Neutrino flavor instabilities in dense astrophysical environments

Julien Froustey

N3AS Fellow, UC San Diego / UC Berkeley

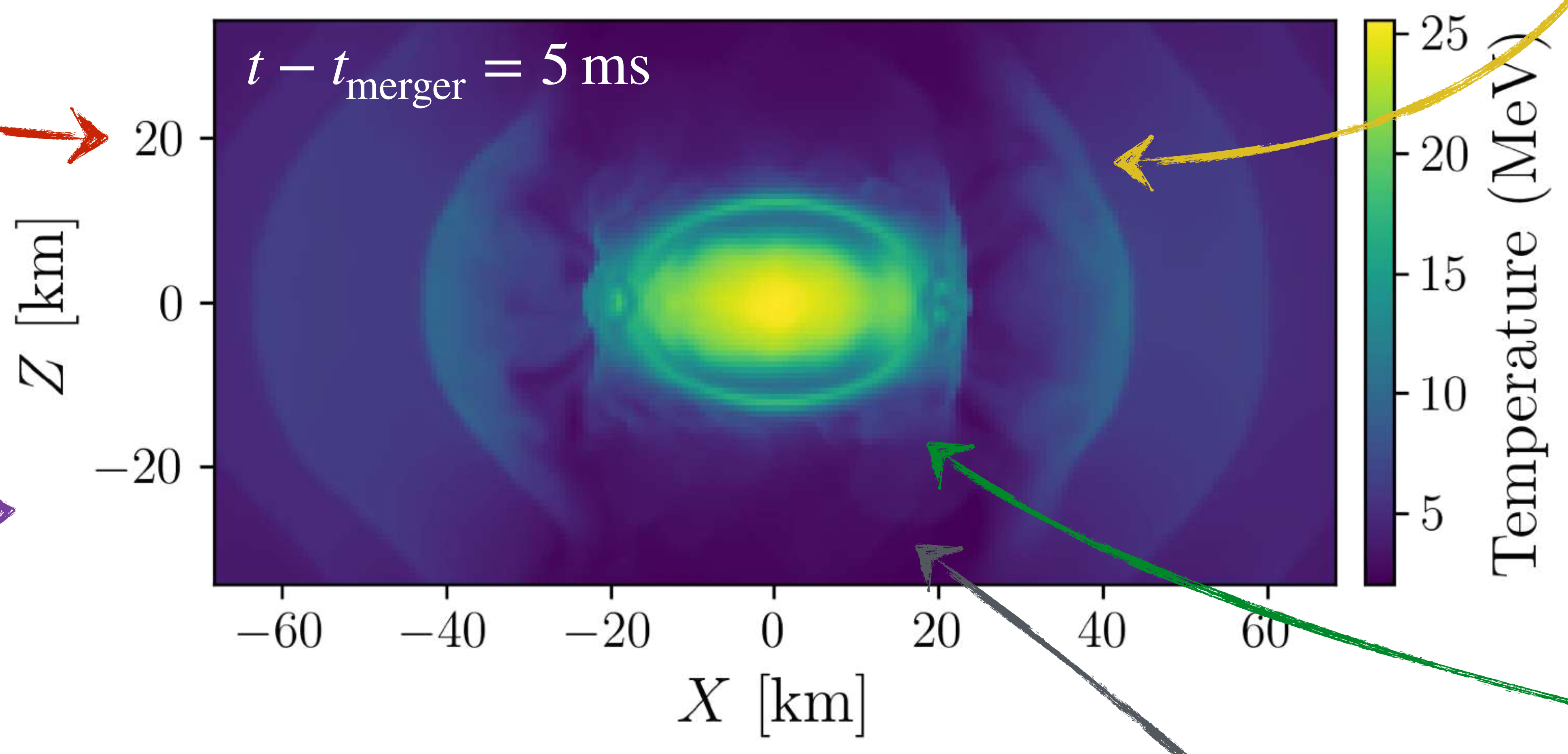
Invisibles25 Workshop — 01/09/2025 —

Large-scale simulations: what's included and what's not

General relativity

Hydrodynamics

Merger of two $1.2\,M_\odot$ neutron stars



F. Foucart *et al.* [1607.07450]

Neutrino transport

Equation of state

...

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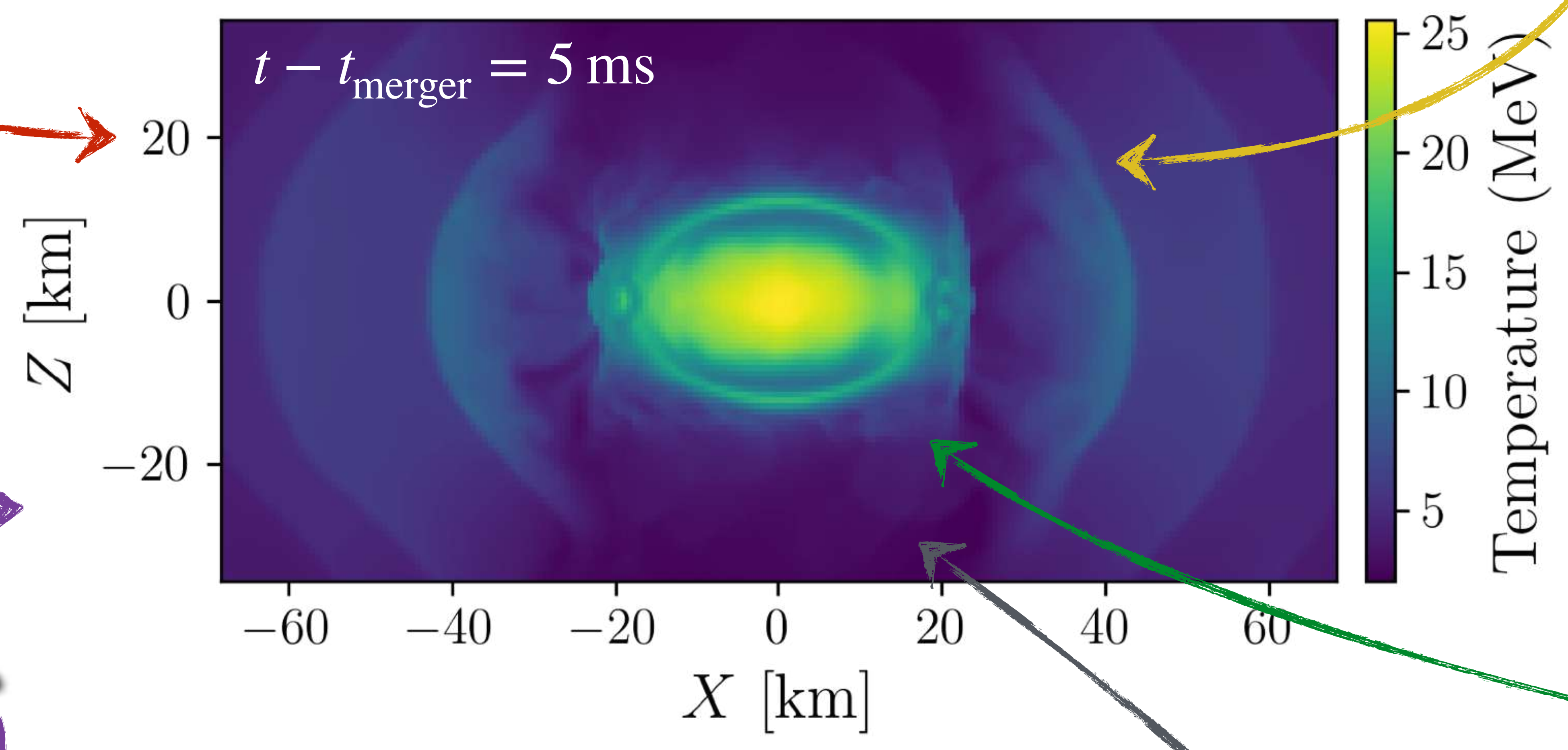

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* without neutrino flavor oscillations

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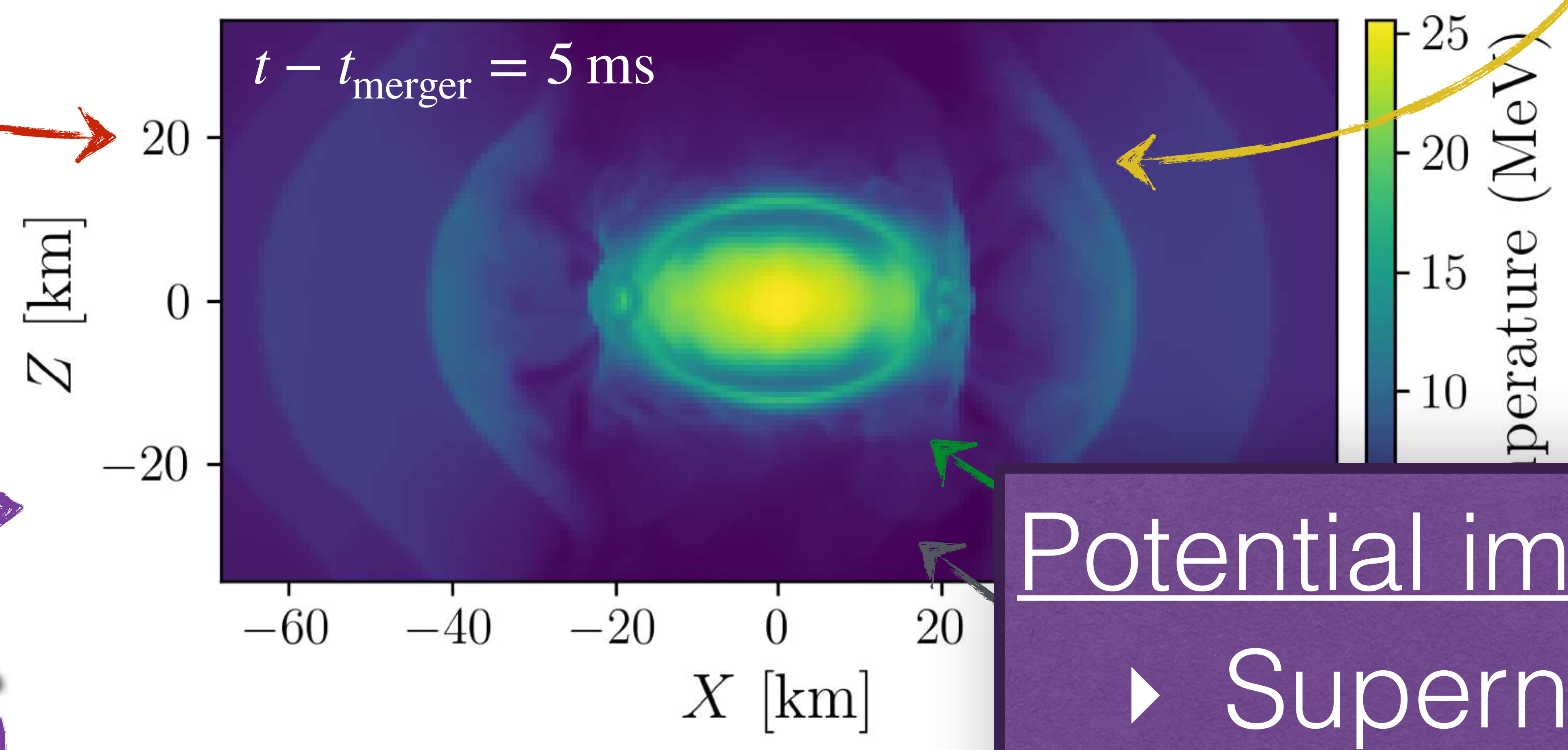

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Neutrino transport*

ate

Potential impact:

- Supernova explosion
- Nucleosynthesis
- Neutrino signal
- GW signal
- …

* without neutrino flavor oscillations

Classical and quantum transport

- Classical neutrino transport:

Distribution function → f_{ν_α}; Collision term → C_{ν_α}

Boltzmann's equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right) f_{\nu_\alpha} = C_{\nu_\alpha}$$

- Quantum neutrino transport:

One-body density matrix

$$\varrho = \begin{pmatrix} \varrho_{ee} & \varrho_{ex} \\ \varrho_{ex}^{*} & \varrho_{xx} \end{pmatrix} = \begin{pmatrix} \text{Distribution function of } \nu_e & \text{Flavor coherence} \\ \text{Flavor coherence} & \text{Distribution function of } \nu_x \end{pmatrix}$$

Quantum Kinetic Equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right) \varrho = -i\left[\mathcal{H}, \varrho\right] + \mathcal{C}$$

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Quantum Kinetic Equation $$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right) \varrho = -i\,[\mathcal{H}, \varrho] + \mathcal{C}$$ Collisions

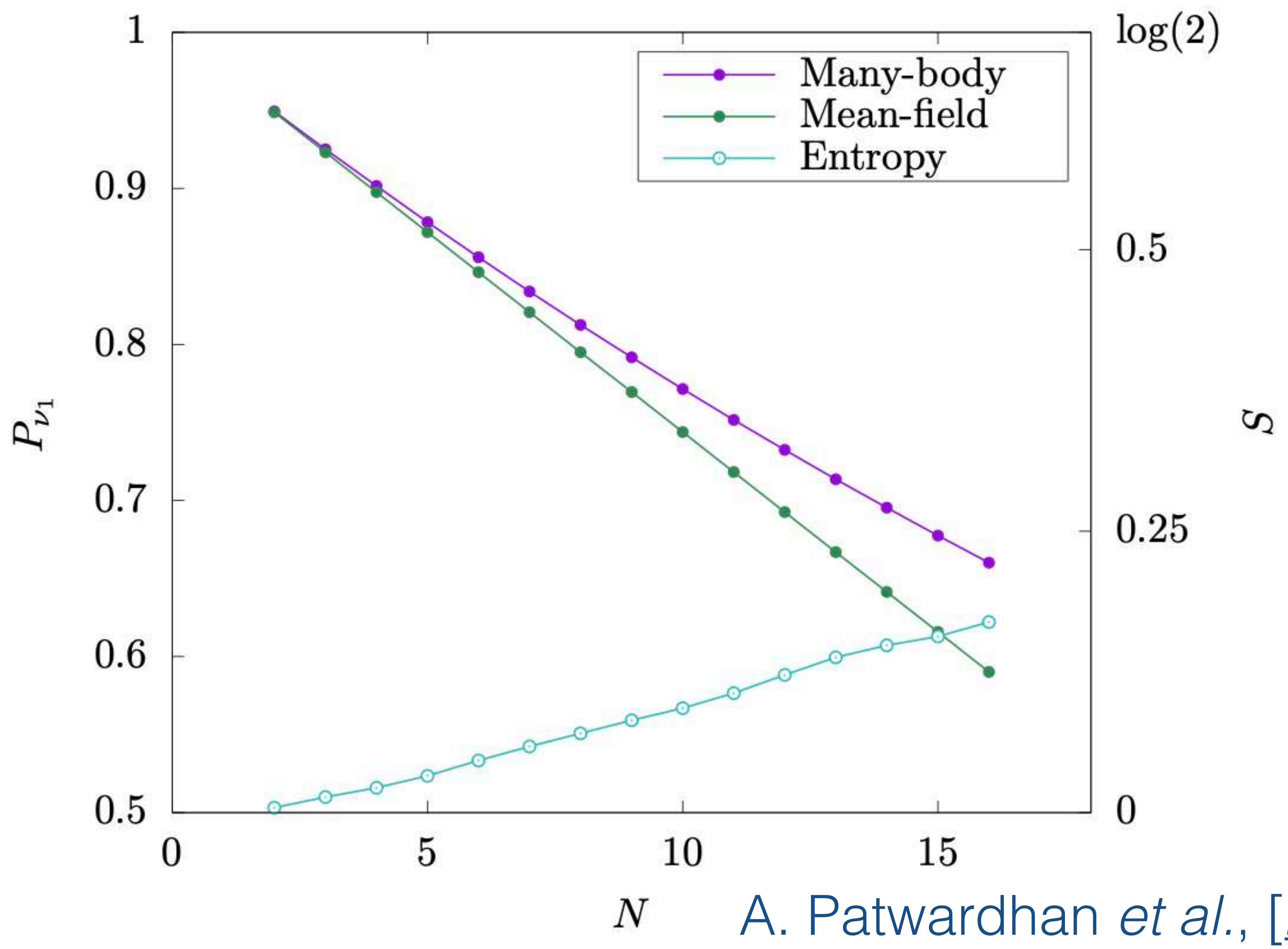
Vacuum term
+
mean-field potentials

Quantum many-body approaches

- Growing literature on possible signatures of **many-body effects**, discarded in the QKE (mean-field) approach

  Rrapaj [1905.13335], Martin+ [2112.12686], Cervia+ [2202.01865], Patwardhan+ [2301.00342], Balantekin+ [2305.01150], Siwach+ [2411.05169]…

  Impact on heavy-element nucleosynthesis? Balantekin+ [2311.02562]



A. Patwardhan *et al.*, [2109.08995]

$$\mathrm{i}\frac{\mathrm{d}|\Psi_{(N)}\rangle}{\mathrm{d}t} = \hat{H}|\Psi_{(N)}\rangle$$

→ *N*-body quantum state

$$\mathrm{i}\frac{\mathrm{d}\varrho_{(1)}}{\mathrm{d}t} = \left[\mathcal{H}^{(\mathrm{MF})}, \varrho_{(1)}\right]$$

→ *1*-body density matrix

Quantum many-body approaches

- *Quantum computers* are a promising tool to solve this quantum many-body problem.

Hall+ [2102.12556], Yeter-Aydeniz+ [2104.03273], Illa+ [2202.12340, 2210.08656], Amitrano+ [2207.03189], Siwach+ [2308.09123], Spagnoli+ [2503.00607], …

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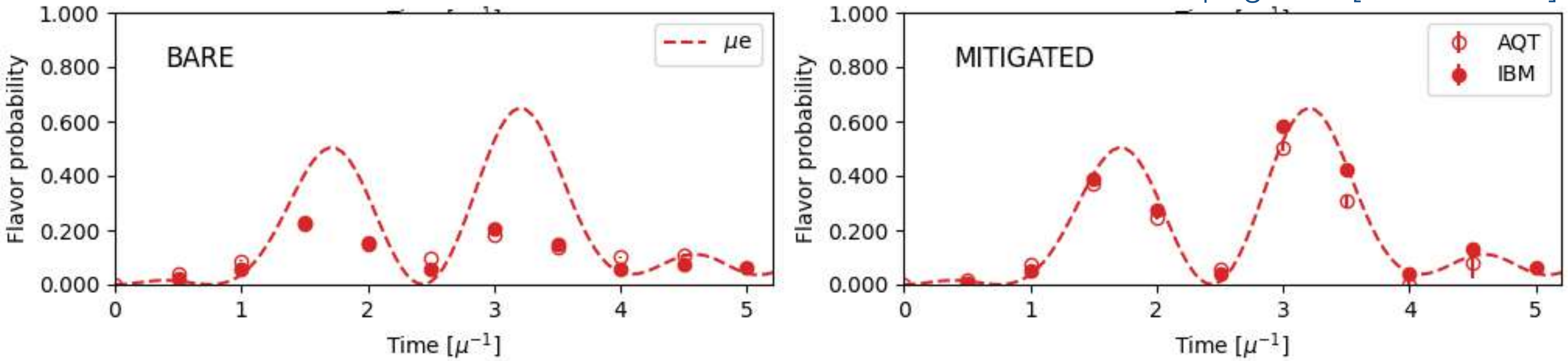

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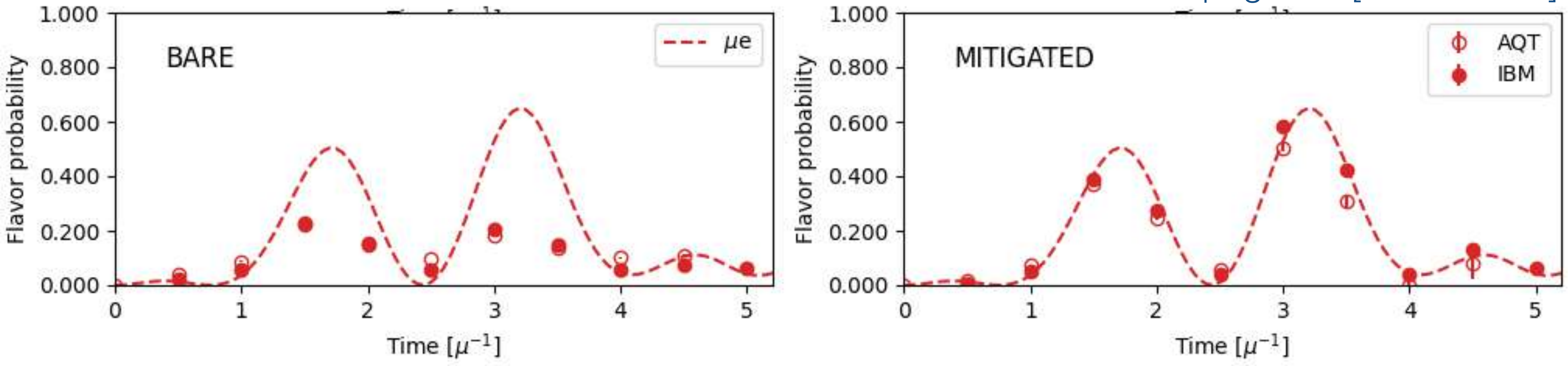

- But these “many-body studies” **may not be applicable** in actual environments *(separation of scales not satisfied)*

Do we have enough evidence to invalidate the mean-field approximation adopted to model collective neutrino oscillations?

Shashank Shalgar and Irene Tamborra

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Phys. Rev. D **107**, 123004 – **Published 5 June, 2023**

International Journal of Modern Physics A | Vol. 39, No. 30, 2450122 (2024) | Research Paper

Neutrino many-body correlations

Lucas Johns

Shalgar & Tamborra [2304.13050], Johns [2305.04916]

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$$\varrho = \begin{pmatrix} \varrho_{ee} & \varrho_{ex} \\ \varrho_{ex}^{*} & \varrho_{xx} \end{pmatrix} = \Big(\text{Distrib} \;\; \text{Flavor co}$$

In the following, we assume the Quantum Kinetic Equation accurately describes neutrino evolution in astrophysical environments

Quantum Kinetic Equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right) \varrho = -i\left[\mathcal{H}, \varrho\right] + \mathcal{C}$$

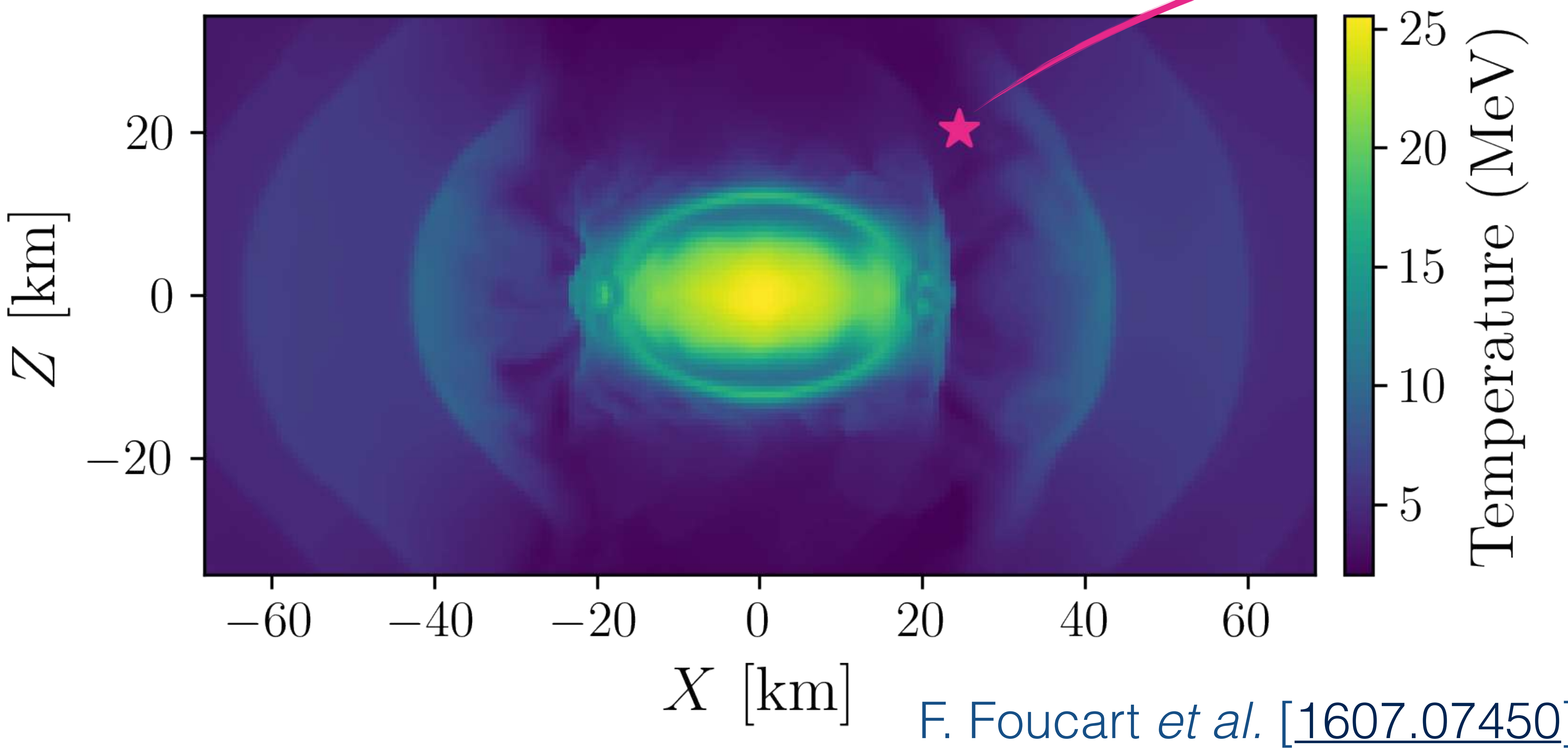
Collisions

Vacuum term
+
mean-field potentials

Flavor instabilities in dense astrophysical environments

- We can check if classical simulations are self-consistent.
- Take the classical results, and solve the QKE.

Merger of two $1.2\,M_\odot$ neutron stars



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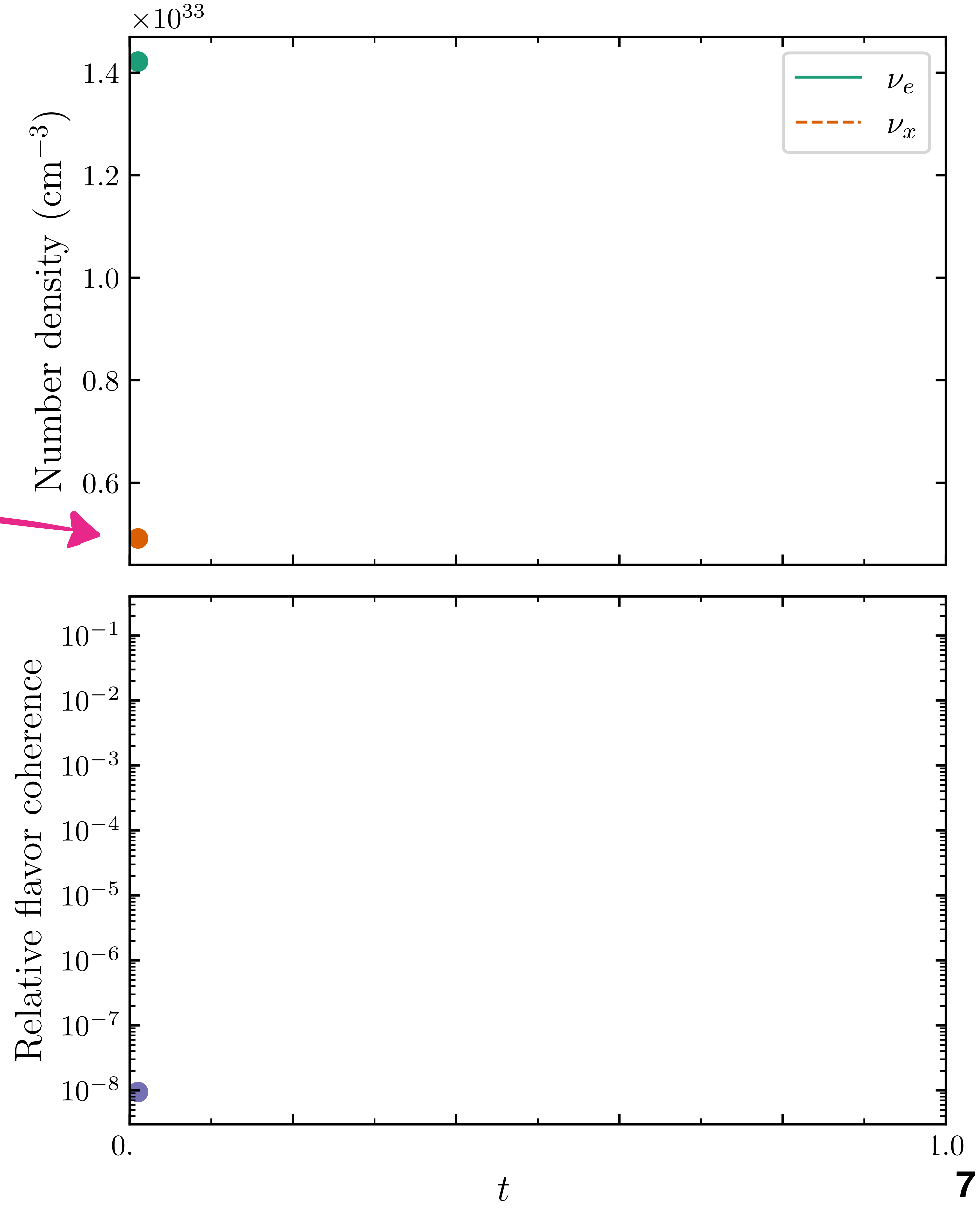

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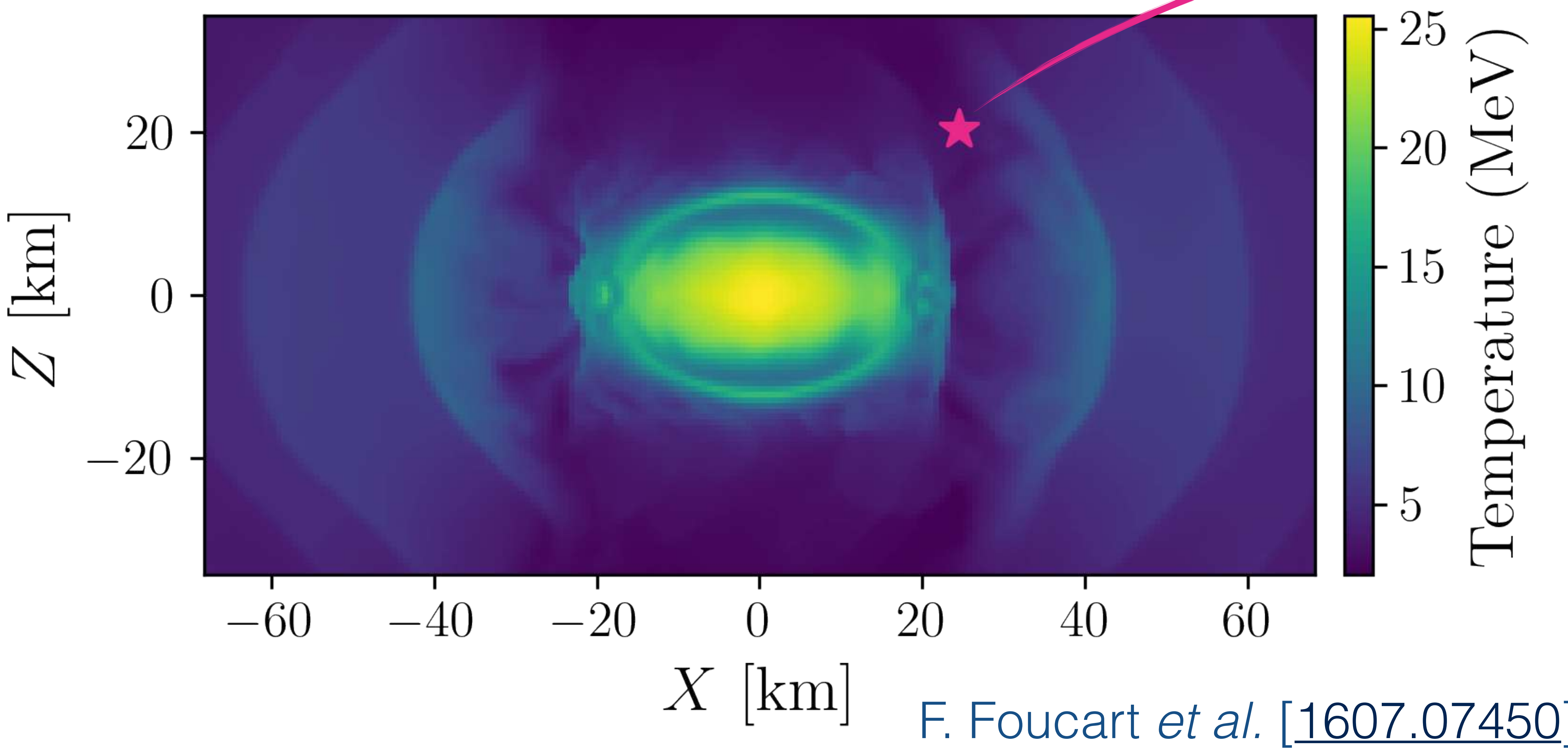

