# *On the Gamma-Ray Conversion pdf: Towards a Direct 5D Event Generation*

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29rd Geant4 Collaboration Meeting

EM WG

Plan of activity for 2024

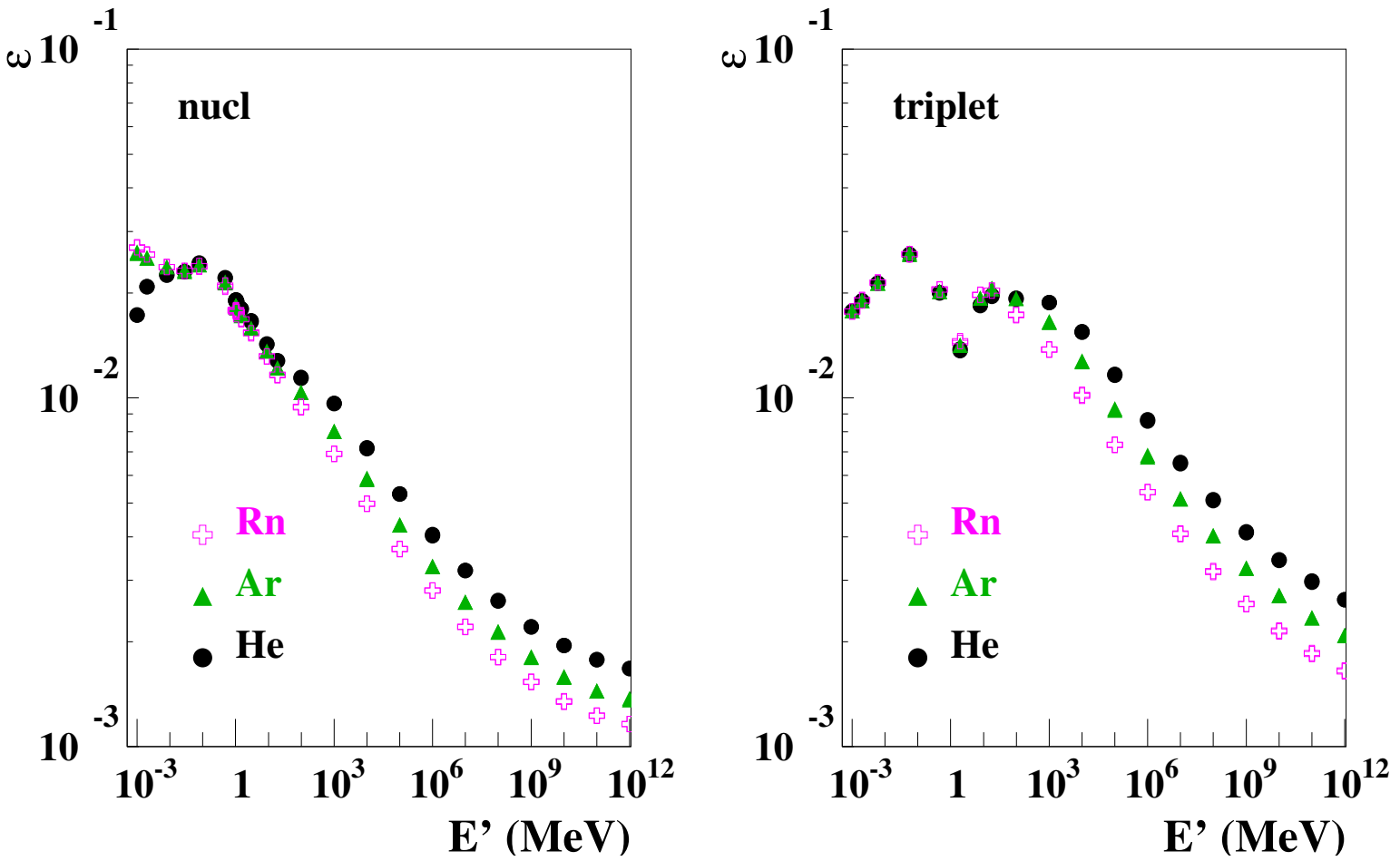
Development of a new 5D angular generator for e+e- pair production.

# *A Critical View on G4BetheHeitler5DModel*

- Slow

- Approximations
  - LPM effect on the final state pdf not taken into account
  - Nucleus finite size not taken into account
    (no high-$q^2$ suppression in $\gamma \to \mu^+\mu^-$)
  - Coulomb interaction effects on the final state pdf not taken into account

# *Generator efficiency*




Generator efficiency as a function of photon $E' = E$-threshold.

