Non-singular solutions to the Boltzmann equation with a fluid *Ansatz*

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Modeling the evolutions of bubbles in cosmological First Order Phase Transitions

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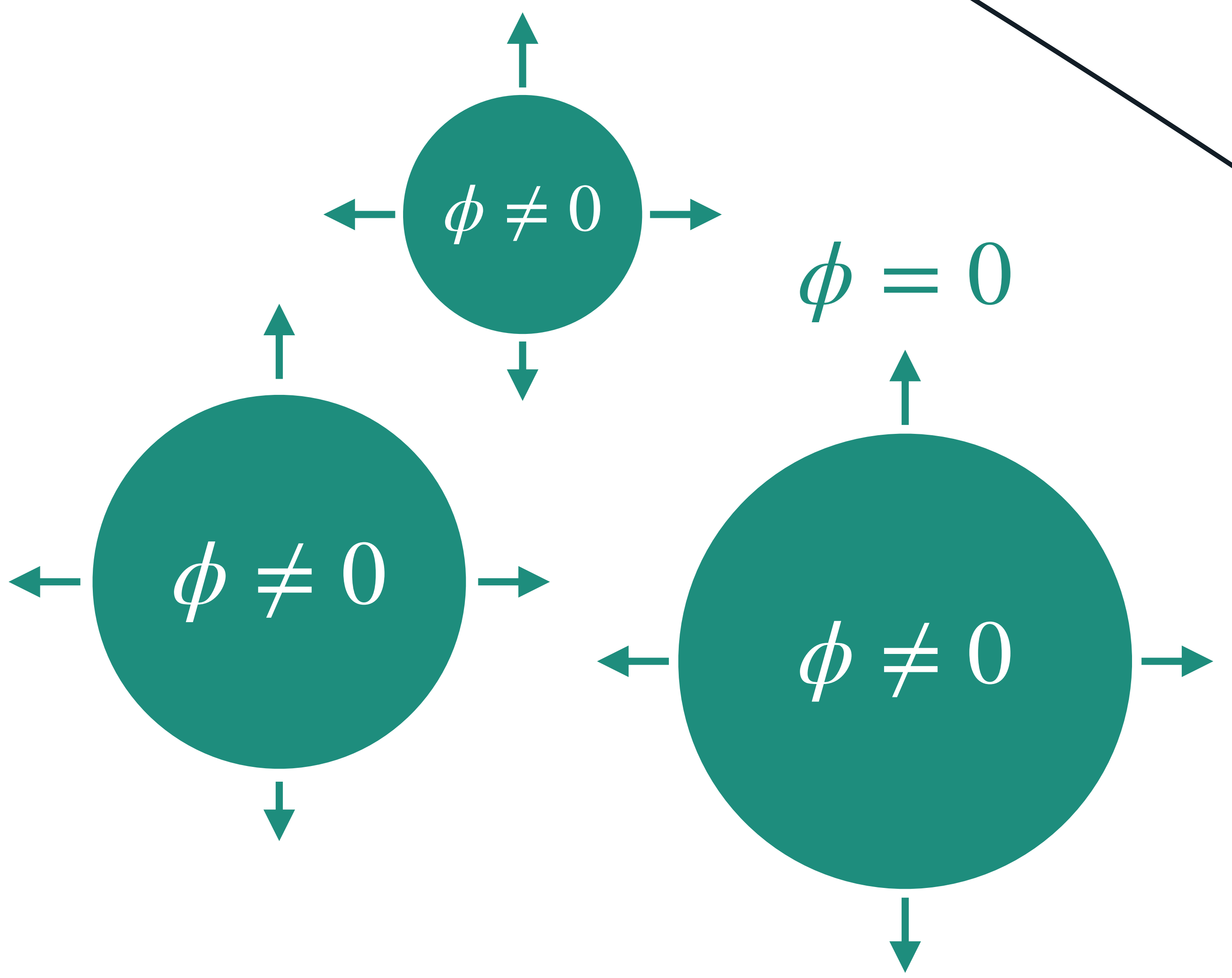
Why Cosmological First Order Phase Transitions (FOPT)?

Bubble nucleation



- Baryogenesis
- Primordial magnetic fields
- Stochastic background of gravitational waves
- Particle Dark Matter

Why Cosmological First Order Phase Transitions (FOPT)?

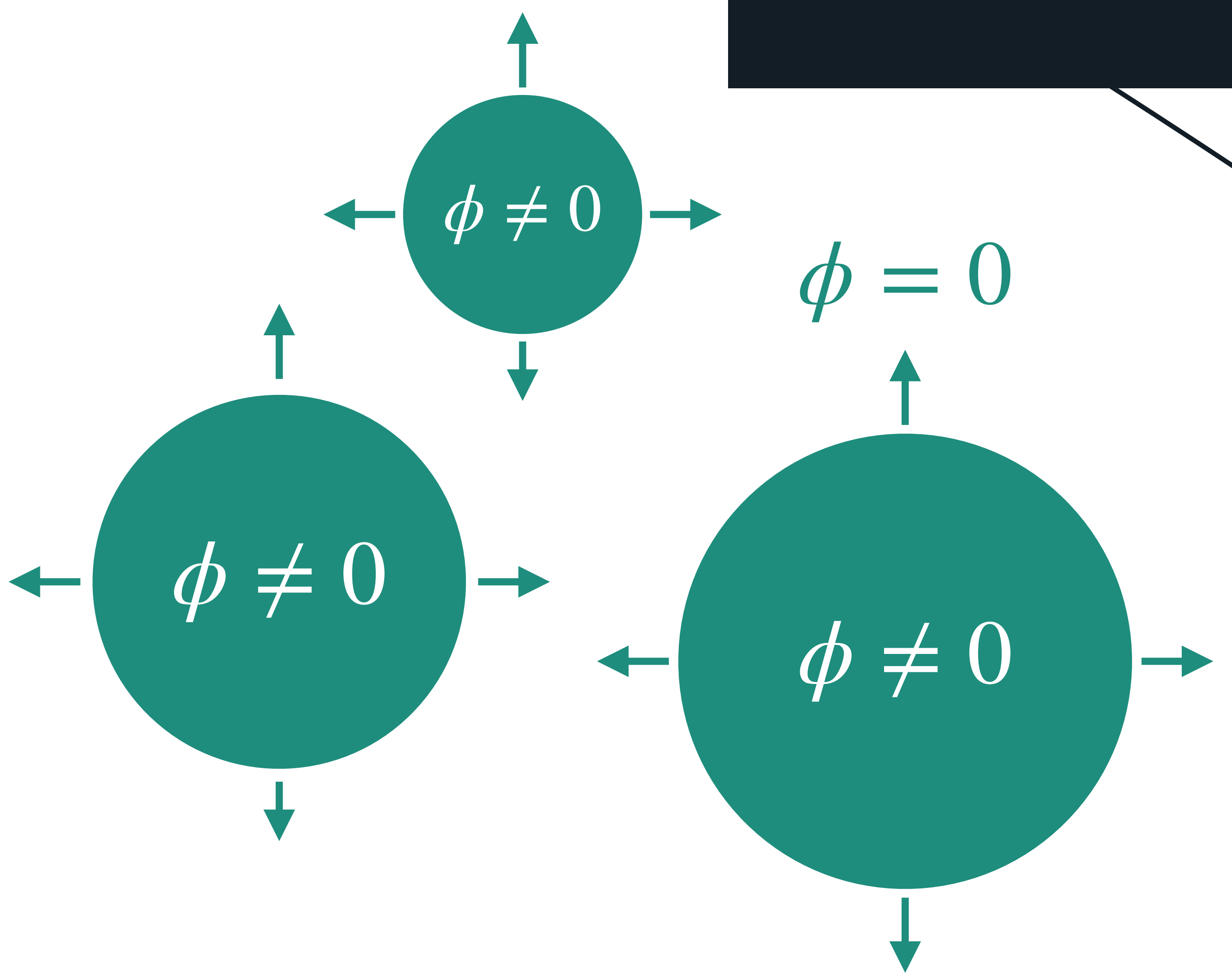
Bubble nucleation

Baryogenesis

Strong dependance of these processes on the velocity of the expanding bubble, ξ_w

Stochastic background of gravitational waves

Particle Dark Matter



The wall velocity ξ_w

The parameter ξ_w is closely connected to the friction on the expanding wall ϕ, and it can be estimated in different way.

