Ultimate limit on beam brightness

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Other Science Opportunities at the FCC-ee, CERN Geneva

November 28, 2024

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DESY.

Why would one want a light source at high energy?

Many contemporary light sources (will) operate at 3-6 GeV

Parameters of operating and planed 4th generation light sources

Machine	Energy	Circumference	Hor. emittance
P4	6 GeV	2.3 km	20 pm
ESRF-EBS	6 GeV	844 m	110 pm
APS-U	6 GeV	1.1 km	65 pm
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SLS-II	2.7 GeV	288 m	131 pm
Diamond-II	3.5 GeV	562 m	162 pm
SOLEIL-II	2.75 GeV	354 km	83 pm
MAX-IV	3 GeV	528 m	200 pm
*FCC-ee Booster**	*20 GeV (inj.)*	*90.7 km*	*15 pm*

** S. Casalbuoni, F. Zimmermann, FCC Week'21*

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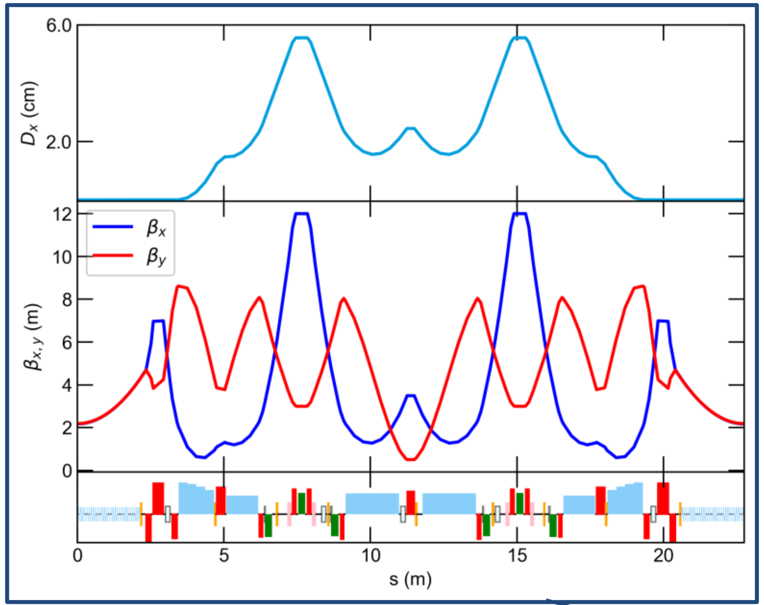
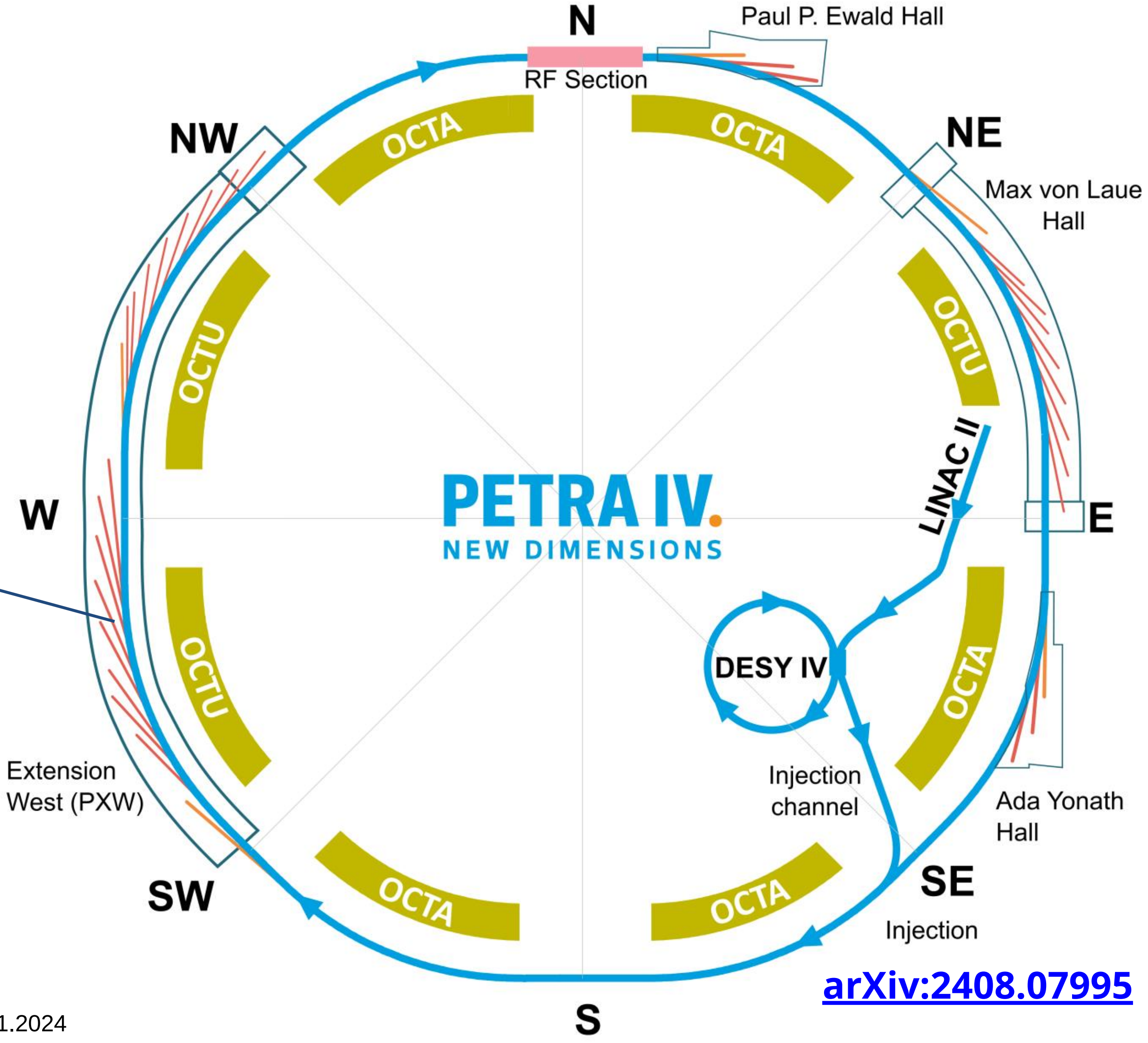
PETRA IV: Germany's future flagship light source

