# Pixelization of Quantum Correlation Functions

MARCO ZACCHEDDU – JEFFERSON LAB

IN COLLABORATION WITH: A.FREESE, J. QIU, N. SATO AND Z. YU

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- Pixelization of Quantum Correlation Functions
  - Main idea, features and advantages
- Extraction of GPDs from Compton Form Factors

# Pixelization of Quantum Correlation Functions

In pQCD, Quantum Correlation Functions are necessary to understand the hadron structure and fragmentation mechanism

QCFs:

| | |
|---|---|
| Collinear PDF/FF | $f_1(x;Q^2)$ |
| Transverse momentum dependent (TMD) PDF/FF | $\tilde{f}_1(x,b_T;Q^2)$<br>$\tilde{D}_1(z,b_T;Q^2)$ |
| Generalized PDF (GPDs) | $H(x,\xi,t;Q^2)$ |

# Pixelization of Quantum Correlation Functions

In pQCD, Quantum Correlation Functions are necessary to understand the hadron structure and fragmentation mechanism

Standard approach: extract parameters

QCFs:

Parametrization

| | | Parametrization |
| --- | --- | --- |
| Collinear PDF/FF | $f_1(x;Q^2)$ | $x^\alpha(1-x)^\beta$ |
| Transverse momentum dependent (TMD) PDF/FF | $\tilde{f}_1(x,b_T;Q^2)$<br>$\tilde{D}_1(z,b_T;Q^2)$ | $f(x)e^{-b_T^2/w_f}$<br>$D(z)e^{-b_T^2/w_D}$ |
| Generalized PDF (GPDs) | $H(x,\xi,t;Q^2)$ | $GK(\alpha,\beta,t)$ |

# Pixelization of Quantum Correlation Functions

NEW APPROACH:

- QCFs as D-dimensional ‘’pictures’’ (or tensors)
- Discretize using a grid
- Fit/tune each pixels of the grid

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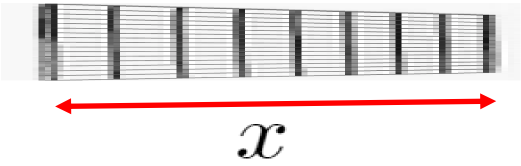
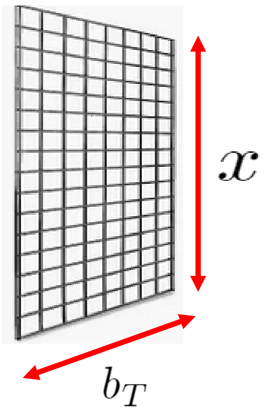
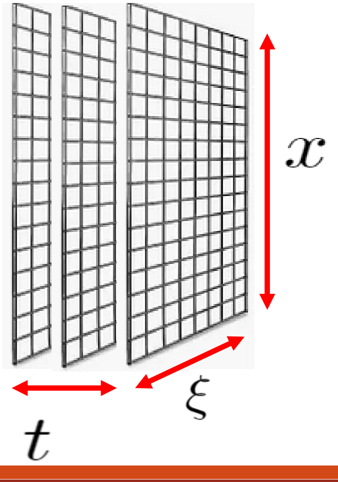
$$f_1(x; Q^2)$$




$$\tilde{f}_1(x, b_T; Q^2)$$
$$\tilde{D}_1(z, b_T; Q^2)$$




$$H(x, \xi, t; Q^2)$$



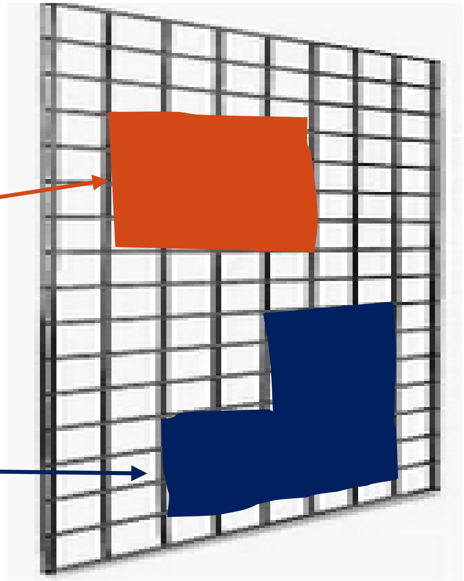
# Pixelization of Quantum Correlation Functions

- Very flexible approach → We need to use constraints to have a physical object
- All operations are performed using tensor multiplication.
  - Evolution equations $$\frac{\mathrm{d}H_i(\xi,Q^2)}{\mathrm{d}\log(Q^2)} = \sum_{j=1}^{N_x} K_{ij}(\xi,Q^2)H_j(\xi,Q^2)$$
- Leverage PyTorch/GPU parallelization to accelerate computations

We can calculate the variation of the $\mathrm{X}^2$ w.r.t. the variation of each single pixel

$$\frac{\Delta\chi^2}{\Delta\text{pixel}} \begin{cases} = 0 \\ \neq 0 \end{cases}$$

no sensitivity



# GPDs Extraction from Compton Form Factors

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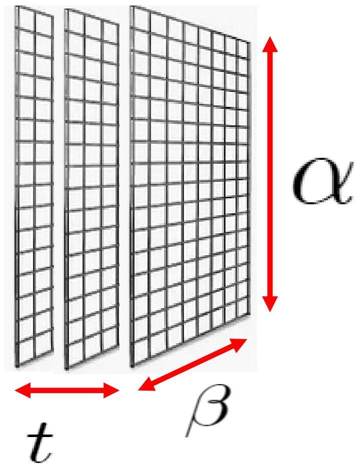
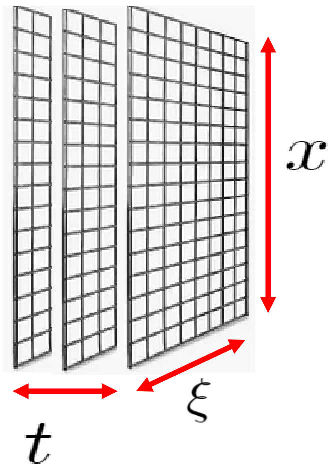
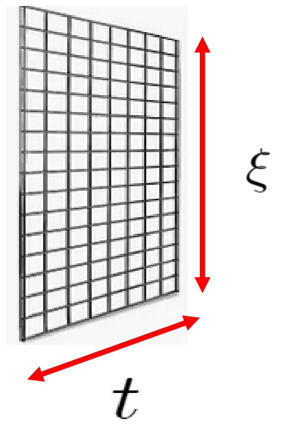
## Double Distribution

