Mixing of flavor-singlet light meson, charmonium and gluonic operators with optimal distillation profiles

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Motivation

Why is this mixing important?

? Relevant for calculating the low-lying flavor-singlet meson spectrum.

? States of theoretical and experimental interest are in this channel, e.g (recent) glueball candidates M. Ablikim *et al.* (BESIII), Phys. Rev. Lett., 132 (2024), exotics, hybrids, etc...

? Understanding the mixing dynamics between light meson, charmonium and gluonic operators can give some insights on the composition of these states of interest.

Challenges:

! Signal-to-noise problem, particularly in *disconnected* correlations.

! How to saturate the spectrum? ”Physical”-looking operators, multi-particle operators, etc...

! Presence of dynamical quarks degrades the efficacy of gluonic operators: what is a glueball in this setup?

Mixing of flavor singlets

We work with $N_f = 3+1$ QCD $\rightarrow$ SU(3) light flavor symmetry.

- Flavor singlet: f_0, η_c, glueballs, singlet 2-pion states, etc...
- Flavor octet: π, a_0, octet 2-pion states, etc...

Can we study charmonia, light mesons and glueballs separately? **No**. All flavor singlets are in the **same** lattice symmetry channel! **Mixing!**

$\langle \chi_{c0} | \mathcal{O}^\dagger_{\bar{c}c} | \Omega \rangle \neq 0 \rightarrow$ This is expected!

$\langle f_0 | \mathcal{O}^\dagger_{\bar{c}c} | \Omega \rangle \neq 0 \rightarrow$ This is the issue!

$$C_{\bar{c}c}(t) \overset{t\rightarrow\infty}{\approx} | \langle f_0 | \mathcal{O}^\dagger_{\bar{c}c} | \Omega \rangle |^2 e^{-E_{f_0} t} \neq | \langle \chi_{c0} | \mathcal{O}^\dagger_{\bar{c}c} | \Omega \rangle |^2 e^{-E_{\chi_{c0}} t}$$

Energies obtained from charmonium operators ***do not*** *necessarily correspond to charmonium energy eigenstates.*

This mixing can be **artificially** removed to some extent by **ignoring charm annihilation effects** but doing so is **not** strictly correct.

Can we gain some insight on the nature of the different states?

Flavor SU(3) states are labeled by $|D, Y, I, I_3\rangle$ and we are interested in $|1, 0, 0, 0\rangle$:

- Meson interpolators: $\mathcal{O}_c = \bar{c}c$, $\mathcal{O}_l = \frac{1}{\sqrt{3}}\left(\bar{u}u + \bar{d}d + \bar{s}s\right)$
- Gluonic interpolators: $\mathcal{O}_g =$ , , etc...
- 2-pion interpolators: $\mathcal{O}_{2\pi}$ built from $8 \otimes 8 = \mathbf{1} \oplus 8 \oplus 8' \oplus 10 \oplus \bar{10} \oplus 27$ via SU(3) Clebsch-Gordan coefficients. P. McNamee & F. Chillton, Rev. Mod. Phys 36 (1964)

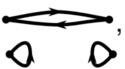
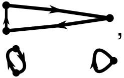
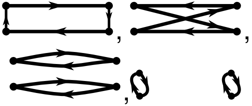
We build a **mixing** correlation matrix

$$\begin{pmatrix} \langle \mathcal{O}_l(t)\bar{\mathcal{O}}_l(0)\rangle & \langle \mathcal{O}_l(t)\bar{\mathcal{O}}_c(0)\rangle & \langle \mathcal{O}_l(t)\bar{\mathcal{O}}_{2\pi}(0)\rangle & \langle \mathcal{O}_l(t)\bar{\mathcal{O}}_g(0)\rangle \\ * & \langle \mathcal{O}_c(t)\bar{\mathcal{O}}_c(0)\rangle & \langle \mathcal{O}_c(t)\bar{\mathcal{O}}_{2\pi}(0)\rangle & \langle \mathcal{O}_c(t)\bar{\mathcal{O}}_g(0)\rangle \\ * & * & \langle \mathcal{O}_{2\pi}(t)\bar{\mathcal{O}}_{2\pi}(0)\rangle & \langle \mathcal{O}_{2\pi}(t)\bar{\mathcal{O}}_g(0)\rangle \\ * & * & * & \langle \mathcal{O}_g(t)\bar{\mathcal{O}}_g(0)\rangle \end{pmatrix}$$

