Generalised Parton Distributions

DPS Workshop 2025

Aussois, France

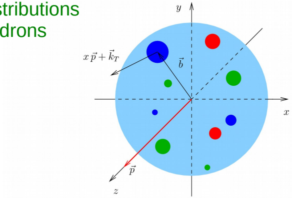
Saad Nabeebaccus

University of Manchester

January 12, 2025

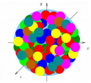
6D/5D

Wigner distributions for hadrons



$W(x,\vec{b},k_T)$

Experimentally inaccessible directly



perturbative Regge limit

3D

uPDFs (gluons)

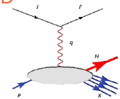
Unintegrated parton distributions

$\int d^3\vec{b}$

=

$\int d^2k_T \int d\,b_z$

$\int d^2k_T \int Fourier(\vec{b})$



TMDs $f(x,k_T)$

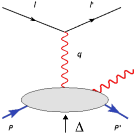
Transverse momentum dependent distributions

$f(x,b_T)$ $b_T \Leftrightarrow \Delta$ $\longleftrightarrow$ $H(x,0,t)$, $t=-\Delta^2$ $\longleftarrow$ $\xi=0$

Impact parameter distributions

GPDs $H(x,\xi,t)$

generalised parton distributions

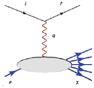


Semi-inclusive processes

exclusive processes

$\int d^2k_T$ $\int d^2b_T$ $t=0$ $\int dx$ $\int dx\, x^{n-1}$

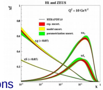
1D

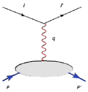


inclusive and semi-inclusive processes

PDFs $f(x)$

parton distributions

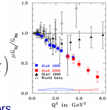




elastic processes

FFs $G_{E,M}(t)$

form factors

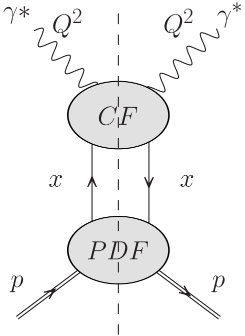


GFFs

generalized form factors

lattices

- Deep Inelastic Scattering DIS: inclusive process

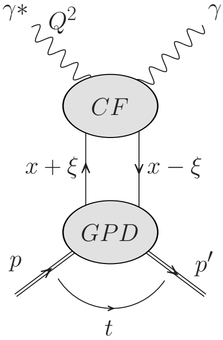


$\Rightarrow$ 1-dimensional structure

$\Rightarrow$ Collinear factorisation at the *cross section* level

$$\sigma = \text{Coefficient Function} \otimes \text{Parton Distribution Function}$$

DVCS: exclusive process (non forward amplitude)



Fourier transf.: $t \leftrightarrow$ impact parameter
$\Rightarrow$ 3-dimensional structure

Collinear factorisation at the amplitude level:

$$\mathcal{A} = \text{Coefficient Function} \otimes \text{Generalised Parton Distribution}$$

x: *Average* mom. fraction of the nucleon carried by the parton
ξ: Mom. fraction of the nucleon *transferred* to hard part

- Collinear factorisation proved (all-orders, leading twist):

  [X. Ji: hep-ph/9609381]

  [A. Radyushkin: hep-ph/9604317, hep-ph/9704207]

  [J. Collins, A. Freund: hep-ph/9801262]

  [D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, J. Horejsi: hep-ph/9812448]

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  [V. Braun, Y. Ji, A. Manashov, S. Moch, J. Schoenleber: 2007.06348, 2106.01437, 2310.05724]

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- Also, *collinear factorisation at twist-3* recently proved using SCET:
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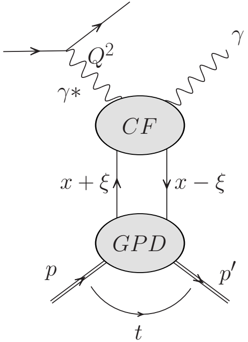
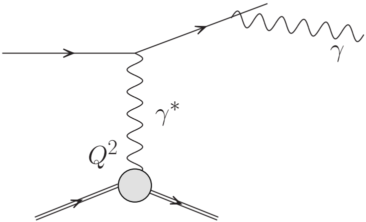
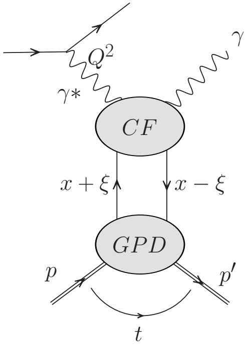
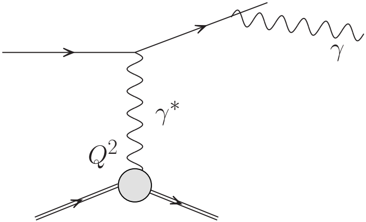
- One of the most extensively measured exclusive process (including on neutron target): JLab, CERN (COMPASS), HERA





Bethe-Heitler background (simple FF contribution) messes observable.





Bethe-Heitler background (simple FF contribution) messes observable.

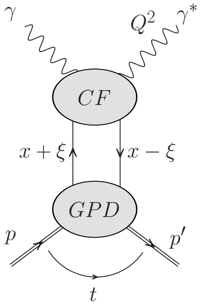
BUT, exploit BH contributions in interference terms:
⇒Calculate asymmetries.

Get reasonable statistics, and can extract GPD-sensitive information.

TCS: Cross incoming and outgoing photon in DVCS.

Same *Collinear* factorisation:

$$\mathcal{A} = \mathrm{CF} \otimes \mathrm{GPD}$$



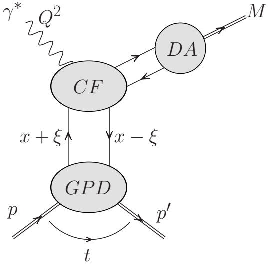
- First proposed by E. Berger, M. Diehl, B. Pire [hep-ph/0110062]
- NLO corrections: B. Pire, L. Szymanowski, J. Wagner [1101.0555]
- (Recently) measured at JLab! [CLAS collaboration: 2108.11746]
- Can also be measured in UPCs at hadron colliders: [B. Pire, L. Szymanowski, J. Wagner: 0811.0321]

DVMP: Outgoing γ in DVCS replaced by light meson $\rho,\ \pi, \cdots$

Collinear factorisation now given by

$$\mathcal{A} = \text{CF} \otimes \text{GPD} \otimes \text{Distribution Amplitude}$$

[J. Collins, L. Frankfurt, M. Strikman: hep-ph/9611433]
[A. Radyushkin: hep-ph/9704207]
proofs valid only for some restricted cases



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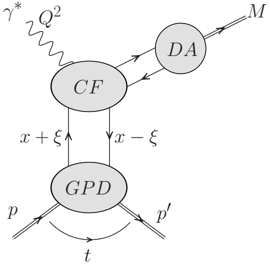

- NLO CF: A. Belitsky, G. Duplancic, D. Ivanov, D. Muller, G. Krasnikov, K. Passek-K, L. Szymanowski [hep-ph/0105046, hep-ph/0407207, 1612.01937]
- Twist-3: G. Duplancic, P. Kroll, K. Passek-Kumericki, L. Szymanowski [2312.13164]

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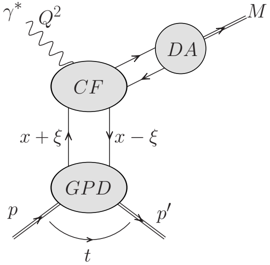

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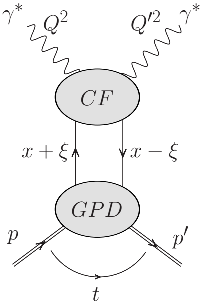
- Twist-3: G. Duplancic, P. Kroll, K. Passek-Kumericki, L. Szymanowski [2312.13164]
- DVρP (vector) measured at HERA, DVπP measured at JLab, and also at COMPASS.

More on exclusive quarkonium production later...

