Limits of EFTs at finite temperature for strong phase transitions

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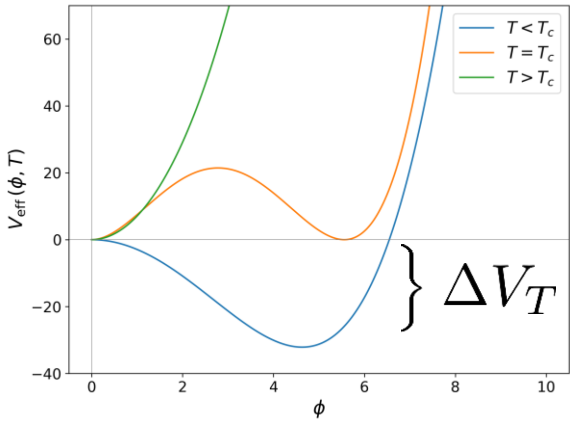
Effective Potential



At critical point **thermal corrections** are as large as **zero** temperature contribution

$$\frac{\Delta V_T}{V_{\rm LO}} \sim 1$$

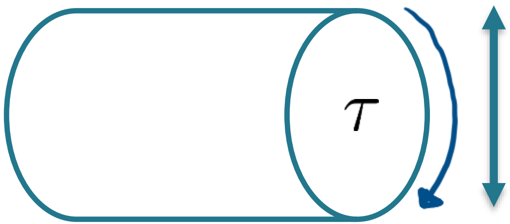
large theoretical uncertainties

$$\frac{\Delta V_T}{V_{\rm LO}} \sim \frac{g^2 T^2}{m^2} \sim 1 \longrightarrow T \gg m$$

Imaginary time formalism

$$\mathcal{Z}_{\rm th} = {\rm Tr}\left(e^{-\beta \mathcal{H}}\right) = \sum_{\Phi} \langle \Phi | \, e^{-\beta \mathcal{H}} \, | \Phi \rangle$$

$$= \mathcal{N} \int_{\rm b.c.} \mathcal{D}\Phi \, \exp(-S^E_\beta)$$



$$\sim \beta = 1/T$$

$$\mathbb{R}^3 \times S^1$$

The EFT approach

Matsubara modes ($\sim$ Kaluza-Klein modes) $$M_n^2 = m^2 + 4\pi^2 n^2 T^2$$

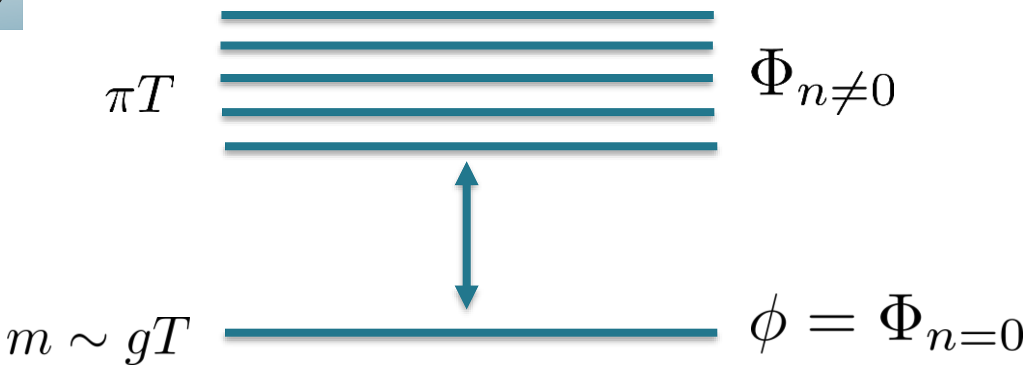
(tree level) **bosons' masses**

Idea! : **integrate out hard modes**

$$\mathcal{L}_{4d}(\Phi(x,\tau))$$

DIMENSIONAL REDUCTION (DR)

