Orthopositronium $\rightarrow 3\gamma$

Decay at Rest

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Polarization ?

– Orthopositronium spin-one triplet state

– Photons "vector" particles

Unpolarized Ore and Powell [1] May 2024 talk

– randomly polarized Orthopositronium

– sum on the polarization state of the final photons

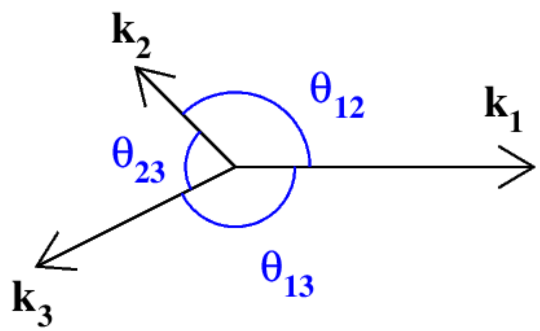
⇒ G4OrePowellAtRestModel , 11.3-beta-01 , Igor & Denis

Polarized Ore and Powell [1], Drisko [2] July 2024 EM meeting

⇒ G4PolarizedOrePowellAtRestModel , , Igor & Denis

Random Polarization: Differential decay rate

Sum on the Ps triplet states and on the γ polarizations,



Probability per unit time,

$$P \propto \left[(1-\cos(\theta_{23}))^2 + (1-\cos(\theta_{31}))^2 + (1-\cos(\theta_{12}))^2\right] \mathrm{d}k_1 \mathrm{d}k_2 \mathrm{d}\Omega_1 \quad (1)$$

A. Ore and J. L. Powell [1]

Inclusive photon spectrum

Momentum and energy conservation

$$\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0 \tag{2}$$

$$k_1 + k_2 + k_3 = 2m \tag{3}$$

m, electron mass. Integrating on k_2, inclusive photon spectrum:

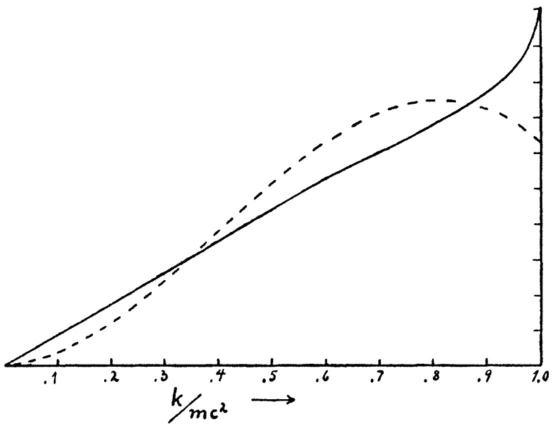
$$F(k_1) = 2\left[\frac{k_1(m-k_1)}{(2m-k_1)^2} - \frac{2m(m-k_1)^2}{(2m-k_1)^3}\ln\frac{m-k_1}{m} + \frac{2m-k_1}{k_1} + \frac{2m(m-k_1)}{k_1^2}\ln\frac{m-k_1}{m}\right] \tag{4}$$

that is, with $x \equiv k_1/m$,

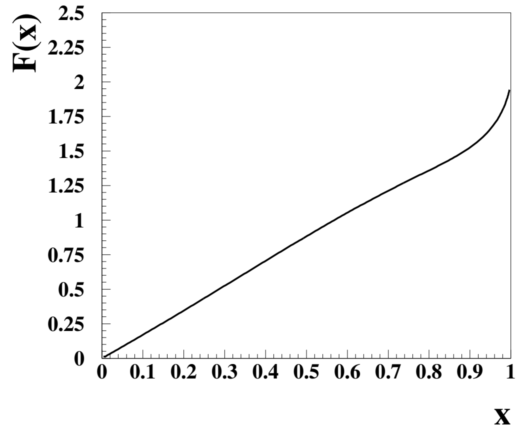
$$F(x) = 2\left[\frac{x(1-x)}{(2-x)^2} - \frac{2(1-x)^2}{(2-x)^3}\ln(1-x) + \frac{2-x}{x} + \frac{2(1-x)}{x^2}\ln(1-x)\right] \tag{5}$$