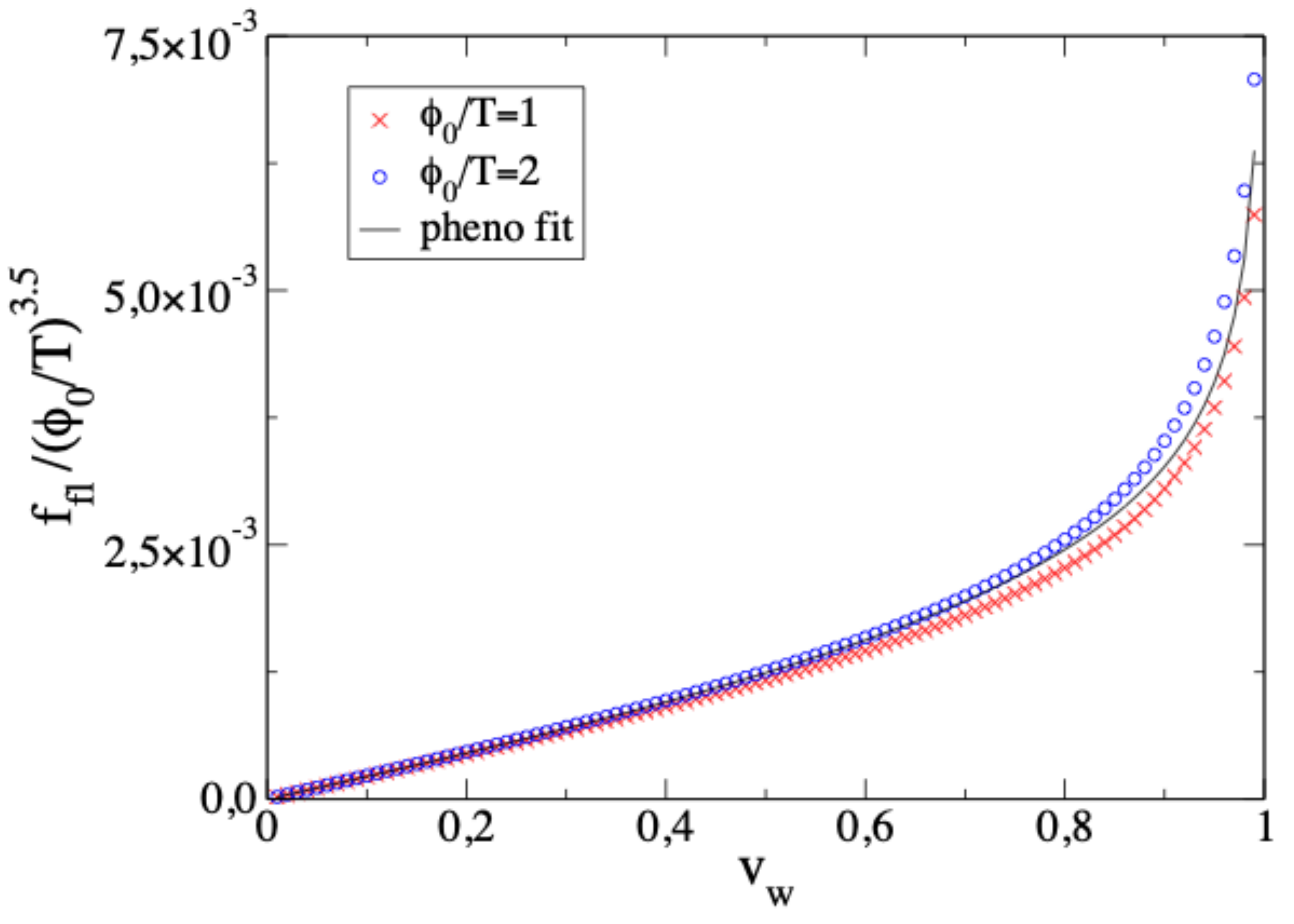
One approach consists in to solving the coupled system of the Klein-Gordon (KG) equation for ϕ and the Boltzmann Equations (BEs) for the particles in the plasma.

$$\Box\phi + \frac{dV_0}{d\phi} + \sum_i \frac{dm_i^2}{d\phi}\int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_i} f_i(p^\mu, x^\mu) = 0$$