6 GeV, 2.3 km, 20 pm



Highly optimized achromatic cell to approach diffraction limit at 10 keV



arXiv:2408.07995

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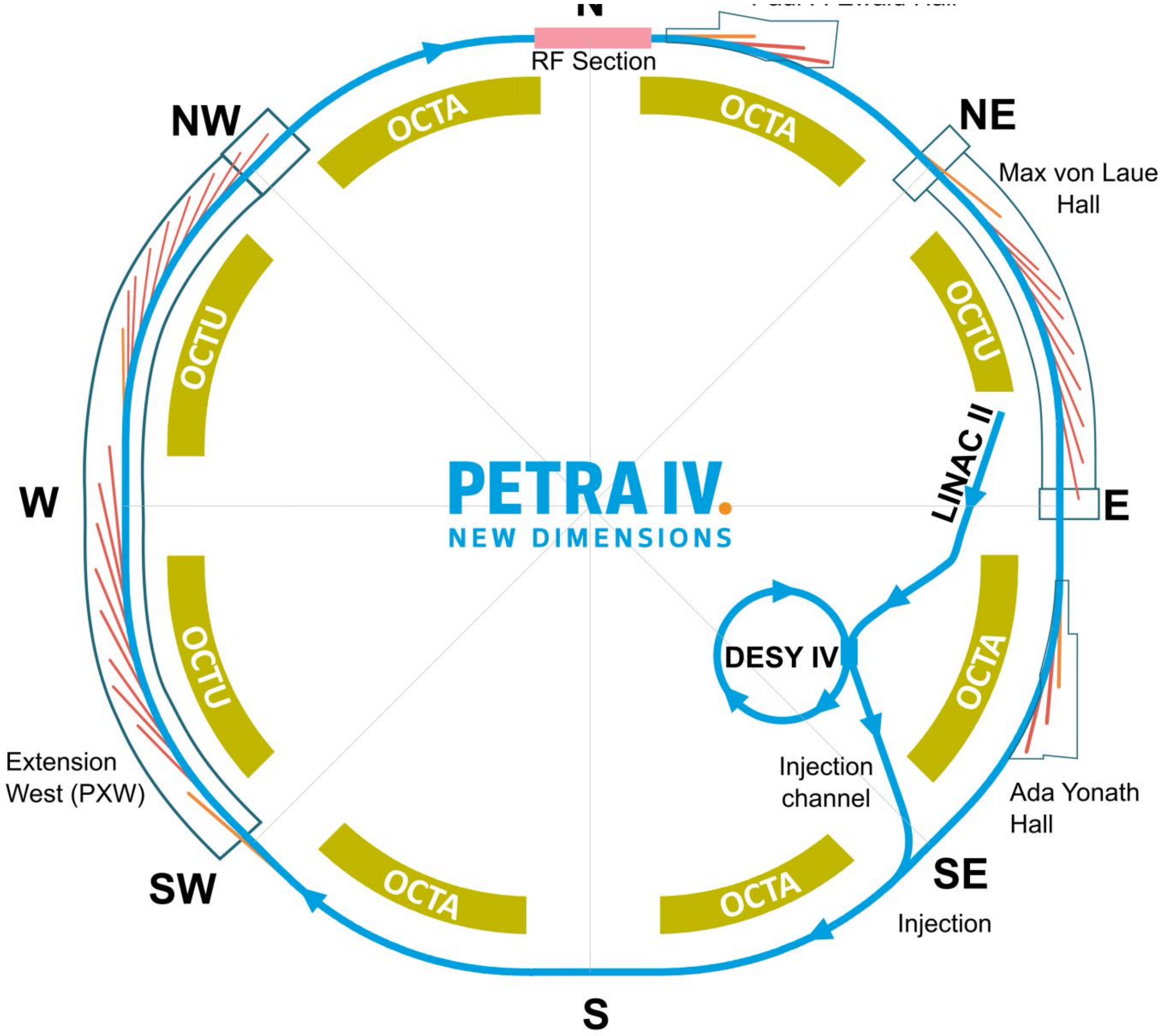
Space charge tune shift

