

Non-singular solutions to the Boltzmann equation with a fluid *Ansatz*



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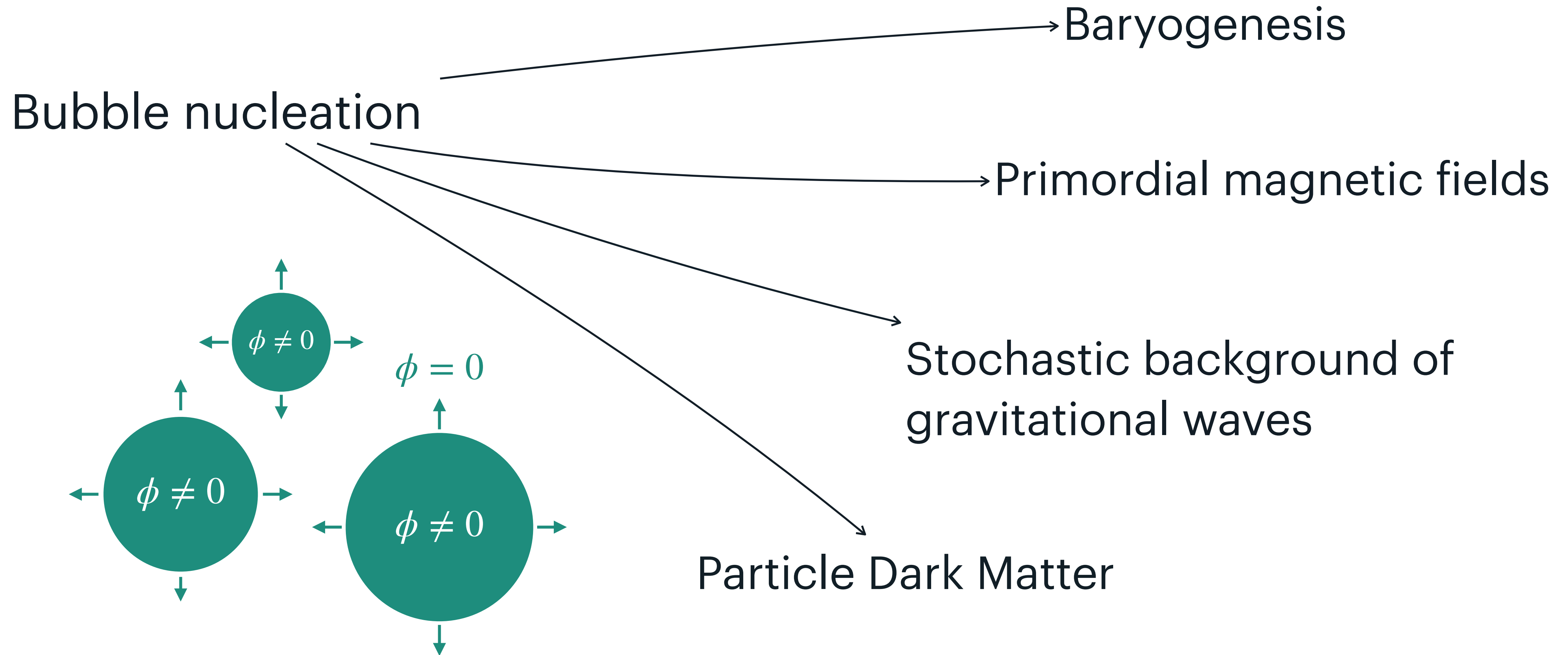
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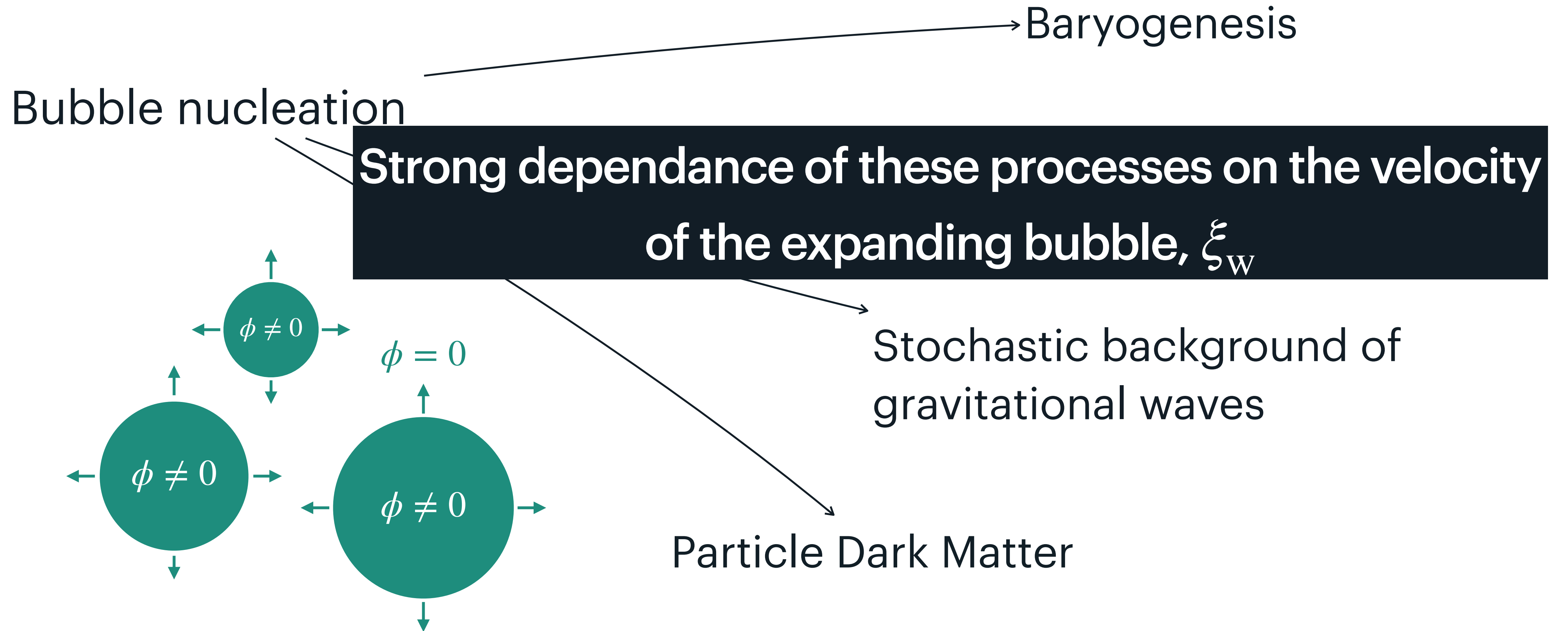
Modeling the evolutions of bubbles in cosmological First Order Phase Transitions



Why Cosmological First Order Phase Transitions (FOPT)?



