# Simulation of non-Markovian dynamics in the spin boson model via quantum algorithm

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# Introduction

Markovian master equation ⋯ Lindblad equation

$$\frac{d\rho_S(t)}{dt} = -i[H, \rho_S(t)] + \sum_k \left( V_k \rho_S(t) V_k^\dagger - \frac{1}{2}\{V_k^\dagger V_k, \rho_S(t)\} \right)$$

non-Markovian process is important

1. Markovian process is still approximation
2. Markovian process neglects the memory effect

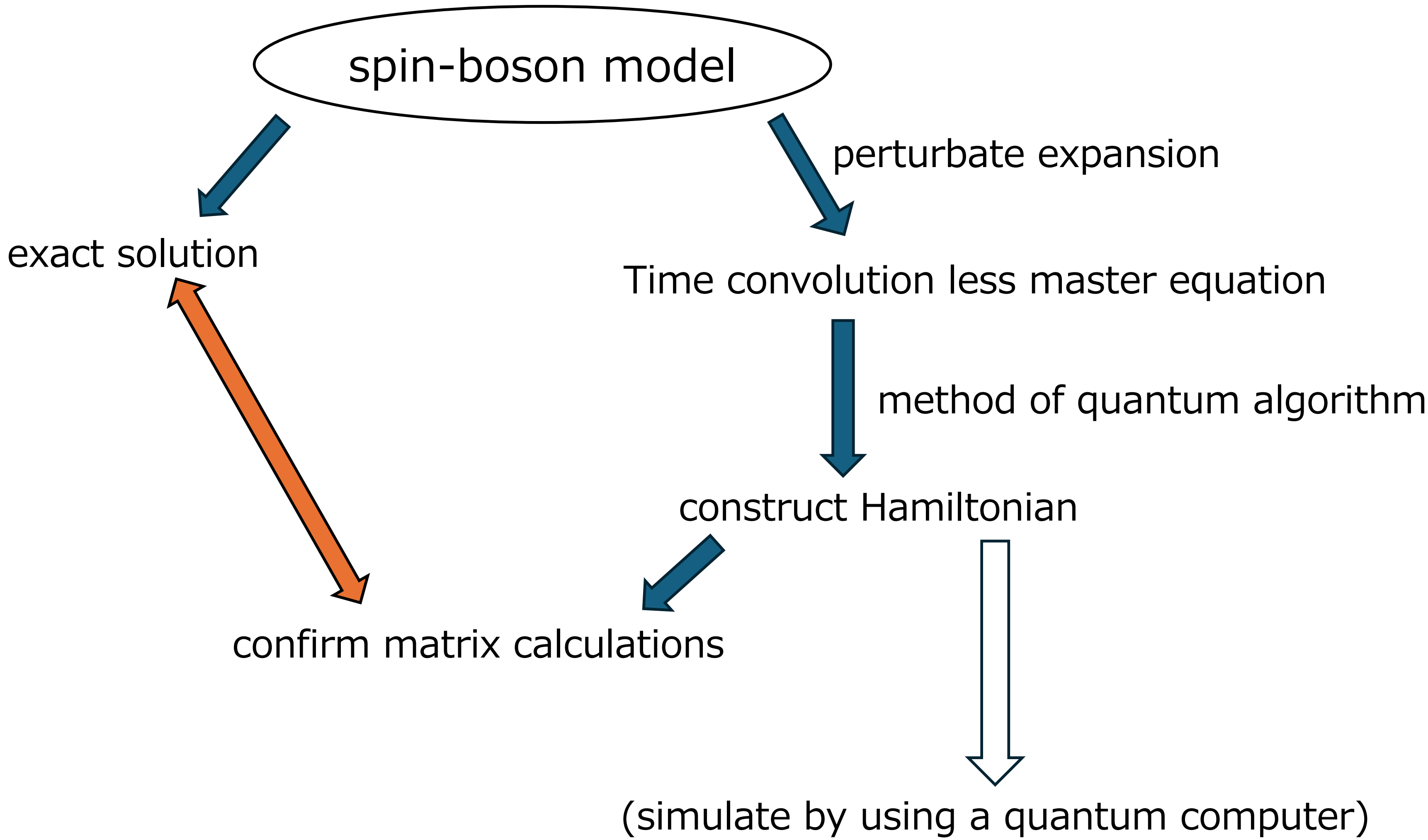
**Research topic**

simulate a non-Markovian open system by using a quantum algorithm

an example of a master equation describing a non-Markovian process
⋯ Time convolution-less master equation

$$\frac{d\rho_S(t)}{dt} = -i[H(t), \rho_S(t)] + \sum_k \left( V_k(t) \rho_S(t) V_k^\dagger(t) - \frac{1}{2}\{V_k^\dagger(t) V_k(t), \rho_S(t)\} \right)$$

# What I did


spin-boson model
perturbate expansion
exact solution
Time convolution less master equation
method of quantum algorithm
construct Hamiltonian
confirm matrix calculations
(simulate by using a quantum computer)

# Spin boson model

[1] H. -P. Breuer, et. al., *The Theory of Open Quantum Systems,*(2002).
[3] B. Vacchini, H.P. Breuer, Phys. Rev. A **81,** 042103 (2010).