$$\mathcal{L}_{3d}(\phi(x))$$



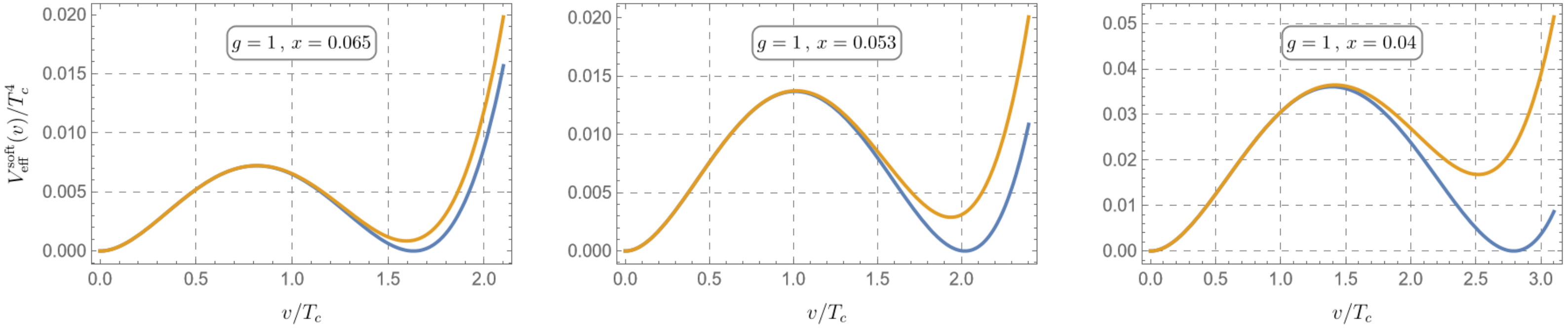
Resummation of large logarithms $\sim \log\left(\frac{m}{T}\right)$
reduces theoretical uncertainties

Limits of the EFTs

The **EFT** is valid as far as we have convergence in the operator expansion

What's the size of **Higher Dimensional Operators** ?

$$V_{\rm eff}^{\rm softer}(v) = \frac{1}{2}\bar{\mu}_3^2\, v^2 - \frac{1}{6\pi}\,\bar{g}_3^3\, v^3 + \frac{1}{4}\bar{\lambda}_3\, v^4 + \frac{1}{8}\bar{c}_6\, v^6, \qquad v^2 \equiv \phi\,\phi^\dagger/2,$$



Valid EFT

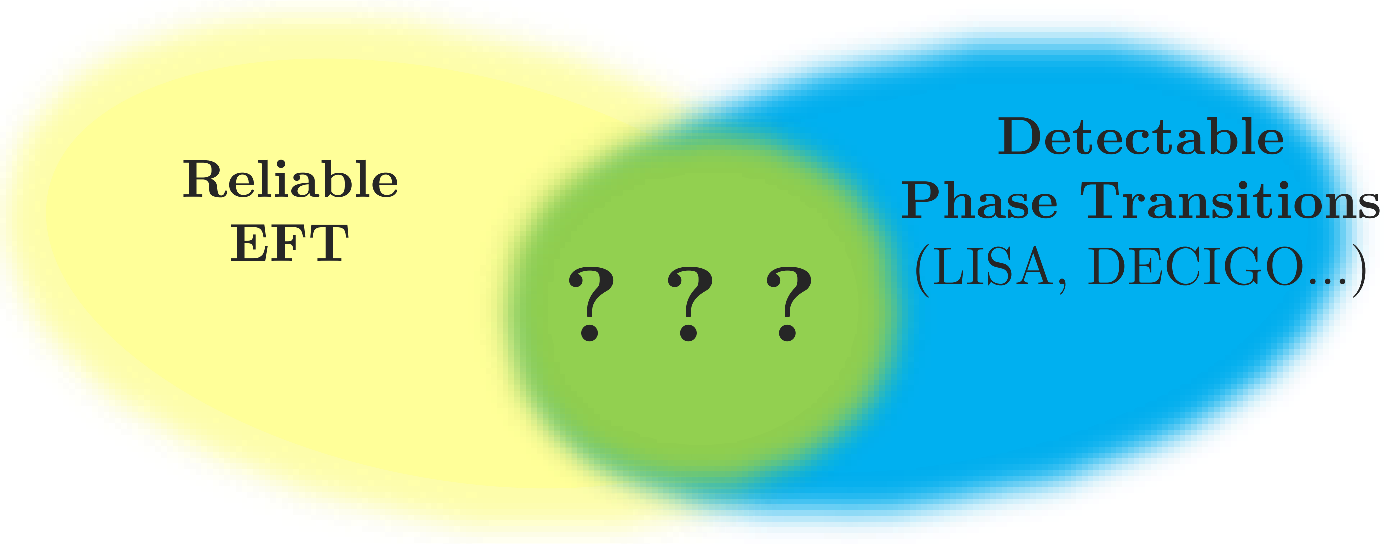
without $c_6 v^6$ — with $c_6 v^6$

Invalid EFT (?)