A. Ore and J. L. Powell [1]

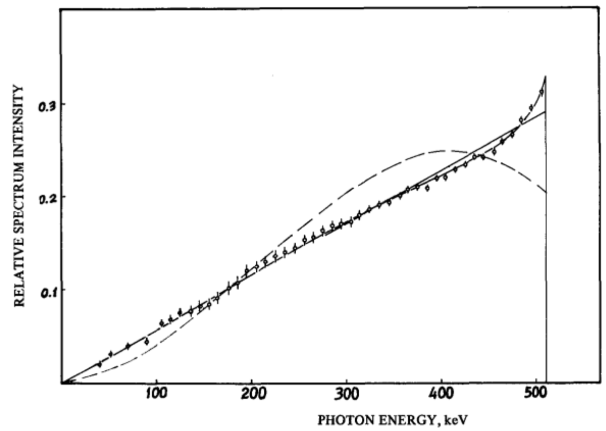
Inclusive γ energy spectrum



Ore & Powell [1].



$F(x)$ (eq. (5)).



Phys. Lett. B **157** (1985) 357

Right: comparison of experimental energy spectrum measured by Chang et al. Phys. Lett. B **157** (1985) 357 with Ore & Powell [1].

Event Generation; 1

– Three coplanar photons; decay computed in that plane

eq. (1)

$$P = c\left[(1-\cos(\theta_{23}))^2 + (1-\cos(\theta_{31}))^2 + (1-\cos(\theta_{12}))^2\right] \mathrm{d}k_1\mathrm{d}k_2\mathrm{d}\Omega_1$$

– k_1 **along** $\vec{x}$, so eq. (1) reduces to a twice differential equation (of k_1, k_2).

– **Take k_1 and k_2 at random, flat.**

– $k_3 = 2m - k_1 - k_2$. Request $k_3 > 0$.

Event Generation; 2

– Momentum conservation, eq. (2), projecting $\parallel$ and $\perp$ to $\vec{x}$, that is, to $\vec{k}_1$.

$$k_1 + k_2\, c_{12} + k_3\, c_{13} = 0$$

$$0 + k_2\, s_{12} + k_3\, s_{13} = 0$$

with $c_{ij} \equiv \cos\theta_{ij}$, $\quad s_{ij} \equiv \sin\theta_{ij}$, $\quad \theta_{ij} \equiv (\vec{k}_i, \vec{k}_j)$. Obtain cosines from energies

$$c_{12} = \frac{k_3^2 - k_1^2 - k_2^2}{2k_1k_2} \quad , \quad c_{13} = \frac{k_2^2 - k_1^2 - k_3^2}{2k_1k_3}.$$

– **Request** $|c_{1i}| \le 1$, $i = 2, 3$. $\qquad$ k_i found to be in the range $[0, m]$

– Define sign convention as $\theta_{12} \ge 0, \quad \theta_{13} \le 0$.

– Compute θ_{23} from the scalar product of $\vec{k}_2$ and $\vec{k}_3$,

$$c_{23} = c_{12}\, c_{13} + s_{12}\, s_{13} \tag{6}$$

– Generate events from Ore & Powell factor eq. (1)