6 GeV, 2.3 km, 20 pm

$$|\Delta\nu_{x,y}^{SC}| = \frac{N\, r_e C}{(2\pi)^{\frac{3}{2}}\gamma^3\sigma_z}\left\langle\frac{\beta_{x,y}}{\sigma_{x,y}(\sigma_x+\sigma_y)}\right\rangle$$

$1/10^{12}$



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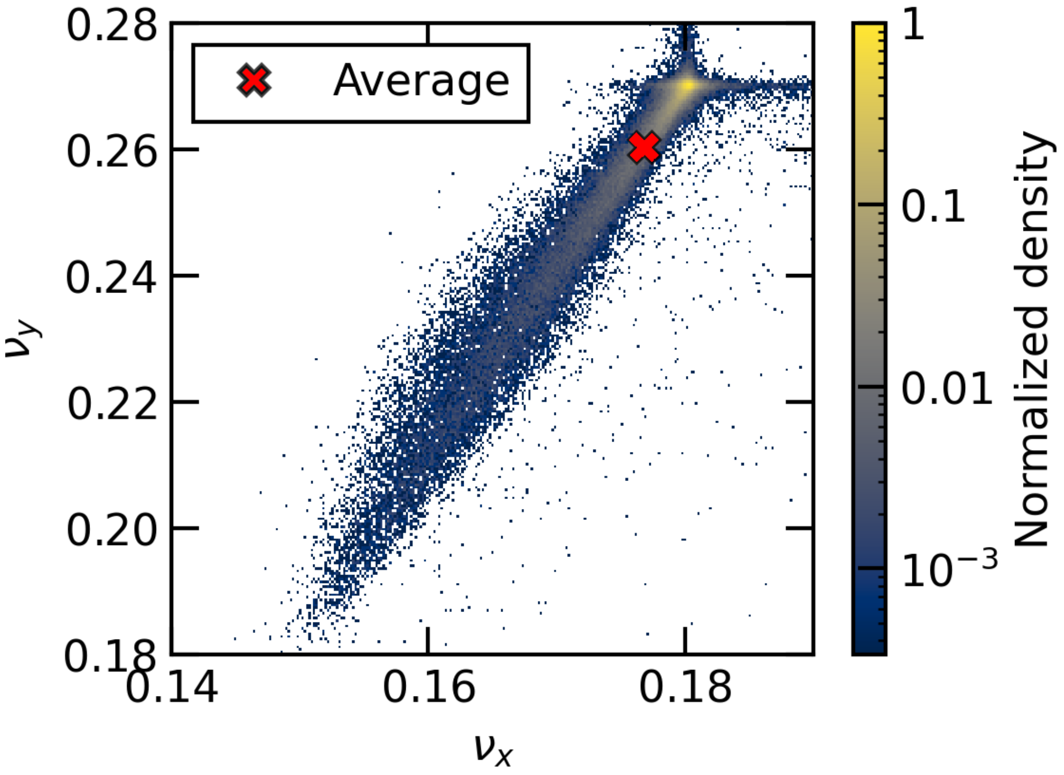

