Phase-space Distributions of Nuclear Short-Range Correlations

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Joint Session CNM / DPS-3D Structure
Jan 14, 2025

- Based on `WC, J. Ryckebusch 2106.01249, PLB('21)`
- Wigner distributions of SRC nucleons
- Interest for heavy ion community?
  - medium modification
  - centrality

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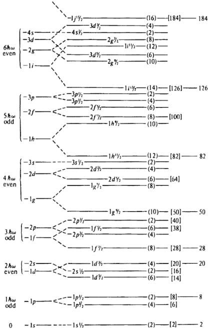
Independent Particle Model (IPM)



- Solve 1b Schrodinger equation in a **mean-field** potential
- Nucleons have an identity: $\alpha_i(\boldsymbol{n_i}, \boldsymbol{l_i}, \boldsymbol{j_i}, \boldsymbol{m_i}, \boldsymbol{t_i})$ and $\psi_{\alpha_i}(\vec{r})$
- Average quantities: $\langle T_p \rangle, \langle U_{pot} \rangle, \langle \rho \rangle, \ldots$

Nuclei in all their facets: IPM, SRC, LRC

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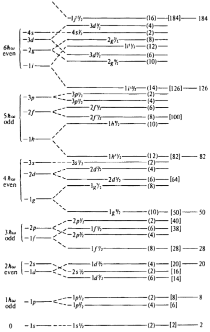

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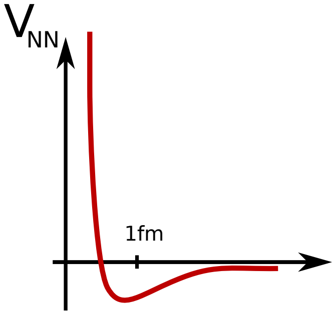
Long Range Correlations (LRC)

- Nucleons lose their identity
- Spatio-temporal fluctuations: $\Delta T_p, \Delta U_{pot}, \Delta\rho, \ldots$
- "Most" nucleons get involved ($\sim R_A$)
- Energy scale $\Delta E \approx$10 MeV
- Exp. observed, th. understood [giant resonances in $\gamma^{(*)}(A, X)$]

Short Range Correlations (SRC)

- Nucleons lose their identity
- Spatio-temporal fluctuations: $\Delta T_p, \Delta U_{pot}, \Delta\rho, \ldots$
- "Few" nucleons get involved ($\sim R_N$)
- Energy scale $\Delta E \approx$100 MeV
- Exp. observed, th. understood [2N knockout in $A(e, e'X)$]

Nuclear short-range correlations (SRC)



Warning: reductive picture!!

- *NN*-force: intermediate-range attraction, short-range repulsion ("hard core")
- Induce high-momentum tails in momentum distributions
- Universal across the nuclear mass range (**local** character of SRC)
- In experiments, one-body and two-body momentum distributions are **not directly observable** and the obtained information on SRC is indirect
- f.i. $A(e, e'p)$ cross section only **factorizes** in non-relativistic plane-wave (=no final-state interactions) approximation

$$d\sigma_A^{(e,e'p)} = K\sigma^{ep}\rho(\vec{p}_m)$$

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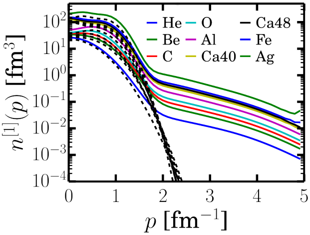