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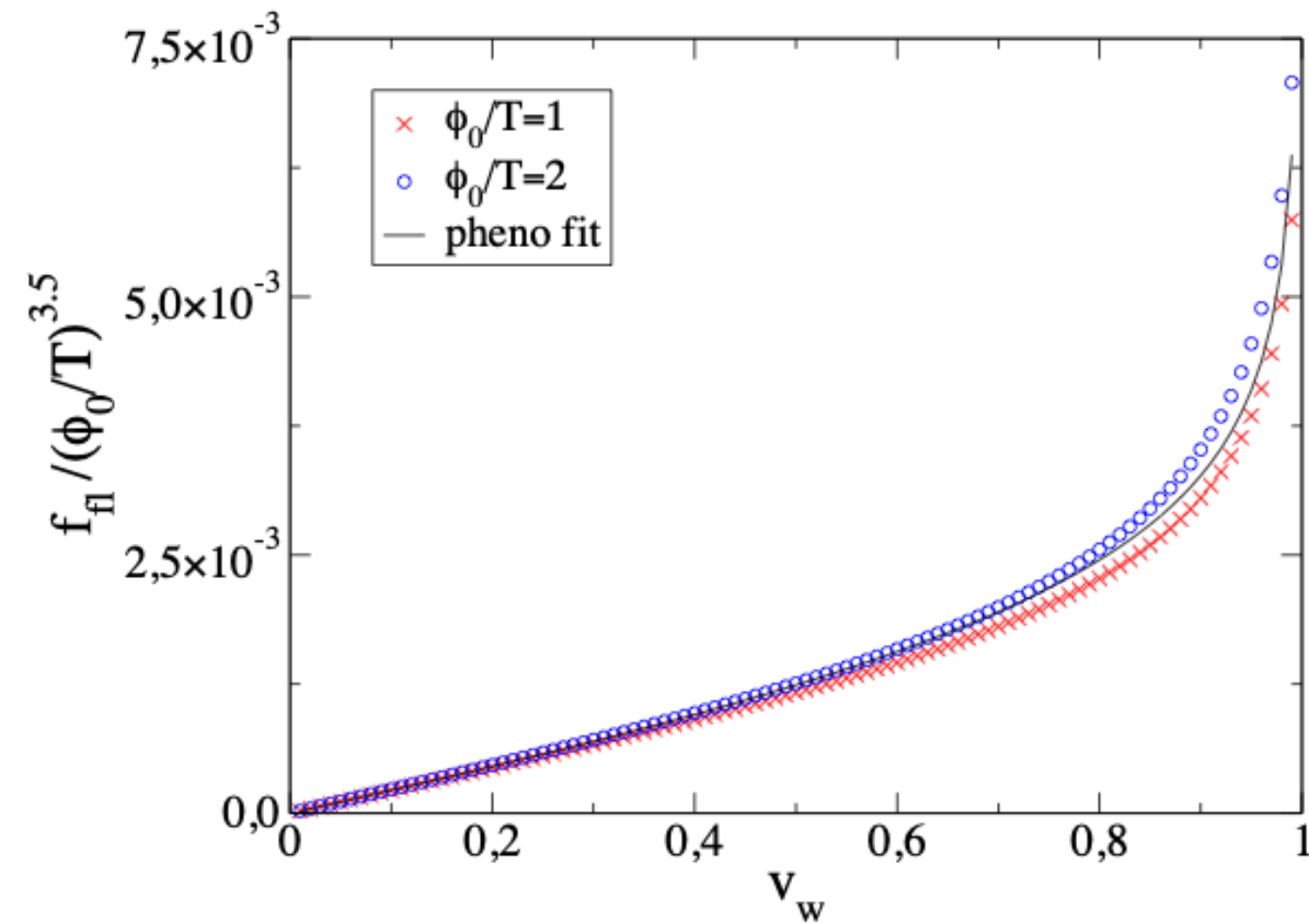
The wall velocity ξ_w

The parameter ξ_w is closely connected to the friction on the expanding wall ϕ , and it can be estimated in different way.

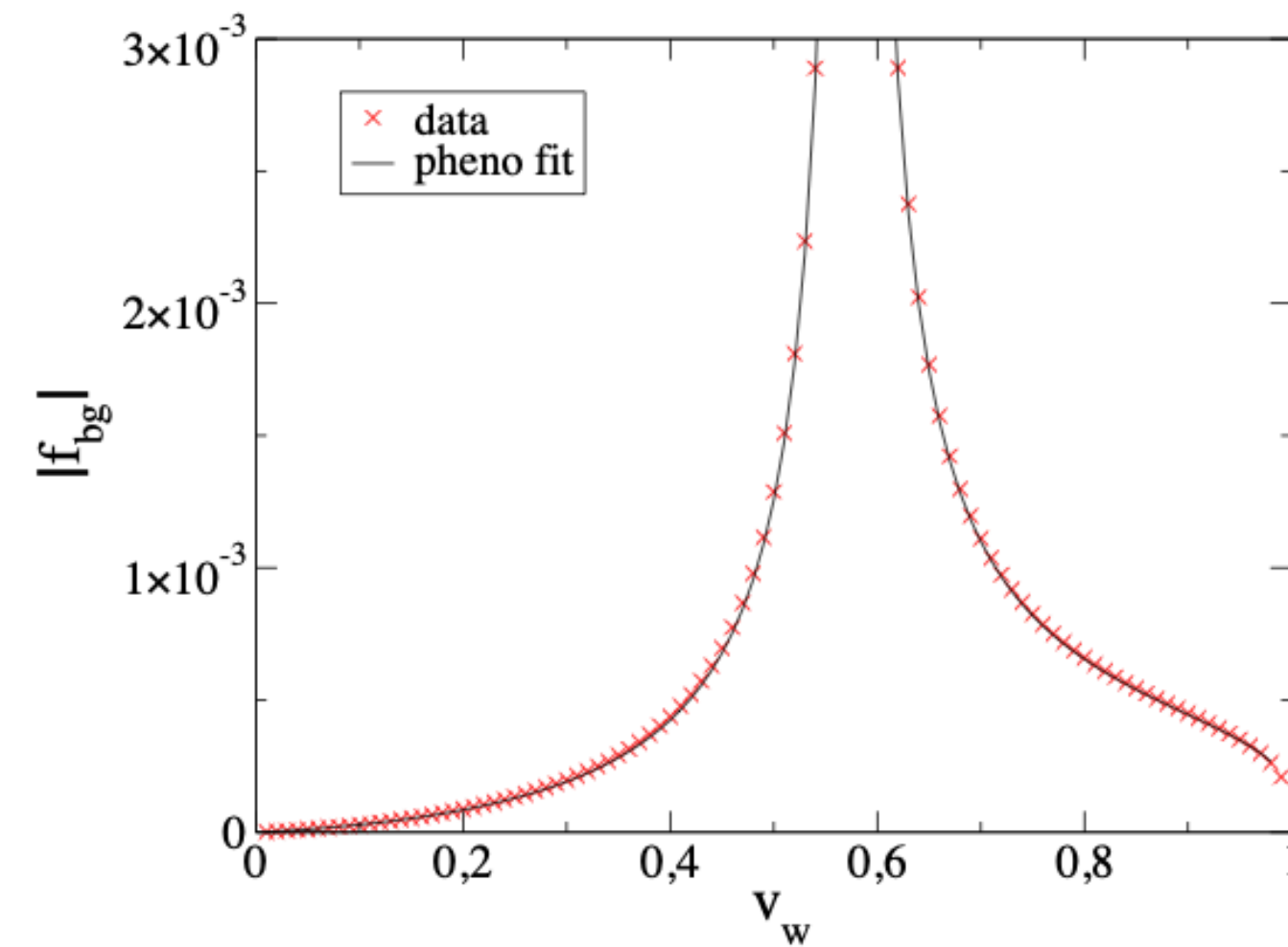
One approach consists in to solving the coupled system of the Klein-Gordon (KG) equation for ϕ and the Boltzmann Equations (BEs) for the particles in the plasma.

$$\square \phi + \frac{dV_0}{d\phi} + \sum_i \frac{dm_i^2}{d\phi} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_i} f_i(p^\mu, x^\mu) = 0$$
$$+ p^\mu \partial_\mu f_i(x^\mu, p^\mu) + \frac{1}{2} \partial_\mu m^2 \partial_{p^\mu} f_i(x^\mu, p^\mu) + \mathcal{C}_i = 0$$

A “sonic boom” in the friction



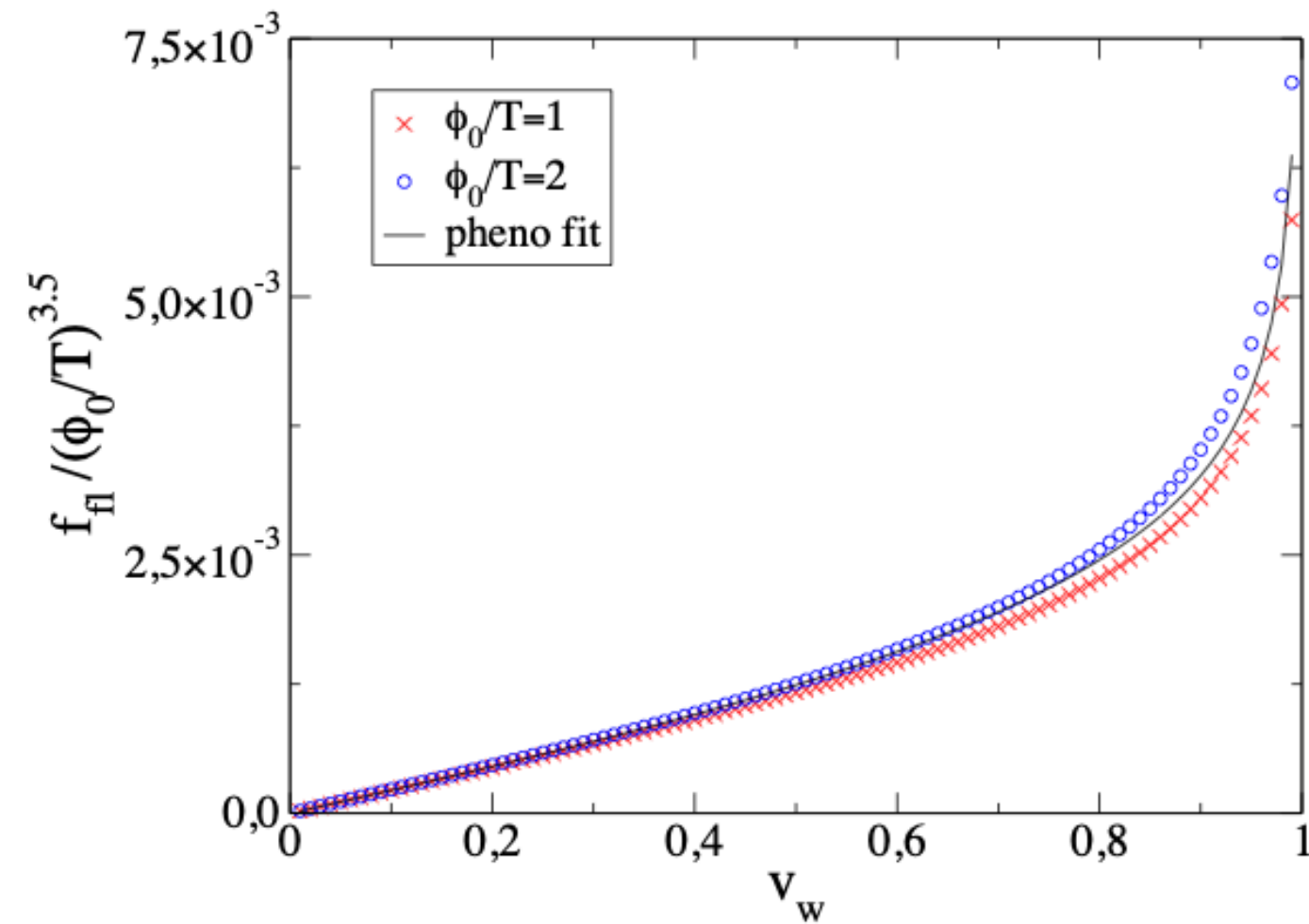
[1407.3132] Konstandin et al.