and from the GEVP we obtain an *optimal* interpolator

$$\tilde{\mathcal{O}}^{(n)} = \mathbf{a}_\mathbf{1}^\mathbf{n}\mathcal{O}_c + \mathbf{a}_\mathbf{2}^\mathbf{n}\mathcal{O}_l + \mathbf{a}_\mathbf{3}^\mathbf{n}\mathcal{O}_g + \mathbf{a}_\mathbf{4}^\mathbf{n}\mathcal{O}_{2\pi},\ a_i^n \in \mathbb{R}$$

which maximizes $\langle n|\,\hat{\tilde{\mathcal{O}}}^{(n)\dagger}\,|\Omega\rangle$.

	$\mathcal{O}_l$	$\mathcal{O}_c$	$\mathcal{O}_{2\pi}$	$\mathcal{O}_g$
$\mathcal{O}_l$				
$\mathcal{O}_c$	-			
$\mathcal{O}_{2\pi}$	-	-		
$\mathcal{O}_g$	-	-	-	

Disconnected correlations are **necessary** for flavor-singlets but have **large** statistical errors.

- **Explicit mixing:** including flavor-mixing off-diagonals.
- **Implicit mixing:** including disconnected contributions in diagonals.

Optimal distillation profiles

Distillation M. Peardon et al. Phys. Rev. D 80, 054506 (2009)

- Project quark fields onto low-dimensional subspace of smooth, gauge-covariant fields $\rightarrow$ **Smearing**.
- $\psi(t) \rightarrow V[t]V[t]^\dagger\psi(t)$ with $V[t]$ the low-modes of the 3D gauge-covariant Laplacian operator.
- Perambulators: $\tau[t_1, t_2] = V[t_1]^\dagger D^{-1} V[t_2]$
- Elementals: $\Phi[t] = V[t]^\dagger \Gamma V[t]$, $\Gamma = \gamma_5, \gamma_i, \nabla_i, ...$

Example two-point meson correlation:

$$C(t) = -\left\langle \text{Tr}\left(\Phi[t]\tau[t,0]\bar{\Phi}[0]\tau[0,t]\right)\right\rangle_{\text{gauge}} + \left\langle \text{Tr}\left(\Phi[t]\tau[t,t]\right)\text{Tr}\left(\bar{\Phi}[0]\tau[0,0]\right)\right\rangle_{\text{gauge}}$$

High inversion cost but matrices have manageable sizes and can be reused for different choices of correlations, e.g one-particle, two-particle, etc...

Improved Distillation J. A. Urrea-Niño, F. Knechtli, T. Korzec & M. Peardon. Phys. Rev. D 106, 034501 (2022)

- Exploit further freedom: $V[t]V[t]^\dagger \to V[t]J[t]V[t]^\dagger$ with quark distillation profile $J[t]_{ij} = \delta_{ij} g\left(\lambda_i[t]\right)$.
- Build a GEVP using different quark profiles:

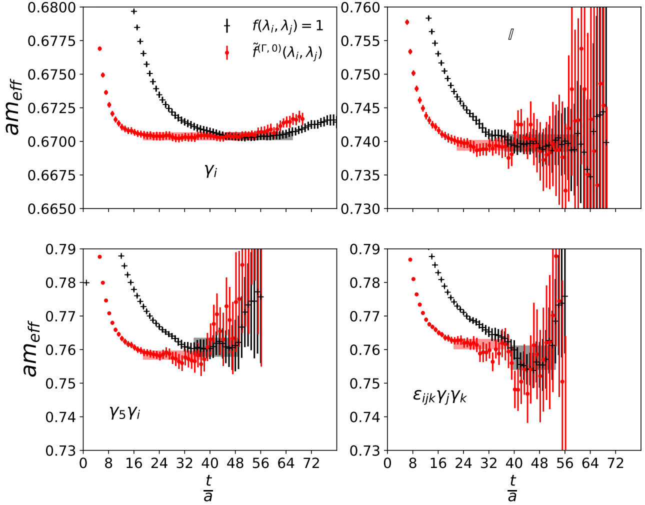
$$\psi_k(t) = V[t]J_k[t]V[t]^\dagger \psi(t) \to \text{ Different quark profiles...}$$
$$\mathcal{O}_k(t) = \bar{\psi}_k(t)\Gamma\psi_k(t) \to \text{ ...define different meson operators...}$$
$$\Phi^k[t] = J_k[t]^\dagger \Phi[t] J_k[t] \to \text{ ... with different elementals...}$$
$$C_{ab}(t) = \left\langle \mathcal{O}_a(t)\bar{\mathcal{O}}_b(0)\right\rangle \to \text{ ... for a correlation matrix.}$$

- E.g optimal meson profile

$$f^{(\Gamma,n)}\left(\lambda_i[t], \lambda_j[t]\right) = \sum_k a_k^{(\Gamma,n)} g_k\left(\lambda_i[t]\right)^* g_k\left(\lambda_j[t]\right)$$

✓ **No** additional inversion cost for improvement.
✓ One optimal profile **for each** Γ and energy level n.
✓ Profiles introduced at contraction level, which can be made **efficient** via BLAS, Numpy, etc...

Standard vs **Improved** distillation in connected-only charmonium correlations.



Gauge ensembles

Wilson fermion action with non-perturbatively determined clover improvement + Lüscher-Weisz gauge action. R. Höllwieser et al. Eur. Phys. J. C 80, 349. P. Fritzsch et al. J. High Energ. Phys. 2018, 25 (2018)

A1	A1h
96×32^3	96×32^3
$a \approx 0.054$ fm	$a \approx 0.069$ fm
$m_\pi \approx 420$ MeV	$m_\pi \approx 800$ MeV
$N_v^{\text{charm}} = 200$	$N_v^{\text{charm}} = 200$
$N_v^{\text{light}} = 100$	$N_v^{\text{light}} = 200$

Our setup is **very convenient:**

! Quenched lattice QCD predicts a 0^{++} glueball at $\approx$ 1800 MeV. C. Morningstar and M. Peardon, Phys. Rev. D 60, 034509

We can restrict pion decays in each ensemble:

- A1: Glueball $\rightarrow \pi\pi$, $\pi\pi\pi\pi$
- A1h: Glueball $\rightarrow \pi\pi$

Hadron creation operators

We have to saturate the spectrum as best as possible.

One-meson operators: $\bar{c}\,\mathbb{I}\,c$, $\frac{1}{\sqrt{3}}\sum_{q=u,d,s}\bar{q}\,\mathbb{I}\,q$

- Build correlation matrix with 7 Gaussian profiles.
- Prune via SVD to 3(5) charmonium(light meson) operators. J. Balog et al., Phys. Rev. D 60, 094508, F. Niedermayer et al., Nuclear Physics B 597, 413–450

✓ Pruned operators are almost ”aligned” with energy eigenstates.

Gluonic operators: Sum of Laplacian eigenvalues. C. Morningstar et al. Phys. Rev. D 88, 014511

✓ Consistent with 3D Wilson loops but slightly cleaner signal.

Two-pion operators: $\pi\,(\vec{p}=0)\,\pi\,(-\vec{p}=0)$ with **standard** distillation.

We account for most expected states but improvement is possible...

What will I show?

