Multiple Parton Scattering: Symmetrising PYTHIA's Model of mPDFs

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Workshop on Double Parton Scattering and the 3D Structure of Hadrons

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Overview

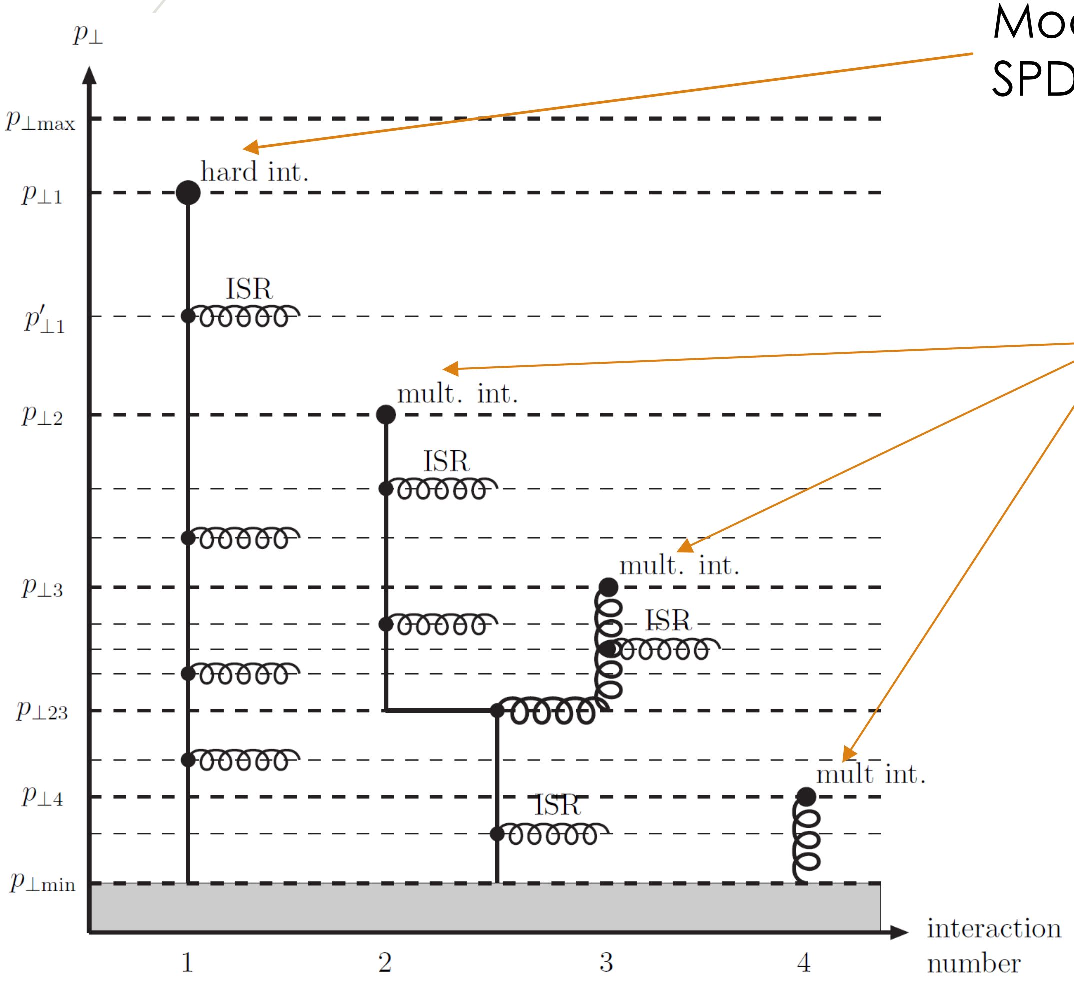
PYTHIA's Model of mPDFs
- Approach
- Structure

The Gaunt-Stirling (GS) Sum Rules
- General Specification
- Conflict with PYTHIA

The "X-Ordered" mPDF prescription
- Improving Symmetrisation
- Effects on Sum Rules
- Effects on Simple Observables

PYTHIA Model of MPI

How Does PYTHIA approach MPI?



Models the Hardest Interaction first, with SPDFs $f_j^r(x,Q)$ as in single parton scattering

"Subsequent" interactions modelled with modified SPDFs $f_{j_n}^{m \leftarrow j_1 x_1, \ldots, j_{n-1} x_{n-1}}(x,Q)$

ISR handled for each of these interactions as it would be done in SPS

Sjostrand, Skands, hep-ph/0402078, hep-ph/0408302

PYTHIA Model of mPDFs

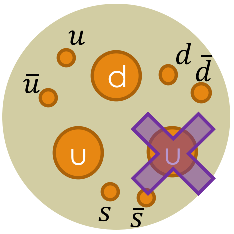
Four Principal Modifications:

(I) MOMENTUM "SQUEEZING": $f_{j_n}^{m \leftarrow j_1, x_1 \ldots j_{n-1}, x_{n-1}}(x, Q) = \frac{1}{X} f_i^r\left(\frac{x}{X}, Q\right)$ $\left[X = 1 - \sum_{j=1}^{n-1} x_j\right]$

Ensures $\sum_{jn} \int dx\, x\ f_{j_n}^{m \leftarrow j_1, x_1 \ldots j_{n-1}, x_{n-1}}(x, Q) = X$.

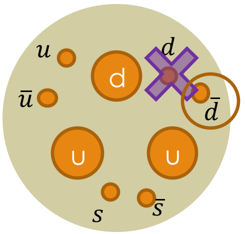
(II) VALENCE NUMBER SUBTRACTION:

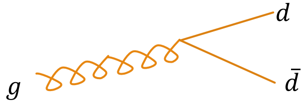
$$f_{i_v}^{m \leftarrow j_1, x_1 \ldots j_{n-1}, x_{n-1}}(x, Q) = \frac{N_{i_v n}}{N_{i_v 0}} \frac{1}{X} f_{i_v}^r\left(\frac{x}{X}, Q\right)$$



(III) COMPANION QUARK ADDITION:

$$q_c(x, x_s) = C(x_s) P_{g \to q\bar{q}}\left(\frac{x_s}{x_s + x}\right) \frac{g(x_s + x)}{x_s + x}$$





(IV) SEA QUARK AND GLUON RESCALING: Steps (II) and (III) break (I). To fix, rescale all sea quark and gluon distributions by a factor "a".

Sjostrand, Skands, hep-ph/0402078, hep-ph/0408302

PYTHIA Model of MPI

Full expression for for multi-parton PDFs in PYTHIA:

$$F_{j_1 \dots j_n}(x_1 \dots x_n, Q_1 \dots Q_n) = f^r_{j_1}(x_1, Q_1) f^{m \leftarrow j_1, x_1}_{j_2}(x_2, Q_2) \dots f^{m \leftarrow j_1, x_1 \dots j_{n-1}, x_{n-1}}_{j_n}(x_n, Q_n)$$

PYTHIA thus uses the mPDFs, instead of y_i-dependent mPDs

$$F_{j_1 \dots j_n}(x_1 \dots x_n, Q_1 \dots Q_n) \equiv \int d^2 y_1 \dots d^2 y_{n-1} \Gamma(x_1 \dots x_n, \boldsymbol{y_1} \dots \boldsymbol{y_{n-1}}, Q_1 \dots Q_n)$$

Can these objects be constrained by theory?

