# Tensor Networks and the simulation of LGT

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#### tensor network



#### tensor network



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TNS: entanglement-based ansatzes for quantum many-body states

$$|\Psi\rangle = \sum_{i_j} c_{i_1...i_N} |i_1...i_N\rangle$$

$$|\Psi\rangle = \sum_{i_j} c_{i_1\dots i_N} |i_1\dots i_N\rangle$$

## TNS: restricted family







random state not close to product



## random state not close to product



## random state not close to product





## random state not close to product





area law  $S_{A\max} \propto |\delta A|$ ground states of local Hamiltonians thermal equilibrium Hastings 2007 Calabrese, Cardy 2004

Wolf, Verstraete, Hastings, Cirac, 2008







#### otherTNS

#### critical ID correlations



otherTNS

critical ID correlations





a side comment

tensor networks may also describe partition functions (observables)



Nishino, JPSJ 1995 Levin & Wen PRL 2008 Xie et al PRL2009; Zhao et al PRB 2010 a side comment

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formal approach

classify tensors (symmetries)

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efficient algorithms for GS, low excited states, thermal, dynamics

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entanglement: crucial ingredient to understand QMB systems using TNS for QFT



Non-perturbative for Hamiltonian systems

Extremely practical (and successful) for ID systems (MPS)

Promising improvements for higher dimensions

ground states low-lying excitations thermal states time evolution



## there is long way to go until LQCD

## journey begins with I+ID steps

Photo by Maria Teneva on Unsplash

#### early works with DMRG/TNS

Byrnes PRD2002; Sugihara NPB2004 Tagliacozzo PRB2011; Sugihara JHEP2005 Meurice PRB2013

MCB, K. Cichy 1910.00257 QTFLAG Collab.1911.00003

#### early works with DMRG/TNS

Byrnes PRD2002; Sugihara NPB2004 Tagliacozzo PRB2011; Sugihara JHEP2005 Meurice PRB2013

#### Schwinger model U(1) in ID precise equilibrium simulations, feasibility of QSim

MCB et al JHEP11(2013)158; Rico et al PRL 2014; Buyens et al. PRL 2014; Kühn et al., PRA 90,042305 (2014); MCB et al PRD 2015, Buyens et al. PRD 2016; Pichler et al. PRX 2016; review Dalmonte, Montangero, Cont. Phys. 2016 MCB, Cichy, Cirac, Jansen, Kühn, arXiv:1810.12838

MCB, K. Cichy 1910.00257 QTFLAG Collab.1911.00003

#### 3+1 dimensions

Magnifico et al. Nat. Com. 12, 3600 (2021)

#### 2+1 dimensions

Felser et al. PRX10, 041040 (2020) Robaina et al. PRL126, 050401 (2021) Emonts et al. PRD102, 074501 (2020)

### Non-Abelian in ID

#### string breaking dynamics

S. Kühn et al., JHEP 07 (2015) 130; Silvi et al., Quantum 2017 S. Kühn et al. PRX 2017

> SU(3)QLM Silvi et al, PRD 2019

#### finite density

S. Kuehn et al, PRL118 (2017) 071601

#### finite density

#### spectrum

entropy

## I + I D RESULTS

thermal equilibrium

time evolution

#### finite density

#### spectrum





## I + I D RESULTS

thermal equilibrium

time evolution

#### spectrum

## 

continuum limit: very precise masses for U(1), SU(2) different techniques demonstrate suitability of the ansatz

MCB, Cichy, Cirac, Jansen JHEP11 (2013) 158 Buyens et al. PRL113 (2014) 091601 MCB, Cichy, Cirac, Jansen, Kühn PRX 7, 041046 (2017)



## thermal equilibrium

time evolution

#### finite density

Schwinger model with several flavours and chemical potentials (continuum)

S. Kühn et al, PRLI18 (2017) 071601



#### finite density



Schwinger model with several flavours and chemical potentials (continuum)

S. Kühn et al, PRLI18 (2017) 071601



phase diagram of SU(2) and SU(3) QLM at finite density Silvi et al, Quantum 1, 9 (2017) PRD 100, 074512 (2019) some results in 2+1, 3+1 U(1)


#### thermal equilibrium

time evolution

#### gauge constraints not purely local $\Rightarrow$ not all entropy physical

Casini et al 2014; Gosh et al JHEP 2015 Soni,Trivedi JHEP 2016; van Acoleyen et al PRL 2016

entropy

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entropy

#### divergence in the continuum limit

$$S \propto \frac{c}{6} \log_2 \frac{\xi}{a}$$

Calabrese, Cardy JStatMech 2004

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#### Kühn PRX7, 041046 (2017)