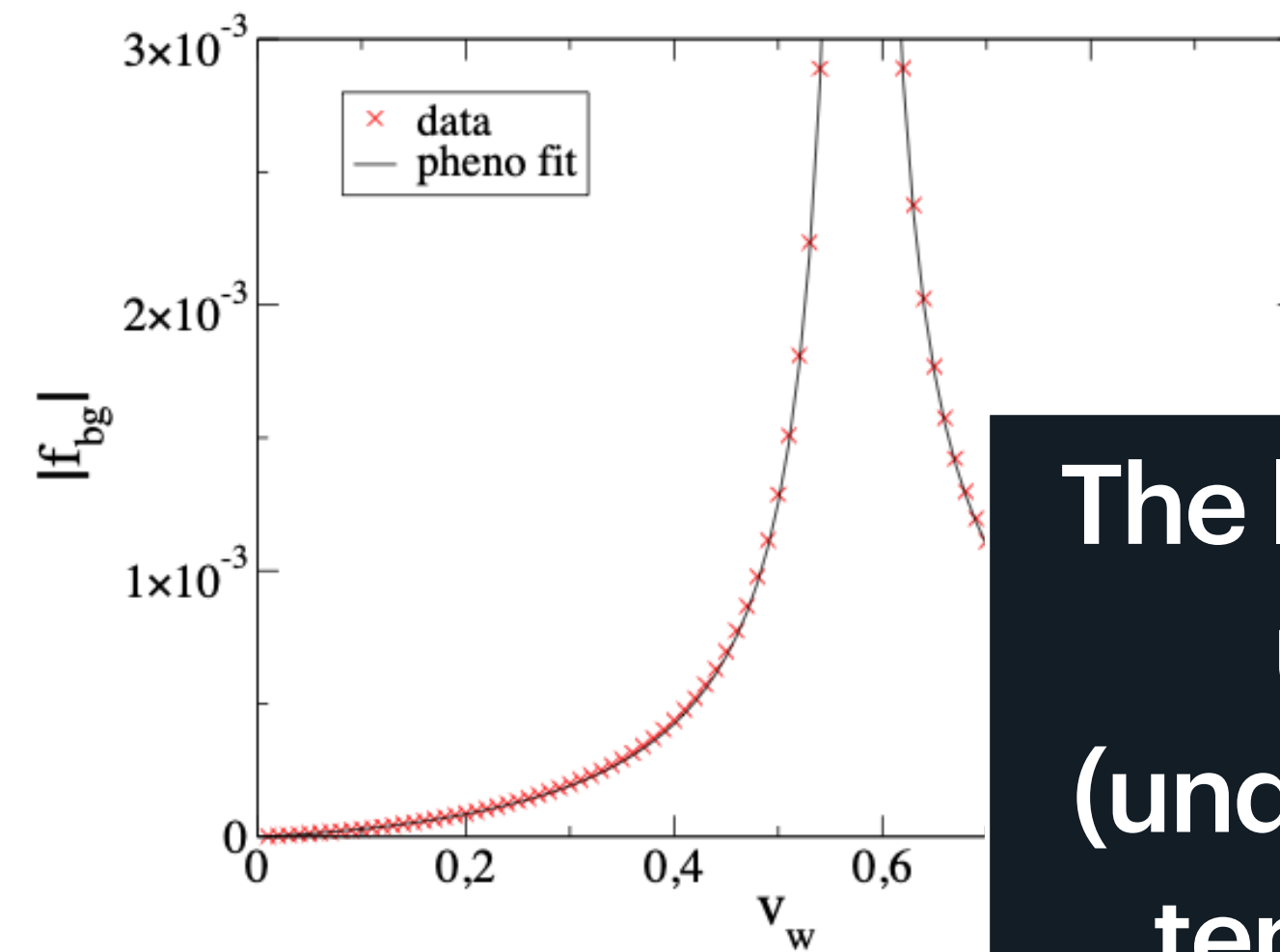
The singularity arises because of an interplay between the energy-momentum conservation in the BE and the linearisation procedure.

Divergent friction from massless particles at the speed of sound!

A “sonic boom” in the friction



[1407.3132] Konstandin et al.



The singularity arises because of an interplay between the energy-
The light degrees of freedom undergo a collective (undamped) shift in the local temperature and velocity

in the BE procedure.

Divergent friction from massless particles at the speed of sound!