J. Ryckebusch et al.,JPG42 055104 ('15)

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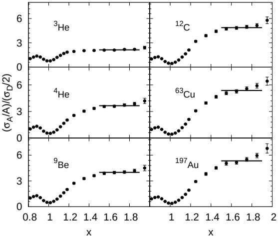
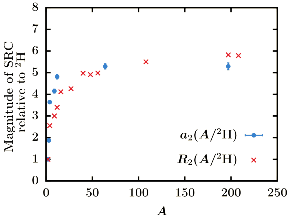


data: Fomin et al. (JLab Hall C), PRL108 092502



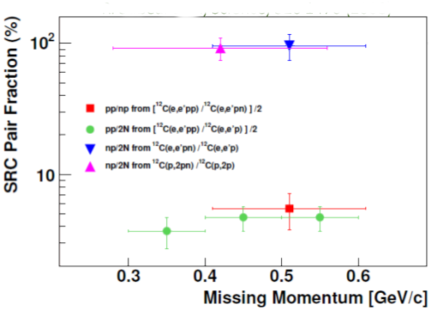
Vanhalst, W.C, Ryckebusch, JPG'15

- SRC **universality**: Cross section ratios to the deuteron show **scaling** for 1.4<x<2
- $\sigma^A = a_2 \frac{A}{2} \sigma^D \rightarrow a_2$ is **measure** for the relative amount of correlated pairs in nucleus A to the deuteron $\rightarrow$ **soft scaling!**

  $\rightarrow$ Mean-Field quantity!!!
- Compared to deuteron correlated pair in nucleus A also has
  - Binding energy
  - Center of mass motion
  - Final-state interactions with nuclear medium
- a_2 are correlated with the size of the **EMC effect**
  $\rightarrow$Hen et al.,Int.J.Mod.Phys. E22 (2013) 1330017

Exclusive $A(e, e'pp)$

2N correlations in ^{12}C$(e, e'pp)$ / ^{12}C$(e, e'p)$ JLAB Hall A

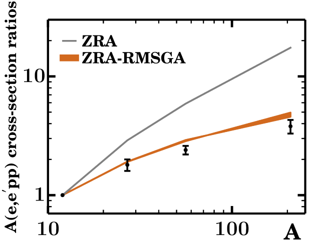


R. Subedi et al., Science 320 ('08)
R. Shneor et al., PRL99 ('07)

- Detector setup covering very small phase space: tuned to initial back-to-back nucleons
- Assumption A(e,e'p)=A(e,e'pp)+A(e,e'pn) to extract SRC fractions

- 20% of the nucleons are in a SRC pair
- 90% of the correlated pairs are *np* pairs → **tensor force** dominance for these initial momenta

Mass dependence of pp cross section ratio



C.Colle et al. PRC92 024604 ('15)

- $\frac{\sigma[A(e,e'pN)]}{\sigma[^{12}C(e,e'pN)]} \approx \frac{\int \mathrm{d}^3\vec{P}_{12} F^D_{\mathrm{A}}(\vec{P}_{12})}{\int \mathrm{d}^3\vec{P}_{12} F^D_{12_{\mathrm{C}}}(\vec{P}_{12})}$
- Data from data mining initiative for the Jefferson Lab CLAS collaboration (4π detector, **huge phase space**)
- Calculations performed for ^{12}C,^{27}Al,^{56}Fe and ^{208}Pb.
- Cross section ratios scale much softer than $Z(Z-1)$
- Final-state interactions soften the mass dependence further
- Charge-exchange effects in final-state interactions also taken into account

Motivation & framework for Wigner study

- Distribution of SRCs in phase space?
  - generate a high-momentum tail in the 1b momentum distribution
  - where in the nuclear medium?
  - phase-space correlations?