- Finite volume energy spectrum of the scalar channel **taking into account** mixing between mesonic and gluonic operators.
- Effect of including **different types** of operators on the spectrum.
- **Overlaps** between energy eigenstates and the different types of operators.

The **overlaps** are calculated as

$$\langle n| \hat{\mathcal{O}}_i^\dagger |\Omega\rangle \propto [C(t_0)\vec{\omega}_n(t,t_0)]_i$$

to see which type of operator contributes most to each state. J.J. Dudek et al. Phys. Rev. D 77, 034501 (2008)

We obtain a **finite volume** spectrum and a scattering study requires the **Lüscher formalism**. M. Lüscher, Nucl. Phys. B 354, 531 (1991)

0^{++} flavor-singlet correlation matrix at $t = a$

$C_{ij}(t) \to C_{ij}(t)/\sqrt{C_{ii}(a)C_{jj}(a)}$.

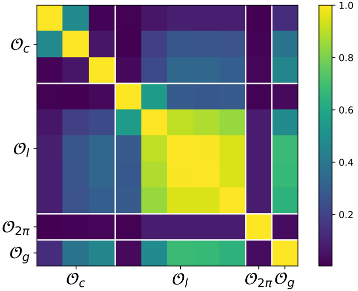


Figure: A1 ensemble

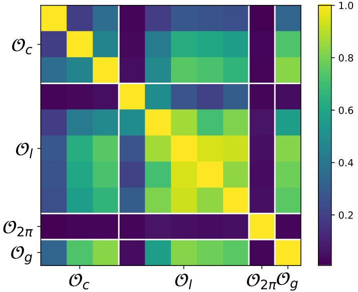
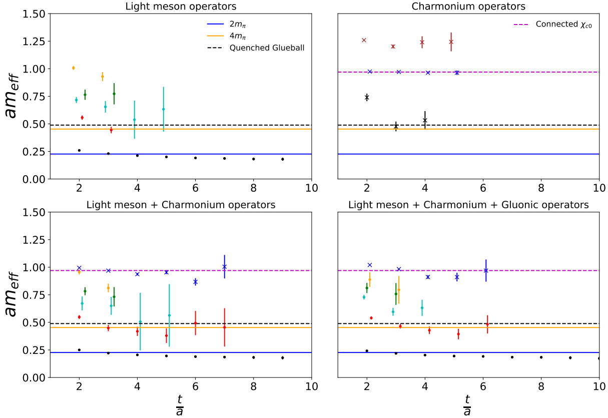


Figure: A1h ensemble

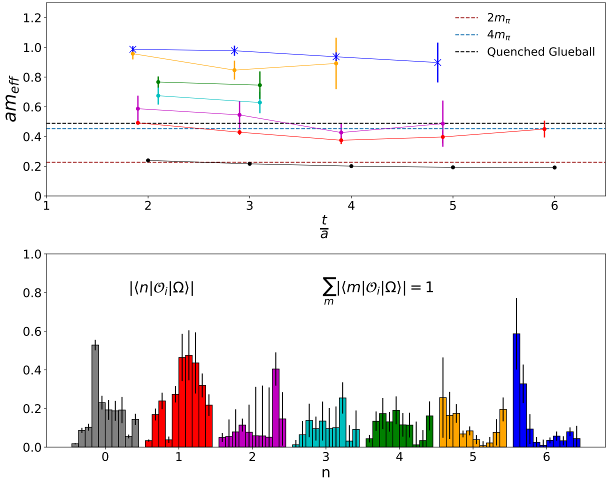
- Charmonium, light meson and gluonic operators have **non-zero** correlations: **mixing!**
- 2-pion operator mixes **very little**: creates a mostly multi-particle state.