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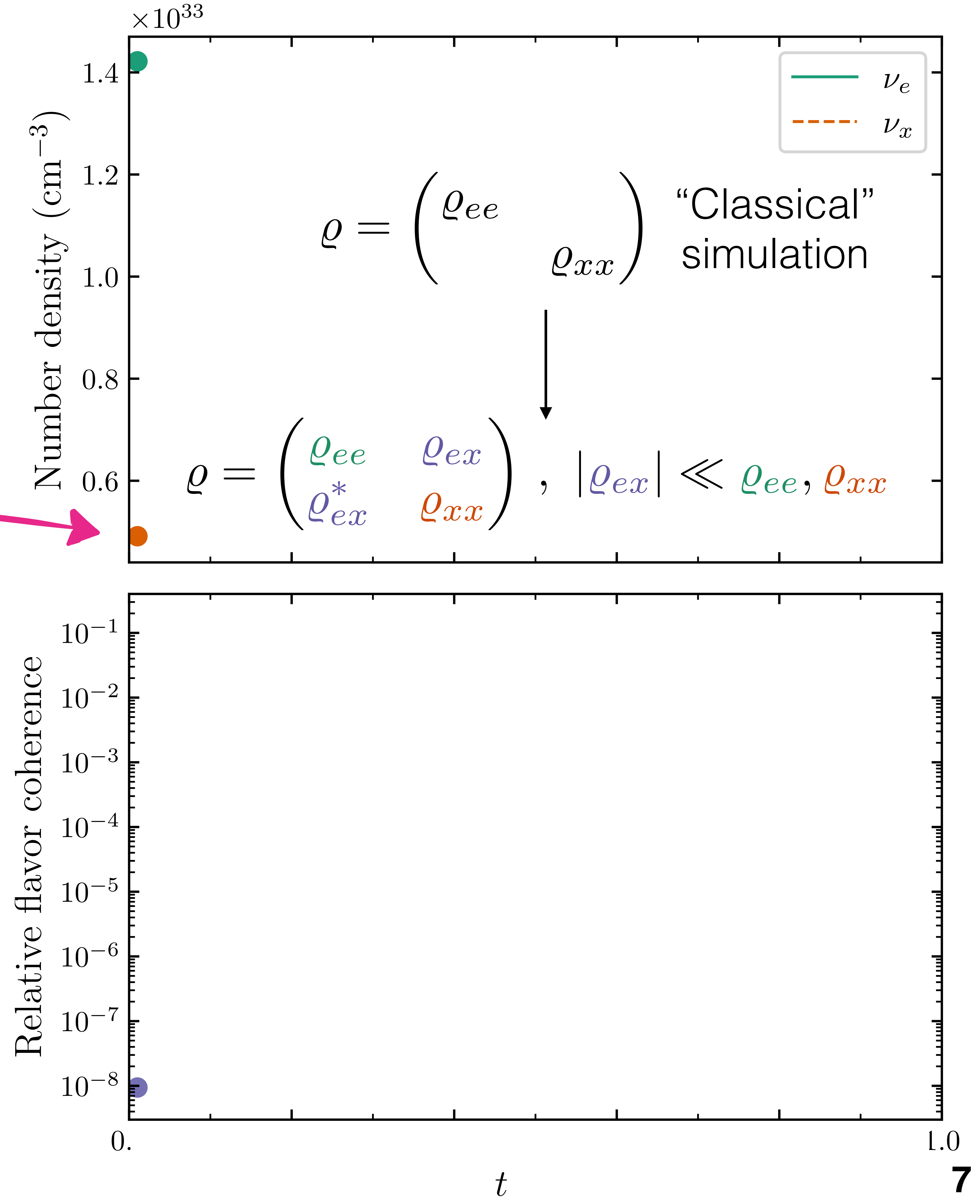

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$$\varrho = \begin{pmatrix} \varrho_{ee} & \varrho_{ex} \\ \varrho_{ex}^* & \varrho_{xx} \end{pmatrix}, \quad |\varrho_{ex}| \ll \varrho_{ee}, \varrho_{xx}$$

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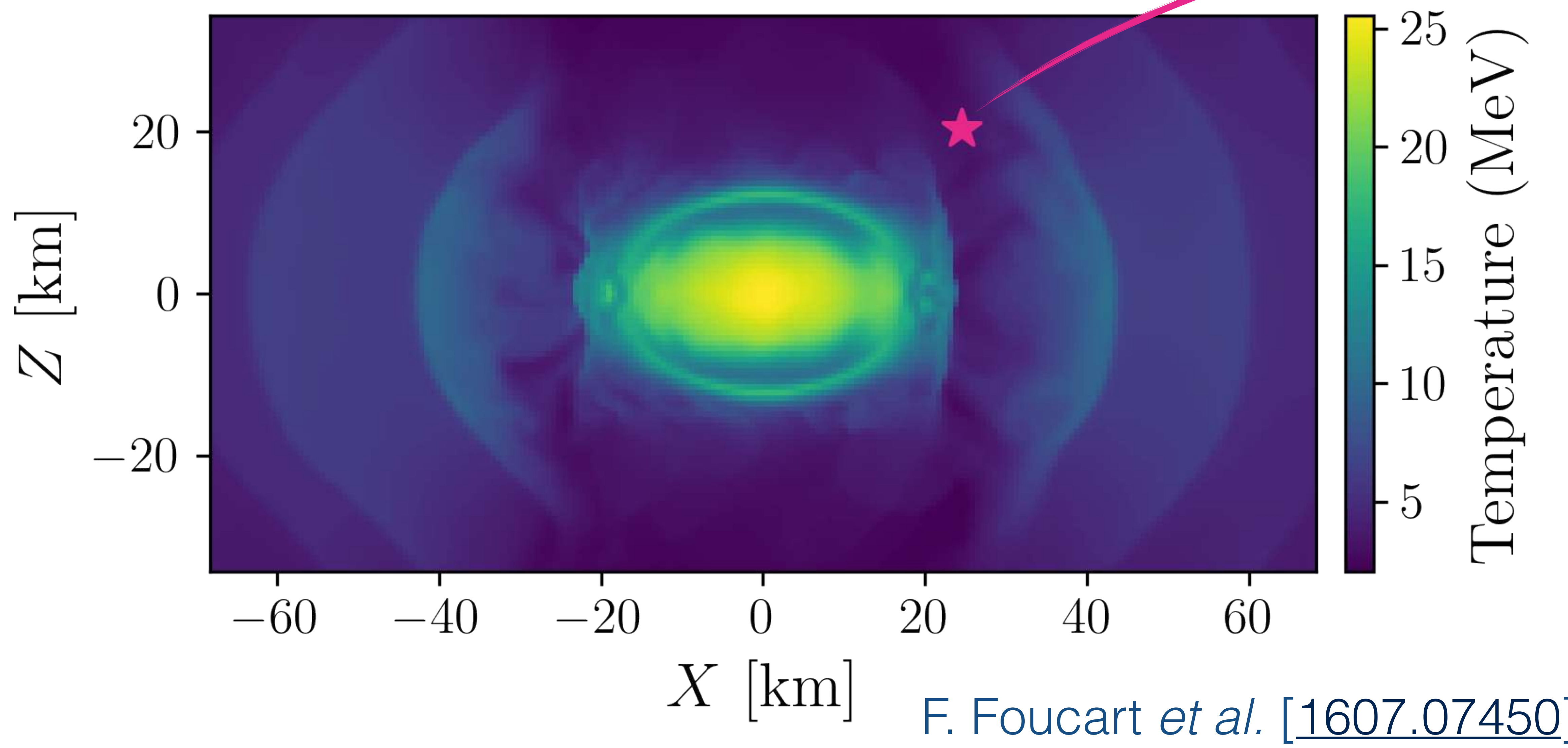

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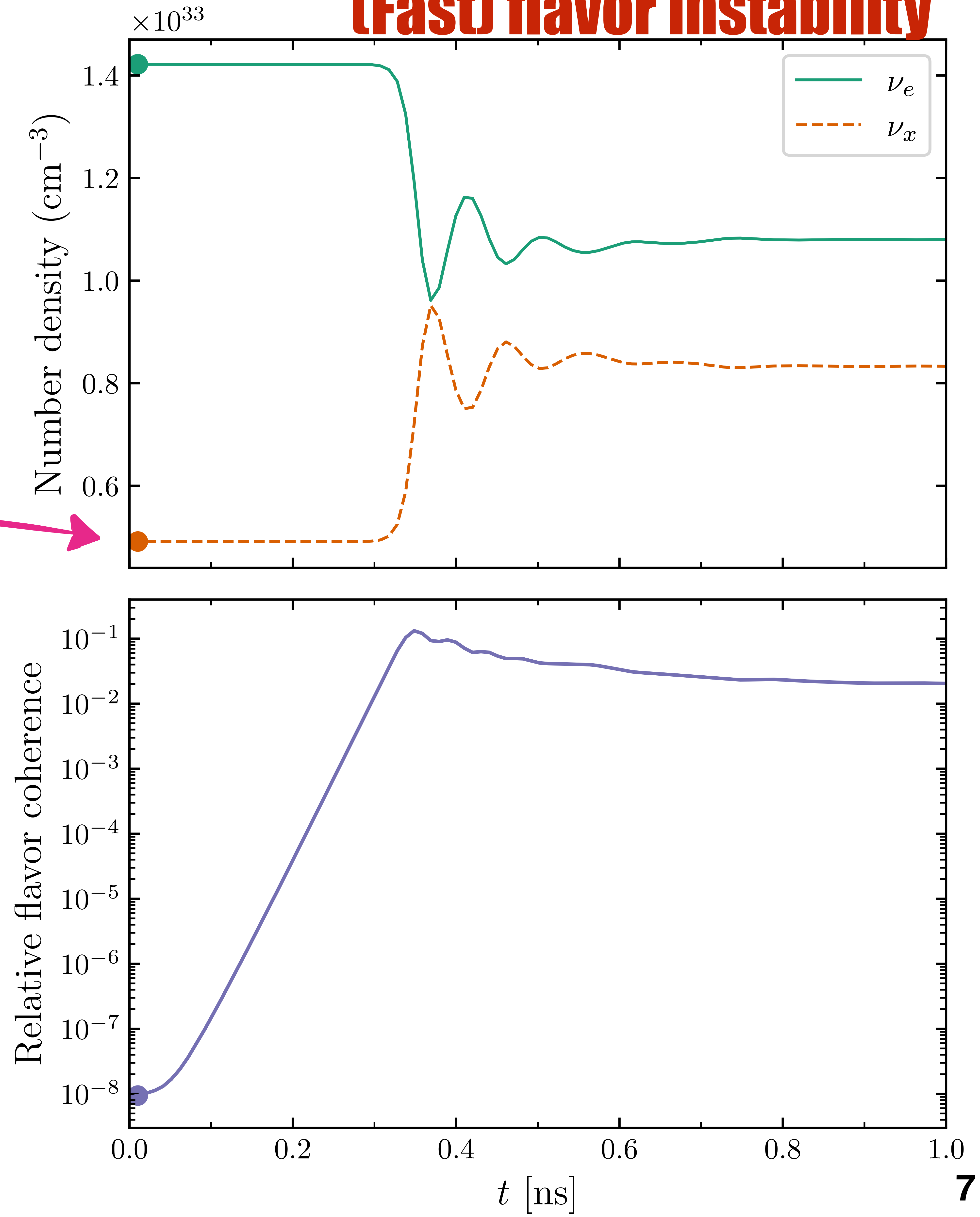
(Fast) flavor instability



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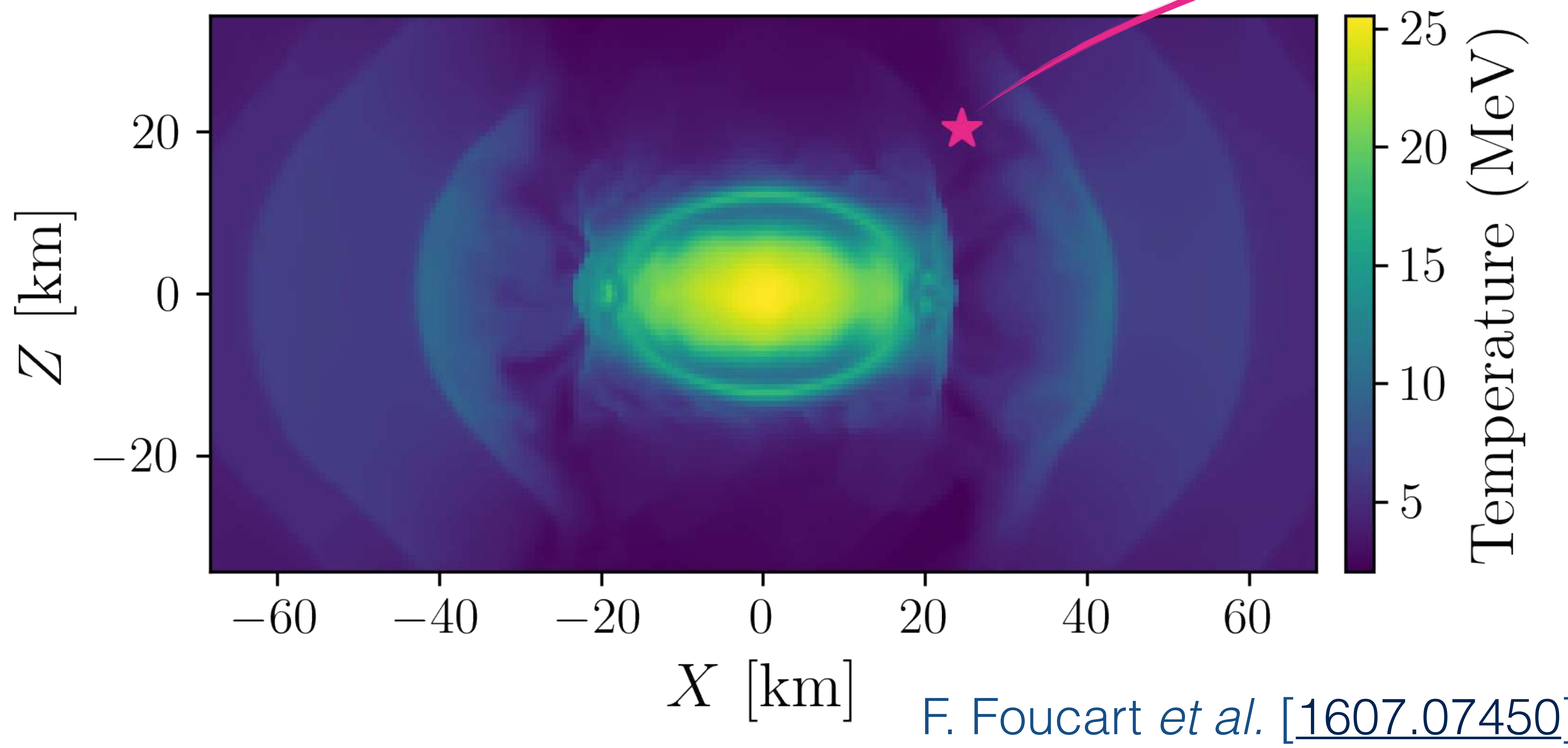

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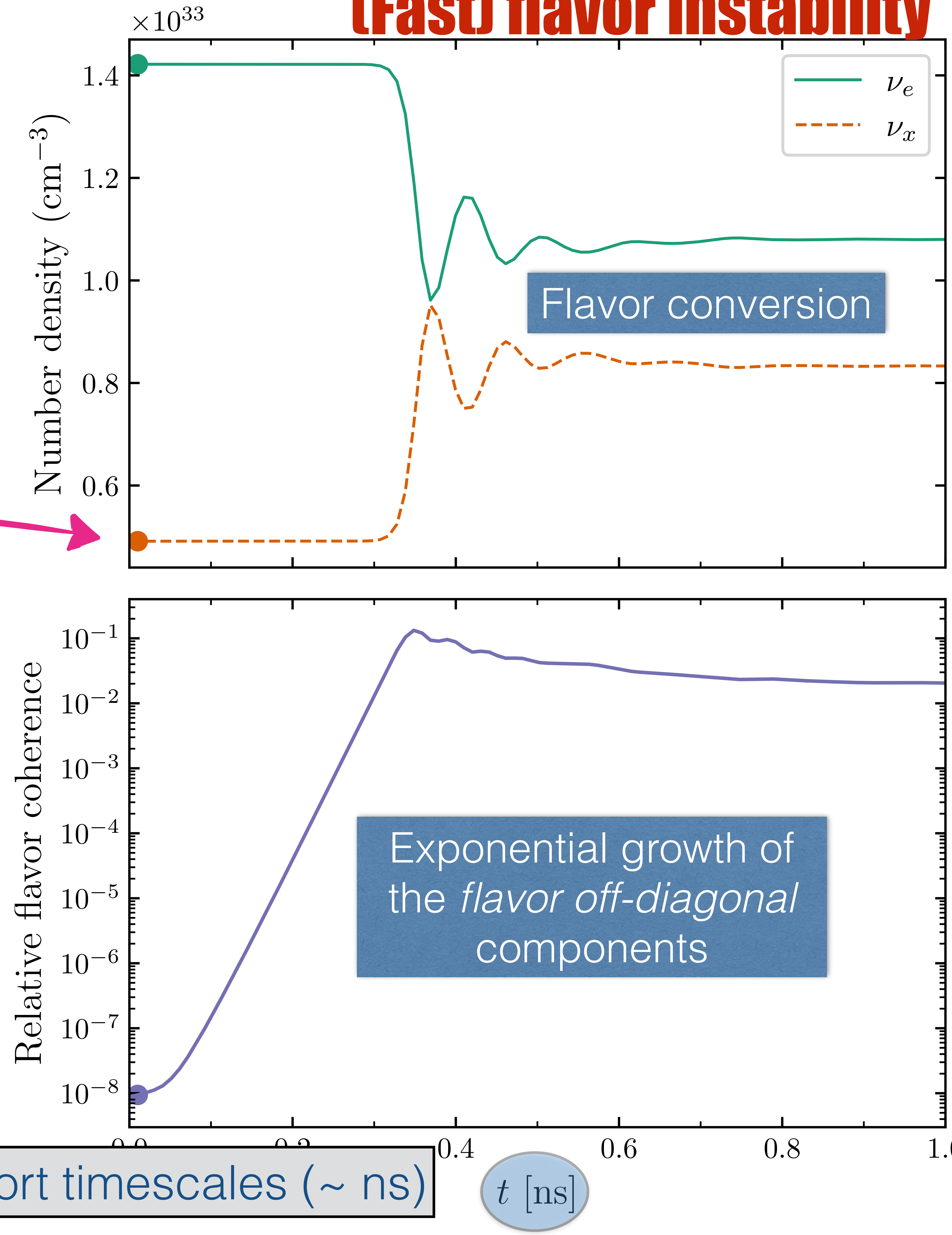
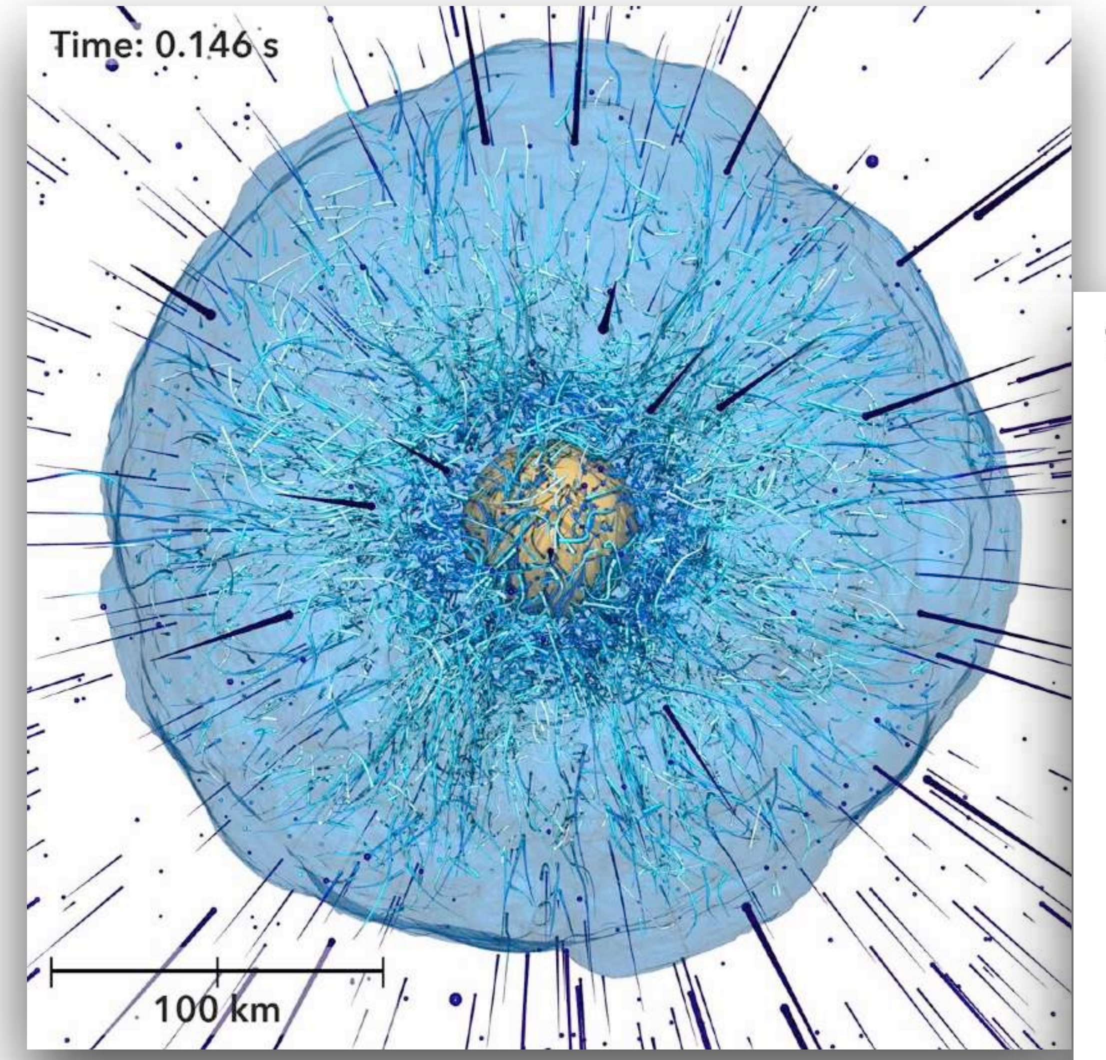


How to include neutrino flavor conversion in large-scale simulations?



A. Burrows & D. Vartanyan, [2009.14157]

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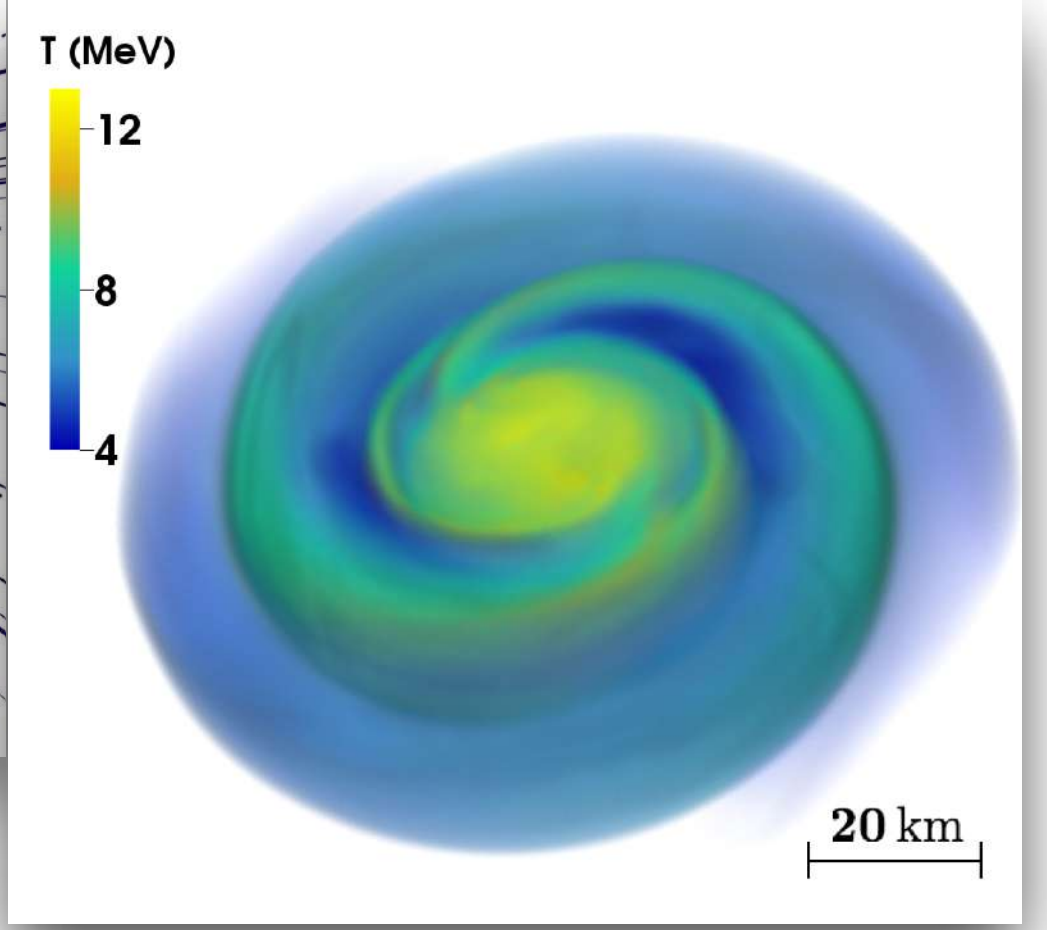

$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}\right)\varrho(\vec{x},\vec{p},t) = -i\left[\mathcal{H}\,,\,\varrho\right] + \mathcal{C}(\varrho,\bar{\varrho})$$

1 Direct simulation

$$t_{\mathcal{H}} \ll t_{\mathrm{coll}}, t_{\mathrm{adv}}$$

but hugely different scales! $\sim 0.1\,\mathrm{ns}$ $\sim \mu\mathrm{s}$

How to reduce the computational cost?

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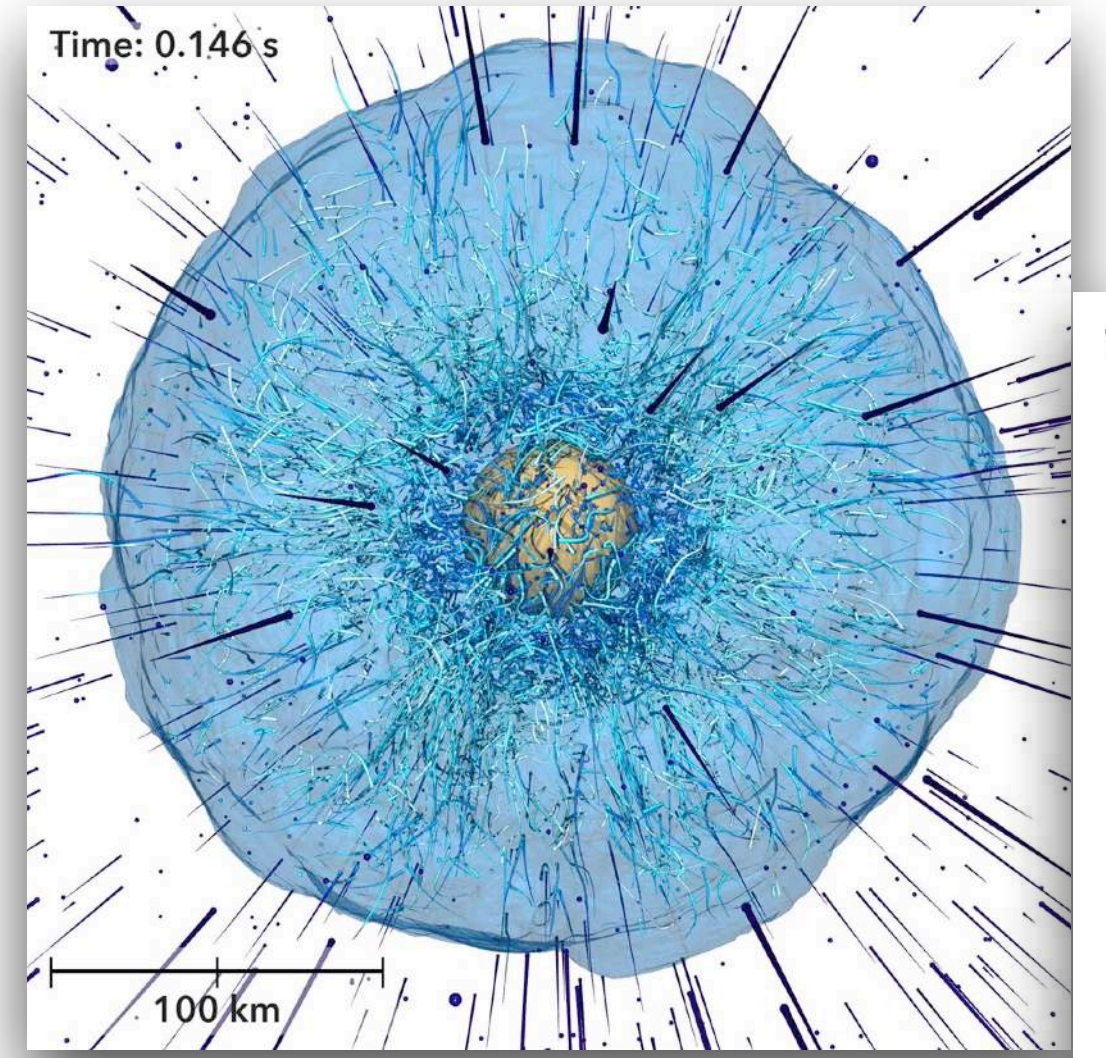

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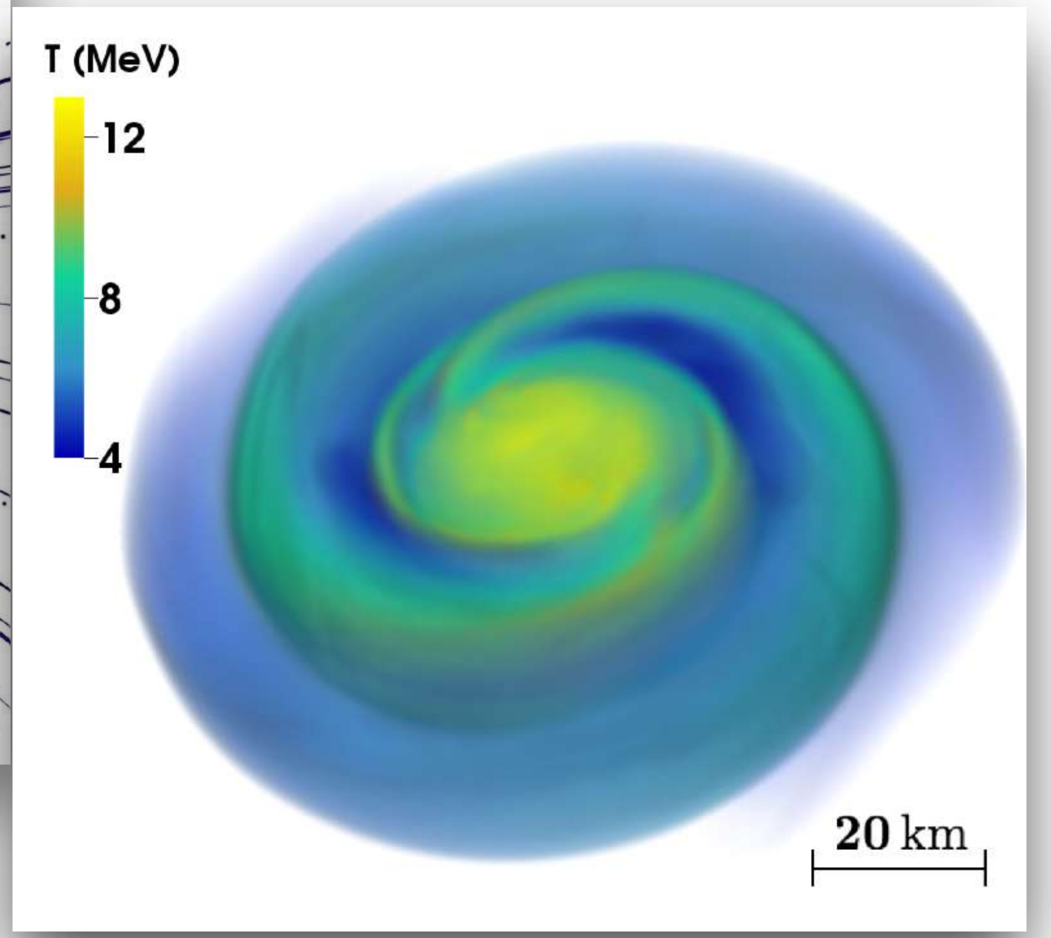

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}\right) \varrho(\vec{x}, \vec{p}, t) = -i\left[\eta \mathcal{H},\ \varrho\right] + \mathcal{C}(\varrho, \bar{\varrho})$$

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How to reduce the computational cost?

