

Vector-like quark doublets, weak-basis invariants and CP violation

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Vector-like quark doublets, weak-basis invariants and CP violation

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Motivation

Why Vector-Like Quarks?

CP violation in the SM quark sector is not enough to explain **baryon asymmetry**

$$\text{tr} \left[y_u y_u^\dagger, y_d y_d^\dagger \right]^3 \propto J \ll 1$$

New **complex couplings** may arise if we **introduce new quark fields**



New Sources of CP Violation?

A **fourth chiral quark** generation is **ruled out**, but **VLQs*** are **still experimentally viable**.

* **“Vector-like”** \Rightarrow LH and RH components have the same quantum numbers

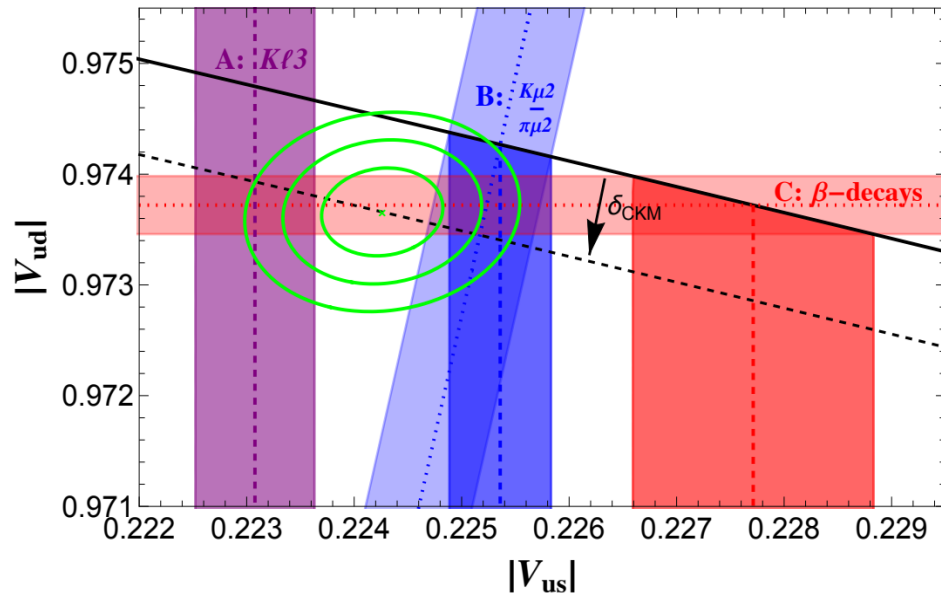
Motivation

Why Vector-Like Quark **Doublets**?

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix} \quad Q_R = \begin{pmatrix} U_R \\ D_R \end{pmatrix}$$

Models with $SU(2)_L$ doublets $(\mathbf{3}, \mathbf{2})_{1/6}$ are the **avored candidates in explaining the Cabibbo Angle Anomalies***.

Belfatto & Trifinopoulos [2302.14097]



A $K\ell 3$: $|V_{us}|_A = 0.22308(55)$

B $K\mu 2/\pi\mu 2$: $|V_{us}|/|V_{ud}|_B = 0.23126(48)$

C β -decays: $|V_{ud}|_C = 0.97372(26)$

(**A+B**) vs **C** at 3σ

* CAA1: $\delta_{\text{CKM}} \equiv 1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2 \approx 1.60(0.53) \times 10^{-3}$

A vs **B** at 3.1σ

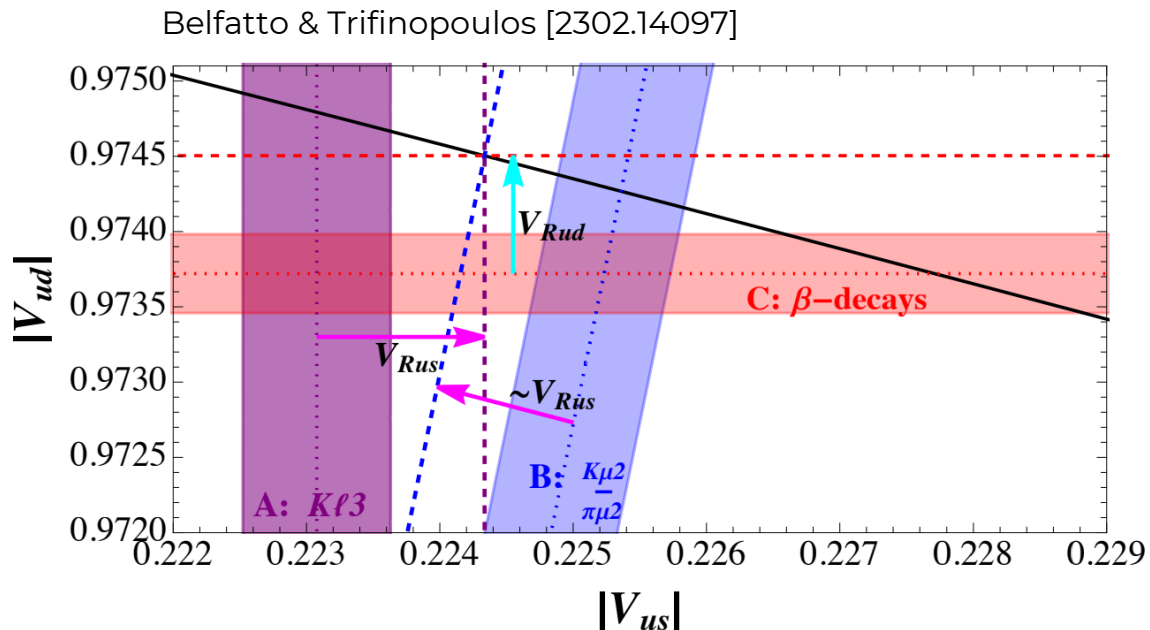
* CAA2: $|V_{us}|_A = 0.22308(55)$ $|V_{us}|_B = 0.225313(47)$