Single-particle operator mixing in A1 $(m_\pi \approx 420$ MeV$)$



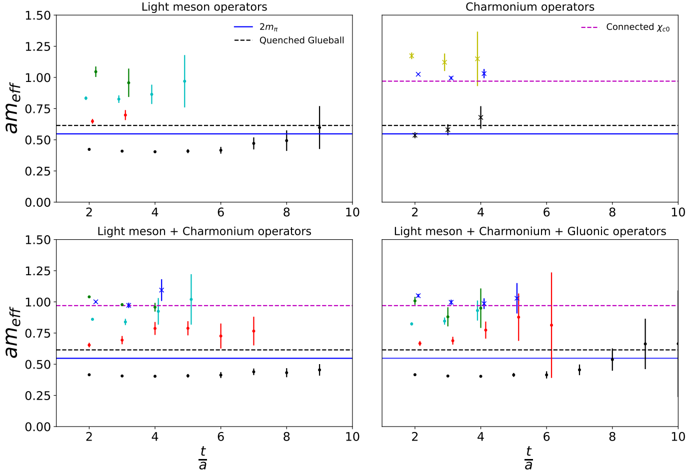
- Charmonium operators **alone** see a light state: **charm annihilation (disconnected) effects!**
- Including $\mathcal{O}_g$ does not change the low-lying spectrum. Similar to R. Brett et al. 1909.07306.

Full operator mixing in A1 $(m_\pi \approx 420$ MeV$)$



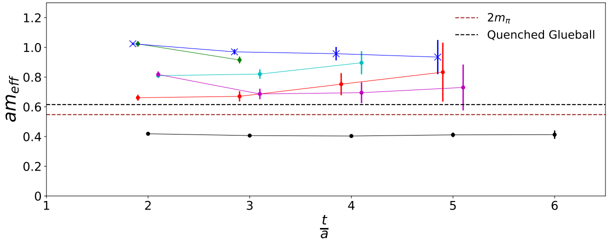
- 10 operators: $3 \times \mathcal{O}_c$, $5 \times \mathcal{O}_l$, $\mathcal{O}_{2\pi}$ and $\mathcal{O}_g$.
- 2-pion operator introduces an additional state.

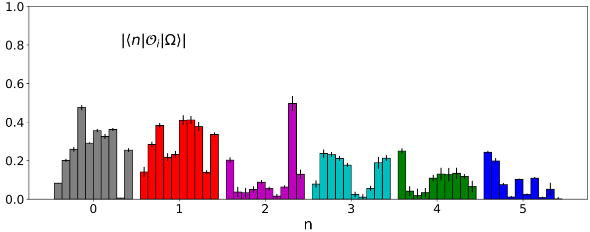
Single-particle operator mixing in A1h $(m_\pi \approx 800$ MeV$)$



- Charm annihilation effects are again important!
- Including $\mathcal{O}_g$ does not change the low-lying spectrum.

Full operator mixing in A1h $(m_\pi \approx 800$ MeV$)$





- 10 operators: $3 \times \mathcal{O}_c$, $5 \times \mathcal{O}_l$, $\mathcal{O}_{2\pi}$ and $\mathcal{O}_g$.
- 2-pion operator introduces an additional state.

What have we learned from the spectrum?

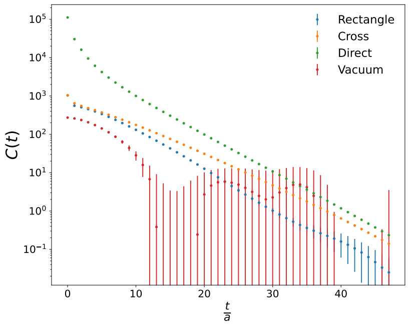
- ✓ Including flavor-mixing of operators is fundamental to map out the energy spectrum.
- ✓ Including multi-particle operators is necessary to avoid **missing** existing states.
- ✓ Including purely gluonic operators does **not** introduce a new state.

What have we learned from the overlaps?

- ✓ All flavor-singlet operators ”talk” to each other to some degree.
- ✓ Charmonium operators ”see” light states and viceversa: charm annihilation effects are important!
- ✓ 2-pion operators overlap strongly with a state somewhat separate from the 1-particle ones.