PT too weak to be detected

PT strong enough to be detected

Limits of the EFTs



Does it exists a regime where **we can use EFT** to study Phase Transitions which are **strong enough to be detected**?

Abelian Higgs Model

$$\mathcal{L}^{4d}_{\rm AH} = \frac{1}{4}F_{\mu\nu}F_{\mu\nu} + (D_\mu\phi)(D_\mu\phi^\dagger) + \mu^2(\phi\phi^\dagger) + \lambda(\phi\phi^\dagger)^2$$

$$B_\mu = \begin{pmatrix} B_0 \\ B_i \end{pmatrix}$$

→ B_0 **temporal scalar**

→ B_i **3d gauge vector**

$$\frac{1}{2}m_D^2\,B_0^2$$

Debye mass

$$\mathcal{L}^{3d}_{\rm soft\ (DR)} = \mathcal{L}^{3d}_{\rm AH} + \frac{1}{2}(\partial_i B_0)^2 + \frac{1}{2}m_D^2 B_0^2 + \kappa_3(\phi_3^\dagger\phi_3)B_0^2 + h_3 B_0^4$$

$$+ \text{ Higher Dimensional operators}$$

ϕ : Higgs

B_μ : Abelian gauge field

g : gauge coupling

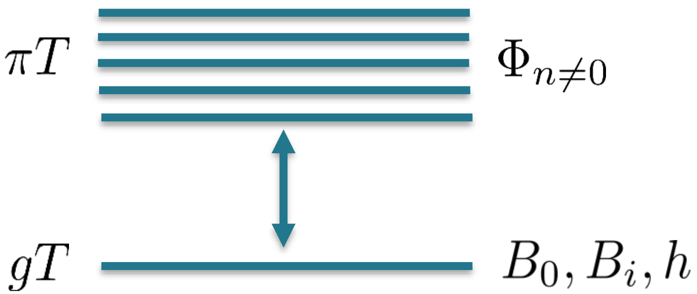
λ : Higgs quartic coupling

$x \equiv \lambda/g^2$

A second EFT

Masses (at T_c) $\quad m^2_{B_0} = m^2_D \sim g^2 T^2 \quad m^2_{B_i} \sim g_3^2 v_3^2 \quad m^2_h \sim \lambda_3 v_3^2$

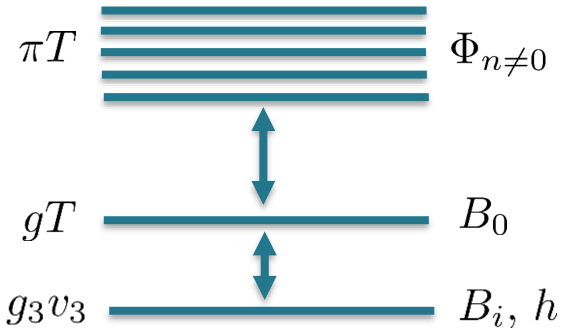
SOFT (DR)



if $v_3 \ll T^{1/2}$

$$m_{B_i}\,,\, m_h \ll m_{B_0}$$

SOFTER (DR+B_0)