Motivation

Why Vector-Like Quarks **Doublets**?

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix} \quad Q_R = \begin{pmatrix} U_R \\ D_R \end{pmatrix}$$

Models with $SU(2)_L$ doublets $(\mathbf{3}, \mathbf{2})_{1/6}$ are the **avored candidates in explaining the Cabibbo Angle Anomalies.**



$$\mathcal{L}_{cc} \supset -\frac{g}{2\sqrt{2}} W_\mu^+ \bar{u} [\gamma^\mu \boxed{(V_L + V_R)} - \gamma^\mu \gamma_5 \boxed{(V_L - V_R)}] d$$

Vector coupling Axial coupling

Vector coupling $Kl3 : |V_{us}|_A = |V_{us}^L + V_{us}^R|$

Axial coupling $K\mu 2/\pi\mu 2 : \left| \frac{V_{us}}{V_{ud}} \right|_B = \left| \frac{V_{us}^L - V_{us}^R}{V_{ud}^L - V_{ud}^R} \right|$

Vector coupling $\beta - \text{decays} : |V_{ud}|_C = |V_{ud}^L + V_{ud}^R|$

New Sources of CPV?

New CP-odd **Weak Basis Invariants***?

*Quantities blind to arbitrary choice of WB

In the SM, CP violation hinges solely on one WBI $\text{tr} [h_u, h_d]^3 \propto J$

Now we have larger mass matrices and more Hermitian “building blocks”

$$h_q = m_q m_q^\dagger = \mathcal{H}_q - H$$

$$H = M M^\dagger$$

$$\mathcal{H}_q = \mathcal{M}_q \mathcal{M}_q^\dagger$$

$$\mathcal{M}_q = \left(\begin{array}{c|c} \overbrace{\quad}^3 & \overbrace{\quad}^N \\ \hline m_q & M \end{array} \right) \Bigg\}^{3+N}$$

Yukawa couplings Bare-mass terms

Relevant WBI structures fit into 4 structures:

3-blocks: $I(n + m, \mathbf{3}) \equiv \text{ImTr} (\mathcal{H}_u^n \mathcal{H}_d^m H)$

4-blocks: $I(n + m + p, \mathbf{4})_u \equiv \text{ImTr} (\mathcal{H}_u^n \mathcal{H}_d^p \mathcal{H}_u^m H)$
 $I(n + m + p, \mathbf{4})_d \equiv \text{ImTr} (\mathcal{H}_d^n \mathcal{H}_u^p \mathcal{H}_d^m H)$

5-blocks: $I(n + m + p + q, \mathbf{5}) \equiv \text{ImTr} (\mathcal{H}_u^n \mathcal{H}_d^p \mathcal{H}_u^m \mathcal{H}_d^q H)$

SM - like: $I(12, \mathbf{4}) \equiv \text{ImTr} (\mathcal{H}_u^2 \mathcal{H}_d^2 \mathcal{H}_u \mathcal{H}_d)$

CP invariance would require the vanishing of up to 23 invariants

New Sources of CPV?

Even at extremely high energies?

In the **limit of extremely high energies** (extreme chiral limit) where $\mathbf{m}_u = \mathbf{m}_c = \mathbf{m}_d = \mathbf{m}_s = \mathbf{0}$, there is **no CPV in the SM**.

With **one VLQ doublet**, in this limit the mass matrices reduce to the minimal form:

$$\mathcal{M}_u = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \hat{y}_t v & 0 \\ 0 & z_c v & z_t v & M_Q \end{pmatrix} \quad \mathcal{M}_d = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & y_{23} v & 0 \\ 0 & 0 & y_{33} v & 0 \\ 0 & z_s v & \boxed{z_b v} & M_Q \end{pmatrix}$$

complex

CP violation may be present in this limit, as in general **one CP-odd WBI survives**

$$\frac{1}{M_Q^6} \text{ImTr} [\mathcal{H}_u \mathcal{H}_d H] = \frac{v^4}{M_Q^4} \text{Im} (\hat{y}_t y_{33} z_t z_b^*) \simeq \frac{m_t m_b}{M_Q^2} \text{Im} (V_{tb}^L V_{tb}^{R*})$$

New Sources of CPV?

Imprints on observables?