DDVCS: Both photons virtual

- CF now known to two loops:
  [V. Braun, H-Y. Jiang, A. Manashov, A. Manteuffel: 2411.14985]
- Resummation of threshold logs: [J. Schoenleber: 2411.11686]
- As with DVCS, collinear factorisation also valid at twist-3: [J. Schoenleber, R. Szafron: 2407.09263]
- Note: Three-scale process (Q^2, Q'^2 and $s_{\gamma p}$)



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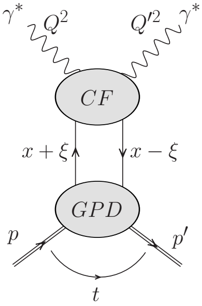


Revived recently by pheno analyses performed by K. Deja, V. Martinez-Fernandez, B. Pire, P. Sznajder, J. Wagner [2303.13668]

Also, plans on measuring DDVCS at JLab! M. Boër, D. Biswas [2403.02605]

Quark GPDs at twist 2 [M. Diehl: hep-ph/0307382]

without helicity flip (chiral-even Γ matrices): 4 chiral-even GPDs: (Note: $\Delta = p' - p$)

$$F^q = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p'|\, \bar{q}(-\tfrac{1}{2}z)\, \gamma^+ q(\tfrac{1}{2}z)\, |p\rangle \Big|_{z^+=0,\, z_\perp=0}$$
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\, \bar{u}(p')\gamma^+ u(p) + E^q(x,\xi,t)\, \bar{u}(p') \frac{i\,\sigma^{+\alpha}\Delta_\alpha}{2m} u(p) \right],$$

$$\tilde{F}^q = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p'|\, \bar{q}(-\tfrac{1}{2}z)\, \gamma^+\gamma_5\, q(\tfrac{1}{2}z)\, |p\rangle \Big|_{z^+=0,\, z_\perp=0}$$
$$= \frac{1}{2P^+}\left[\tilde{H}^q(x,\xi,t)\, \bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}^q(x,\xi,t)\, \bar{u}(p') \frac{\gamma_5\, \Delta^+}{2m} u(p) \right].$$

$H^q \xrightarrow{\xi=0,t=0}$ PDF q $\qquad$ $\tilde{H}^q \xrightarrow{\xi=0,t=0}$ polarised PDF Δq

with helicity flip (chiral-odd Γ matrices): 4 chiral-odd GPDs:

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p'|\, \bar{q}(-\tfrac{1}{2}z)\, i\,\sigma^{+i}\, q(\tfrac{1}{2}z)\; |p\rangle\Big|_{z^+=0,\, z_\perp=0}$$

$$= \frac{1}{2P^+}\,\bar{u}(p')\left[H_T^q\, i\sigma^{+i} + \tilde{H}_T^q\, \frac{P^+\Delta^i - \Delta^+P^i}{m^2} + E_T^q\, \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m} + \tilde{E}_T^q\, \frac{\gamma^+P^i - P^+\gamma^i}{m}\right] u(p)\,,$$

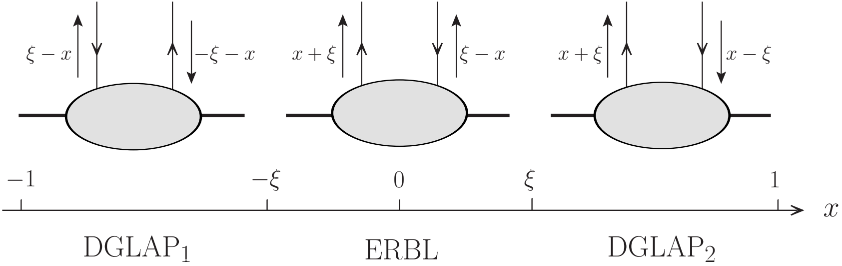
$$H_T^q \xrightarrow{\xi=0,t=0} \text{quark transversity PDFs } \delta q$$

Note: $\tilde{E}_T^q(x,-\xi,t) = -\tilde{E}_T^q(x,\xi,t)$

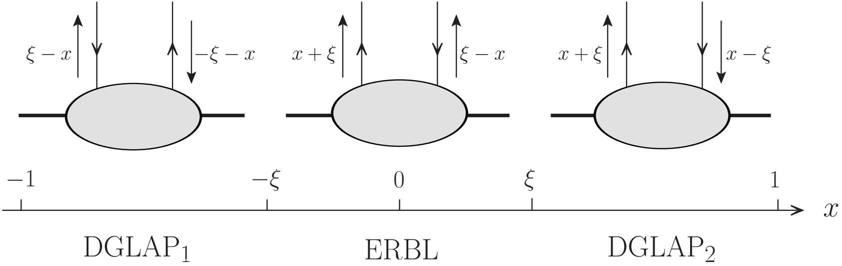
Unpolarised gluon GPDs at twist 2 [M. Diehl: hep-ph/0307382]

$$F^g = \frac{1}{P^+}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'|G^{+\mu}(-\frac{z}{2})G_\mu^+(\frac{z}{2})|p\rangle\Big|_{z^+=0,z_\perp=0}$$
$$= \frac{1}{2P^+}\left[H^g(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E^g(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right]$$

Forward limit: $H^g \xrightarrow{\xi=0,t=0}$ PDF $xg(x)$



Evolution equations different in ERBL/DGLAP regions.



Evolution equations different in ERBL/DGLAP regions.

"Breakpoints" at $x = \pm\xi \implies$ Parton stops being collinear, and becomes soft.

- 3D parton tomography [M. Burkardt hep-ph/0207047; M. Diehl hep-ph/0205208]: In the $\xi \to 0$ limit, the Fourier transform of the GPD over $\Delta_\perp$ gives the *impact parameter dependent* PDF, $q(x, b_\perp)$:

$$q(x, b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H^q(x, \xi = 0, \Delta_\perp^2) e^{ib_\perp \cdot \Delta_\perp}$$

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- Form Factors:
Integration over x gives the *electromagnetic* FF.
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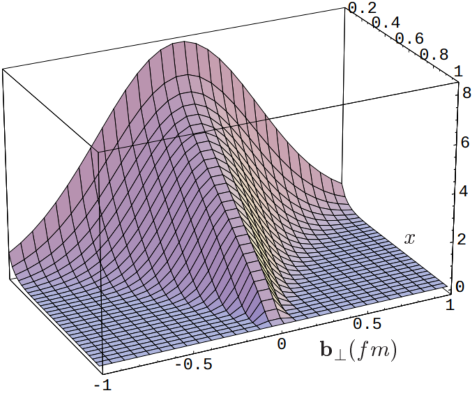
- Connection to nucleon spin:
  Ji sum rule [X. Ji hep-ph/9603249]

$$2J^q = \int_{-1}^{1} dx\, x \left(H^q(x, \xi, 0) + E^q(x, \xi, 0) \right)$$



Taken from M. Burkardt [hep-ph/0207047]

$$H_i(x,\xi,t) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(\beta + \xi\alpha - x) \times \left[d_i(\beta,\alpha,t) + \xi\delta(\beta) D(\alpha,t) \right] .$$

- Based on *double distributions (DDs)* [A. Radyushkin: hep-ph/9704207].

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- DDs $d_i(\beta,\alpha,t)$ even in α. $\Longrightarrow$ *polynomiality* property of GPDs

$$\int_{-1}^{1} dx\, x^n H_i(x,\xi,t) = \sum_{j=0,\,\mathrm{even}}^{n} (2\xi)^j A^i_{n+1,j}(t) + \mathrm{mod}(n,2)\,(2\xi)^{n+1} C^i_{n+1}(t).$$

- Consequence of Lorentz invariance [X. Ji: hep-ph/9807358].

Modelling GPDs
Double distributions

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- Consequence of Lorentz invariance [X. Ji: hep-ph/9807358].
- Usually fix $t = t_{\mathrm{min}}$.
- Also, usually neglect D-terms [M. Polyakov, C. Weiss: hep-ph/990241].