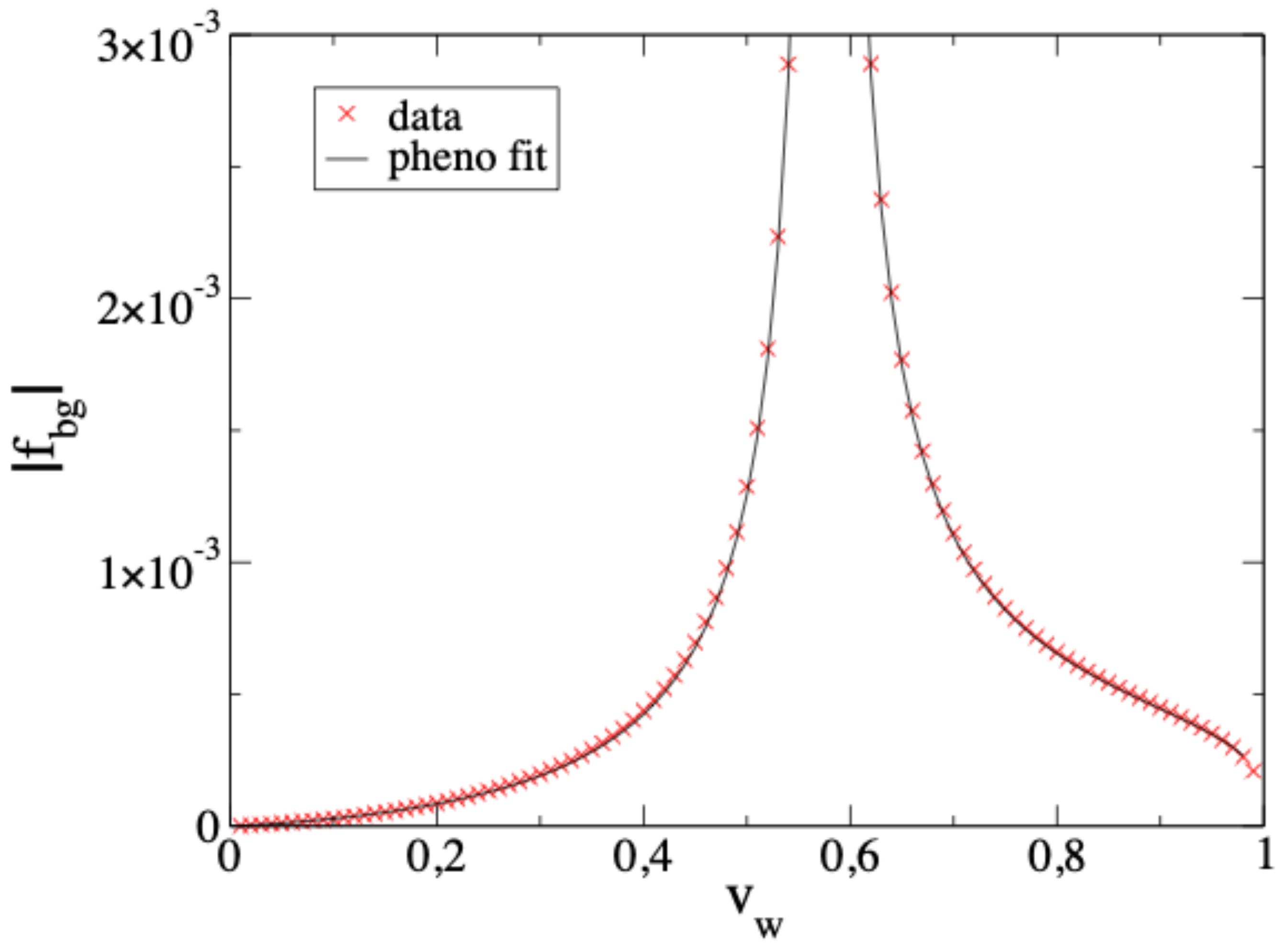
+

$$p^\mu \partial_\mu f_i\left(x^\mu, p^\mu\right) + \frac{1}{2}\partial_\mu m^2 \partial_{p^\mu} f_i\left(x^\mu, p^\mu\right) + \mathscr{C}_i = 0$$

A “sonic boom” in the friction



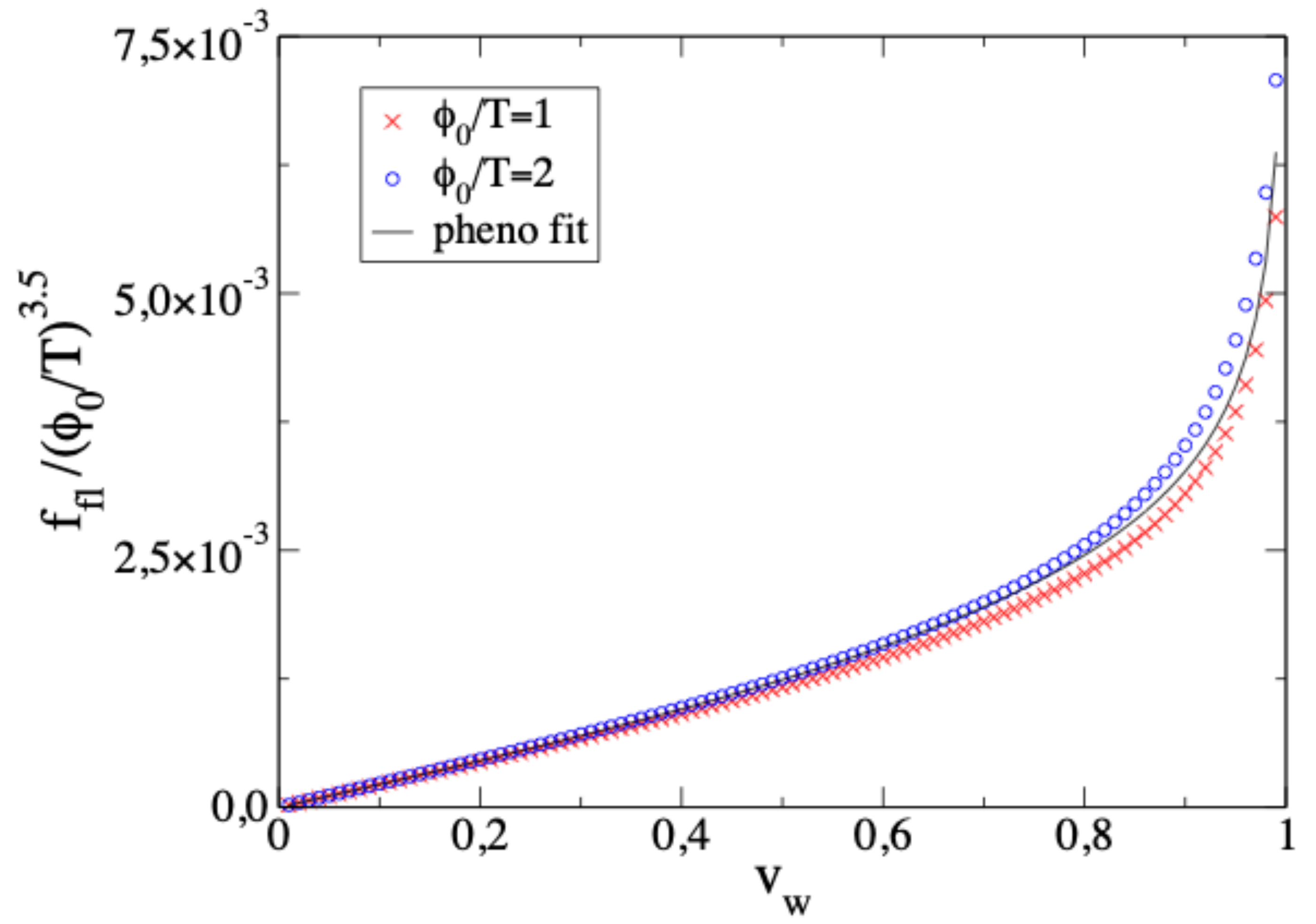
[1407.3132] Konstandin et al.



The singularity arises because of an interplay between the energy-momentum conservation in the BE and the linearisation procedure.

Divergent friction from massless particles at the speed of sound!

