Inferring Cosmological Parameters from the Cross-Correlation of Gravitational Wave and Electromagnetic Observations

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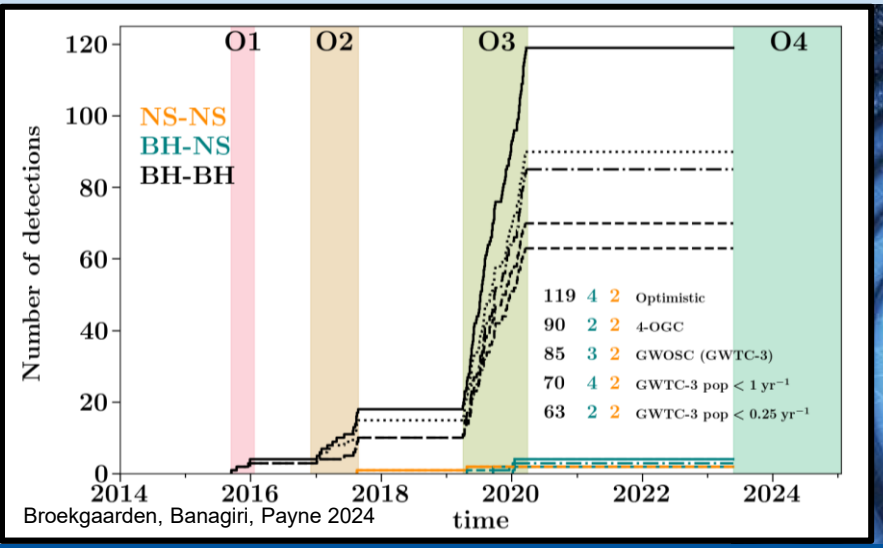
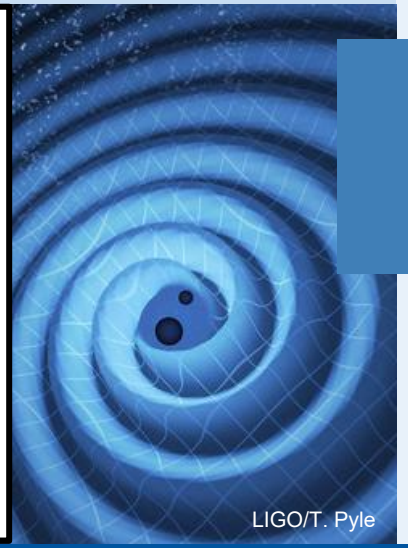
Roadmap

Dark sirens as probe of cosmology:

- Gravitational waves observations are rapidly growing in number;
- Statistical methods with dark sirens can constrain cosmological parameters.



Broekgaarden, Banagiri, Payne 2024


LIGO/T. Pyle

Procedure:

- Identify complementary LSS tracers:
  - ISW effect;
  - Galaxy clustering;

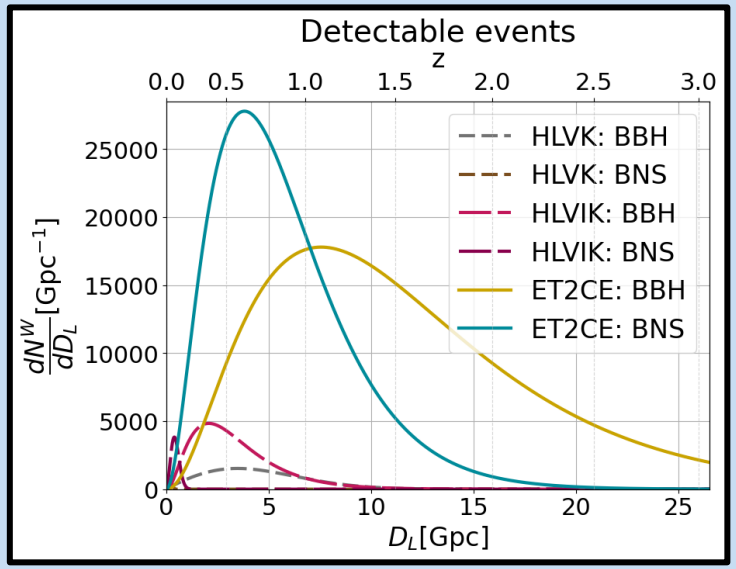
$$D_L = a(t_0)(1+z)\int_0^z \frac{cdz'}{a(t_0)H(z')}$$

- Compute angular power spectrum C_ℓ;
- **Cross-correlation** for C_ℓ-based likelihood;
- Run MCMC chains to infer cosmological parameters.