Forward limits of GPDs: factorisation of the double distributions:

$$d_i(\beta,\alpha) = f_i(\beta) \times h_i(\beta,\alpha)$$

such that the profile function $h_i(\beta,\alpha)$ satisfies

$$\int_{-1+|\beta|}^{1-|\beta|} d\alpha \, h_i(\beta,\alpha) = 1$$

To reproduce the correct forward limits,

$$f_g(\beta) = |\beta| g(|\beta|)\,,$$

$$f_q^{\mathrm{val}}(\beta) = \theta(\beta) q_{\mathrm{val}}(|\beta|)\,, \qquad f_q^{\mathrm{sea}}(\beta) = \mathrm{sgn}(\beta) q_{\mathrm{sea}}(|\beta|)\,,$$

Modelling GPDs
Double distributions

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For the profile function [A. Radyushkin: hep-ph/9805342, hep-ph/9810466]

$$h_i(\beta, \alpha) = \frac{\Gamma(2n_i + 2)}{2^{2n_i+1}\Gamma^2(n_i + 1)} \frac{\left((1 - |\beta|)^2 - \alpha^2\right)^{n_i}}{(1 - |\beta|)^{2n_i+1}} \,.$$

$n_i \leftrightarrow$ width of the profile function (generates *skewness*):

$n_i \to \infty \implies$ no ξ dependence in GPDs

Another property GPDs are expected to satisfy is positivity, which is in turn a consequence of the positivity of a Hilbert space norm

[A.V. Radyushkin: hep-ph/9805342]

[B. Pire, J. Soffer, O. Teryaev: hep-ph/9804284]

[M. Diehl, T. Feldmann, R. Jakob, P. Kroll: hep-ph/0009255]

$$\left| H(x,\xi,t) - \frac{\xi^2}{1-\xi^2} E(x,\xi,t) \right| \leq \sqrt{\frac{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}{1-\xi^2}}$$

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$$\left| H(x,\xi,t) - \frac{\xi^2}{1-\xi^2} E(x,\xi,t) \right| \leq \sqrt{\frac{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}{1-\xi^2}}$$

Positivity is *not automatically satisfied* by double distribution models!

[H. Dutrieux, O. Grocholski, H. Moutarde, P. Sznajder: 2112.10528]

Deconvolution problem

Collinear factorisation occurs at the *amplitude level*:
$\Longrightarrow$ So is the convolution over the GPD.

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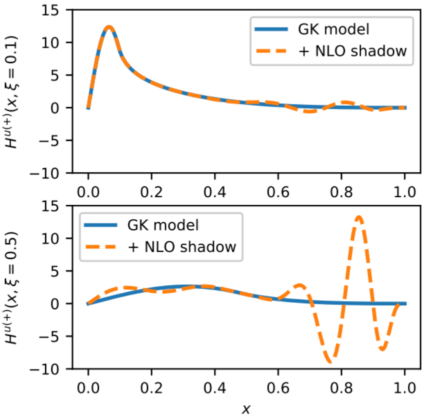
$\Longrightarrow$ Shadow GPDs: They satisfy the property that

$$\int_{-1}^{1} dx \, \mathrm{CF}(x) \, \mathrm{GPD}(x) = 0$$

Deconvolution problem



Taken from [V. Bertone, H. Dutrieux, C. Mezrag, H. Moutarde, P. Sznajder: 2104.03836]

Both the blue and dashed orange lines give the *same NLO CFFs*!
(although NLO corrections reduce the shadow space)

What lattice QCD can tell us

GPDs are defined from matrix elements separated along the *lightcone* $\Longrightarrow$ naively inaccessible on the lattice.

Two main techniques:

1. quasi-distributions [X. Ji: 1305.1539]
   - Non-local operator separated on a *purely spacelike separation* instead (quasi-distribution).
   - By boosting to the infinite momentum limit (Large-momentum effective theory, LaMET), one recovers the lightcone separated one.
   - Limitation: Lattice sets a limit on the maximum achievable hadron momentum. Errors of the order $\Lambda_{\mathrm{QCD}}^2/(x^2P^2)$, $\Lambda_{\mathrm{QCD}}^2/((1-x)^2P^2)$.

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2. pseudo-distributions [A. Radyushkin: 1612.05170, 1702.01726, 1705.01488]
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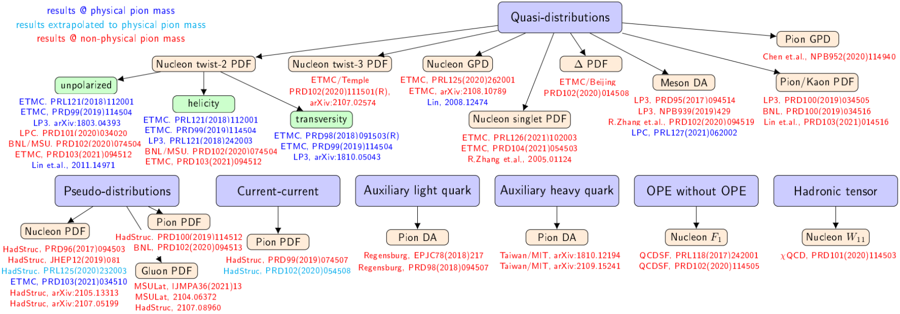
Note: Many details skipped (renormalisation, matching procedure, etc)

What lattice QCD can tell us



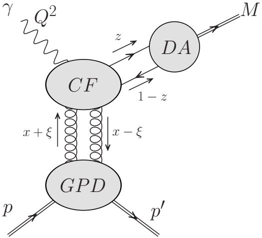
Taken from K. Cichy [2110.07440]

$\Longrightarrow$ Can help fill the gaps in the x-dependence of GPDs!

Exclusive quarkonium photoproduction

$$\mathcal{A} = \int_{-1}^{1} dx \int_{0}^{1} dz\, H(x)\phi(z)C(x,z)$$

$H(x)$: Generalised parton distribution (GPD)
$\phi(z)$: Distribution amplitude (DA)
$C(x,z)$: Coefficient function (CF)



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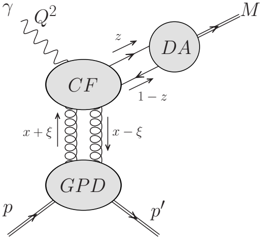

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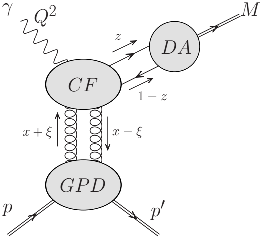

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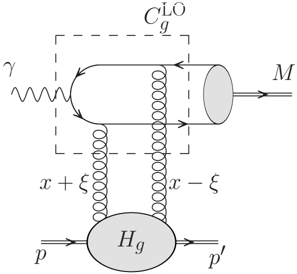
Generalised to electroproduction in [C. Flett, J. Gracey, S. Jones, T. Teubner: 2105.07657] *See talk by Chris on Tuesday*

- Exclusive J/ψ photoproduction probes gluon GPDs only at LO.
- Employ *static limit* (NRQCD): $\Longrightarrow \phi(z) \sim \delta(z - 1/2)$.



$$\mathcal{A} = \epsilon_\gamma^\mu \epsilon_M^{*\nu} \mathcal{T}^{\mu\nu}$$

$$\mathcal{T}_{\mathrm{LO}}^{\mu\nu} = -g_\perp^{\mu\nu} \int_{-1}^{1} \frac{dx}{x^2} C_g^{LO}\left(\frac{\xi}{x}\right) F_g(x, \xi, t; \mu_F)\,,$$

$$C_g^{\mathrm{LO}}\left(\frac{\xi}{x}\right) = \frac{F_{\mathrm{LO}}}{\left[1 + \frac{\xi}{x} - i\delta\,\mathrm{sgn}(x)\right]\left[1 - \frac{\xi}{x} + i\delta\,\mathrm{sgn}(x)\right]}$$

$$F_{\mathrm{LO}} = 4\pi\alpha_s e e_Q R_Q(0)/(m_Q^{\frac{3}{2}}\sqrt{2\pi N_c})\,, \qquad \xi = \frac{M^2}{2W_{\gamma p}^2 - M^2} \sim \frac{M^2}{2W_{\gamma p}^2}$$

Large $W_{\gamma p}$ (small x in inclusive physics) $\leftrightarrow$ *small* ξ

$$\mathcal{T}^{\mu\nu}_{\text{NLO}} \supset i\pi \frac{g_\perp^{\mu\nu} F_{LO}}{\xi} \left[F_g(\xi,\xi) + \frac{\alpha_s(\mu_R) C_A}{\pi} \ln\left(\frac{M^2}{4\mu_F^2}\right) \int_\xi^1 \frac{dx}{x} F_g(x,\xi) \right.$$
$$\left. + \frac{\alpha_s(\mu_R) C_A}{\pi} \frac{C_F}{C_A} \ln\left(\frac{M^2}{4\mu_F^2}\right) \int_\xi^1 dx \left(F_q(x,\xi) - F_q(-x,\xi)\right) \right]$$

$F_g(x,\xi) \sim \mathrm{const}$, as $x \to \xi$ for small ξ
$\Longrightarrow$ appearance of $\ln \xi$ (high-energy logs).