# divergence in the continuum limit $S\propto \frac{c}{6}\log_2\frac{\xi}{a}$

Calabrese, Cardy JStatMech 2004

entropy Schwinger



Buyens PRX6, 041040 (2016)



chiral condensate at finite T: analytical for m/g=0



Sachs, Wipf 92

chiral condensate at finite T: analytical for m/g=0



smooth
restoration of
∞ chiral symmetry

Sachs, Wipf 92

chiral condensate at finite T: analytical for m/g=0



smooth restoration of chiral symmetry Sachs, Wipf 92



PRD 92,034519 (2015); PRD 93,094512 (2016)

chiral condensate at finite T: analytical for m/g=0



 $T \rightarrow 0$ smoothrestoration of $T \rightarrow \infty$ chiral symmetry



PRD 92, 034519 (2015); PRD 93, 094512 (2016)



Sachs, Wipf 92

## beyond ID



area law by construction

Verstraete, Cirac, 2004



efficient numerical algorithms (small spatial dimensions) and good theoretical understanding

non-technical review: Annu Rev. CMP 2023 14:1; arXiv:2205.10345

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work well for GS, low energy, thermal equilibrium related to area laws



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but so much for high energy eigenstates, quenches

Osborne, PRL 2006 Schuch et al., NJP 2008 Vidmar et al., PRL 2017

volume law

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volume law

new tools may allow us to access some of these regimes

## real time

#### real time evolution with MPS

TEBD, t-DMRG Vidal, PRL 2003, 2004 Verstraete, García-Ripoll, Cirac, PRL 2004

#### real time evolution with MPS



TEBD, t-DMRG Vidal, PRL 2003, 2004 Verstraete, García-Ripoll, Cirac, PRL 2004 time evolved state approximated by MPS

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## but entanglement grows





### but entanglement grows





required bond for fixed precision

 $D \sim e^{\alpha t}$ 

### but entanglement grows



#### yet many physical situations (in closed and open quantum systems) can be successfully studied!

García-Ripoll, NJP 2006 Wall, Carr NJP 2012 Paeckel et al arXiv:1901.05824

### yet many physical situations (in closed and open quantum systems) can be successfully studied!

short times, adiabatic, low energy can work well

García-Ripoll, NJP 2006 Wall, Carr NJP 2012 Paeckel et al arXiv:1901.05824

# Standard evolution algorithms for LGT

Reliable for moderate times, or in some setups

Useful for quantum simulation

S. Kühn et al., Phys. Rev. A 90, 042305 (2014) S. Kühn et al., JHEP 07 (2015) 130 Buyens et al., PRL 2014; PRX 2016 Rico et al., PRL 2014; NJP 2014; PRX 2016

No full continuum extrapolation yet, but results near the continuum limit

#### string breaking



T. Pichler,<sup>1</sup> M. Dalmonte,<sup>2,3</sup> E. Rico,<sup>4,5,6</sup> P. Zoller,<sup>2,3</sup> and S. Montangero<sup>1</sup>



#### quench scenario

Interacting vacuum (TI)

## switch on background electric field $\alpha$ $\Rightarrow$ pair production iTEBD evolution

PHYSICAL REVIEW D 96, 114501 (2017)

Real-time simulation of the Schwinger effect with matrix product states

Boye Buyens,<sup>1</sup> Jutho Haegeman,<sup>1</sup> Florian Hebenstreit,<sup>2</sup> Frank Verstraete,<sup>1,3</sup> and Karel Van Acoleyen<sup>1</sup>

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#### quench scenario



#### strong field

PHYSICAL REVIEW D 96, 114501 (2017)

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#### scattering in the Schwinger model

with I. Papaefstathiou, J. Knolle, arXiv:2402.18429



#### PHYSICAL REVIEW B 92, 125136 (2015) Scattering particles in quantum spin chains

Laurens Vanderstraeten,<sup>1</sup> Frank Verstraete,<sup>1,2</sup> and Jutho Haegeman<sup>1</sup>

PHYSICAL REVIEW RESEARCH 3, 013078 (2021)

Real-time scattering of interacting quasiparticles in quantum spin chains

Maarten Van Damme,\* Laurens Vanderstraeten, Jacopo De Nardis, Jutho Haegeman, and Frank Verstraete

#### PRX QUANTUM 3, 020316 (2022)

#### **Collisions of False-Vacuum Bubble Walls in a Quantum Spin Chain**

Ashley Milsted,<sup>1,2,3,4,\*,‡</sup> Junyu Liu,<sup>1,2,†</sup> John Preskill,<sup>1,2,4,‡</sup> and Guifre Vidal<sup>3,5</sup>



uMPS formalism provides ansatz for quasiparticles

### scattering in LGT

PHYSICAL REVIEW X 6, 011023 (2016)



#### **Real-Time Dynamics in U(1) Lattice Gauge Theories with Tensor Networks**

T. Pichler,<sup>1</sup> M. Dalmonte,<sup>2,3</sup> E. Rico,<sup>4,5,6</sup> P. Zoller,<sup>2,3</sup> and S. Montangero<sup>1</sup>

PHYSICAL REVIEW D 104, 114501 (2021)