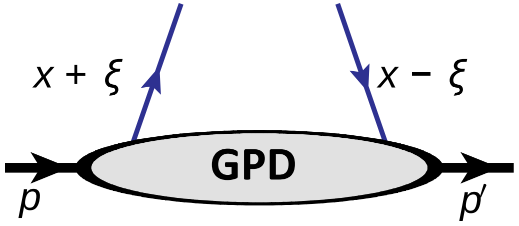



Evolution

$$\frac{dH_i(\xi,Q^2)}{d\log(Q^2)} = \sum_{j=1}^{N_x} K_{ij}(\xi,Q^2) H_j(\xi,Q^2)$$




$f(\beta,\alpha,t)$

DD

$H(x,\xi,t)$

GPD

$\mathcal{H}(\xi,t)$

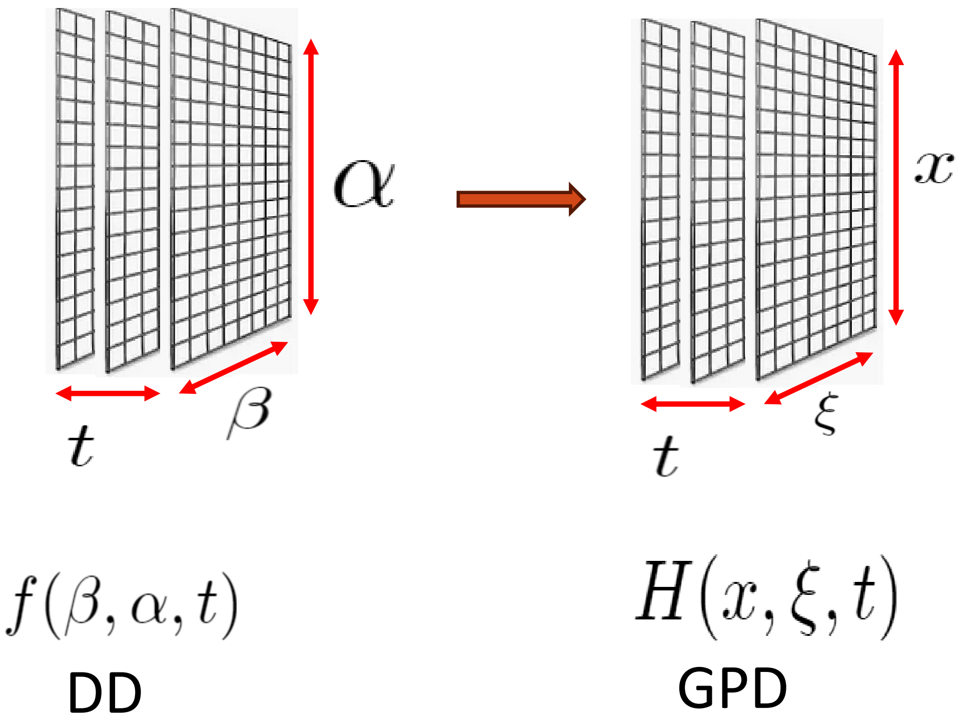
CFF

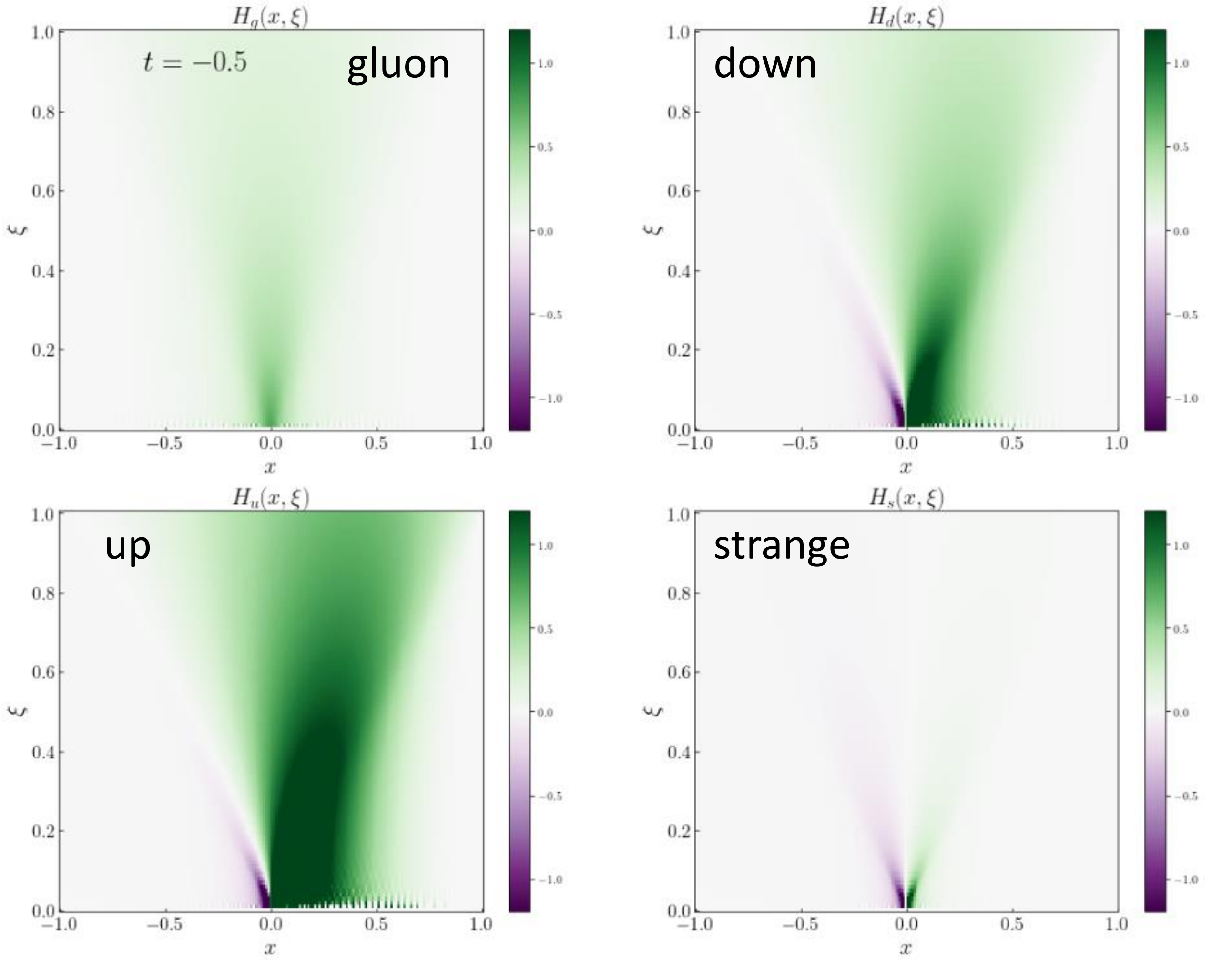
$$\mathcal{H}(\xi,t) = \int_{-1}^{+1} dx\, C^{(-)}(x,\xi)\, H(x,\xi,t)$$

