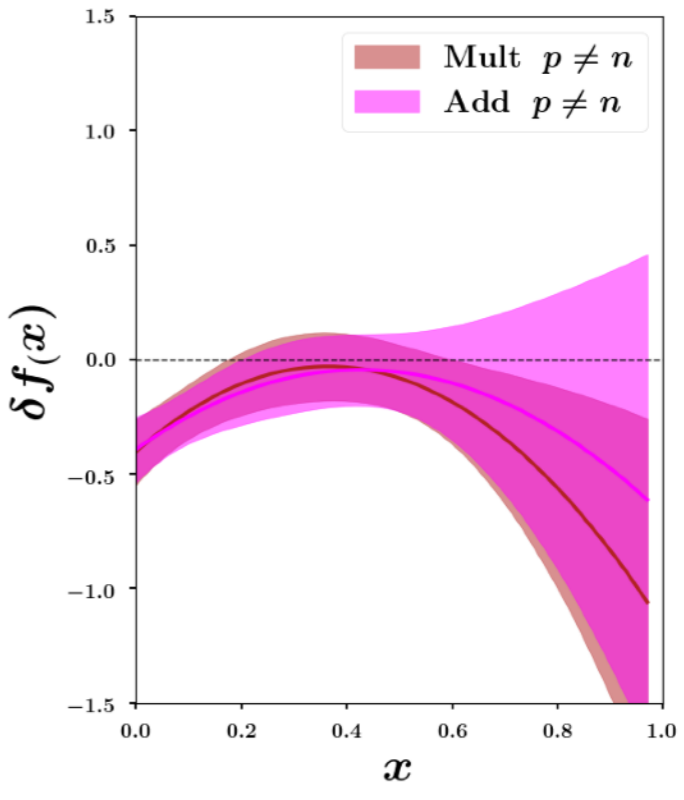
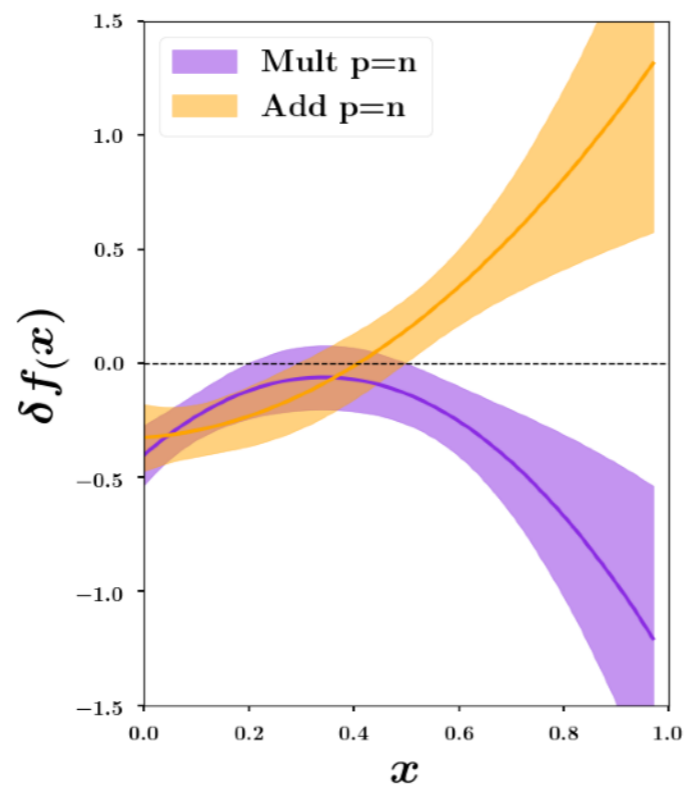
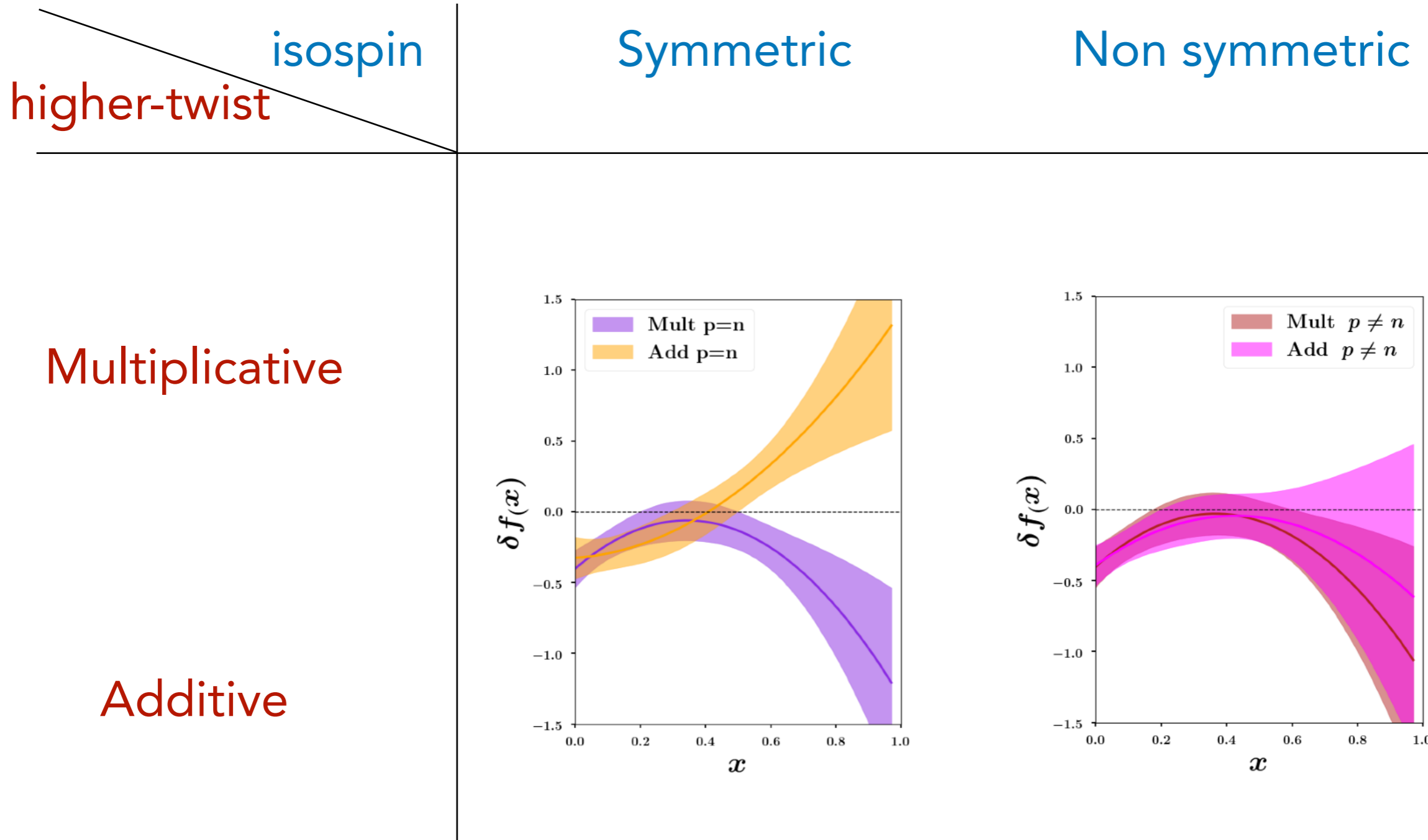
- Model dependent approach:
High precision measurements with deuterium target
- Less model dependent approaches:
3H/3He ratio (MARATHON) - results published (Phys.Rev.Lett. 128 (2022) 13, 132003)
Spectator tagging (BoNus) - new data taken in 2020
- Model independent approach:
Future PVDIS on proton (SoLID)