Tune footprint with SC. Simulation in XSuite.
10 nC bunch, 128 turns, long. wakes included,
single-harmonic RF system

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arXiv:2409.08637

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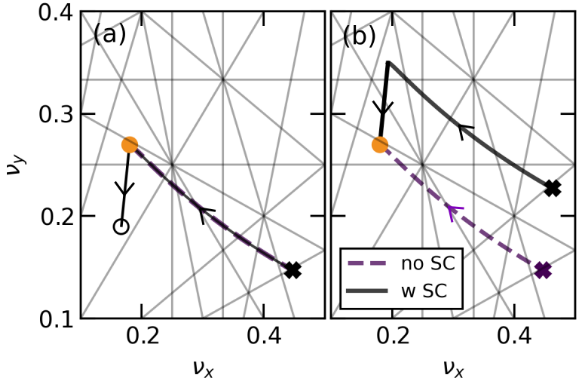
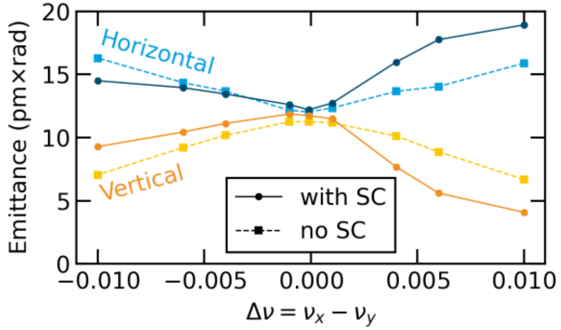
Space charge effects

Both coherent and incoherent

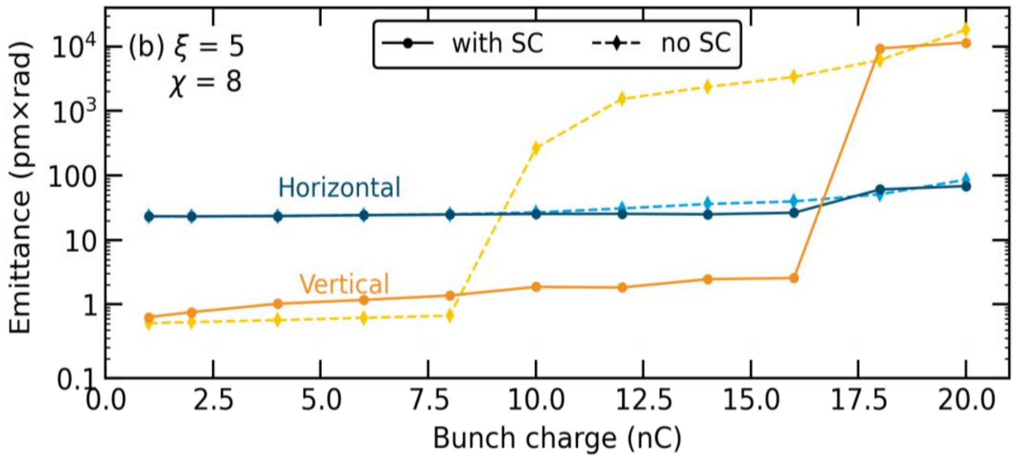
Dynamics of off-axis top-up injection



Emittance ratios at high coupling



Single-bunch intensity limit



[arXiv:2409.08637](https://arxiv.org/abs/2409.08637)

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SC will ultimately lead to crossing of the integer resonance

This shall limit the bunch intensity even if there is no beam coupling impedance

$$|\Delta\nu_{x,y}^{SC}| = \frac{N\, r_e C}{(2\pi)^{\frac{3}{2}}\gamma^3\sigma_z}\left\langle\frac{\beta_{x,y}}{\sigma_{x,y}\left(\sigma_x + \sigma_y\right)}\right\rangle \lesssim \frac{1}{2}$$