$$H(x,\xi,t) = \int d\beta d\alpha \delta(x-\beta-\xi\alpha) f(\beta,\alpha,t)$$

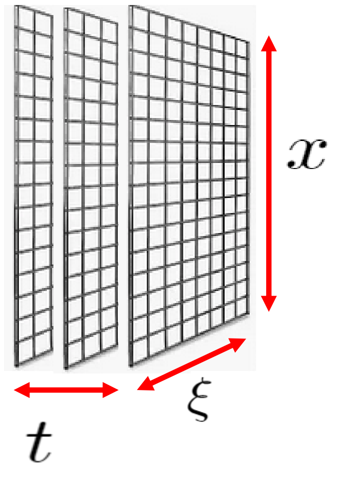
# GPDs Extraction from Compton Form Factors

Double distribution: Goloskokov – Kroll model: arxiv.org/pdf/1210.6975

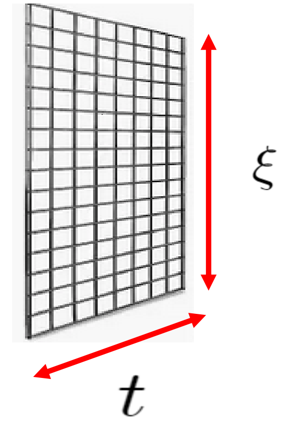


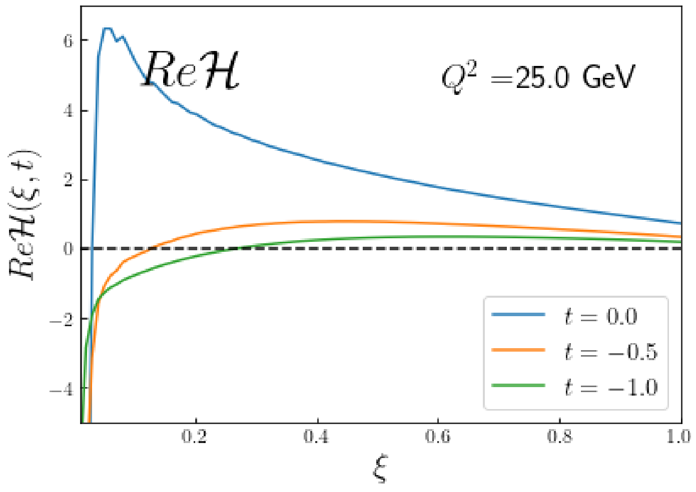
# GPDs Extraction from Compton Form Factors



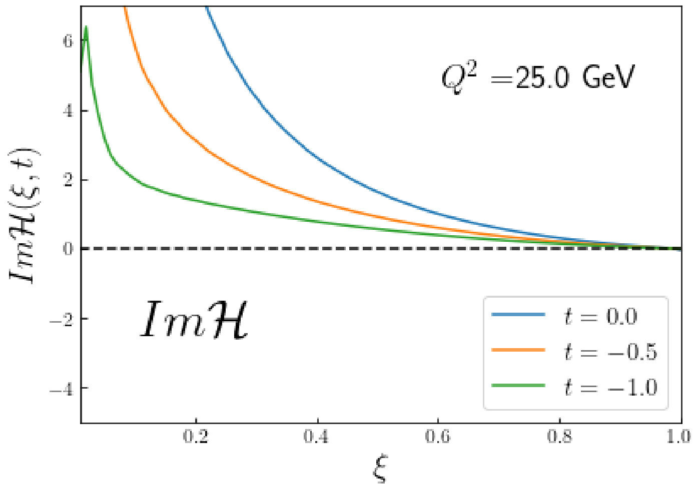

$$H(x, \xi, t)$$




$$\mathcal{H}(\xi, t)$$



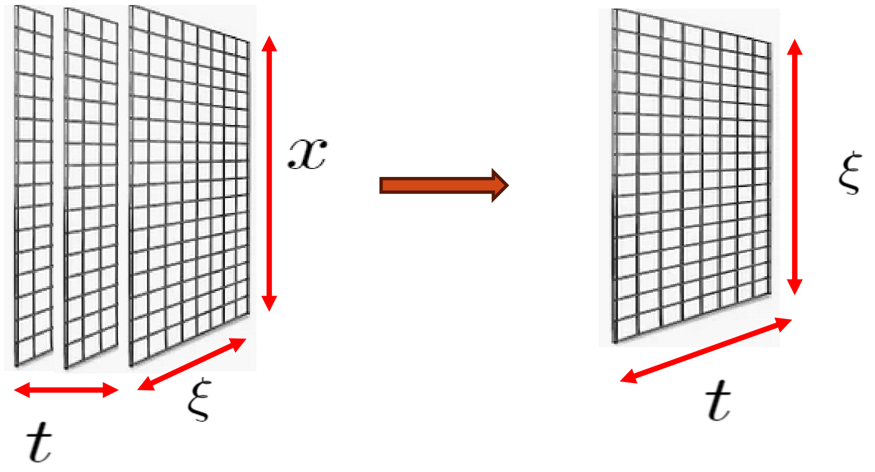

$$Re\,\mathcal{H}(\xi,t) = e_q^2\,\mathcal{P}\int_{-1}^{+1} dx\,\frac{H^+(x,\xi,t)}{\xi - x}$$