A "sonic boom" in the friction



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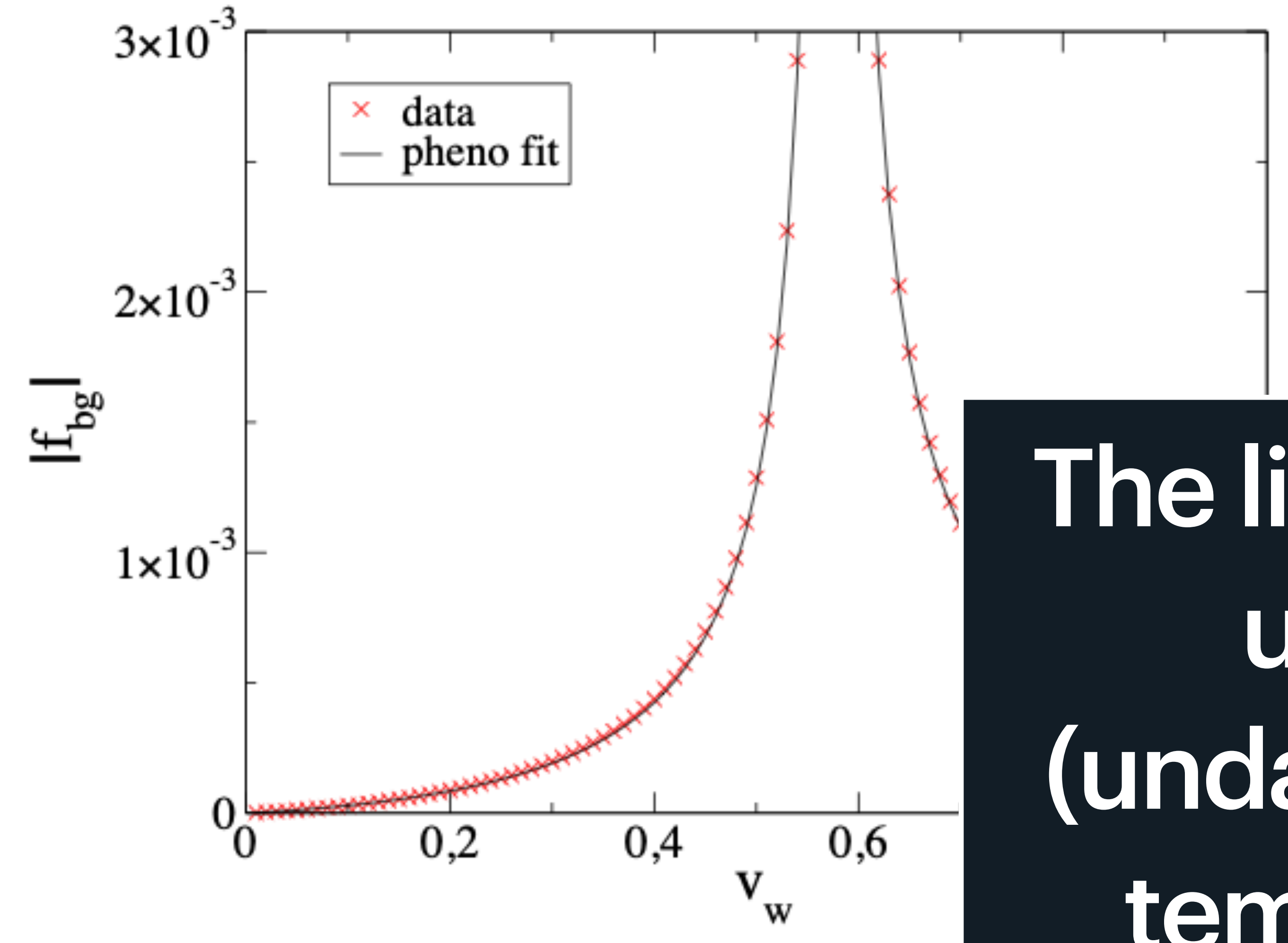

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cedure.

The light degrees of freedom undergo a collective (undamped) shift in the local temperature and velocity

Divergent friction from massless particles at the speed of sound!

A new way for solving Boltzmann Equations

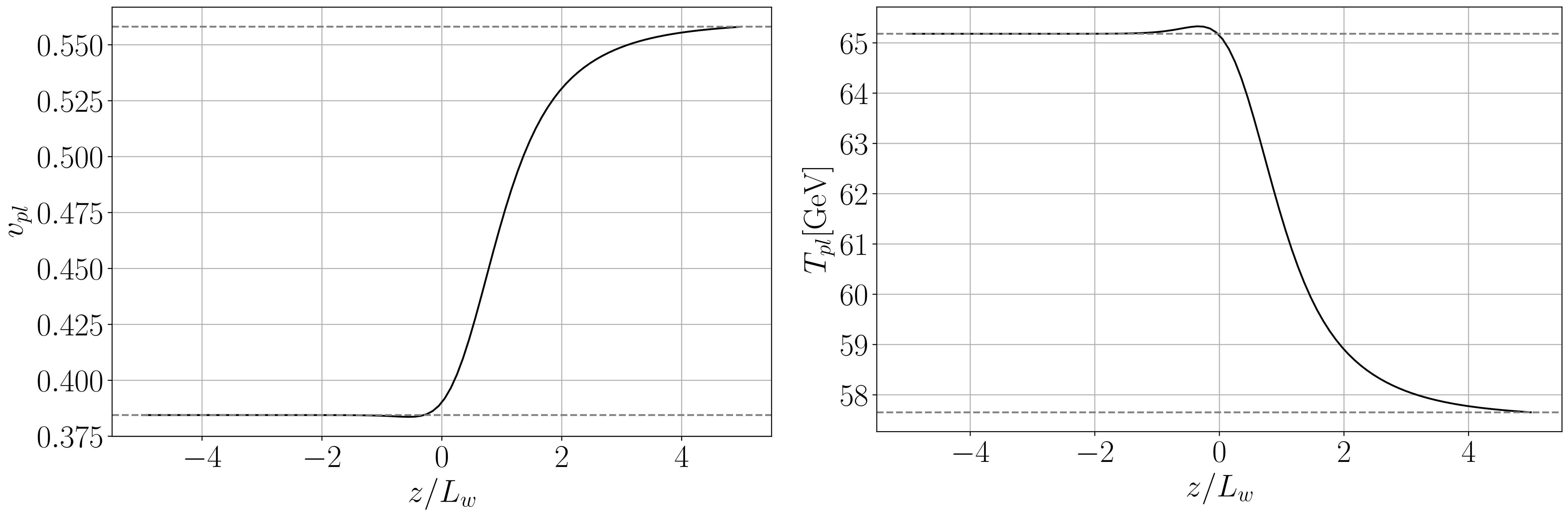
We define the background by imposing the conservation of its energy-momentum across the PT wall. This means solving

$$v_{\mathrm{bg}}^2\gamma_{\mathrm{bg}}^2\omega_{\mathrm{bg}} - \mathscr{F}_{\mathrm{bg}} + \frac{1}{2}(\partial_z\phi)^2 = \mathrm{k}_1,$$

$$v_{\mathrm{bg}}\gamma_{\mathrm{bg}}^2\omega_{\mathrm{bg}} = \mathrm{k}_2\,.$$

Scalar field contribution

Modified matching conditions for $\mathrm{v_{bg}(z)}$ and $\mathrm{T_{bg}(z)}$



A new way for solving Boltzmann Equations

Going back to the BEs: $$p^\mu \partial_\mu f_i\left(x^\mu, p^\mu\right) + \frac{1}{2}\partial_\mu m^2 \partial_{p^\mu} f_i\left(x^\mu, p^\mu\right) + \mathscr{C}_i = 0$$

Old approach	Our approach
$\mathrm{T_i(z) = \bar{T}_{bg} + \delta T_i(z)}$	$\mathrm{T_i(z) = T_{bg}(z) + \delta T_i(z)}$
$\mathrm{v_i(z) = \bar{v}_{bg} + \delta v_i(z)}$	$\mathrm{v_i(z) = v_{bg}(z) + \delta v_i(z)}$
$\mu_i(z) = \mu_{\mathrm{bg}} + \delta\mu_i(z)$	$\mu_i(z) = \mu_{\mathrm{bg}}(z) + \delta\mu_i(z)$

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$\mathrm{T_i(z)} = \bar{\mathrm{T}}_{\mathrm{bg}} + \delta \mathrm{T_i(z)}$	$\mathrm{T_i(z)} = \mathrm{T_{bg}(z)} + \delta \mathrm{T_i(z)}$
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This conceptual difference implies the presence of a new term (one for every particle) in the BEs!