Key Assumptions:

- Linear SC model
- Integer resonance cannot be crossed
- The largest tune shift is in the vertical plane
- Dispersion contribution to beam size is small, on average
- Coupling is sufficiently small

Approximate SC limit

Crossing of the integer resonance

$$|\Delta\nu_{x,y}^{SC}| = \frac{N\, r_e C}{(2\pi)^{\frac{3}{2}}\gamma^3\sigma_z}\left\langle\frac{\beta_{x,y}}{\sigma_{x,y}(\sigma_x + \sigma_y)}\right\rangle \lesssim \frac{1}{2}$$

$$\left|\Delta\nu_y^{SC}\right| \approx \frac{N\, r_e C}{(2\pi)^{\frac{3}{2}}\gamma^3\sigma_z}\left\langle\frac{\beta_y}{\sqrt{\beta_y\epsilon_y}\left(\sqrt{\beta_y\epsilon_y} + \sqrt{\beta_x\epsilon_x}\right)}\right\rangle$$

$$\left\langle\frac{\beta_y}{\sqrt{\beta_y\epsilon_y}\left(\sqrt{\beta_y\epsilon_y}+\sqrt{\beta_x\epsilon_x}\right)}\right\rangle = \frac{1}{\epsilon_x\kappa}\left\langle\frac{\sqrt{\kappa}\sqrt{\frac{\beta_y}{\beta_x}}}{1+\sqrt{\kappa}\sqrt{\frac{\beta_y}{\beta_x}}}\right\rangle \approx \frac{1}{\epsilon_x\kappa}\left\langle\sqrt{\kappa}\sqrt{\frac{\beta_y}{\beta_x}}\left(1 - \sqrt{\kappa}\sqrt{\frac{\beta_y}{\beta_x}}\right)\right\rangle \sim \frac{1}{4\epsilon_x\kappa}\ {}^{*}$$

$$\kappa = \epsilon_y/\epsilon_x$$

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- Linear SC model
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** to be checked later*

Approximate SC limit

Crossing of the integer resonance

$$|\Delta\nu_{x,y}^{SC}| = \frac{N\, r_e C}{(2\pi)^{\frac{3}{2}}\gamma^3\sigma_z}\left\langle\frac{\beta_{x,y}}{\sigma_{x,y}(\sigma_x+\sigma_y)}\right\rangle \lesssim \frac{1}{2}$$

$$|\Delta\nu_x^{SC}| \sim \frac{N\, r_e C}{4(2\pi)^{\frac{3}{2}}\gamma^3\sigma_z\epsilon_x\kappa} \lesssim \frac{1}{2}$$

$$|\Delta\nu_y^{SC}| \approx \frac{N\, r_e C}{(2\pi)^{\frac{3}{2}}\gamma^3\sigma_z}\left\langle\frac{\beta_y}{\sqrt{\beta_y\epsilon_y}\left(\sqrt{\beta_y\epsilon_y}+\sqrt{\beta_x\epsilon_x}\right)}\right\rangle$$

$$B = \frac{2I}{\pi^2\epsilon_x\epsilon_y}$$

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$$\kappa = \epsilon_y/\epsilon_x$$

$$B \times \epsilon_x \lesssim \frac{8\gamma^3 I_A}{\pi C}$$

Approximate SC limit

Crossing of the integer resonance

$$|\Delta\nu_{x,y}^{SC}| = \frac{N\, r_e C}{(2\pi)^{\frac{3}{2}}\gamma^3\sigma_z}\left\langle\frac{\beta_{x,y}}{\sigma_{x,y}(\sigma_x+\sigma_y)}\right\rangle \lesssim \frac{1}{2}$$