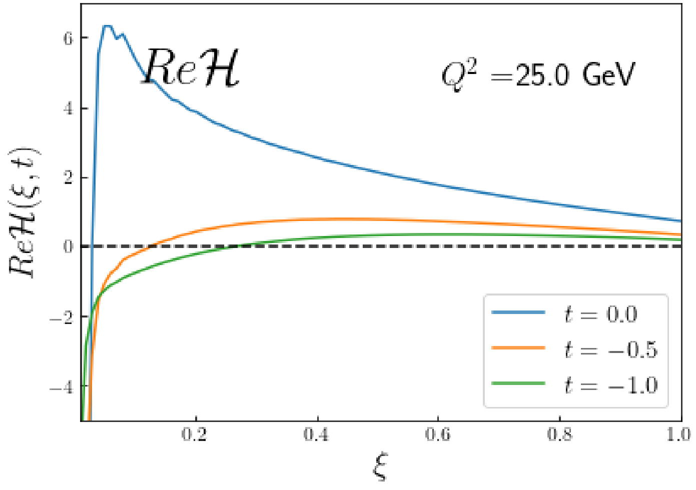

$$Im\,\mathcal{H}(\xi,t) = e_q^2 \pi H^+(\xi,\xi,t)$$

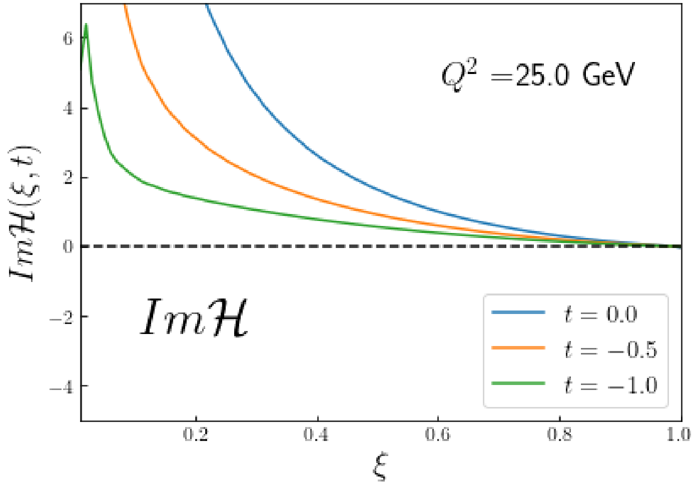
$$H^+(\xi,\xi,t) = H(\xi,\xi,t) - H(-\xi,\xi,t)$$

# GPDs Extraction from Compton Form Factors



$H(x, \xi, t)$ $\qquad\qquad$ $\mathcal{H}(\xi, t)$



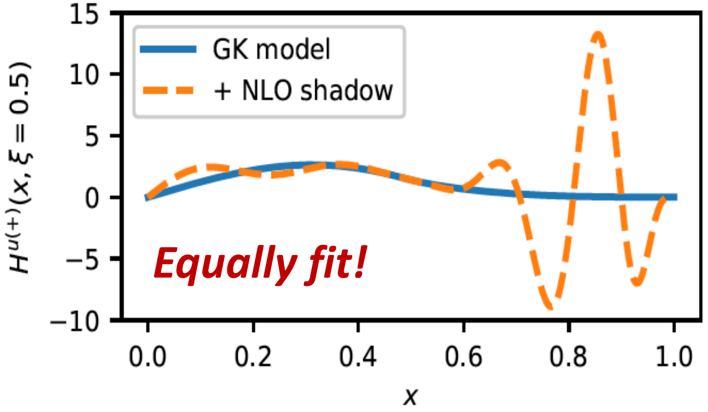




$$Re\,\mathcal{H}(\xi,t) = e_q^2\,\mathcal{P}\int_{-1}^{+1} dx\,\frac{H^+(x,\xi,t)}{\xi - x}$$

$$Im\,\mathcal{H}(\xi,t) = e_q^2 \pi H^+(\xi,\xi,t)$$

$$H^+(\xi,\xi,t) = H(\xi,\xi,t) - H(-\xi,\xi,t)$$

Inversion Problem: CFFs depend only on $\xi$.
We cannot recover the x dependence of GPDs
''Shadow GPDs''

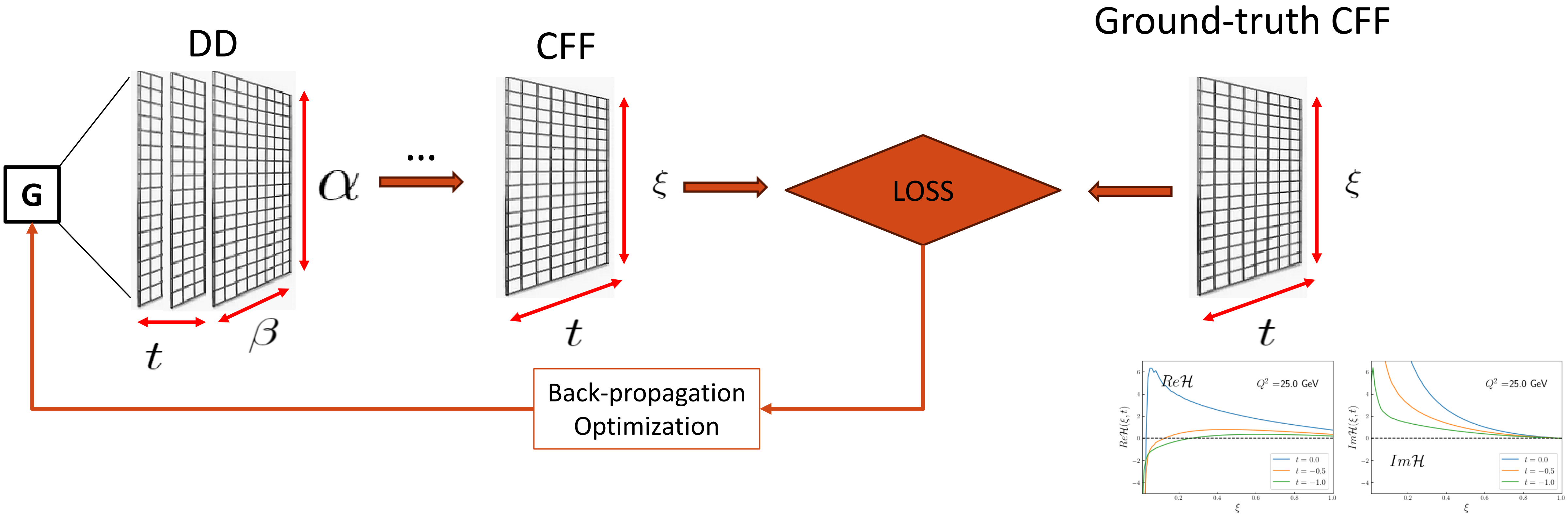



[Bertone et al. PRD `21]