The Sum Rules

For DPS ($n = 2$), we know for $Q_1 = Q_2 = Q$ the **sum rules**:

Gaunt, Stirling, 0910.4347
Blok, Dokshitzer, Frankfurt, Strikman, 1306.3763
Diehl, Plößl, Schäfer, 1811.00289

Momentum rule: $\sum_{j_2} \int dx_2\, x_2\, D_{j_1 j_2}(x_1, x_2, Q) = (1 - x_1)\, f_{j_1}(x_1, Q)$

Available momentum after "taking out" j_1

Number rule: $\int dx_2\, D_{j_1 j_{2v}}(x_1, x_2, Q) = \left(N_{j_{2v}} - \delta_{j_1 j_2} + \delta_{j_1 \bar{j}_2}\right) f_{j_1}(x_1, Q)$

Number of j_2 quarks - number of $\bar{j}_2$ quarks after "taking out" j_1

In TPS ($n = 3$), it has been shown that similar rules hold:

Gaunt, Fedkevych, 2208.08197

Momentum rule: $\sum_{j_3} \int dx_3\, x_3\, T_{j_1 j_2 j_3}(x_1, x_2, x_3, Q) = (1 - x_1 - x_2)\, D_{j_1 j_2}(x_1, x_2, Q)$

Number rule:

$$\int dx_3\, T_{j_1 j_2 j_{3v}}(x_1, x_2, x_3, Q) = \left(N_{j_{3v}} - \delta_{j_1 j_3} - \delta_{j_2 j_3} + \delta_{j_1 \bar{j}_3} + \delta_{j_2 \bar{j}_3}\right) D_{j_1 j_2}(x_1, x_2, Q)$$

Similar rules have been proven for the QPDS (n=4) case.

How well do the Pythia double and triple PDFs satisfy these?

Pythia mPDFs and the Sum Rules

Sum rules satisfied by construction when integrating over final parton

$$F_{j_1 \dots j_n}(x_1 \dots x_n, Q) = f^r_{j_1}(x_1, Q) f^{m \leftarrow j_1, x_1}_{j_2}(x_2, Q) \dots f^{m \leftarrow j_1, x_1 \dots j_{n-1}, x_{n-1}}_{j_n}(x_n, Q)$$

Rules violated when integration conducted over any other x_i

In an mPDF we expect $\{x_i, j_i\} \leftrightarrow \{x_k, j_k\}$ symmetry

$$F_{j_1 \dots j_i \dots j_k \dots j_n}(x_1 \dots x_i \dots x_k \dots x_n\,, Q) = F_{j_1 \dots j_k \dots j_i \dots j_n}(x_1 \dots x_k \dots x_i \dots x_n, Q)$$

A symmetry not maintained by the PYTHIA model

Pythia mPDFs and the Sum Rules

Can construct ‘naïve’ symmetrisation for arbitrary mPDF:

$$F_{j_1 \dots j_n}^{symm}(x_1 \dots x_n, Q) = \frac{1}{n!} \sum_{\{1,\dots,n\}} F_{j_1 \dots j_n}(x_1 \dots x_n, Q)$$

Gaunt, Fedkevych, 2208.08197

DPDF (n=2)

$$D_{j_1 j_2}^{sym}(x_1, x_2, Q) = \frac{1}{2!} \sum_{\{1,2\}} D_{j_1 j_2}(x_1, x_2, Q)$$

TPDF (n=3)

$$T_{j_1 j_2 j_3}^{sym}(x_1, x_2, x_3, Q) = \frac{1}{3!} \sum_{\{1,2,3\}} T_{j_1 j_2 j_3}(x_1, x_2, x_3, Q)$$

Pythia mPDFs and the Sum Rules

Symmetrised DPDF : satisfies sum rules to ~10-25% level across most x_1, but very large deviations elsewhere

x_1	Momentum sum rule (MSR) ($j_1 = u$). Should = 1.	$u\bar{u}$ number sum rule (NSR). Should = -1.	$\bar{u}u$ NSR. Should = 3.
10^{-6}	0.988	-0.932	3.252
10^{-3}	0.986	-0.860	3.321
0.1	1.023	-0.921	3.459
0.2	1.057	-0.922	3.590
0.4	1.143	-0.881	4.505
0.8	1.684	-0.748	42.620

Connected to companion quark mechanism when both quarks have large x

Symmetrised TPDF ($x_2 = 10^{-4}$): broadly similar trends, extreme values of x_1 problematic

x_1	$j_1 = j_2 = u$ MSR. Should = 1.	uuu NSR. Should = 0.	$u\bar{u}u$ NSR. Should = 2.
10^{-6}	0.972	-0.169	2.219
10^{-3}	0.969	-0.228	2.156
0.1	1.002	-0.206	2.165
0.2	1.034	-0.219	2.163
0.4	1.119	-0.289	2.134
0.8	1.695	-0.540	2.057

Gaunt, Fedkevych, 2208.08197

Improving Symmetrisation

Companion quark asymmetry

$$D_{j_1 j_2}^{comp}(x_1, x_2, Q) = f_{j_1}^{s}(x_1, Q) q_c(x_2, x_1) = f_{j_1}^{s}(x_1, Q) C(x_1) P_{g\to q\bar{q}}\left(\frac{x_1}{x_1 + x_2}\right) \frac{g(x_1 + x_2)}{x_1 + x_2}$$

Companion term asymmetric

Proposed Modification

Change companion function:

$$q_c(x, x_s) = \frac{C(x_s + x)}{f_{j_1}^{s}(x_s)} \left\langle P_{g\to q\bar{q}}\left(\frac{x_s}{x_s + x}\right)\right\rangle \frac{g(x_s + x)}{x_s + x} = \frac{-[\partial_y f_{j_1}^{s}(y)]\big|_{y=x+x_s}}{f_{j_1}^{s}(x_s)}$$

Gaunt, Stirling, 0910.4347

Gives a symmetric companion contribution:

$$D_{j_1 j_2}^{comp}(x_1, x_2, Q) = f_{j_1}^{s}(x_1, Q) q_c(x_2, x_1) = -[\partial_y f_{j_1}^{s}(y)]\big|_{y=x_2+x_1}$$

Improving Symmetrisation

HIGH-x_1 DEVIATION

x_1	$j_1 = u$ MSR. Should = 1.	$u\bar{u}$ NSR. Should = -1.	$\bar{u}u$ NSR. Should = 3.
0.8	1.684	-0.748	42.620

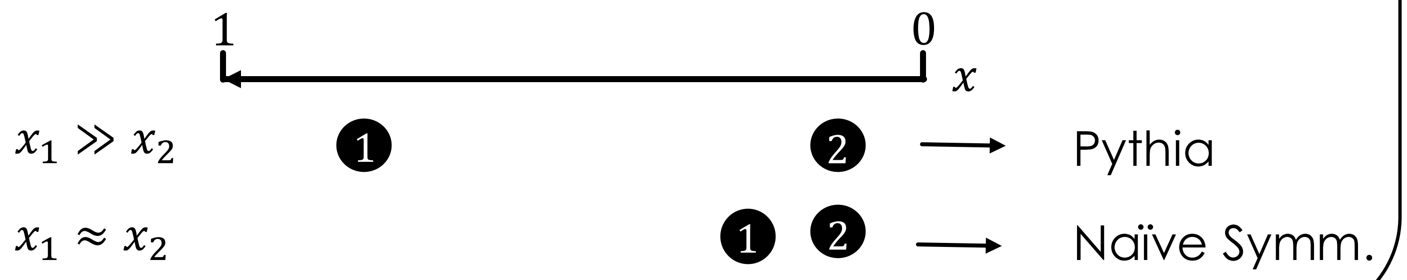
Large deviation from expectation when x_1 large