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Large $\ln \xi$ contributions are purely imaginary and come from the DGLAP region $(\xi < |x| < 1)$.

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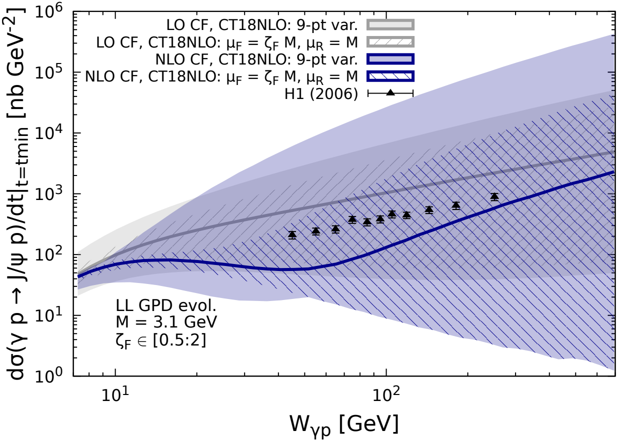
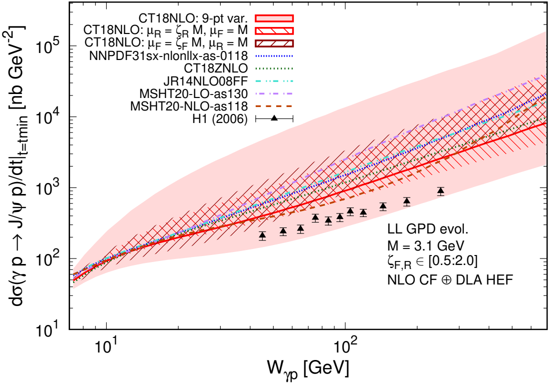
Opposite sign to LO for $\mu_F > M/2$.

Exclusive quarkonium production

Issue at NLO



Matching with high-energy result: [C. Flett, J-P. Lansberg, **S.N.**, M. Nefedov, P. Sznajder, J. Wagner: 2409.20544]



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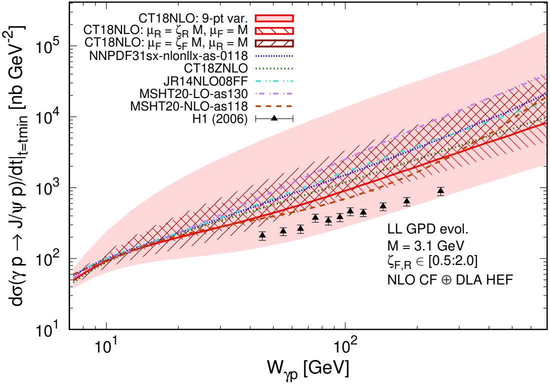

Higher twist corrections? Relativistic corrections? PDF/GPD model?

Exclusive photon-meson photoproduction

Original motivation: Extraction of chiral-odd GPDs at *leading* twist.

- $\gamma N \to \rho_T^0 \pi^+ N'$:
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  - $M = \rho^0$: R. Boussarie, B. Pire, L. Szymanowski, S. Wallon: [1609.03830]
  - $M = \pi^\pm$: G. Duplančić, K. Passek-Kumerički, B. Pire, L. Szymanowski, S. Wallon: [1809.08104]
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Richer kinematics of 3-body final state processes allows the sensitivity of GPDs wrt x to be probed (beyond moment-type dependence, e.g. in DVCS)
J. Qiu, Z. Yu: [2305.15397]

Exclusive photon-meson photoproduction

$$\gamma(q) + N(p_1) \to \gamma(k) + M(p_M) + N'(p_2)$$

$$\mathcal{A} = \int_{-1}^{1} dx \int_{0}^{1} dz \; T(x, \xi, z) \; H(x, \xi, t) \; \Phi_M(z)$$

- *Fully differential* cross-section differential covering $S_{\gamma N}$ from $\sim 4\,\mathrm{GeV}^2$ to $20000\,\mathrm{GeV}^2$.
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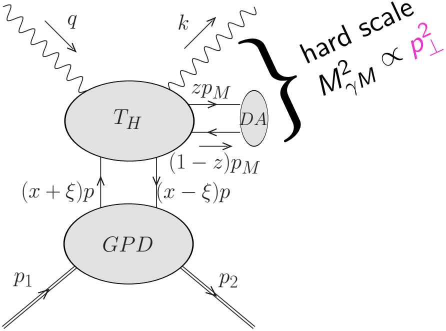
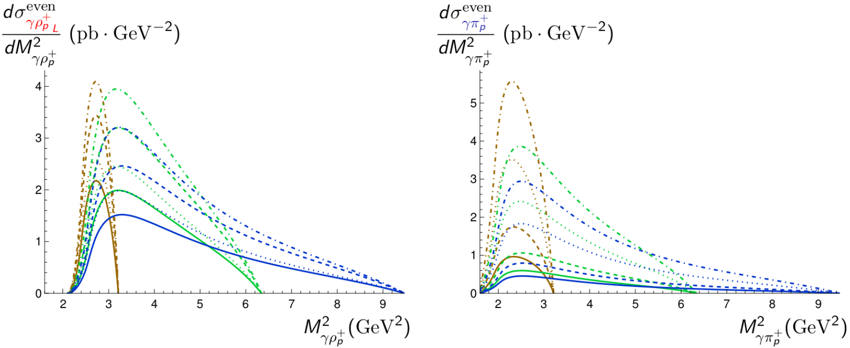
- Good statistics at various experiments, particularly at *JLab*.
- Polarisation asymmetries also sizeable.
- *Small* ξ limit of quark GPDs can be studied at collider experiments.



Exclusive photon-meson photoproduction

Results: Single differential cross-section: $\gamma\rho^+_{p\,L}$ vs $\gamma\pi^+_p$



$S_{\gamma N} = 8,\ 14,\ 20\ \text{GeV}^2$

Dashed: Holographic DA non-dashed: Asymptotical DA

Dotted: standard scenario non-dotted: valence scenario

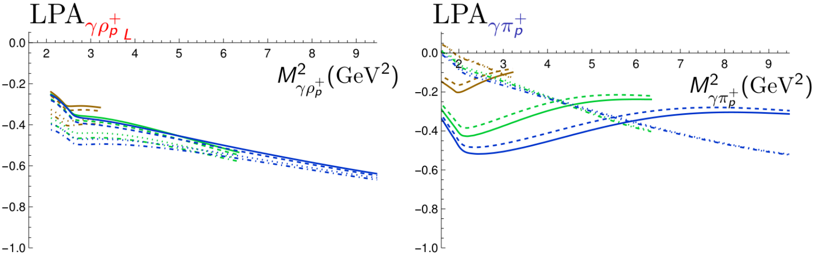
$\Longrightarrow$ Effect of GPD model more important on π^+_p than on ρ^+_p

We consider an unpolarised target, and determine polarisation asymmetries wrt the incoming photon.

- Circular polarisation asymmetry $= 0$. (QCD/QED invariance under parity)
- Linear polarisation asymmetry, LPA $= \frac{d\sigma_x - d\sigma_y}{d\sigma_x + d\sigma_y}$, where x is the direction defined by $p_\perp$ (direction of outgoing photon in the transverse plane).
- In fact,

$$\mathrm{LPA_{Lab}} = \mathrm{LPA}\cos(2\theta)\ ,$$

  where θ is the angle between the lab frame x-direction and $p_\perp$.
- Both asymmetries zero in chiral-odd case!



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$\Longrightarrow$ GPD model changes behaviour of LPA completely in π^+_p case!