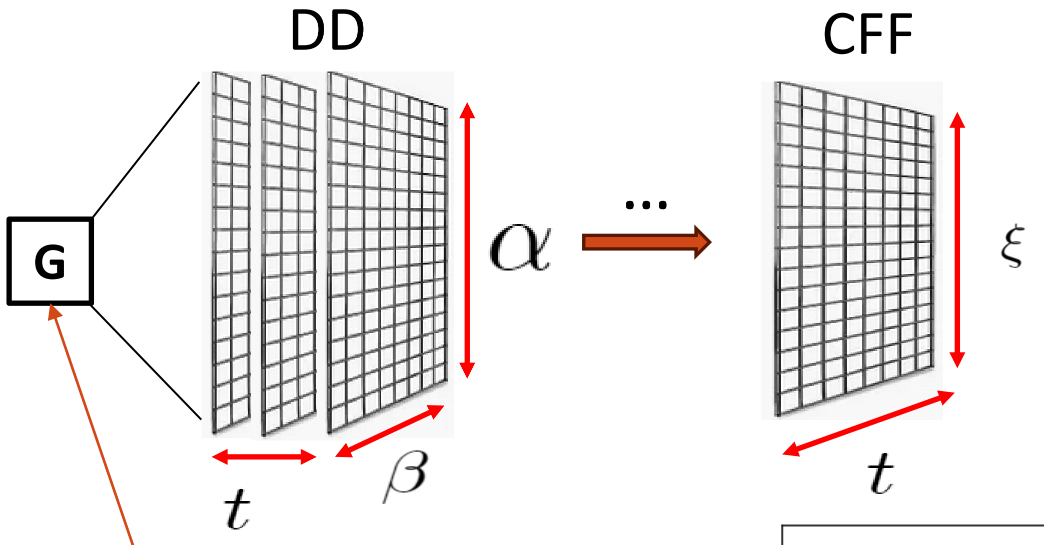
# GPDs Extraction from Compton Form Factors

The exercise consists of trying to reconstruct GPDs by analyzing CFFs.

Construct GPDs from the most flexible pixelization method

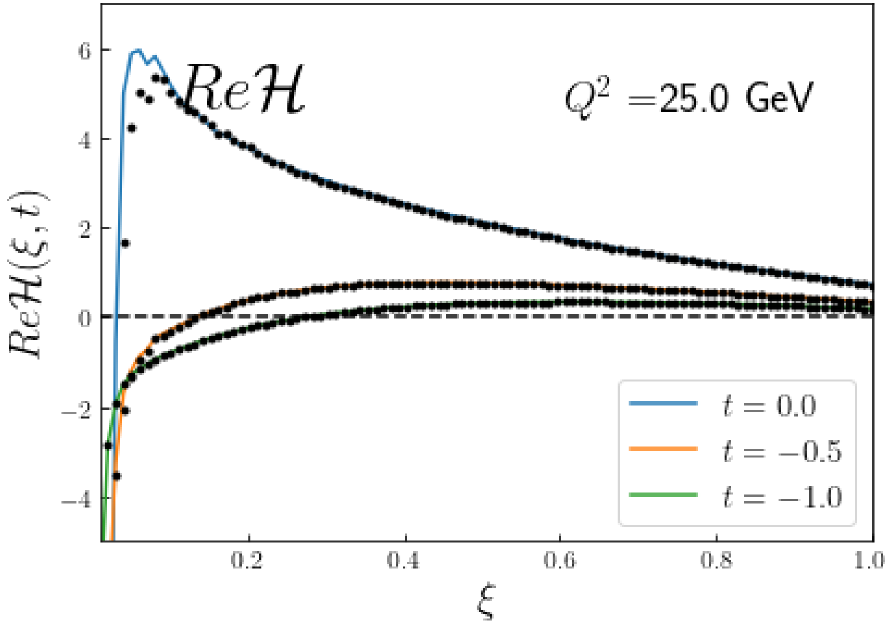


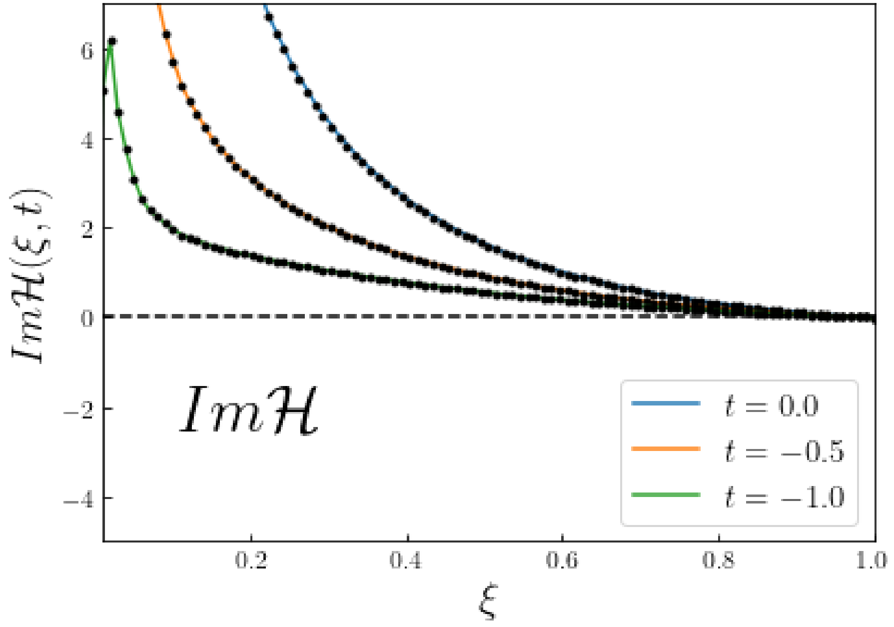
# GPDs Extraction from Compton Form Factors