Efficiency small, but mild decrease with energy

Nucl. Instrum. Meth. A 899 (2018) 85

# *Bethe-Heitler differential cross section: divergences*

- **Fully linearly polarised** photons

  Berlin and Madansky, Phys. Rev. 78 (1950) 623 and May, Phys. Rev. 84 (1951) 265.

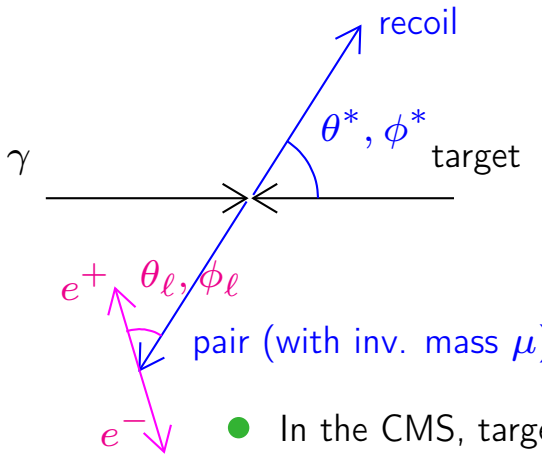
$$
\begin{aligned}
\mathrm{d}\sigma = & \frac{-\alpha Z^2 r_0^2 m^2}{(2\pi)^2 \omega^3} \mathrm{d}E_+ \mathrm{d}\Omega_+ \mathrm{d}\Omega_- \frac{|p_-||p_+|}{|\vec{q}|^4} \left[ \left( 2E_+ \frac{p_- \sin\theta_- \cos\varphi_-}{E_- - p_- \cos\theta_-} + 2E_- \frac{p_+ \sin\theta_+ \cos\varphi_+}{E_+ - p_+ \cos\theta_+} \right)^2 \right. \\
& - q^2 \left( \frac{p_- \sin\theta_- \cos\varphi_-}{E_- - p_- \cos\theta_-} - \frac{p_+ \sin\theta_+ \cos\varphi_+}{E_+ - p_+ \cos\theta_+} \right)^2 \\
& \left. - E^2 \frac{(p_+ \sin\theta_+)^2 + (p_- \sin\theta_-)^2 + 2p_+ p_- \sin\theta_+ \sin\theta_- \cos(\varphi_+ - \varphi_-)}{(E_- - p_- \cos\theta_-)(E_+ - p_+ \cos\theta_+)} \right] .
\end{aligned}
$$

  with: $|\vec{q}|^2 = |\vec{p_+} + \vec{p_-} - \vec{k}|^2$.

- Divergences:
  - For $e^+$ and $e^-$ $\dfrac{1}{(E - p\cos\theta)}$ forward divergence at high energies
  - $\dfrac{1}{q^4}$ small recoil divergence

# *G4BetheHeitler5DModel: Sampling Method*

- Perform each step in appropriate Lorentz frame
  - Center-of-mass system (CMS) boost determined from photon energy $E$ and target mass $M$.
  - Five variables are taken at random, ($\ell$: lepton, that is, $e^+$ or $e^-$)




| variable | name | Lorentz frame |
|---|---|---|
| $\theta^*$ | target and pair polar angle | CMS |
| $\phi^*$ | target and pair azimuthal angle | CMS |
| $\mu$ | $e^+e^-$ invariant mass | |
| $\theta_\ell$ | electron and positron polar angle | pair frame |
| $\phi_\ell$ | electron and positron azimuthal angle | pair frame |

  - In the CMS, target (mass $M$) and pair (mass $\mu$) are back-to-back with opposite momenta.
  - "Decay" of the pair to an electron and a positron performed in the pair Lorentz frame.
  - The lepton 4-vectors are boosted "back" to the CMS.
  - The three final particle 4-vectors are boosted "back" to the laboratory Lorentz frame.
  - The Bethe-Heitler variables are obtained from the 4-vectors.
  - The probability density function (pdf) is computed.

- Final-state phase space normalization for this set of cascade decays:

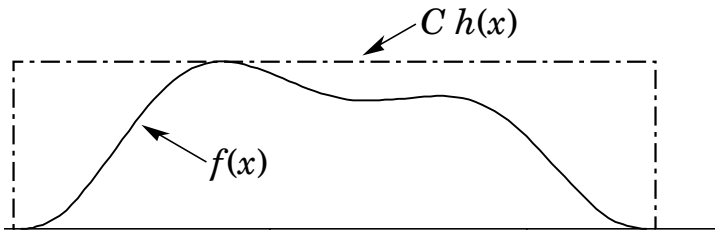
  Review of Particle Physics (Particle Data Group) — See eqs. (1)-(3) in NIM A 899 (2018) 85