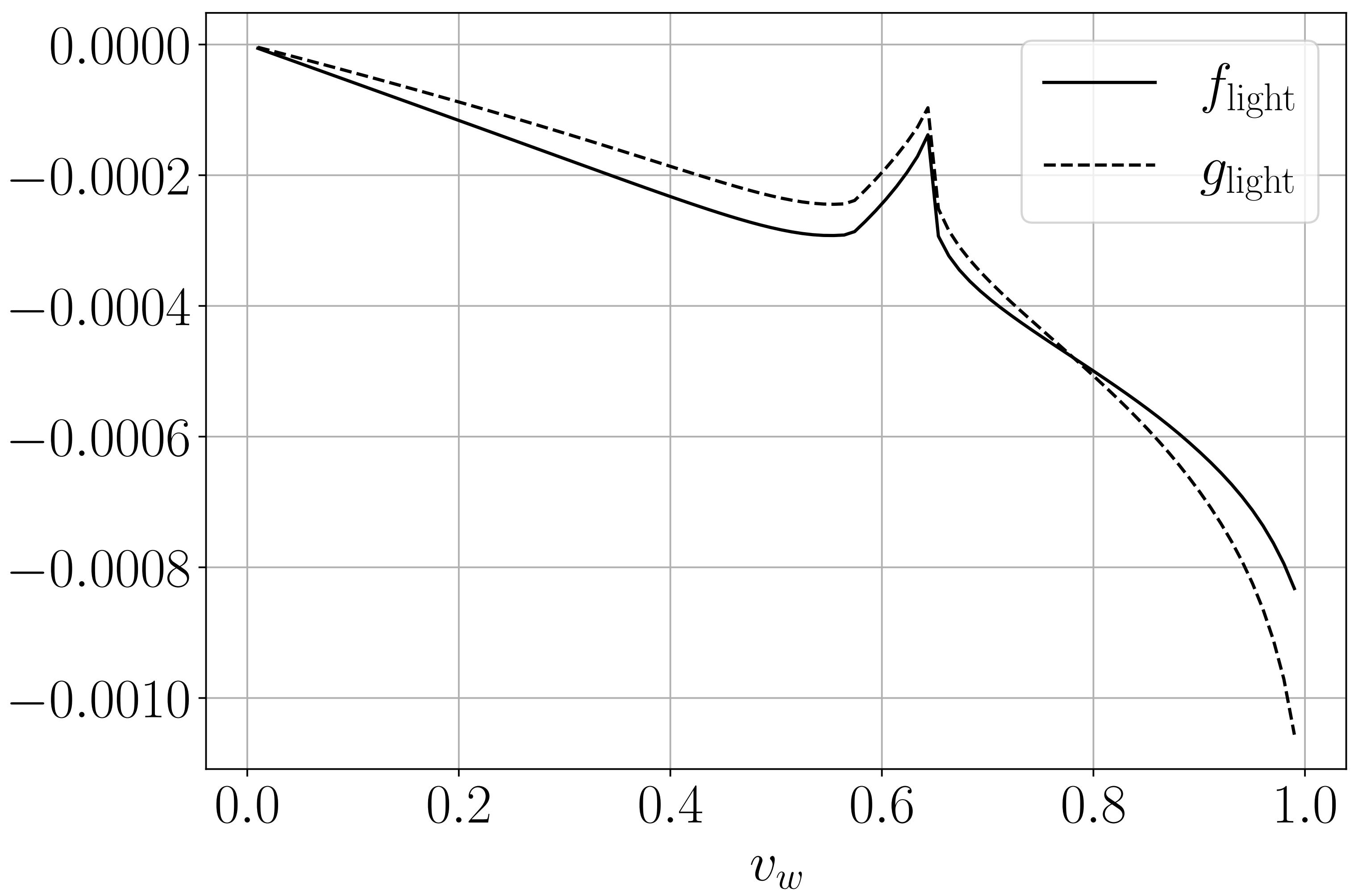
$$p^\mu \partial_\mu f_i^{\mathrm{bg}}(x, p) \supset (f_i^{\mathrm{bg}})' \frac{p^\mu p^\nu}{T}\left(u_\nu \frac{\partial_\mu T}{T} - \partial_\mu u_\nu\right)$$

This new "source" term is fundamental to ensure energy-momentum conservation at BEs level

Results

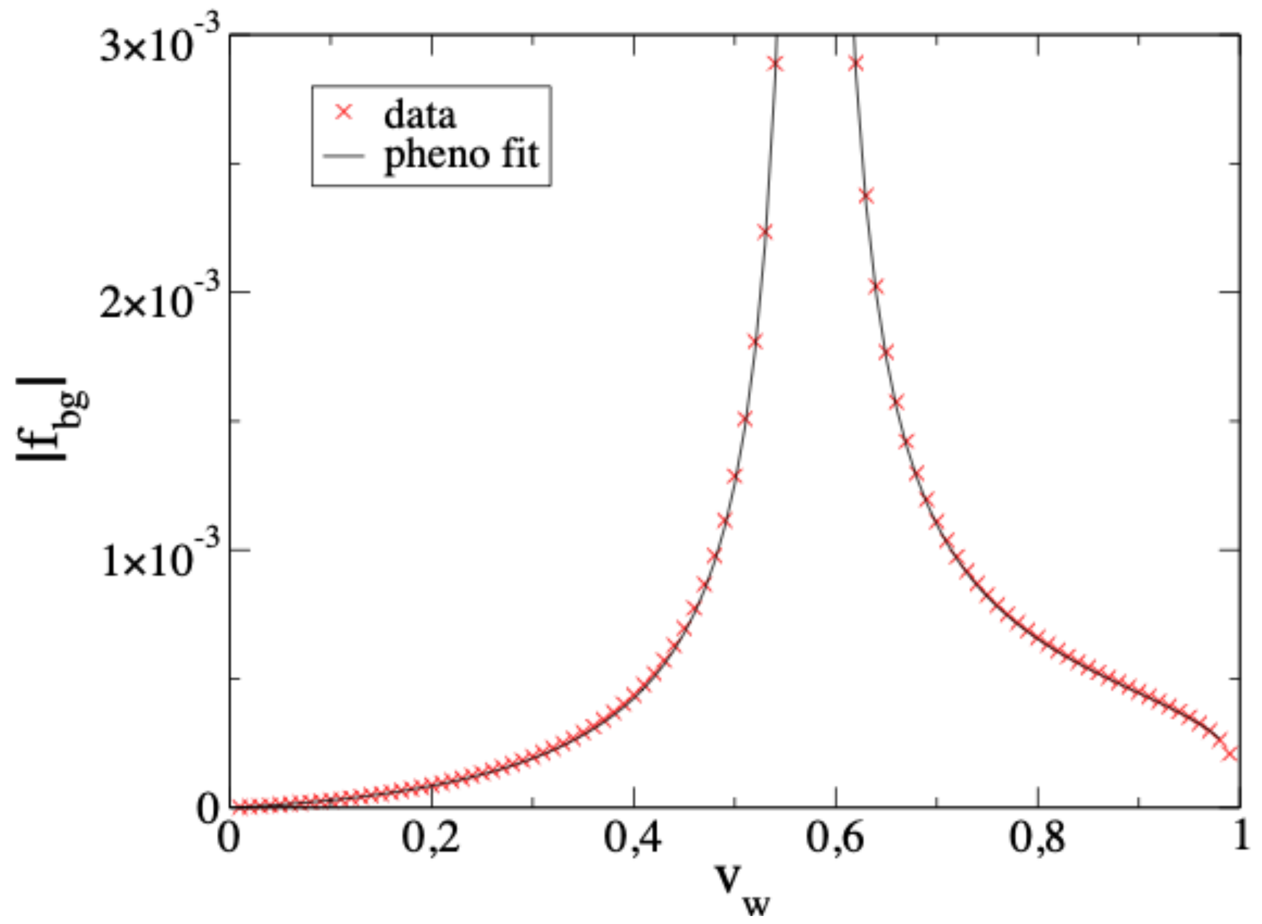
Benchmark model: SM with low cutoff

$$\mathscr{F}(\phi,T) = V_0(\phi) - \frac{a}{3}T^4 + \frac{c}{2}\phi^2T^2, \quad V_0(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\tilde{\lambda}}{4}\phi^4 + \frac{1}{8\Lambda^2}\phi^6$$