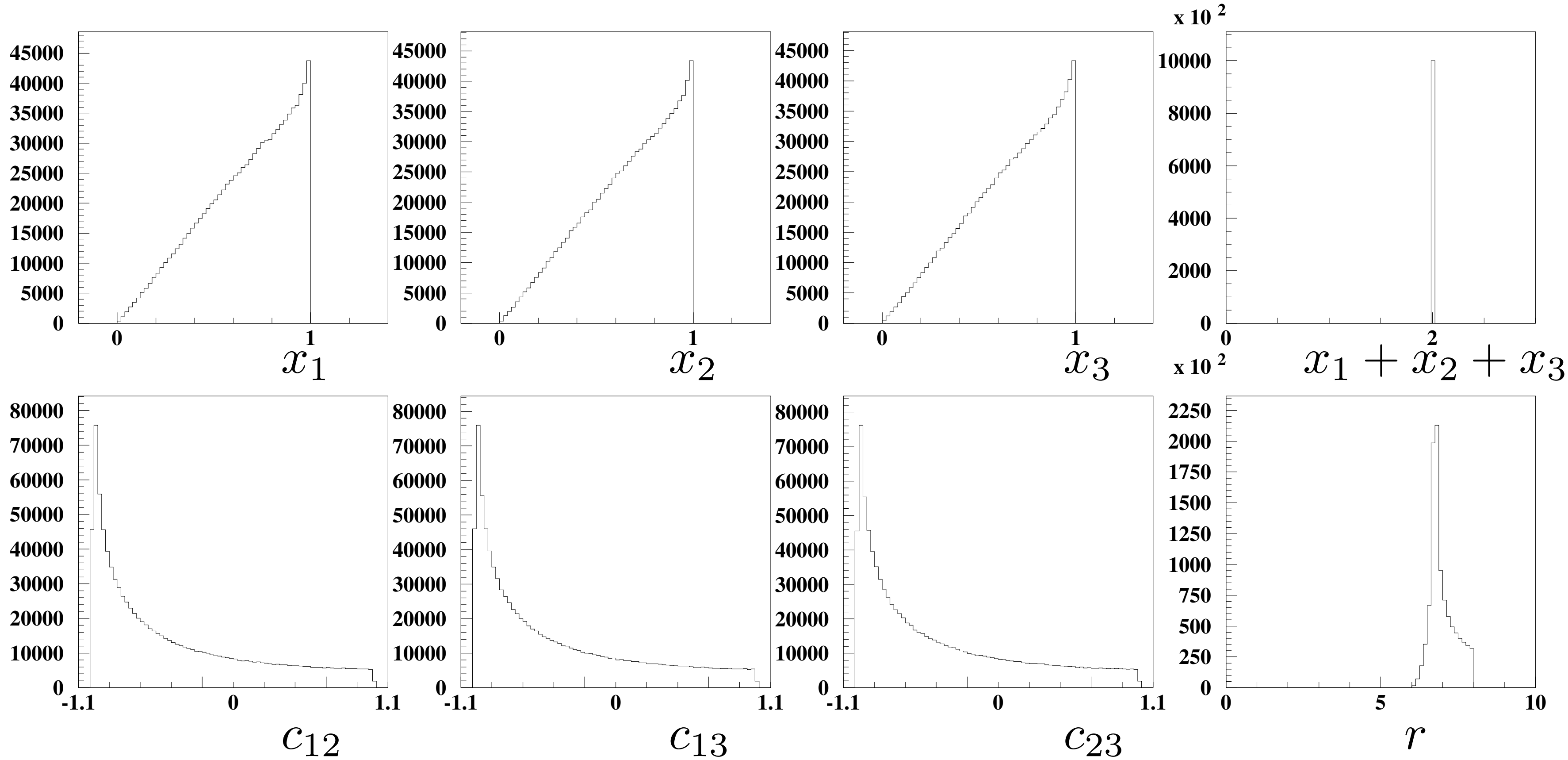
$$r = \left[(1 - \cos(\theta_{23}))^2 + (1 - \cos(\theta_{31}))^2 + (1 - \cos(\theta_{12}))^2 \right] \tag{7}$$

Comments

– No need to compute the limits of the Dalitz area

– No divergences

Random Polarization: Results



Inclusive photon spectra OK.

- $x_i \leq 1 \quad (k_i \leq m)$
- $r \leq 8.$

Polarized Model

P becomes proportional to $\left|(\vec{t}_1 + \vec{t}_2 + \vec{t}_3) \cdot \vec{u}\right|^2 \mathrm{d}k_1 \mathrm{d}k_2 \mathrm{d}\Omega_1$,

- $\vec{u}$, positronium polarization, complex unit vector
- $\vec{t}_1 = \vec{a}_1(\vec{a}_2.\vec{a}_3) - \vec{a}_2(\vec{a}_3.\vec{a}_1) - \vec{a}_3(\vec{a}_1.\vec{a}_2) + \vec{a}_1(\vec{a}\,'_2.\vec{a}\,'_3) - \vec{a}\,'_2(\vec{a}\,'_3.\vec{a}_1) - \vec{a}\,'_3(\vec{a}_1.\vec{a}\,'_2)$,

  $\vec{t}_2$ and $\vec{t}_3$ obtained by permutation of the indices.