- In contrast to Bethe-Heitler we **do conserve energy momentum**, $E = E_- + E_+ + E_r$

# *Change of variables; Acceptance-rejection method*

| | name | | | Jacobian | $x_i$ range |
|---|---|---|---|---|---|
| $\theta^*$ | target and pair polar angle | CMS | $\cos\theta^* = \dfrac{y-1}{1+y},\ y = \exp(x_1)$ | $\dfrac{y}{(1+y)^2}$ | $[x_{1l}, x_{1u}]$ |
| $\mu$ | $e^+e^-$ invariant mass | | $\mu = \mu_{\min} \times (\mu_r)^{x_2^2}$ | $2x_2 \log(\mu_r)\mu$ | $[0,1]$ |
| $\theta_\ell$ | electron and positron polar angle | pair frame | $\cos\theta_\ell = x_3$ | $\lvert\sin\theta_\ell\rvert$ | $[0,\pi]$ |
| $\phi_\ell$ | electron and positron azimuthal angle | pair frame | $\phi_\ell = x_4$ | 1 | $[-\pi,\pi]$ |
| $\phi^*$ | target and pair azimuthal angle | CMS | $\phi^* = x_5$ | 1 | $[-\pi,\pi]$ |

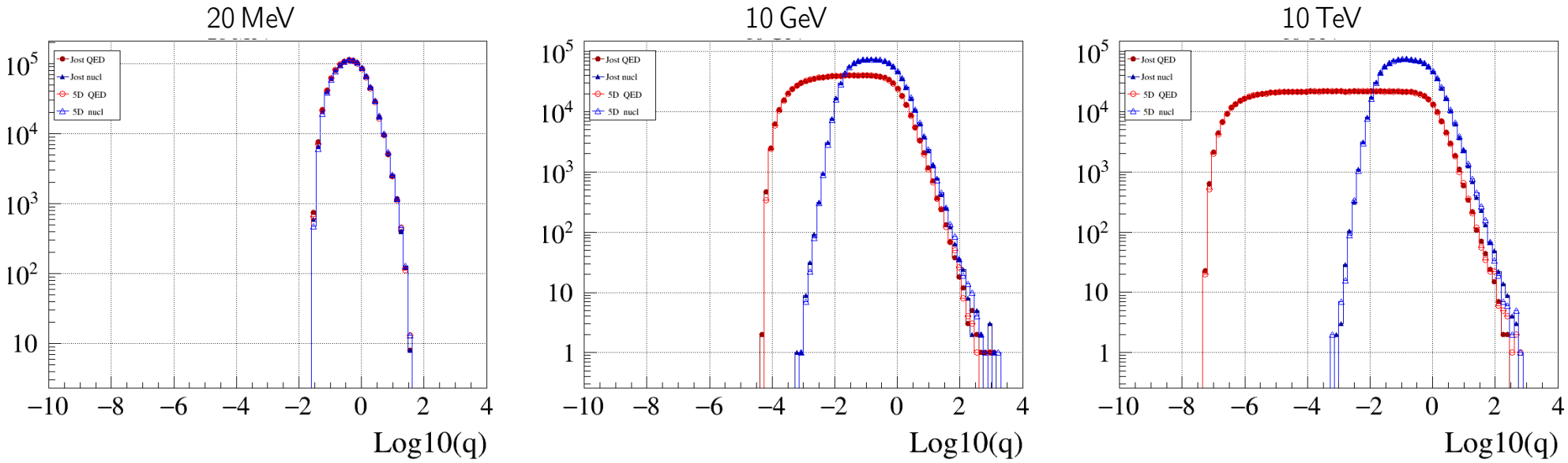
- $x_i$, $i = 1 \cdots 5$ shot flat, this defines a pdf $h(\vec{x})$




- Then use acceptance-rejection method (Von Neumann) PDG
  - Shoot $u$ flat within $[0,1]$
  - If $u \times C \times h(x) \leq f(x)$, keep; else reject.
- $C$ depends on $E$ and target, must be provided

$C \times h(x) \geq f(x)$ must hold for all x !