- Attenuation of the Hamiltonian (then extrapolation of results)

Nagakura+ [2206.04097, 2211.01398], Xiong+ [2402.19252]…

$\eta < 10^{-2}$

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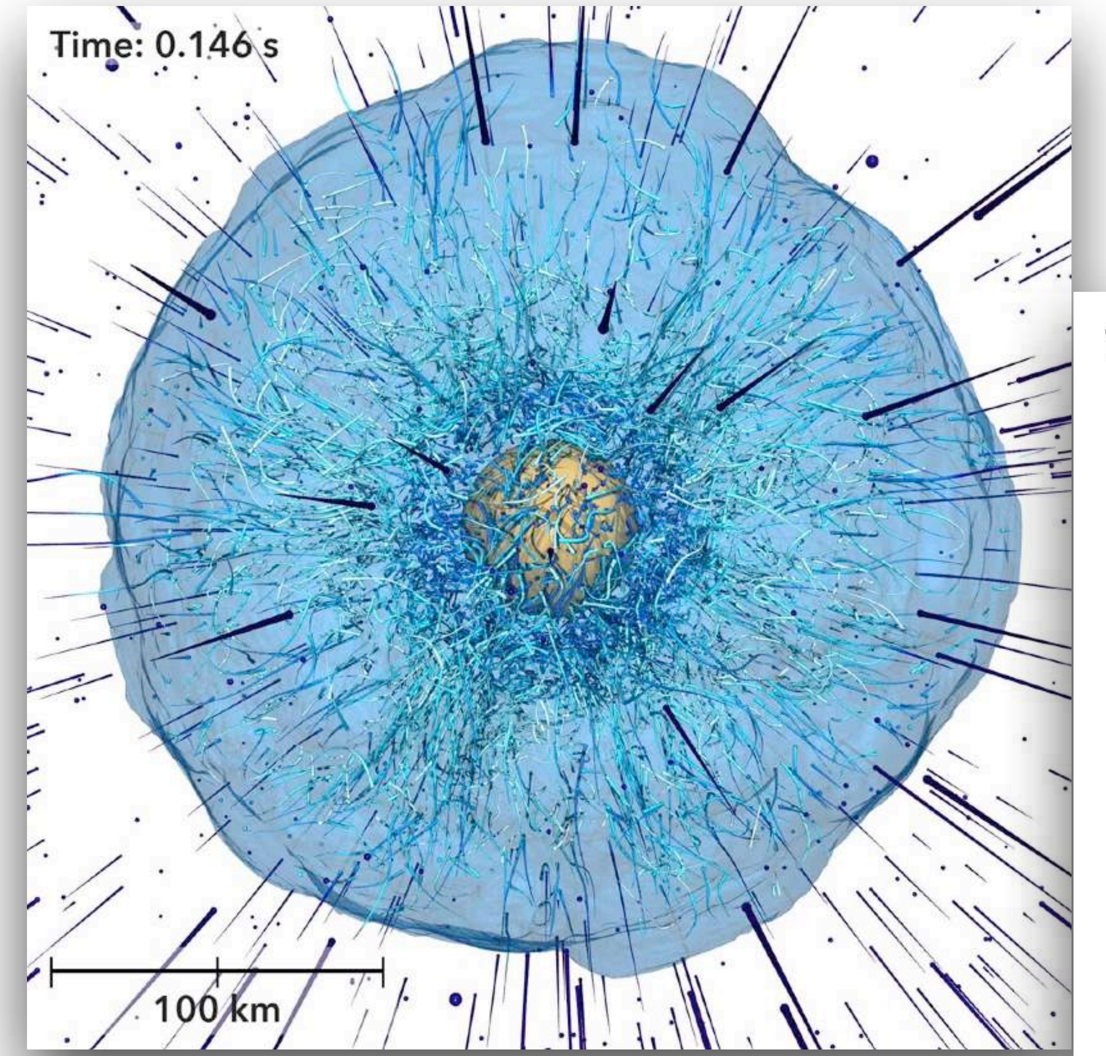

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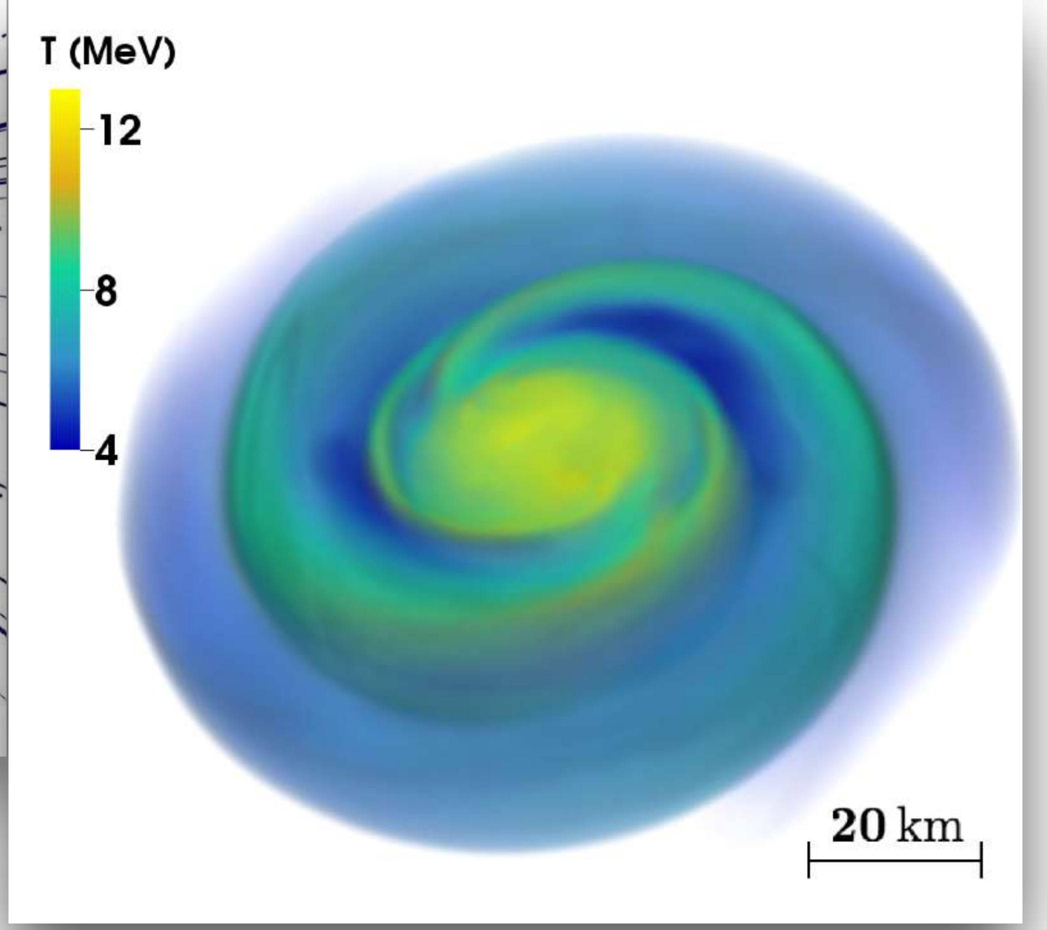

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7-dimensional problem

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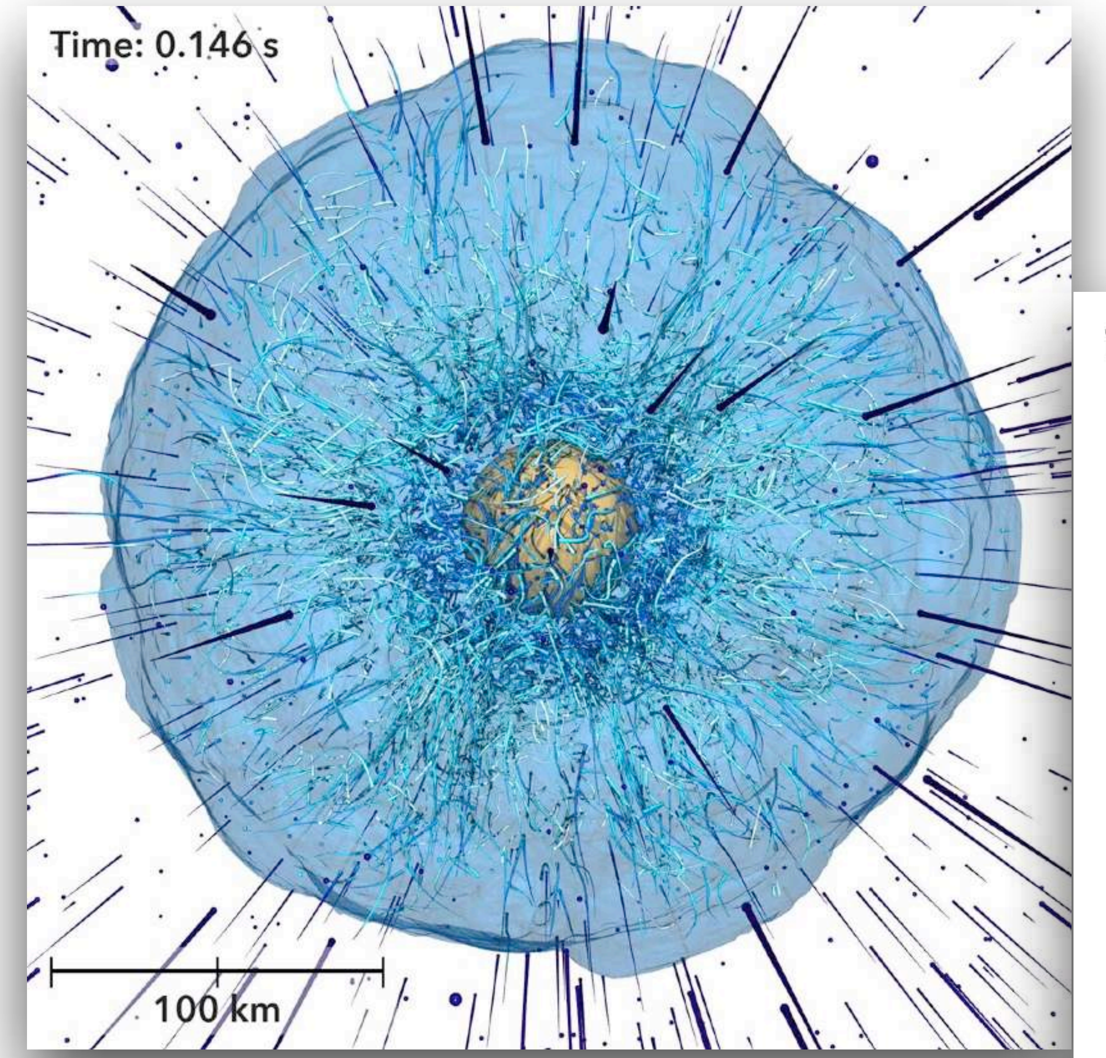

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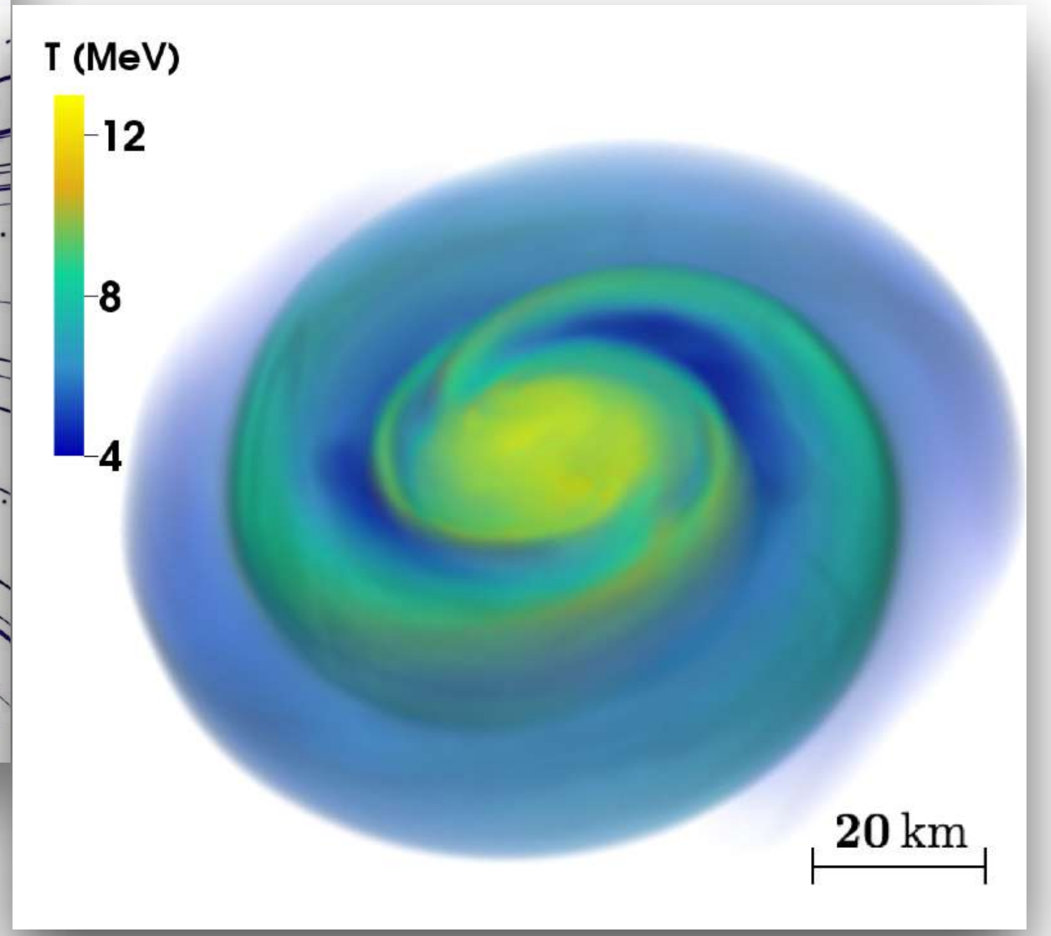

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7-dimensional problem

$$\int \mathrm{d}\Omega_{\vec{p}}\,\cdots\,, \qquad \int \mathrm{d}\Omega_{\vec{p}}\,\frac{\vec{p}}{|\vec{p}|}\,\cdots$$

$\varrho \longrightarrow$ Angular moments

1 Direct simulation

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- Angular moment approaches

  Grohs+ [2207.02214, 2309.00972, 2501.05740], Froustey+ [2311.11968, 2409.05807], Kneller+ [2410.00719]

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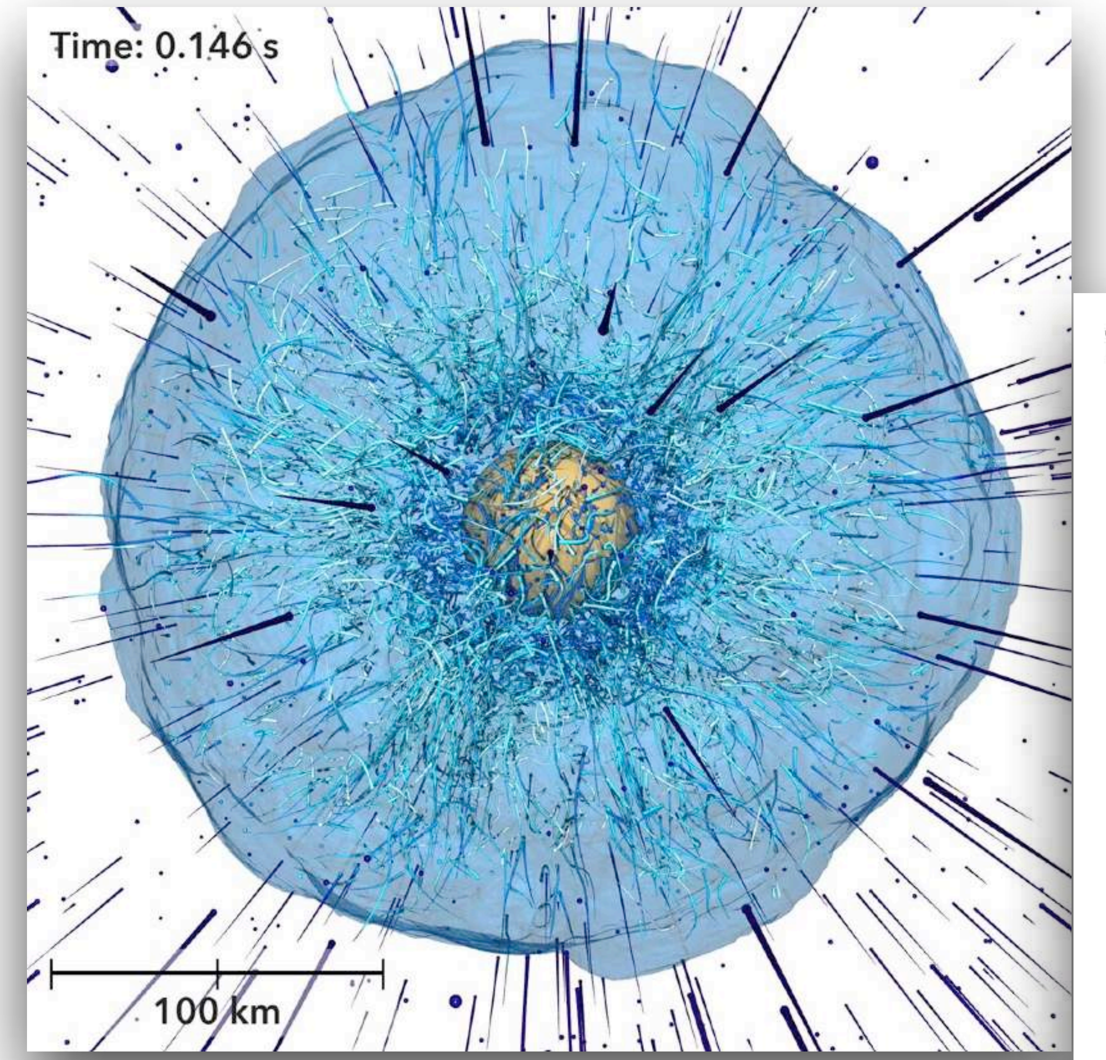

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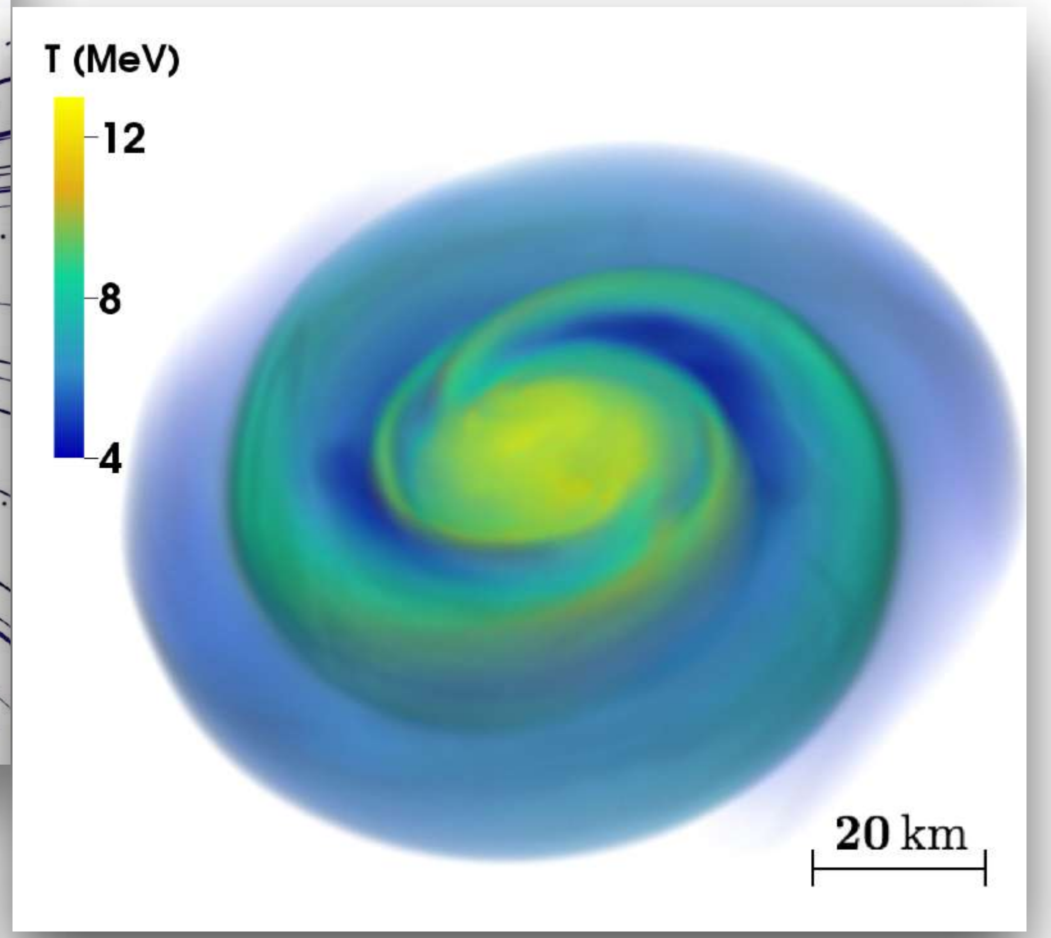

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1 **Direct simulation**

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→ Attenuation of the Hamiltonian, moment methods…

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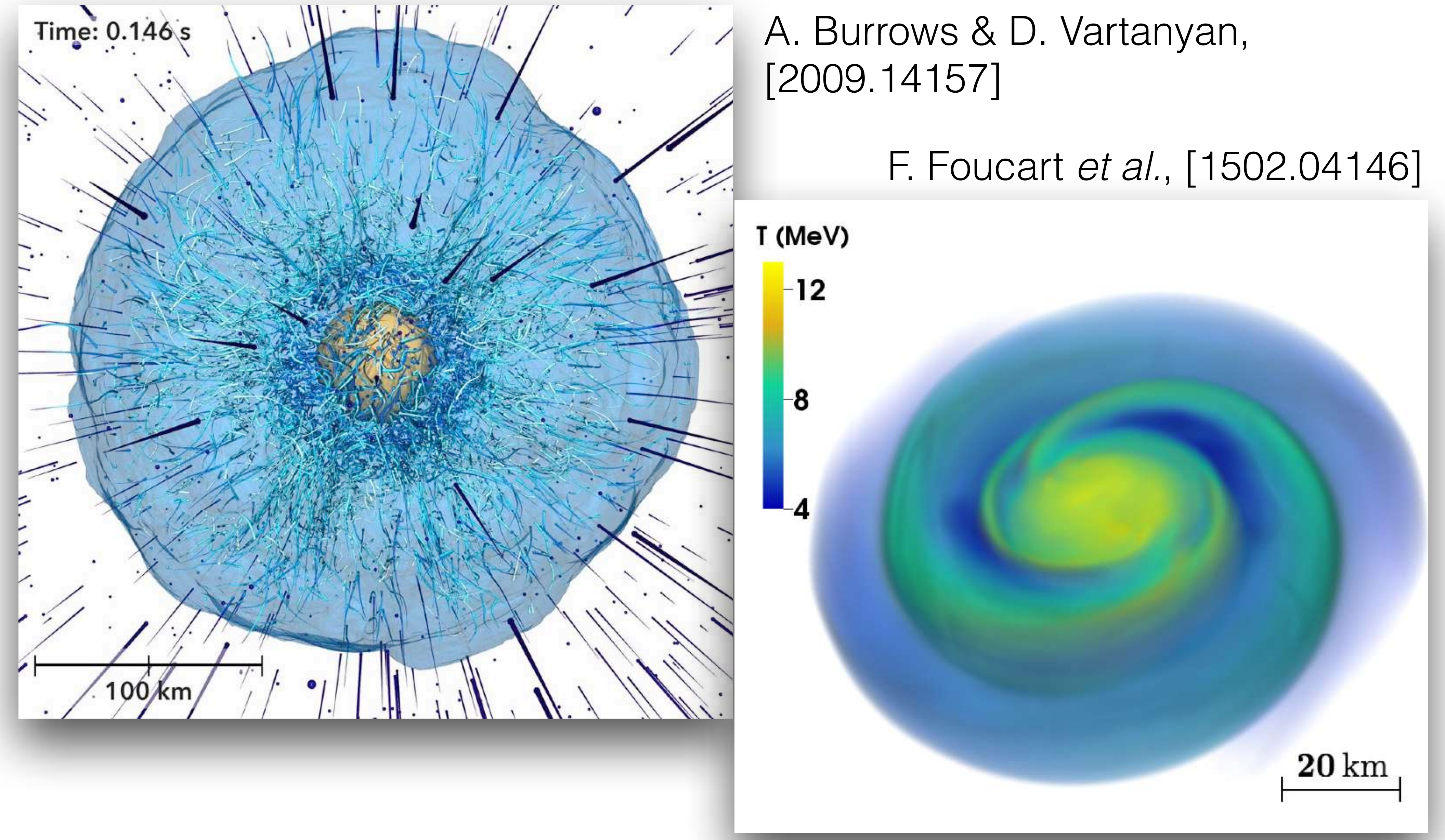

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2 Phenomenological models

e.g., full flavor equilibration in some regions

Li & Siegel [2103.02616], Just+ [2203.16559]…

Included in some supernova models (Ehring+ [2301.11938, 2305.11207], Mori+ [2501.15256] → impact on explosion), and

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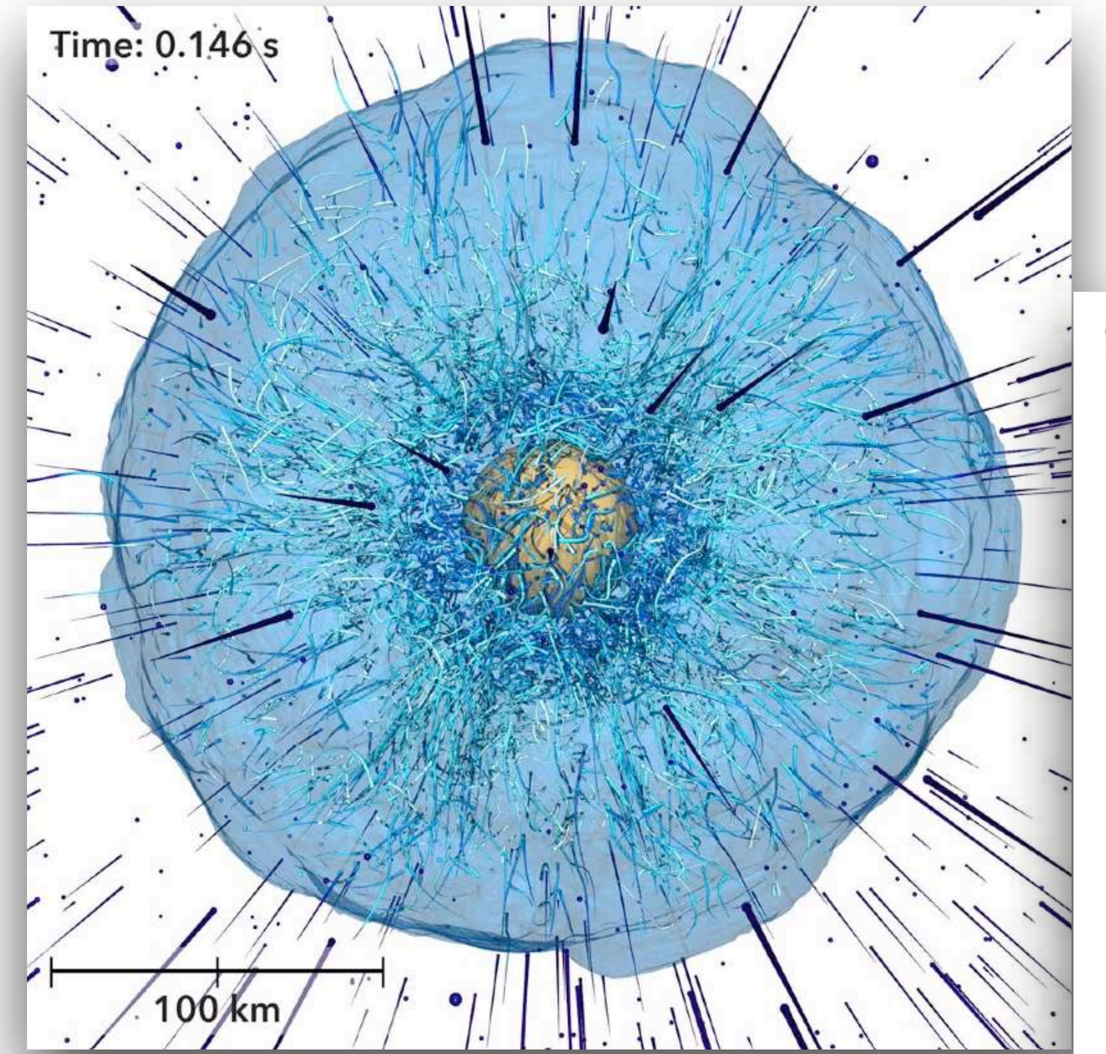