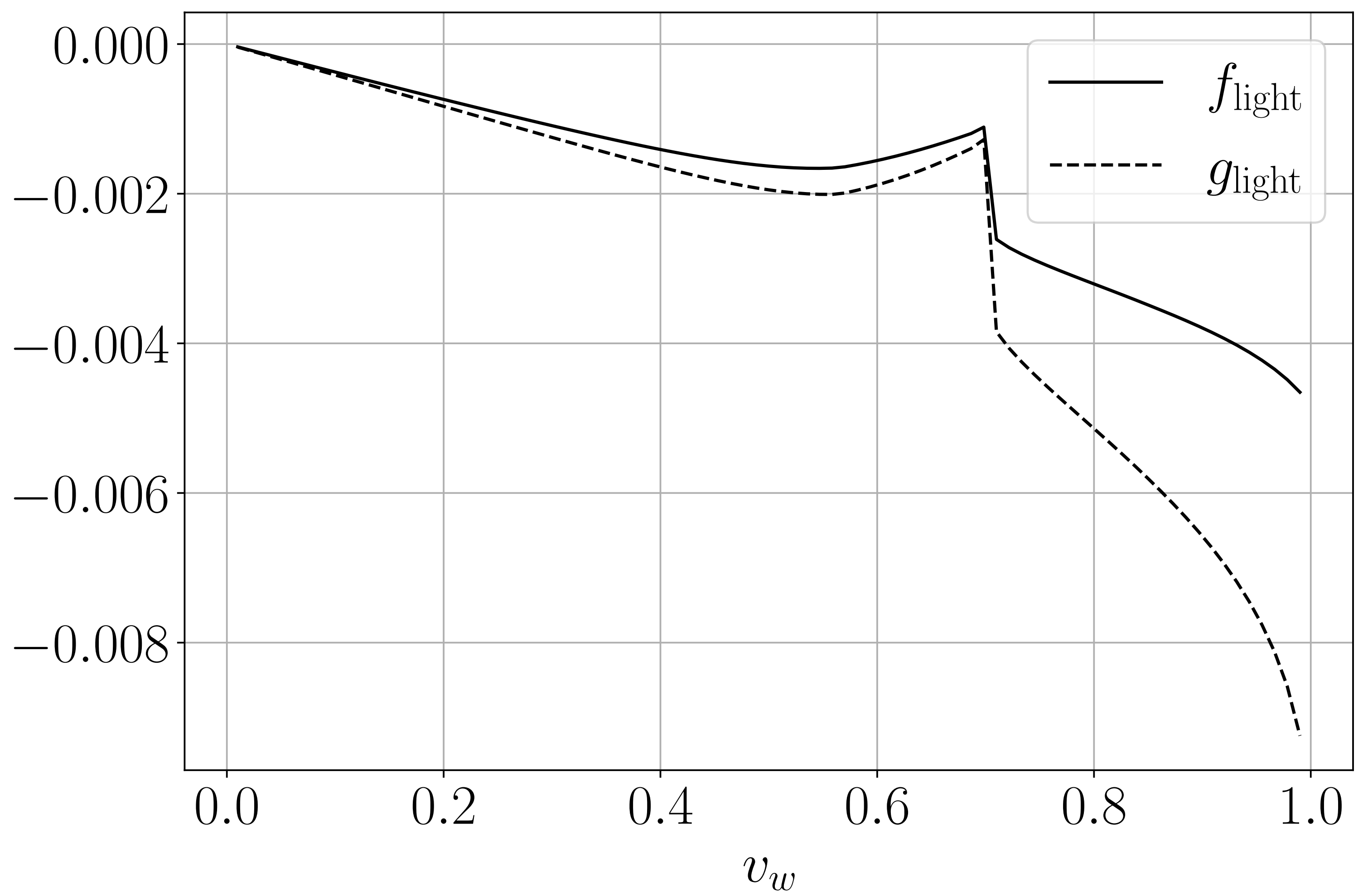


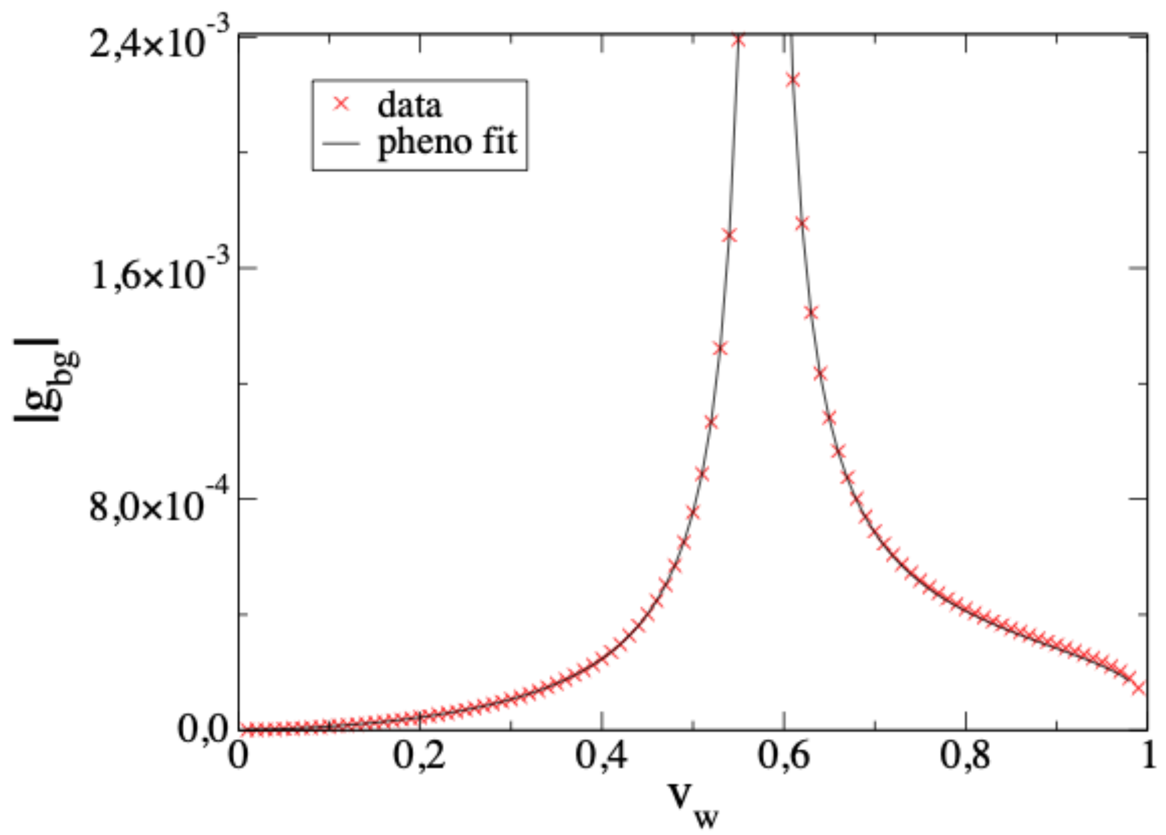
New

Old



For the "light" particles the new source term takes away the divergence!