# $q$ Distribution



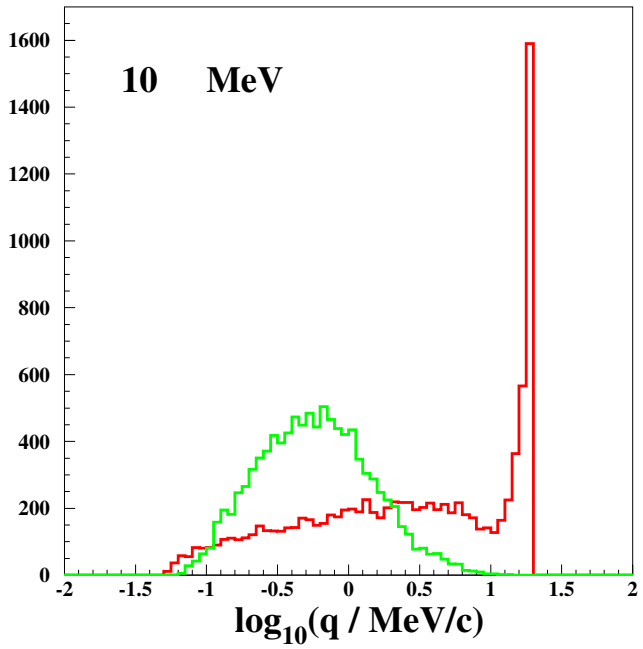

$\log_{10}(q/(\mathrm{MeV}/c))$ distributions for $\gamma$-ray conversions on Argon

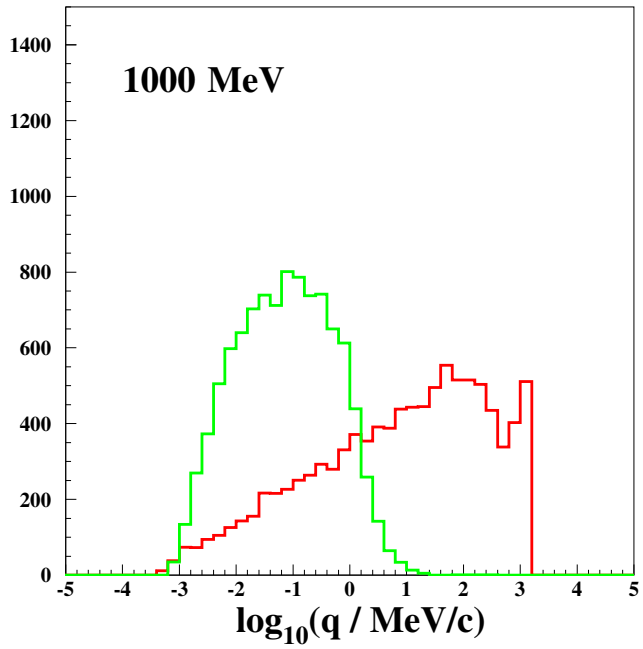
- isolated nucleus, low $q$ divergence further at higher energies
- atom, low-$q$ suppression

Normalization: same number of events.

Semeniouk and Bernard, 24th Geant4 Collaboration meeting, 23-27 Sept. 2019, JLab

# $q$ *Spectra;* $h()$ *vs* $f()$




All events

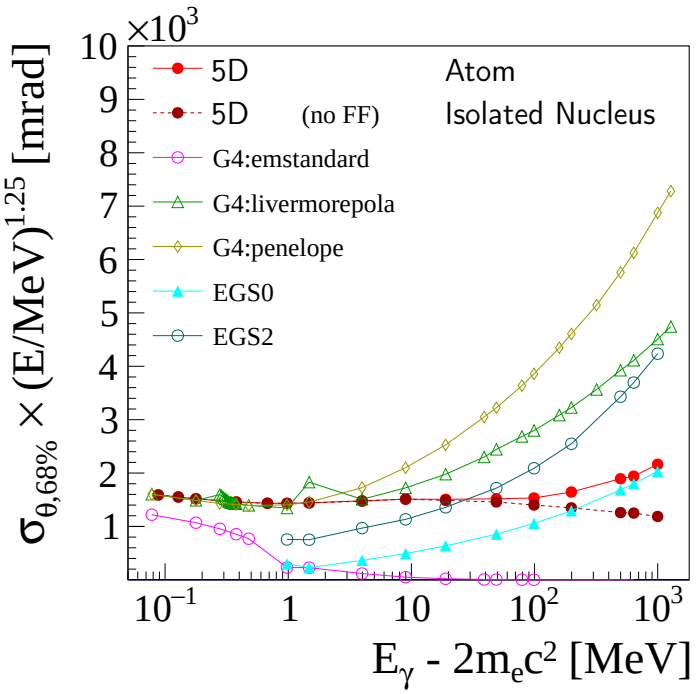
Acceptance-Rejection Method Applied

The pdf "proposed" by $h(x)$ fails to show the low-$q$ divergence of $f(x) \Rightarrow$ low efficiency