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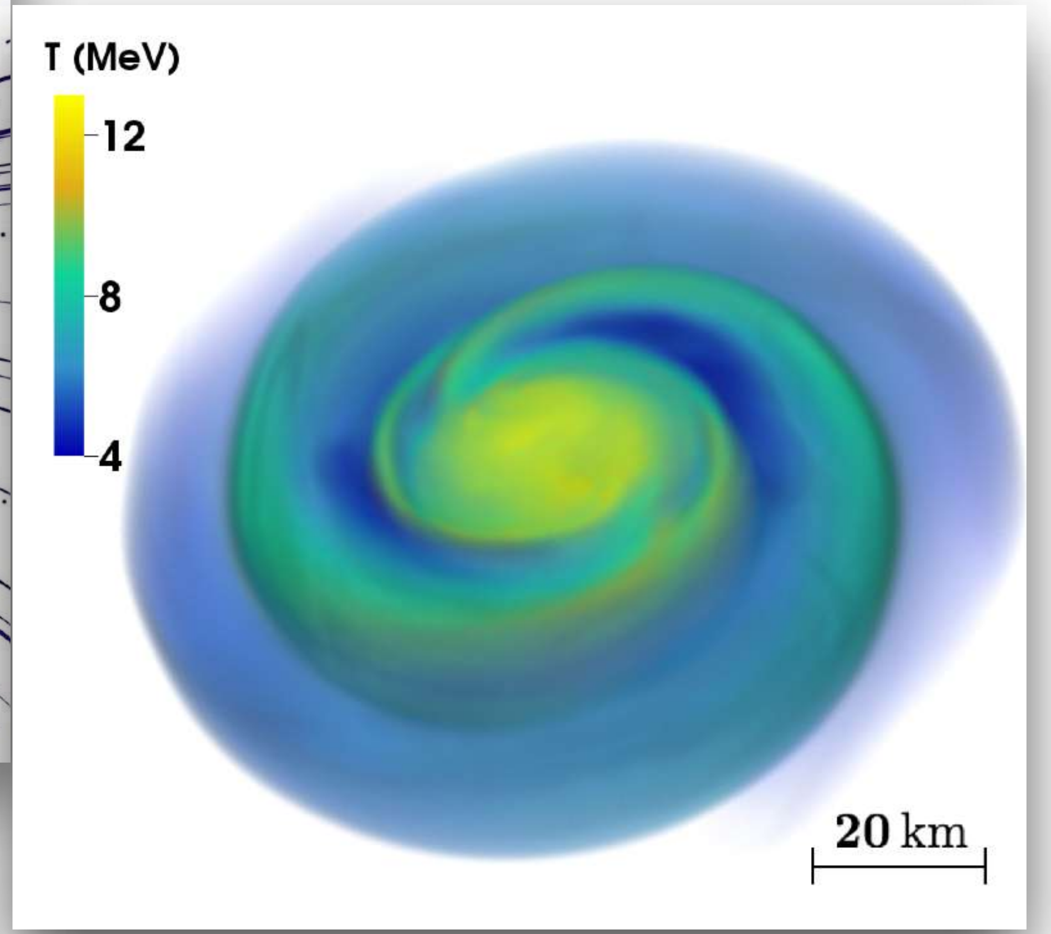

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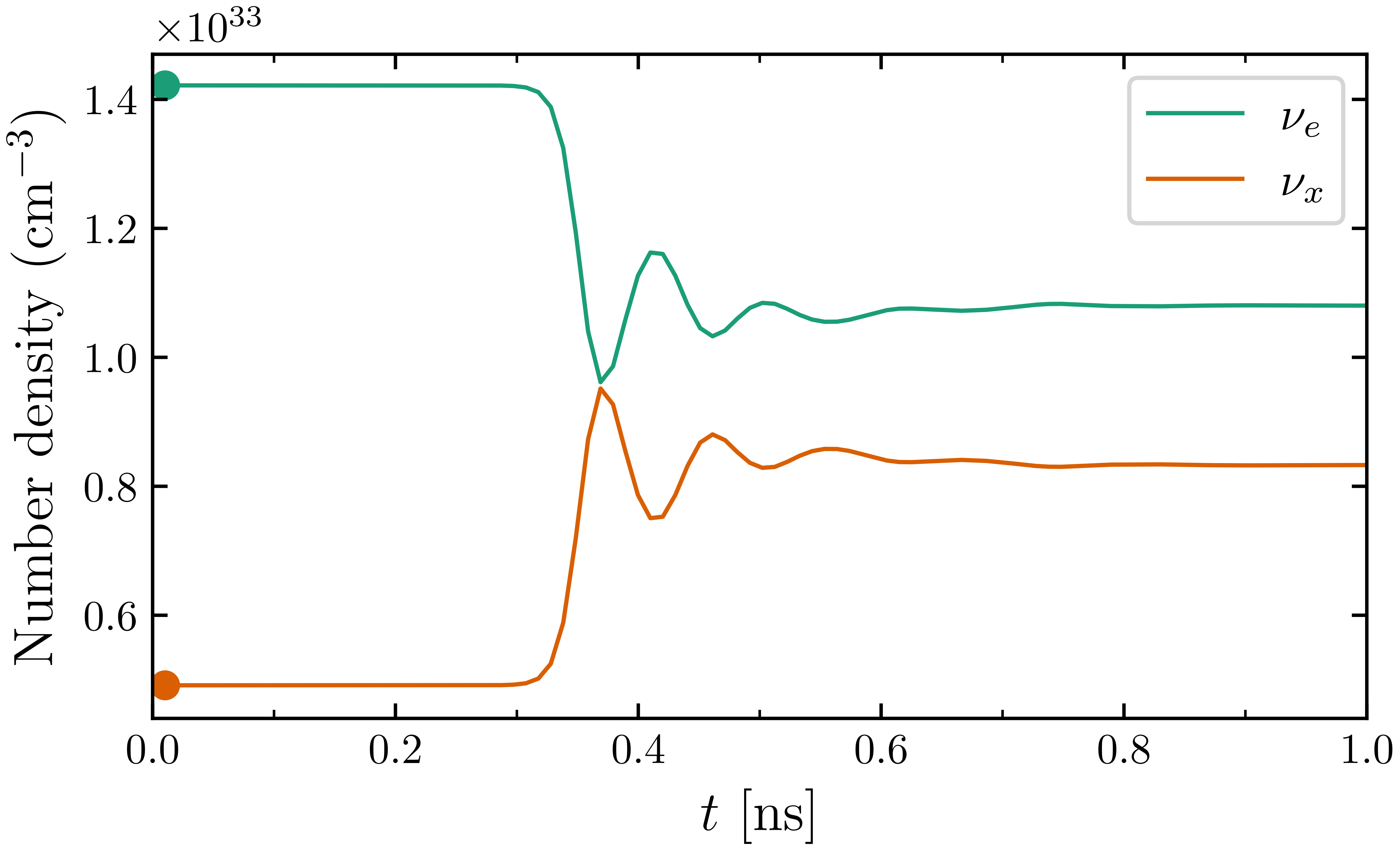
3 Subgrid models

Subgrid models

- BGK approach:

H. Nagakura *et al.*, [2312.16285]



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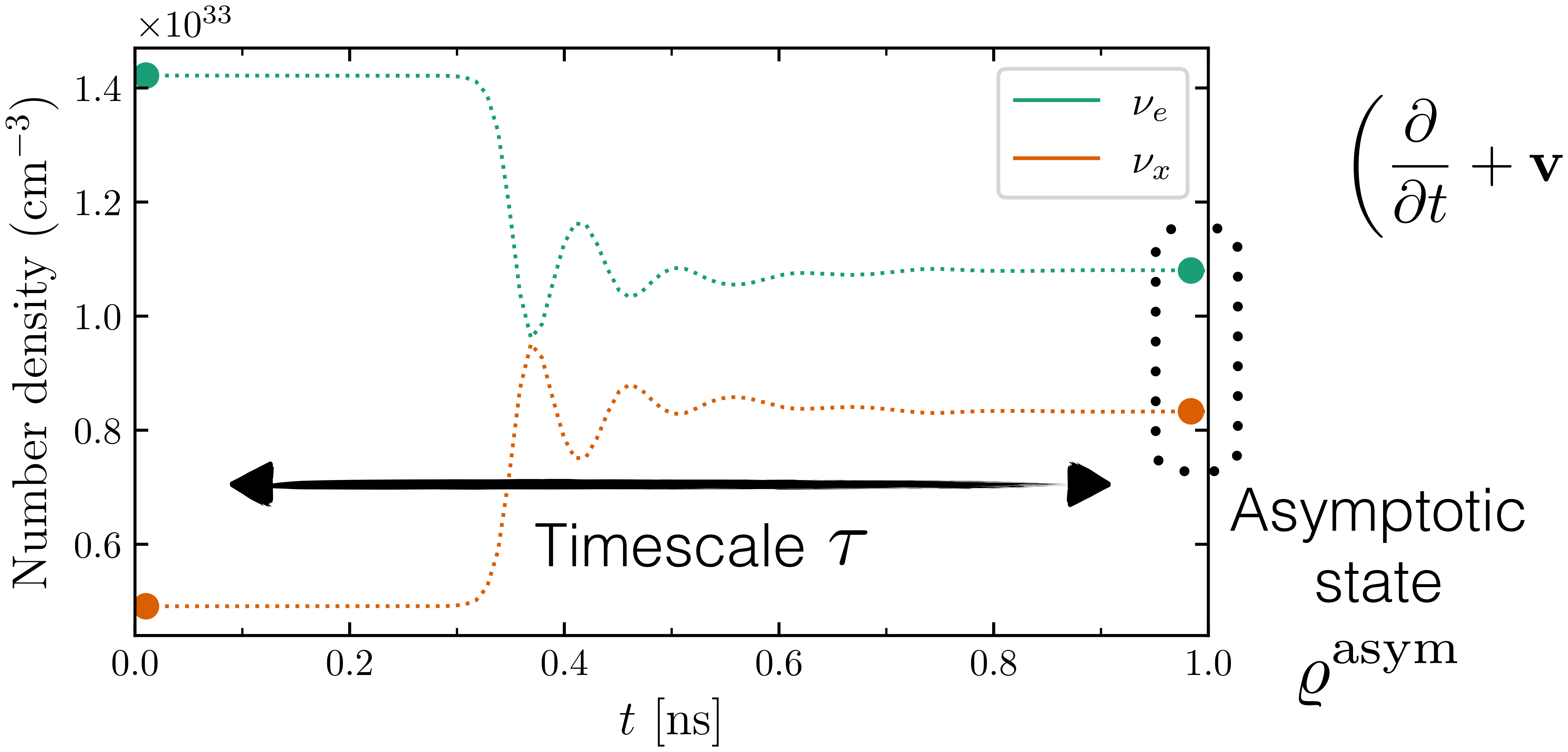

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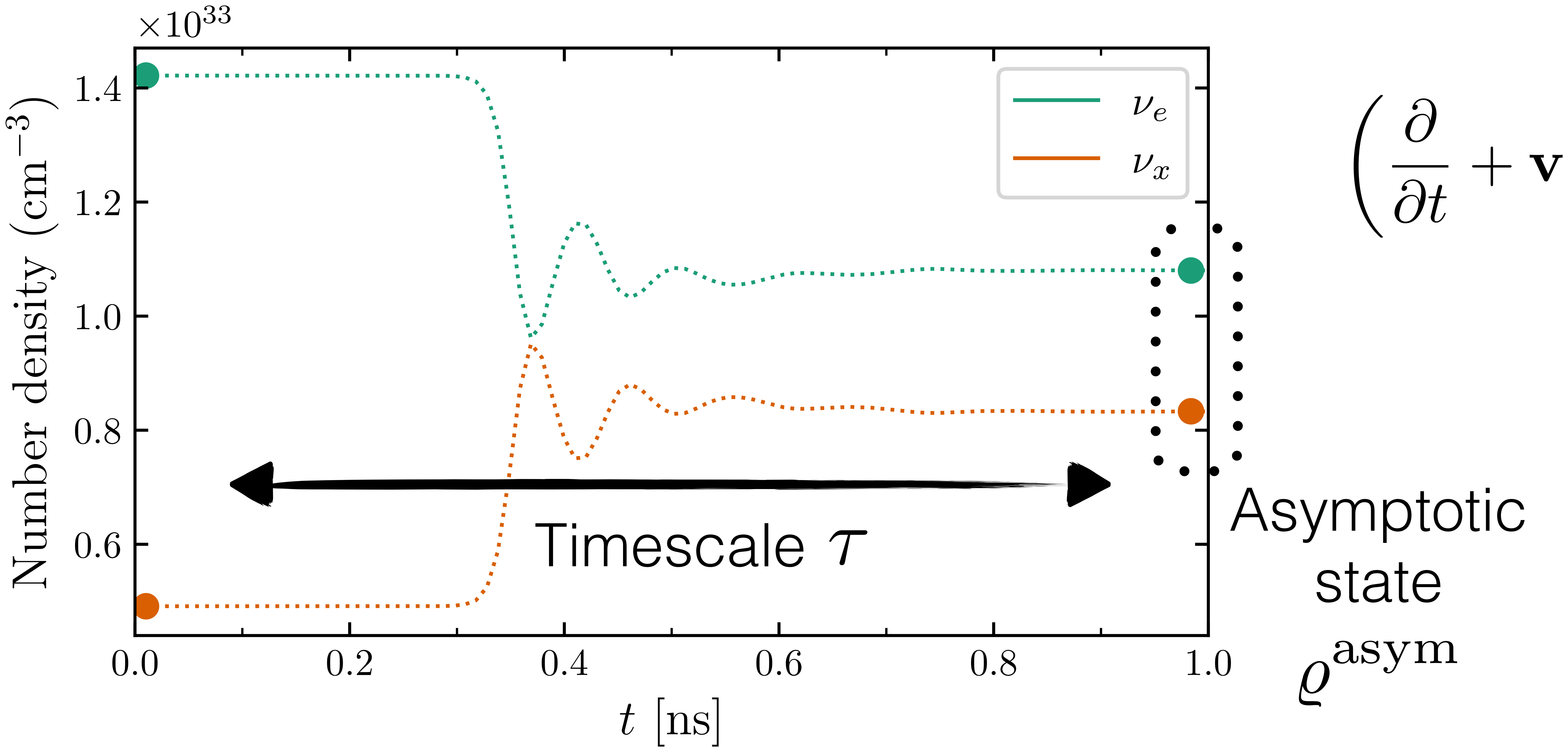

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$$\downarrow$$

$$-\frac{1}{\tau}\left(\varrho - \varrho^{\mathrm{asym}}\right)$$

- Timescale: *linear stability analysis* (Banerjee *et al.* [1107.2308], Izaguirre *et al.* [1610.01612], Froustey *et al.*, [2311.11968], Fiorillo and Raffelt [2505.20389], and many many others)

- **Asymptotic state?**

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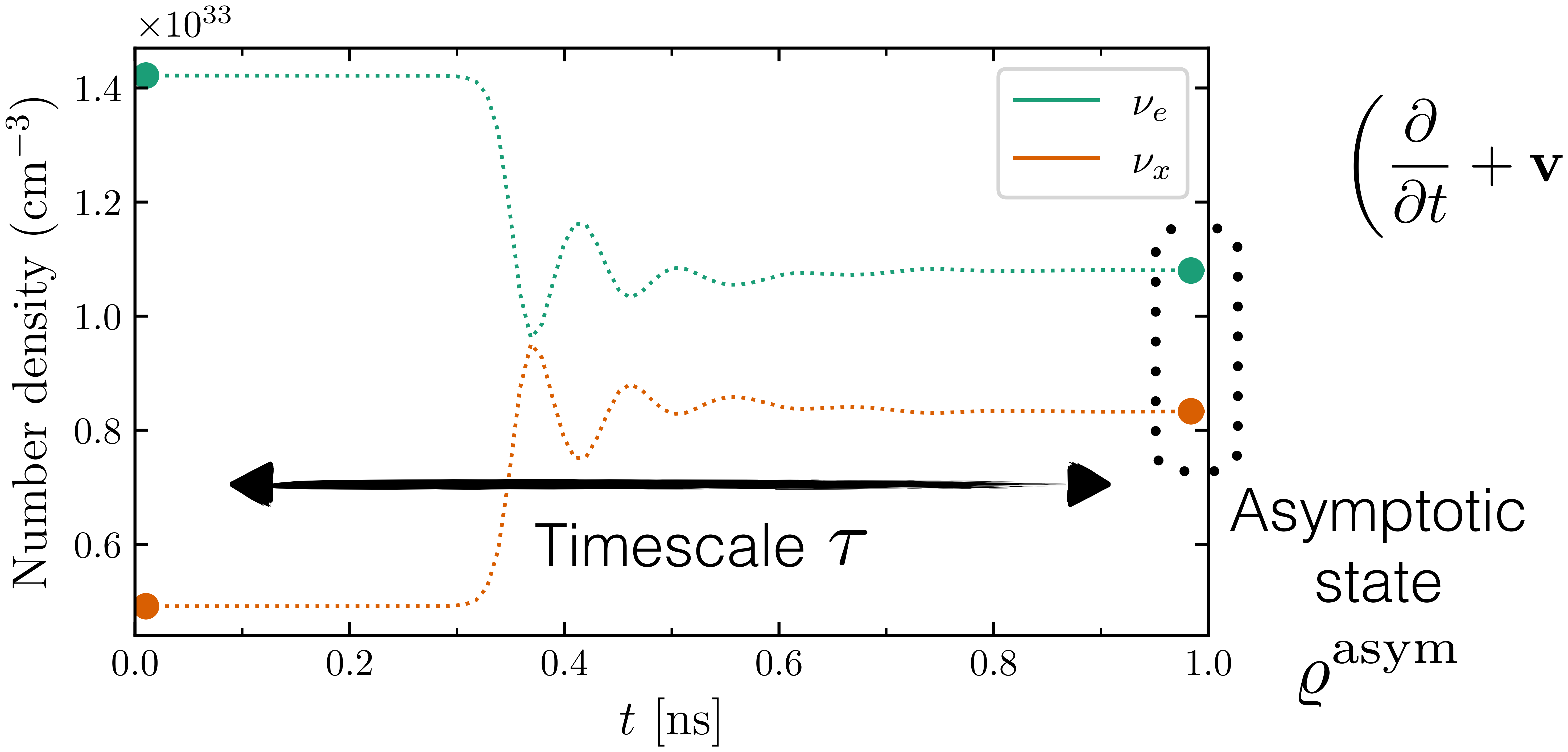

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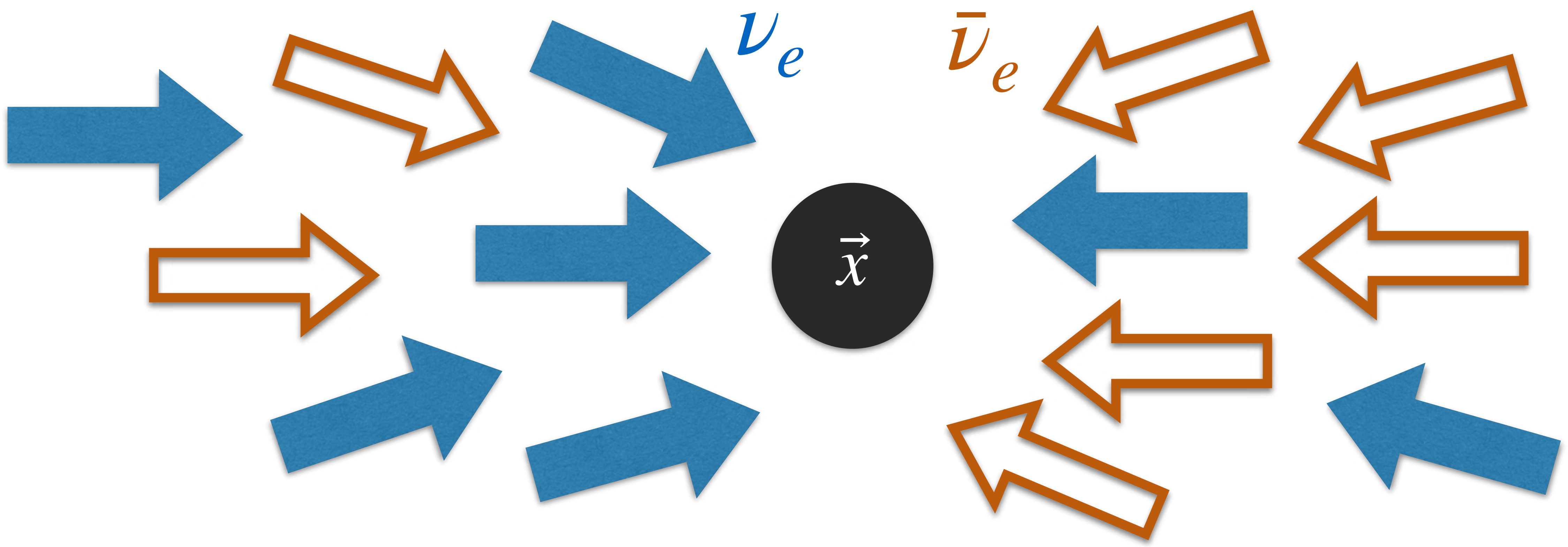
- **Asymptotic state?**

Method: local (cm-scale) high-resolution QKE calculations with periodic boundary conditions

Some flavor instabilities

- **Fast flavor instability (FFI)** *R. Sawyer*, [hep-ph/0503013]

  related to an ***angular crossing*** between neutrino and antineutrino distributions

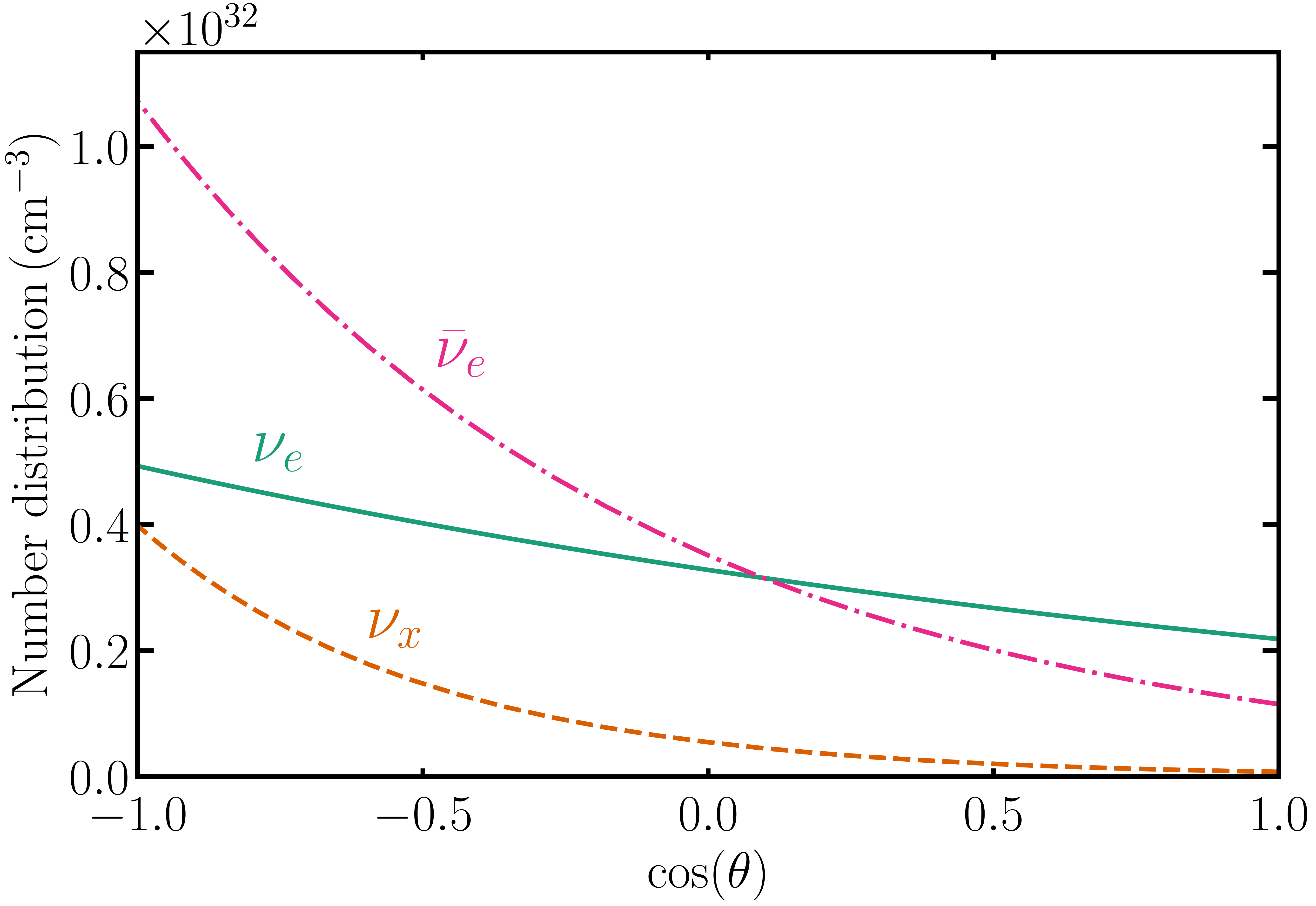


$$f_{\nu_e}(\vec{x}, p, \theta_1) - f_{\bar{\nu}_e}(\vec{x}, p, \theta_1) > 0$$
and
$$f_{\nu_e}(\vec{x}, p, \theta_2) - f_{\bar{\nu}_e}(\vec{x}, p, \theta_2) < 0$$

- **Collisional flavor instability (CFI)** *L. Johns*, [2104.11369]

  related to ***different collision rates*** between neutrinos and antineutrinos

Asymptotic state of FFI — Axisymmetric case



Data: *E. Urquilla* with the particle-in-cell code `Emu`

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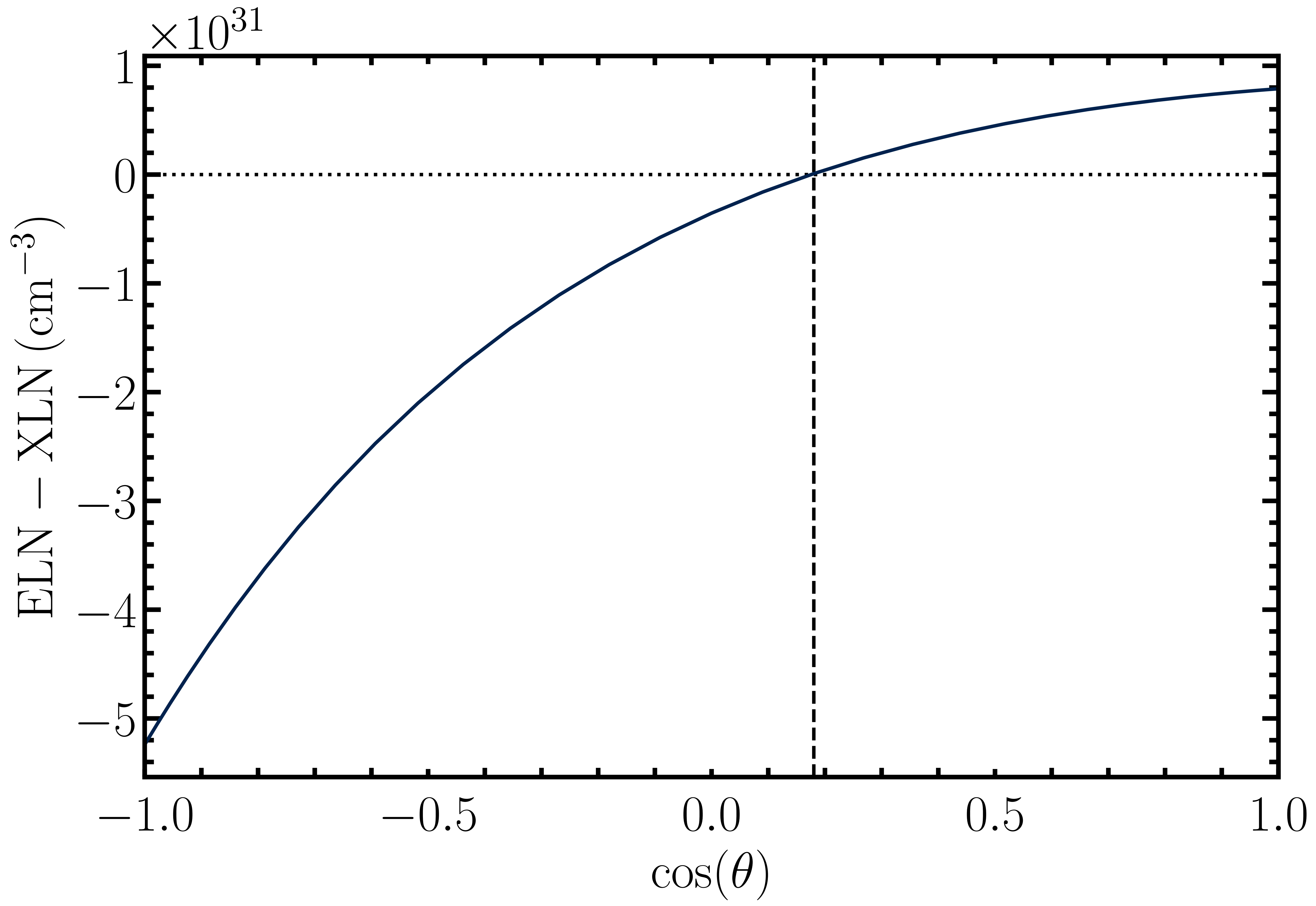