  The first two terms cancel upon the sum $(\vec{t}_1 + \vec{t}_2 + \vec{t}_3)$, so $\vec{t}_1$ can be computed as

  $\vec{t}_1 = -\vec{a}_3(\vec{a}_1.\vec{a}_2) + \vec{a}_1(\vec{a}\,'_2.\vec{a}\,'_3) - \vec{a}\,'_2(\vec{a}\,'_3.\vec{a}_1) - \vec{a}\,'_3(\vec{a}_1.\vec{a}\,'_2)$.
- $\vec{a}_i$ is the polarization vector of photon i,
- $\vec{a}\,'_i = \vec{a}_i \times \vec{n}_i, \quad \vec{n}_i = \vec{k}_i / k_i$.

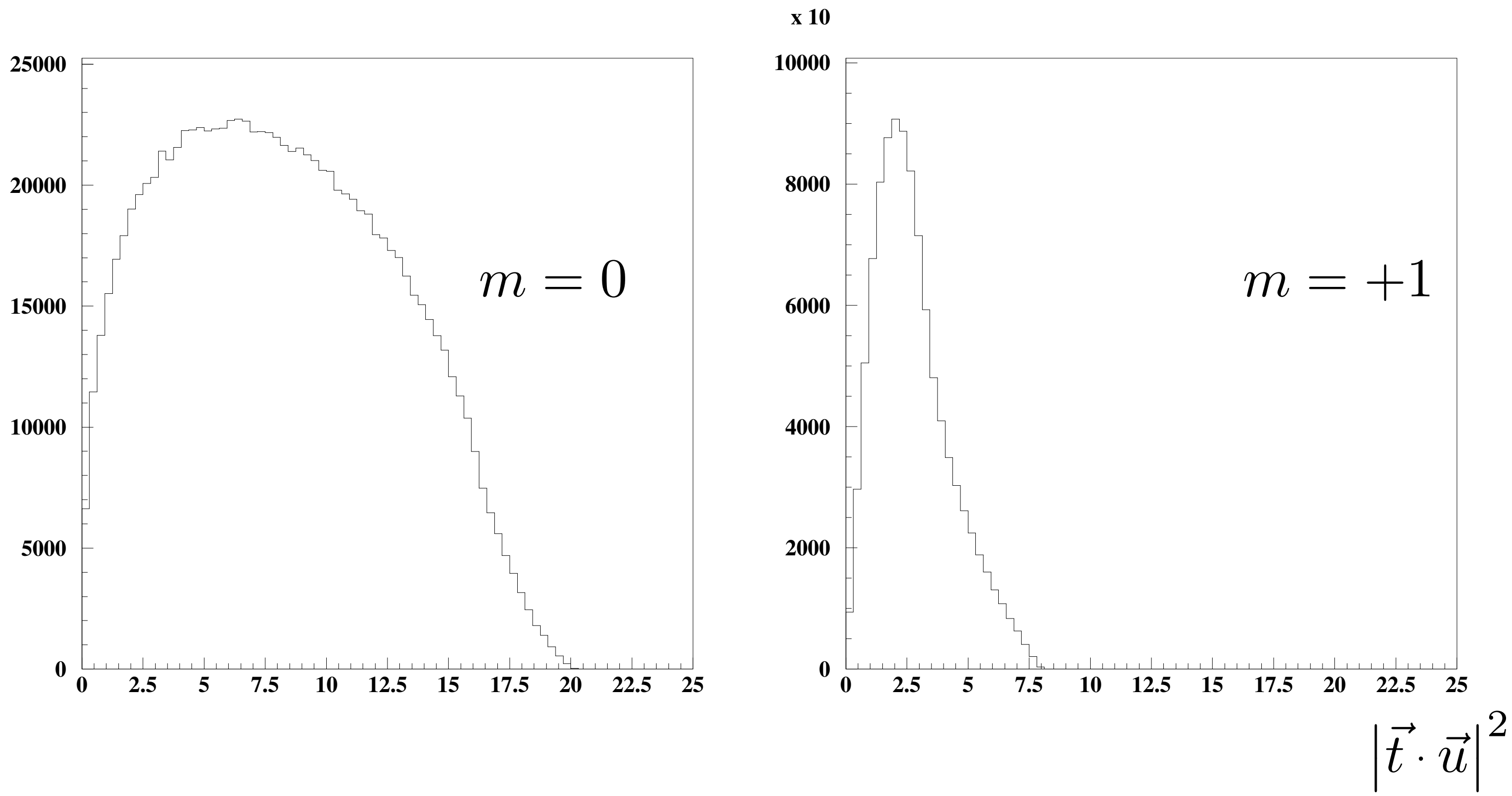
If quantization axis perpendicular to decay plane,

- $m = 0$, $\vec{u} = (0, 0, 1)$
- $m = +1$, $\vec{u} = (1, i, 0)/\sqrt{2}$
- $m = -1$, $\vec{u} = (1, -i, 0)/\sqrt{2}$.

