Cosmological gravitational particle production: Starobinsky vs Bogolyubov

University of Helsinki

HELSINGIN YLIOPISTO
HELSINGFORS UNIVERSITET
UNIVERSITY OF HELSINKI

Duarte Feiteira

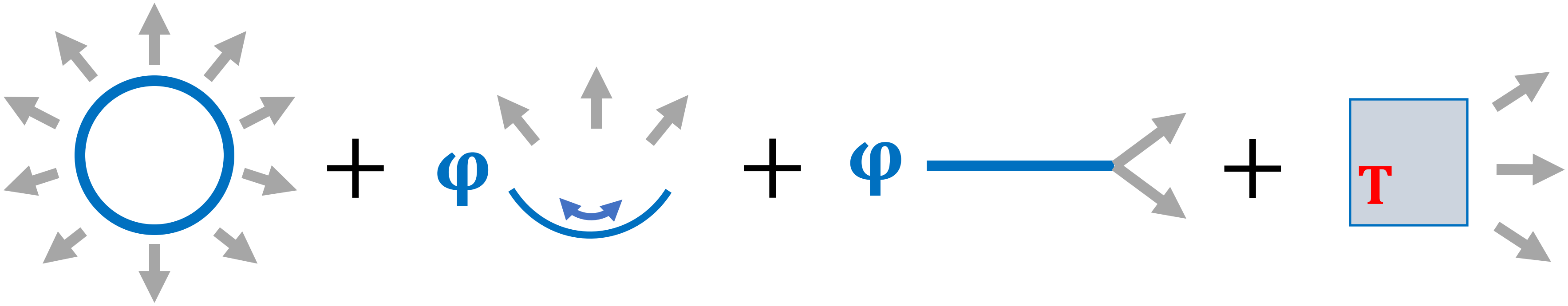
Supervisor: Prof. Oleg Lebedev

D. Feiteira and O. Lebedev; *Cosmological gravitational particle production: Starobinsky vs Bogolyubov, uncertainties, and issues*; arXiv:2503.14652

Motivation

Lebedev 2210.02293

- Gravitational particle production is crucial for **non-thermal dark matter** studies.
- **Add up** all production mechanisms:



During inflation

Via inflaton oscillations

Inflaton decay

Thermal emission (freeze-in)

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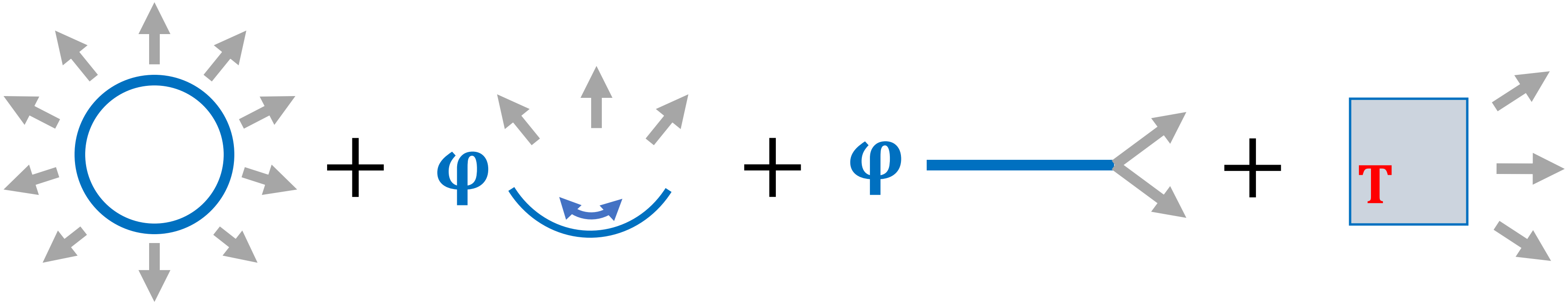

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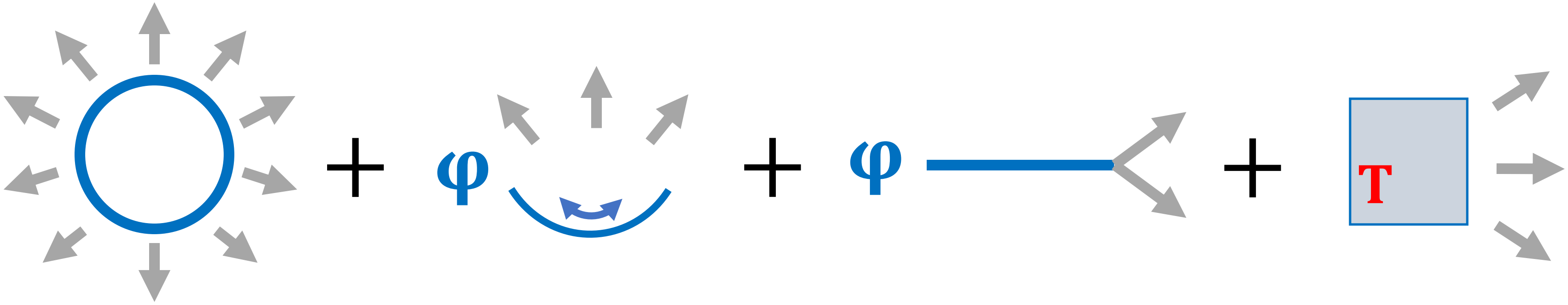