# *Previously existing generators*

- Direct generation of the final state (example, $\theta_+$, $\theta_-$, $\phi_+$, $\phi_-$, $E_+/E$)
  - $\theta$, polar angle
  - $\phi$, azimuthal angle
  - $E$, energy
  - $p$, momentum
  - $+$, positron; $-$, electron; $r$, recoil
- No Acceptance-Rejection Method

# *Nucleus Recoil: Photon Polar Angle Resolution*




Cannot measure nucleus recoil:

induced 68 % containment photon polar angle resolution for various event generators.

adapted from P. Gros et al. Astropart. Phys. **88** (2017) 60

# *On Other Models*

| | | | |
|---|---|---|---|
| BetheHeitler | $\theta$, $E$ from Urban - Tsai | coplanar | energy conservation |
| Livermore | $\theta$, $E$ from Urban - Tsai | coplanar | HE momentum conservation |
| Penelope | not coplanar<br>$p(\cos\theta) = (1 - \beta\cos\theta)^{-2}$ | $\theta_+$ $\theta_-$ $\phi_+$ $\phi_-$ sampled independantly | |

Not the full (5D) Bethe-Heitler differential cross section

Either no recoil generated (BetheHeitler, Livermore),

or not distributed as from QED (Penelope)

# *On the Generation of lepton polar angle, $\theta$*

"Leading order multiplicative term of the Sauter-Gluckstern-Hull expression"

A. F. Bielajew, NRCC Report: PIRS-0287R (1994).

$$\frac{\mathrm{d}P}{\mathrm{d}\theta_\pm} = \frac{\sin\theta_\pm}{2p_\pm(E_\pm - p_\pm\cos\theta_\pm)^2}$$

Sampled from $x$, random number uniform on $[0,1]$

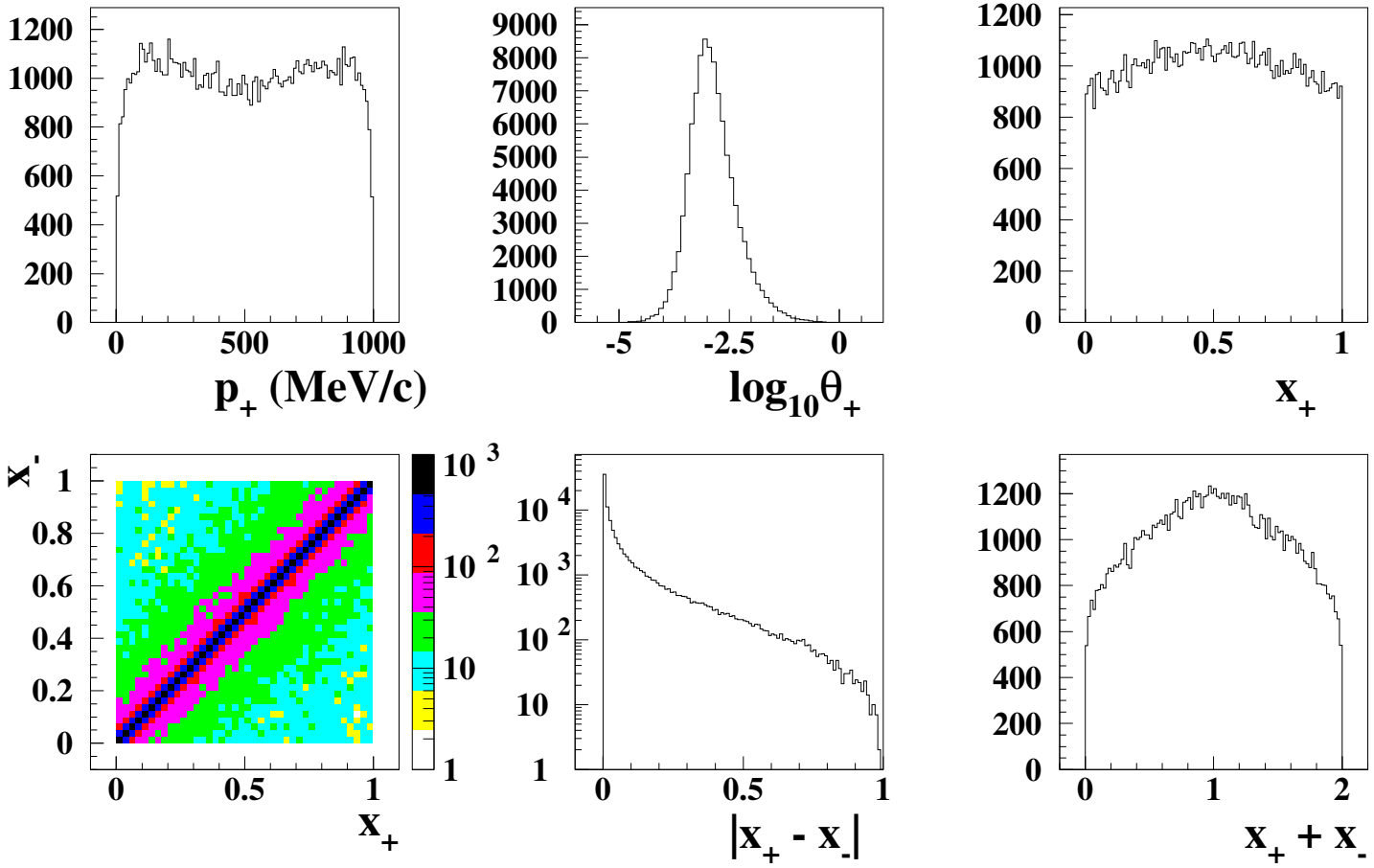
$$\sin\theta_\pm = \frac{\sqrt{x(1-x)}}{p_\pm(2x-1)+E_\pm},\ \cos\theta_\pm = \frac{E_\pm(2x-1)+p_\pm}{p_\pm(2x-1)+E_\pm}$$