- For SRCs, what regions of r and/or p influence bulk properties (T, r_{rms})

- LCA: lowest-order correlation operator approximation [Ghent group 2010+]
  - approximate flexible method across the whole mass range

    → Wigner distribution results for deuteron considered in [Neff, Feldmeier 2016]
  - include essential SRC operators (central,tensor,spin-isospin), no LRCs
  - learn about SRC physics (nuclear structure AND reactions) in a unified framework
  - excellent agreement with extracted quantities from data and ab initio results

- Inputs: HO parameters, radial correlation functions

- Systematic study yielded robust results

Nuclear correlation operators (I)

- Correlated nuclear wave funtion Ψ: act with **correlation operators** $\hat{\mathcal{G}}$ (short-range structure) on Φ (mean-field quantum numbers + long-range structure)

$$
\mid \Psi \rangle = \frac{1}{\sqrt{\mathcal{N}}} \widehat{\mathcal{G}} \mid \Phi \rangle \qquad \text{with,} \qquad \mathcal{N} \equiv \langle \Phi \mid \widehat{\mathcal{G}}^{\dagger} \widehat{\mathcal{G}} \mid \Phi \rangle
$$

  in our case $\mid \Phi \rangle$ is an IPM single Slater determinant

- Nuclear correlation operator $\widehat{\mathcal{G}}$ contains two-nucleon correlation operators $\hat{l}(i,j)$ (A-body operator):

$$
\widehat{\mathcal{G}} \approx \widehat{\mathcal{S}} \left(\prod_{i<j=1}^{A} \left[1 - \hat{l}\,(i,j) \right] \right) ,
$$

- Major source of correlations: central (Jastrow), tensor ($t\tau$) and spin-isospin ($\sigma\tau$)

$$
\hat{l}\,(i,j) = -g_c(r_{ij}) + f_{t\tau}(r_{ij}) \widehat{S}_{ij} \vec{\tau}_i \cdot \vec{\tau}_j + f_{\sigma\tau}(r_{ij}) \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\tau}_i \cdot \vec{\tau}_j \ .
$$

Nuclear correlation operators (II)

- Expectation values between **correlated states** Ψ can be turned into expectation values between **uncorrelated states** Φ

$$\langle \Psi \mid \widehat{\Omega} \mid \Psi \rangle = \frac{1}{\mathcal{N}} \langle \Phi \mid \widehat{\Omega}^{\text{eff}} \mid \Phi \rangle$$

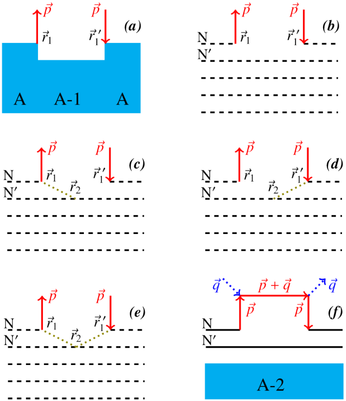
- "Conservation Law of Misery": multi(A)-body operators

$$\widehat{\Omega}^{\text{eff}} = \widehat{\mathcal{G}}^{\dagger}\, \widehat{\Omega}\, \widehat{\mathcal{G}} = \left(\prod_{i<j=1}^{A} \left[1 - \hat{l}(i,j) \right] \right)^{\dagger} \widehat{\Omega} \left(\prod_{k<l=1}^{A} \left[1 - \hat{l}(k,l) \right] \right)$$

- Low-order correlation operator approximation (LCA): cluster expansion truncated at lowest order
- LCA: N-body operators receive SRC-induced $(N+1)$-body corrections

Dominant contribution to SRC-sensitive matrix elements stems from **relative $n = 0, l = 0$ pairs** in the IPM wf [strength at $r \to 0$]