Data: *E. Urquilla* with the particle-in-cell code **Emu**

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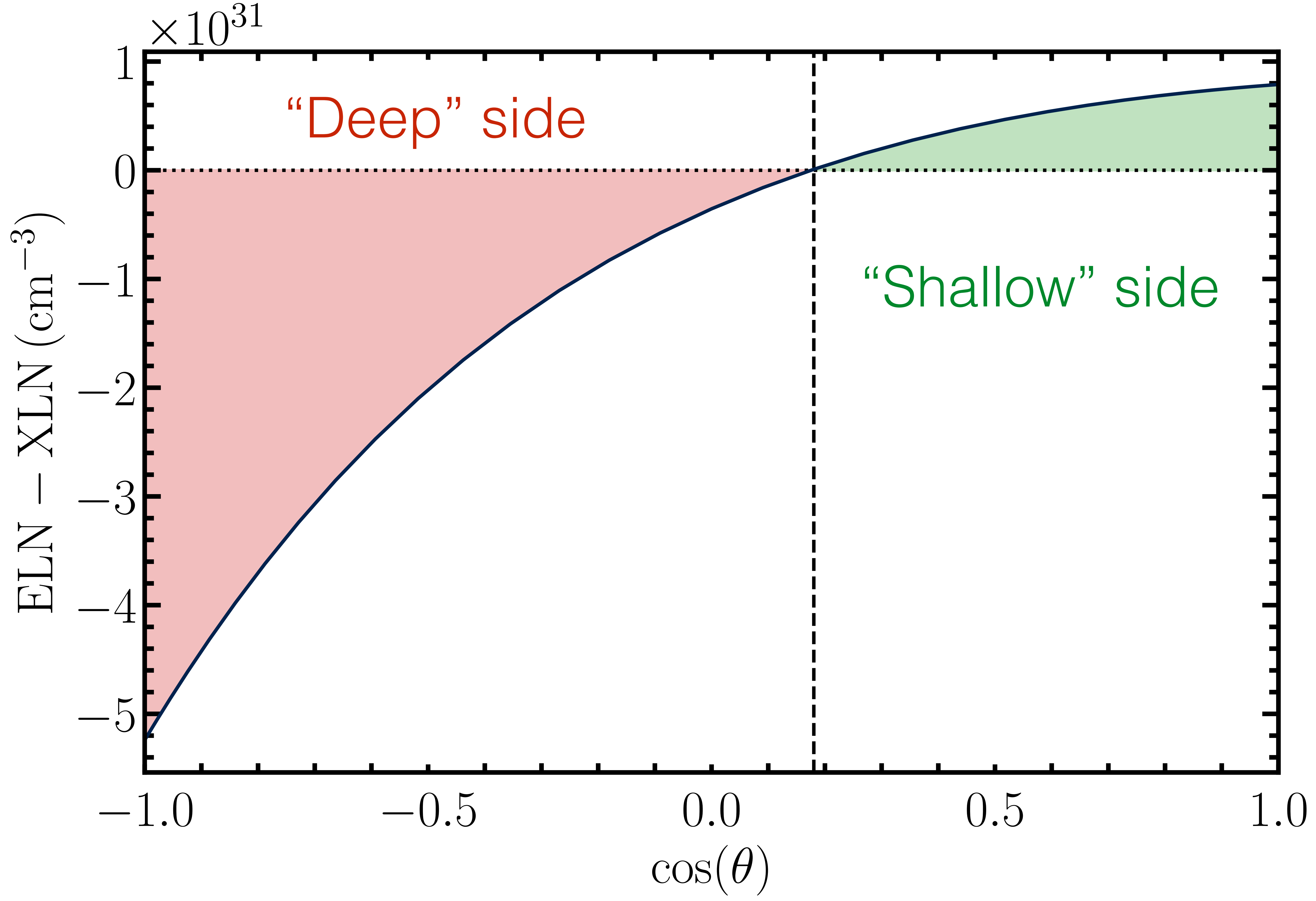
$$\mathrm{ELN} = \varrho_{ee}(\theta) - \bar{\varrho}_{ee}(\theta)$$
$$\mathrm{XLN} = \varrho_{xx}(\theta) - \bar{\varrho}_{xx}(\theta)$$

$$\text{Instability} \iff \text{Crossing in ELN} - \text{XLN}$$

Asymptotic state of FFI — Axisymmetric case



Data: *E. Urquilla* with the particle-in-cell code `Emu`

S. Richers *et al.*, [2101.02745]

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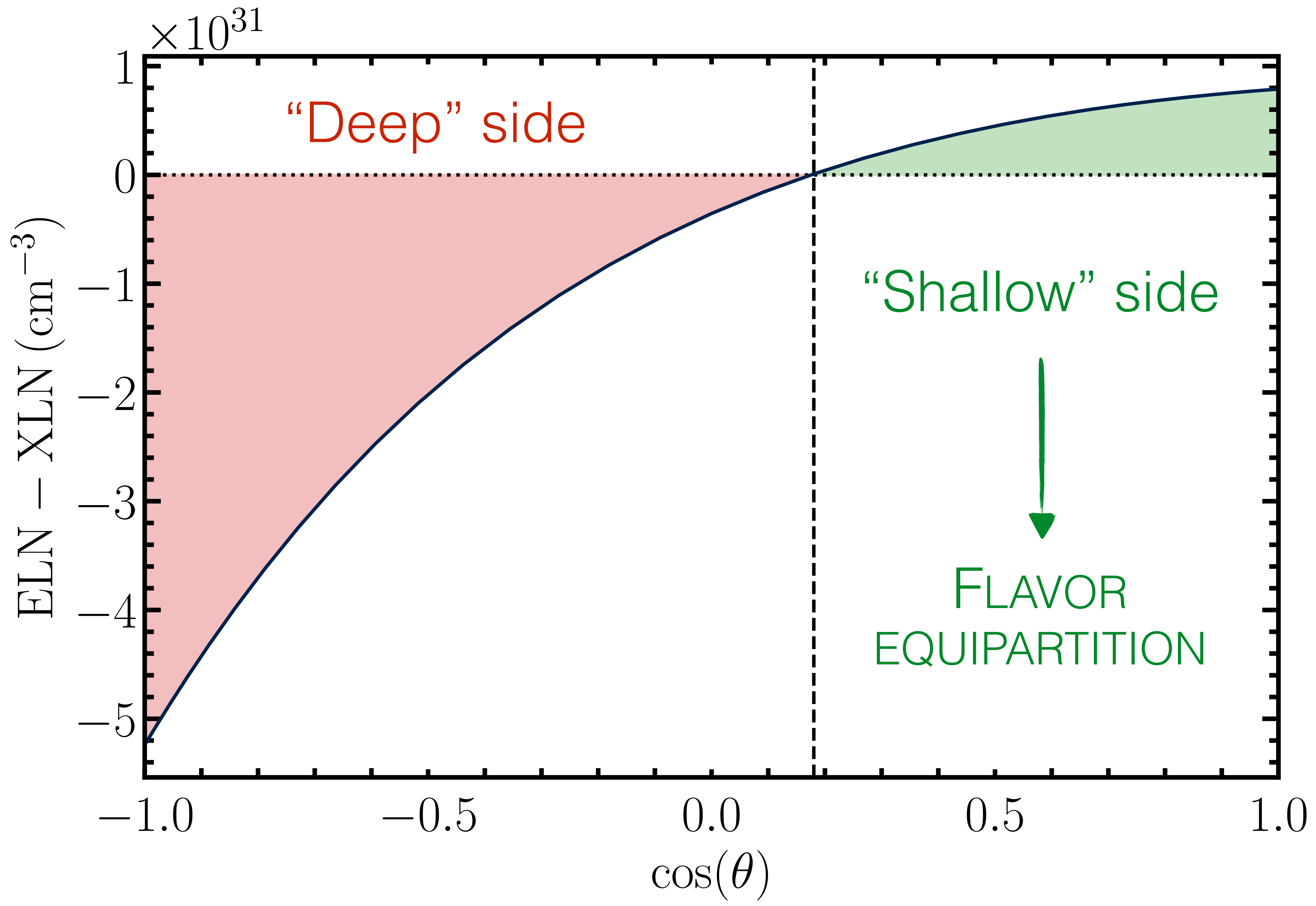

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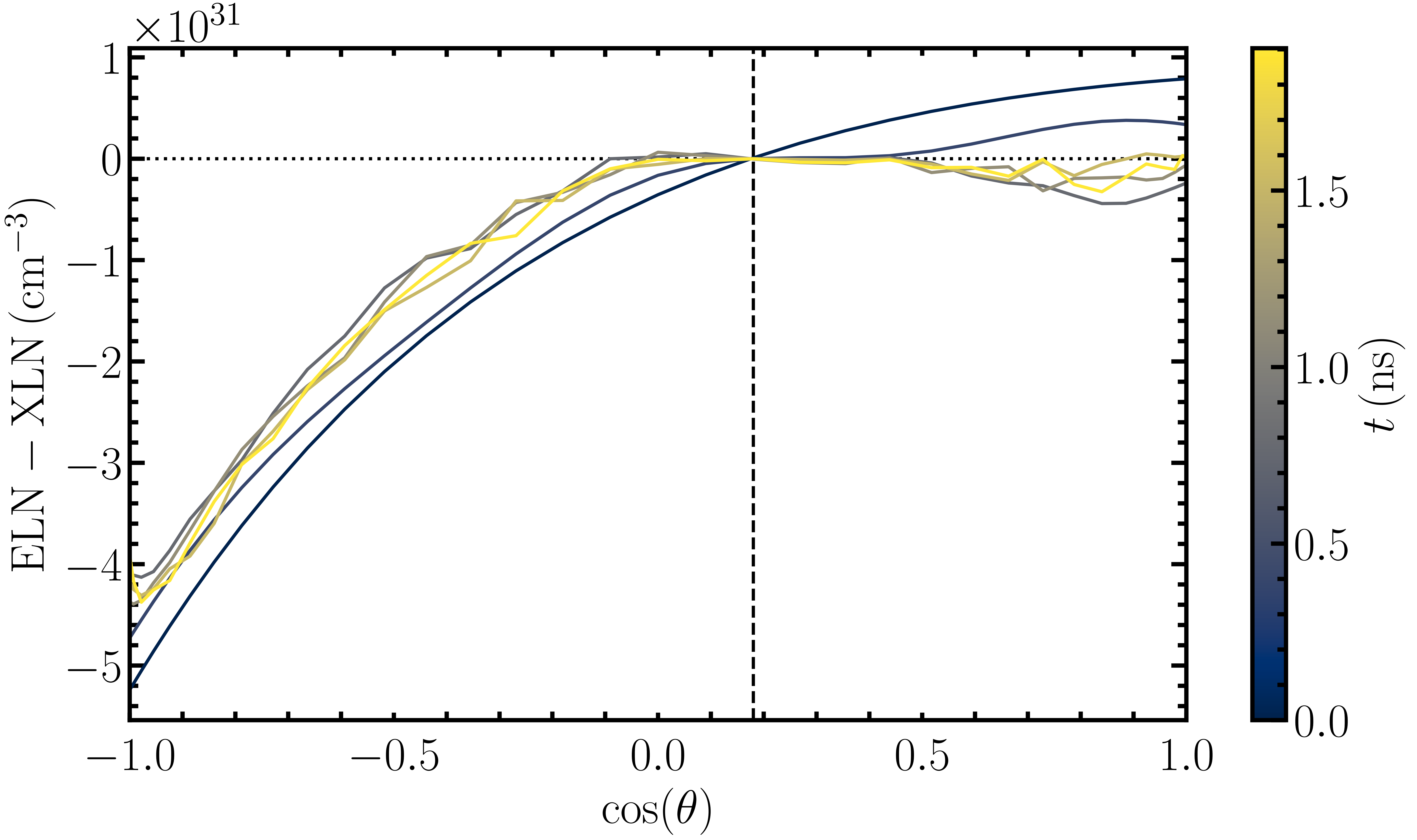

<u>Data:</u> *E. Urquilla* with the particle-in-cell code `Emu`

S. Richers *et al.*, [2101.02745]

This "localized crossing erasure" can be expressed as a survival probability.

M. Zaizen and H. Nagakura, [2211.09343]

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Box3D scheme

S. Richers, **JF**, S. Ghosh, F. Foucart, J. Gomez, [2409.04405]

ELN-XLN distribution

$$G(\mathbf{n}) \equiv [f_{\nu_e}(\mathbf{n}) - f_{\bar{\nu}_e}(\mathbf{n})] - [f_{\nu_x}(\mathbf{n}) - f_{\bar{\nu}_x}(\mathbf{n})]$$

Positive and negative parts

$$I_+ = \left| \int_{G(\mathbf{n})>0} \mathrm{d}\mathbf{n}\, G(\mathbf{n}) \right| ,$$

$$I_- = \left| \int_{G(\mathbf{n})<0} \mathrm{d}\mathbf{n}\, G(\mathbf{n}) \right| ,$$

$$I_> = \max\{I_+, I_-\} ,$$

$$I_< = \min\{I_+, I_-\} .$$

Recall:
instability = crossing =
both integrals are non-zero

Box3D scheme

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Survival probability (3 flavors)

$$\mathbb{P}(\mathbf{n}) = \begin{cases} \dfrac{1}{3} & \text{for } \mathbf{n} \in \Omega_< , \\ 1 - \dfrac{2I_<}{3I_>} & \text{for } \mathbf{n} \in \Omega_> . \end{cases}$$

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→

Asymptotic state

$$f'_{\nu_e}(\mathbf{n}) = \mathbb{P}(\mathbf{n}) f_{\nu_e}(\mathbf{n}) + [1 - \mathbb{P}(\mathbf{n})]\, f_{\nu_x}(\mathbf{n}) ,$$
$$f'_{\bar{\nu}_e}(\mathbf{n}) = \mathbb{P}(\mathbf{n}) f_{\bar{\nu}_e}(\mathbf{n}) + [1 - \mathbb{P}(\mathbf{n})]\, f_{\bar{\nu}_x}(\mathbf{n}) ,$$
$$f'_{\nu_x}(\mathbf{n}) = \frac{1}{2}[1 - \mathbb{P}(\mathbf{n})]\, f_{\nu_e}(\mathbf{n}) + \frac{1}{2}[1 + \mathbb{P}(\mathbf{n})]\, f_{\nu_x}(\mathbf{n}) ,$$
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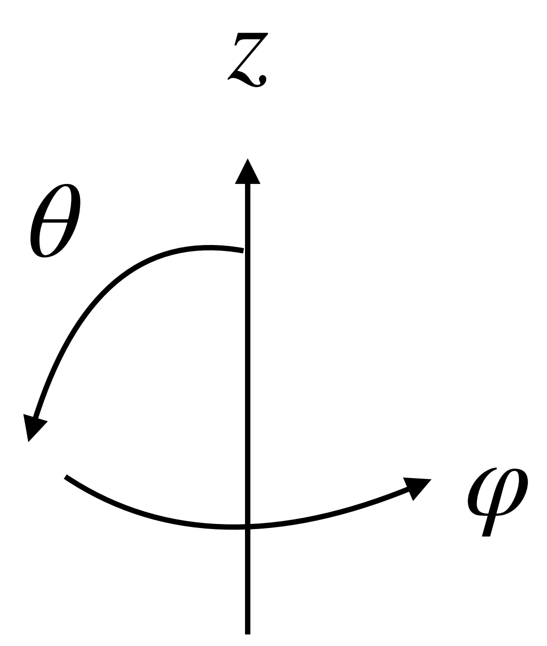
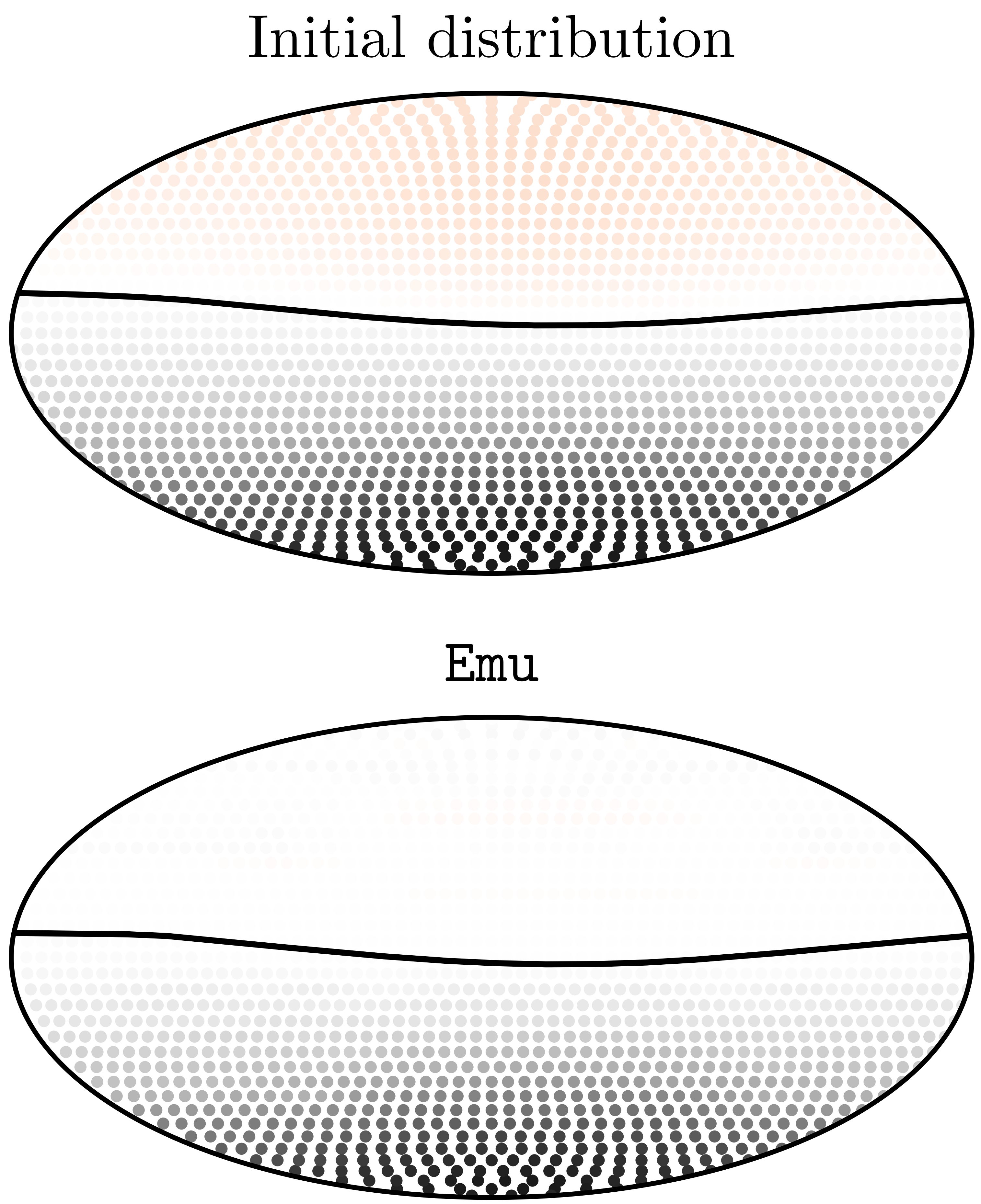
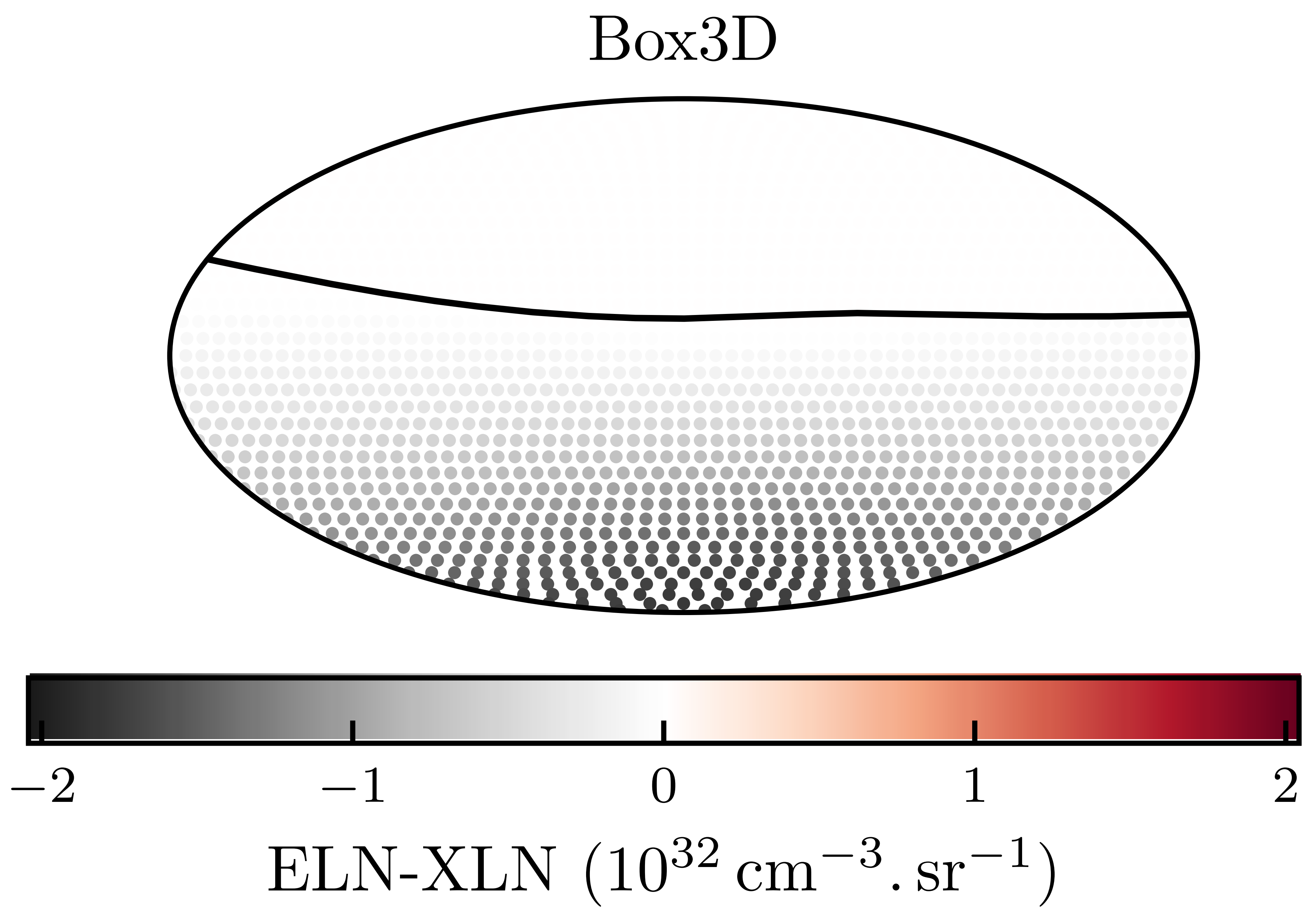
Three-dimensional asymptotic state





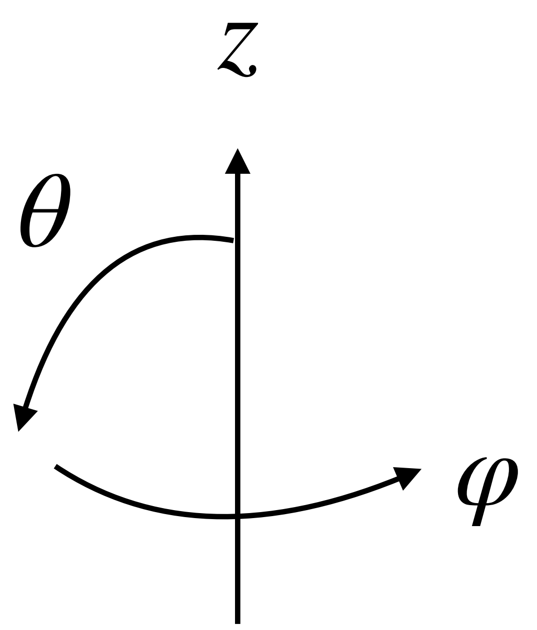
Final state obtained with **`Emu`**

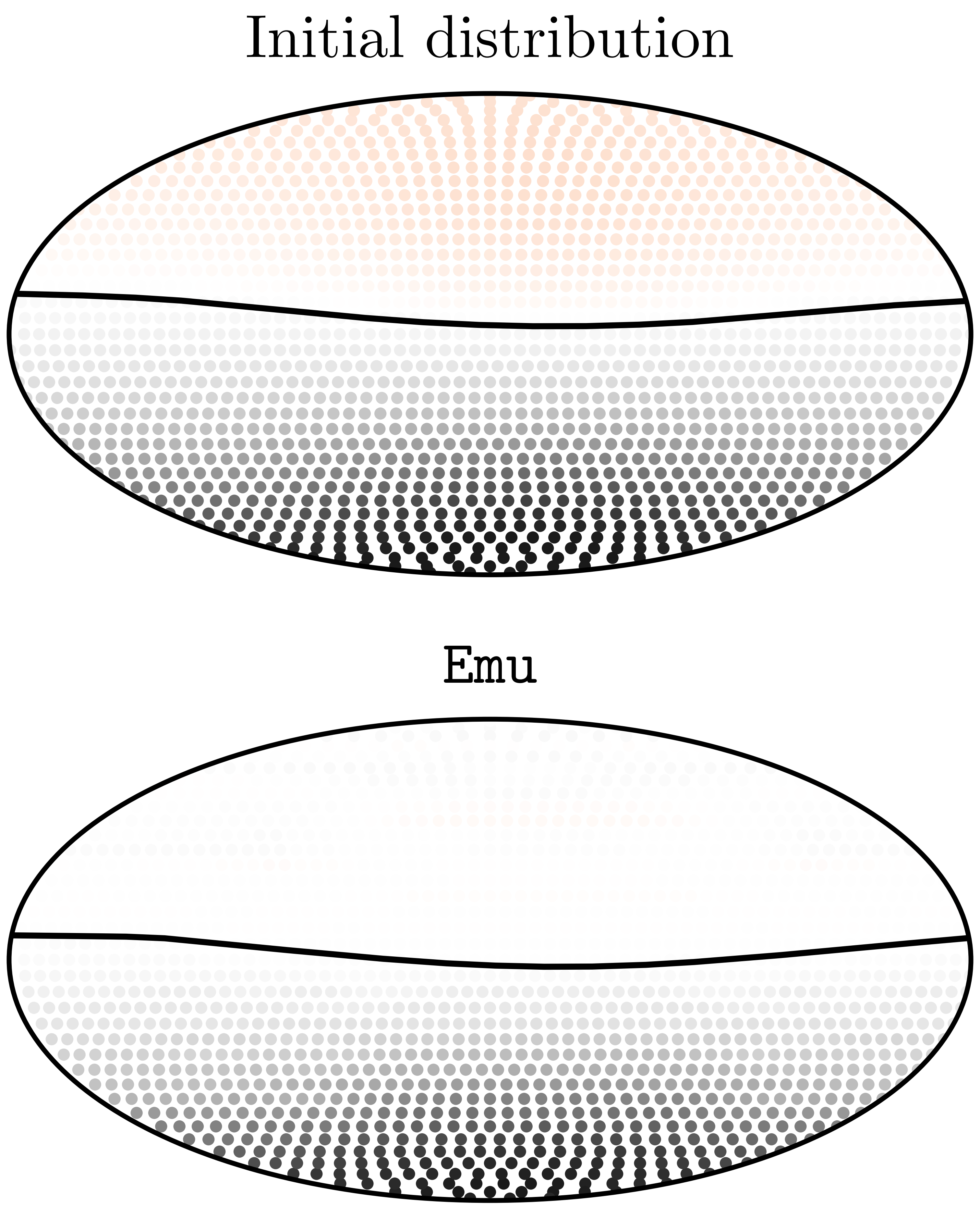
Three-dimensional generalization of ELN-XLN disappearance



S. Richers, **JF**, S. Ghosh, F. Foucart, J. Gomez, [2409.04405]

Three-dimensional asymptotic state





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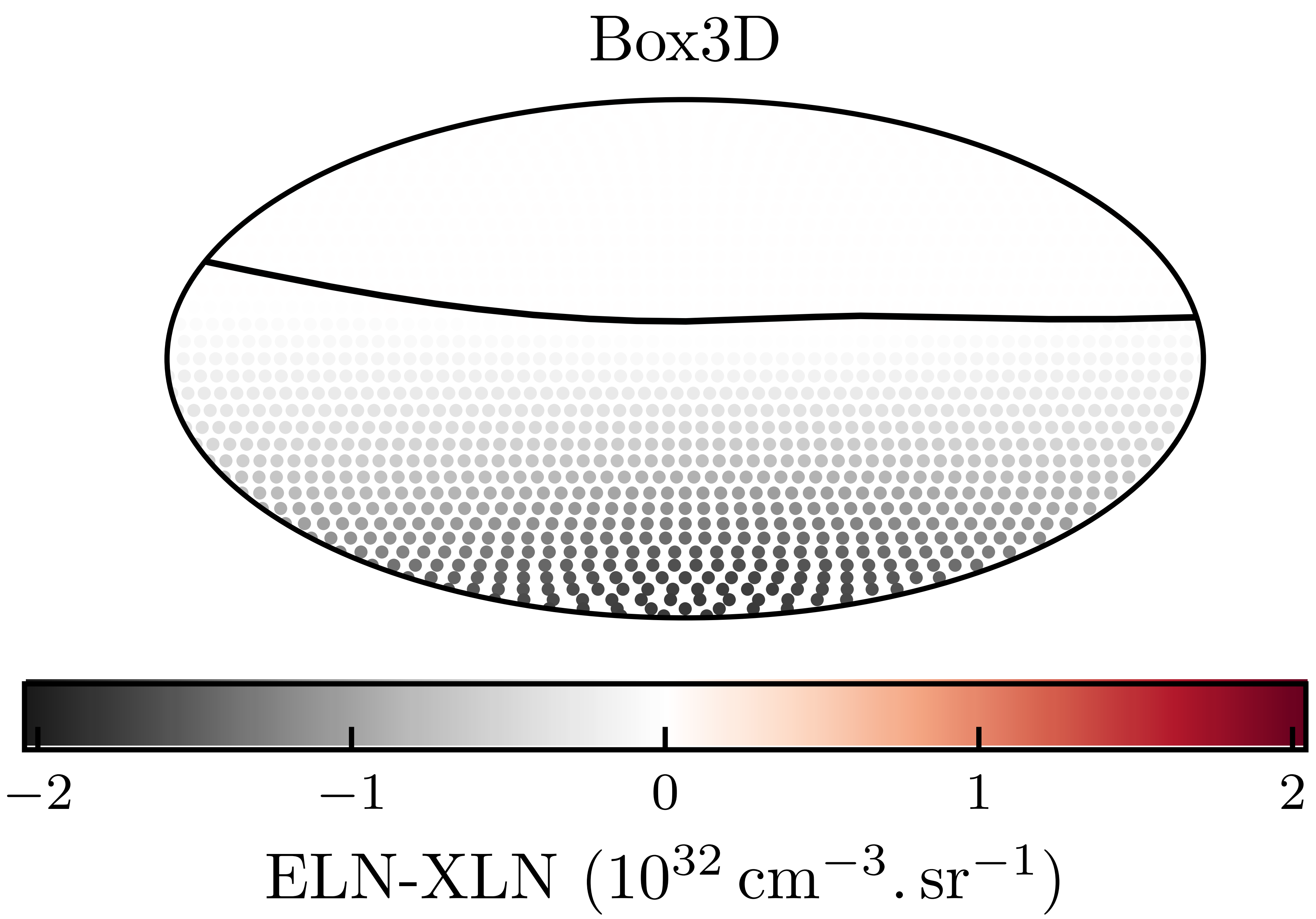
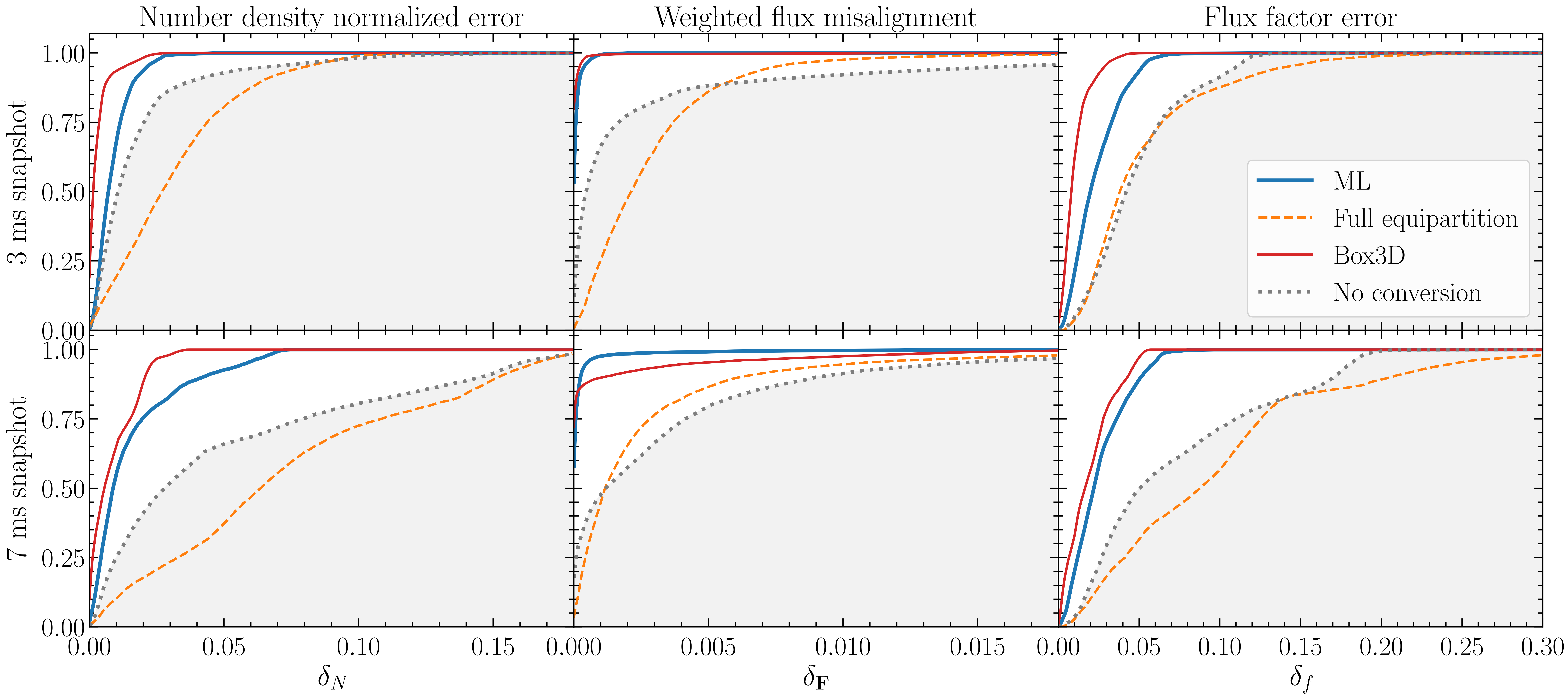