$$\text{that is, } x = \left(1 - \frac{\cos\theta_\pm E_\pm - p_\pm}{\cos\theta_\pm p_\pm - E_\pm}\right)/2$$

$$\text{that is, } x = \frac{m^2(\cos\theta_\pm+1)(\cos\theta_\pm p_\pm + E_\pm)}{2(E_\pm+p_\pm)((m\cos\theta_\pm)^2+(E_\pm\sin\theta_\pm)^2)}$$

# *Distributions from 5D Model*

1 GeV $\gamma$-ray conversions on Argon nuclei



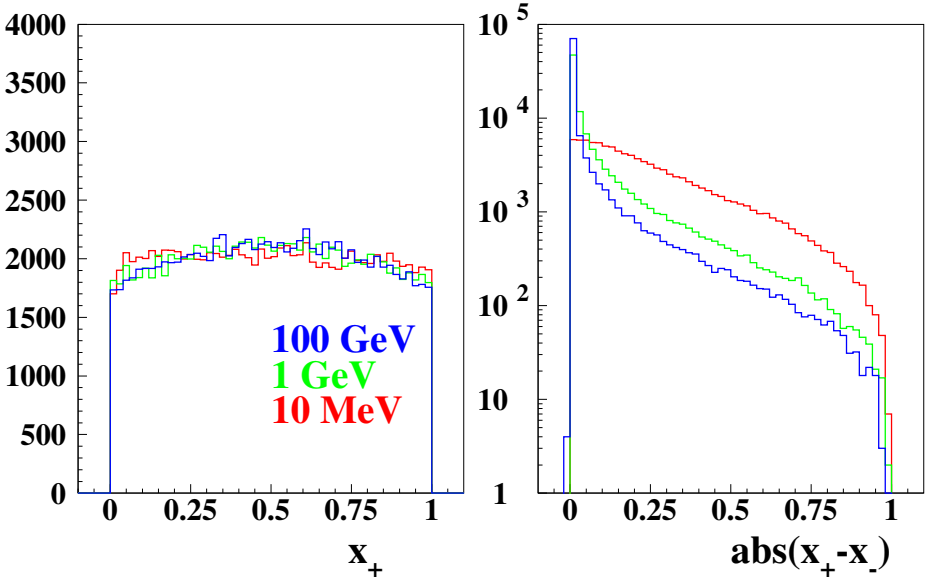

– $x_+$ distribution almost flat

– $x_+$ and $x_-$ strongly correlated.

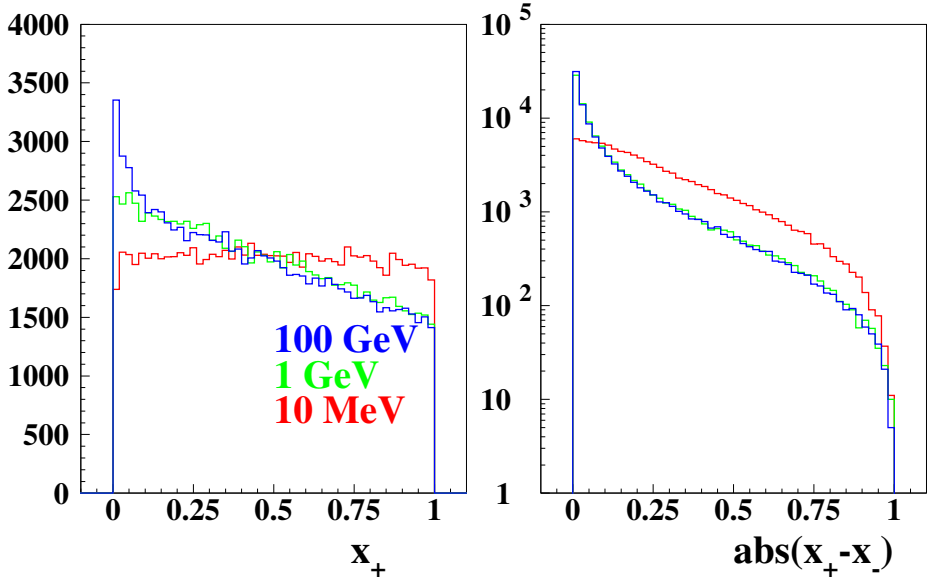
"High efficiency" 5D model must take that $\theta_+, \theta_-$ correlation into account