**total system**

**system**

two level system

$$H_S = \omega_0 \sigma_+ \sigma_-$$

**environment**

boson

$$H_E = \sum_k \omega_k b_k^\dagger b_k$$

**interaction Hamiltonian**

$$H_I = \sum_k (g_k \sigma_+ \otimes b_k + g_k^* \sigma_- \otimes b_k^\dagger)$$

spectrum function : Distribution of coupling strengths with respect to the frequencies of the environmental system

$$J(\omega) = \frac{1}{2\pi} \frac{\gamma_0 \lambda^2}{(\omega_0 - \Delta - \omega)^2 + \lambda^2}$$

exact solution of the survival probability

$$\langle 1 | \rho_S(t) | 1 \rangle = \left| e^{-(\lambda + i\Delta)\frac{t}{2}} \left( \cosh\left(\frac{Dt}{2}\right) + \frac{\lambda - i\Delta}{D} \sinh\left(\frac{Dt}{2}\right) \right) \right|^2 \qquad D = \sqrt{(\lambda - i\Delta)^2 - 2\gamma_0 \lambda}$$

# Master equation

Time convolution less master equation (spin-boson model)

$$\frac{d\rho_S(t)}{dt} = -\frac{i}{2}[S(t)\sigma_+\sigma_-, \rho_S(t)] + \gamma(t)\left(\sigma_-\rho_S(t)\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho_S(t)\}\right)$$

Derive $S(t)$ and$\gamma(t)$ through perturbative expansion with respect to the coupling strength

$$H = H_0 + \alpha H_I$$

$\alpha$ : coupling strength

$$\frac{d\rho_S(t)}{dt} = -i\mathrm{Tr_E}[H_I(t), \rho_{tot}(t)]$$

$$\rho_S(t) = \rho_S(0) + (i\alpha)^2\mathrm{Tr_E}\int_0^t dt_1 \int_0^{t_1} dt_2[H_I(t_1), [H_I(t_2), \rho_S(0)\otimes\rho_E]] + \cdots$$

approximate by 2nd and 4th order

# Quantum algorithm

[4]Z. Ding, X. Li, L. Lin, PRX Quantum **5**, 020332 (2024).

Method of simulating Lindblad master equation

Lindblad equation $$\frac{d\rho_S(t)}{dt} = -i[H, \rho_S(t)] + \sum_k \left( V_k \rho_S(t) V_k^\dagger - \frac{1}{2}\{V_k^\dagger V_k, \rho_S(t)\} \right)$$

↓

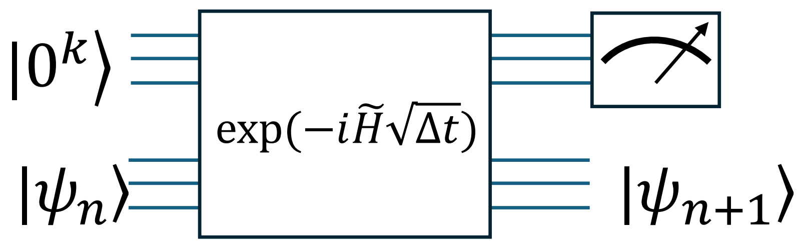
equivalent Stochastic-Schrodinger equation

↓ using Euler Murayama method, approximate by $\Delta t$ n-th order

using Stinespring representation, construct $\widetilde{H}$ including $H$ and $V_k$

$$\rho_{n+1} = \mathrm{Tr}_\mathrm{A} \left( \exp(-i\sqrt{\Delta t}\tilde{H}) \left|0\right\rangle \left\langle 0\right| \otimes \rho_n \exp(-i\sqrt{\Delta t}\tilde{H}) \right)$$

this algorithm can expand into time dependent $H$ and $V_k$



$$\rho_{n+1} = \mathrm{Tr}_{\mathrm{A}} \left( \exp(-i\sqrt{\Delta t}\tilde{H}) \left|0\right\rangle \left\langle 0\right| \otimes \rho_n \exp(-i\sqrt{\Delta t}\tilde{H}) \,\right)$$

$$\tilde{H}(t) = \begin{pmatrix} \frac{1}{2}\sqrt{\Delta t} S(t) & 0 & 0 & \gamma(t) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \gamma(t) & 0 & 0 & 0 \end{pmatrix}$$

time evolution operator $U(t) = e^{-i\sqrt{\Delta t}\tilde{H}(t)}$

$$\mathrm{Tr}_{\mathrm{A}}(U(t_n) \left|0\right\rangle \left\langle 0\right| \otimes \rho(t_n) U^{\dagger}(t_n)) = \rho(t_{n+1})$$