Single-nucleon momentum distributions in LCA

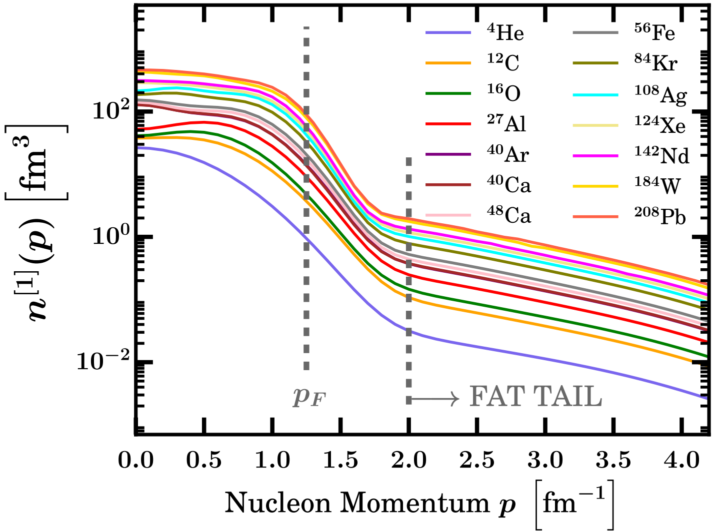


- Single-nucleon momentum distribution $n^{[1]}(p)$
- Universal correlation operators

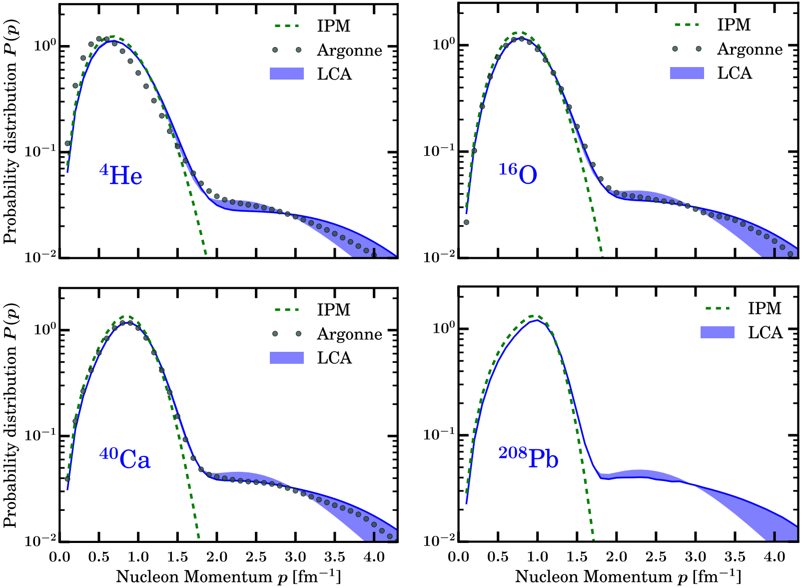
$$|\Psi\rangle = \widehat{\mathcal{G}}\,|\Phi\rangle \,/\sqrt{\langle\Phi|\,\widehat{\mathcal{G}}^{\dagger}\widehat{\mathcal{G}}\,|\Phi\rangle}\ ,$$

- $\mathcal{G}$: Central, spin-isospin, tensor
- Truncation at $\mathcal{O}\left(\mathcal{G}^2\right)$: SRC part of $n^{[1]}(p)$ = 2-body contributions
- Quantify the *pp*, *nn*, *pn* and *np* contribution to $n^{[1]}(p)$
- Capture essential SRC physics and study trends