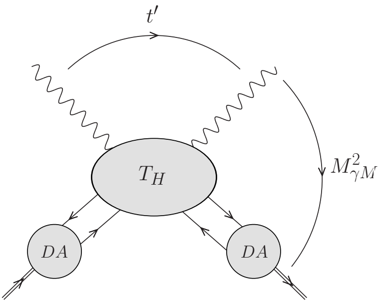
Good statistics: For example, at JLab Hall B:

- untagged incoming $\gamma \Rightarrow$ Weizsäcker-Williams distribution
- with an expected luminosity of $\mathcal{L} = 100 \ \mathrm{nb}^{-1} s^{-1}$, for 100 days of run:
  - ρ_L^0 (on p) : $\approx 2.4 \times 10^5$
  - ρ_T^0 (on p) : $\approx 4.2 \times 10^4$ (Chiral-odd)
  - ρ_L^+ : $\approx 1.4 \times 10^5$
  - ρ_T^+ : $\approx 6.7 \times 10^4$ (Chiral-odd)
  - π^+ : $\approx 1.8 \times 10^5$
- No problem in detecting outgoing photon at JLab.

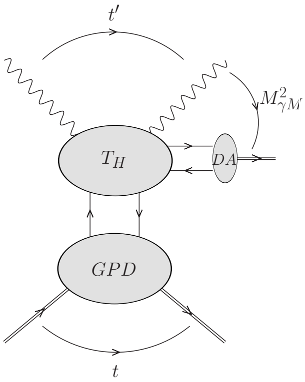
- At the future EIC, with an expected integrated luminosity of $10\,\mathrm{fb}^{-1}$ (about 100 times smaller than JLab):
  - ρ^0_L (on p) : $\approx 2.4 \times 10^4$
  - ρ^0_T (on p) : $\approx 2.4 \times 10^3$ (Chiral-odd)
  - ρ^+_L : $\approx 1.5 \times 10^4$
  - ρ^+_T : $\approx 4.2 \times 10^3$ (Chiral-odd)
  - π^+ : $\approx 1.3 \times 10^4$
- Small ξ study:
  $300 < S_{\gamma N}/\,\mathrm{GeV}^2 < 20000$ ($5 \cdot 10^{-5} < \xi < 5 \cdot 10^{-3}$):
  - ρ^0_L (on p) : $\approx 1.2 \times 10^3$
  - ρ^0_T (on p) : ≈ 6.5 (Chiral-odd) (tiny)
  - ρ^+_L : $\approx 9.3 \times 10^2$
  - π^+ : $\approx 5.0 \times 10^2$

Collinear factorisation

Is collinear factorisation really justified in 2 → 3 processes?



large angle factorisation
à la Brodsky Lepage



We thus argue *collinear factorisation* of the amplitude at large $M^2_{\gamma M}$, t', u', and small t.

$$t = (p_2 - p_1)^2 , \qquad u' = (p_M - q)^2 ,$$

$$t' = (k - q)^2 , \qquad S_{\gamma N} = (q + p_1)^2 .$$

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Is collinear factorisation really justified in $2 \to 3$ processes?

- Recently, factorisation has been proved for the process $\pi N \to \gamma\gamma N'$ by J. Qiu, Z. Yu [2205.07846].
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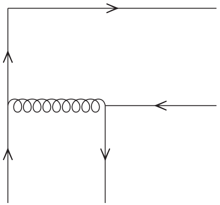
- Also, NLO computation for $\gamma\gamma \to \pi^+\pi^-$ by crossing symmetry G. Duplancic, B. Nizic: [hep-ph/0607069].

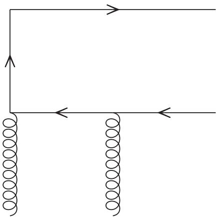
- Because of the quantum numbers of π^0 ($J^{PC} = 0^{-+}$), the exclusive photoproduction of $\pi^0\gamma$ is also sensitive to *gluon GPD contributions*.

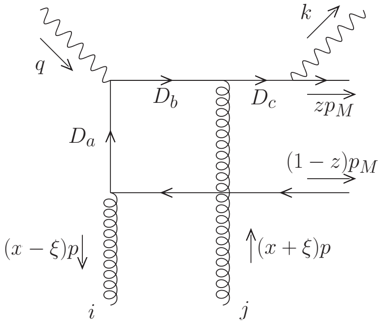
Collinear factorisation

Factorisation breaking effects in $\pi^0\gamma$ photoproduction: Gluon GPD contributions

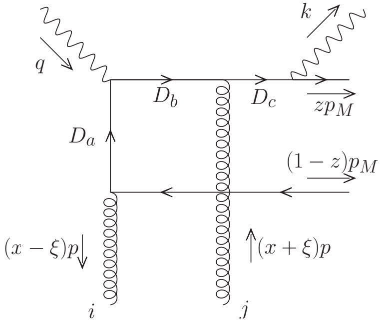
- Because of the quantum numbers of π^0 ($J^{PC} = 0^{-+}$), the exclusive photoproduction of $\pi^0\gamma$ is also sensitive to *gluon GPD contributions*.
- A total of 24 diagrams contribute in this case (compared to 20 diagrams from quark GPD contributions), with 6 groups of 4 related by symmetries ($x \to -x$ and $z \to 1 - z$ separately).
- Diagrams amount to connecting photons to the following two topologies.







$$\mathcal{A} \sim \int_{-1}^{1} dx \frac{\ln(x-\xi-i\epsilon)}{[x-\xi+i\epsilon]} \quad \Longrightarrow \text{ divergent imaginary part!}$$



$$\mathcal{A} \sim \int_{-1}^{1} dx \int_{0}^{1} dz \frac{1}{[(x-\xi)+A\bar{z}-i\epsilon]\,[x-\xi+i\epsilon]}$$

$\Longrightarrow$ The "*pinching*" is caused by propagators D_a and D_b.

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- In $\gamma\gamma \to MM$, only ERBL region exists, no poles are crossed, and endpoint contributions are suppressed by DAs.
- Indication of problem with naive collinear factorisation?
  At twist-2??

Collinear factorisation

Factorisation breaking effects in $\pi^0\gamma$ photoproduction

- In sum of all diagrams, the problem still persists (actually it gets worse)
- In $\gamma\gamma \to MM$, only ERBL region exists, no poles are crossed, and endpoint contributions are suppressed by DAs.
- Indication of problem with naive collinear factorisation?
  At twist-2??
- Collinear factorisation is indeed broken:
  [**S. N.**, J. Schönleber, L. Szymanowski, S. Wallon: 2311.09146, 2409.16067]
- Issue: *Glauber pinch* at leading power present in gluon channel.

Conclusions

- GPDs can help us answer many of nature's questions, including the *3D structure of hadrons*, contributions to the *nucleon spin*, as well as understanding the *charge, energy and pressure distributions in hadrons*.
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- Careful with QCD collinear factorisation: Could create discrepancies between what we think we are fitting...

BACKUP SLIDES

Very useful *Sudakov decomposition* of a generic 4-vector v in lightcone directions n_+ and n_-:

$$v^\mu = v^+ n_+^\mu + v^- n_-^\mu + v_\perp^\mu$$

with

$$\begin{aligned} n_+^2 = n_-^2 &= 0 \\ n_+ \cdot n_- &= 1 \\ v^\pm &= \frac{v^0 \pm v^3}{\sqrt{2}} \\ v^2 &= 2v^+ v^- + v_\perp^2 \end{aligned}$$

In other words, n_+^μ (n_-^μ) defines a *lightlike* 4-vector with spatial components purely in the positive (negative) z-direction

Quark GPDs are parametrised in terms of Double Distributions
[A. Radyushkin: hep-ph/9805342]

For polarised PDFs Δq (and hence transversity PDFs δq), two scenarios are proposed for the parameterization:

- "standard" scenario, with flavor-symmetric light sea quark and antiquark distributions.
- "valence" scenario with a completely flavor-asymmetric light sea quark densities.