Summary

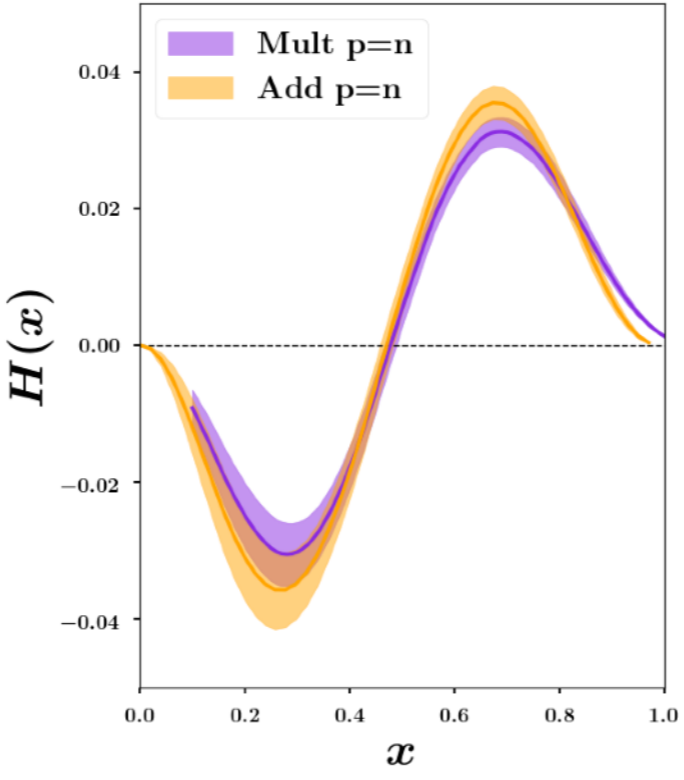
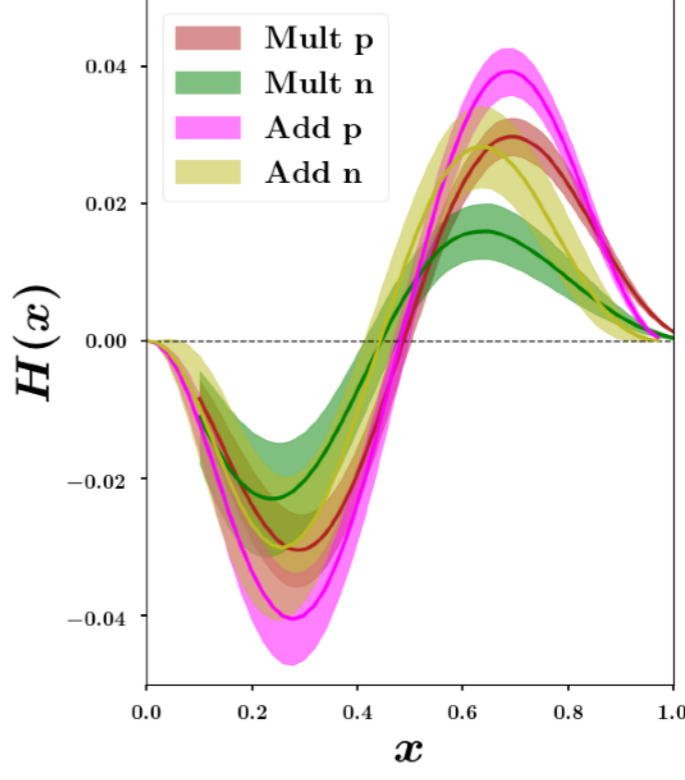
- Implementation of HT correction and its correlation with off-shell correction introduces a bias in extraction of d/u PDFs and structure function calculations
- The bias and systematic uncertainty can be significantly reduced by releasing the assumption of $p=n$ for HT
- Resulting systematic uncertainty needs to be considered. How to properly take into account the systematic uncertainty?
- New experimental data will provide better understanding on d/u, n/p and off-shell corrections as well as their interconnection