Performed matrix calculations numerically

# Comparison between the results and the exact solution

plot of the time evolution of the survival probability of the excited state $\rho_{11}(t) = \langle 1|\rho_S(t)|1\rangle$

$\gamma_0 = 3 \quad \lambda = 0.3\gamma_0 \quad \Delta = 8\lambda$

$\Delta t$ : 0.01

$$J(\omega) = \frac{1}{2\pi}\frac{\gamma_0\lambda^2}{(\omega_0 - \Delta - \omega)^2 + \lambda^2}$$

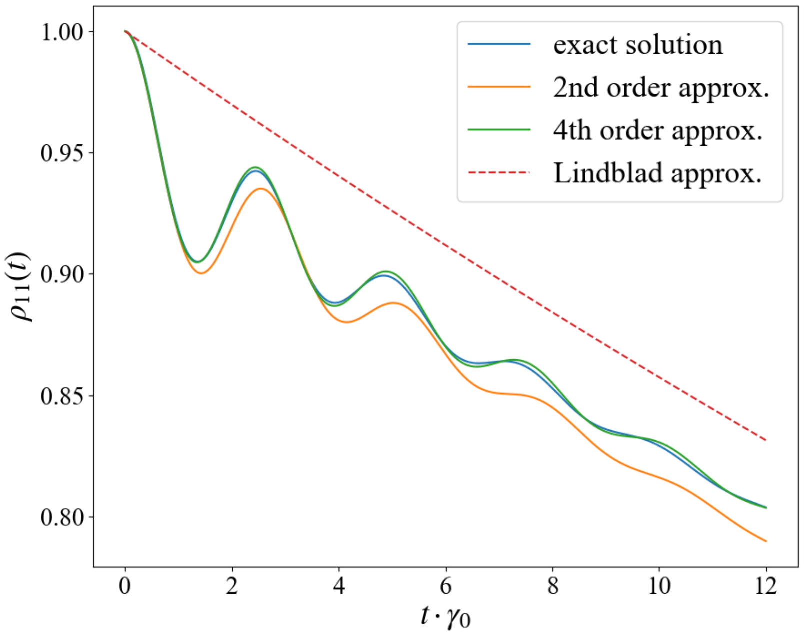



Figure 1. the time evolution of the survival probability

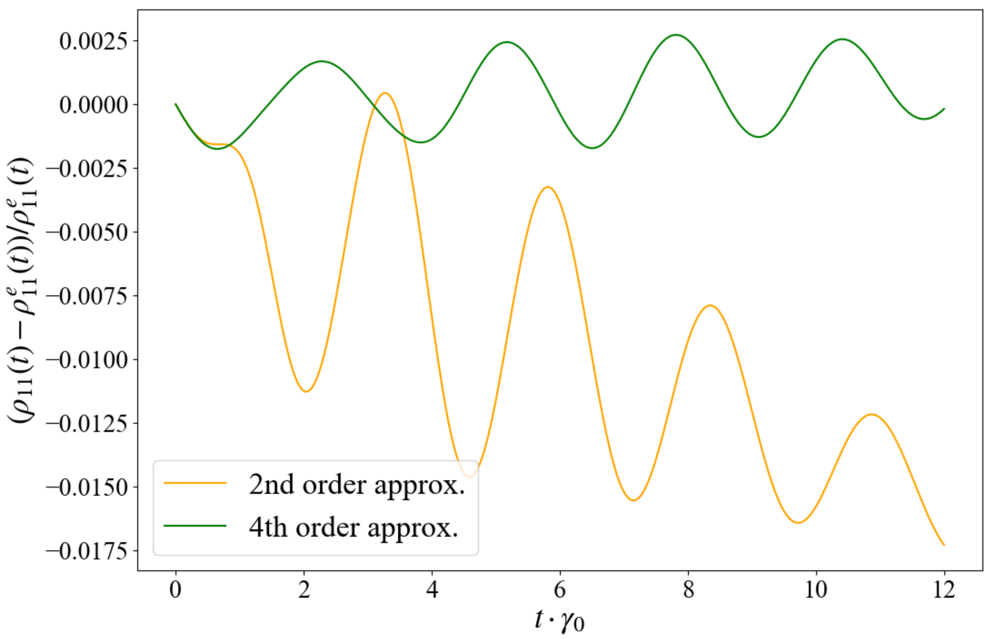



Figure 2. error with respect to the exact solution

# Summary

➢The master equation obtained by the fourth-order perturbative expansion reproduced the exact solution with good accuracy

➢For next step, I want to perform simulations using an actual quantum computer (or a simulator)

system size : two qubit
depth : not clear