$n^{[1]}(p)$ in LCA: from light to heavy nuclei



Probability distribution $P(p) \sim p^2 n^{[1]}(p)$



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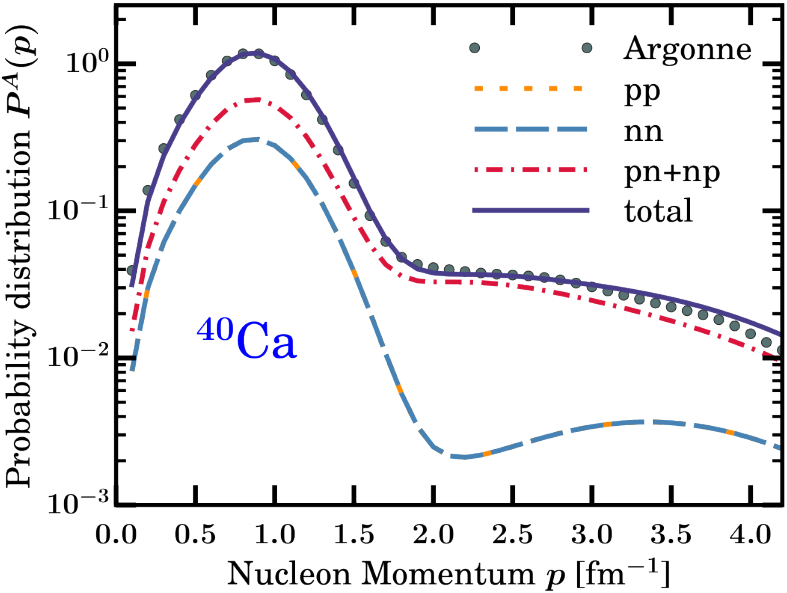
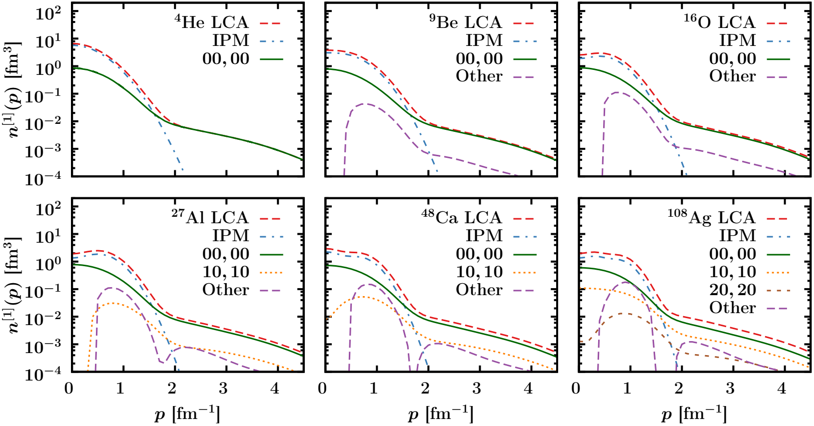


Quantum numbers of SRC-susceptible IPM pairs?

$n^{[1],\mathrm{corr}}$ stems from correlation operators acting on IPM pairs.

$$\sum_{n'l'} n^{[1],\mathrm{corr}}_{nl,n'l'}(p) = n^{[1],\mathrm{corr}}(p)$$



Major source of SRC: correlations acting on ($n = 0$ $l = 0$) **IPM pairs**

Wigner distributions

- Phase-space formulation of QM

$$\langle \widehat{F} \rangle = \iint d\boldsymbol{r} d\boldsymbol{k}\; w(\boldsymbol{r},\boldsymbol{k}) f(\boldsymbol{r},\boldsymbol{k})$$

- Wigner quasidistribution $w(r,k) = \langle \Psi | \, \widehat{w}(\boldsymbol{r},\boldsymbol{k}) \, |\Psi\rangle$

$$\widehat{w}(\boldsymbol{r},\boldsymbol{k}) = \frac{1}{(2\pi)^3} \int d\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \left|\boldsymbol{r} - \frac{\boldsymbol{x}}{2}\right\rangle\left\langle \boldsymbol{r} + \frac{\boldsymbol{x}}{2}\right| = \frac{1}{(2\pi)^3} \int d\boldsymbol{q}\; e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \left|\boldsymbol{k} + \frac{\boldsymbol{q}}{2}\right\rangle\left\langle \boldsymbol{k} - \frac{\boldsymbol{q}}{2}\right|$$

$$\hat{\rho}(\boldsymbol{r}) = \int d\boldsymbol{k}\; \widehat{w}(\boldsymbol{r},\boldsymbol{k}) = |\boldsymbol{r}\rangle\langle\boldsymbol{r}| \qquad \hat{n}(\boldsymbol{k}) = \int d\boldsymbol{r}\; \widehat{w}(\boldsymbol{r},\boldsymbol{k}) = |\boldsymbol{k}\rangle\langle\boldsymbol{k}|\,,$$