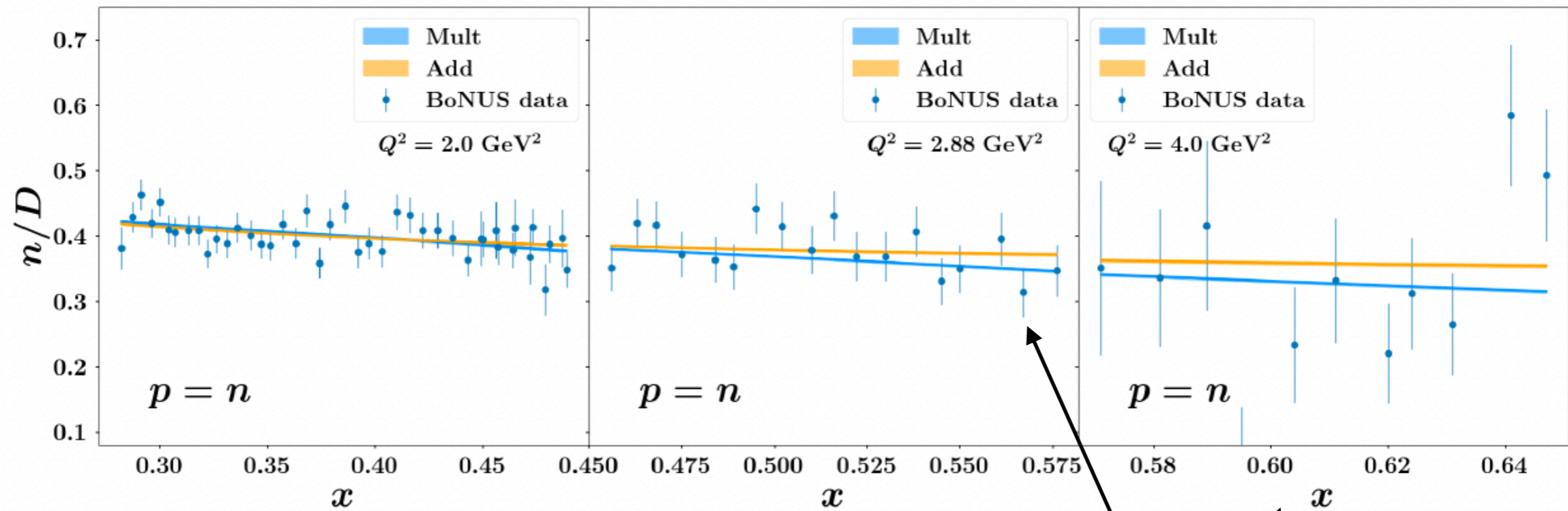
Backup

Off-shell table



Higher-Twist table

	isospin	Symmetric	Non symmetric
higher-twist			
Multiplicative		$\tilde{H} = F_{2,N}(x, Q^2) H_{\text{Mult}}(x)$ 	$\delta\tilde{H} = F_{2,N}(x, Q^2) \delta H_{\text{Mult}}(x)$ 
Additive			