Event Generation

- Depending on m, fill $\vec{u}$.
- Browse the Dalitz plane as for the non polarized case
- Photon i;
  - compute $\vec{n}_i$, direction vector
  - compute $\vec{m}_i$, perpendicular to $\vec{n}_i$ in the horizontal plane,
  - generate polarization vector $\vec{a}_i = \cos(\phi_i)\vec{m}_i + \sin(\phi_i)\vec{z}$, ϕ_i at random flat
  - compute $\vec{a}\,'_i = \vec{a}_i \times \vec{n}_i$;
- Compute $\vec{t} = (\vec{t}_1 + \vec{t}_2 + \vec{t}_3)$, $\quad \left|\vec{t} \cdot \vec{u}\right|^2$;
- Shoot events from pdf proportional to $\left|\vec{t} \cdot \vec{u}\right|^2$.

pdfs

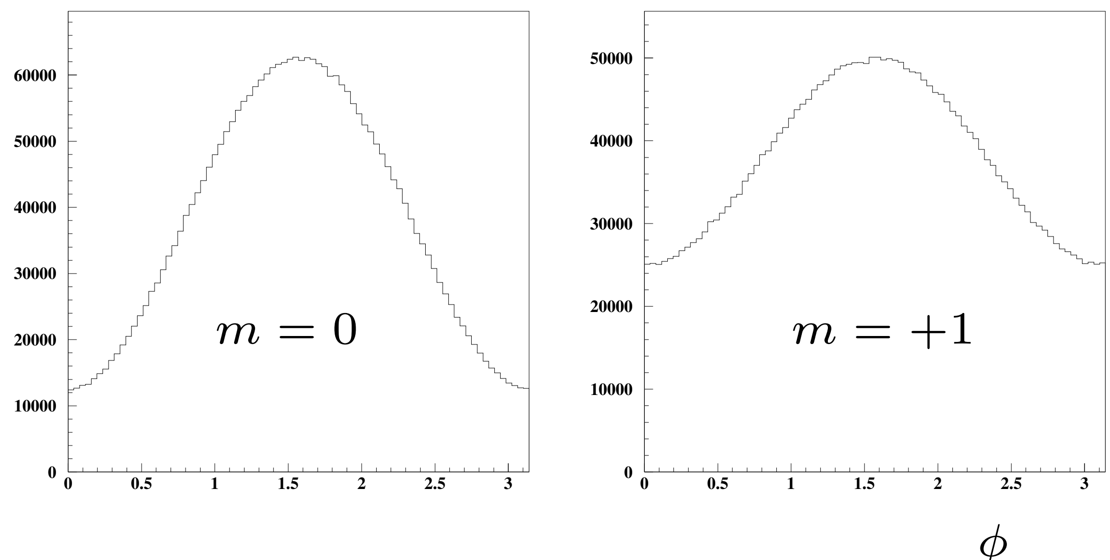

x 10
25000
20000
15000
10000
5000
0
0 2.5 5 7.5 10 12.5 15 17.5 20 22.5 25
$m = 0$
10000
8000
6000
4000
2000
0
0 2.5 5 7.5 10 12.5 15 17.5 20 22.5 25
$m = +1$
$\left|\vec{t} \cdot \vec{u}\right|^2$

Validation: (1) quantization axis normal to decay plane; at equipartition

$\theta_{12} = \theta_{23} = \theta_{13} = 120^\circ$, $k_1 = k_2 = k_3 = 2m/3$,

Drisko [2]: $\dfrac{\text{number of quanta polarized perpendicular to the plane of the quanta}}{\text{number polarized in the plane of detection of the quanta}}$ is

- 5:1 when the positronium atom annihilates from the triplet $m = 0$ state,
- 2:1 for the $m = \pm 1$ states, and
- 3:1 for unpolarized positronium