we can **integrate out** B_0

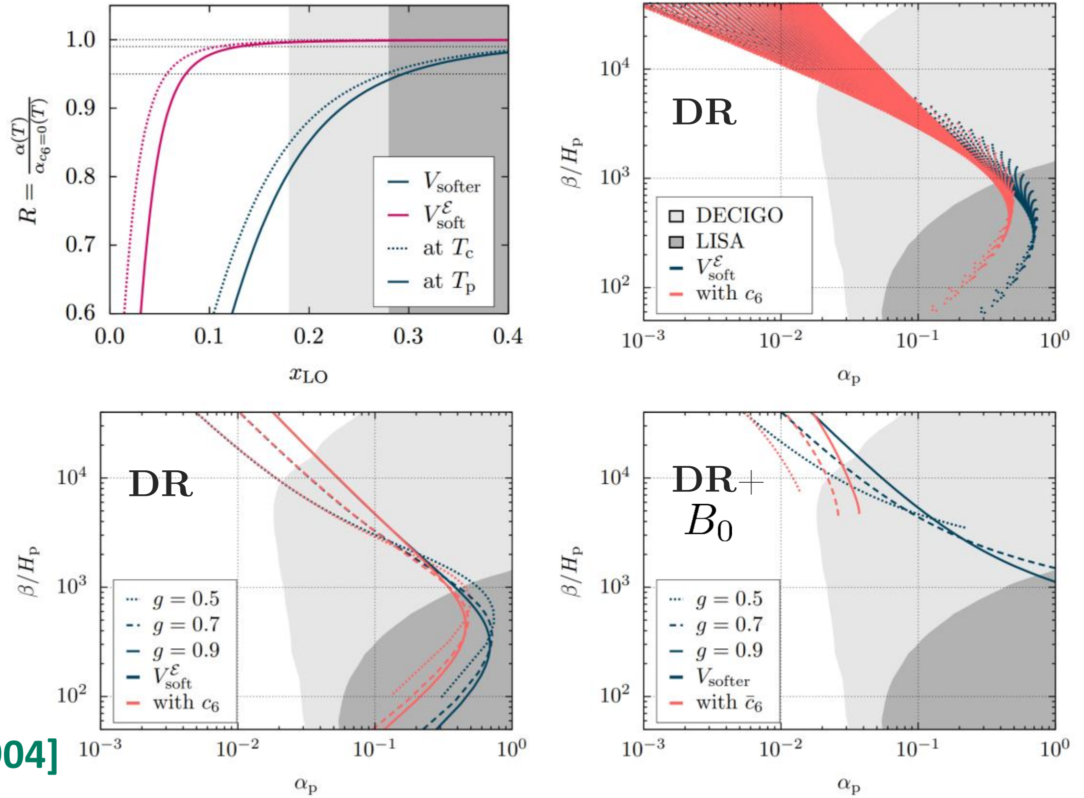
$$\mathcal{L}^{3d}_{\text{softer (DR+ } B_0)} = \mathcal{L}^{3d}_{\text{AH}} + \text{Higher Dimensional operators}$$

Results

EFTs **can fail** in describing **stronger PT**, which are the ones that can generate a GW signal intense enough to be detected (for example with **LISA**)

DR approach still valid but requires **Higher Dimensional Operators**

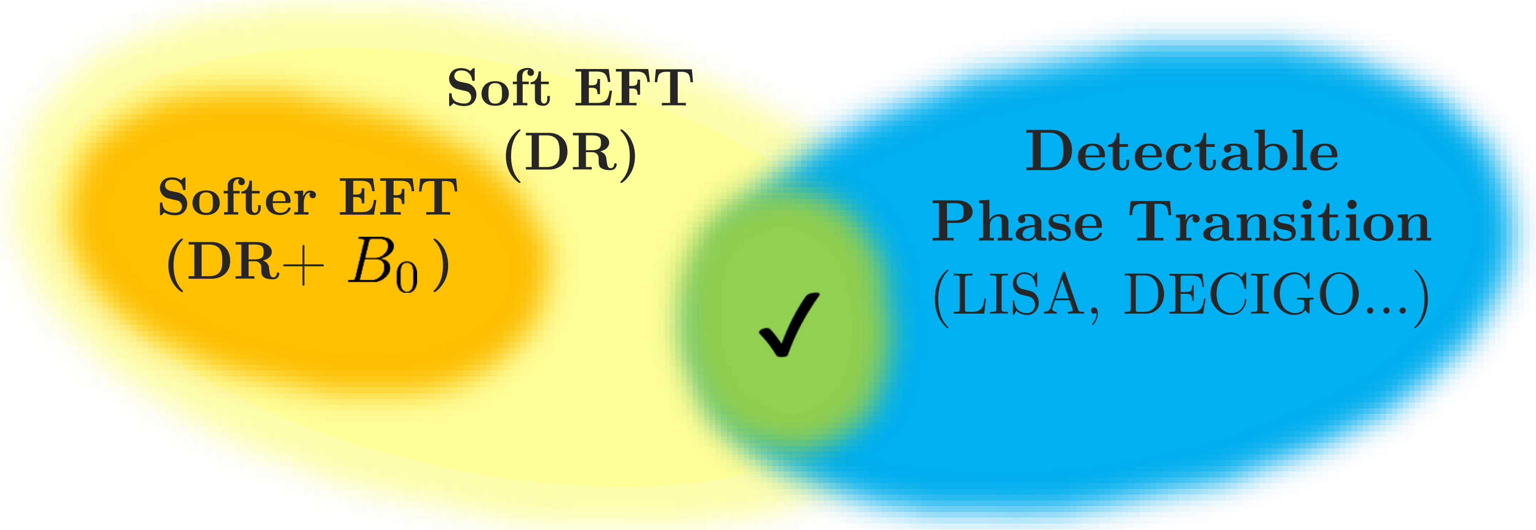
DR+B_0 approach **invalid** for **strong PT**