Implies $D^{sym}_{j_1 j_2}(x_1, x_2, Q)$ behaving unlike an accurate PDF

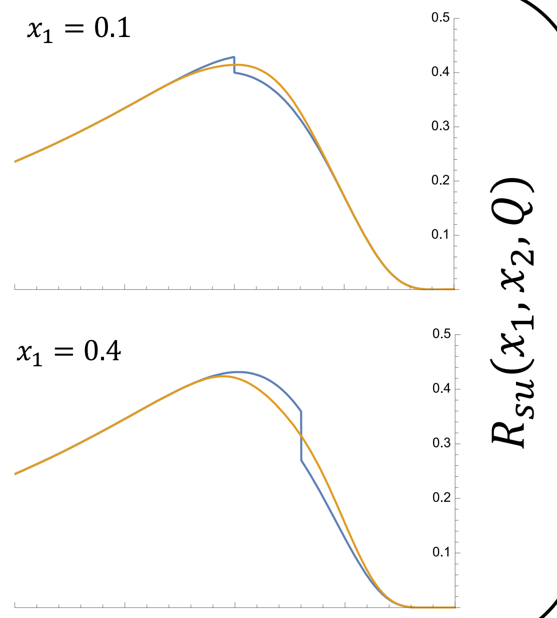
INTRODUCE 'X-ORDERING'

Step 1) Force the largest x 'first'

Step 2) Smoothly interpolate between the PYTHIA and naïvely symmetrised terms away from problem areas



SMOOTH VS. STRICT



Modified mPDFs and the sum rules

Please Note!

All plots & numbers here and beyond are preliminary!

Modified DPDF : satisfies sum rules to <10% level across most x_1, introduced >10% deviations at low x_1.

x_1	$(j_1 = u)$ MSR. Should = 1.	$u\bar{u}$ NSR. Should = -1.	$\bar{u}u$ NSR. Should = 3.
10^{-6}	0.974	-1.137	3.134
10^{-3}	0.968	-1.085	3.089
0.1	1.023	-1.003	2.928
0.2	1.022	-0.996	2.923
0.4	1.007	-0.994	2.965
0.8	1.000	-0.997	2.934

x_1	$j_1 = s$ MSR. Should = 1.	$s\bar{s}$ NSR. Should = 1.	$\bar{s}s$ NSR. Should = 1.
10^{-6}	0.974	-0.999	0.999
10^{-3}	0.967	-1.000	1.000
0.1	0.957	-1.000	1.000
0.2	0.976	-1.000	1.000
0.4	0.986	-1.000	1.000
0.8	1.014	-0.964	0.964

Improving Symmetrisation

Low-x_1 Deviation Post-Modification

x_1	$u\bar{u}$ NSR. Should = -1.	$\bar{u}u$ NSR. Should = 3.
10^{-6}	-1.137	3.134

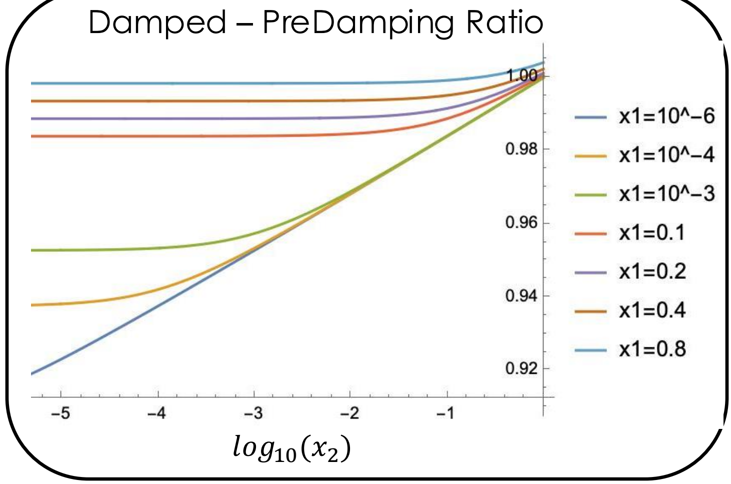
Introduce Damping Factor

Weight the quark components by $(x_1 + x_2)^{\alpha}$, $0 < \alpha \ll 1$, reducing overcontributions when x_1 and x_2 are both low

Modified DPDF, $\alpha = 0.007$: satisfies sum rules to <10% level across all sampled x_1

x_1	$j_1 = u$ MSR. Should = 1.	$u\bar{u}$ NSR. Should = -1.	$\bar{u}u$ NSR. Should = 3.
10^{-6}	0.965	-1.074	3.072
10^{-3}	0.960	-1.033	3.035
0.1	1.019	-0.987	2.902
0.2	1.020	-0.986	2.904
0.4	1.006	-0.988	2.953
0.8	1.001	-0.996	2.995

Damping Effect



Improving Symmetrisation

These modifications also greatly improve most ($x_2 = 10^{-4}$) TPDF sum rules, satisfying to within 10%.

x_1	$j_1 = j_2 = u$ MSR. Should = 1.	uuu NSR. Should = 0.	$u\bar{u}u$ NSR. Should = 2.
10^{-6}	0.928	0.010	2.143
10^{-3}	0.924	0.004	2.100
0.1	0.973	-0.011	2.020
0.2	0.971	-0.007	2.014
0.4	0.957	-0.007	2.010
0.8	0.948	-0.001	2.001

However, 5 of 196 NS rules exhibit low-x1 deviation >10%

x_1	guu NSR. Should = 1.	ggu NSR. Should = 2.
10^{-6}	1.198	2.249
10^{-4}	1.171	2.220
10^{-3}	1.137	2.185

Only occurs when interactions involve a gluon

10 of 196 NS rules exhibit high-x1 (x1~=0.4) deviation >10%

x_1	$\bar{s}\bar{d}s$ NSR. Should = 1.	$\bar{s}ss$ NSR. Should = 0.
0.4	1.112	0.104

Only occurs when $f_1 = f_3 = s$

Effects on Observables

Effects of modifications:

- Sum Rule level – clear
- Predictions (X-sec/Observables) – less clear

Considered two toy processes:

- Same-sign W production – two W bosons from p-p
  - Considered both W+ and W-
- Double Z production
  - No photon interference

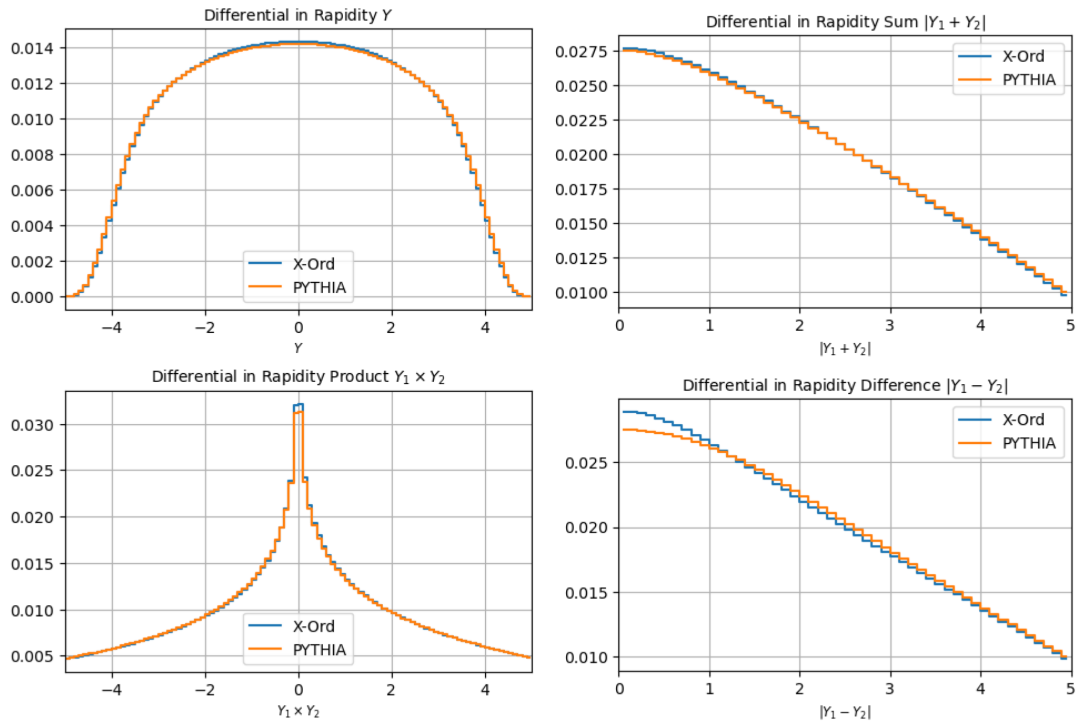
Considered a few observables:

- Asymmetry

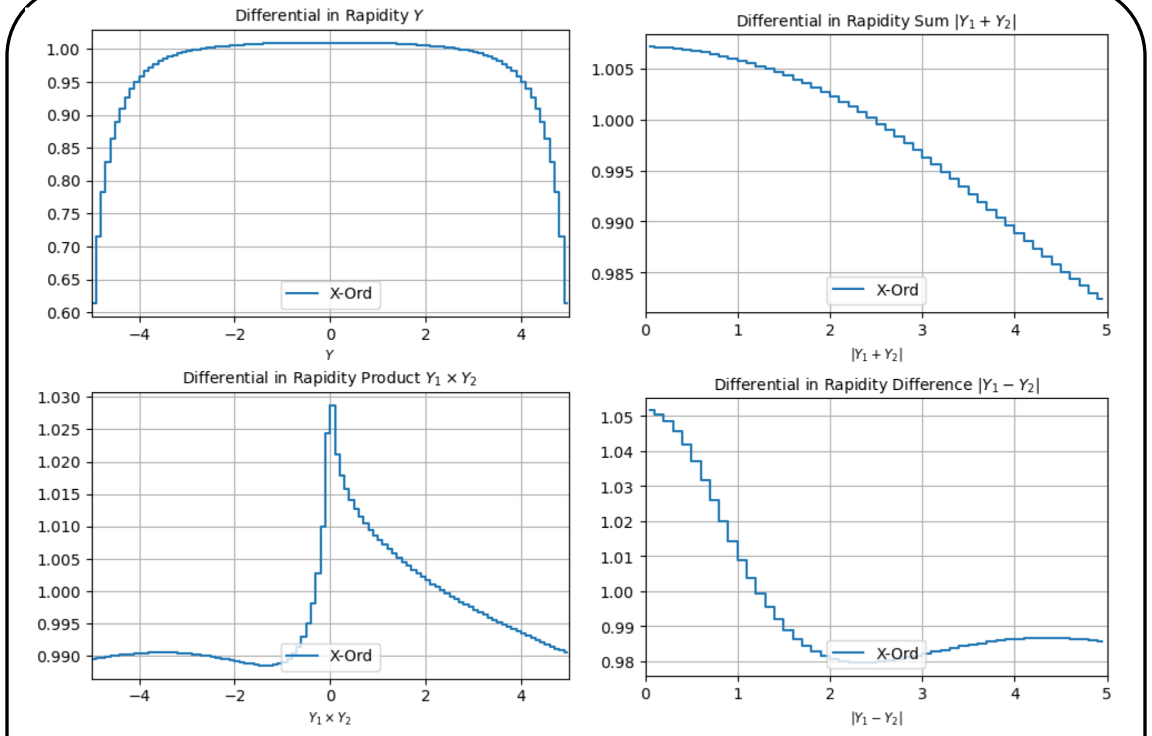
$$A = \frac{\sigma(Y_1 \times Y_2 < 0) - \sigma(Y_1 \times Y_2 > 0)}{\sigma(Y_1 \times Y_2 < 0) + \sigma(Y_1 \times Y_2 > 0)}$$

- Differential cross-sections, in
  - Rapidity of individual bosons Y_i
  - Rapidity product $Y_1 \times Y_2$
  - Absolute rapidity sum $|Y_1 + Y_2|$
  - Absolute rapidity difference $|Y_1 - Y_2|$