A new way for solving Boltzmann Equations

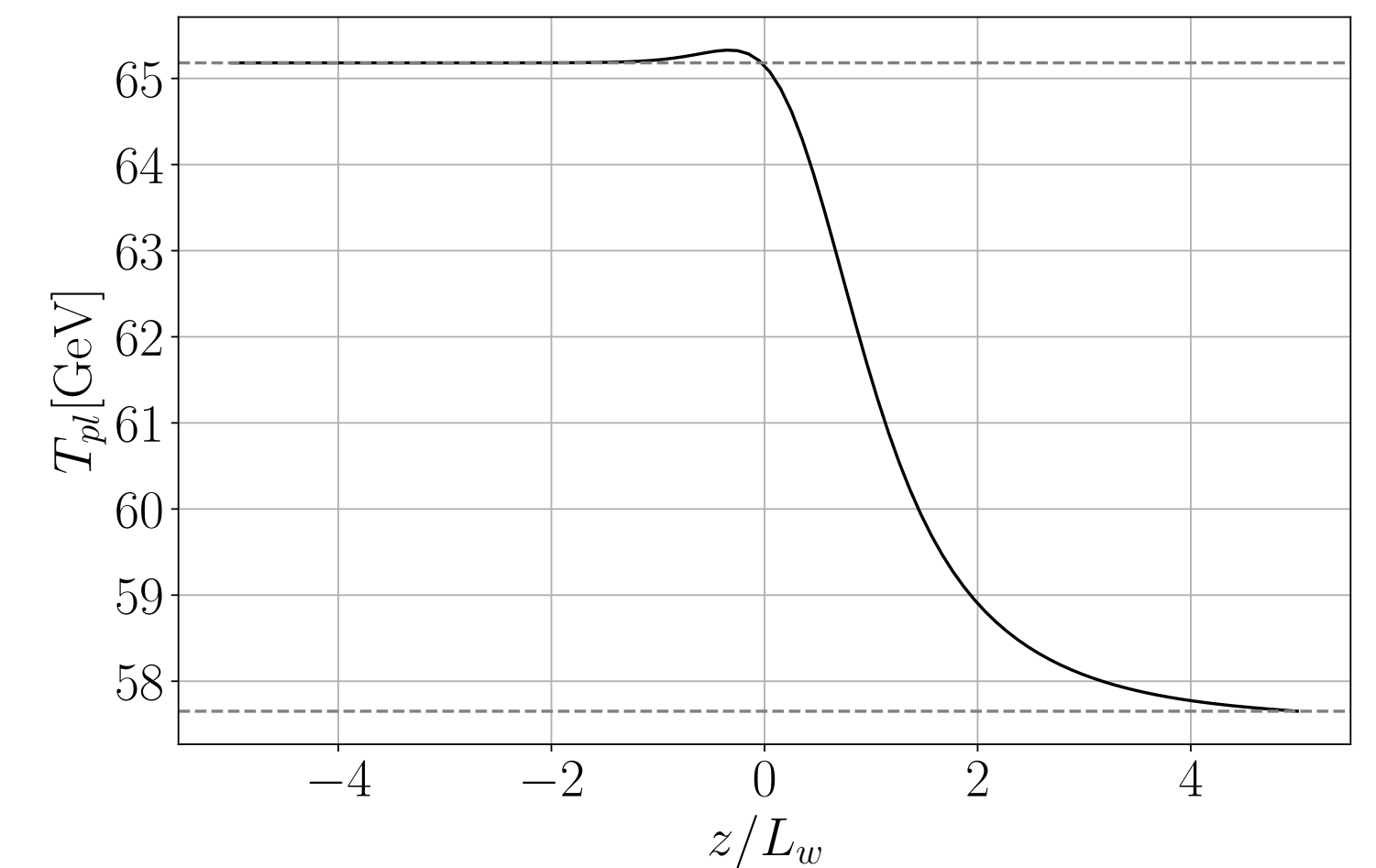
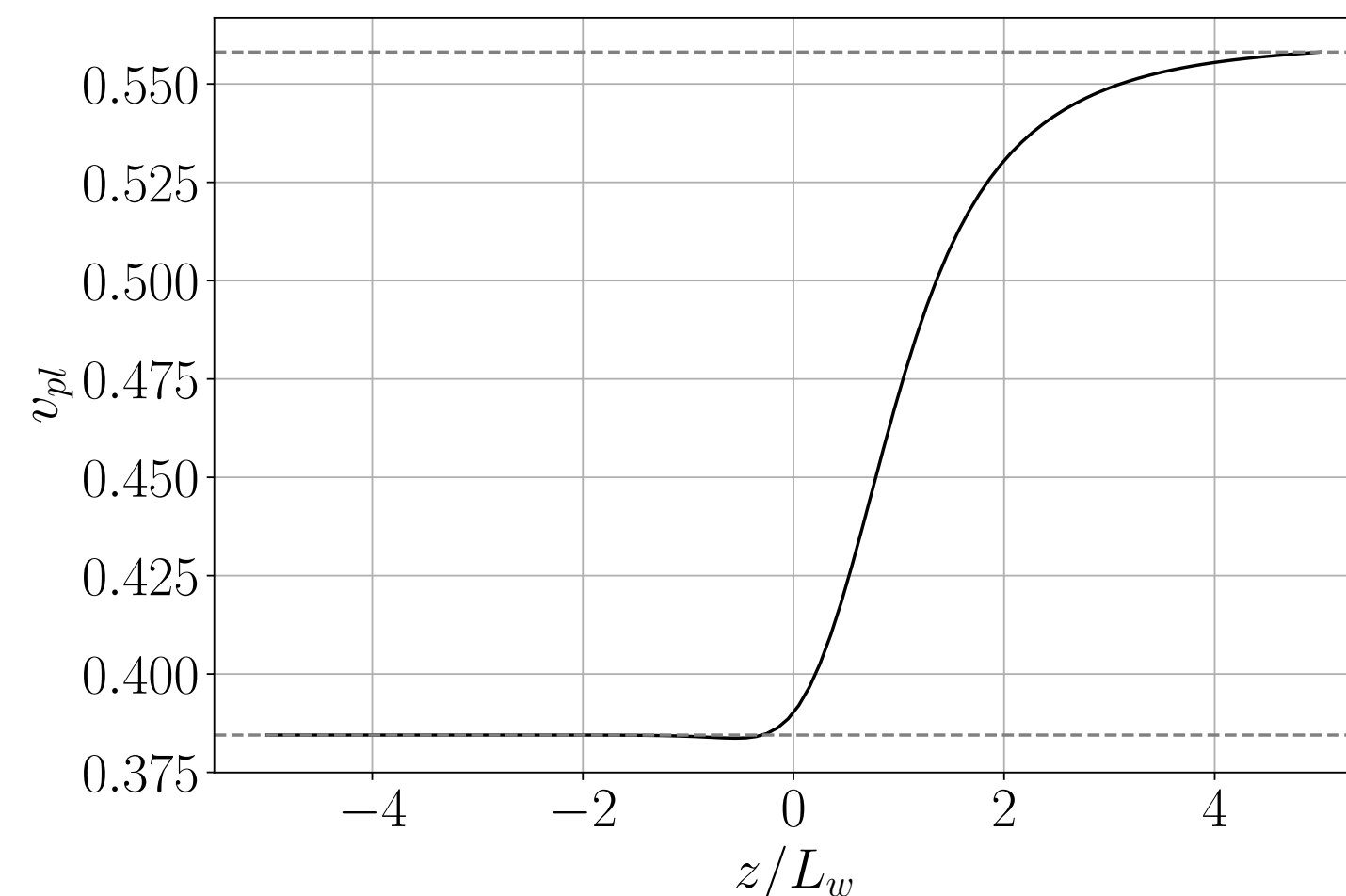
We define the background by imposing the conservation of its energy-momentum across the PT wall. This means solving

$$v_{\text{bg}}^2 \gamma_{\text{bg}}^2 \omega_{\text{bg}} - \mathcal{F}_{\text{bg}} + \frac{1}{2} (\partial_z \phi)^2 = k_1,$$

$$v_{\text{bg}} \gamma_{\text{bg}}^2 \omega_{\text{bg}} = k_2.$$

Scalar field contribution

Modified matching conditions
for $v_{\text{bg}}(z)$ and $T_{\text{bg}}(z)$



A new way for solving Boltzmann Equations

Going back to the BEs:

$$p^\mu \partial_\mu f_i(x^\mu, p^\mu) + \frac{1}{2} \partial_\mu m^2 \partial_{p^\mu} f_i(x^\mu, p^\mu) + \mathcal{C}_i = 0$$

Old approach	Our approach
$T_i(z) = \bar{T}_{\text{bg}} + \delta T_i(z)$	$T_i(z) = T_{\text{bg}}(z) + \delta T_i(z)$
$v_i(z) = \bar{v}_{\text{bg}} + \delta v_i(z)$	$v_i(z) = v_{\text{bg}}(z) + \delta v_i(z)$
$\mu_i(z) = \mu_{\text{bg}} + \delta \mu_i(z)$	$\mu_i(z) = \mu_{\text{bg}}(z) + \delta \mu_i(z)$