The **structure of CP-odd WBIs** are uniquely connected **types of effective* rephasing invariants**

3-blocks: $\frac{1}{M_Q^6} \text{ImTr}(\mathcal{H}_u \mathcal{H}_d H) \simeq \frac{v^2}{M_Q^2} y_\alpha y_i \text{Im}(\hat{V}_{\alpha i}^L \hat{V}_{\alpha i}^{*R})$ **bilinear** *effective \equiv involving only SM quarks

4-blocks: $\frac{1}{M_Q^{10}} \text{ImTr}(\mathcal{H}_d \mathcal{H}_u \mathcal{H}_d^2 H) \simeq (\dots) + \frac{v^8}{M_Q^8} y_\alpha^4 y_i^3 y_j \text{Im}(\hat{V}_{\alpha i}^{L*} \hat{V}_{\alpha j}^{L*} \hat{F}_{ij}^d)$ **trilinear** $F^d \equiv V_R^\dagger V_R$

5-blocks: $\frac{1}{M_Q^{10}} \text{ImTr}(\mathcal{H}_d \mathcal{H}_u \mathcal{H}_d \mathcal{H}_u H) \simeq (\dots) + \frac{v^6}{M_Q^6} y_\alpha y_\beta^2 y_i y_j^2 \text{Im}(\hat{V}_{\alpha i}^L \hat{V}_{\beta j}^L \hat{V}_{\beta i}^{R*} \hat{V}_{\alpha j}^{L*})$ **exotic-quartet**

SM-like: $\frac{1}{M_Q^{12}} \text{ImTr}(\mathcal{H}_d^2 \mathcal{H}_u^2 \mathcal{H}_d \mathcal{H}_u) \sim (\dots) + \frac{v^{12}}{M_Q^{12}} \text{Im}(\hat{V}_{\alpha i}^L \hat{V}_{\beta j}^L \hat{V}_{\beta i}^{L*} \hat{V}_{\alpha j}^{L*})$ **SM-like-quartet**

We can connect **rephasing invariants** to **CP-violating observables**



Summary

- Extension with VLQs doublets provide the most promising solutions to the CAAs.
- A wide variety of possible new sources of CPV arise in these models. This is manifested via the appearance of new types of rephasing and weak basis invariants.
- Some of these sources may allow for CPV at extremely high energies.
- We established a direct and unambiguous connection between WBIs and effective rephasing invariants and thus to flavor observables.

Thank You!

Extensions with VLQ iso-doublets

Introducing N VLQ iso-doublets with q.n.s $(\mathbf{3}, \mathbf{2})_{1/6}$

$$Q_{L\alpha}^0 = \begin{pmatrix} U_{L\alpha}^0 \\ D_{L\alpha}^0 \end{pmatrix}, \quad Q_{R\alpha}^0 = \begin{pmatrix} U_{R\alpha}^0 \\ D_{R\alpha}^0 \end{pmatrix} \quad (\alpha = 1, \dots, N)$$

leads to

$$\begin{aligned}
 -\mathcal{L} = & \underbrace{(Y_u)_{ij} \bar{q}_{Li}^0 \tilde{\Phi} u_{Rj}^0 + (Y_d)_{ij} \bar{q}_{Li}^0 \Phi d_{Rj}^0 + \text{h.c.}}_{\text{Yukawa couplings}} \left. \vphantom{\begin{aligned} -\mathcal{L} = \\ + \\ + \end{aligned}} \right\} \text{SM} \\
 & + (Z_u)_{\alpha j} \bar{Q}_{L\alpha}^0 \tilde{\Phi} u_{Rj}^0 + (Z_d)_{\alpha j} \bar{Q}_{L\alpha}^0 \Phi d_{Rj}^0 + \text{h.c.} \left. \vphantom{\begin{aligned} -\mathcal{L} = \\ + \\ + \end{aligned}} \right\} \text{NP} \\
 & + (\bar{M})_{i\beta} \bar{q}_{Li}^0 Q_{R\beta}^0 + (M_Q)_{\alpha\beta} \bar{Q}_{L\alpha}^0 Q_{R\beta}^0 + \text{h.c.}, \\
 & \underbrace{\hspace{10em}}_{\text{Bare-mass terms (sector independent)}}
 \end{aligned}$$

or using $Q_L^0 = \begin{pmatrix} q_L^0 \\ Q_L^0 \end{pmatrix}$

$$-\mathcal{L} = \underbrace{\bar{Q}_L^0 \tilde{\Phi} \mathcal{Y}_u u_R^0 + \bar{Q}_L^0 \Phi \mathcal{Y}_d d_R^0}_{\text{Yukawa couplings}} + \underbrace{\bar{Q}_L^0 M Q_R^0}_{\text{Bare-mass terms}} + \text{h.c.}$$