Mock data

Dark sirens detections over 10 years:

- HLVK: LIGO + Virgo + Kagra;
- HLVIK: HLVK + LIGO India;
- ET2CE: Einstein Telescope + 2 Cosmic Explorer.



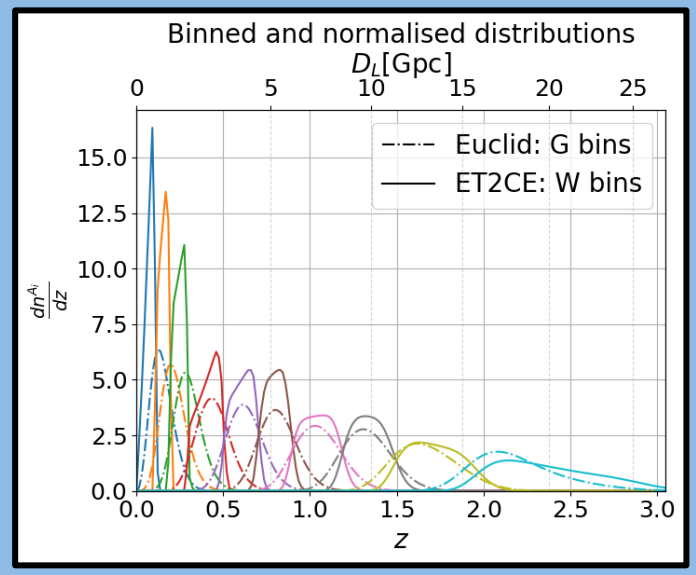
Galaxy clustering:

- Euclid photometric forecasted data

Tomographic approach:

- Binned distribution;
- Normalization with distribution error.

$$\frac{dn^{G_i}}{dz}, \frac{dn^{W_i}}{dD_L}$$



C_ℓ-based Likelihood

Angular power spectrum: $\delta^A(\vec{k}, z) = b^A(z)\,\delta_m(\vec{k}, z)$ $\quad W^{A_i}(z) = \dfrac{dn^{A_i}}{dz}\,H(z)\,b^A(z)$

$$C^{A_iB_j}(\ell) = \int_{z_{\min}}^{z_{\max}} dz \frac{W^{A_i}(z)W^{B_j}(z)}{H(z)r^2(z)} \times P_{\delta\delta}\left[k = \frac{\ell + 1/2}{r(z)}, z\right] + N^{A_iB_j}(\ell)$$

Likelihood:

$$-2\ln\mathcal{L}(\vec{D}|\vec{\theta}) = \sum_{\ell=\ell_{\min}}^{\ell_{\max}} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)^T \mathcal{C}_\ell^{-1} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)$$

- $\vec{D}_\ell$: Data vector;
- $\vec{T}_\ell$: Theory vector for parameters $\vec{\theta}$;

$$\mathrm{Cov}\left[C^{A_iB_j}(\ell), C^{C_kD_n}(\ell')\right] = \frac{\delta_{\ell\ell'}}{(2\ell+1)f_{\mathrm{fov}}\Delta\ell}\left(C^{A_iC_k}(\ell)C^{B_jD_n}(\ell) + C^{A_iD_n}(\ell)C^{B_jC_k}(\ell)\right)$$

Cross-correlation

$$-2\ln\mathcal{L}(\vec{D}|\vec{\theta}) = \sum_{\ell=\ell_{\min}}^{\ell_{\max}} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)^T \mathcal{C}_\ell^{-1} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)$$

$$\mathrm{Cov}\left[C^{AB}(\ell), C^{CD}(\ell)\right] = (\overrightarrow{AB}_\ell, \overrightarrow{CD}_\ell)$$

$$\mathcal{C}_\ell = \begin{pmatrix} (\overrightarrow{GG}_\ell, \overrightarrow{GG}_\ell) & (\overrightarrow{GG}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{GG}_\ell, \overrightarrow{WW}_\ell) \\ (\overrightarrow{WG}_\ell, \overrightarrow{GG}_\ell) & (\overrightarrow{WG}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{WG}_\ell, \overrightarrow{WW}_\ell) \\ (\overrightarrow{WW}_\ell, \overrightarrow{GG}_\ell) & (\overrightarrow{WW}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{WW}_\ell, \overrightarrow{WW}_\ell) \end{pmatrix}$$

Cross-correlation

$$-2\ln\mathcal{L}(\vec{D}|\vec{\theta}) = \sum_{\ell=\ell_{\min}}^{\ell_{\max}} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)^T \mathcal{C}_\ell^{-1} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)$$

$$\mathrm{Cov}\left[C^{AB}(\ell), C^{CD}(\ell)\right] = (\overrightarrow{AB}_\ell, \overrightarrow{CD}_\ell)$$