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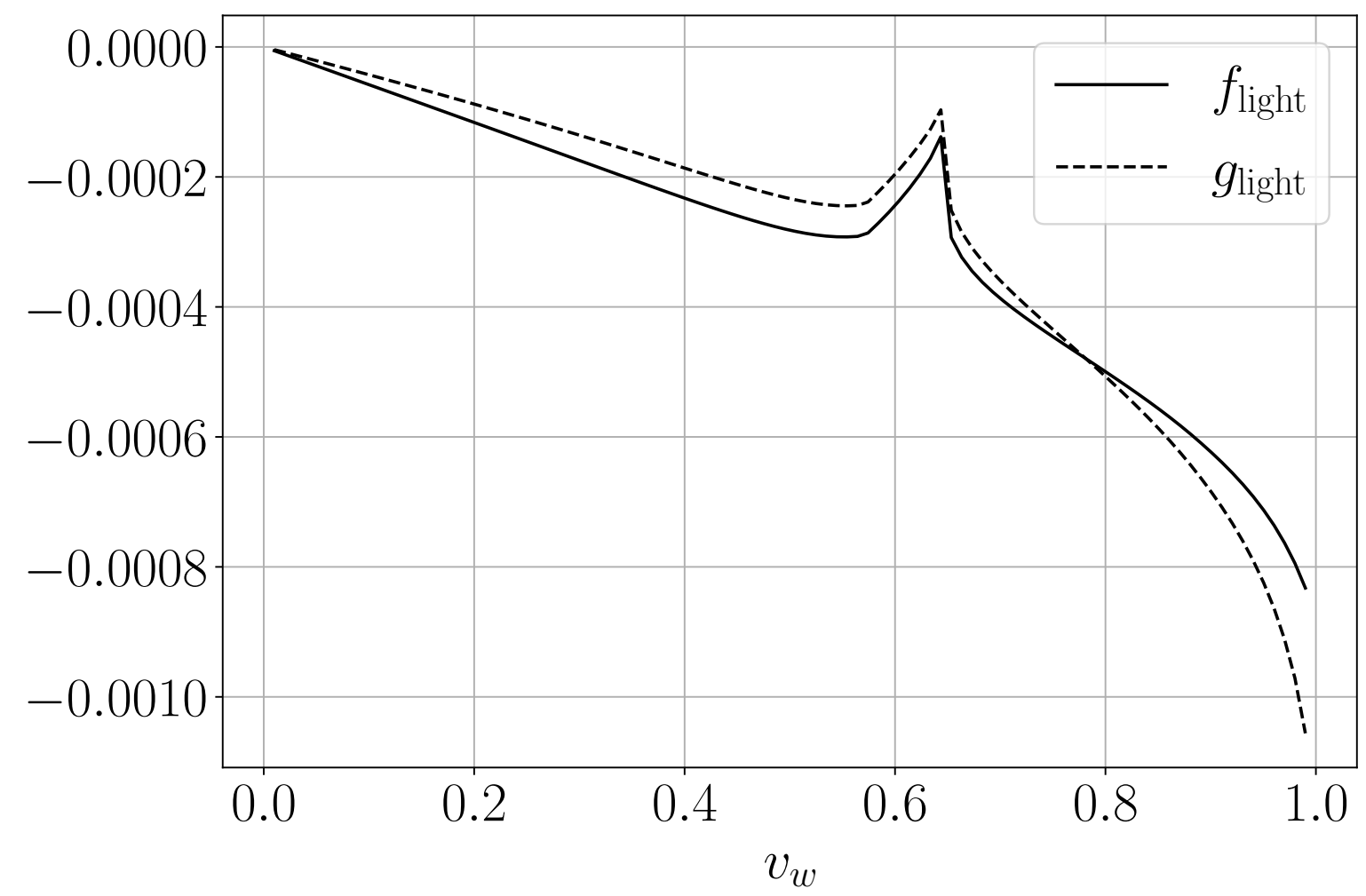
This conceptual difference implies the presence of a new term (one for every particle) in the BEs!

$$p^\mu \partial_\mu f_i^{\text{bg}}(x, p) \supset (f_i^{\text{bg}})' \frac{p^\mu p^\nu}{T} \left(u_\nu \frac{\partial_\mu T}{T} - \partial_\mu u_\nu \right)$$

This new “source” term is fundamental to ensure energy-momentum conservation at BEs level

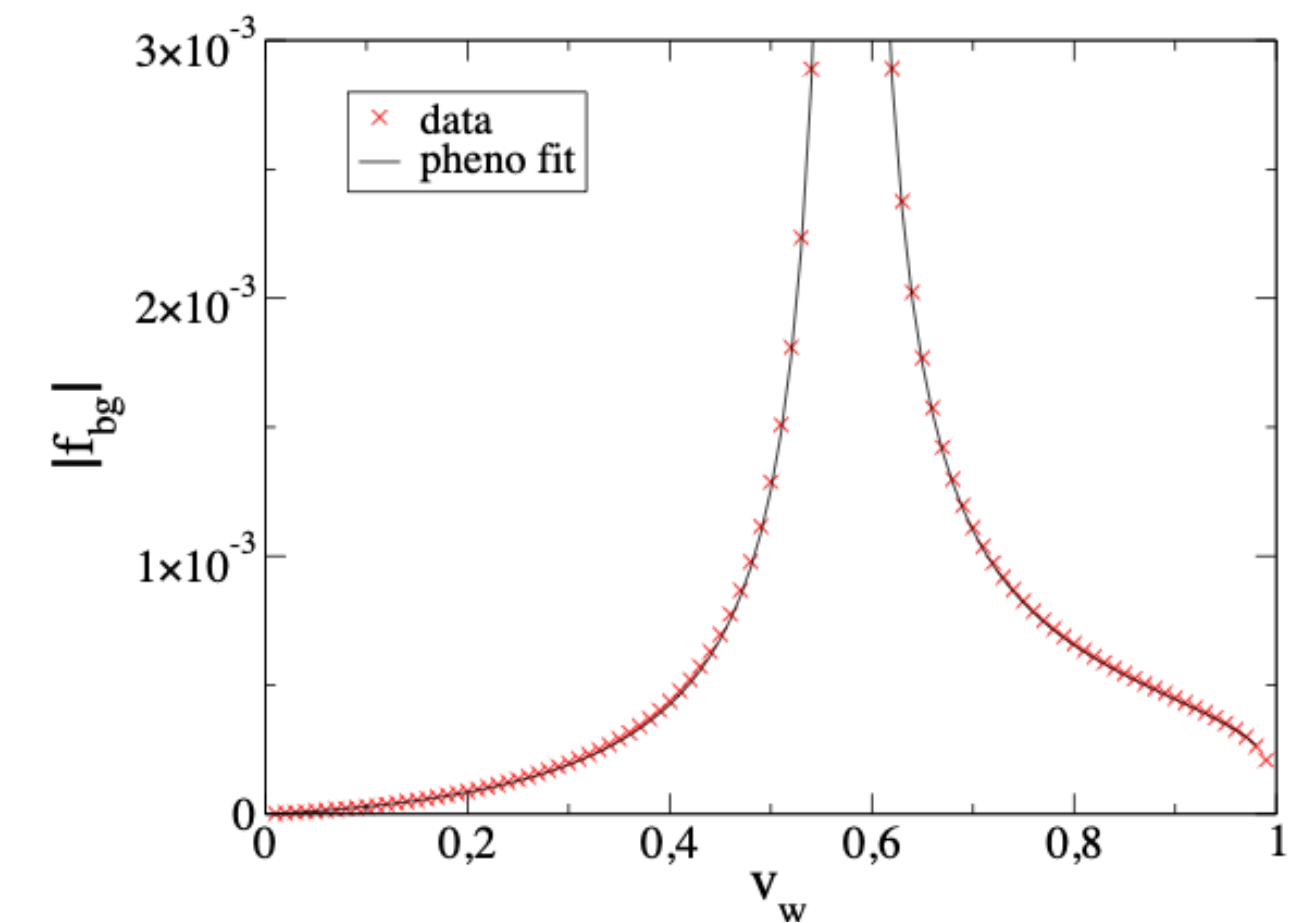
Results

$$\mathcal{F}(\phi, T) = V_0(\phi) - \frac{a}{3}T^4 + \frac{c}{2}\phi^2T^2, \quad V_0(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\tilde{\lambda}}{4}\phi^4 + \frac{1}{8\Lambda^2}\phi^6$$