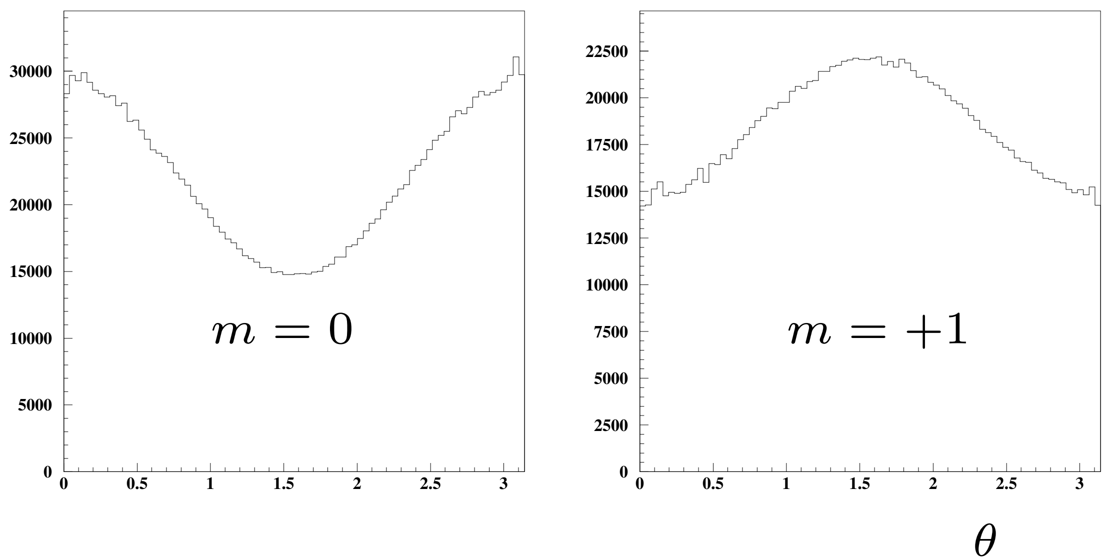
3:1 corresponds to a photon polarization fraction
$P = (N_+ - N_-)/(N_+ + N_-) = (3 - 1)/(3 + 1) = 0.5$, (Tab. 2 of [4], Tab. 1 of [3]).

Validation: (2) With Arbitrary Orientation

Distribution of θ, angle between the normal to the plane and the quantization axis,

divided by $\sin\theta$.

$$\frac{1}{\sin\theta}\frac{\mathrm{d}N}{\mathrm{d}\theta}$$



m	$\frac{16\pi}{3}\frac{1}{\Gamma}\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega}$
± 1	$(3 - \cos^2\theta)/2$
0	$1 + \cos^2\theta$

, with $\mathrm{d}\Omega = \sin\theta\,\mathrm{d}\theta$.

(Table 4 of W. Bernreuther and O. Nachtmann Z.Phys.C 11 (1981) 235)

Conclusion

- Generation of Orthopositronium $\rightarrow 3\,\gamma$ at rest explored with mock-up code.
  - Random polarization
  - Polarized
- Validation exercices include check
  - Inclusive energy spectra
  - Angle distributions (polarized case)
- G4 Physics Models developed (Igor & Denis)
  - G4OrePowellAtRestModel (11.3-beta-01)
  - G4PolarizedOrePowellAtRestModel
    (m generated random $(-1, 0, 1)$ with 1:1:1 proportion at the moment)

  Same validation exercices performed during development.

Back-up

CPU time per event

On a DELL PRECISION 7520 with Fortran (Ubuntu 7.5.0-3ubuntu1 18.04)

Randomly polarized		0.22	μs
Polarized	$m = 0$	3.17	
	$m = 1$	6.21	

pdf normalization factors used in the acceptance-rejection method

- $f = 8.1$ (randomly polarized)
- $f = 22.$ (polarized).

References

[1] A. Ore and J. L. Powell, "Three photon annihilation of an electron - positron pair," Phys. Rev. **75** (1949) 1696

[2] "Spin and Polarization Effects in the Annihilation of Triplet Positronium", R. M. Drisko, Phys. Rev. 102, 1542 1956

[3] "Measurement of photon polarization from the three-photon annihilation of orthopositronium", JB Ye, BZ Yang, XW Tang, Q Zhu, Physics Letters A 133, 1988, 309

[4] "Polarization effects in the decay of orthopositronium to three photons", Frank M. Abel, Gregory S. Adkins, and Theodore J. Yoder" Phys. Rev. A 83, 062502 2011 ,