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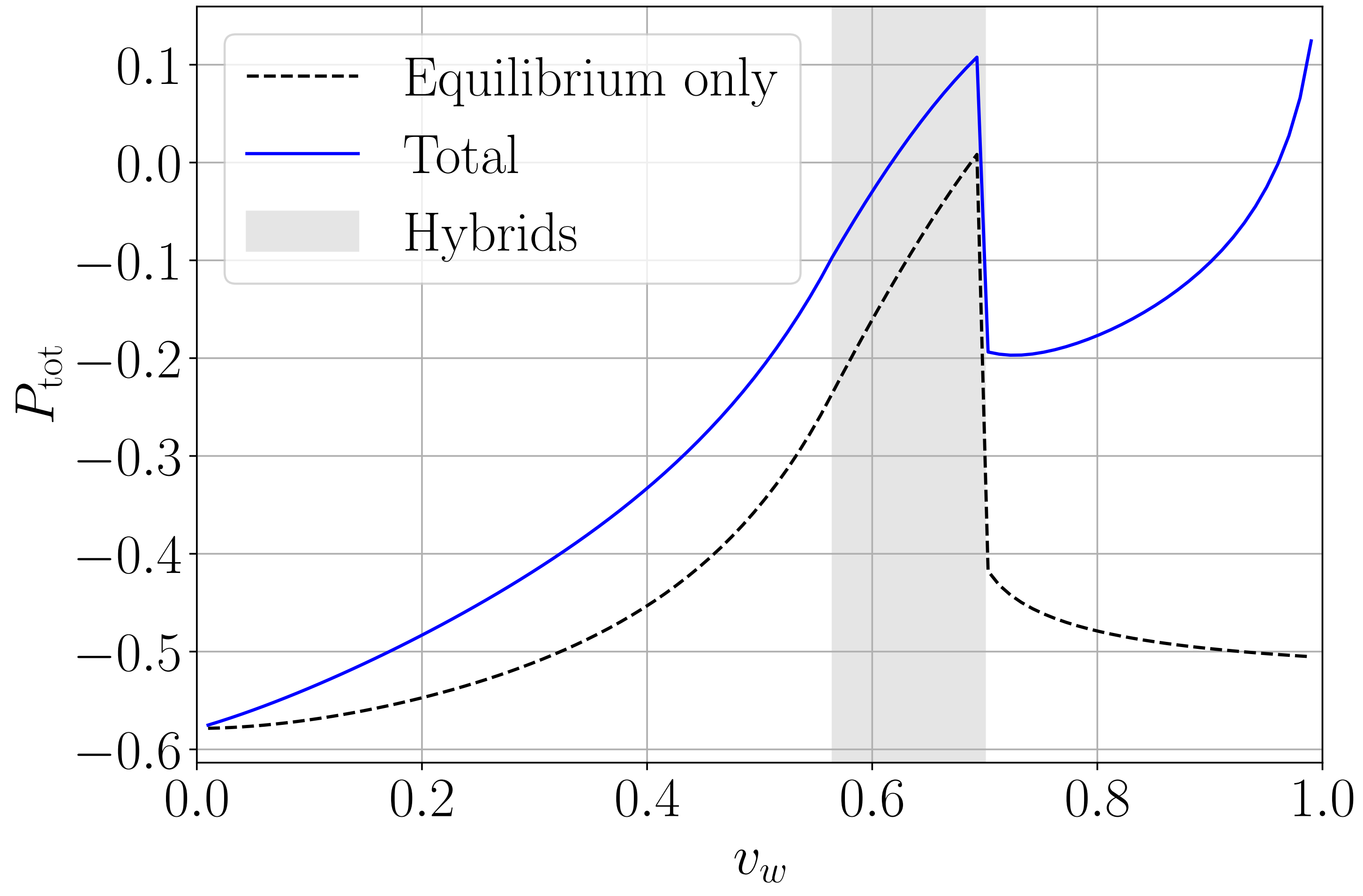
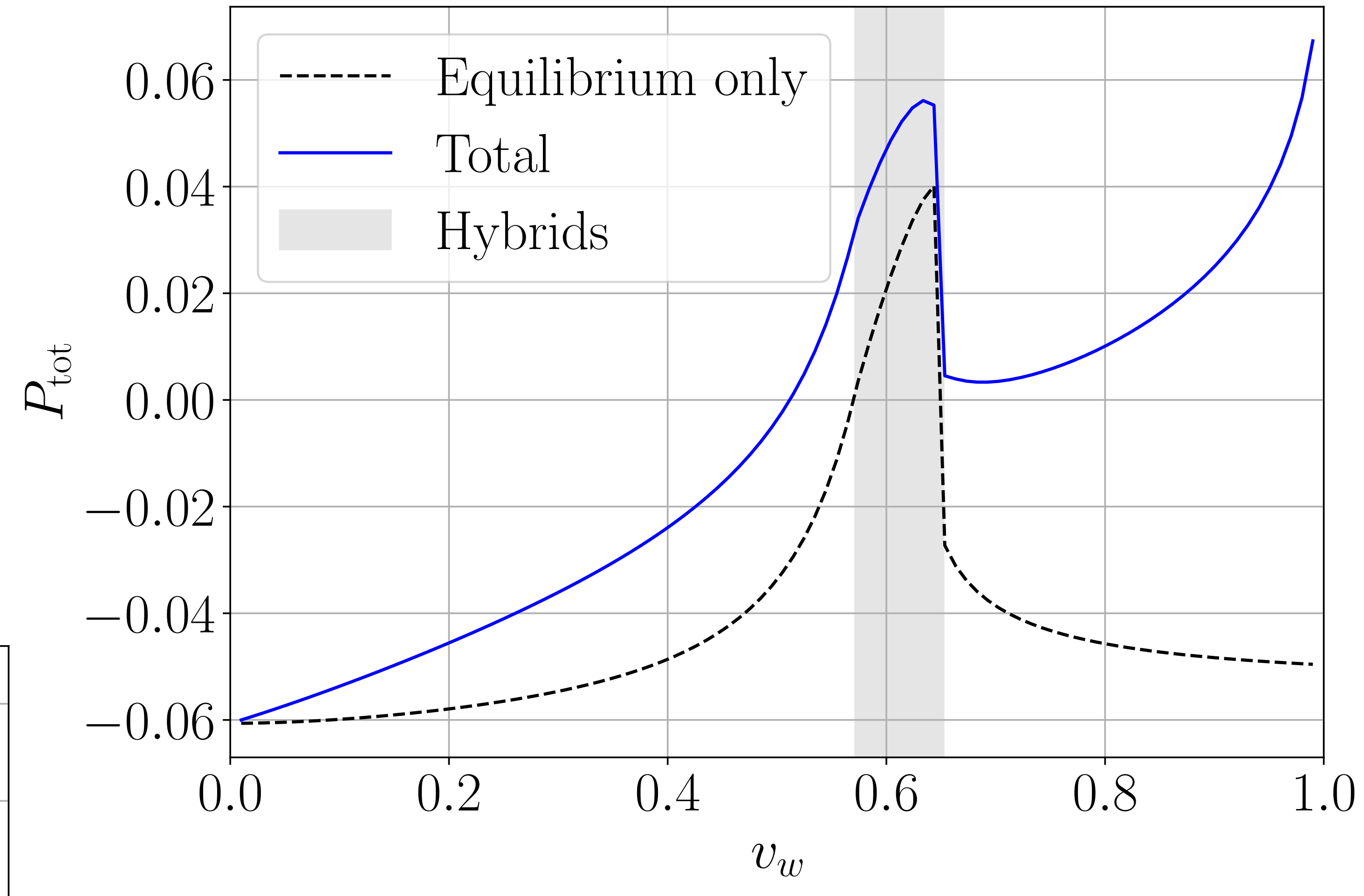
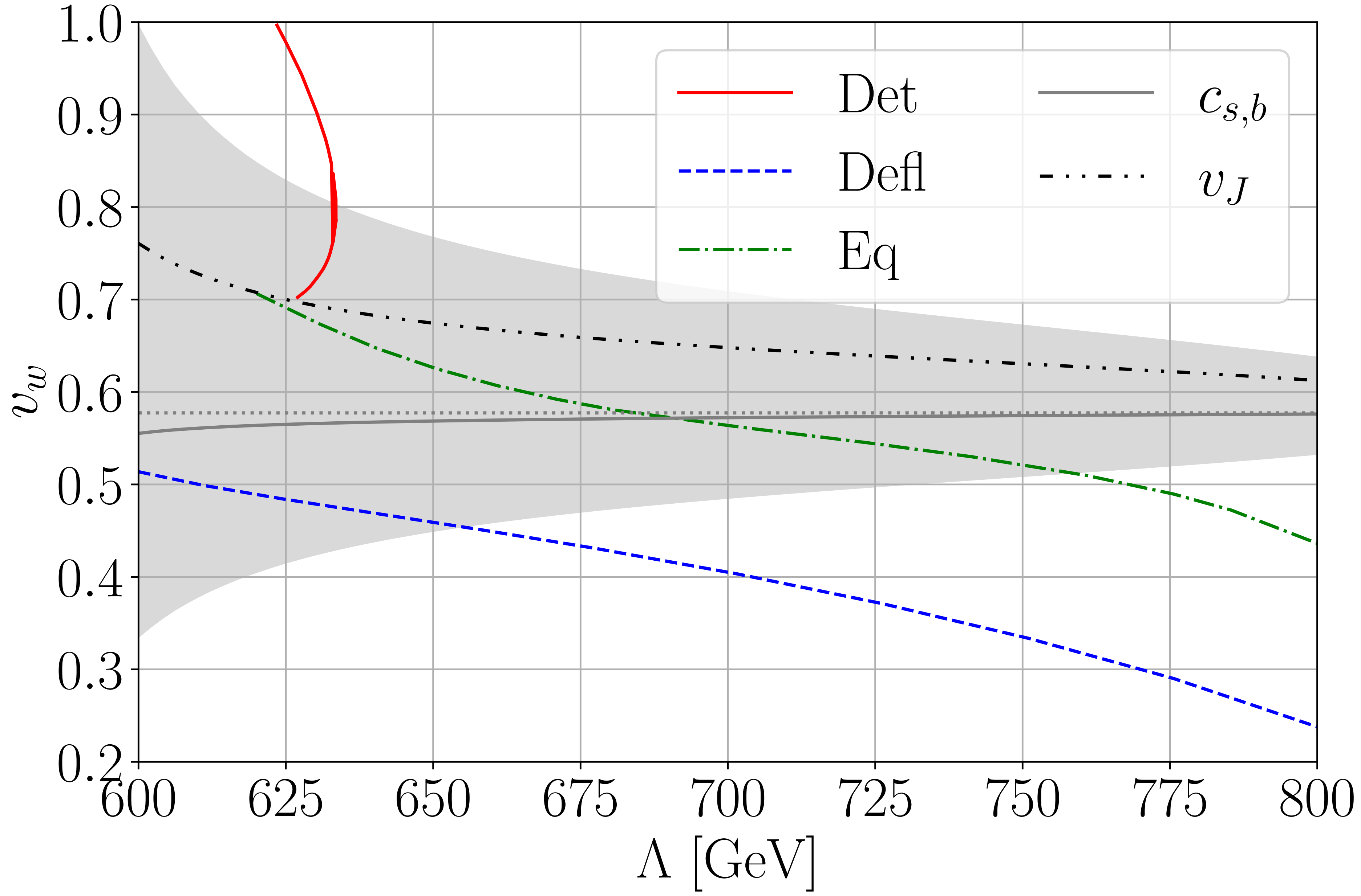
Pressure acting on the expanding wall for $\Lambda = 625$ GeV and $\Lambda = 690$ GeV