S. Richers, **JF**, S. Ghosh, F. Foucart, J. Gomez, [2409.04405]

Scheme tested and validated on more than 15,000 **Emu** calculations

Global performance on unstable configurations

Richers+ [2409.04405]



$$\delta_N = \max_\alpha \frac{|N_\alpha^{(\text{pred})} - N_\alpha^{(\text{true})}|}{N_{\text{tot}}}$$

$$\delta_{\mathbf{F}} = \max_\alpha \left[\frac{1}{2} \left(1 - \frac{\mathbf{F}_\alpha^{(\text{pred})} \cdot \mathbf{F}_\alpha^{(\text{true})}}{|\mathbf{F}_\alpha^{(\text{pred})}||\mathbf{F}_\alpha^{(\text{true})}|} \right) \times \frac{\mathbf{F}_\alpha^{(\text{true})}}{N_\alpha^{(\text{true})}} \right]$$

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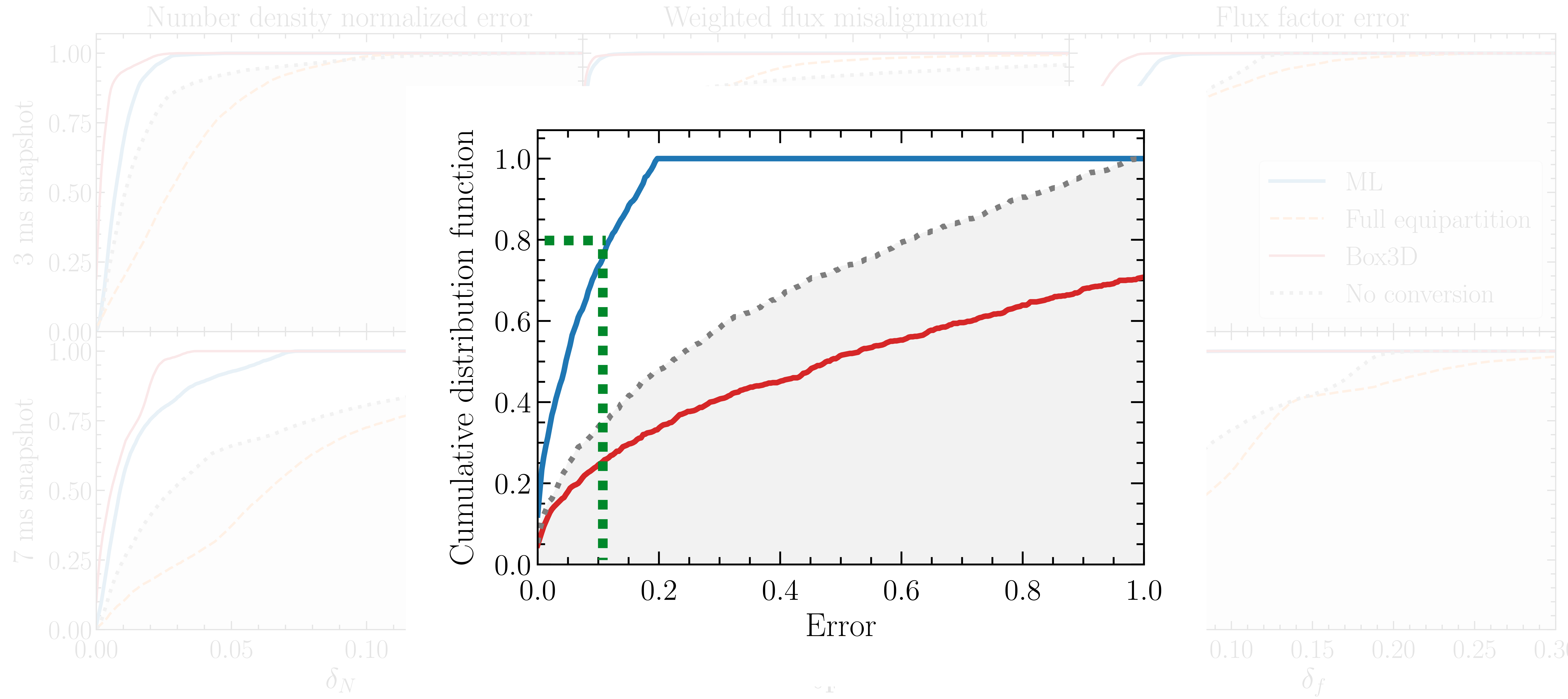

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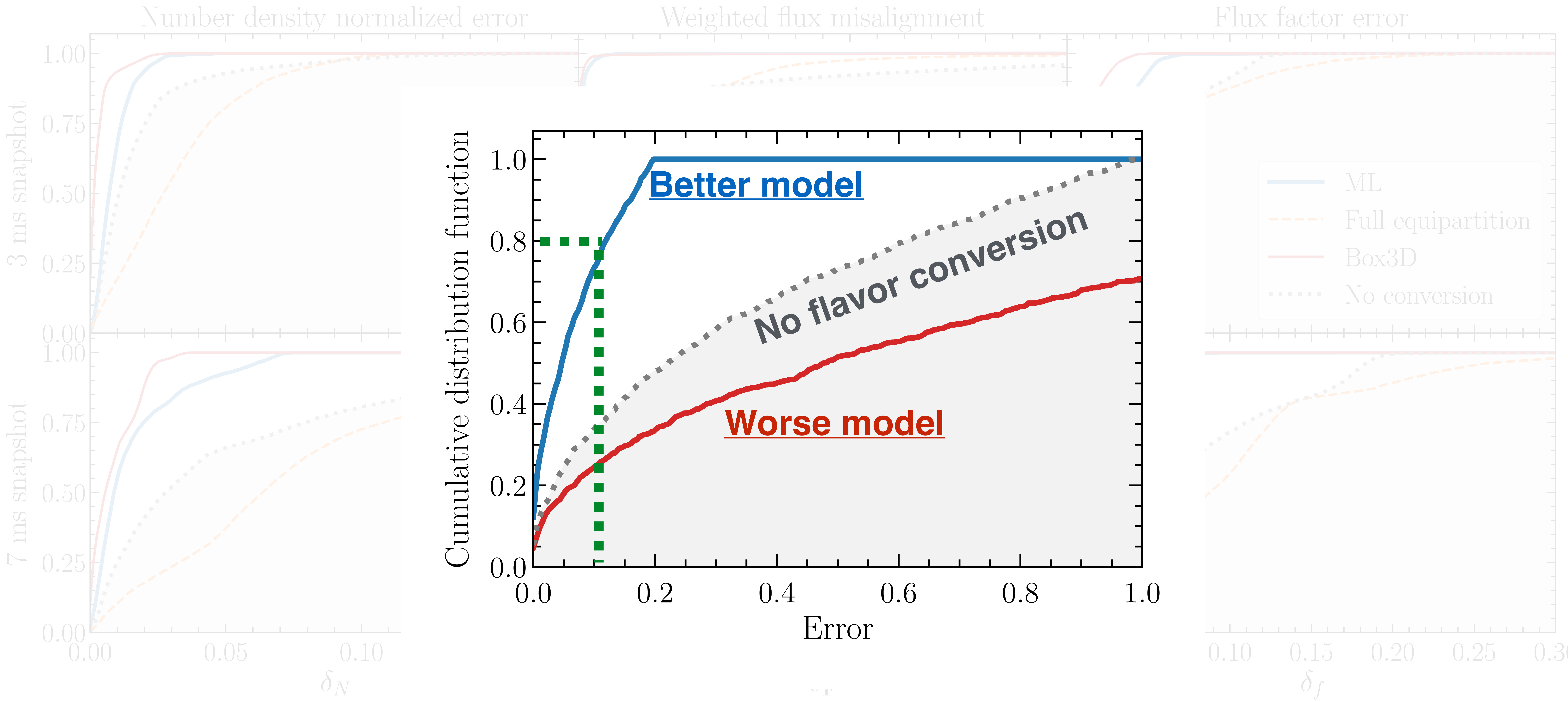

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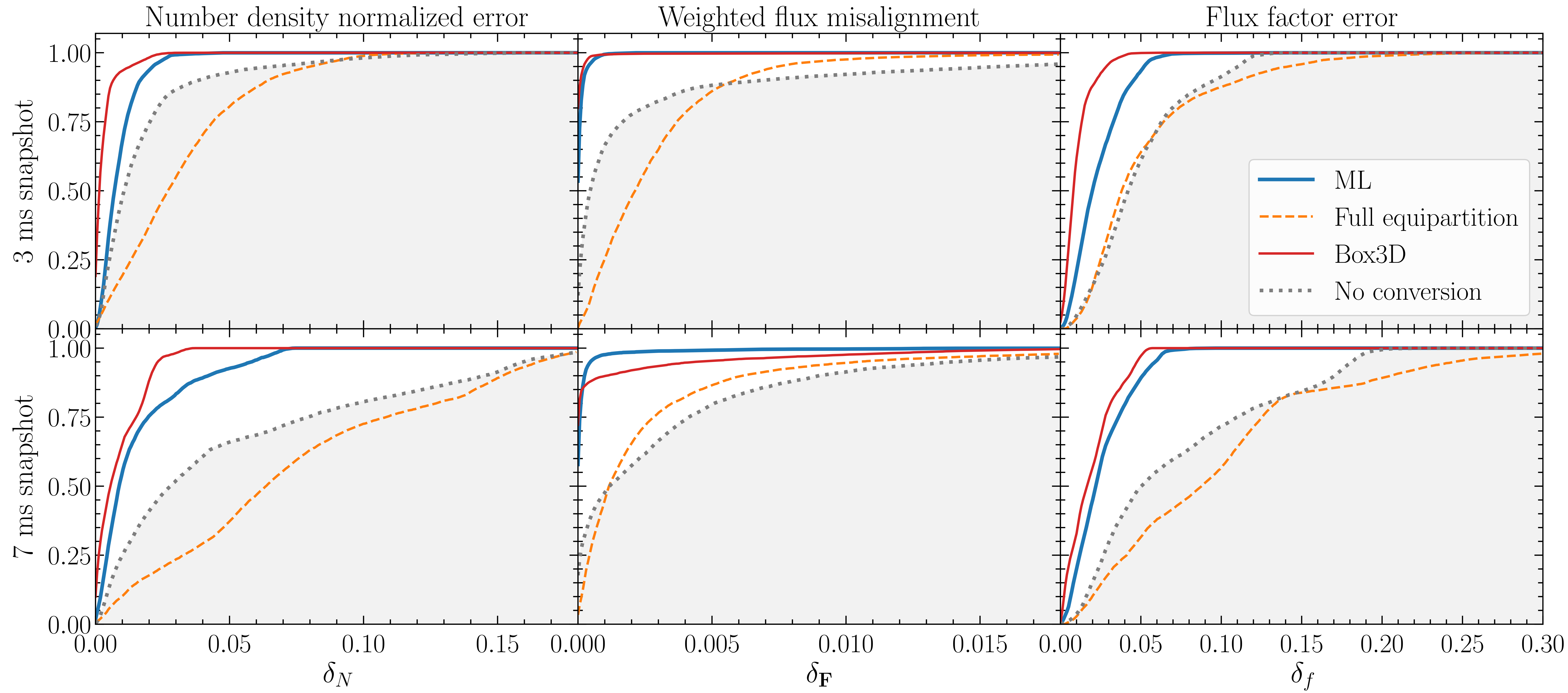

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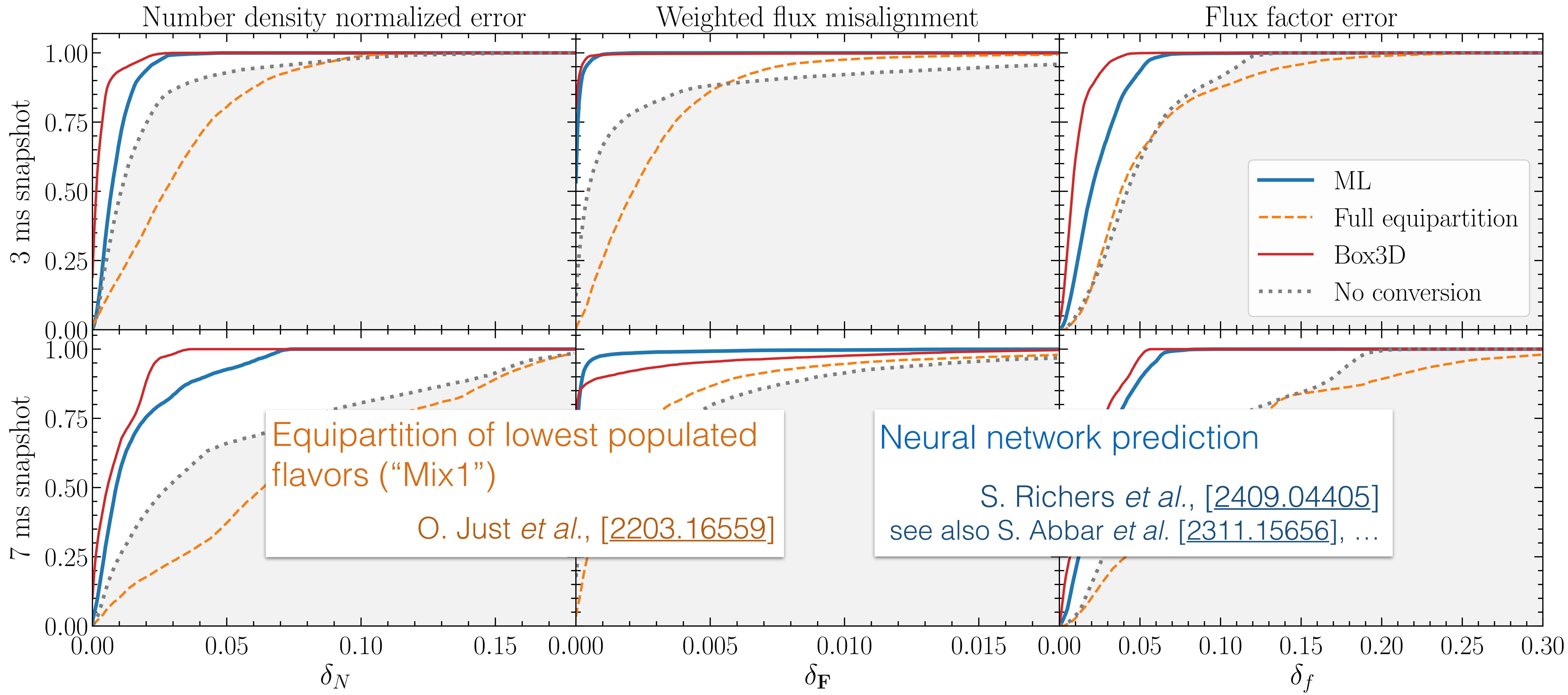

Equipartition of lowest populated flavors (“Mix1”)

O. Just *et al.*, [2203.16559]

Neural network prediction

S. Richers *et al.*, [2409.04405]

see also S. Abbar *et al.* [2311.15656], …

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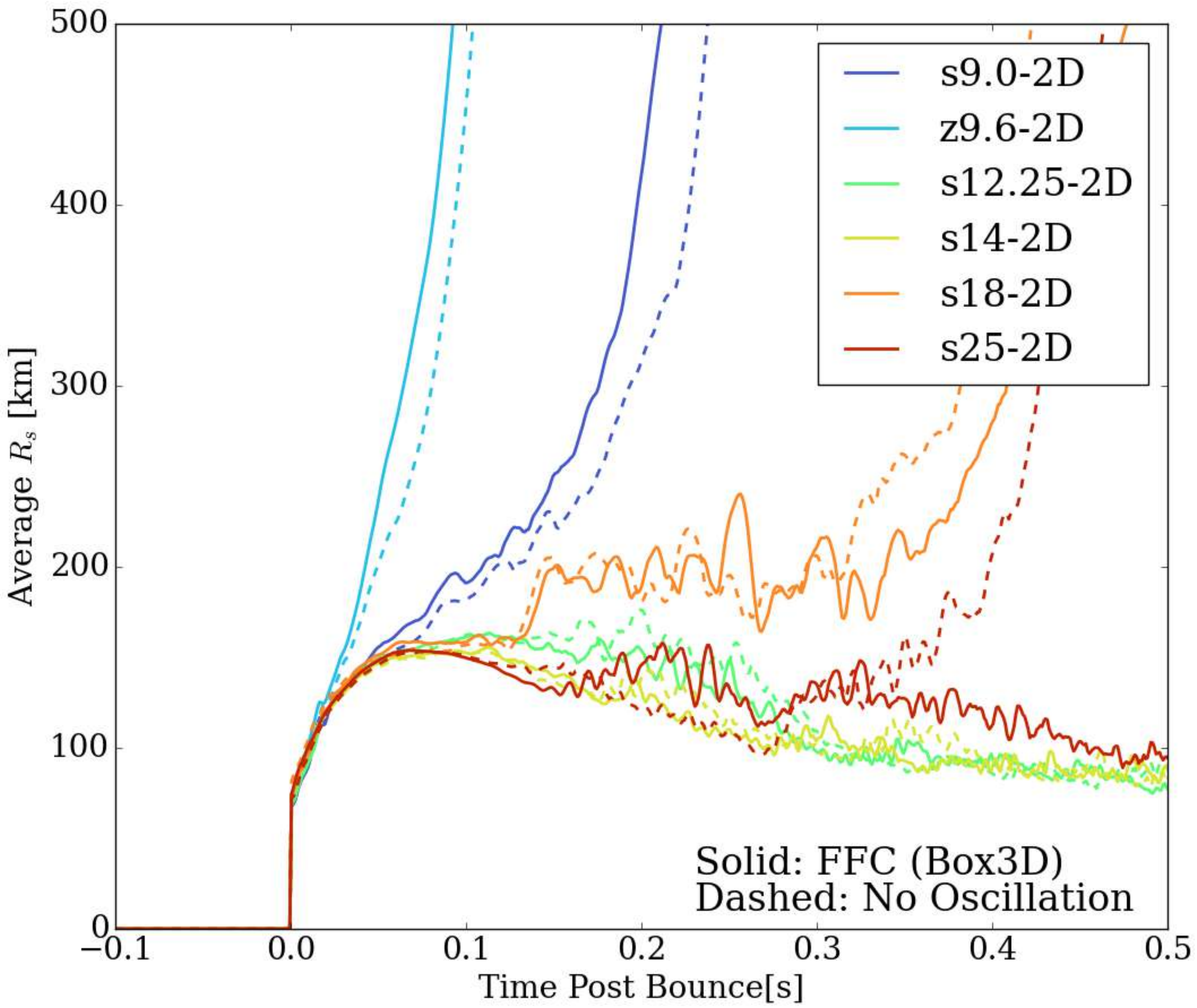
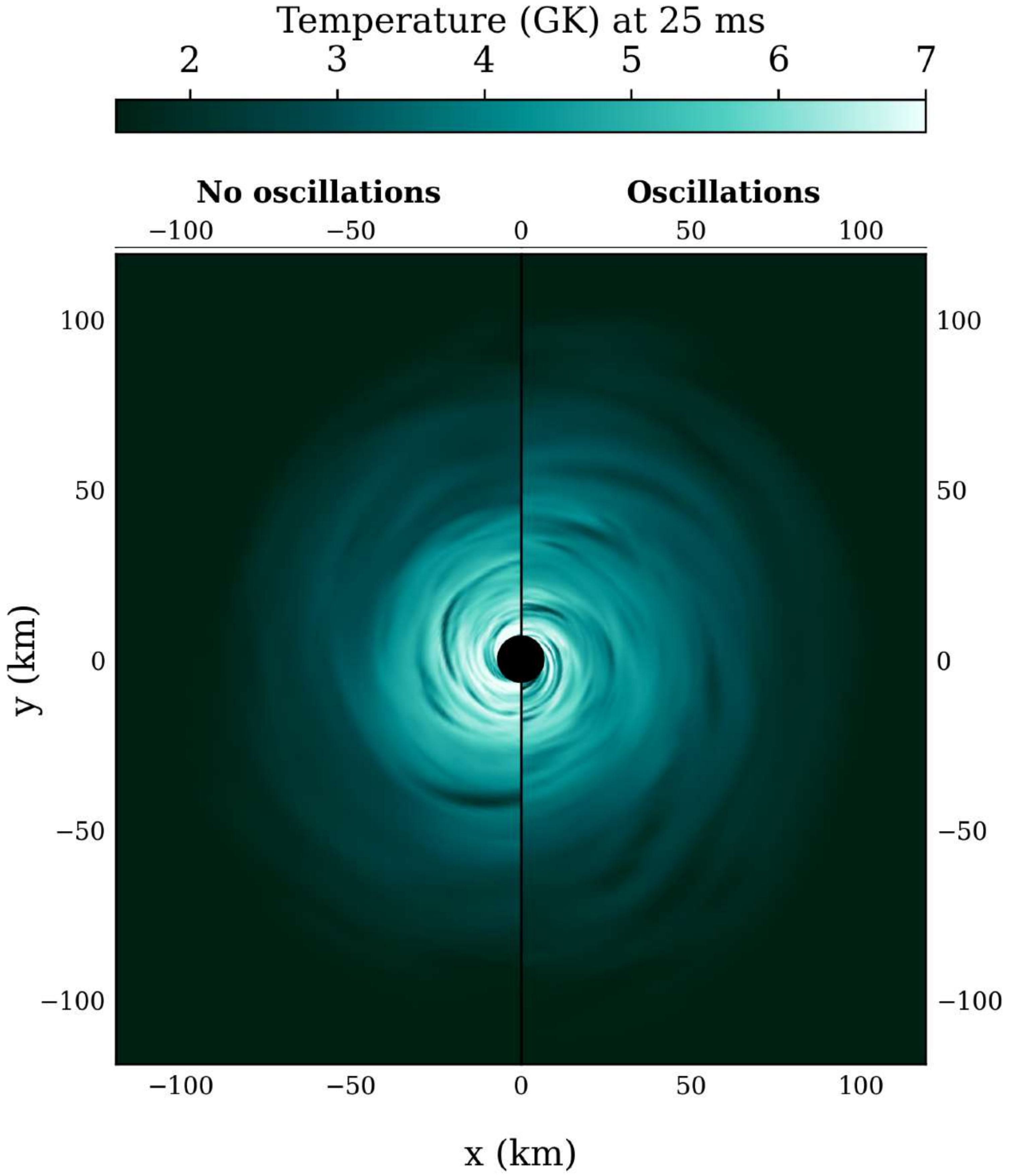
Inclusion of FFIs in large-scale simulations

T. Wang and A. Burrows, *The effect of the fast-flavor instability on core-collapse supernova models*, [2503.04896]

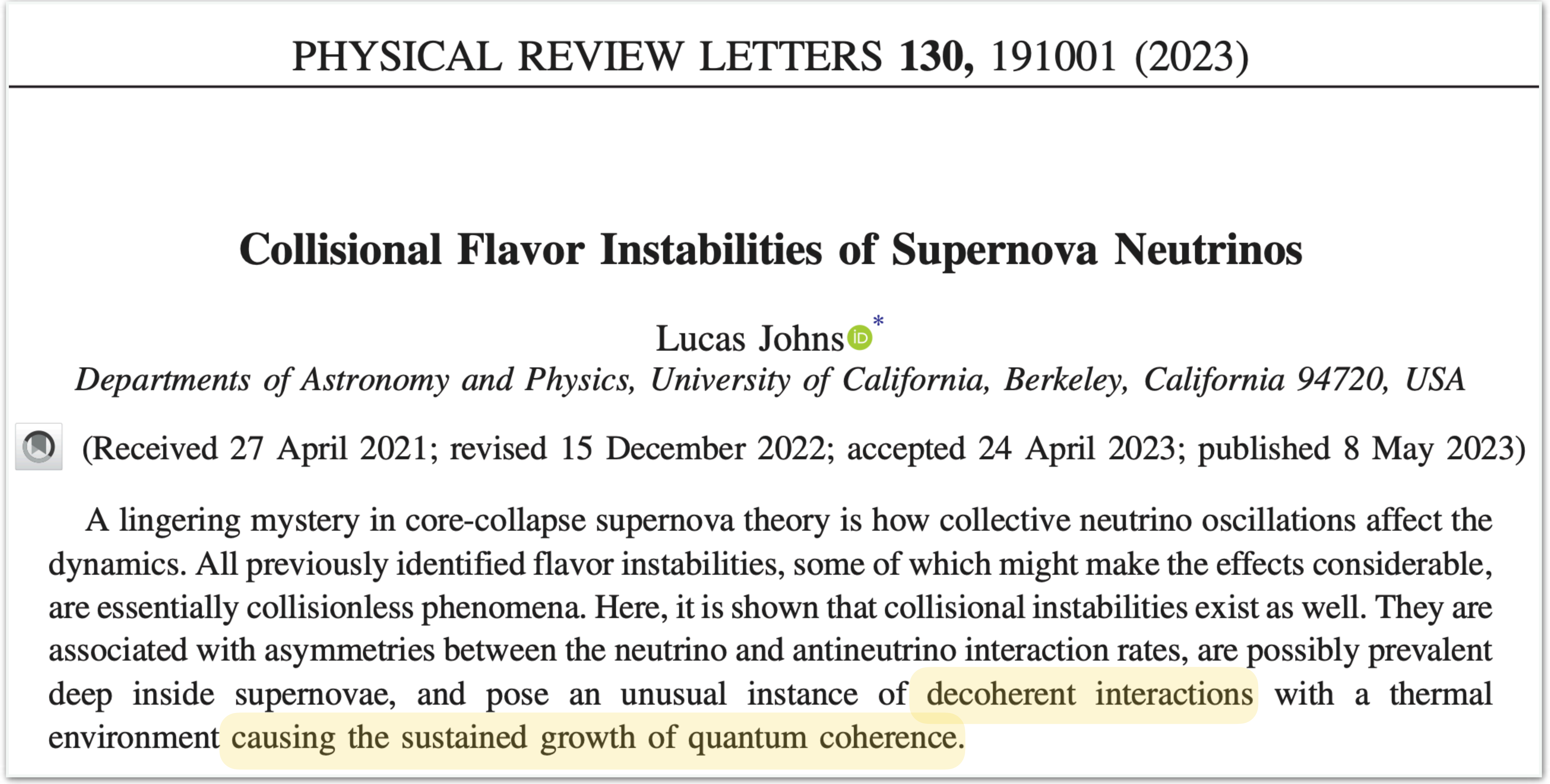


K. Lund et al., *Angle-dependent in-situ fast flavor transformations in post-neutron star merger disks*, [2503.23727]



Collisional flavor instabilities (CFIs)

- Collisions are generally expected to **damp** flavor coherence.
- In some regimes, a discrepancy between the neutrino and antineutrino reaction rates can actually **amplify** coherence *through the non-linear self-interaction term*.

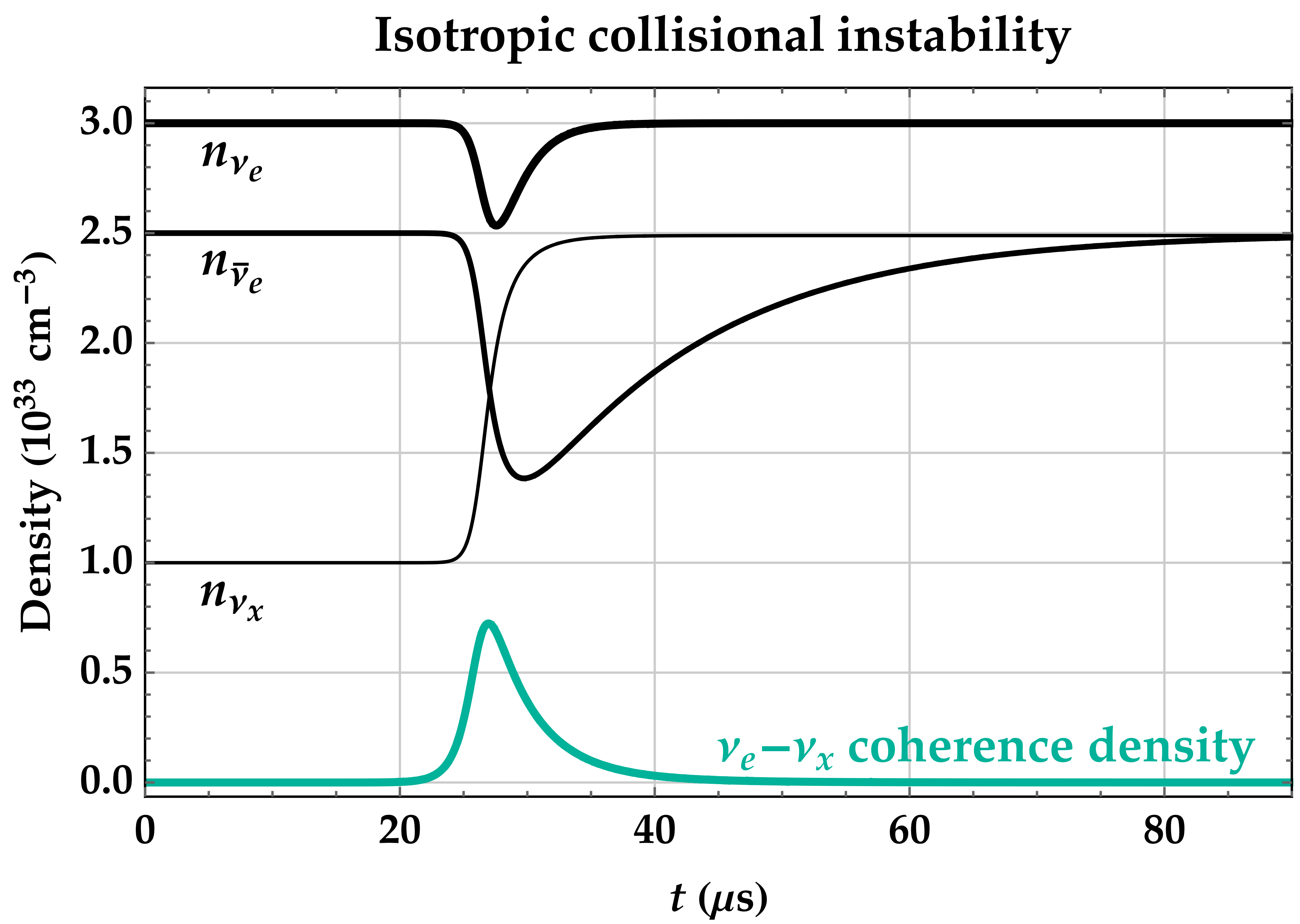

PHYSICAL REVIEW LETTERS **130,** 191001 (2023)

Collisional Flavor Instabilities of Supernova Neutrinos

Lucas Johns*
Departments of Astronomy and Physics, University of California, Berkeley, California 94720, USA

(Received 27 April 2021; revised 15 December 2022; accepted 24 April 2023; published 8 May 2023)

A lingering mystery in core-collapse supernova theory is how collective neutrino oscillations affect the dynamics. All previously identified flavor instabilities, some of which might make the effects considerable, are essentially collisionless phenomena. Here, it is shown that collisional instabilities exist as well. They are associated with asymmetries between the neutrino and antineutrino interaction rates, are possibly prevalent deep inside supernovae, and pose an unusual instance of decoherent interactions with a thermal environment causing the sustained growth of quantum coherence.