# *Distributions from 5D Model: $x$, $|x_+ - x_-|$*

Isolated nuclei




Argon atoms

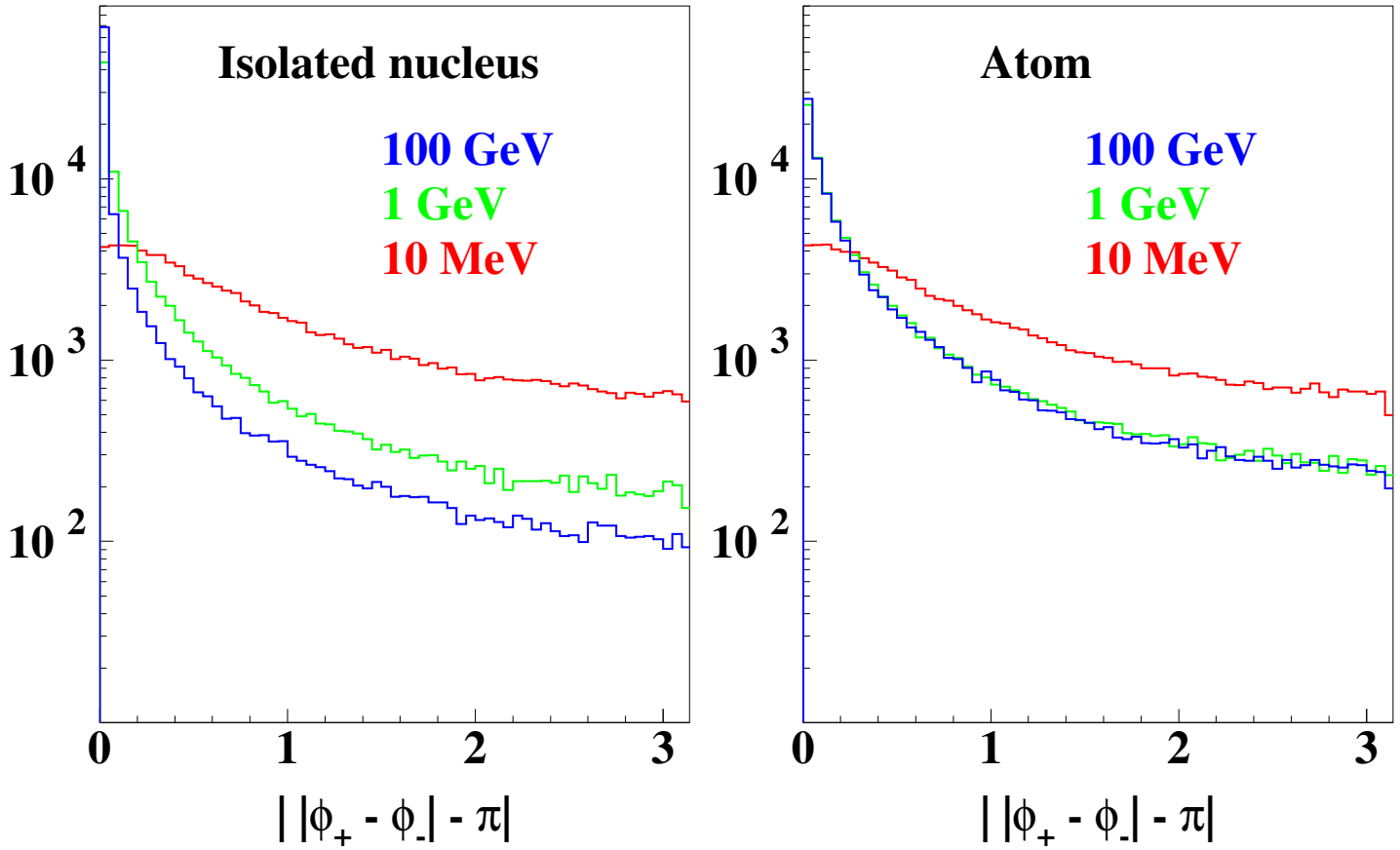



- Isolated nuclei: "universal" $x_+$ distribution, energy-independant
- Atoms: $x_+$ distribution decreases at high energy

  "universal" $|x_+ - x_-|$ high-energy distribution , due to low $q$ suppression.