Galaxy clustering auto-correlation only (GG)

$$\mathcal{C}_\ell = \begin{pmatrix} (\overrightarrow{GG}_\ell, \overrightarrow{GG}_\ell) & (\overrightarrow{GG}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{GG}_\ell, \overrightarrow{WW}_\ell) \\ (\overrightarrow{WG}_\ell, GG_\ell) & (\overrightarrow{WG}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{WG}_\ell, \overrightarrow{WW}_\ell) \\ (\overrightarrow{WW}_\ell, GG_\ell) & (\overrightarrow{WW}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{WW}_\ell, \overrightarrow{WW}_\ell) \end{pmatrix}$$

Cross-correlation

$$-2\ln\mathcal{L}(\vec{D}|\vec{\theta}) = \sum_{\ell=\ell_{\min}}^{\ell_{\max}} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)^T \mathcal{C}_\ell^{-1} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)$$

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Gravitational Waves auto-correlation only (WW)

Cross-correlation

$$-2\ln\mathcal{L}(\vec{D}|\vec{\theta}) = \sum_{\ell=\ell_{\min}}^{\ell_{\max}} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)^T \mathcal{C}_\ell^{-1} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)$$

$$\mathrm{Cov}\left[C^{AB}(\ell), C^{CD}(\ell)\right] = (\overrightarrow{AB}_\ell, \overrightarrow{CD}_\ell)$$

$$\mathcal{C}_\ell = \begin{pmatrix} (\overrightarrow{GG}_\ell, \overrightarrow{GG}_\ell) & (\overrightarrow{GG}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{GG}_\ell, \overrightarrow{WW}_\ell) \\ (\overrightarrow{WG}_\ell, \overrightarrow{GG}_\ell) & (\overrightarrow{WG}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{WG}_\ell, \overrightarrow{WW}_\ell) \\ (\overrightarrow{WW}_\ell, \overrightarrow{GG}_\ell) & (\overrightarrow{WW}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{WW}_\ell, \overrightarrow{WW}_\ell) \end{pmatrix}$$

Cross-Correlation only (XC)

Cross-correlation

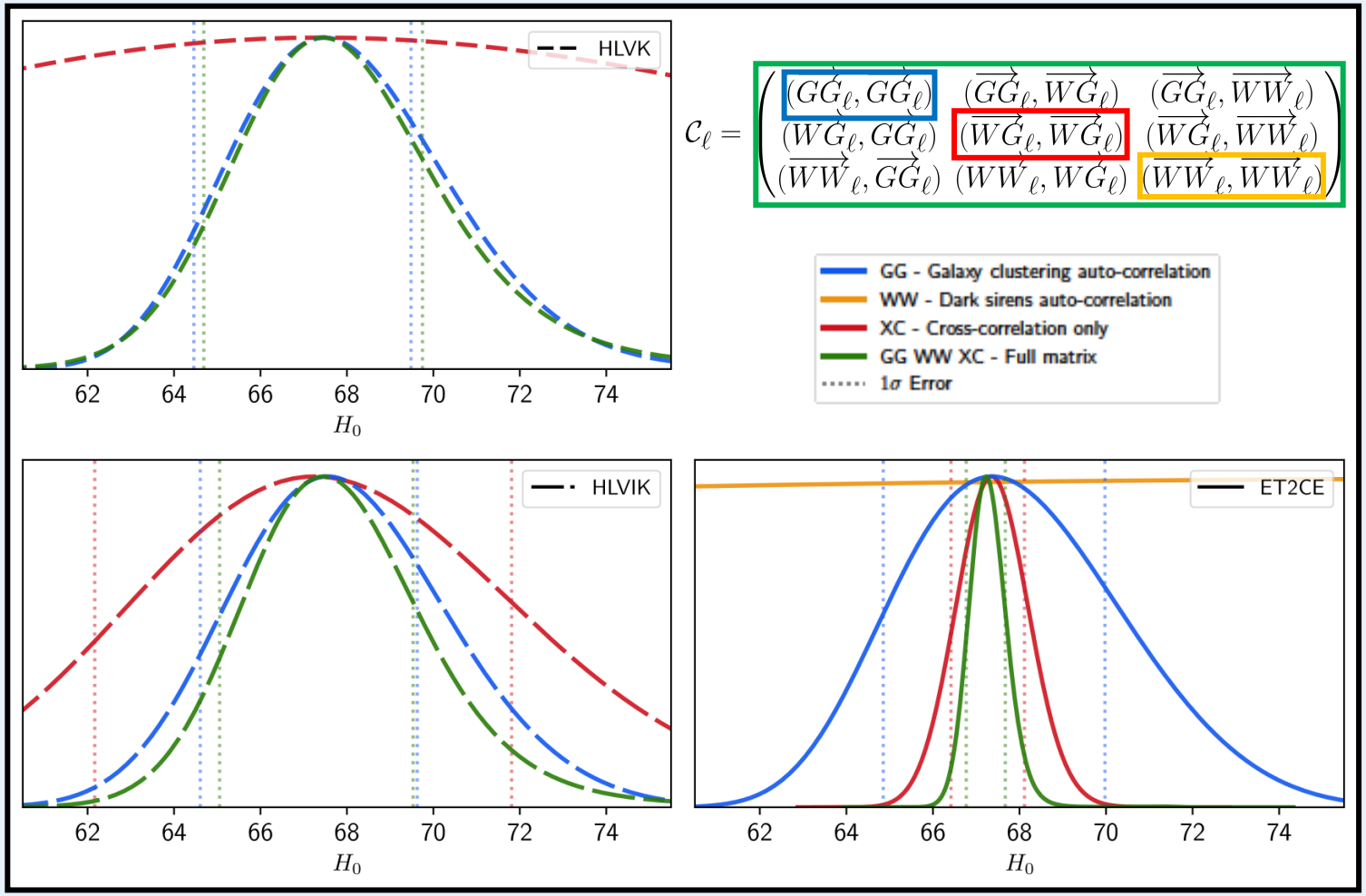
$$-2\ln\mathcal{L}(\vec{D}|\vec{\theta}) = \sum_{\ell=\ell_{\min}}^{\ell_{\max}} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)^T \mathcal{C}_\ell^{-1} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)$$