Trained Generator

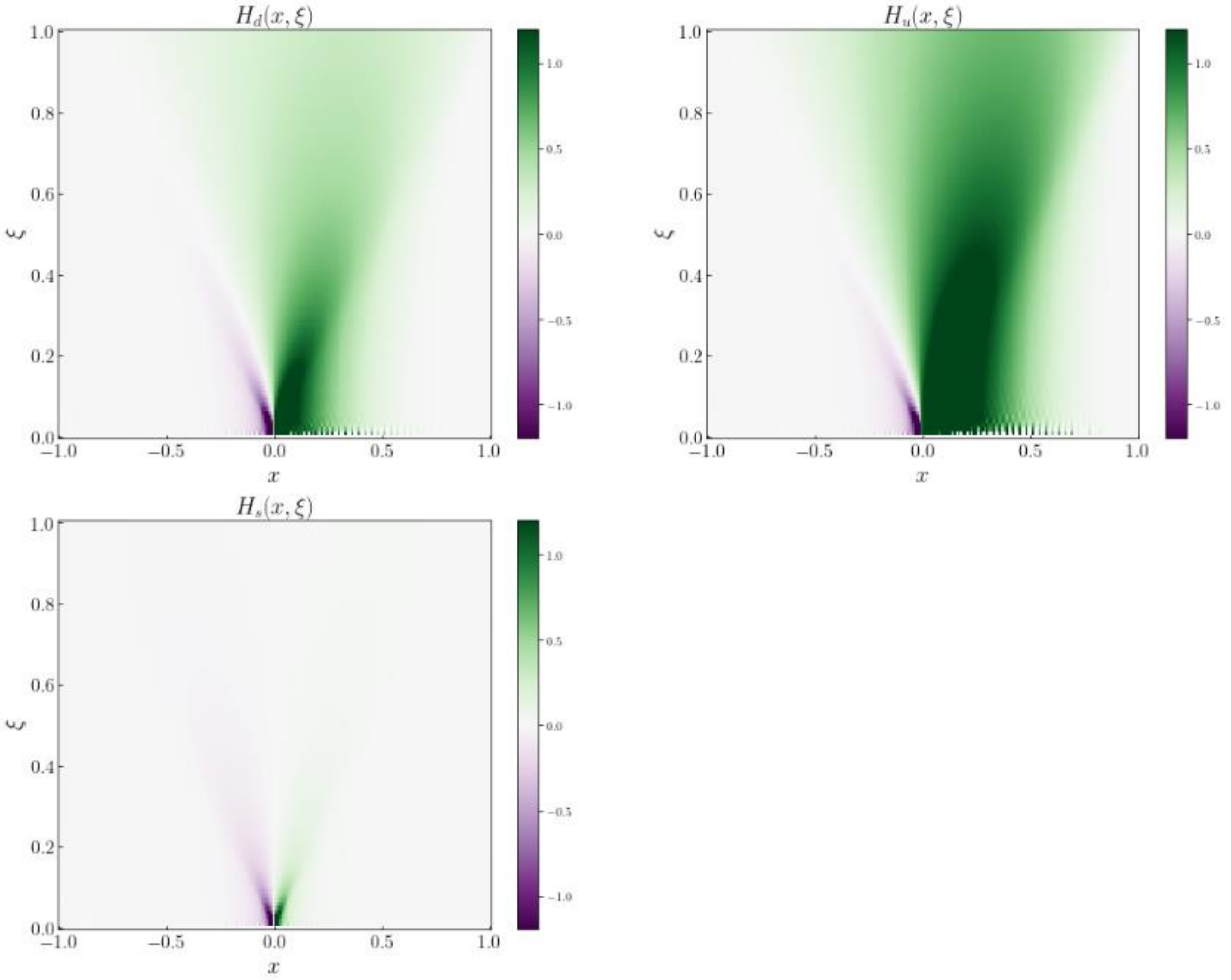
It is possible to reconstruct the CFFs.



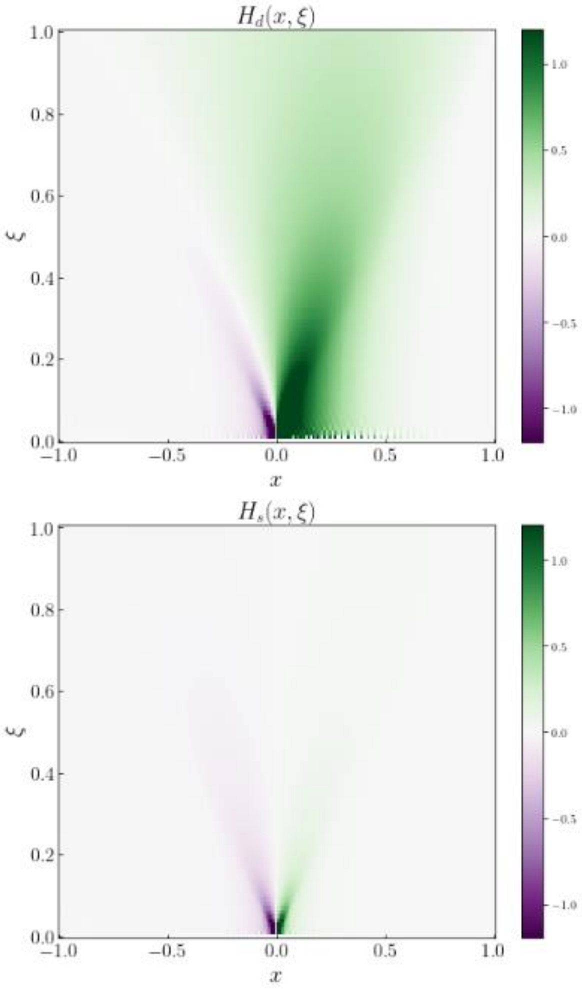
# GPDs Extraction from Compton Form Factors