The grey region represents the limit where the old approach is not trustable anymore

Solutions of ξ_w for different Λ



Thank you for your attention!

The wall velocity ξ_w

Solving the Boltzmann Equation (BE)

The dynamics of the particles in the plasma can be described by the BE

$$p^\mu \partial_\mu f_i\left(x^\mu, p^\mu\right) + \frac{1}{2}\partial_\mu m^2 \partial_{p^\mu} f_i\left(x^\mu, p^\mu\right) + \mathscr{C}_i = 0,$$

The source term drives particles out of equilibrium

Collision terms, couples the different species in the plasma

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The wall velocity $\xi_{\rm w}$

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We transform this integro-differential equation into a set of ODEs by:

1. *Ansatz* on distributions: $f_i(p^\mu, x^\mu) = f_{\rm eq,i} + \delta f_i(p^\mu, x^\mu)$ and linearisation

2.Taking momenta: $\int \frac{d^3p}{(2\pi)^3 E}, \int \frac{d^3p}{(2\pi)^3 E} p_\mu u^\mu, \int \frac{d^3p}{(2\pi)^3 E} p_\mu \bar{u}^\mu, \ldots$

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The wall velocity ξ_w

The fluid *Ansatz*

$\delta\mu$ *repr. chemical pot. fluctuations*

δu_μ *repr. velocity fluctuations*

δT *repr. temperature fluctuations*

In the case of the *fluid Ansatz* we can write $\delta f = f'\delta_p$, with $\delta_p = \delta\mu + p^\mu(\delta u_\mu - u_\mu \delta T/T)$ and solve these BEs for the 3 fluctuations.

Inserting the fluctuations in the linearized version of the Klein-Gordon equation we can find the values of the wall velocity ξ_w and of its width L_w.

$$-\phi'' + \frac{\partial \mathcal{F}}{\partial \phi} + \sum_i \frac{dm_i^2}{d\phi} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_i} \delta f_i(p, x) = 0$$

1. $$\int dz [\text{l.h.s of KG }] \times \phi' = 0$$

$$f_{\text{fl}} \equiv \frac{N_t}{2T_+^2} \int dz \, \frac{dm_t^2}{dz} \left(c_{f1}\delta\mu_f + c_{f2}\delta\tau_f\right) + \frac{N_W}{2T_+^2} \int dz \, \frac{dm_W^2}{dz} \left(c_{b1}\delta\mu_b + c_{b2}\delta\tau_b\right)$$

$$f_{\text{light}} \equiv \frac{N_t}{2T_+^2} \int dz \, \frac{dm_t^2}{dz} \, c_{f2} \, \delta\tau_{\text{light}} + \frac{N_W}{2T_+^2} \int dz \, \frac{dm_W^2}{dz} \, c_{b2} \, \delta\tau_{\text{light}}$$

2. $$\int dz [\text{l.h.s of KG}] \times \phi'(2\phi - \phi_0) = 0$$