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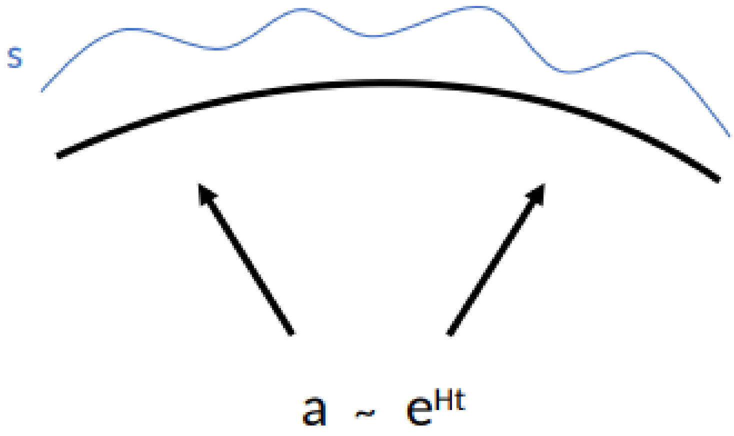
Irreducible background for other production mechanisms.

- **Dark relics:** if particle number is conserved (free or very weakly interacting particles), dark relics produced in inflation survive up to the present day.

Bogolyubov coefficient approach

Parker, Grib,
Zeldovich,
Starobinsky.
60-70’s



- Condensate of a **scalar spectator field** with fluctuations.

$$\phi(\eta, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left[a_{\mathbf{k}}\, \chi_k(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + a_{\mathbf{k}}^{\dagger}\, \chi_k^*(\eta) e^{-i\mathbf{k}\cdot\mathbf{x}}\right]$$

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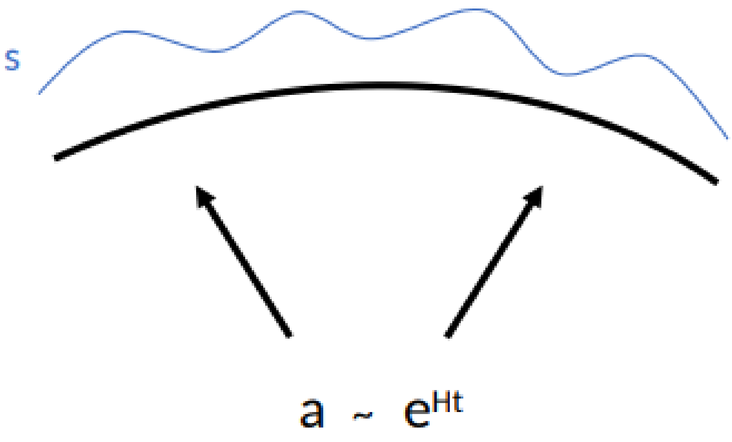

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- Equation of motion for $\chi_k(\eta)$:

$$\chi_k{}''(\eta) + \omega_k^2(\eta)\, \chi_k(\eta) = 0, \qquad \omega_k^2(\eta) = k^2 + a^2(\eta)\, m^2 + \left(\frac{1}{6} - \xi\right) a^2(\eta) R(\eta)$$

Bogolyubov coefficient approach

- **Important assumption: Bunch-Davies vacuum** (no particles initially):

$$a_{\mathbf{k}}^{\mathrm{in}}|0^{\mathrm{in}}\rangle = 0$$

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- **Analytical results:**

Inflation followed by **radiation dominated** epoch:

$$\boldsymbol{a^3 n = \frac{3\kappa^2 a_e^3}{4\pi^2}\boxed{\frac{H_e^{11/2}}{m^{5/2}}}}$$

$\downarrow$

Very large number

Field size at the end of inflation: $\langle \Phi^2 \rangle = \frac{3}{8\pi^2}\frac{H^4}{m^2}$

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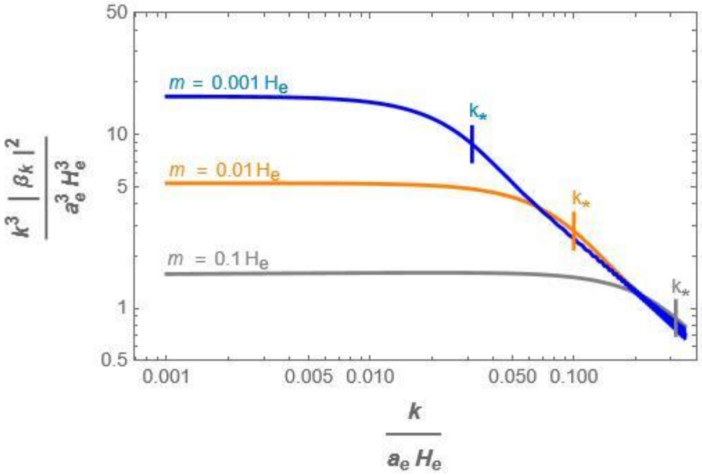