Extensions with VLQ iso-doublets

After SSB, one obtains

$$-\mathcal{L}_m = \overline{U}_L^0 \mathcal{M}_u U_R^0 + \overline{\mathcal{D}}_L^0 \mathcal{M}_d \mathcal{D}_R^0 + \text{h.c.} \quad u_{L,R}^0 = \begin{pmatrix} u_{L,R}^0 \\ U_{L,R}^0 \end{pmatrix}, \quad \mathcal{D}_{L,R}^0 = \begin{pmatrix} d_{L,R}^0 \\ D_{L,R}^0 \end{pmatrix},$$

with

$$\mathcal{M}_q = \left(\begin{array}{c|c} \overbrace{\quad}^3 & \overbrace{\quad}^N \\ \hline m_q & M \end{array} \right) \Bigg\}^{3+N} \quad m_q \equiv v \mathcal{Y}_q = v \begin{pmatrix} Y_q \\ Z_q \end{pmatrix}, \quad M \equiv \begin{pmatrix} \overline{M} \\ M_Q \end{pmatrix}$$

We can now diagonalize the mass matrices by transforming into the mass eigenstates

$$u_{L,R}^0 = \mathcal{V}_{L,R}^u u_{L,R} = \mathcal{V}_{L,R}^u \begin{pmatrix} u_{L,R} \\ U_{L,R} \end{pmatrix}, \quad \mathcal{D}_{L,R}^0 = \mathcal{V}_{L,R}^d \mathcal{D}_{L,R} = \mathcal{V}_{L,R}^d \begin{pmatrix} d_{L,R} \\ D_{L,R} \end{pmatrix}$$

$$-\mathcal{L}_m = \overline{u}_L \mathcal{D}_u u_R + \overline{\mathcal{D}}_L \mathcal{D}_d \mathcal{D}_R + \text{h.c.}$$

$$\mathcal{V}_L^{q\dagger} \mathcal{M}_q \mathcal{V}_R^q = \mathcal{D}_q \equiv \text{diag}(m_1^q, m_2^q, m_3^q, M_1^q, \dots, M_N^q)$$

$$\mathcal{V}_{L,R}^q = \left(\begin{array}{c} A_{L,R}^q \\ \hline B_{L,R}^q \end{array} \right) \Bigg\} \begin{array}{l} 3 \\ N \\ \hline 3+N \end{array}$$

Extensions with VLQ iso-doublets

As for the gauge interactions we have

$$\begin{aligned} \mathcal{L}_W &= -\frac{g}{\sqrt{2}} W_\mu^+ \left[\overline{U}_L^0 \gamma^\mu \mathcal{D}_L^0 + \overline{U}_R^0 \gamma^\mu D_R^0 \right] + \text{h.c.} && \left. \vphantom{\mathcal{L}_W} \right\} \text{flavor basis} \\ &= -\frac{g}{\sqrt{2}} W_\mu^+ \left[\overline{U}_L \gamma^\mu V_L \mathcal{D}_L + \overline{U}_R \gamma^\mu V_R \mathcal{D}_R \right] + \text{h.c.}, && \left. \vphantom{\mathcal{L}_W} \right\} \text{mass basis} \end{aligned}$$

$$\mathcal{V}_{L,R}^q = \left(\begin{array}{c} A_{L,R}^q \\ \dots \\ B_{L,R}^q \end{array} \right) \left. \vphantom{\mathcal{V}_{L,R}^q} \right\} \begin{array}{l} 3 \\ N \end{array}$$

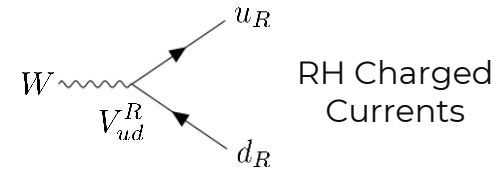
3+N

$$V_L = \mathcal{V}_L^{u\dagger} \mathcal{V}_L^d$$

Unitary

$$V_R = B_R^{u\dagger} B_R^d = \mathcal{V}_R^{u\dagger} \text{diag}(0, 0, 0, 1, \dots, 1) \mathcal{V}_R^d$$

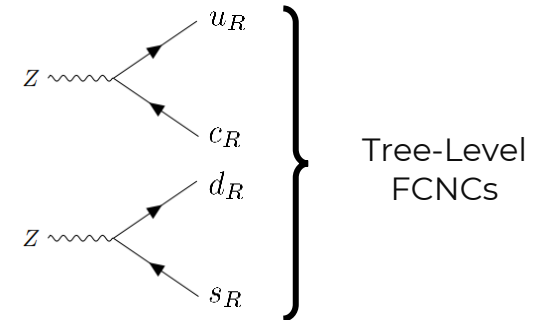
Non-Unitary



$$\begin{aligned} \mathcal{L}_Z &= -\frac{g}{2c_W} Z_\mu \left[\overline{U}_L^0 \gamma^\mu U_L^0 - \overline{\mathcal{D}}_L^0 \gamma^\mu \mathcal{D}_L^0 + \overline{U}_R^0 \gamma^\mu U_R^0 - \overline{D}_R^0 \gamma^\mu D_R^0 - 2s_W^2 J_{\text{em}}^\mu \right] && \left. \vphantom{\mathcal{L}_Z} \right\} \text{flavor basis} \\ &= -\frac{g}{2c_W} Z_\mu \left[\overline{U}_L \gamma^\mu U_L - \overline{\mathcal{D}}_L \gamma^\mu \mathcal{D}_L + \overline{U}_R \gamma^\mu F_u U_R - \overline{\mathcal{D}}_R \gamma^\mu F_d \mathcal{D}_R - 2s_W^2 J_{\text{em}}^\mu \right] && \left. \vphantom{\mathcal{L}_Z} \right\} \text{mass basis} \end{aligned}$$