## Ground-truth GPDs

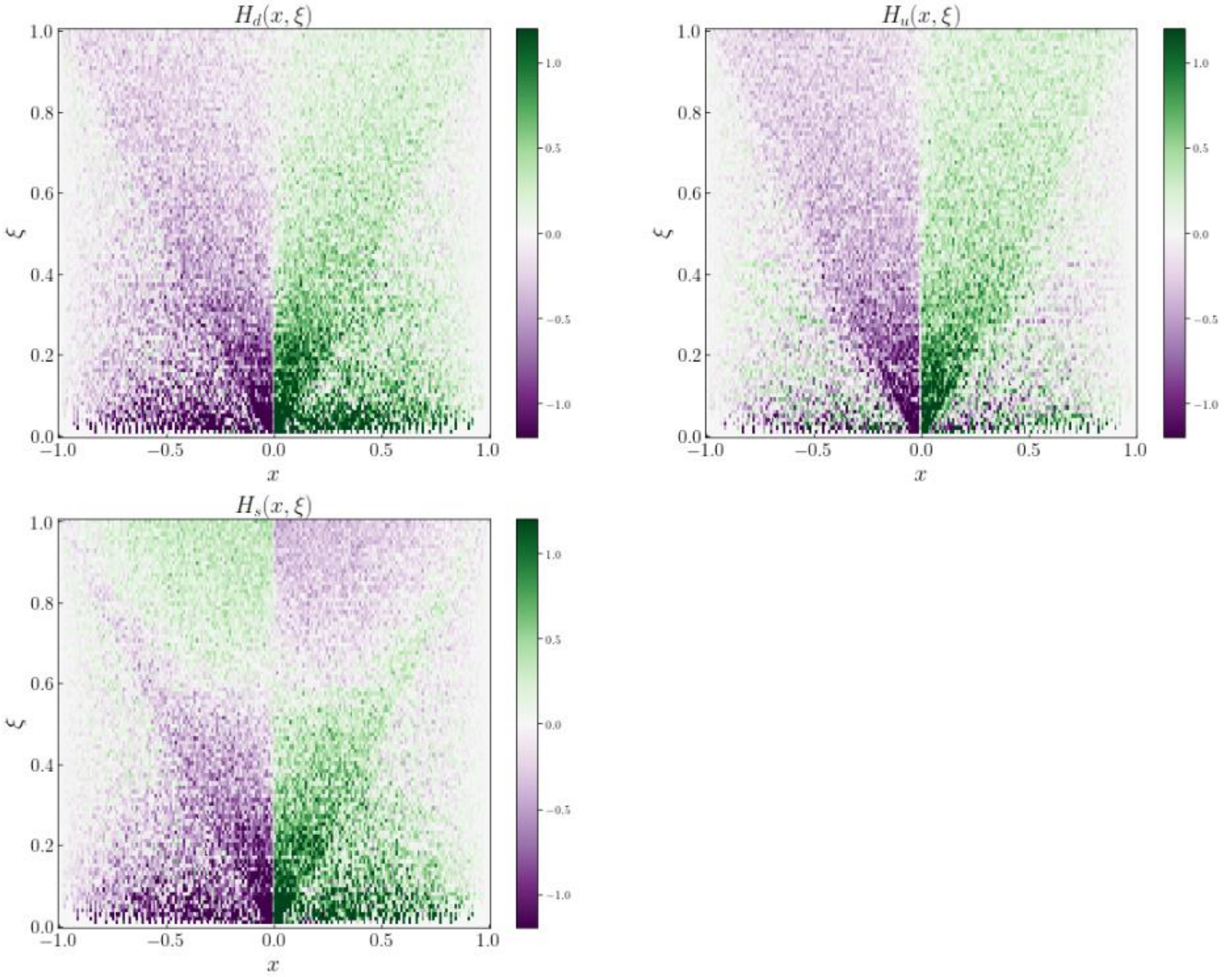


# GPDs Extraction from Compton Form Factors

## Ground-truth GPDs




## Extracted GPDs

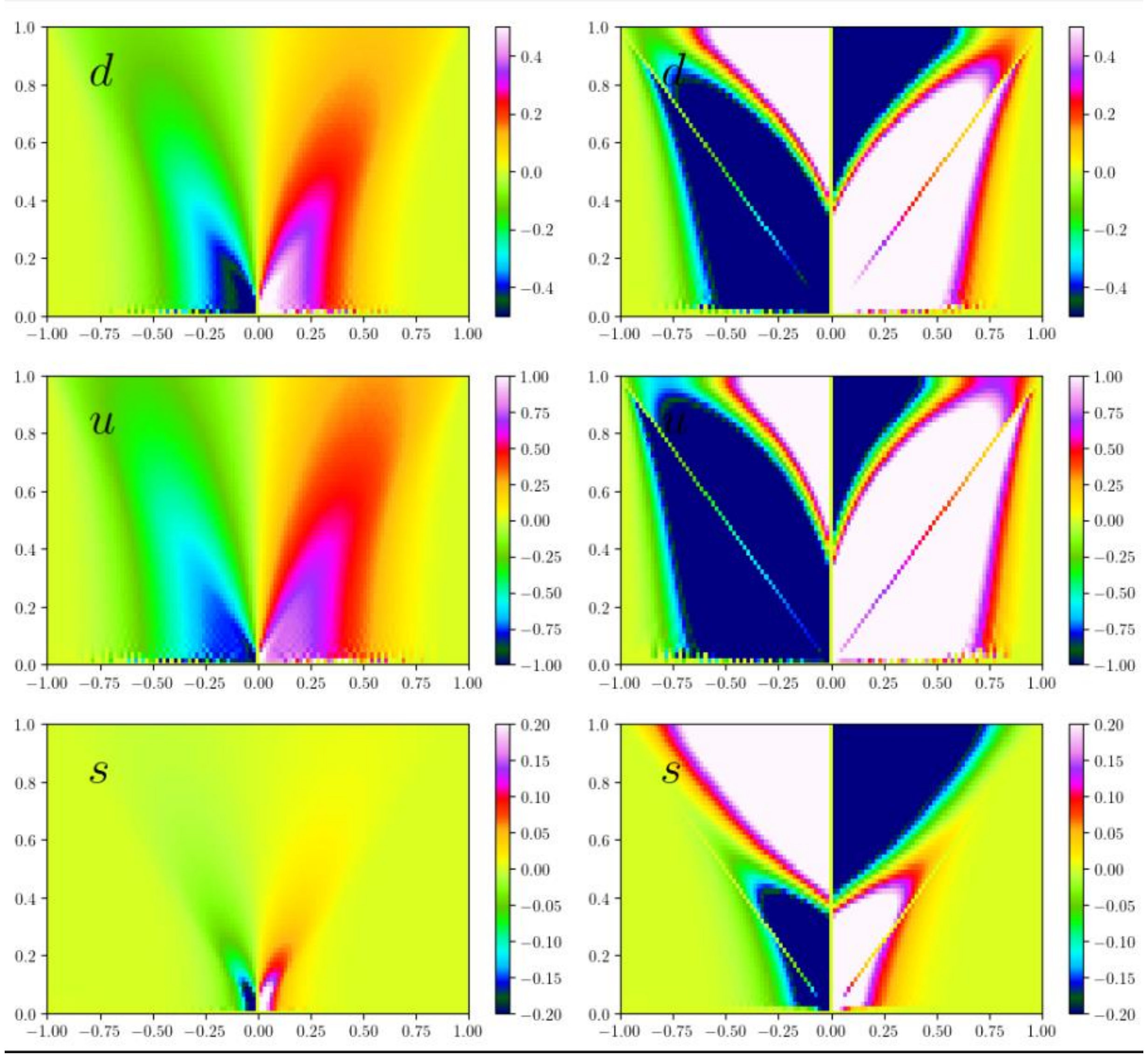


# GPDs Extraction from Compton Form Factors

The CFF is defined as the integral of:

$$H^{+}(x, \xi, t)$$

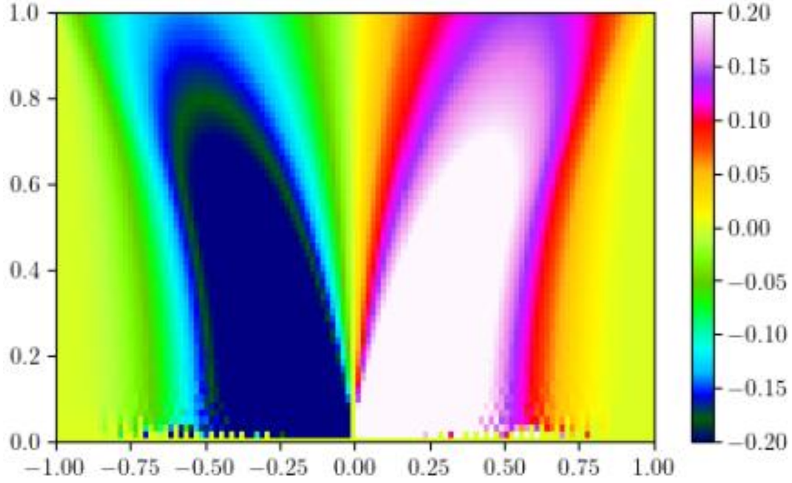
and there is a sum over all flavors.



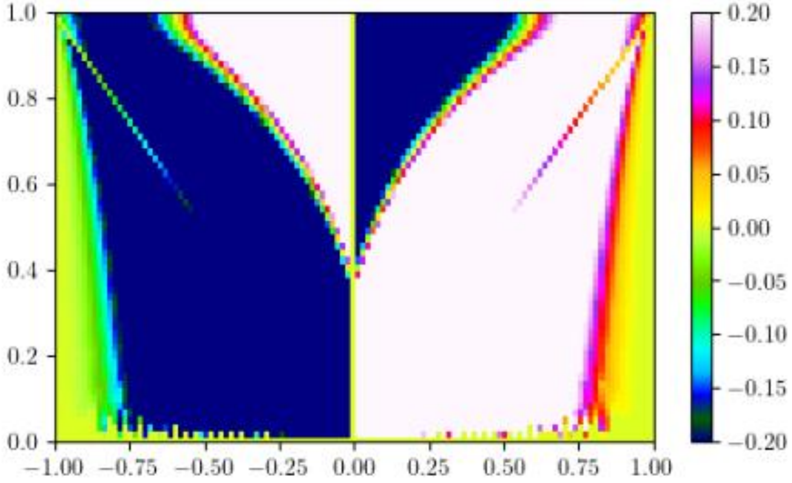
# GPDs Extraction from Compton Form Factors

$$\sum_q e_q^2 H_q^+(x, \xi, t)$$

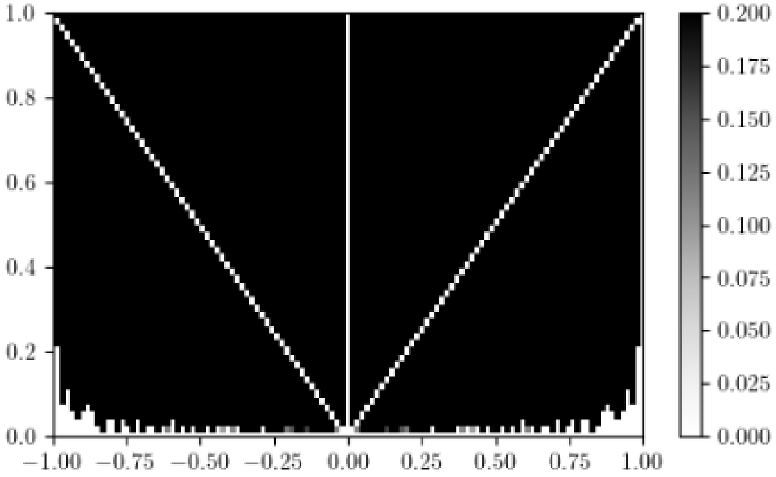
Ground-truth GPD




Extracted GPD




$$|(H^+_{true} - H^+_{extracted})/H^+_{true}| * 100$$




As expected, we can only perfectly reconstruct pixels when $x = \xi$

# Conclusions and Outlook

- Introduced a novel approach for extracting QCFs.
- The approach allows for a highly flexible parametrization.
- Shown how to visualize various QCFs as images or tensors.
- Examples and applications for extracting GPDs.
- Reconstruction of GPDs from CFFs and the associated challenges.
- Future developments: applications for TMDs.