Starobinsky approach

Starobinsky ‘86

$$\Phi(t,\mathbf{x}) = \underset{\substack{\big\downarrow \\ \text{Long wavelength}}}{\bar{\Phi}(t,\mathbf{x})} + \underbrace{\int \frac{d^3\mathbf{k}}{(2\pi)^3}\,\theta(k-\epsilon a(t)H)\left[a_{\mathbf{k}}\,\chi_k(\eta)e^{i\mathbf{k}\cdot\mathbf{r}} + a_{\mathbf{k}}^{\dagger}\,\chi_k^{*}(\eta)e^{-i\mathbf{k}\cdot\mathbf{r}}\right]}_{\text{Short wavelength}}$$

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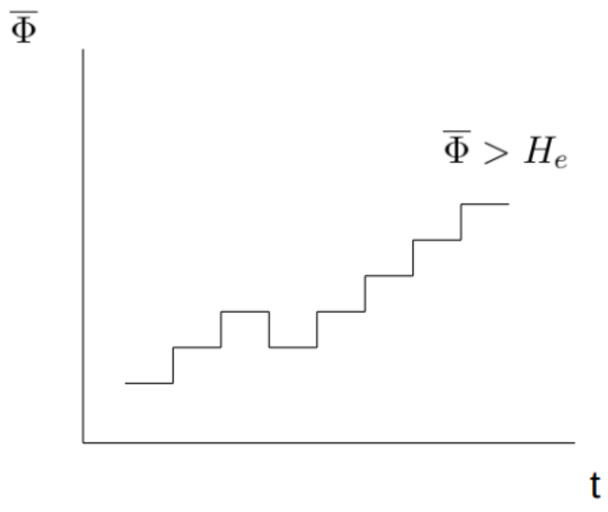
- Random walk:



$$\langle\bar{\Phi}^2\rangle = \frac{3}{8\pi^2}\frac{H^4}{m^2} + \left(\underset{\substack{\Big\downarrow \\ \text{Pre-inflationary} \\ \text{initial conditions}}}{\langle\bar{\Phi}^2\rangle_0} - \frac{3}{8\pi^2}\frac{H^4}{m^2}\right)\underset{\substack{\Big\downarrow \\ \text{Duration of} \\ \text{inflation}}}{e^{-\frac{2m^2}{3H}(t-t_0)}}$$

Infinitely long inflation: agrees with Bogolyubov approach.
Finite inflation: strong dependence on the initial conditions.

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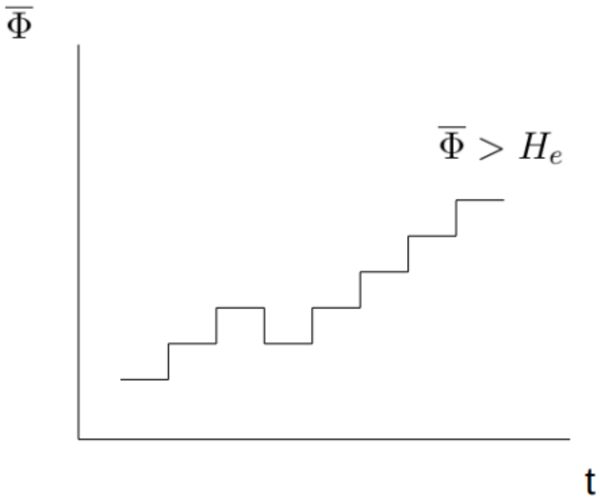

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- When $H \sim m$: condensate $\bar{\Phi}$ is **converted into particles**: $\boldsymbol{a^3 n = a_e^3 \frac{H_e^{3/2}}{2m^{1/2}} \bar{\Phi}^2}$

- **Issues**: Inflation has a finite duration; scalars can have non-trivial initial conditions.

Constraints on dark relics

- Abundance of stable particles produced by inflation cannot exceed that of dark matter.

- Particle number remains constant after reheating and is bounded by the dark matter abundance:

- **Radiation domination:** $\bar{\Phi} < \frac{5\times10^{9}}{(m/\mathrm{GeV})^{1/4}}\ \mathrm{GeV}$ ⇨ $\boxed{m \ll \mathrm{eV}}$

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- **Matter domination:** $\bar{\Phi} < \Delta^{1/2}\frac{5\times10^9}{(H_e/\mathrm{GeV})^{1/4}}\ \mathrm{GeV}$ ⇨ $\boxed{T_R \lesssim \mathrm{GeV}}$

$$\Delta \equiv \sqrt{\frac{H_e}{H_R}} \simeq \frac{T_{inst}}{T_R} \gg 1$$