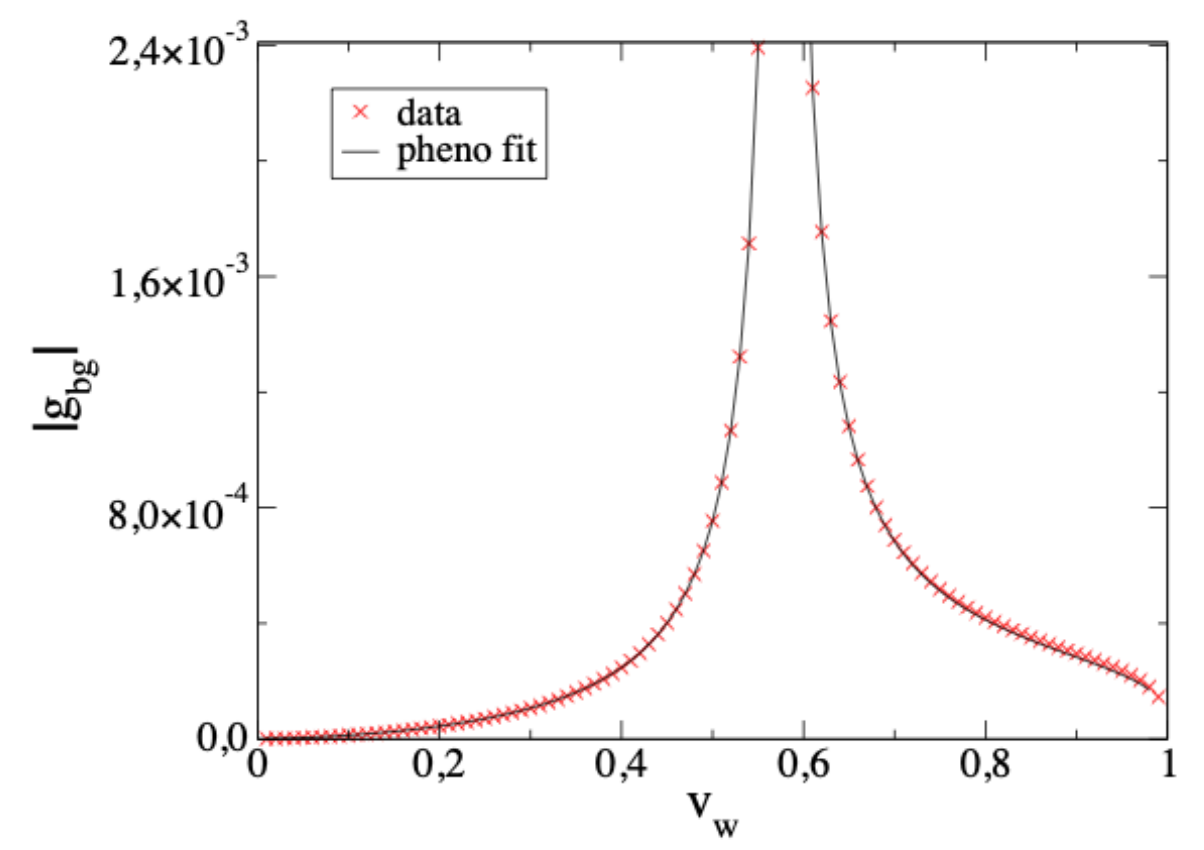
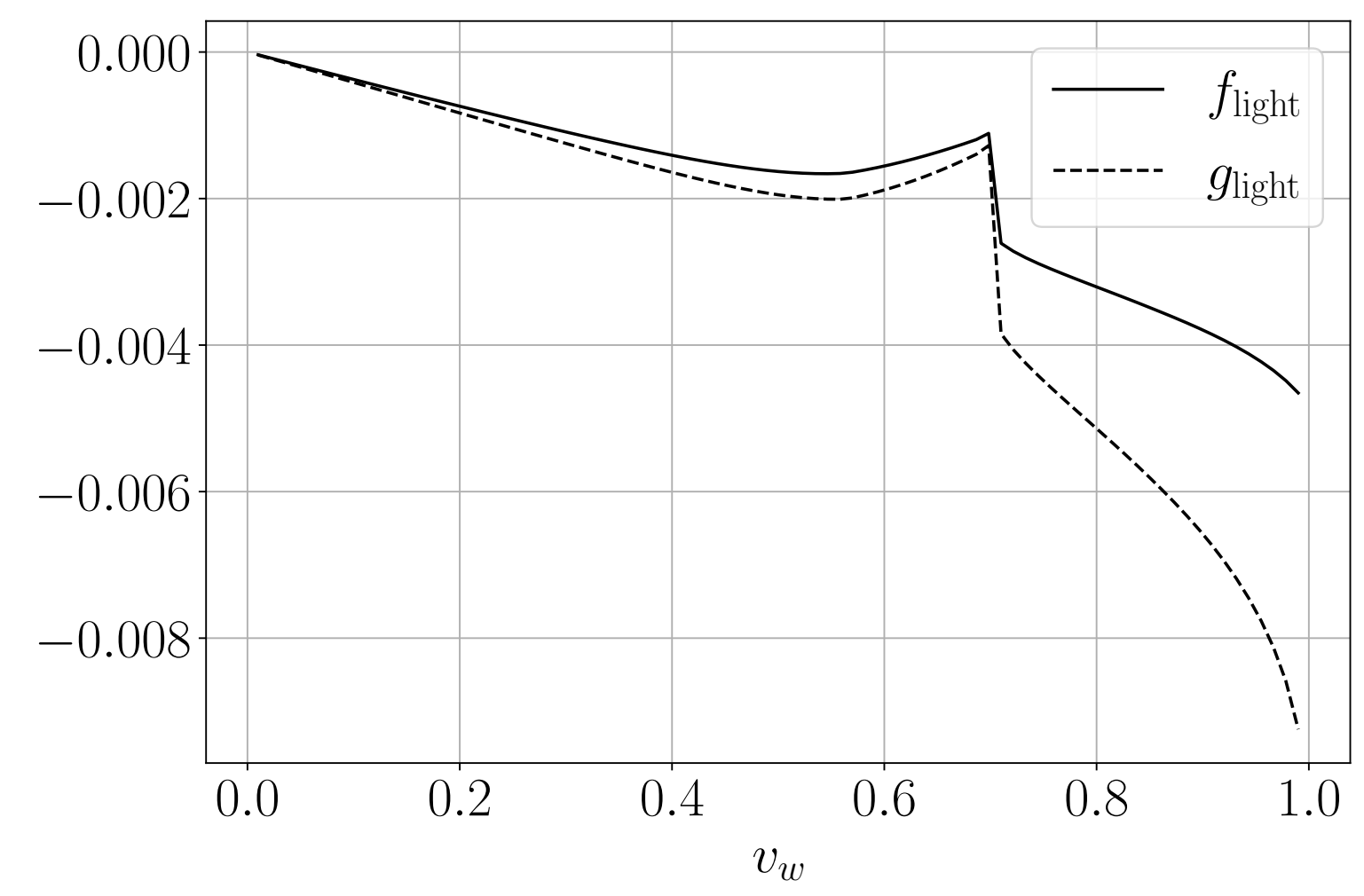


New

Old



For the “light” particles the new source term takes away the divergence!