Double-Z Differential Cross-Sections

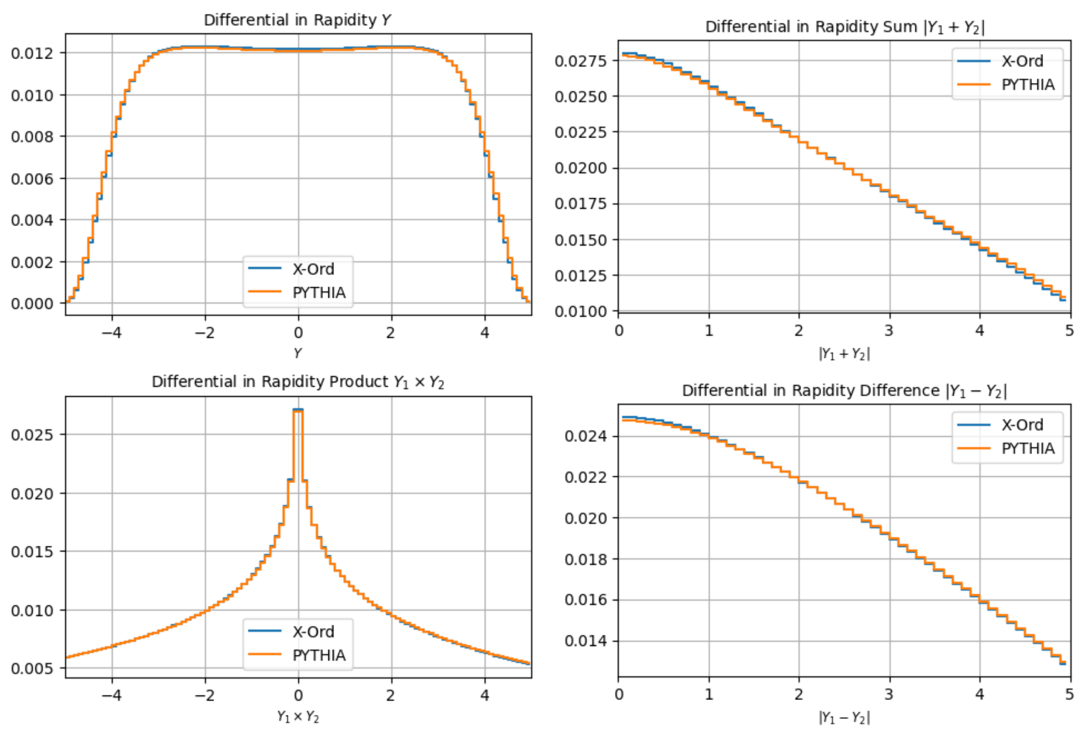


DZ Differential Cross-Sections

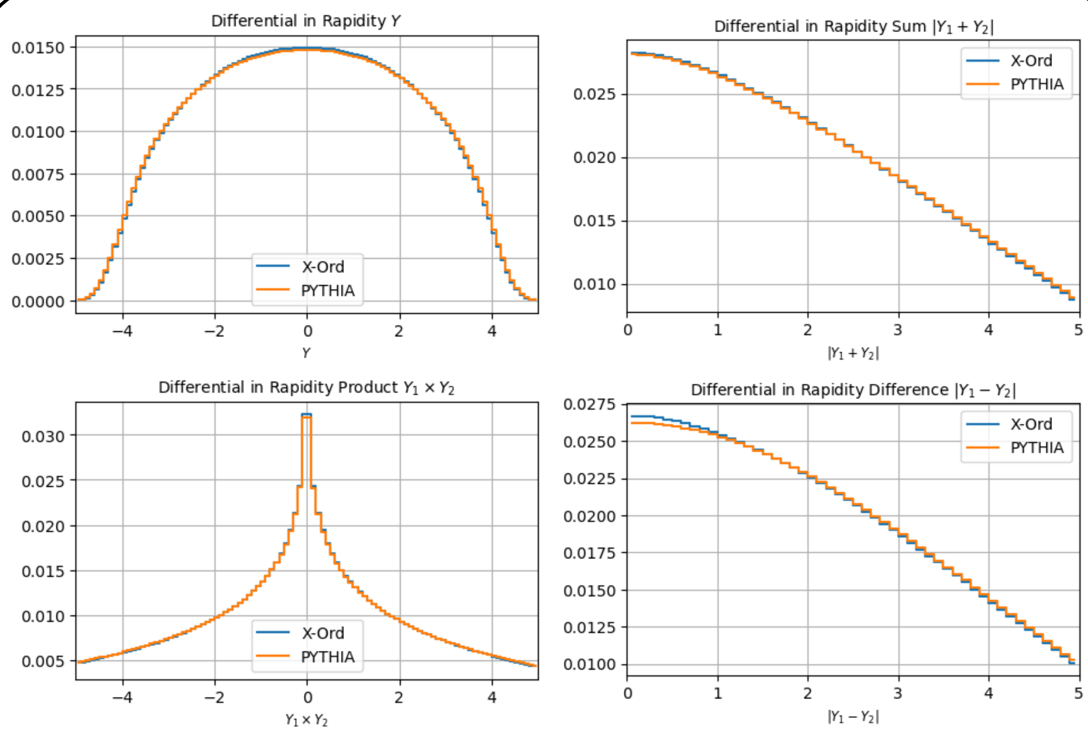


DZ Diff. Cross-Section ratios to PYTHIA

Double-W Differential Cross-Sections

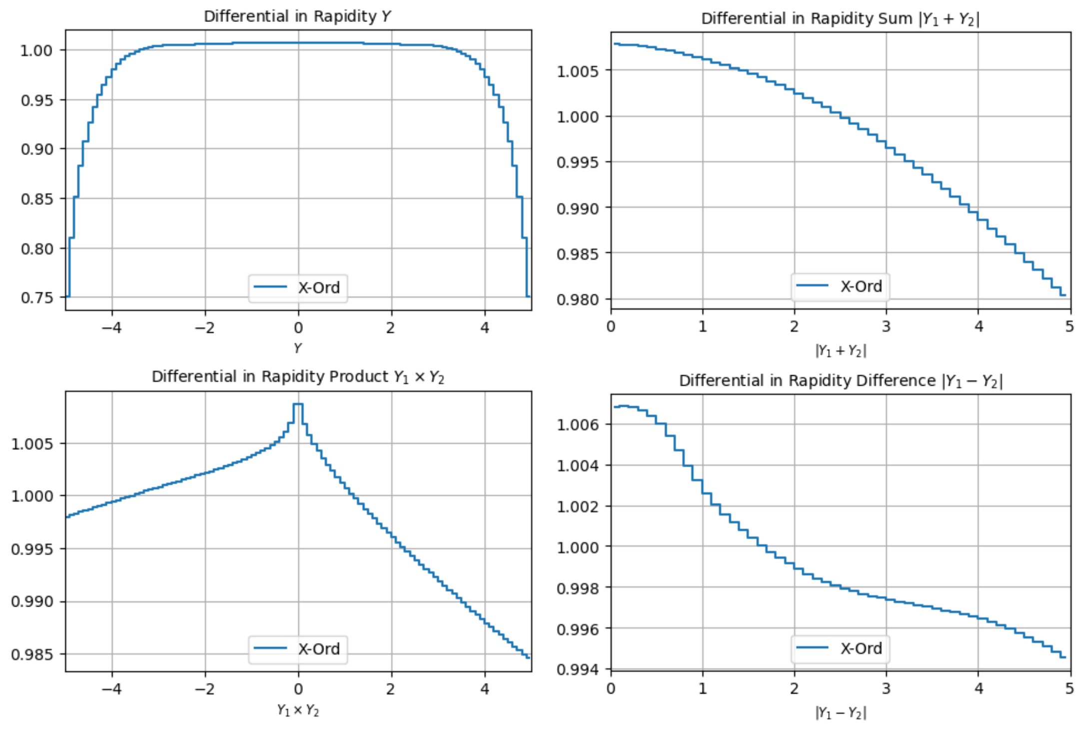


DW+ Differential Cross-Sections

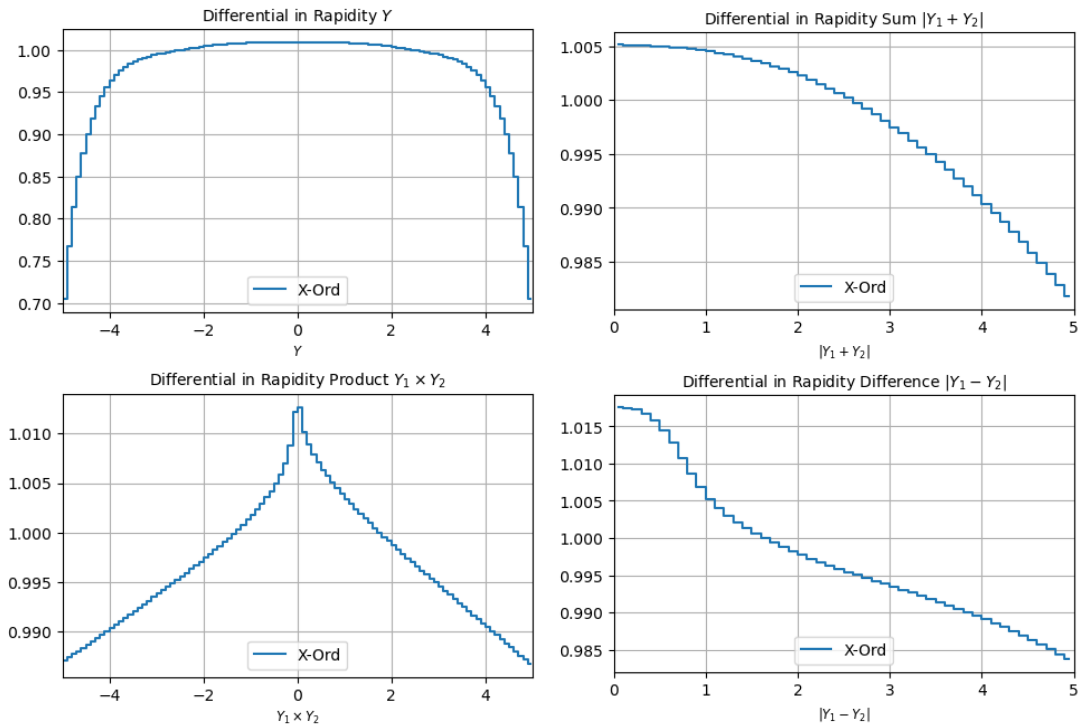


DW- Differential Cross-Sections

Double-W Differential Cross-Sections