- Quasi-expectation values (do **NOT** integrate to total T, r_{rms})

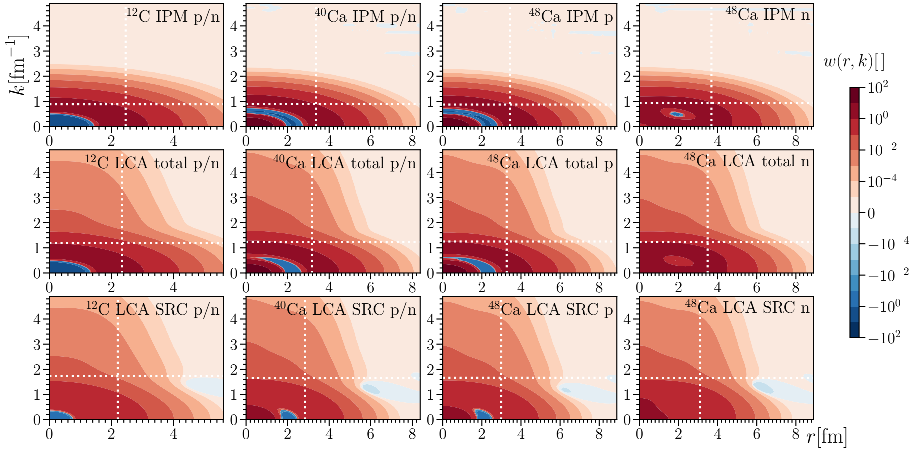
$$T(r) = \left\langle \widehat{T}(r) \right\rangle = \frac{\int k^2 dk\; \frac{k^2}{2m}\; w(r,k)}{\int k^2 dk\; w(r,k)}\,, \qquad r_{\mathrm{rms}}(k) \equiv \sqrt{\langle \hat{r}^2(k)\rangle} = \sqrt{\frac{\int r^2 dr\; r^2 w(r,k)}{\int r^2 dr\; w(r,k)}}\,.$$

- Densities (**DO** integrate to total T, r_{rms})

$$\rho_T(r) \equiv \frac{r^2 \int k^2 dk\; \frac{k^2}{2m}\; w(r,k)}{\int r^2 dr \int k^2 dk\; w(r,k)}\,, \qquad \int dr \rho_T(r) = \left\langle \widehat{T} \right\rangle = T\,;$$

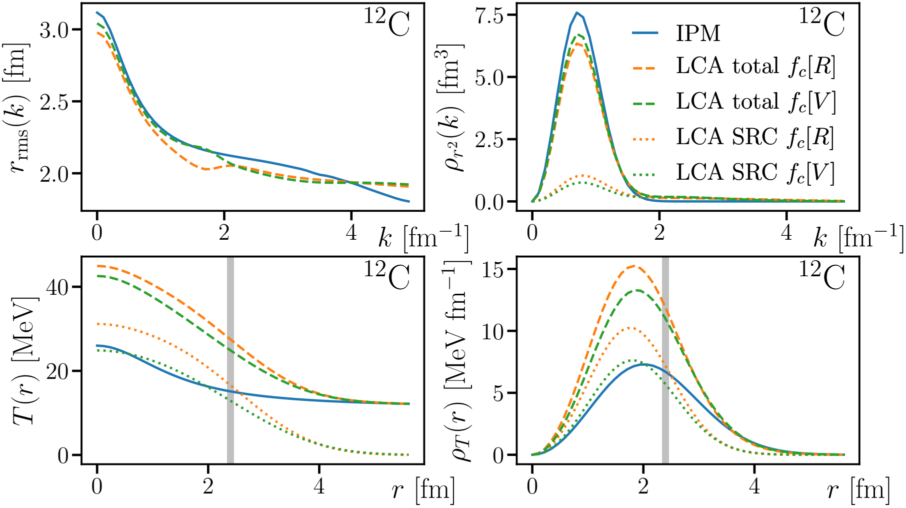
$$\rho_{r^2}(k) \equiv \frac{k^2 \int r^2 dr\; r^2 w(r,k)}{\int k^2 dk \int r^2 dr\; w(r,k)}\,, \qquad \int dk \rho_{r^2}(k) = \langle \hat{r}^2 \rangle = r^2_{\mathrm{rms}}$$