FCNCs

$$\begin{aligned} F_u &= B_R^{u\dagger} B_R^u = V_R V_R^\dagger = \mathcal{V}_R^{u\dagger} \text{diag}(0, 0, 0, 1, \dots, 1) \mathcal{V}_R^u \\ F_d &= B_R^{d\dagger} B_R^d = V_R^\dagger V_R = \mathcal{V}_R^{d\dagger} \text{diag}(0, 0, 0, 1, \dots, 1) \mathcal{V}_R^d \end{aligned}$$



Approximate masses and mixing

The RH mixings can be written as:

$$V_R = \begin{pmatrix} z_u^* z_d \frac{v^2}{M_Q^2} & z_u^* z_s \frac{v^2}{M_Q^2} & z_u^* z_b \frac{v^2}{M_Q^2} & -z_u^* \frac{v}{M_Q} \\ z_c^* z_d \frac{v^2}{M_Q^2} & z_c^* z_s \frac{v^2}{M_Q^2} & z_c^* z_b \frac{v^2}{M_Q^2} & -z_c^* \frac{v}{M_Q} \\ z_t^* z_d \frac{v^2}{M_Q^2} & z_t^* z_s \frac{v^2}{M_Q^2} & z_t^* z_b \frac{v^2}{M_Q^2} & -z_t^* \frac{v}{M_Q} \\ -z_d \frac{v}{M_Q} & -z_s \frac{v}{M_Q} & -z_b \frac{v}{M_Q} & 1 - \frac{1}{2} (|z_u|^2 + |z_d|^2) \frac{v^2}{M_Q^2} \end{pmatrix} + \mathcal{O}\left(\frac{v^3}{M_Q^3}\right)$$

$$\hat{V}_{\alpha i}^R \equiv \frac{v^2}{M_Q^2} z_\alpha^* z_i$$

For the 3x3 submatrices

$$F_u = \begin{pmatrix} |z_u|^2 \frac{v^2}{M_Q^2} & z_u^* z_c \frac{v^2}{M_Q^2} & z_u^* z_t \frac{v^2}{M_Q^2} & -z_u^* \frac{v}{M_Q} \\ z_c^* z_u \frac{v^2}{M_Q^2} & |z_c|^2 \frac{v^2}{M_Q^2} & z_c^* z_t \frac{v^2}{M_Q^2} & -z_c^* \frac{v}{M_Q} \\ z_t^* z_u \frac{v^2}{M_Q^2} & z_t^* z_c \frac{v^2}{M_Q^2} & |z_t|^2 \frac{v^2}{M_Q^2} & -z_t^* \frac{v}{M_Q} \\ -z_u \frac{v}{M_Q} & -z_c \frac{v}{M_Q} & -z_t \frac{v}{M_Q} & 1 - |z_u|^2 \frac{v^2}{M_Q^2} \end{pmatrix} + \mathcal{O}\left(\frac{v^3}{M_Q^3}\right)$$

$$\hat{F}_{\alpha i}^u \equiv \frac{v^2}{M_Q^2} z_\alpha^* z_\beta$$

$$\hat{F}_{ij}^d \equiv \frac{v^2}{M_Q^2} z_i^* z_j$$

Approximate masses and mixing

While for the LH mixing we have:

$$V_L \simeq \begin{pmatrix} \mathbb{1}_{3 \times 3} & -\hat{Y}_u z_u^\dagger \frac{v^2}{M_Q^2} \\ z_u \hat{Y}_u \frac{v^2}{M_Q^2} & 1 \end{pmatrix} \begin{pmatrix} \hat{V}_L & \\ & 1 \end{pmatrix} \begin{pmatrix} \mathbb{1}_{3 \times 3} & \hat{Y}_d z_d^\dagger \frac{v^2}{M_Q^2} \\ -z_d \hat{Y}_d \frac{v^2}{M_Q^2} & 1 \end{pmatrix} + \mathcal{O}\left(\frac{v^4}{M_Q^4}\right)$$

$$\simeq \begin{pmatrix} \boxed{\hat{V}_L \simeq \hat{U}_{uL}^\dagger U_{dL}} & \mathcal{O}\left(yz \frac{v^2}{M_Q^2}\right) \\ \mathcal{O}\left(yz \frac{v^2}{M_Q^2}\right) & 1 \end{pmatrix}$$

For the 3x3 submatrix

$$\boxed{V_{\alpha i}^L = \hat{V}_{\alpha i}^L + \mathcal{O}\left(y^2 z^2 \frac{v^4}{M_Q^4}\right)}$$

\approx the SM CKM mixing

Contrary to the VLQ iso-singlet case, here the LH mixing is unitary and the **couplings outside the 3x3 block are suppressed naturally**.