[**FB,** Klose, Schicho, Tenkanen ’25- **2503.18904**]

Results

Which other approaches are **still valid** for **detectable Phase Transitions**? (e.g. 3d/4d Lattice, Partial Dressing, No high-T expansion?)



THANKS FOR YOUR ATTENTION

BACKUP SLIDES

Gauge dependence in higher dimensional operators

Naive Correlators Matching leads to gauge dependent Wilson Coefficients

[Croon, Gould, Schicho, Tenkanen, White et al. ’20 - **2009.10080**]

$$\widehat{\alpha}_{\phi^6}(\phi_3^\dagger\phi_3)^3$$

$$\widehat{\alpha}_{\phi^6} = \frac{4}{3}\left[dg^6 - 3\xi g^2\lambda^2 + 28\lambda^3\right]\mathcal{I}_3 T^2$$

Apparent gauge dependence is related to the **redundancy** of the EFT operator basis

Field redefinitions

$$\alpha_{\phi^6} = \frac{4}{15}\left[5dg^6 - (5d+3)g^4\lambda + 75g^2\lambda^2 + 100\lambda^3\right]\mathcal{I}_3 T^2$$

Gauge Independence is **manifest** once the redundancy is removed

[**FB,** Klose, Schicho, Tenkanen ’25- **2503.18904**]

[Chala, Guedes ’25-**2503.20016**] for EW sector of SM

FIELD REDEFINITIONS

1. **Gauge dependence** in Higher Dimensional Operators

dimension-six Redundant operator basis	
$F_{ij}F_{ij}B_0^2$	$(\partial_i F_{ij})^2$
$F_{ij}F_{ij}\phi^\dagger\phi$	$(\partial^2 B_0)^2$
$(D_i\phi^\dagger D_i\phi)(\phi^\dagger\phi)$	$B_0^3\partial^2 B_0$
$(D_i\phi^\dagger D_i\phi)B_0^2$	$(D^2\phi^\dagger)(D^2\phi)$
B_0^6	$(\phi^\dagger\phi)(\phi^\dagger D^2\phi + h.c.)$
$B_0^4(\phi^\dagger\phi)$	$(\partial_i F_{ij})i\phi^\dagger(D_j\phi)$
$B_0^2(\phi^\dagger\phi)^2$	$(\phi^\dagger\phi)B_0\partial^2 B_0$
$(\phi^\dagger\phi)^3$	$(\phi^\dagger D^2\phi)B_0^2 + h.c.$

Field redefinitions →

dimension-six Physical basis
$F_{ij}F_{ij}B_0^2$
$F_{ij}F_{ij}\phi^\dagger\phi$
$(D_i\phi^\dagger D_i\phi)(\phi^\dagger\phi)$
$(D_i\phi^\dagger D_i\phi)B_0^2$
B_0^6
$B_0^4(\phi^\dagger\phi)$
$B_0^2(\phi^\dagger\phi)^2$
$(\phi^\dagger\phi)^3$

$$\widehat{\alpha}_{\phi^6} = \frac{4}{3}\left[dg^6 - 3\xi g^2\lambda^2 + 28\lambda^3\right]\mathcal{I}_3\, T^2 \longrightarrow \alpha_{\phi^6} = \frac{4}{15}\Big[5dg^6 - (5d+3)g^4\lambda + 75g^2\lambda^2 + 100\lambda^3\Big]\mathcal{I}_3\, T^2$$