$w(r,k)$ numerical results



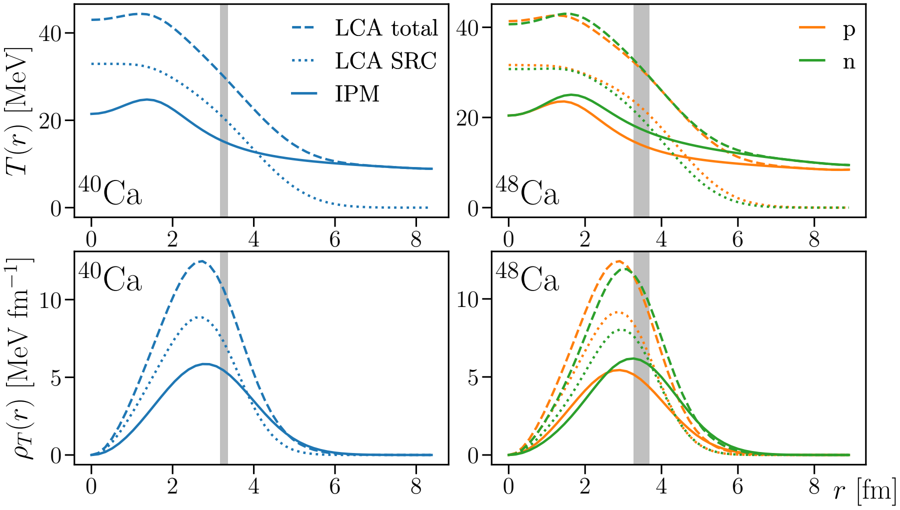
- High-momentum SRCs restricted to interior
  $\rightarrow$ generated from IPM relative S-pairs

^{12}C rms radius and kinetic energy



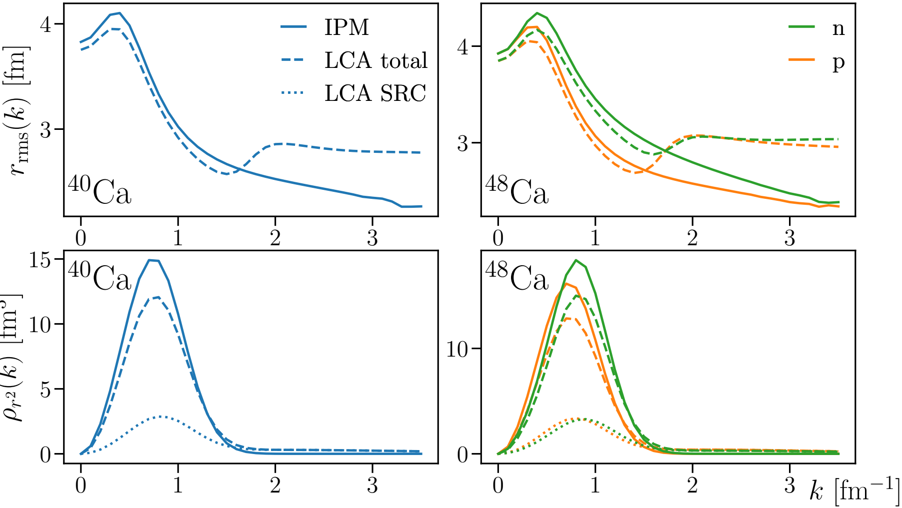
- rms radius dominated by $k < 2\text{fm}^{-1}$
- For $r < r_{\rm rms}$ 60–70% of T due to SRC