The **CAAs cannot be addressed** solely with the **LH currents**.

Weak Basis Invariants

We find a larger number and variety of independent CP-odd WBIs, signaling **many more sources of CP violation**.

In the SM, CP violation hinges solely on one WBI $\text{tr} [h_u, h_d]^3 \propto J$

With **one VLQ doublet** alone we find a **much more complex situation**:

3-blocks: $I(n + m, \mathbf{3}) \equiv \text{ImTr} (\mathcal{H}_u^n \mathcal{H}_d^m H)$

4-blocks: $I(n + m + p, \mathbf{4})_u \equiv \text{ImTr} (\mathcal{H}_u^n \mathcal{H}_d^p \mathcal{H}_u^m H)$

$I(n + m + p, \mathbf{4})_d \equiv \text{ImTr} (\mathcal{H}_d^n \mathcal{H}_u^p \mathcal{H}_d^m H)$

5-blocks: $I(n + m + p + q, \mathbf{5}) \equiv \text{ImTr} (\mathcal{H}_u^n \mathcal{H}_d^p \mathcal{H}_u^m \mathcal{H}_d^q H)$

SM - like: $I(12, \mathbf{4}) \equiv \text{ImTr} (\mathcal{H}_u^2 \mathcal{H}_d^2 \mathcal{H}_u \mathcal{H}_d)$

$(q, q' = u, d \quad q \neq q')$

$\#z_{q_i}$	$\#z_{q'_\alpha}$	$\#\delta$	$\#\text{I}(M \geq 6, 3)$	$\#\text{I}(M \geq 8, 4)_q$	$\#\text{I}(M \geq 8, 4)_{q'}$	$\text{I}(10, 5)$	$\text{I}_{(12,4)}^{\text{SM-like}}$
3	3	6	9				
3	2	5	7				
2	2	4	4				
3	1	4	3	1			
2	1	3	2	1			
3	0	3	= 0	6	= 0	= 0	
2	0	2	= 0	2	= 0	= 0	
1	1	2	1			1	
1	0	1	= 0	= 0	= 0	= 0	1

Number and structure of **WBIs whose vanishing is needed for CPI** in each possible scenario

Rephasing Invariants

Under **general rephasing transformations** $\mathcal{D}_i \rightarrow \mathcal{D}_i e^{i\varphi_i}$ and $\mathcal{U}_\alpha \rightarrow \mathcal{U}_\alpha e^{i\varphi_\alpha}$ we have

$$V_{\alpha i}^L \rightarrow e^{-i(\varphi_\alpha - \varphi_i)} V_{\alpha i}^L$$

$$V_{\alpha i}^R \rightarrow e^{-i(\varphi_\alpha - \varphi_i)} V_{\alpha i}^R$$

$$F_{\alpha\beta}^u \rightarrow e^{-i(\varphi_\alpha - \varphi_\beta)} F_{\alpha\beta}^u$$

$$F_{ij}^d \rightarrow e^{-i(\varphi_i - \varphi_j)} F_{ij}^d$$

and we can build the following **complex rephasing invariants (RIs)**

Bilinears: $\mathcal{B}_{\alpha i} = V_{\alpha i}^L V_{\alpha i}^{R*}$

Trilinears: $\mathcal{T}_{i,\alpha\beta} = V_{\alpha i}^{L*} V_{\beta i}^{L*} F_{\alpha\beta}^u$

$\mathcal{T}_{\alpha,ij} = V_{\alpha i}^{L*} V_{\alpha j}^{L*} F_{ij}^d$

Quartets: $\mathcal{Q}_{\alpha i\beta j} = V_{\alpha i}^L V_{\beta j}^R V_{\alpha j}^{L*} V_{\beta i}^{L*}$

$\mathcal{Q}_{\alpha i\beta j} = V_{\alpha i}^L V_{\beta j}^L V_{\alpha j}^{L*} V_{\beta i}^{L*}$

SM-Like

For the 3x3
sector of SM

\simeq

Effective RIs

$$\hat{\mathcal{B}}_{\alpha i} \equiv \frac{v^2}{M_Q^2} z_\alpha^* z_i \hat{V}_{\alpha i}^{L*} \equiv \hat{V}_{\alpha i}^R \hat{V}_{\alpha i}^{L*}$$

$$\hat{\mathcal{T}}_{i,\alpha\beta} \equiv \frac{v^2}{M_Q^2} z_\alpha^* z_\beta \hat{V}_{\alpha i}^{L*} \hat{V}_{\beta i}^L \equiv \hat{F}_{\alpha\beta}^u \hat{V}_{\alpha i}^{L*} \hat{V}_{\beta i}^L$$

$$\hat{\mathcal{T}}_{\alpha,ij} \equiv \frac{v^2}{M_Q^2} z_i^* z_j \hat{V}_{\alpha i}^L \hat{V}_{\alpha j}^{L*} \equiv \hat{F}_{ij}^d \hat{V}_{\alpha i}^L \hat{V}_{\alpha j}^{L*}$$

$$\hat{\mathcal{Q}}_{\alpha i\beta j} \equiv \frac{v^2}{M_Q^2} z_\alpha^* z_i \hat{V}_{\beta j}^L \hat{V}_{\alpha j}^{L*} \hat{V}_{\beta i}^{L*} \equiv \hat{V}_{\alpha i}^R \hat{V}_{\beta j}^L \hat{V}_{\alpha j}^{L*} \hat{V}_{\beta i}^{L*}$$

$$\hat{\mathcal{Q}}_{\alpha i\beta j} \equiv \hat{V}_{\alpha i}^L \hat{V}_{\beta j}^L \hat{V}_{\alpha j}^{L*} \hat{V}_{\beta i}^{L*}$$