- simplistic factorised ansatz for the t-dependence:

$$H^q(x,\xi,t) = H^q(x,\xi,t=t_{\mathrm{min}}) \times F_H(t)$$

with $F_H(t) = \frac{(t_{\mathrm{min}}-C)^2}{(t-C)^2}$ a standard dipole form factor ($C = 0.71\mathrm{GeV}^2$)

Leading twist DAs

- Helicity conserving (vector) DA at twist 2: ρ_L

$$\langle 0|\bar{u}(0)\gamma^\mu u(x)|\rho_L^0(p)\rangle = \frac{p^\mu}{\sqrt{2}} f_\rho \int_0^1 du\ e^{-iup\cdot x}\phi_\rho(u)$$

- Helicity flip (tensor) DA at twist 2: ρ_T

$$\langle 0|\bar{u}(0)\sigma^{\mu\nu} u(x)|\rho_T^0(p,s)\rangle = \frac{i}{\sqrt{2}}(\epsilon_\rho^\mu p^\nu - \epsilon_\rho^\nu p^\mu) f_\rho^\perp \int_0^1 du\ e^{-iup\cdot x}\ \phi_\rho(u)$$

- Helicity conserving (axial) DA at twist 2: $\pi^\pm$

$$\langle 0|\bar{u}(0)\gamma^\mu\gamma^5 d(x)|\pi(p)\rangle = i p^\mu f_\pi \int_0^1 du\ e^{-iup\cdot x}\phi_\pi(u)$$

Leading twist DAs

- We take the simplistic asymptotic form of the DAs

$$\phi_{\mathrm{as}}(z) = 6z(1-z)\,.$$

- We also investigate the effect of using a holographic DA:

$$\phi_{\mathrm{hol}}(z) = \frac{8}{\pi}\sqrt{z(1-z)}\,.$$

Suggested by

- AdS/QCD correspondence [S. Brodsky, G. de Teramond: hep-ph/0602252],
- dynamical chiral symmetry breaking on the light-front [C. Shi, C. Chen, L. Chang, C. Roberts, S. Schmidt, H, Zong: 1504.00689],
- recent lattice results. [X. Gao, A. Hanlon, N. Karthik, S. Mukherjee, P. Petreczky, P. Scior, S. Syritsyn, Y. Zhao: 2206.04084]

- Transverse spin content of the proton:

$$\begin{array}{ccc} |\uparrow\rangle_{(x)} & \sim & |\rightarrow\rangle + |\leftarrow\rangle \\ |\downarrow\rangle_{(x)} & \sim & |\rightarrow\rangle - |\leftarrow\rangle \\ \text{spin along } x & & \text{helicity states} \end{array}$$

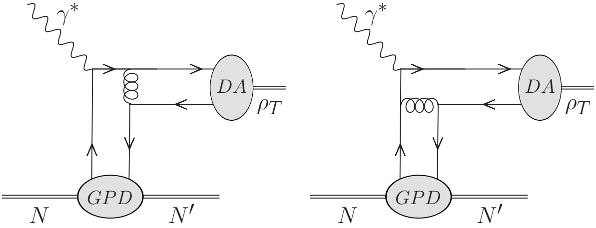
- Observables which are sensitive to helicity flip thus give access to transversity PDFs. Poorly known.

- Transversity GPDs are completely unknown experimentally.
- For massless (anti)particles, chirality = (-)helicity
- Transversity GPDs can thus be accessed through chiral-odd Γ matrices.
- Since (in the massless limit) QCD and QED are chiral-even $(\gamma^\mu, \gamma^\mu\gamma^5)$, the chiral-odd quantities $(1, \gamma^5, [\gamma^\mu, \gamma^\nu])$ which one wants to measure should appear in pairs.

Exclusive photon-meson photoproduction

Why γ-meson pair? Can we probe quark transversity GPDs in DVMP?

- the leading DA (twist 2) of ρ_T is chiral-odd ($\sigma^{\mu\nu}$ coupling)
- unfortunately $\gamma^* N \to \rho_T N' = 0$, since such a process would require a helicity transfer of 2 from a photon. [M. Diehl, T. Gousset, B. Pire: hep-ph/9808479], [J. Collins, M. Diehl: hep-ph/9907498]
- lowest order diagrammatic argument:



$$\gamma^\alpha[\gamma^\mu, \gamma^\nu]\gamma_\alpha = 0$$

Exclusive photon-meson photoproduction

Why γ-meson pair? Go to higher twist?

- Vanishing of chiral-odd amplitude in DVMP only occurs at twist 2
- At twist 3 this process does not vanish [S. Ahmad, G. Goldstein, S. Liuti: 0805.3568], [S. Goloskokov, P. Kroll: 1106.4897, 1310.1472]
- However processes involving twist 3 DAs may face problems with factorisation (end-point singularities)

  $\Rightarrow$ can be made safe in the high-energy k_T-factorisation approach

  [I. Anikin, D. Ivanov, B. Pire, L. Szymanowski, S. Wallon: 0909.4090]

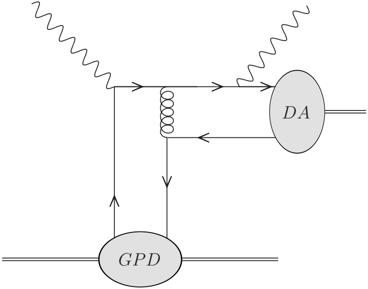
Circumvent this using *3-body* final states:

- $\gamma N \to \rho^0_T \pi^+ N'$:
  M. El Beiyad, B. Pire, M. Segond, L. Szymanowski, S. Wallon: [1001.4491]
- $\gamma N \to \gamma M N'$:
  R. Boussarie, B. Pire, L. Szymanowski, S. Wallon: [1609.03830]
  G. Duplančić, K. Passek-Kumerički, B. Pire, L. Szymanowski, S. Wallon: [1809.08104]
  G. Duplančić, **S.N.**, K. Passek-Kumerički, B. Pire, L. Szymanowski, S. Wallon: [2212.00655, 2302.12026]

Moreover, the richer kinematics of the process allows the sensitivity of GPDs wrt x to be probed (beyond moment-type dependence, e.g. in DVCS): J. Qiu, Z. Yu: [2305.15397]

$$\mathcal{A} = \int_{-1}^{1} dx \int_{0}^{1} dz \; T(x,\xi,z) \; H(x,\xi,t) \; \Phi_M(z)$$

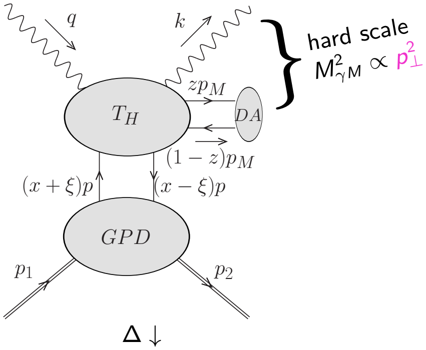
How does it work (at LO)?



Typical non-zero diagram for a transverse ρ meson

the σ matrices (from either the DA or the GPD) do not kill it anymore!

$$\gamma(q) + N(p_1) \to \gamma(k) + M(p_M, \varepsilon_M) + N'(p_2)$$



Useful Mandelstam variables:

$$t = (p_2 - p_1)^2\,,$$

$$u' = (p_M - q)^2\,,$$

$$t' = (k - q)^2\,,$$

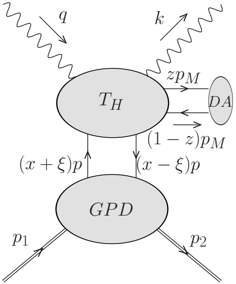
$$S_{\gamma N} = (q + p_1)^2\,.$$

- Factorisation requires:
  $-u' > 1\ \mathrm{GeV}^2$, $-t' > 1\ \mathrm{GeV}^2$ and $(-t)_{\min} \leqslant -t \leqslant .5\ \mathrm{GeV}^2$
  $\implies$ sufficient to ensure large p_T.
- Cross-section differential in $(-u')$ and $M^2_{\gamma M}$, and evaluated at $(-t) = (-t)_{\min}$, covering $S_{\gamma N}$ from $\sim 4\,\mathrm{GeV}^2$ to $20000\,\mathrm{GeV}^2$.