DW+ Diff. Cross-Section ratios to PYTHIA

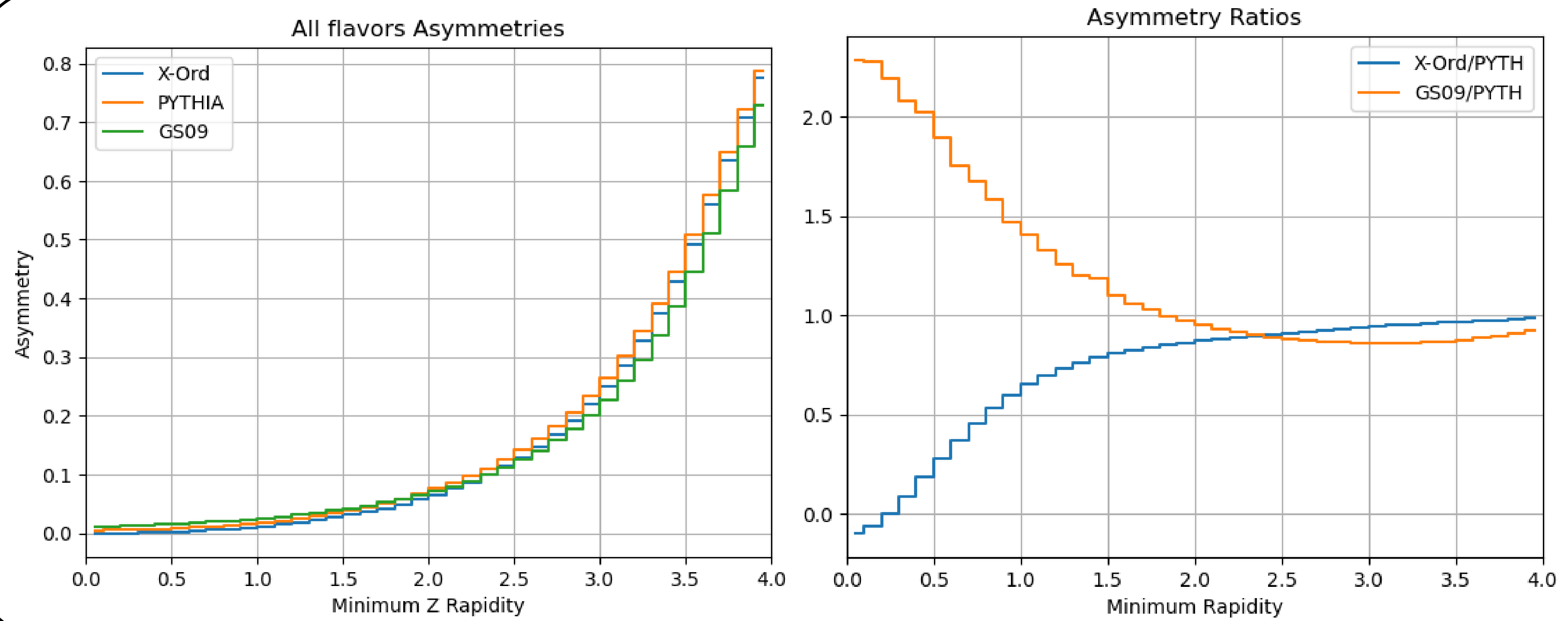


DW- Diff. Cross-Section ratios to PYTHIA

Small but appreciable difference between dPDF prescriptions

- Ratio similarity between Diff. cscs – universal effect from X-ordering procedure
- DZ and DW $Y_1 \times Y_2$ vary strongly – likely a companion

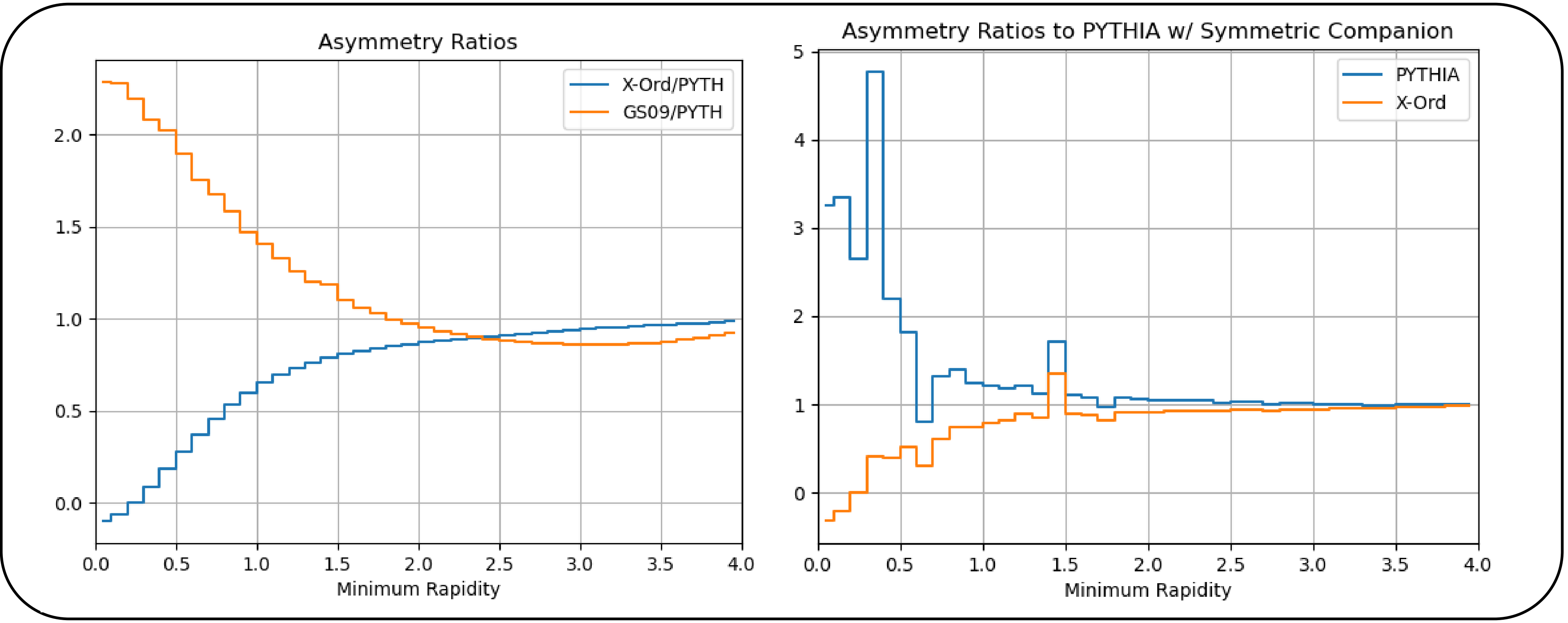
Double Z Asymmetry



Non-negligible difference between the dPDF prescriptions!