Homogeneous and isotropic CFI

JF, [2505.16961]

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -i\left[\mathcal{H}, N\right] + \frac{1}{2}\left\{\begin{pmatrix}\kappa_e & 0\\ 0 & \kappa_x\end{pmatrix}, \begin{pmatrix}N_{ee}^{(\mathrm{cl})} & 0\\ 0 & N_{xx}^{(\mathrm{cl})}\end{pmatrix} - N\right\}$$

$$\frac{\mathrm{d}\overline{N}}{\mathrm{d}t} = -i\left[\overline{\mathcal{H}}, \overline{N}\right] + \frac{1}{2}\left\{\begin{pmatrix}\bar{\kappa}_e & 0\\ 0 & \bar{\kappa}_x\end{pmatrix}, \begin{pmatrix}\overline{N}_{ee}^{(\mathrm{cl})} & 0\\ 0 & \overline{N}_{xx}^{(\mathrm{cl})}\end{pmatrix} - \overline{N}\right\}$$

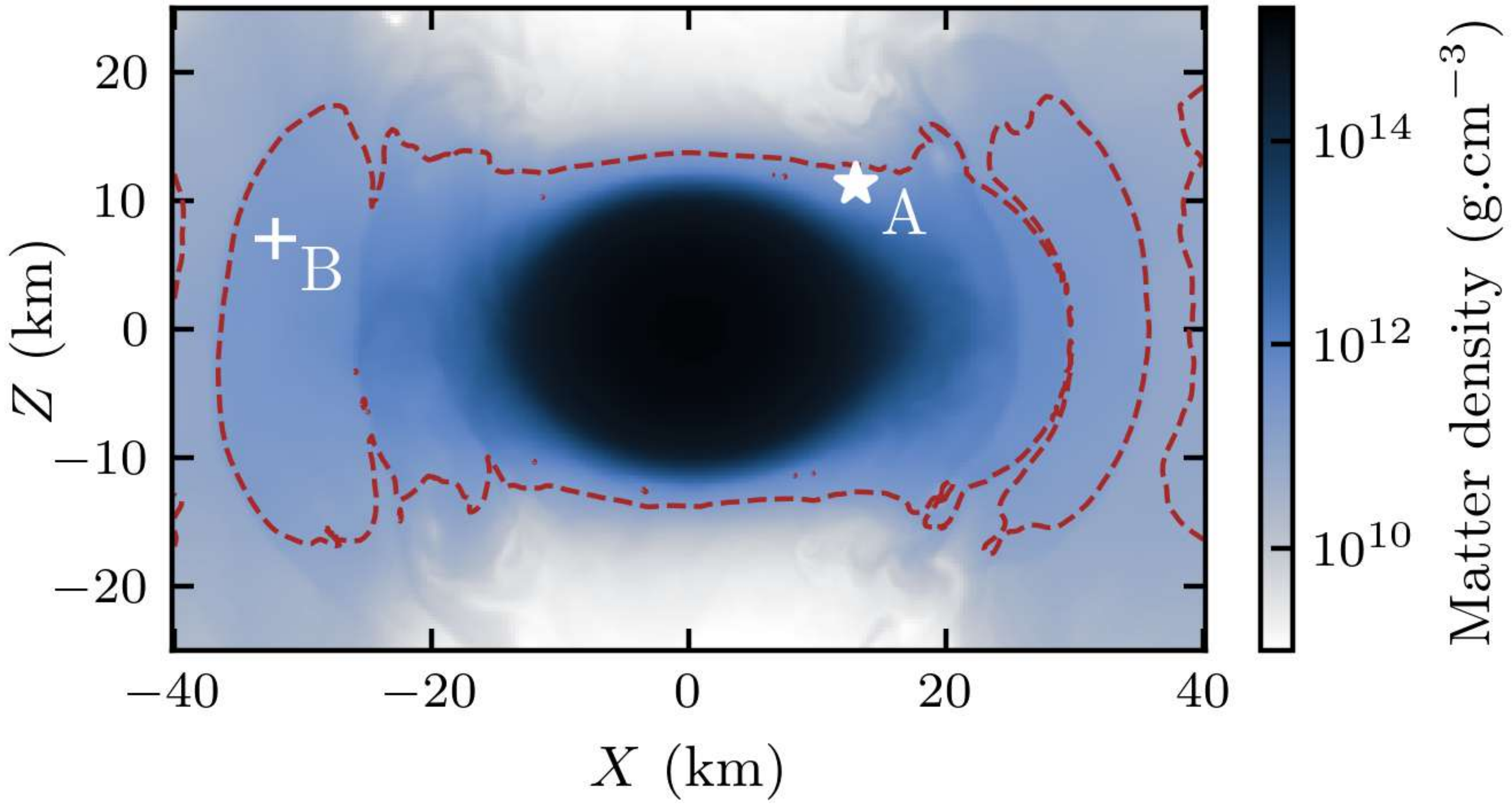
Oscillations **Collisions (classical relaxation)**

- Approximate model: homogeneity, isotropy, mono-energetic system
- Quantum Kinetic Equations with collision-like term, driving the system back to the initial *classical steady-state*

NSM-like configuration

JF, [2505.16961]

- 7 ms post-merger snapshot of NSM simulation by Foucart *et al.* [2407.15989]



- Assume isotropy, number densities from simulation = classical steady-state

- Solve the homogeneous and isotropic QKEs

NSM-like configuration

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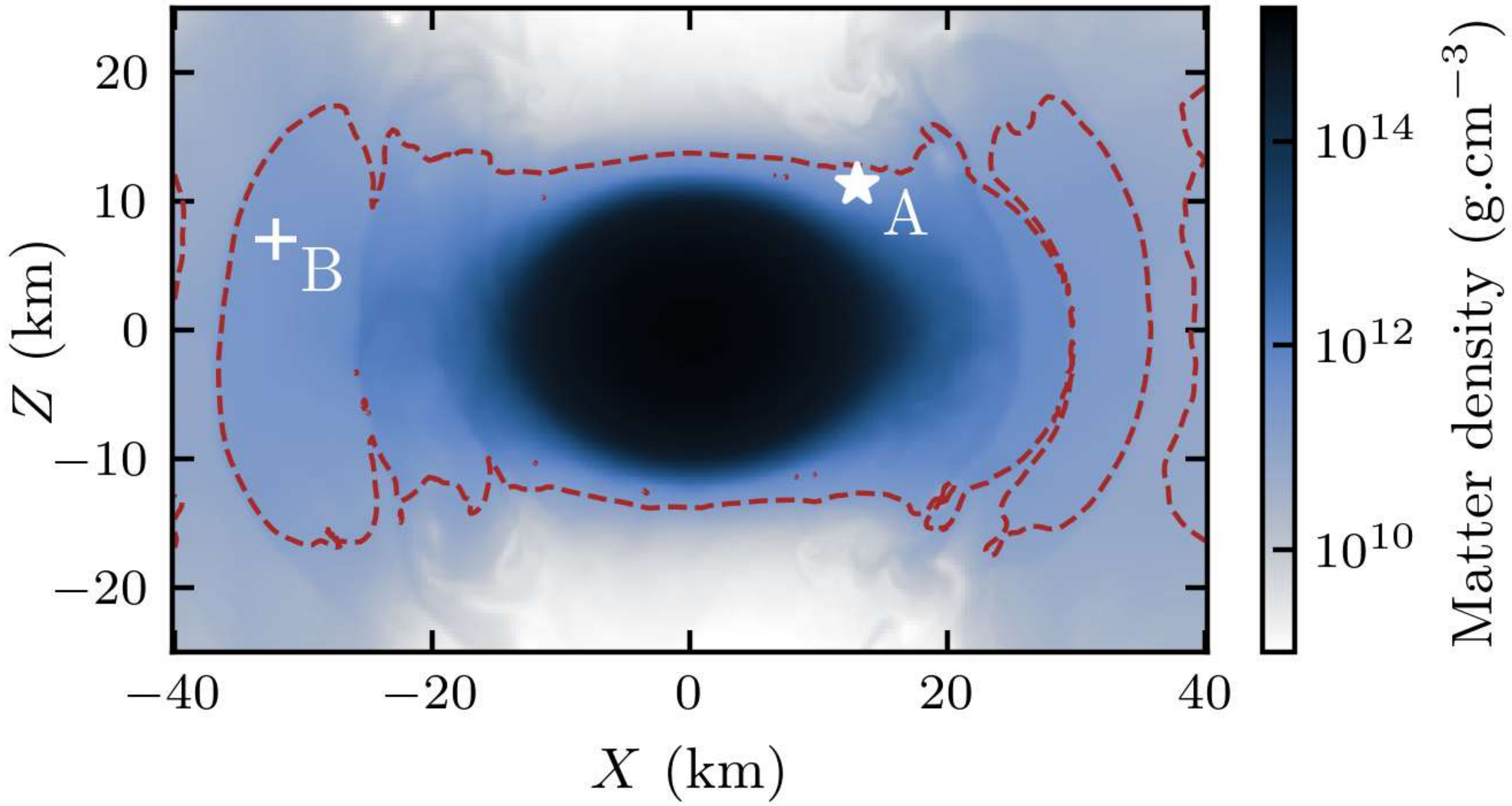

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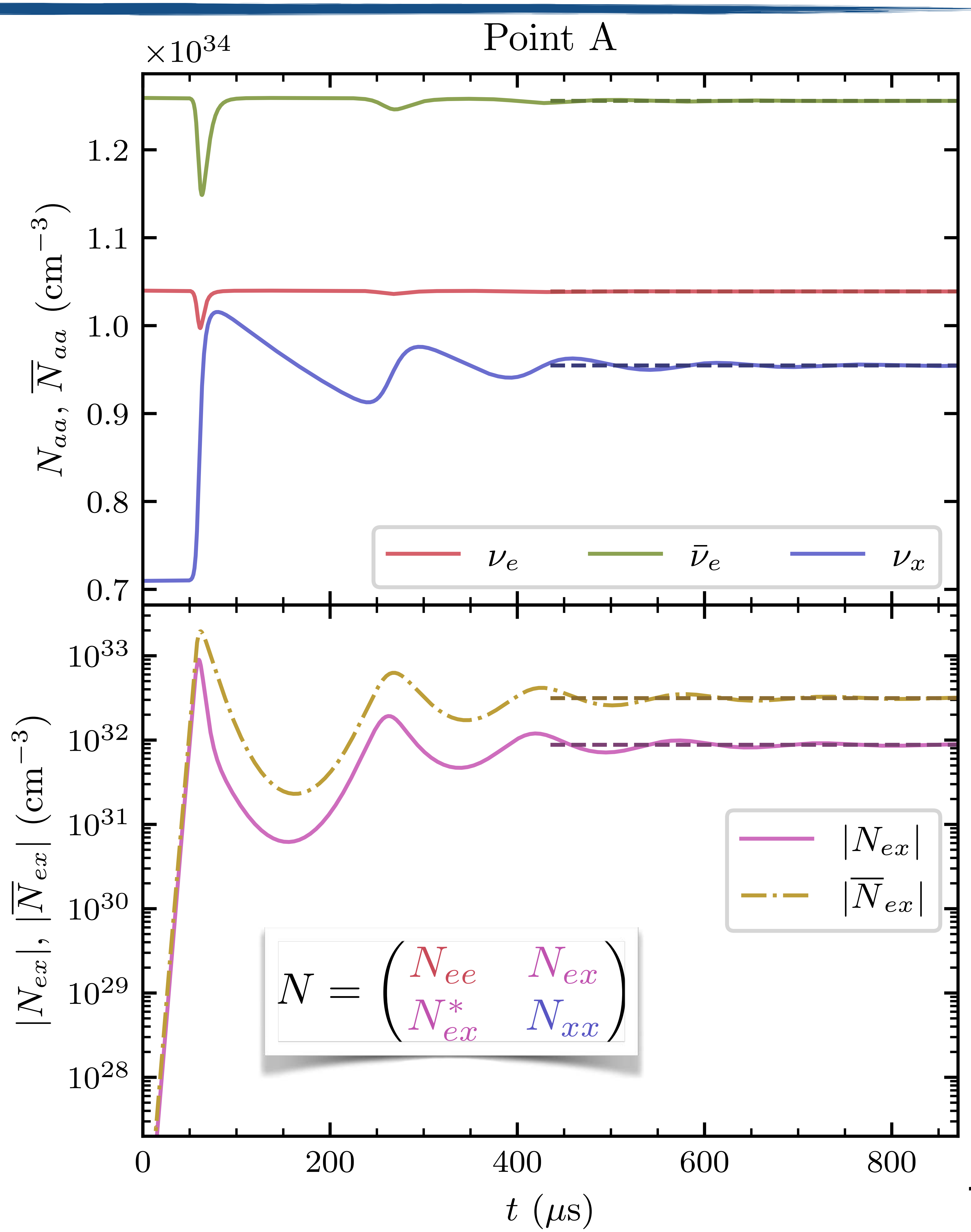
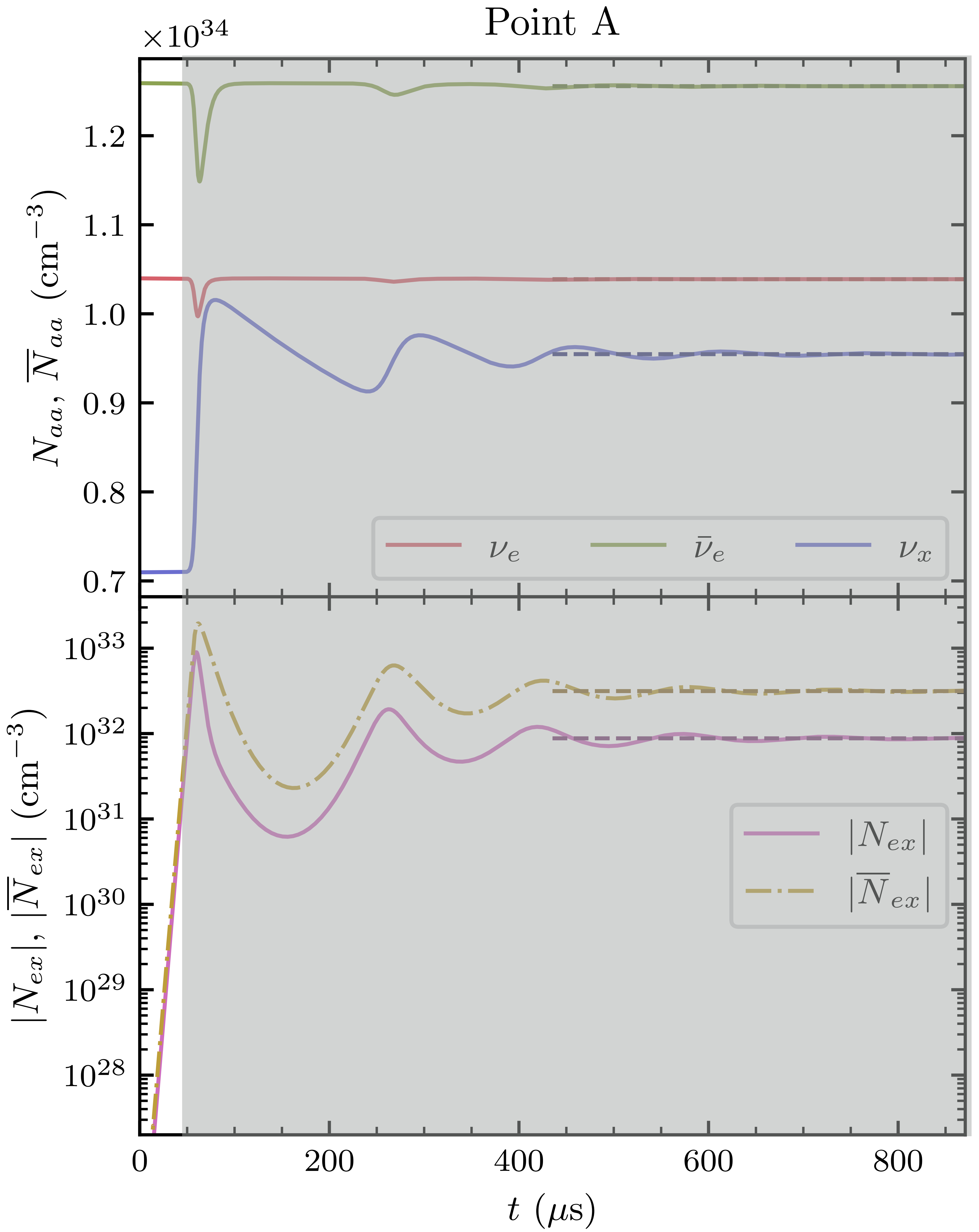


Evolution and asymptotic state

JF, [2505.16961]



$$\frac{\mathrm{d}N}{\mathrm{d}t} = [\textbf{Oscillations}] + [\textbf{Collisions}]$$

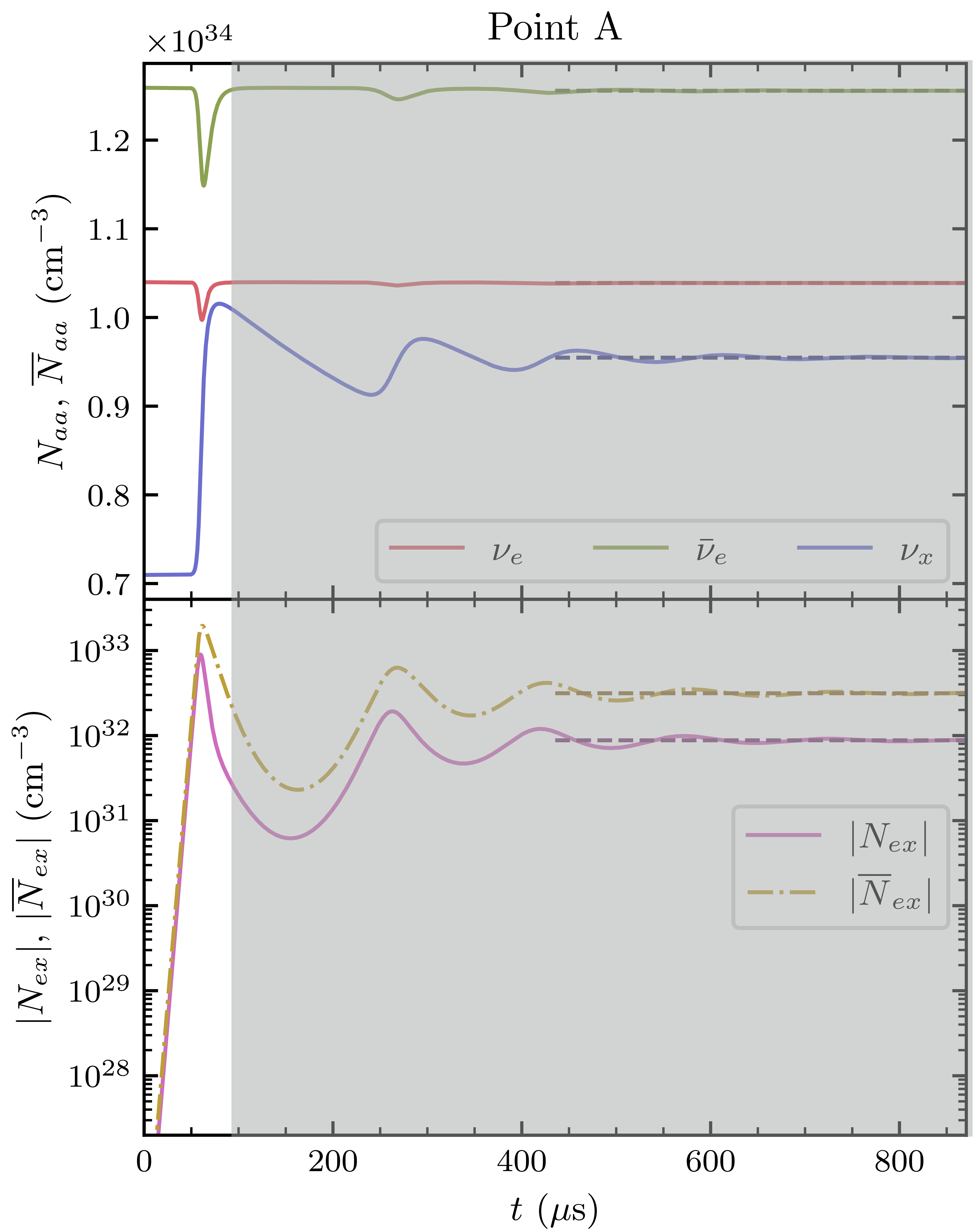
$$\sim -\kappa \left[N - N^{(\mathrm{cl})} \right]$$

- The classical steady-state is unstable

$\Longrightarrow$ instability and flavor conversion

Evolution and asymptotic state

JF, [2505.16961]



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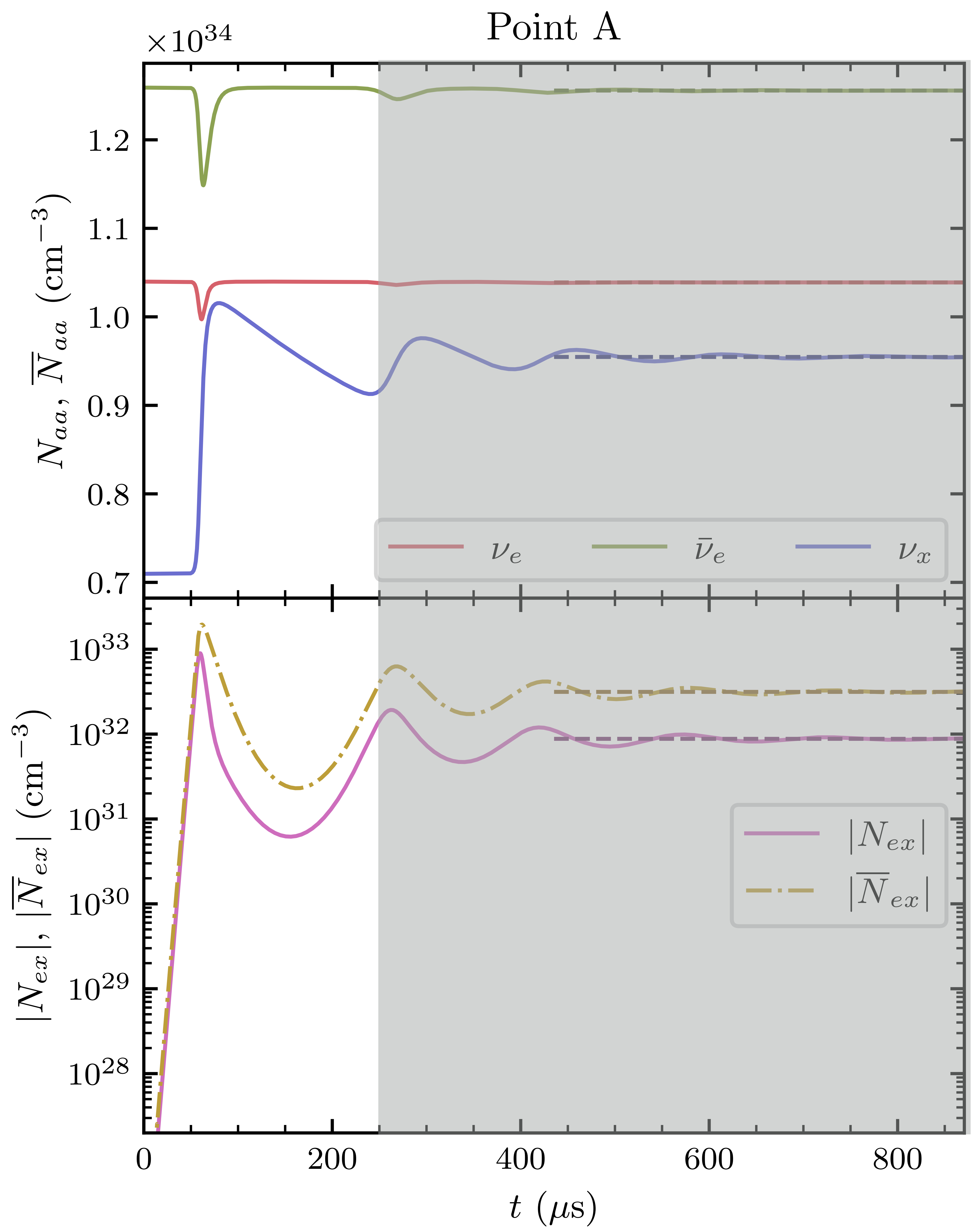

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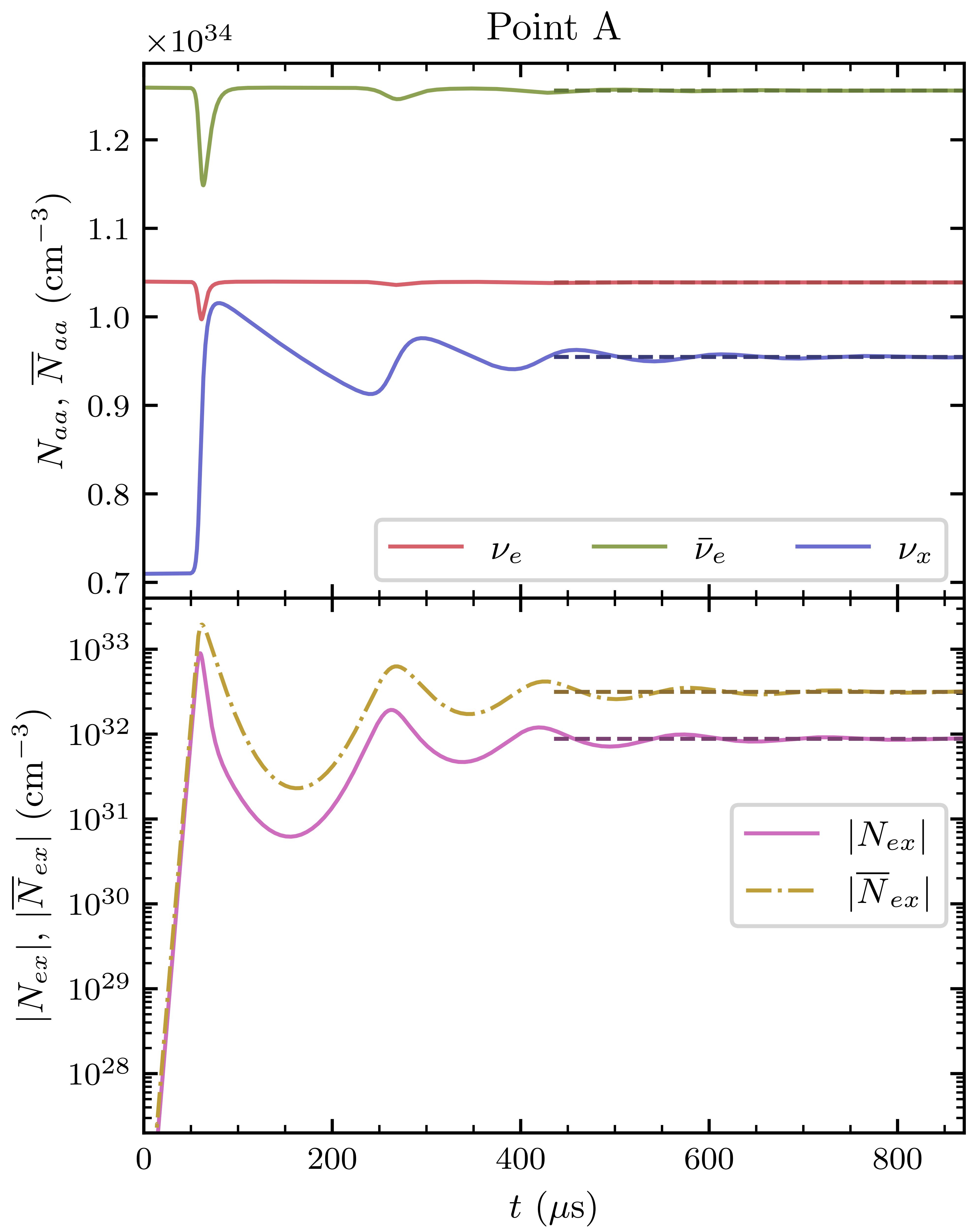
- ν_e and $\bar{\nu}_e$ are quickly brought back to classical equilibrium
- Long-term relaxation of ν_x toward $N_{xx}^{(\mathrm{cl})}$ (difference of timescales $\kappa_x \ll \kappa_e, \bar{\kappa}_e$)
- System unstable again

Evolution and asymptotic state

JF, [2505.16961]



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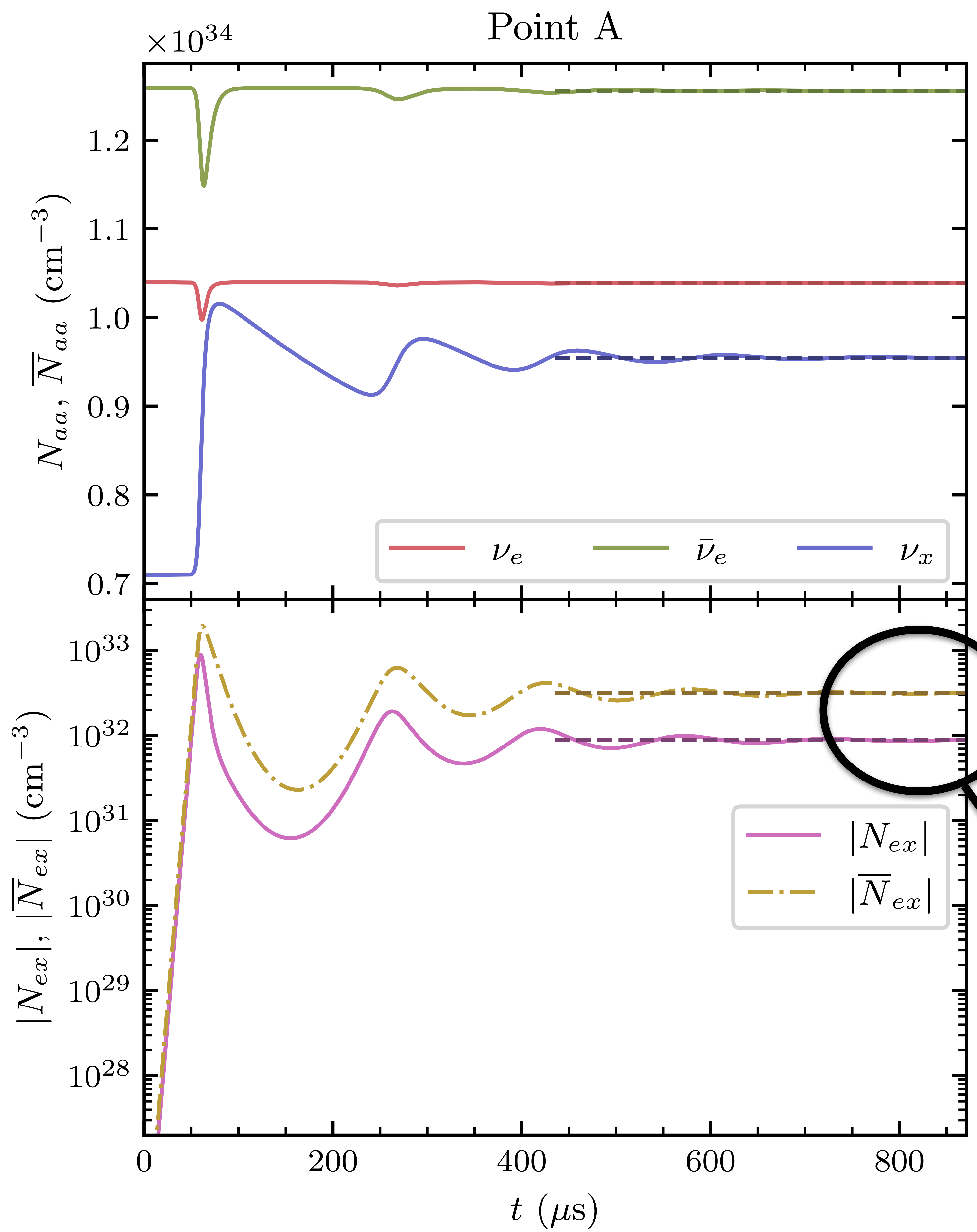
$\Longrightarrow$ instability and (smaller) flavor conversion

- …

Evolution and asymptotic state

JF, [2505.16961]



$$\frac{\mathrm{d}N}{\mathrm{d}t} = [\textbf{Oscillations}] + [\textbf{Collisions}]$$

$$\sim -\kappa \left[N - N^{(\mathrm{cl})}\right]$$

- The classical steady-state is unstable

$\Longrightarrow$ instability and flavor conversion

- ν_e and $\bar{\nu}_e$ are quickly brought back to classical equilibrium

- Long term relaxation of ν_x toward $N_{xx}^{(\mathrm{cl})}$ (difference of ...)