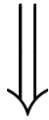
Bounds from Phenomenology

Bounds on **CP violating RIs** from flavor observables in the **limit where the mixing can address the CAAs**

$$|V_{ud}|_\beta = |\hat{V}_{ud}^L + \hat{V}_{ud}^R| \simeq |\hat{V}_{ud}^L| + \frac{\text{Re}\hat{\mathcal{B}}_{ud}}{|\hat{V}_{ud}^L|}$$

$$|V_{us}|_{K\ell 3} = |\hat{V}_{us}^L + \hat{V}_{us}^R| \simeq |\hat{V}_{us}^L| + \frac{\text{Re}\hat{\mathcal{B}}_{us}}{|\hat{V}_{us}^L|}$$

$$\left| \frac{V_{us}}{V_{ud}} \right|_{\frac{K\mu 2}{\pi\mu 2}} = \frac{|\hat{V}_{us}^L - \hat{V}_{us}^R|}{|\hat{V}_{ud}^L - \hat{V}_{ud}^R|} \simeq \frac{|\hat{V}_{us}^L|}{|\hat{V}_{ud}^L|} \left(1 - \frac{\text{Re}\hat{\mathcal{B}}_{us}}{|\hat{V}_{us}^L|^2} + \frac{\text{Re}\hat{\mathcal{B}}_{ud}}{|\hat{V}_{ud}^L|^2} \right)$$



$$\frac{\text{Re}\hat{\mathcal{B}}_{ud}}{|\hat{V}_{ud}^L|} = -0.79(27) \times 10^{-3}$$

$$\frac{\text{Re}\hat{\mathcal{B}}_{us}}{|\hat{V}_{us}^L|} = -1.24(37) \times 10^{-3}$$

Conventional name	Bounds on VLQ invariants
$ \omega $	$\frac{ \hat{V}_{us}^L \text{Re}\hat{\mathcal{B}}_{ud}}{ \hat{V}_{ud}^L } - \frac{ \hat{V}_{ud}^L \text{Re}\hat{\mathcal{B}}_{us}}{ \hat{V}_{us}^L } \lesssim 10^{-3}$
$\text{Re}\frac{\epsilon'}{\epsilon}$	$\left \frac{ \hat{V}_{us}^L \text{Im}\hat{\mathcal{B}}_{ud}}{ \hat{V}_{ud}^L } + \frac{ \hat{V}_{ud}^L \text{Im}\hat{\mathcal{B}}_{us}}{ \hat{V}_{us}^L } - \frac{\text{Im}\hat{\mathcal{T}}_{u,ds} \cos^2\theta_W}{ \hat{V}_{us}^L \hat{V}_{ud}^L } \right \lesssim 6 \times 10^{-7}$
$d_{n,p}$	$\frac{\text{Im}\hat{\mathcal{B}}_{ud}}{ \hat{V}_{ud}^L }, \frac{\text{Im}\hat{\mathcal{B}}_{us}}{ \hat{V}_{us}^L } \lesssim 3 - 6 \times 10^{-6}$
$\Gamma(Z \rightarrow \text{had})$	$\hat{F}_{\alpha\alpha}^u, \hat{F}_{ii}^d \lesssim 5 \times 10^{-3}$ (for $\alpha \neq t$)
$ \epsilon , \Delta m_K$	$\frac{ \hat{\mathcal{T}}_{u,ds} }{ \hat{V}_{ud}^L \hat{V}_{us}^L } < 6 \times 10^{-7} - 2 \times 10^{-4}$
$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})$	$\frac{ \text{Im}\hat{\mathcal{T}}_{u,ds} }{ \hat{V}_{ud}^L \hat{V}_{us}^L } < 2 \times 10^{-5}$
$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$	$\frac{ \hat{\mathcal{T}}_{u,ds} }{ \hat{V}_{ud}^L \hat{V}_{us}^L } < (3 - 8) \times 10^{-6}$

Couplings to the Higgs:

$$\begin{aligned}\mathcal{L}_h &= -\frac{h}{\sqrt{2}} \left[\overline{\mathcal{U}_L^0} \begin{pmatrix} Y_u & 0 \\ Z_u & 0 \end{pmatrix} \mathcal{U}_R^0 + \overline{\mathcal{D}_L^0} \begin{pmatrix} Y_d & 0 \\ Z_d & 0 \end{pmatrix} \mathcal{D}_R^0 \right] + \text{h.c.} \\ &= -\frac{h}{\sqrt{2}} \left[\overline{\mathcal{U}_L} \frac{\mathcal{D}_u}{v} (\mathbf{1} - F_u) \mathcal{U}_R + \overline{\mathcal{D}_L} \frac{\mathcal{D}_d}{v} (\mathbf{1} - F_d) \mathcal{D}_R \right] + \text{h.c.},\end{aligned}$$