Ca kinetic energy



- Kinetic energy inversion in interior ^{48}Ca

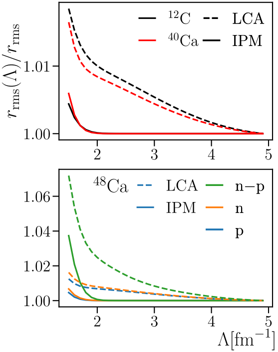
Ca rms radius



- In ^{48}Ca, due to tensor force $r_p \approx r_n$ for $k > 2\text{fm}^{-2}$

Influence of SRC on bulk properties

Model	^{12}C $T_{p,n}$	^{12}C $r_{p,n}$	^{40}Ca $T_{p,n}$	^{40}Ca $r_{p,n}$	^{48}Ca T_p	^{48}Ca T_n	^{48}Ca T_n-T_p	^{48}Ca r_p	^{48}Ca r_n	^{48}Ca r_n-r_p
IPM $\hbar\omega[d]$	16.1	2.46	16.5	3.36	15.7	18.0	2.2	3.44	3.68	0.237
LCA $\hbar\omega[d]/f_c[R]$	29.7	2.34	31.9	3.17	32.5	31.6	-1.0	3.27	3.48	0.216
LCA $\hbar\omega[d]/f_c[V]$	26.8	2.40	28.9	3.22	29.5	28.7	-0.8	3.31	3.53	0.221
IPM $\hbar\omega[f]$	18.3	2.30	17.2	3.29	16.2	18.5	2.3	3.39	3.63	0.234
LCA $\hbar\omega[f]/f_c[R]$	30.4	2.31	30.1	3.28	30.5	29.5	-0.9	3.39	3.62	0.226
LCA $\hbar\omega[f]/f_c[V]$	28.7	2.32	27.8	3.28	28.2	27.5	-0.8	3.39	3.62	0.227



- Introduce momentum cutoff Λ in phase-space integral for $r_{\rm rms}$

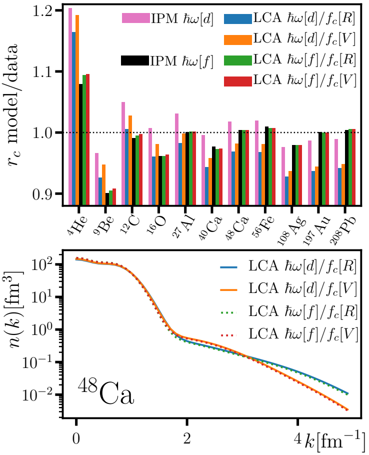
$$\langle r^2 \rangle_\Lambda = \frac{\int^\Lambda k^2 dk \int r^2 dr\; r^2 w(r,k)}{\int^\Lambda k^2 dk \int r^2 dr\; w(r,k)}$$

- Radii and size of neutron skin increase when not accounting for high-momentum SRC consistently

Conclusions

- LCA provides a comprehensive picture of SRC-sensitive observables
- Phase-space formulation yields combined coordinate and momentum space information
- High-momentum SRC confined to **nuclear interior**
- **Large increase** in kinetic energy + inversion for $N > Z$, generated in interior
- **Modest effect** on rms radii and neutron skin, but non-negligible effect

Inputs



IPM $\hbar\omega[d]$
LCA $\hbar\omega[d]/f_c[R]$
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r_c model/data
1.2
1.1
1.0
0.9
^{4}He ^{9}Be ^{12}C ^{16}O ^{27}Al ^{40}Ca ^{48}Ca ^{56}Fe ^{108}Ag ^{197}Au ^{208}Pb
$n(k)[\text{fm}^3]$
10^2
10^1
10^0
10^{-1}
10^{-2}
^{48}Ca
0
2
4
$k[\text{fm}^{-1}]$