In the “quantum” equilibrium, the collision term does not vanish

- System unstable again

Nonzero flavor coherence

$\Longrightarrow$ instability and (smaller) flavor conversion

- …

Summary

- Asymptotic state of **fast flavor instabilities**: *erasure of ELN-XLN crossing in the "shallow" angular domain*
  - Used in large-scale simulations! [Wang & Burrows, 2503.04896], [Lund *et al.*, 2503.23727]
  - Caveat: dependence on the boundary conditions [Zaizen & Nagakura, 2304.05044]

- Study of the outcome of **collisional flavor instabilities**
  - "Compromise" between relaxation to / instability of classical steady-state
  - Flavor coherence cannot necessarily be neglected in the asymptotic state!

Limitations: single-energy, homogeneous calculation (large-scale advection effects?)

- Other instabilities (e.g., "slow"),… → need a *diversity of approaches*, **theoretical** and **numerical**, to address neutrino flavor transformation in astrophysical environments.

Moment methods for classical neutrino transport

- Many large-scale simulations use angular moments:

Number density, **Flux density**, **Pressure tensor**

$$\begin{bmatrix} N \\ F^i \\ P^{ij} \end{bmatrix} = \frac{p^2}{(2\pi)^3} \int \mathrm{d}\Omega_{\vec{p}} \begin{bmatrix} 1 \\ \frac{p^i}{p} \\ \frac{p^i p^j}{p^2} \end{bmatrix} g(t, \vec{x}, \vec{p})$$

$g(t, \vec{x}, \vec{p})$ → distribution function, ≥ 0

- Boltzmann equation:

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla} \right) g(t, \vec{x}, \vec{p}) = \left[\frac{\mathrm{d}g}{\mathrm{d}t} \right]_{\mathrm{coll}}$$

$$\int \mathrm{d}\Omega_{\vec{p}} \cdots, \quad \int \mathrm{d}\Omega_{\vec{p}} \frac{p^i}{p} \cdots$$

Hierarchy of moment equations

$$\frac{\partial N}{\partial t} + \frac{\partial F^j}{\partial x^j} = S_N$$

$$\frac{\partial F^i}{\partial t} + \frac{\partial P^{ij}}{\partial x^j} = S_{F^i}$$

TWO-MOMENT SCHEME $(N, \vec{F})$

Need for a *closure*

$P^{ij}(N, F^k)$

Moment methods for quantum neutrino transport

- Many large-scale simulations use angular moments:

Number density, **Flux density**, **Pressure tensor**

$$\begin{bmatrix} N_{\alpha\beta} \\ F^i_{\alpha\beta} \\ P^{ij}_{\alpha\beta} \end{bmatrix} = \frac{p^2}{(2\pi)^3} \int \mathrm{d}\Omega_{\vec{p}} \begin{bmatrix} 1 \\ \frac{p^i}{p} \\ \frac{p^i p^j}{p^2} \end{bmatrix} \varrho_{\alpha\beta}(t, \vec{x}, \vec{p})$$

2-flavor system

$$\varrho = \begin{pmatrix} \varrho_{ee} & \varrho_{ex} \\ \varrho^*_{ex} & \varrho_{xx} \end{pmatrix}, \quad \varrho_{ex} \in \mathbb{C}$$

Flavor matrix

$\Longrightarrow$ much more complicated closure problem!

- Quantum Kinetic equation:

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}\right) \varrho_{\alpha\beta}(t, \vec{x}, \vec{p}) = -\mathrm{i}\,[\mathcal{H}, \varrho]_{\alpha\beta} + \left[\frac{\mathrm{d}\varrho}{\mathrm{d}t}\right]_{\mathrm{coll},\alpha\beta}$$

$$\int \mathrm{d}\Omega_{\vec{p}} \cdots, \quad \int \mathrm{d}\Omega_{\vec{p}} \frac{p^i}{p} \cdots$$

<u>Hierarchy of moment equations</u>

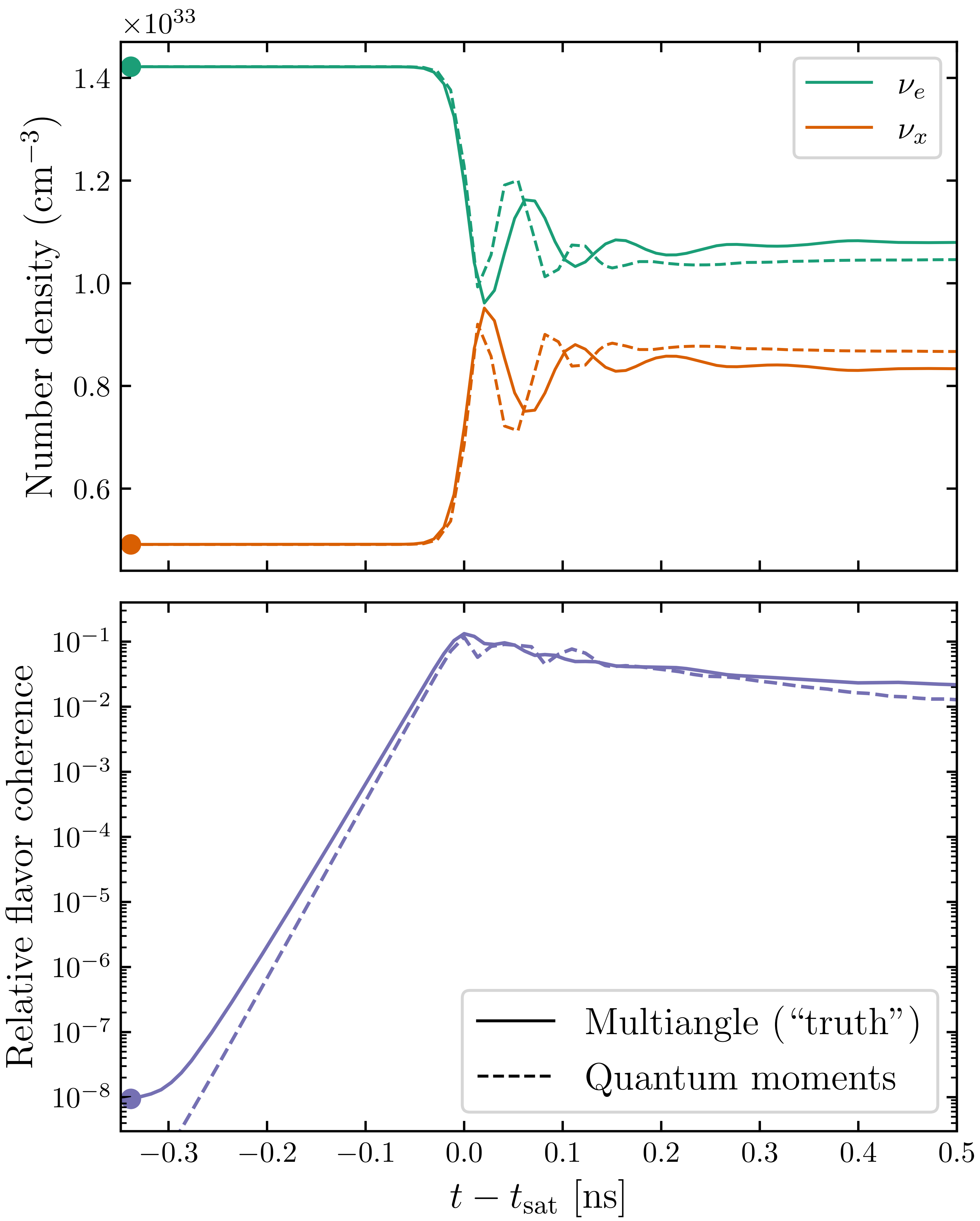
$$\frac{\partial N_{\alpha\beta}}{\partial t} + \frac{\partial F^j_{\alpha\beta}}{\partial x^j} = S_{N,\alpha\beta}$$

$$\frac{\partial F^i_{\alpha\beta}}{\partial t} + \frac{\partial P^{ij}_{\alpha\beta}}{\partial x^j} = S_{F^i,\alpha\beta}$$

Need for a *<u>closure</u>*

$$P^{ij}_{\alpha\beta}(N_{\gamma\delta}, F^k_{\sigma\kappa})$$

Quantum moments and neutrino flavor instabilities

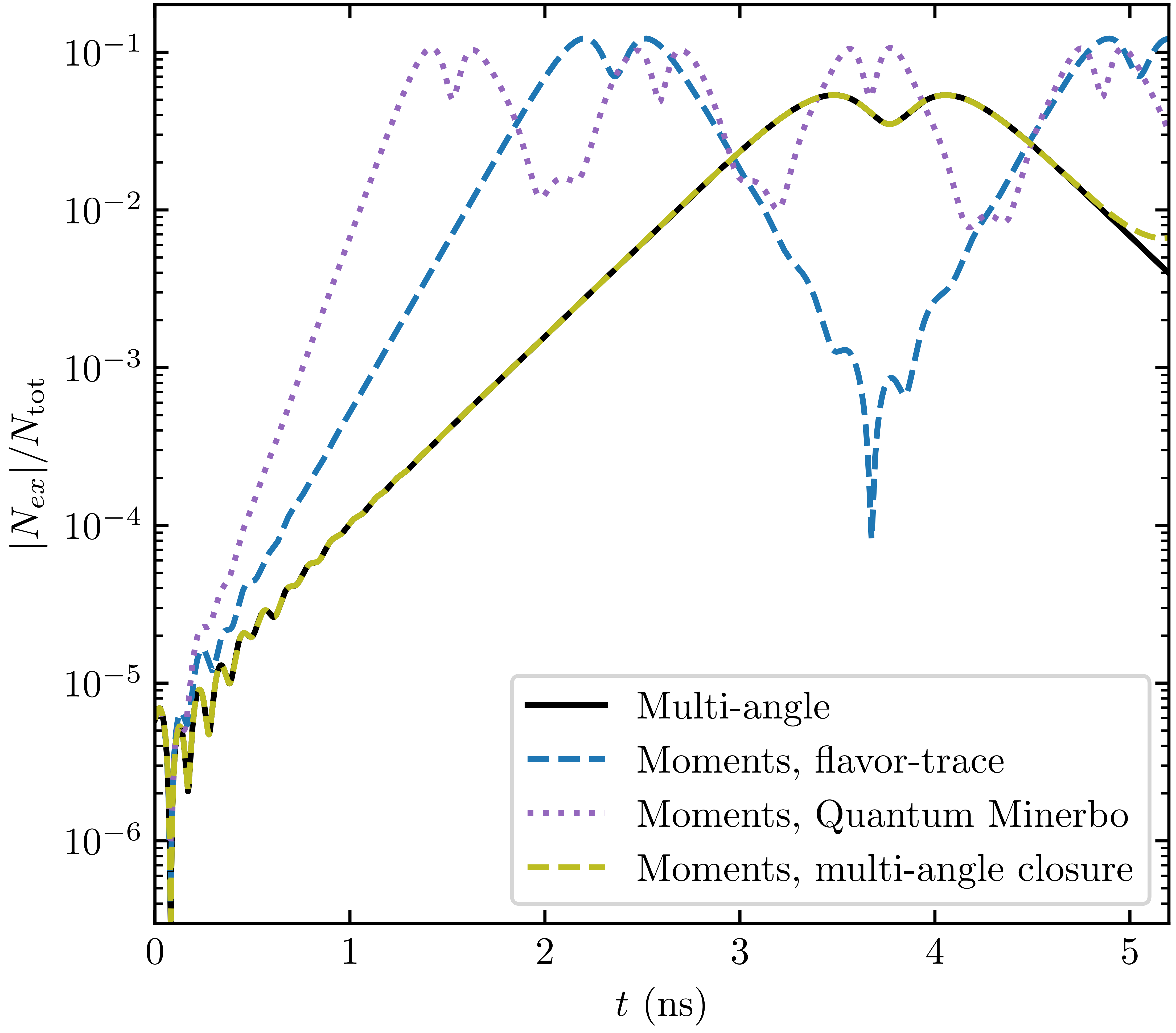


- Moments can describe flavor instabilities!

E. Grohs *et al.* [2207.02214, 2309.00972, 2501.05740]

Quantum moments and neutrino flavor instabilities

Homogeneous FFI from [2410.00719]



- Moments can describe flavor instabilities!

  E. Grohs *et al.* [2207.02214, 2309.00972, 2501.05740]

- Underlying issue: **the "closure" problem**
  - Quantum extension of the *maximum entropy closure*

    J. Froustey *et al.* [2409.05807]

  - General parameterization of quantum closures

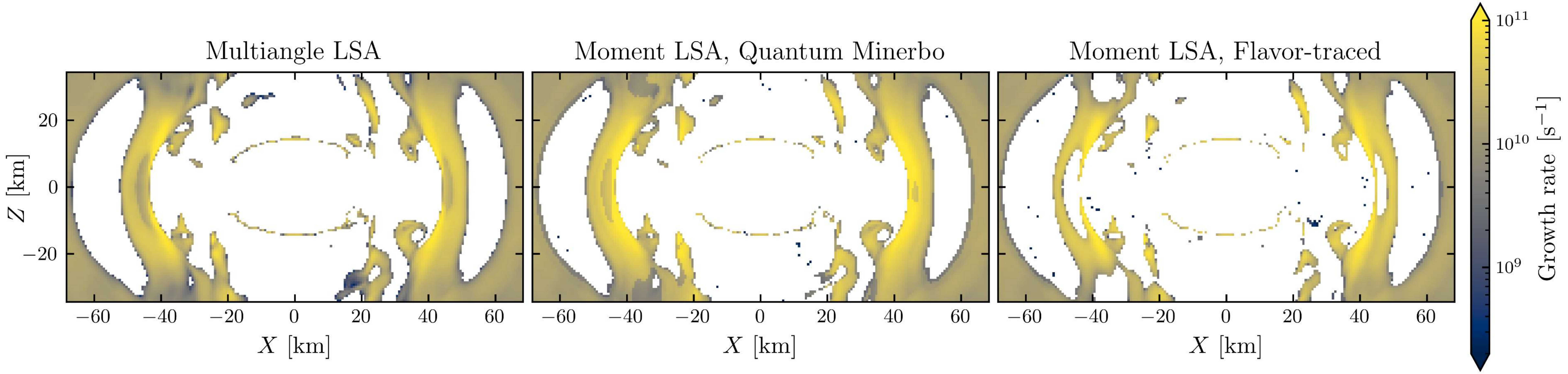
    J. Kneller *et al.* [2410.00719]

  - Exploration needed for other instabilities and large-scale calculations

Quantum maximum entropy closure

JF, J. Kneller and G. McLaughlin [2409.05807]

NSM 5 ms post-merger snapshot from F. Foucart *et al.* [1607.07450]



"Truth", expensive

Quantum maximum entropy closure

Semiclassical, *ad hoc* closure

Better performance of the Quantum Minerbo closure in finding regions of instabilities

Points A and B

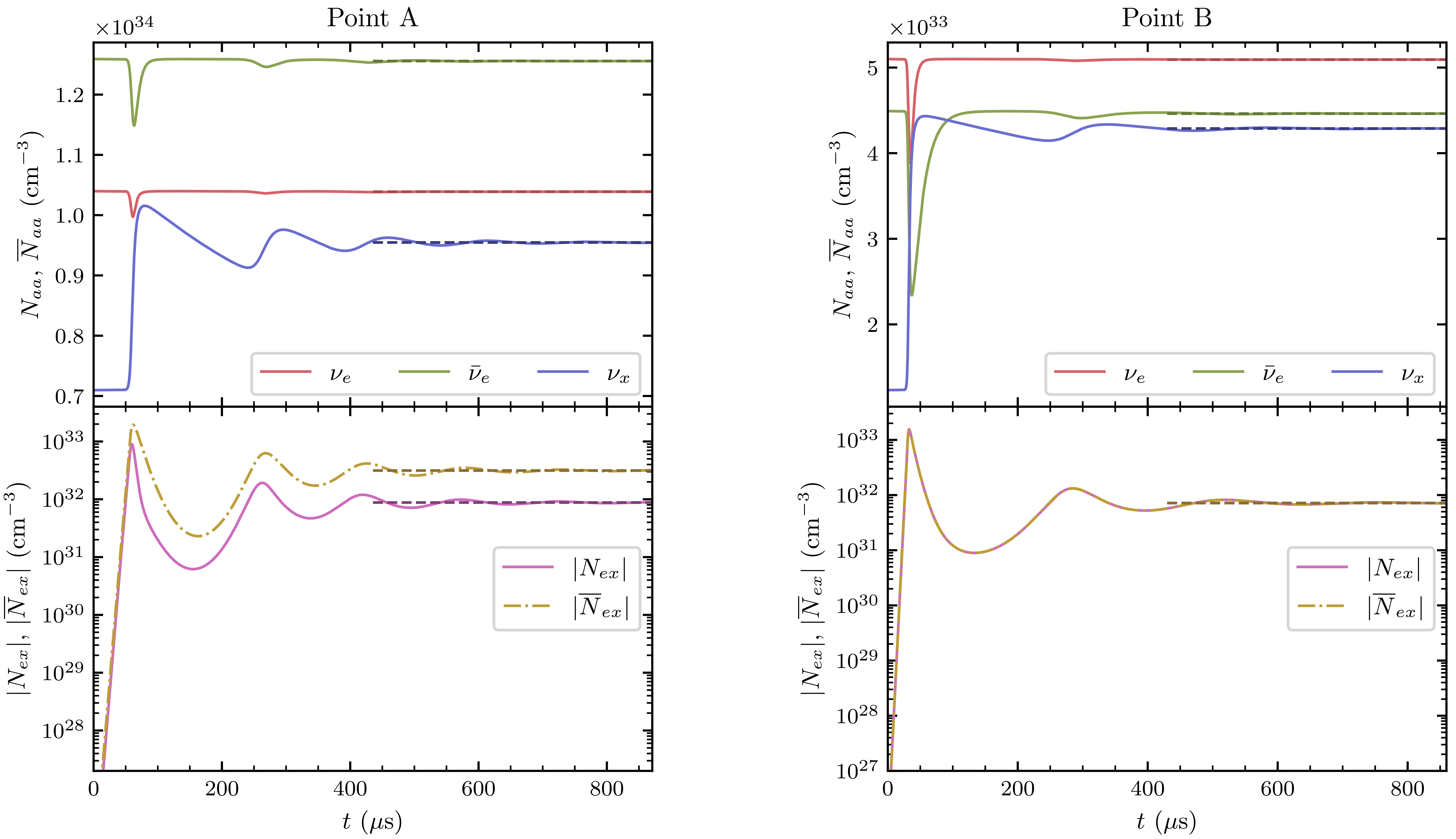
Parameters:

	Point A	Point B
N_{ee} (cm^{-3})	1.04×10^{34}	5.10×10^{33}
$\overline{N}_{ee}$ (cm^{-3})	1.26×10^{34}	4.49×10^{33}
$N_{xx} = \overline{N}_{xx}$ (cm^{-3})	7.10×10^{33}	1.23×10^{33}
ρ ($\mathrm{g.cm}^{-3}$)	2.92×10^{11}	2.50×10^{11}
Y_e	0.298	0.231
$\langle E_\nu \rangle$ (MeV)	35.7	34.6
κ_e (s^{-1})	7.63×10^{5}	2.78×10^{5}
$\bar{\kappa}_e$ (s^{-1})	2.11×10^{5}	5.95×10^{4}
$\kappa_x = \bar{\kappa}_x$ (s^{-1})	2.87×10^{3}	5.82×10^{2}

Timescales:

	Point A	Point B
$\sqrt{2}G_F\lvert N_{ee} - \overline{N}_{ee}\rvert$ (s^{-1})	4.2×10^{11}	1.2×10^{11}
$\sqrt{2}G_F n_e$ (s^{-1})	1.0×10^{13}	6.9×10^{12}
$\Delta m^2/2\langle E_\nu \rangle$ (s^{-1})	5.3×10^{4}	5.5×10^{4}

Points A and B



Amount of flavor coherence

JF, [2505.16961]

- We can also predict the asymptotic size of the off-diagonal components

Modulus-phase

$$N_{ex} = R\, e^{iS}$$

$$\overline{N}_{ex} = \overline{R}\, e^{i\overline{S}}$$

$$\Delta S = S - \overline{S}$$

Steady-state

$$\dot{N}_{ee} = 0$$

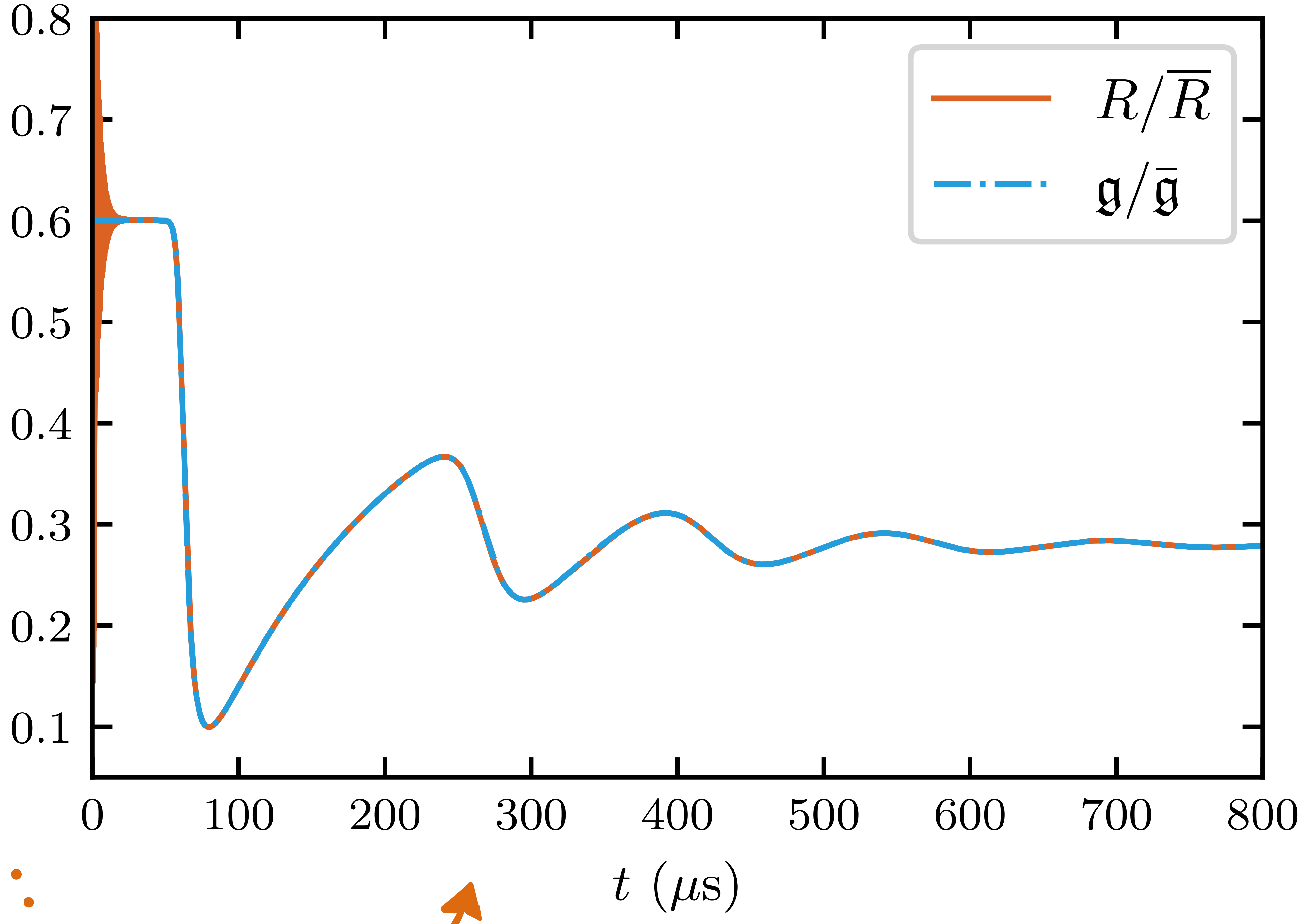
$$\dot{\overline{N}}_{ee} = 0$$

$$\dot{N}_{xx} = 0$$

$$\dot{R} = 0$$

$$\dot{\overline{R}} = 0$$

$$\frac{R}{\overline{R}} = \frac{\mathfrak{g}}{\overline{\mathfrak{g}}} = \frac{N_{ee} - N_{xx}}{\overline{N}_{ee} - \overline{N}_{xx}}$$



Amount of flavor coherence

JF, [2505.16961]

- We can also predict the asymptotic size of the off-diagonal components

$$\kappa_e[N_{ee}^{(\infty)} - N_{ee}^{(\mathrm{cl})}] = -\kappa_x[N_{xx}^{(\infty)} - N_{xx}^{(\mathrm{cl})}]\,,$$

$$\kappa_e[N_{ee}^{(\infty)} - N_{ee}^{(\mathrm{cl})}] = \bar{\kappa}_e[\overline{N}_{ee}^{(\infty)} - \overline{N}_{ee}^{(\mathrm{cl})}]\,,$$

$$\kappa_e[N_{ee}^{(\infty)} - N_{ee}^{(\mathrm{cl})}] = -2\sqrt{2}G_F\, R^{(\infty)}\, \overline{R}^{(\infty)}\, \sin[\Delta S^{(\infty)}]\,,$$

$$\Gamma\, R^{(\infty)} = \sqrt{2}G_F[N_{ee}^{(\infty)} - N_{xx}^{(\infty)}]\overline{R}^{(\infty)} \sin[\Delta S^{(\infty)}]\,,$$

$$\overline{\Gamma}\, \overline{R}^{(\infty)} = \sqrt{2}G_F[\overline{N}_{ee}^{(\infty)} - N_{xx}^{(\infty)}]R^{(\infty)} \sin[\Delta S^{(\infty)}]\,,$$

ratio

$$+\ \frac{R^{(\infty)}}{\overline{R}^{(\infty)}} = \frac{N_{ee}^{(\infty)} - N_{xx}^{(\infty)}}{\overline{N}_{ee}^{(\infty)} - N_{xx}^{(\infty)}}$$

$$\Gamma[N_{ee}^{(\infty)} - N_{xx}^{(\infty)}] = \overline{\Gamma}[\overline{N}_{ee}^{(\infty)} - N_{xx}^{(\infty)}]$$

Edge of instability, $\mathrm{Im}(\Omega\_) = 0$

Amount of flavor coherence

- We can also predict the asymptotic size of the off-diagonal components

$$\kappa_e[N_{ee}^{(\infty)} - N_{ee}^{(\mathrm{cl})}] = -\kappa_x[N_{xx}^{(\infty)} - N_{xx}^{(\mathrm{cl})}]\,,$$

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$$\kappa_e[N_{ee}^{(\infty)} - N_{ee}^{(\mathrm{cl})}] = -2\sqrt{2}G_F\, R^{(\infty)}\,\overline{R}^{(\infty)}\,\sin[\Delta S^{(\infty)}]\,,$$

$$\Gamma\, R^{(\infty)} = \sqrt{2}G_F[N_{ee}^{(\infty)} - N_{xx}^{(\infty)}]\overline{R}^{(\infty)}\,\sin[\Delta S^{(\infty)}]\,,$$

$$\overline{\Gamma}\,\overline{R}^{(\infty)} = \sqrt{2}G_F[\overline{N}_{ee}^{(\infty)} - N_{xx}^{(\infty)}]R^{(\infty)}\,\sin[\Delta S^{(\infty)}]\,,$$

$$\frac{R^{(\infty)}}{\overline{R}^{(\infty)}} = \frac{N_{ee}^{(\infty)} - N_{xx}^{(\infty)}}{\overline{N}_{ee}^{(\infty)} - N_{xx}^{(\infty)}}$$

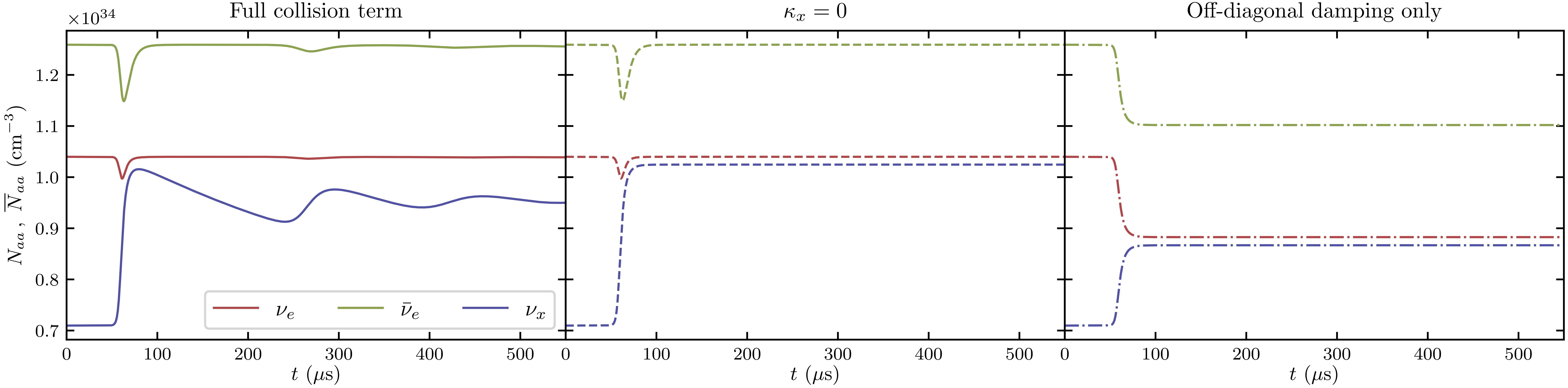
Deviation from classical steady-state because of flavor coherence

Treatment of collisions

JF, [2505.16961]

Collision term:

$$\begin{pmatrix} -\kappa_e\left[N_{ee}-N_{ee}^{(\mathrm{cl})}\right] & -\dfrac{\kappa_e+\kappa_x}{2}N_{ex} \\ -\dfrac{\kappa_e+\kappa_x}{2}N_{xe} & -\kappa_x\left[N_{xx}-N_{xx}^{(\mathrm{cl})}\right]\end{pmatrix} \qquad \begin{pmatrix} -\kappa_e\left[N_{ee}-N_{ee}^{(\mathrm{cl})}\right] & -\dfrac{\kappa_e}{2}N_{ex} \\ -\dfrac{\kappa_e}{2}N_{xe} & 0\end{pmatrix} \qquad \begin{pmatrix} 0 & -\dfrac{\kappa_e+\kappa_x}{2}N_{ex} \\ -\dfrac{\kappa_e+\kappa_x}{2}N_{xe} & 0\end{pmatrix}$$



⟶ Equipartition is an artifact of neglecting the repopulation of ν_e, $\bar{\nu}_e$