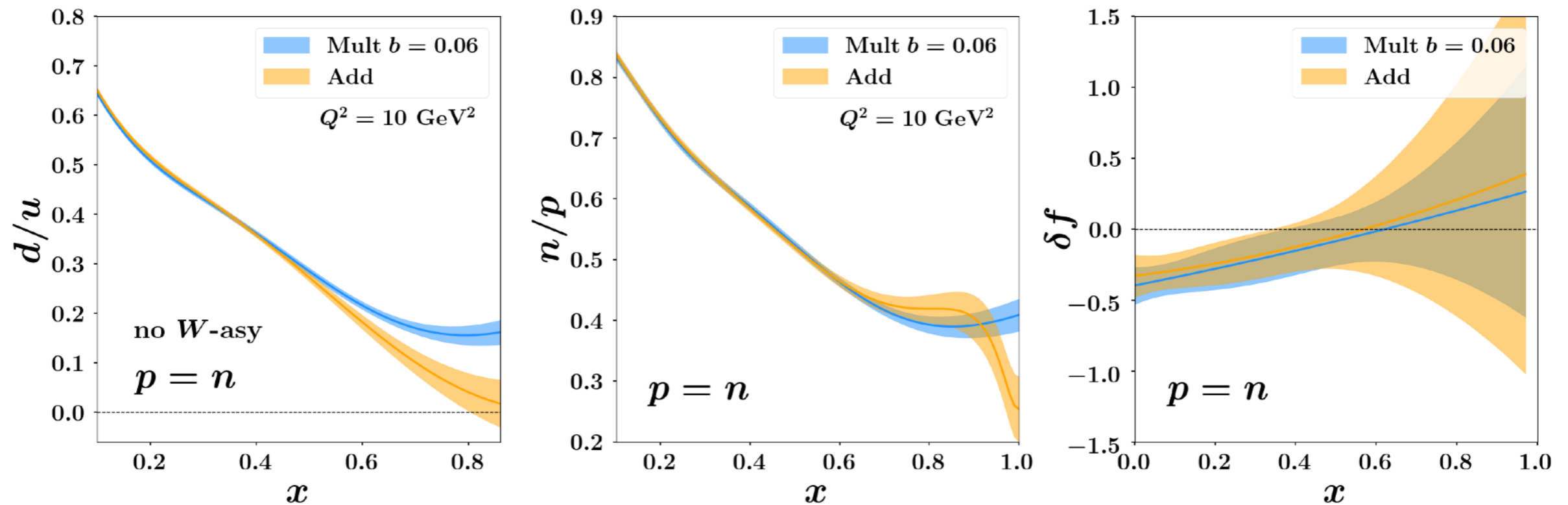


- New precision data could provide a better look on the two implementations

Other results

Case 1: isospin-symmetric HT