- X-ordered Asymmetry negative at low rapidity
- Uniformly lower than PYTHIA overall

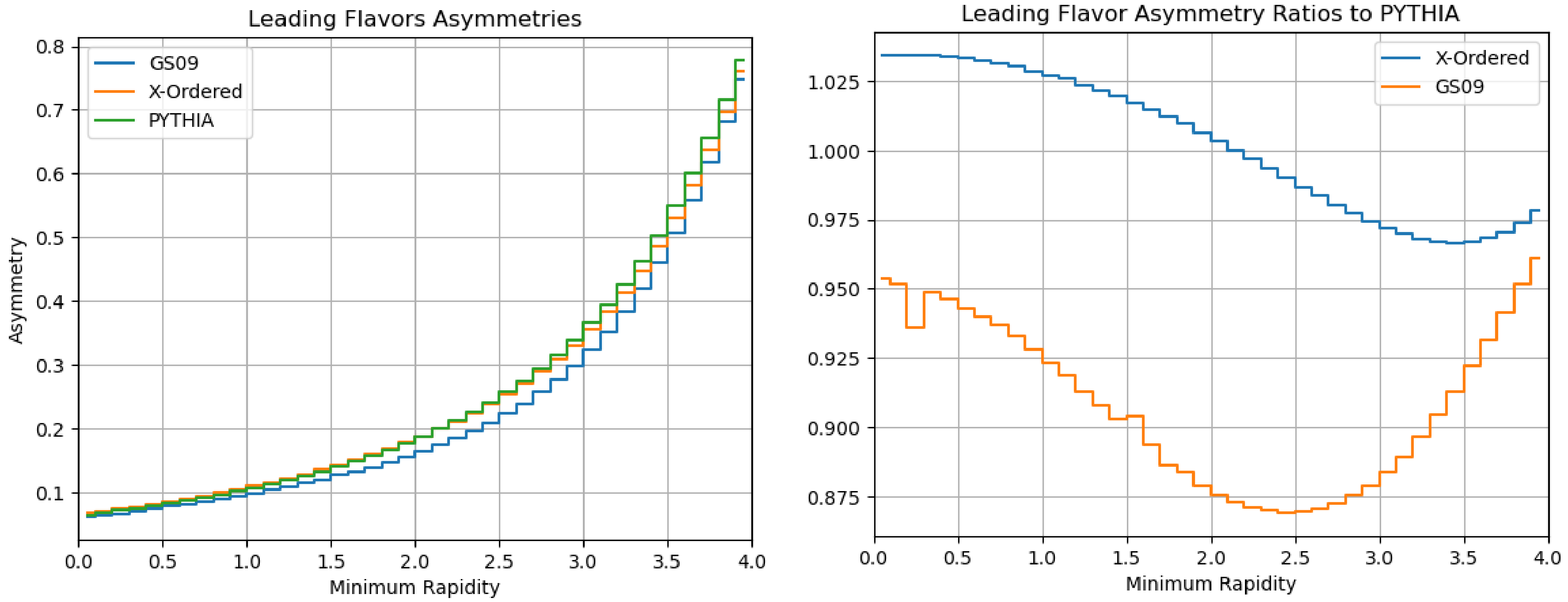
Double Z Asymmetry



Evaluated PYTHIA Asymmetry with new companion quark term

- Clear driver of lower asymmetry for Double Z
- Only appears in 5/25 considered flavor combinations
  - How do the other modifications affect the Asymmetry?