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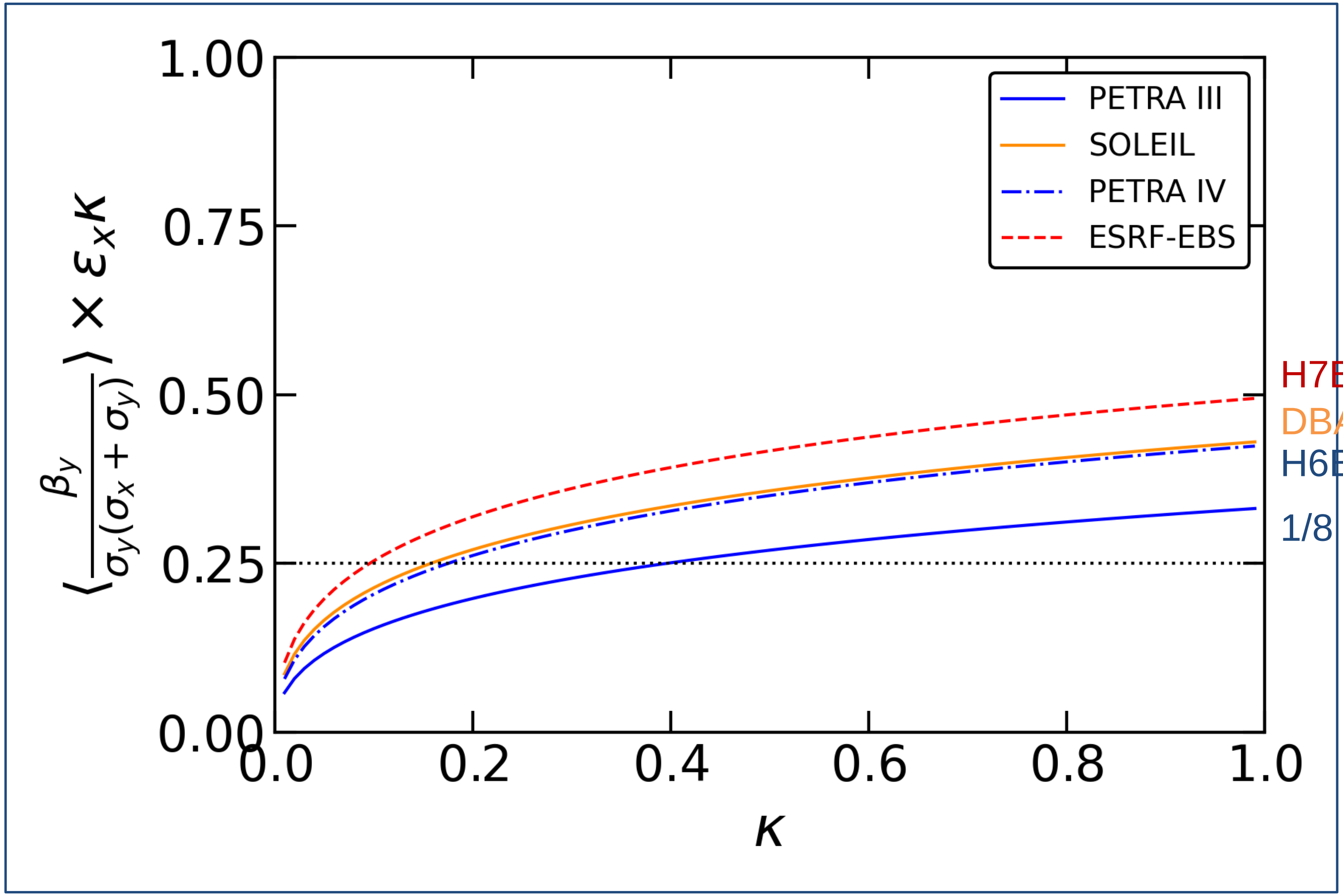
$$\kappa = \epsilon_y/\epsilon_x$$

Example: PETRA IV, high charge Timing mode		
Lorentz factor	γ	11.7×10^3
Circumference	C	2.3 km
Bunch charge	Ne	8 nC
Hor. emittance	ϵ_x	20 pm
Bunch length	σ_z	20 mm
Design brightness	B_{Timing}	$2.4 \times 10^{23}\,\mathrm{A/m^2}$
SC limit	$\boldsymbol{B_{SC}}$	$\boldsymbol{1.4 \times 10^{24}\ A/m^2}$

$$B \times \epsilon_x \lesssim \frac{8\gamma^3 I_A}{\pi C}$$

Approximation holds in a wide parameter range

Within a factor 2 for any conceivable coupling ratio, any machine



H7BA

DBA

H6BA

1/8 DBA + 7/8 FODO

Horizontal emittance at high energy

Governed by a balance of synchrotron radiation and quantum excitation

$$\left(\frac{d\epsilon_x}{dt}\right)_{IBS} + \left(\frac{d\epsilon_x}{dt}\right)_{SR} + \left(\frac{d\epsilon_x}{dt}\right)_{QE} = 0$$