d/u artificially increased



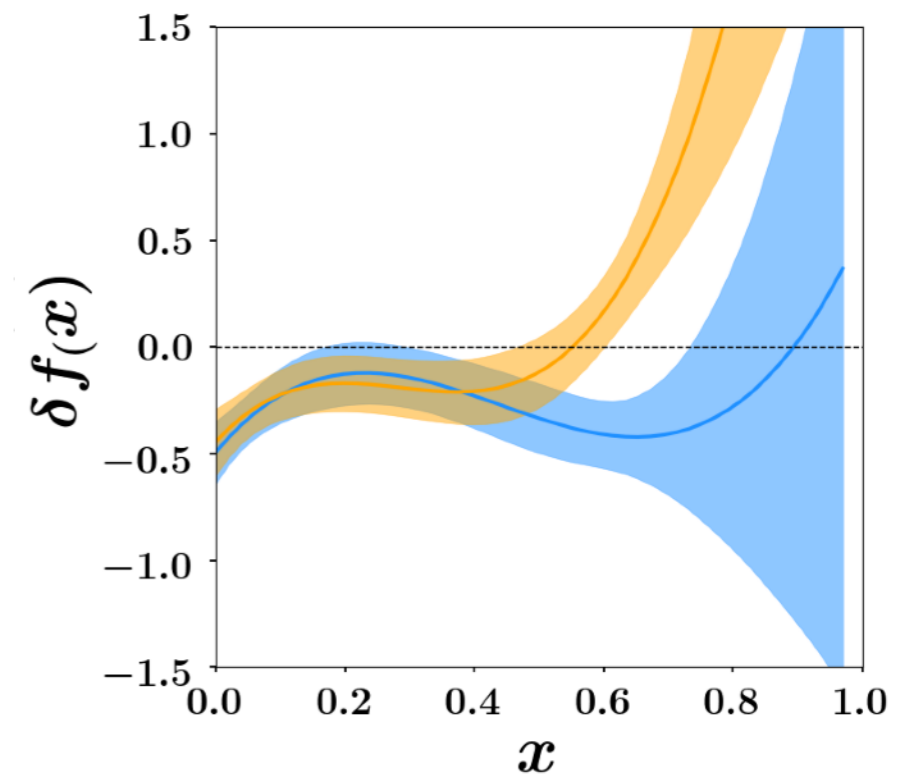
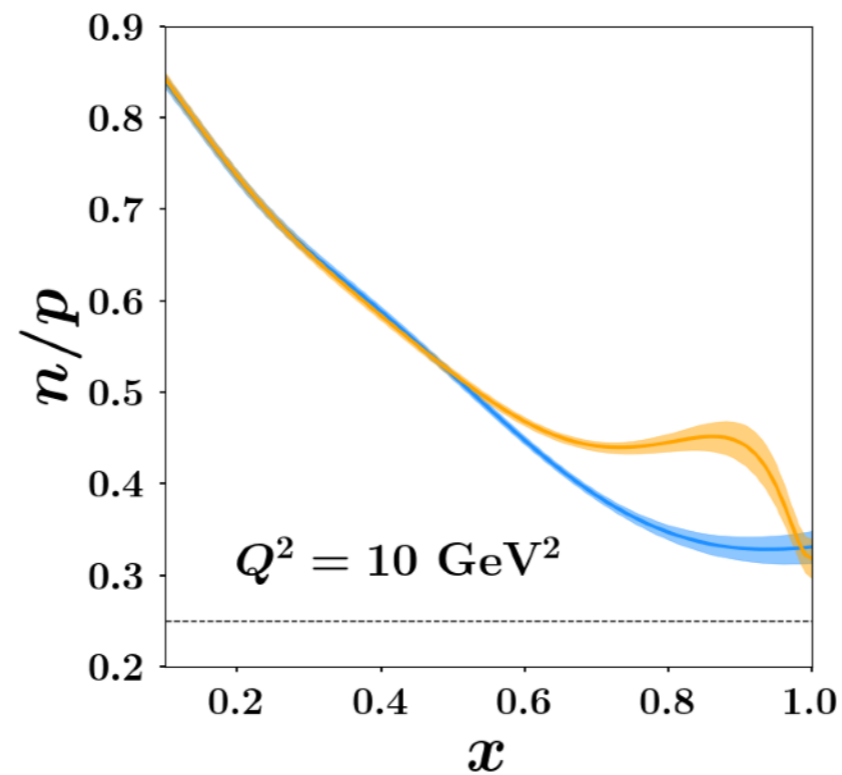
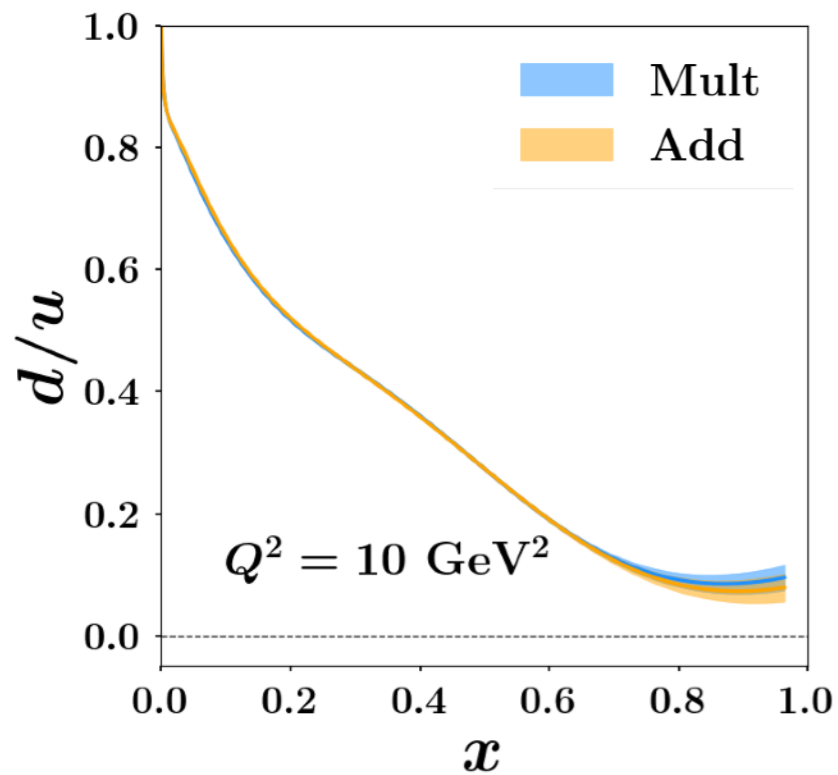
Higher d/u to absorb bias