Where can we improve?

Tackle the signal-to-noise problem for *disconnected*-like correlations, e.g gluonic operators or meson loops.



Multi-level sampling was recently systematically studied for 3D Wilson loops in SU(3) pure gauge theory. L. Barca *et. al.* Phys. Rev. D 110, 054515 (2024)

Application to case with dynamical quarks + distillation is currently under study. Talk by L. Barca at Lattice 2024

This *algorithmic* improvement happens at the level of generating the gauge configurations.

Improve our operator basis.

One-meson operators:

- ► Include multiple Γ with profiles.
- ✓ Derivative-based operators can better sample spatial structure or additional gluonic content.
- ✓ E.g $\mathbb{B}_i = \epsilon_{ijk} \nabla_j \nabla_k$ is very sensitive to gluonic background.

Two-pion operators:

- ► Include non-zero back-to-back momentum in $\pi(\vec{p})\,\pi(-\vec{p})$.
- ► Include distillation profiles to each pion.
- ✓ Profiles have been shown to help also with one-meson operators at non-zero momentum.

These *operator* improvements happen at contraction time.

FOR5269: Future methods for studying confined gluons in QCD

Spokesperson: Prof. Dr. Francesco Knechtli

Collaboration between **physics** and **applied mathematics** at BUW, DESY Zeuthen and Trinity College Dublin.

Some main pillars:

- Contributions from charm-annihilation effects.
- Glueballs and their mixing in dynamical QCD.
- Improved numerical methods, e.g for distillation.
- Multi-level sampling with/without dynamical quarks.
- String breaking in hybrid potentials.
- New schemes for molecular dynamics.

We are tackling *physical, mathematical and numerical* problems particularly relevant for hadron spectroscopy.

Thank you for your attention!

There are computationally intensive steps worth optimizing.

Eigenvector calculation:

- ► N_v lowest eigenvectors of $\nabla^2[t]$ are calculated for every t.
- ✓ Thick-Restart Lanczos with periodic reorthogonalization and Chebyshev acceleration.
- ✓ C + MPI + LAPACK code written in the *qcdlib* library of T. Korzec.

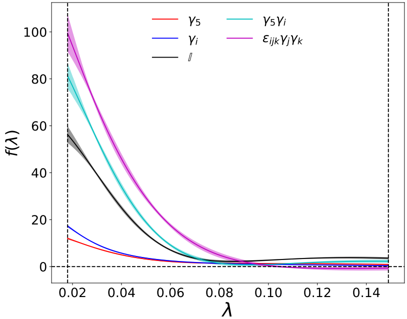
Perambulator calculation:

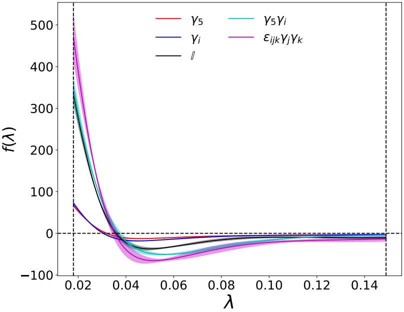
- ► $\tau[t_1, t_2]$ requires solving $4 \times N_v \times N_t$ linear systems $Dx = b$.
- ✓ C + MPI code in *qcdlib* which calls the openQCD linear solver.

Matrix contractions:

- ► $\tau[t_1, t_2]$ and $\Phi[t]$ have size $4N_v \times 4N_v$.
- ✓ Numpy is flexible and efficient for such "small" matrices.
- ✓ Sequences Tr $(\Phi[t_1]\tau[t_1, t_2]\Phi[t_2]\tau[t_2, t_3]...\Phi[t_M]\tau[t_M, t_1])$ can be efficiently calculated, e.g via Numpy's *einsum* + mpi4py.

Example profiles for connected-only charmonium





- Suppression of higher modes for ground states.
- Non-constant profiles.

- Nodes in excited states.
- Non-trivial structure.