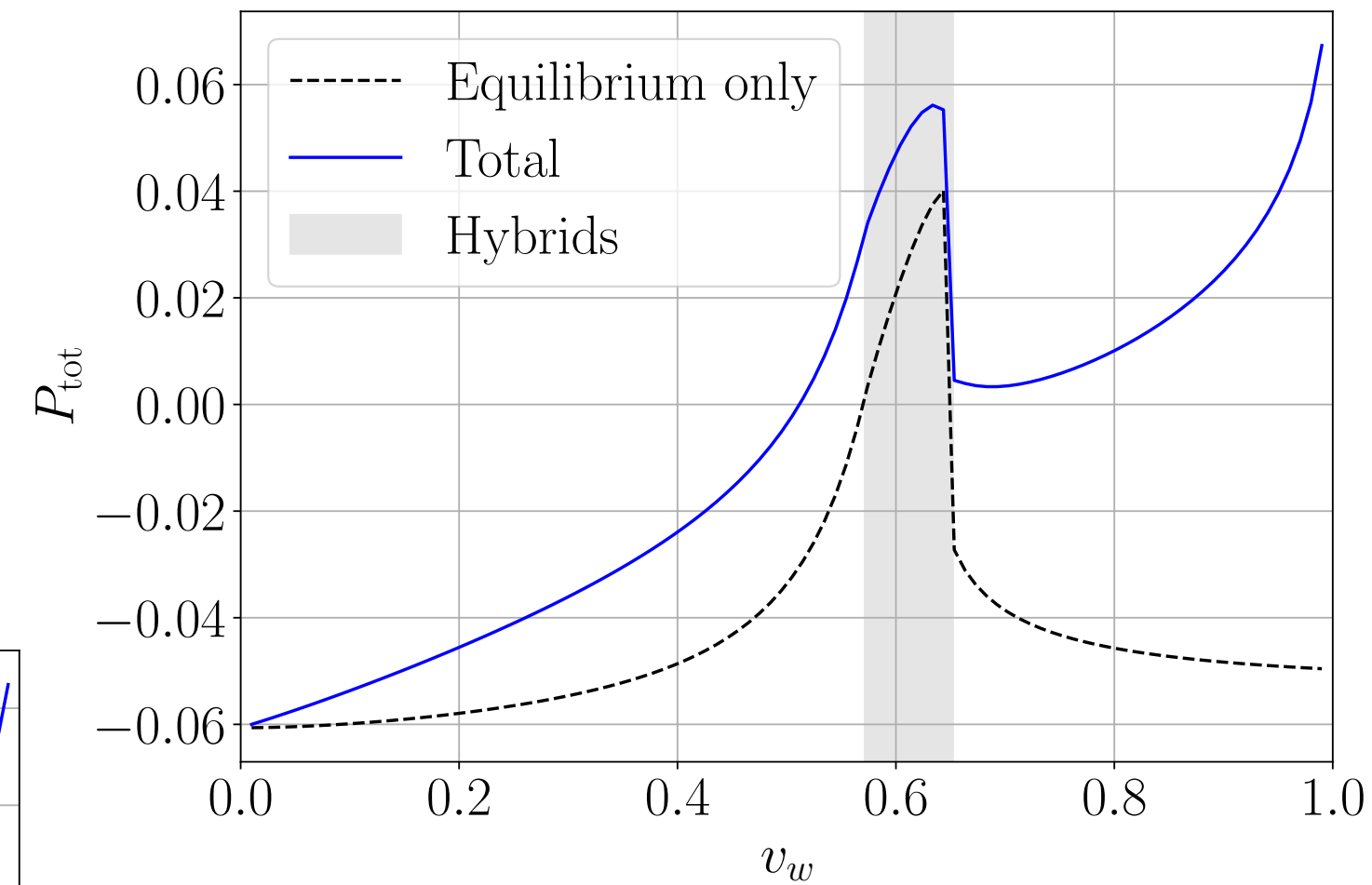
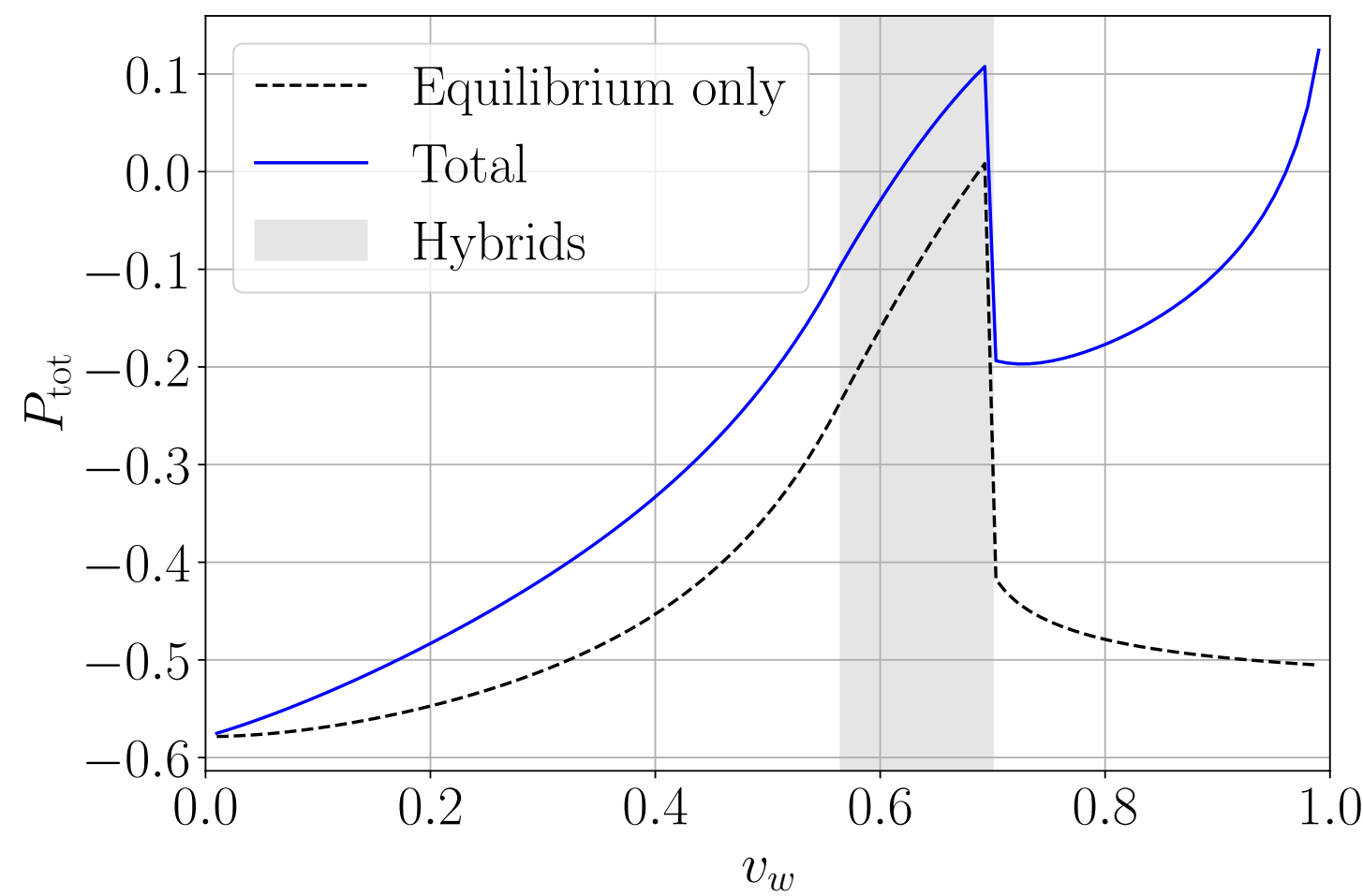


Results

Benchmark model: SM with low cutoff

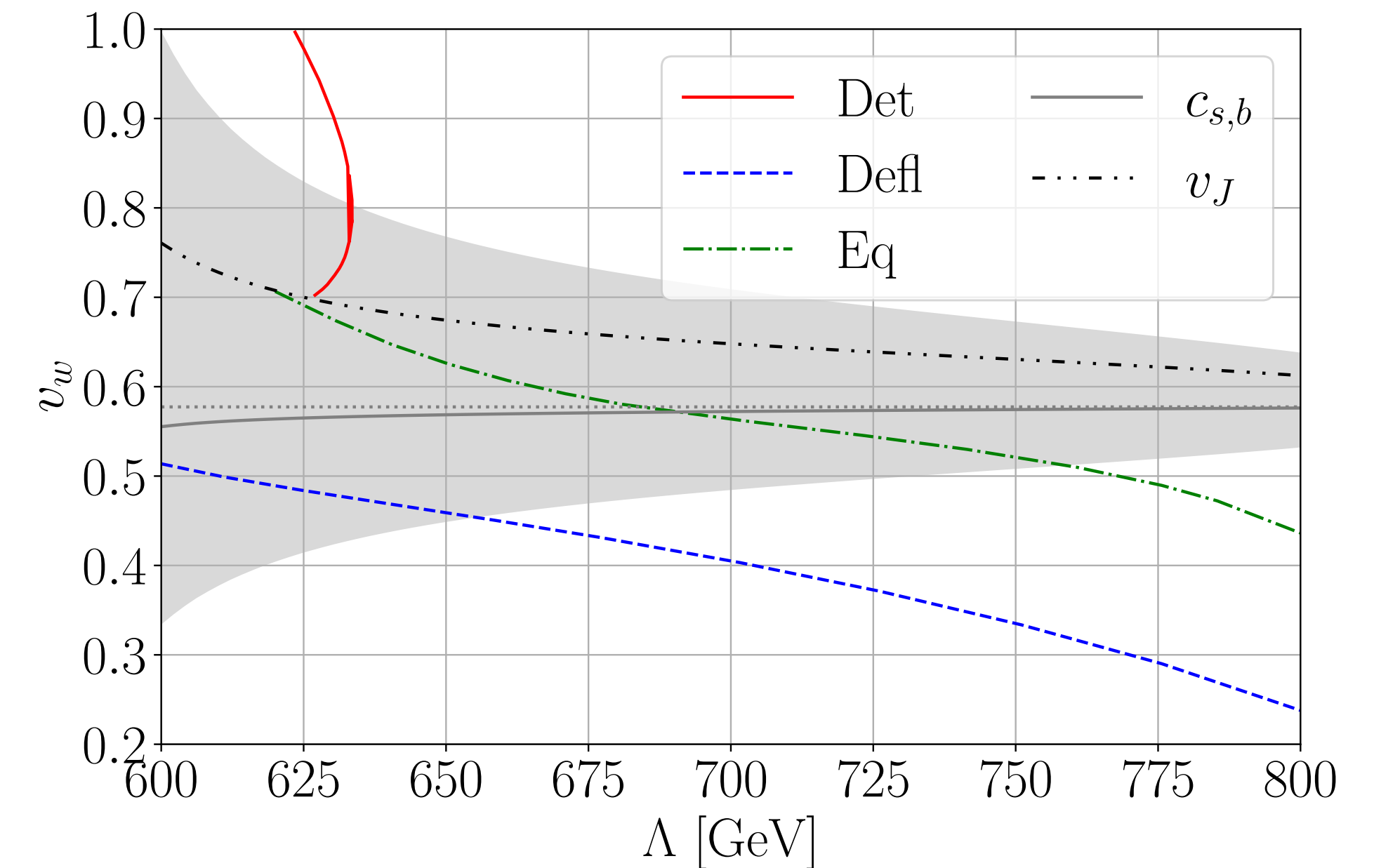
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Pressure acting on the expanding wall for $\Lambda = 625$ GeV and $\Lambda = 690$ GeV



The grey region represents the limit where the old approach is not trustable anymore

Solutions of ξ_w for different Λ



Thank you for your attention!

Non-singular solutions to the Boltzmann equation with a fluid Ansatz

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Motivation

Cosmological first-order phase transitions can produce potential signals (GWs, baryogenesis) via bubble nucleation and plasma dynamics. These signals strongly depend on the terminal velocity of these bubbles.