$$\mathrm{Cov}\left[C^{AB}(\ell), C^{CD}(\ell)\right] = (\overrightarrow{AB}_\ell, \overrightarrow{CD}_\ell)$$

$$\mathcal{C}_\ell = \begin{pmatrix} (\overrightarrow{GG}_\ell, \overrightarrow{GG}_\ell) & (\overrightarrow{GG}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{GG}_\ell, \overrightarrow{WW}_\ell) \\ (\overrightarrow{WG}_\ell, \overrightarrow{GG}_\ell) & (\overrightarrow{WG}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{WG}_\ell, \overrightarrow{WW}_\ell) \\ (\overrightarrow{WW}_\ell, \overrightarrow{GG}_\ell) & (\overrightarrow{WW}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{WW}_\ell, \overrightarrow{WW}_\ell) \end{pmatrix}$$

Full matrix (GG WW XC)

Results



Posteriors on H_0 by running MCMC chains:

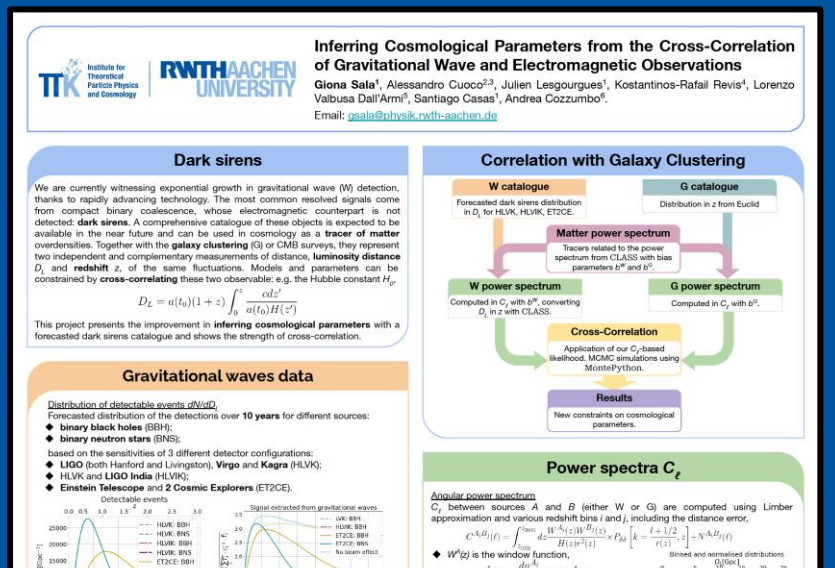
- HLVK and HLVIK barely improve the constraint given by galaxy clustering only;
- Cross-correlation drives the constraint for ET2CE;
- Dark sirens auto-correlation has no constraining power.

Conclusions

- Decomposable full likelihood analysis;
- Only ET2CE will detect a sufficiently large sample of dark sirens to improve H_0 constraint, better than 1% at 1σ;
- Additional cross-correlation analysis, e.g. with CMB, coming soon.

Thank you for your attention!

For more information:



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