- Work in the limit of:
  - $\Delta_\perp \ll p_\perp$
  - $m_N^2,\ m_M^2 \ll M_{\gamma M}^2$
- initial state particle momenta:

$q^\mu = n^\mu,$

$p_1^\mu = (1+\xi)\, p^\mu + \frac{m_N^2}{s(1+\xi)} n^\mu$



} hard scale $M_{\gamma M}^2 \propto p_\perp^2$

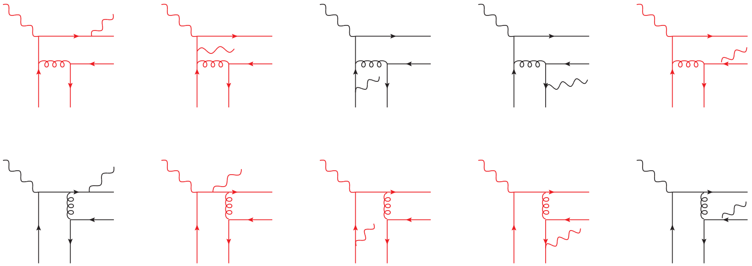
- final state particle momenta:

$$p_2^\mu = (1-\xi)\, p^\mu + \frac{m_N^2 + \vec{p}_t^{\,2}}{s(1-\xi)} n^\mu + \Delta_\perp^\mu$$

$$k^\mu = \alpha\, n^\mu + \frac{(\vec{p}_t - \vec{\Delta}_t/2)^2}{\alpha s} p^\mu + p_\perp^\mu - \frac{\Delta_\perp^\mu}{2}\ ,$$

$$p_M^\mu = \alpha_M\, n^\mu + \frac{(\vec{p}_t + \vec{\Delta}_t/2)^2 + m_M^2}{\alpha_M s} p^\mu - p_\perp^\mu - \frac{\Delta_\perp^\mu}{2}\ ,$$

A total of 20 diagrams to compute



- We compute 10 diagrams: Other half related by $q \leftrightarrow \bar{q}$ (anti)symmetry.
- In fact, by choosing the right gauge, only 4 diagrams can be used to generate all the others by various symmetries (eg. photon exchange).
- Red diagrams cancel in the chiral-odd case

Consider

$$\gamma(q,\lambda_q) + N(p_1,\lambda_1) \to \gamma(k,\lambda_k) + \pi^{\pm}(p_\pi) + N'(p_2,\lambda_2) \ ,$$

where λ_i represent the helicities of the particles.

QED/QCD invariance under parity implies that [C. Bourrely, J. Soffer, E. Leader: Phys.Rept. 59 (1980) 95-297]

$$\mathcal{A}_{\lambda_2\lambda_k\,;\,\lambda_1\lambda_q} = \eta\,(-1)^{\lambda_1-\lambda_q-(\lambda_2-\lambda_k)}\mathcal{A}_{-\lambda_2-\lambda_k\,;\,-\lambda_1-\lambda_q} \ ,$$

where η represents phase factors related to intrinsic spin.

Thus, at the cross-section level, it is clear that circular asymmetry will vanish, since

$$\sum_{\lambda_i,\, i\neq q} \left|\mathcal{A}_{\lambda_2\lambda_k\,;\,\lambda_1+}\right|^2 = \sum_{\lambda_i,\, i\neq q} \left|\mathcal{A}_{\lambda_2\lambda_k\,;\,\lambda_1-}\right|^2$$

For p-Pb UPCs at LHC (integrated luminosity of 1200 nb^{-1}):

- With future data from runs 3 and 4,
  - $\rho_L^0 : \approx 1.6 \times 10^4$
  - $\rho_T^0 : \approx 1.7 \times 10^3$ (Chiral-odd)
  - $\rho_L^+ : \approx 1.1 \times 10^4$
  - $\rho_T^+ : \approx 2.9 \times 10^3$ (Chiral-odd)
  - $\pi^+ : \approx 9.3 \times 10^3$
- $300 < S_{\gamma N}/\,\text{GeV}^2 < 20000$ ($5 \cdot 10^{-5} < \xi < 5 \cdot 10^{-3}$):
  - $\rho_L^0 : \approx 8.1 \times 10^2$
  - $\rho_L^+ : \approx 6.4 \times 10^2$
  - $\pi^+ : \approx 3.4 \times 10^2$

UPCs

- In ultraperipheral collisions (UPCs), hadronic interactions (QCD) are suppressed.

  $\Longrightarrow$ *Interactions between nuclei dominated by photon exchanges.*

- Therefore, we can study p-Pb collisions, with the Pb nucleus acting as the photon source, since it has a much larger charge:

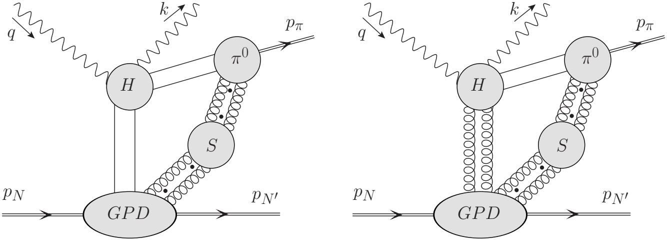
$$\frac{d^3 N_\gamma}{dk d^2 \vec{b}} = \frac{Z^2 \alpha x^2}{\pi^2 k |\vec{b}|^2} K_1^2(x) , \qquad x = \frac{k|\vec{b}|}{\gamma \hbar c}$$

$$\frac{dN_\gamma(k)}{dk} = \int_{b_{\min}}^{b_{\max}} db \, 2\pi b \, \frac{d^3 N_\gamma}{dk d^2 \vec{b}} P_{\mathrm{NOHAD}}(b) ,$$

- $P_{\mathrm{NOHAD}}(b)$ taken from STARlight [S. Klein, J. Nystrand, J. Seger, Y. Gorbunov, J. Butterworth: 1607.03838]

Collinear factorisation

Reduced diagram analysis: Classic Collinear pinch



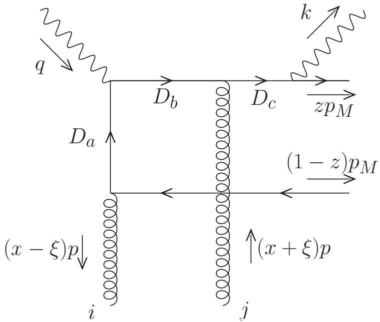
In both of the above cases, the power counting is:

$$\mathcal{A} \sim Q^{-1}\lambda^{\alpha}, \qquad \lambda = \frac{\Lambda_{\rm QCD},\, m_\pi,\, m_N}{Q} \ll 1, \qquad \alpha = 1$$

Collinear factorisation at *all orders* and *leading power* provided:

- the above (classic) collinear pinch diagrams are the *only ones contributing to the leading power of $\alpha = 1$*
- the *soft factor S 'cancels'*
- no Glauber pinches

Collinear factorisation
Reduced diagram analysis: Other leading pinch surfaces?

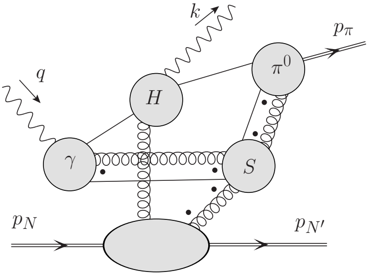


Divergence obtained when $(x - \xi)\, p$ and $(1 - z)\, p_M$ lines become soft:

$\Longrightarrow$ D_a becomes soft and D_b becomes collinear with respect to q.

Is there a *leading* pinch diagram that corresponds to this region?

Yes!



$$\mathcal{A} \sim Q^{-1}\lambda^{\alpha}\,, \qquad \alpha = 1$$

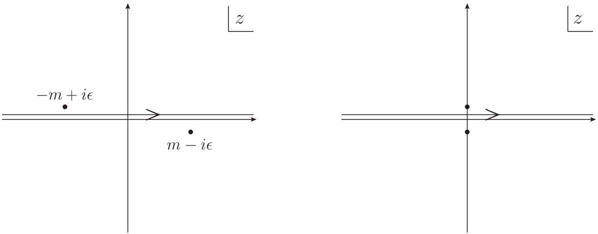
$\Longrightarrow$ power counting is the same as the collinear region!

Note: Corresponding reduced diagram for quark GPD case is power suppressed.

Consider the integral,

$$I = \lim_{\epsilon \to 0^+} \int_{-\infty}^{\infty} dz \frac{1}{(z - m + i\epsilon)(z + m - i\epsilon)}, \quad m > 0$$

Can be represented as a contour integral along the real axis in the complex z-plane: Poles at $z = \pm m \mp i\epsilon$.