IBS is a small correction

$$\left(\frac{d\epsilon_x}{dt}\right)_{SR} + \left(\frac{d\epsilon_x}{dt}\right)_{QE} \approx 0$$

$$\epsilon_x \approx \frac{55\hbar c}{32\sqrt{3}m_e c^2}\frac{\gamma^2}{J_x}\frac{I_5}{I_2}$$

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$$J_x \in [1; 2]$$

$$\frac{I_5}{I_2} = \frac{\oint \mathcal{H}/\rho^3 ds}{\oint 1/\rho^2 ds} \propto 1/C$$

$$\epsilon_x = F \frac{\gamma^2}{C}$$

***F* – scale-independent parameter**

- defined by the ring's optics lattice,
- not its physical scale or energy.
- HMBA lattices: $F = 3 - 7 \times 10^{-16} \mathrm{m}^2$

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$$B \times \epsilon_x \lesssim \frac{8\gamma^3 I_A}{\pi C}$$

→

$$B \lesssim \frac{8\gamma I_A}{\pi F}$$

A 20 GeV-range machine should have a higher brightness limit compared to most 4th generation light sources

Space charge imposes a limit on achievable beam brightness

The way to push further is to aim at higher energies

Space charge can no longer be ignored in state-of-the art light sources

- Emittance reaches the point where it becomes strong enough
- Generates incoherent tune shift and spread affecting beam dynamics

There is a limit on achievable beam brightness

- Due to crossing of the integer resonance
- Consequently, the photon brilliance is limited

The SC limit increases with energy as the interaction weakens

- Potentially, higher photon brilliance at higher beam energies

Extras: emittance scaling factor for different machines

Many state-of-the art HMBA machines have similar performance despite different scales

Machine	Energy	Circumference	Hor. emittance	F
P4 baseline	6 GeV	2.3 km	20 pm	$3.3 \times 10^{-16} \mathrm{m}^2$
HEPS	6 GeV	1.36 km	35 pm	$3.5 \times 10^{-16} \mathrm{m}^2$
P4 no DW	6 GeV	2.3 km	40 pm	$6.6 \times 10^{-16} \mathrm{m}^2$
ESRF-EBS	6 GeV	844 m	110 pm	$6.8 \times 10^{-16} \mathrm{m}^2$
APS-U	6 GeV	1.1 km	65 pm	$5.2 \times 10^{-16} \mathrm{m}^2$
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*FCC-ee Booster**	*20 GeV (inj.)*	*90.7 km*	*15 pm*	$\sim 10 \times 10^{-16} \mathrm{m}^2$

Extras: Intra-beam scattering

A small correction at high energy

Bjorken-Mtingwa model:

$$\sigma_H = \left(\frac{1}{\sigma_p^2} + \frac{\mathcal{H}_x}{\epsilon_x} + \frac{\mathcal{H}_y}{\epsilon_y}\right)^{-1/2}$$

$$a = \frac{\sigma_H}{\gamma}\sqrt{\frac{\beta_x}{\epsilon_x}}, \qquad b = \frac{\sigma_H}{\gamma}\sqrt{\frac{\beta_y}{\epsilon_y}},$$

$$g(x) \approx 2x^{0.021-0.044\ln(x)}$$

$$\alpha_x^{IBS} \approx \frac{r_e^2 c N L_C}{32\gamma^3 \epsilon_x^{\frac{7}{4}} \epsilon_y^{\frac{3}{4}} \sigma_z \sigma_p} \left\langle \mathcal{H}_x \sigma_H g\left(\frac{a}{b}\right) \beta_x^{-1/4} \beta_y^{-1/4} \right\rangle$$

$$\sigma_z \sigma_p \propto U_0^{1/4} \propto \gamma$$

SR damping rate:

$$\alpha_x^{SR} = \gamma^3 \frac{r_e c I_2}{3C}$$

Equilibrium emittance:

$$\epsilon_x = \frac{\epsilon_{x,0}}{1 - \alpha_x^{IBS}/\alpha_x^{SR}}$$

See *K. Bane, SLAC-AP-141*
and *R. Bartolini, IPAC’22*