Disfavored by DIS on deuteron and W-boson asymmetry data

Other results

Case 1: isospin-symmetric HT

Off-shell function: polynomial of 3rd degree



Similar to nominal result

Some implementation differences

Theoretical choices \longrightarrow

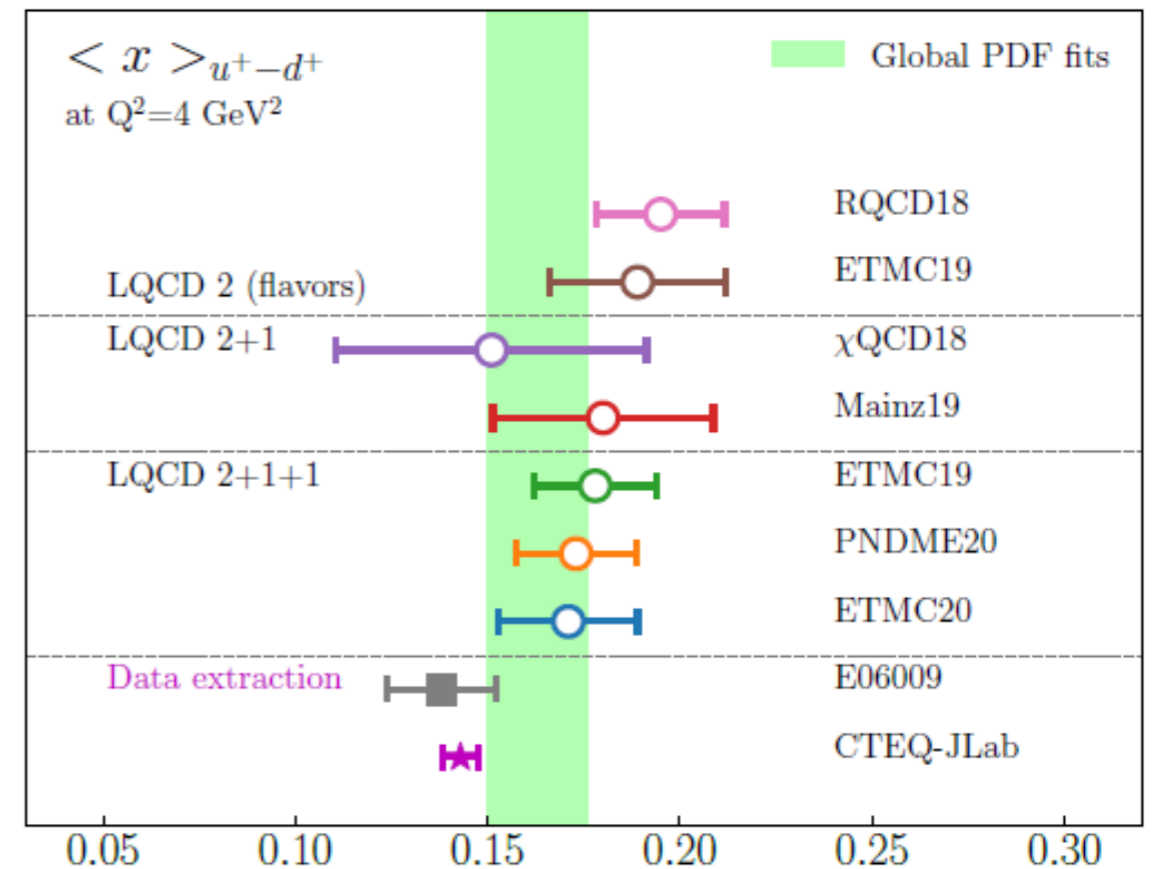
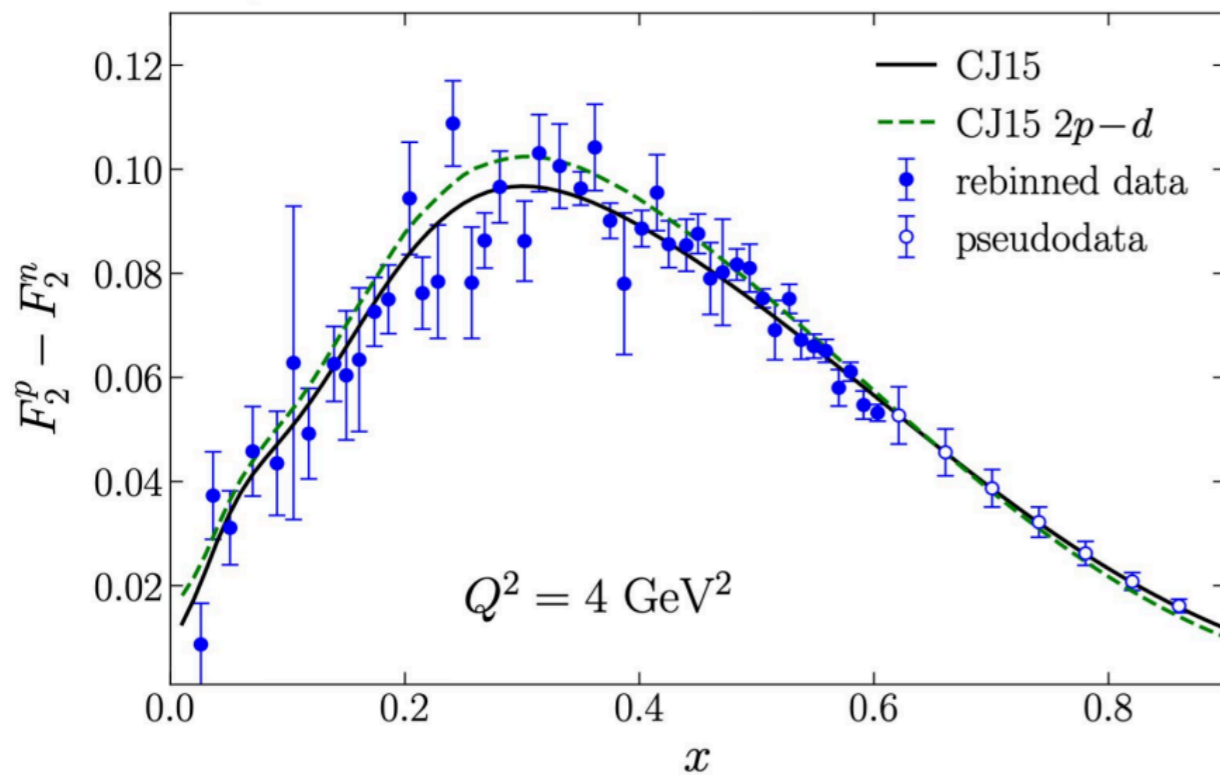
Corrections (increasing-x) \downarrow