- For $m \neq 0$, one can avoid the neighbourhood of the poles through contour deformations.
- For $m = 0$, poles *coalesce*, and there is a *pinch*.

Collinear factorisation

Pinches: Landau conditions

Pinches correspond to regions of loop momentum which cannot be avoided through contour deformations.

They can be identified efficiently through Landau conditions:

$$I(z) = \lim_{\epsilon \to 0^+} \int_{\mathbb{R}^{dL}} d^{dL}\omega \, \frac{N(\omega, z)}{\prod_{j=0}^{n} (D_j(\omega, z) + i\epsilon)} \, .$$

Given z, $\omega_S \in \mathbb{R}^{dL}$ such that the set

$$\mathcal{D} = \{ j \in \{1, ..., n\} \mid D_j(\omega_S, z) = 0 \}$$

is non-empty, we have a pinch at ω_S iff there exist real and non-negative numbers α_j for $j \in \mathcal{D}$ such that

- $\forall i \in \{1, ..., dL\} \; : \; \sum_{j \in \mathcal{D}} \alpha_j \frac{\partial D_j}{\partial \omega_i}(\omega_S; z) = 0.$
- At least one of the α_j is non-zero

Note: Existence of pinch does *not* imply existence of a singularity: Need to also perform *power counting*.

Consider the bubble integral, with massless internal lines:

$$I_1(p^2) = \lim_{\epsilon\to 0^+} \int d^4k \, \frac{1}{(k^2 + i\epsilon)((p-k)^2 + i\epsilon)}.$$

According to the Landau conditions, there is *always* a pinch related to soft momentum k, independent of p.

This is because when $k = 0$, both the propagator $k^2 + i\epsilon$ and its first derivative are zero.

$\implies$ *Landau conditions for a pinch at $k = 0$ are satisfied.*

However, note that the power counting does not give an IR divergence for $p^2 \neq 0$:

$$\implies \frac{[\lambda^4]}{[\lambda^2][1]} \sim \lambda^2$$

Collinear factorisation

What exactly does the pinch surface correspond to?

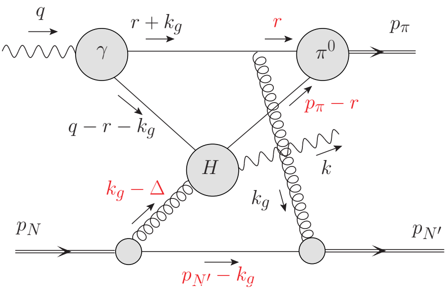
- Use Sudakov basis $(+, -, \perp)$:

$$\text{Collinear} \quad k \sim Q\left(1, \lambda^2, \lambda\right) \quad (\text{or} \quad k \sim Q\left(\lambda^2, 1, \lambda\right))$$

- Need to distinguish between *ultrasoft*, *soft* and *Glauber* gluons:

$$\begin{aligned} &\text{Ultrasoft} && k \sim Q\left(\lambda^2, \lambda^2, \lambda^2\right) \\ &\text{Soft} && k \sim Q\left(\lambda, \lambda, \lambda\right) \\ &\text{Glauber} && k \sim Q\left(\lambda^2, \lambda^2, \lambda\right) \quad (\text{or similar with} \quad |k_\perp^2| \gg k^+ k^-) \end{aligned}$$

- Libby-Sterman power counting formula strictly applies for *ultrasoft gluons* only.
- However, these are typically eliminated by the use of *Ward identities*.
- Glauber gluons *cannot* be eliminated/suppressed by the use of Ward identities.
- Key Question: Is there a Glauber pinch that contributes at leading power?



(Notation: $(+, -, \perp)$)

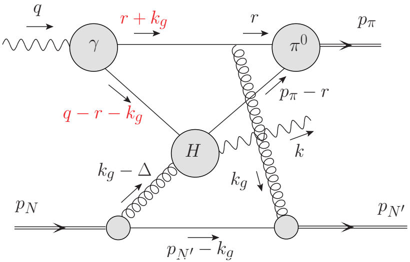
$$p_N,\, p_{N'},\, \Delta \sim Q\left(1, \lambda^2, \lambda\right), \quad \Delta^+ < 0.$$
$$p_\pi \sim Q\left(\lambda^2, 1, \lambda\right)$$
$$q,\, k \sim Q\,(1,1,1)\,, \quad q^2,\, k^2 \sim \lambda^2 Q^2$$
$$[\text{Loop}]\ k_g \sim Q\,(\lambda,\lambda,\lambda)$$
$$[\text{Loop}]\ r \sim Q\,(\lambda,\lambda,\lambda)$$

Recall: Soft loop momenta r and k *always* need to be considered.

- *k_g^- pinch*:

$$(k_g - \Delta)^2 + i0 = -2\Delta^+ k_g^- + \mathcal{O}(\lambda^2) + i0$$
$$\Longrightarrow\ k_g^- = \mathcal{O}(\lambda^2) - i0\,.$$

$$(p_{N'} - k_g)^2 + i0 = -2p_{N'}^+ k_g^- + \mathcal{O}(\lambda^2) + i0$$
$$\Longrightarrow\ k_g^- = \mathcal{O}(\lambda^2) + i0\,.$$



k_g^+ pinch:

$$(q - r - k_g)^2 + i0 = -2q^+ r^- - 2q^- k_g^+ + \mathcal{O}(\lambda) + i0$$

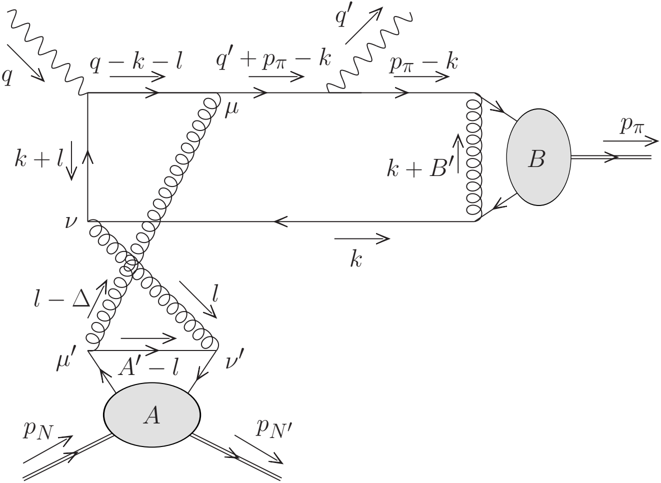
$$\implies k_g^+ = \mathcal{O}(\lambda) + i0\,.$$

$$(r + k_g)^2 + i0 = 2k_g^+ r^- + \mathcal{O}(\lambda^2) + i0$$

$$\implies k_g^+ = \mathcal{O}(\lambda) - \mathrm{sgn}(r^-) i0\,.$$

Conclusion: k_g^+ is pinched to be $\mathcal{O}(\lambda)$, and k_g^- is pinched to be $\mathcal{O}(\lambda^2)$.
$\implies$ Glauber pinch, since $k^+ k^- \ll |k_\perp|^2$.

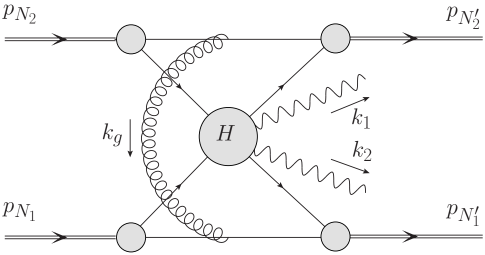
Collinear factorisation
Glauber pinch is leading



Explicit 2-loop analysis shows that the Glauber pinch demonstrated previously is leading, i.e. it scales as λ^α, with $\alpha = 1$.

Very similar to the exclusive double diffractive process, where the Glauber gluon is pinched between the two pairs of incoming and outgoing collinear hadrons.

$$p(p_{N_1}) + p(p_{N_2}) \longrightarrow p(p_{N_1'}) + p(p_{N_2'}) + \gamma(k_1) + \gamma(k_2)$$



Here, the Glauber pinch corresponds to $k_g \sim (\lambda^2, \lambda^2, \lambda)$

Instead, in our case, the Glauber gluon (which corresponds to one of the active partons) is pinched between *a pair of collinear hadrons*, and *a soft line joining the outgoing pion and the incoming photon*.