Relation between LH, RH matrices and FCNCs:

$$\begin{aligned}\mathcal{V}_L^u \mathcal{D}_u B_R^{u\dagger} &= \mathcal{V}_L^d \mathcal{D}_d B_R^{d\dagger} \\ \mathcal{D}_u V_R &= V_L \mathcal{D}_d F_d, \quad \mathcal{D}_d V_R^\dagger = V_L^\dagger \mathcal{D}_u F_u \\ \mathcal{D}_u F_u \mathcal{D}_u V_L &= V_L \mathcal{D}_d F_d \mathcal{D}_d\end{aligned}$$

$$d_n \simeq \text{Im} \left[(1.4 \pm 0.7) \hat{V}_{ud}^{L*} \hat{V}_{ud}^R + (2.7 \pm 1.3) \hat{V}_{us}^{L*} \hat{V}_{us}^R \right] \times 10^{-7} \text{ e fm},$$

$$d_p \simeq \text{Im} \left[- (2.7 \pm 1.3) \hat{V}_{ud}^{L*} \hat{V}_{ud}^R - (3.6 \pm 1.5) \hat{V}_{us}^{L*} \hat{V}_{us}^R \right] \times 10^{-7} \text{ e fm}$$

$$\left| (2.3 \pm 1.1) \frac{\text{Im} \left[\hat{V}_{ud}^{L*} \hat{V}_{ud}^R \right]}{|\hat{V}_{ud}^L|} + (1.0 \pm 0.5) \frac{\text{Im} \left[\hat{V}_{us}^{L*} \hat{V}_{us}^R \right]}{|\hat{V}_{us}^L|} \right| \lesssim 3 \times 10^{-6},$$

$$\left| (3.3 \pm 1.6) \frac{\text{Im} \left[\hat{V}_{ud}^{L*} \hat{V}_{ud}^R \right]}{|\hat{V}_{ud}^L|} + (1.0 \pm 0.4) \frac{\text{Im} \left[\hat{V}_{us}^{L*} \hat{V}_{us}^R \right]}{|\hat{V}_{us}^L|} \right| \lesssim 6 \times 10^{-6},$$

$$M_{12}^{\text{NP}} \simeq -\frac{1}{3}m_K f_K^2 0.43 \left\{ \frac{G_F}{\sqrt{2}} \hat{F}_{ds}^2 + \frac{G_F^2}{4\pi^2} \left[\frac{1}{2} M_Q^2 \hat{F}_{ds}^2 - 3.1 \frac{m_K^2}{(m_d + m_s)^2} \hat{F}_{ds} \hat{V}_{td}^{L*} \hat{V}_{ts}^L m_W^2 f(x_Q, x_t) \right] \right\}$$

$$\frac{\text{Im} \left[M_{12}^{\text{NP}} (\lambda_u)^2 \right]}{|\lambda_u|^2} = \frac{1}{3}m_K f_K^2 0.43 \text{Im} \left\{ \frac{G_F}{\sqrt{2}} \frac{(\hat{T}_{u,ds})^2}{|\lambda_u|^2} + \frac{G_F^2}{4\pi^2} \left[\frac{1}{2} M_Q^2 \frac{(\hat{T}_{u,ds})^2}{|\lambda_u|^2} - 3.1 \frac{m_K^2}{(m_d + m_s)^2} \frac{\hat{T}_{u,ds} \hat{Q}_{tsud}}{|\lambda_u|^2} m_W^2 f(x_Q, x_t) \right] \right\}$$

$$\frac{\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})}{\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{SM}}} \simeq \left| 1 - \frac{\frac{1}{2} \text{Im} \left[\hat{V}_{us}^L \hat{V}_{ud}^{L*} \hat{F}_{sd} \right]}{\text{Im} \left[\hat{V}_{us}^L \hat{V}_{ud}^{L*} \frac{\alpha}{2\pi \sin^2 \theta_W} \left(\hat{V}_{cs}^{L*} \hat{V}_{cd}^L X(x_c) + \hat{V}_{ts}^{L*} \hat{V}_{td}^L X(x_t) \right) \right]} \right|^2$$

$$K^+ \rightarrow \pi^+ \nu \bar{\nu} : \quad 0.98 \lesssim \left| -\frac{\frac{1}{2} \left(\hat{V}_{us}^L \hat{V}_{ud}^{L*} \right) \hat{F}_{sd}}{\frac{\alpha}{2\pi \sin^2 \theta_W} \left(\hat{V}_{us}^L \hat{V}_{ud}^{L*} \right) \left(\hat{V}_{cs}^{L*} \hat{V}_{cd}^L X(x_c) + \hat{V}_{ts}^{L*} \hat{V}_{td}^L X(x_t) \right)} + 1 \right| \lesssim 1.44$$