#### Entanglement generation in (1+1)D QED scattering processes

Marco Rigobello<sup>®</sup>, Simone Notarnicola<sup>®</sup>, Giuseppe Magnifico<sup>®</sup>, and Simone Montangero<sup>®</sup>

notice also: Surace, Lerose, New J. Phys. 23 (2021) 062001 Vovrosh et al. PRX Quantum 3, 040309 (2022) Su, Osborne, Halimeh arXiv:2401.05489



we are interested in simulation of (inelastic) scattering

#### inelastic scattering in the Schwinger model

collision of two vector mesons can produce two scalars



Gaussian wavepacket with momentum k

Papaefstathiou, Knolle, MCB, arXiv:2402.18429
collision of two vector mesons can produce two scalars



### Gaussian wavepacket with momentum k

### probe the inelastic threshold

strong coupling regime

collision of two vector mesons can produce two scalars



inelastic threshold

continuum 
$$2\sqrt{p^2 + M_V^2} = 2\sqrt{M_S^2}$$

collision of two vector mesons can produce two scalars



inelastic threshold

continuum 
$$2\sqrt{p^2 + M_V^2} = 2\sqrt{M_S^2}$$
  
lattice  $k = p/(g\sqrt{x})$   
 $x = 1; N = 100; \mu = 2 \cdot 10^{-5} \Rightarrow k_{\text{thr}} = 1.12$ 

collision of two vector mesons can produce two scalars



#### observables (mostly local)

entropy of two sites4-fermion projector (strong coupling)electric flux correlator (not local)

### below momentum threshold

### entropy of two sites



### below momentum threshold

### entropy of two sites



### above momentum threshold

### entropy of two sites



### above momentum threshold



### above momentum threshold









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# TNS can be a suitable ansatz also for LGT/QFT

- real time is more challenging than equilibrium standard methods limited to short times current results: inelastic scattering Papaefstathiou, arXiv:2402.18429 momentum threshold observed
  - production detected through local observables
- other models: Thirring on lattice DQPT arXiv:2407.11295 further directions
  - challenge: continuum limit









Schwinger model PDFs Schneider, Cichy, MCB, Lin, LATTICE'24, arXiv:2409.16996

real time is more challenging than equ standard methods limited to short tin current results: inelastic scattering Papaefstath momentum threshold observed production detected through local o other models: Thirring on lattice DQF further directions

challenge: continuum limit



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Thanks for your attention!

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### alternative strategies

## Recent work...

# exploring spectral properties of quantum many-body systems

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# exploring spectral properties of quantum many-body systems

spectral properties of a QMB Hamiltonian

> Yang, Iblisdir, Cirac, MCB, PRL124, 100602 (2020)

Lu, MCB, Cirac PRX Quantum 2, 02032 (2021) Çakan, Cirac, MCB PRB 103, 115113 (2021) Yang, Cirac, MCB arxiv:2204.09439

## Recent work...

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spectral properties of a QMB Hamiltonian

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Lu, MCB, Cirac PRX Quantum 2, 02032 (2021) Çakan, Cirac, MCB PRB 103, 115113 (2021) Yang, Cirac, MCB arxiv:2204.09439 generalized density of states

> can be connected to equilibrium and nonequilibrium properties

# DoS of Schwinger model

### large lattice spacing

small lattice spacing



near Gaussian shape

I. Papaefstathiou et al., PRD104, 014514 (2021)

Long-range entanglement and equilibration



# back to quasiparticle intuition



Frías-Pérez, Tagliacozzo, MCB arXiv:2308.04291

# back to quasiparticle intuition



Frías-Pérez, Tagliacozzo, MCB arXiv:2308.04291

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# long-range entanglement $\Rightarrow$ mixture



# long-range entanglement $\Rightarrow$ mixture



# long-range entanglement $\Rightarrow$ mixture



long range contribute as mixture to local observables in neighbouring blocks

# long-range entanglement ⇒ mixture



 $\rho_{L_2R_2} \Rightarrow \rho_{L_2} \otimes \rho_{R_2}$ 





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TNS methods for HEP...





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### TNS methods for HEP...

finite density

- related to equilibrium simulations
- thoroughly tested in I+ID spectrum, thermal equilibrium

Abelian and non-Abelian models





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real time

- in general more challenging
- methods exist
- limitations for out-of-equilibrium: not yet completely understood





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### TNS methods for HEP...

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real time is more challenging

uses in QFT/LGT: DQPTs, pair production scattering PRD100,094504 (2019)

limited to short times

MCOS






MCOS





## To conclude



## TNS can be a suitable ansatz also for LGT/QFT

real time is more

challenging

MCOS

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uses in QFT/LGT: DQPTs, pair production scattering PRD100, 094504 (2019)

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changing entanglement perspective: transforming long-range entanglement into mixture Frías-Pérez, Tagliacozzo, MCB, arXiv:2308.04291







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