	KP	AKP	CJ15	AKP-like
shadowing	yes	yes (which one?)	MST $x < 0.1$	(same)
smearing	Paris	AV18	AV18 $x > 0.1$	(same)
pi-cloud	yes	yes	----	----
TMC	GP O(Q4)?	GP O(Q4)??	GP approx.	(same)
HT	H (p=n ??)	H (p=n)	C (p=n)	H & C, p=n & p!=n
HT(x)	??	5 pt. spline	parametrized	parametrized
off-shell	O(p2-M2)	O(p2-M2)	O(p2-M2)	(same)
df(x)	factorized	polyn. 2nd/3rd	factorized + sum rule	polyn. 2nd/3rd
pi thresh.	yes	yes	----	----

Application: non-singlet moments

$$M_2^{p-n}(Q^2) = \int_0^1 dx \frac{\xi^3}{x^3} \left[\frac{3 + 9r + 8r^2}{20} \right] F_2^{p-n}(x, Q^2)$$

$$\frac{3}{C_2} M_2^{p-n} = \langle x \rangle_{u+-d+} + \text{HT}$$



Li, Accardi, MC, Fernando et al., PRD 109 (2024)

- $x < 0.01$: Regge theory
- $0.01 < x < 0.6$: Exp. data
- $x > 0.6$: CJ15 model

$$\langle x \rangle_{u+-d+} = \int_0^1 dx x [u(x) + \bar{u}(x) - d(x) - \bar{d}(x)]$$