Previous computation of this velocity using Boltzmann equation [1,2]

$$p^\mu \partial_\mu f_i(x^\mu, p^\mu) + m P^\mu \partial_\mu f_i(x^\mu, p^\mu) + C[f_i] = 0$$

and a fluid Ansatz have shown the arising of an unphysical singularity in the friction acting on the bubble wall at the speed of sound c_s .

Why the singularity?

Interplay between energy-momentum conservation and a zero eigenvalue in the kinetic matrices at $v_w = c_s$.

Two linear combinations of moments of the Boltzmann equations correspond to a linearized version of the energy-momentum conservation [3]

$$\chi \cdot A \cdot \Delta q = \chi \int S ds \propto \sum_{\text{particle}} m_i^2 T^2$$

This means that there will be two quantities representing a collective shift in the local temperature and velocity that can be constrained for example using

$$\delta t_{\text{light}} = \sum_i \frac{N_i m_i^2 c_i^2}{2T^2 \sum c_i} \frac{3v_w}{1 - 3c_s^2}$$

Hydrodynamics

The study of the hydrodynamics of cosmological first order phase transition gives us three different classes of solutions [4]:

- Deflagrations:** $v_w < c_s$, shock wave ahead.
- Hybrids:** $c_s < v_w < v_J$, shock wave ahead and rarefaction waves.
- Detonations:** $v_w > v_J$, rarefaction wave only.

Method in a nutshell

Idea: Space-dependent background around the wall defined by its temperature and velocity profiles determined by non-linear hydrodynamics.

Linearization of the Boltzmann equations around this background:

- Background from energy-momentum conservation (including scalar):

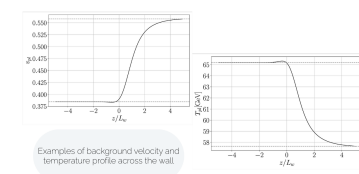
$$v_w^2 \gamma_w^2 \omega_{\text{bg}} - \mathcal{F}_{\text{bg}} + \frac{1}{2} (\partial_x \phi)^2 = k_1,$$

$$v_w \gamma_w^2 \omega_{\text{bg}} = k_2.$$
- New Liouville terms in Boltzmann Equations:

$$p^\mu \partial_\mu f_i^{\text{bg}}(x, p) = f_i^{\text{bg}} \frac{p^\mu p^\nu}{T} \left(u_\nu \frac{\partial_\mu T}{T} - \partial_\mu u_\nu \right)$$

makes energy-momentum tensor unsourced.

- Light-species deviations vanish asymptotically, only genuine non-equilibrium components remain.



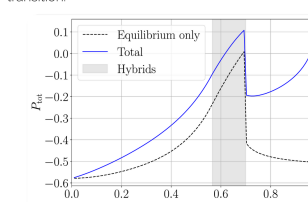
Benchmark model

the SM with a low cutoff:

$$\mathcal{F}(\phi, T) = V_0(\phi) - \frac{c}{2} T^4 + \frac{c'}{2} \phi^2 T^2$$

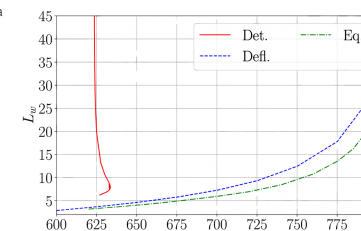
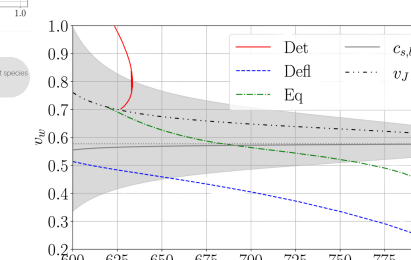
$$V_0(\phi) = -\frac{\mu^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4 + \frac{1}{8\Lambda^4} \phi^8$$

- Λ controls the strength of the PT, α_n and is varied in the range for which the PT is 1st order.
- Only mean field term needed to obtain a 1st order transition.



Main results

- Singularity removed:** energy-momentum is sourced in the Boltzmann equations after treating non-linearly the background and linearising the theory properly around it.
- Physical jump at v_J :** persists due to a change in the boundary conditions, as expected.
- Non-equilibrium effects:** increase pressure on the wall, they reduce Φ_w of 26-45% and increase L_w of 10-25%.
- Deflagrations** are found across all Λ considered.
- Stable detonations** are found, but only in a small range of the cutoff.

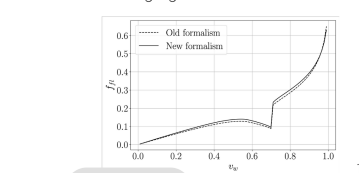
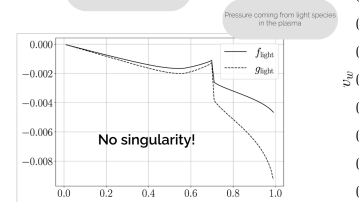
Higgs EOM & Pressure budget

The Higgs EOM


$$\square \phi + \frac{dV}{d\phi} + \sum_i \frac{dm_i^2}{d\phi} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_i} (f_i^m + f_i) = 0$$

is solved for finding the actual value of v_w looking for a pressure balance on the expanding wall.

- Equilibrium backreaction on the wall:** entropy conserving [5].
- Out-of-equilibrium fluctuations:** mainly from top and electroweak gauge bosons.

No singularity!



References:

[1] Konstandin et al., DOI: 10.1088/1475-2875/2013/09/028
 [2] Dorsch et al., DOI: 10.1088/1475-2875/2014/04/020
 [3] Dorsch et al., DOI: 10.1088/1475-2875/2015/05/028
 [4] Espinosa et al., DOI: 10.1088/1475-2875/2010/06/028
 [5] Stebbins et al., DOI: 10.1088/1475-2875/2010/09/028
 [6] Griest et al., DOI: 10.1088/1475-2875/2010/09/028

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The wall velocity ξ_w

Solving the Boltzmann Equation (BE)

The dynamics of the particles in the plasma can be described by the BE

$$p^\mu \partial_\mu f_i(x^\mu, p^\mu) + \frac{1}{2} \partial_\mu m^2 \partial_{p^\mu} f_i(x^\mu, p^\mu) + \mathcal{C}_i = 0,$$

The source term drives particles out of equilibrium

Collision terms, couples the different species in the plasma

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We transform this integro-differential equation into a set of ODEs by:

1. Ansatz on distributions: $f_i(p^\mu, x^\mu) = f_{\text{eq},i} + \delta f_i(p^\mu, x^\mu)$ and linearisation

2. Taking momenta: $\int \frac{d^3 p}{(2\pi)^3 E}$, $\int \frac{d^3 p}{(2\pi)^3 E} p_\mu u^\mu$, $\int \frac{d^3 p}{(2\pi)^3 E} p_\mu \bar{u}^\mu$, ...

[1407.3132] Konstandin et al.

The wall velocity ξ_w

The fluid Ansatz

$\delta\mu$ repr. chemical pot. fluctuations

δu_μ repr. velocity fluctuations

δT repr. temperature fluctuations

In the case of the *fluid Ansatz* we can write $\delta f = f' \delta_p$, with

$\delta_p = \delta\mu + p^\mu (\delta u_\mu - u_\mu \delta T/T)$ and solve these BEs for the 3 fluctuations.

Inserting the fluctuations in the linearized version of the Klein-Gordon equation we can find the values of the wall velocity ξ_w and of its width L_w .

$$-\phi'' + \frac{\partial \mathcal{F}}{\partial \phi} + \sum_i \frac{dm_i^2}{d\phi} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_i} \delta f_i(p, x) = 0$$

$$1. \quad \int dz [\text{l.h.s of KG}] \times \phi' = 0$$

$$f_{\text{fl}} \equiv \frac{N_t}{2T_+^2} \int dz \frac{dm_t^2}{dz} (c_{f1} \delta\mu_f + c_{f2} \delta\tau_f) + \frac{N_W}{2T_+^2} \int dz \frac{dm_W^2}{dz} (c_{b1} \delta\mu_b + c_{b2} \delta\tau_b)$$

$$f_{\text{light}} \equiv \frac{N_t}{2T_+^2} \int dz \frac{dm_t^2}{dz} c_{f2} \delta\tau_{\text{light}} + \frac{N_W}{2T_+^2} \int dz \frac{dm_W^2}{dz} c_{b2} \delta\tau_{\text{light}}$$

$$2. \quad \int dz [\text{l.h.s of KG}] \times \phi' (2\phi - \phi_0) = 0$$