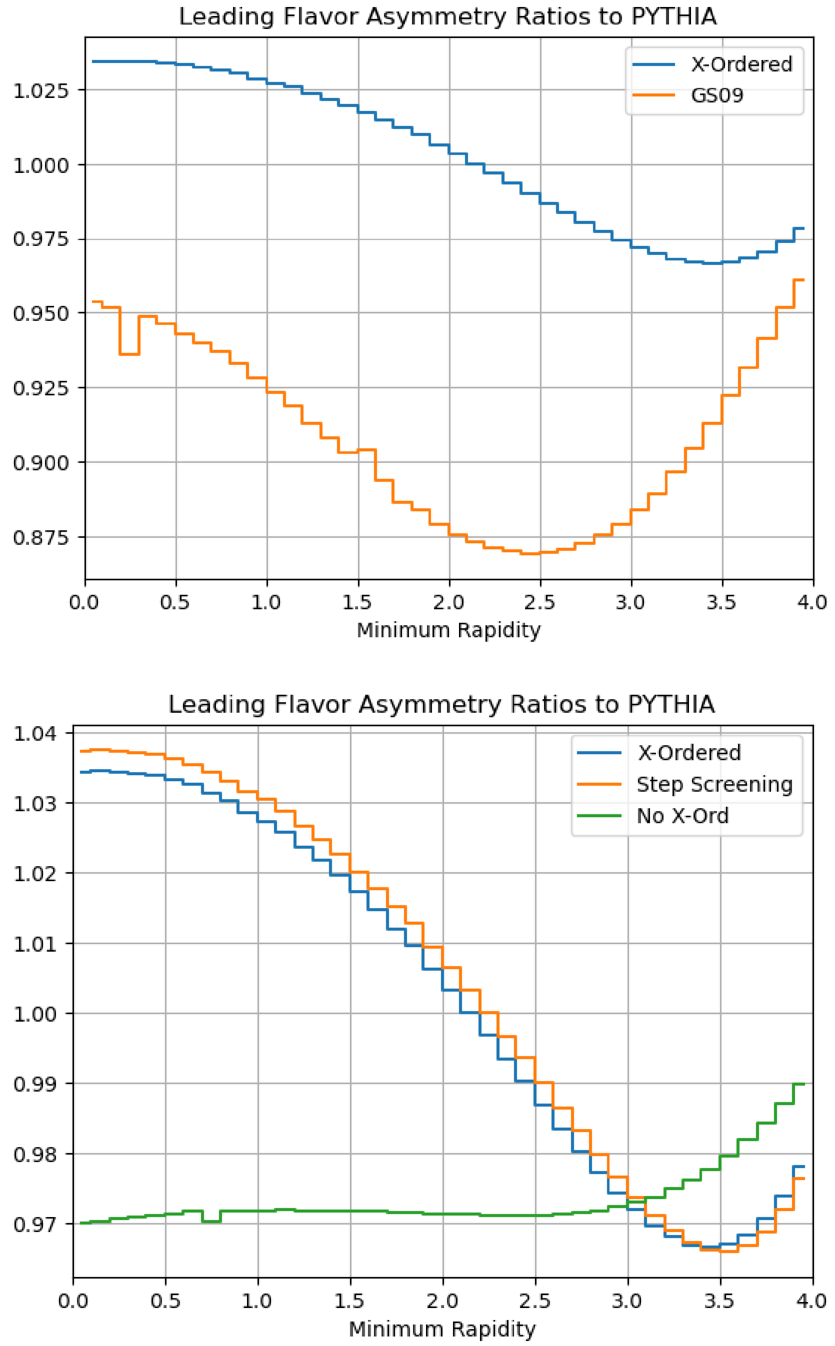
Double W+ Asymmetry



Much smaller difference between prescriptions…

- Not uniform difference
- Similar pattern to GS09

Double W+ Asymmetry

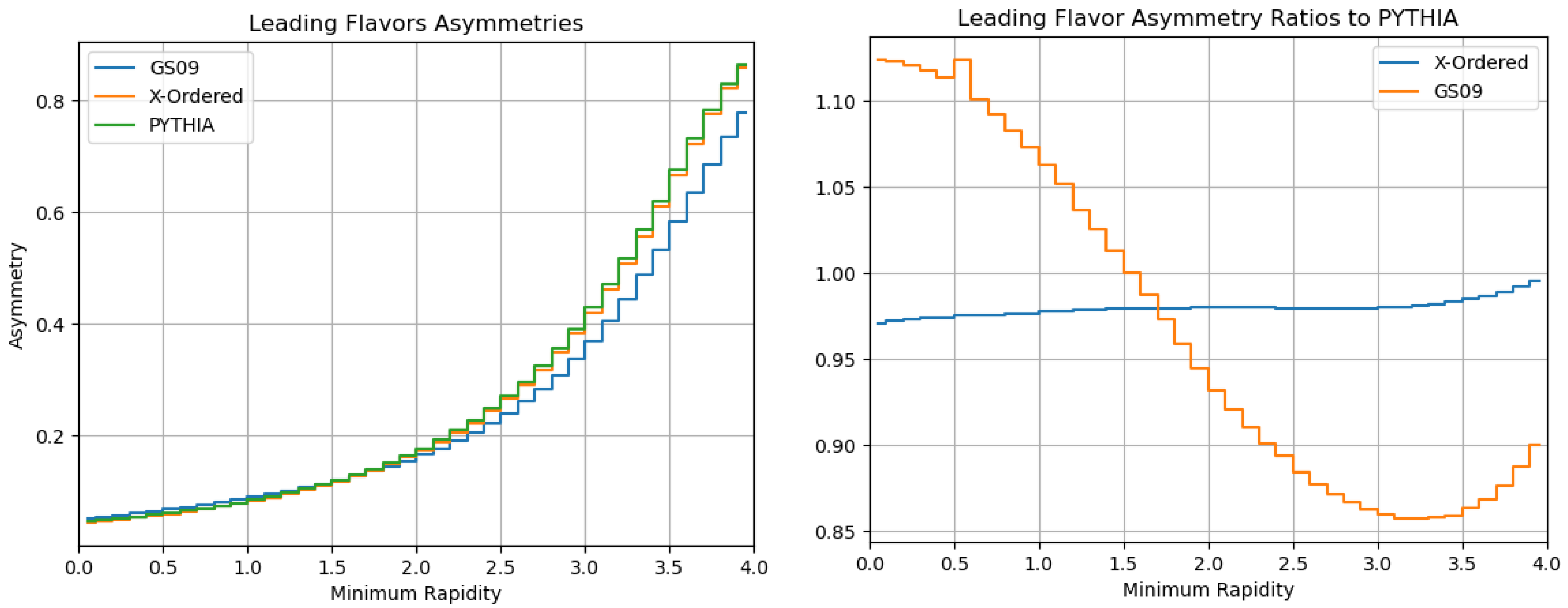


Turn on/off various components of our prescription

- Change the smooth screening function
  - Effects are small, sub-percent
  - Removes "swapped" term
- Turning off the X-Ordering
  - Completely new pattern
  - Small indirect effect of new companion mechanism
  - Inclusion of "swapped" term

X-Ordering procedure responsible for systematic difference to PYTHIA

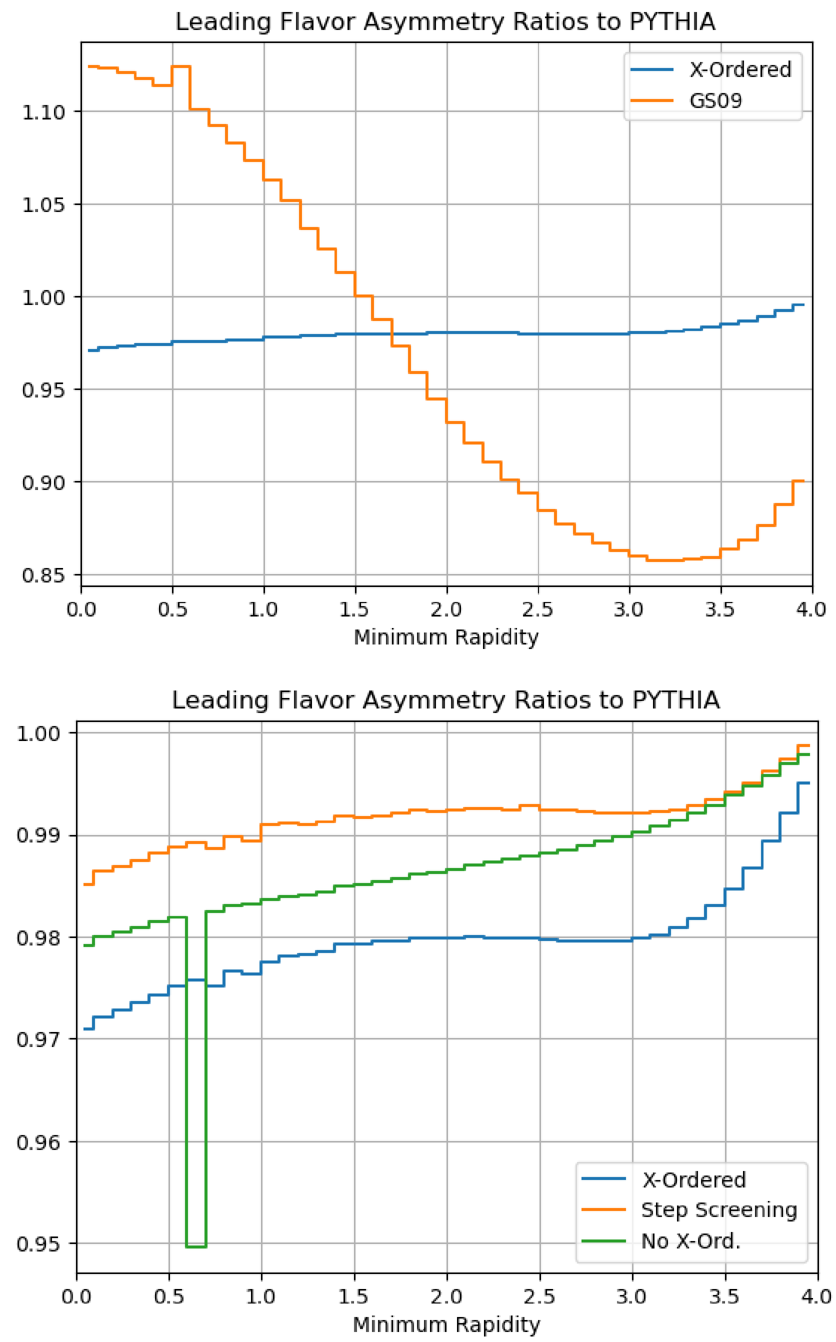
Double W- Asymmetry



Small deviation, as with W+

- Uniformly smaller than PYTHIA
- Much closer to PYTHIA than GS09

Double W- Asymmetry



Turn on/off various components of our prescription

- Change the smooth screening function
  - Larger effect than in W+
  - "Swapped" term more relevant for W- quark flavors

- Turning off the X-Ordering
  - Closes the gap to PYTHIA as in W+

X-Ordering procedure less dominant, impact shared with smooth inclusion of "swapped" term

Summary/Conclusions

- Proposed three modifications to the PYTHIA model of MPI that improve adherence to the GS sum rules:
  - Change the companion quark mechanism to one that is manifestly symmetric.
  - X-order the PDFs instead of naïvely symmetrise to avoid overcontribution from “incorrect” PDFs.
  - Damp out the low x_1, x_2.
- These changes are all symmetric in $\{x_i, j_1\} \leftrightarrow \{x_k, j_k\}$, and have improved the GS sum rule adherence of the symmetrised DPDFs to a <10% deviation from theory.
  - Have additionally allowed for construction of symmetrised TPDFs which have a <10% deviation from theory for the majority (~92%) of flavors.
  - Structure of PYTHIA algorithm hinders improvement

- Lead to change in some simple observables for DPS processes

- Large changes in asymmetry for Double Z
  - New companion mechanism
    - Largest change observed
    - Drives lower asymmetry, even when damped by other flavor combinations

- Asymmetry changed marginally for Double W+/W-
  - X-Ordering
    - Strongest Impact on asymmetry for SSW
    - Effect is dependent on dPDF flavors