# *Distributions from 5D Model: $\phi_+ - \phi_-$*

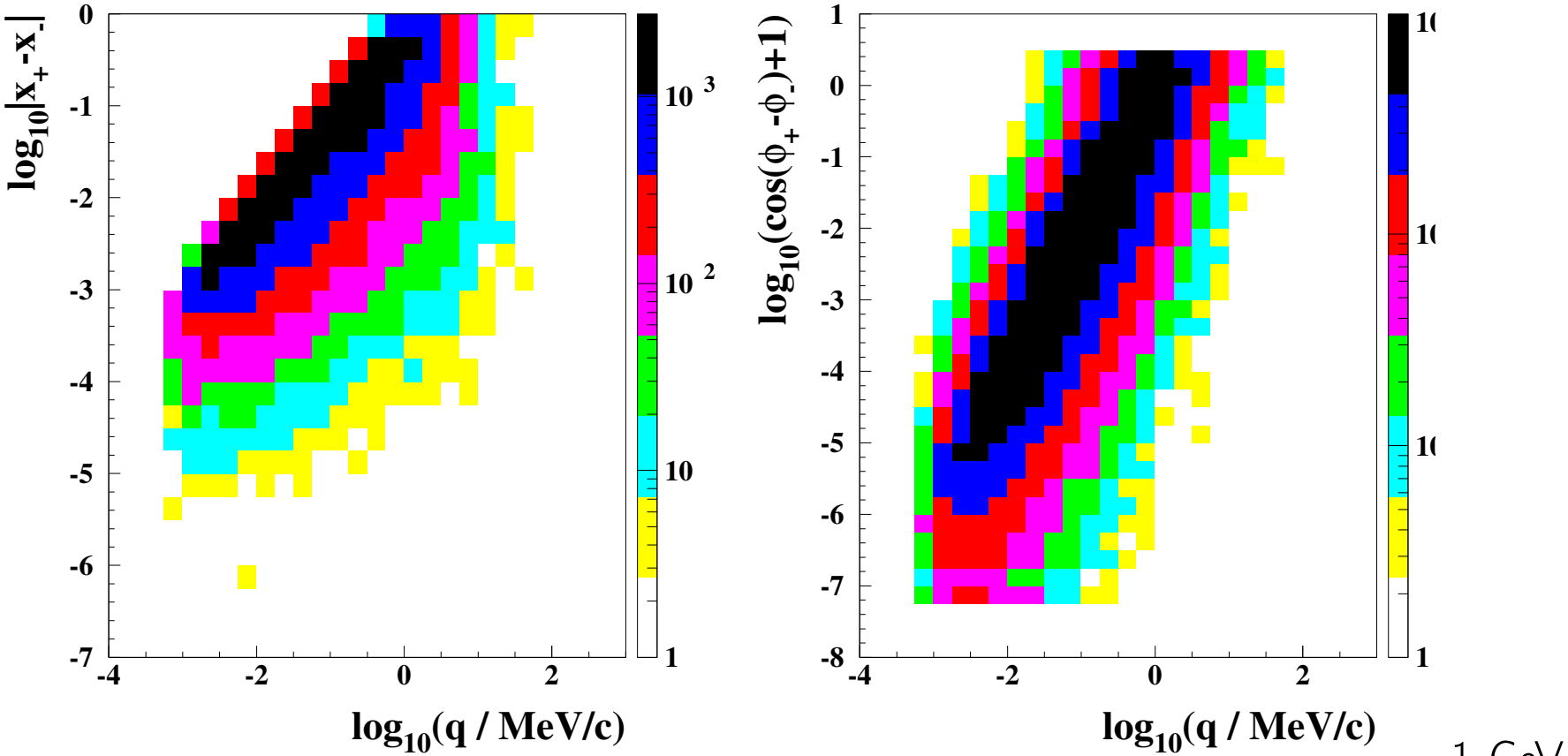
Acoplanarity




– Atoms: "universal" $||\phi_+ - \phi_-| - \pi|$ high-energy distribution , due to low $q$ suppression.

Expected ($\theta_+$ scales like $1/E$, so $p_{\perp,+}$ distribution $E$-invariant, as is $q$ distribution)

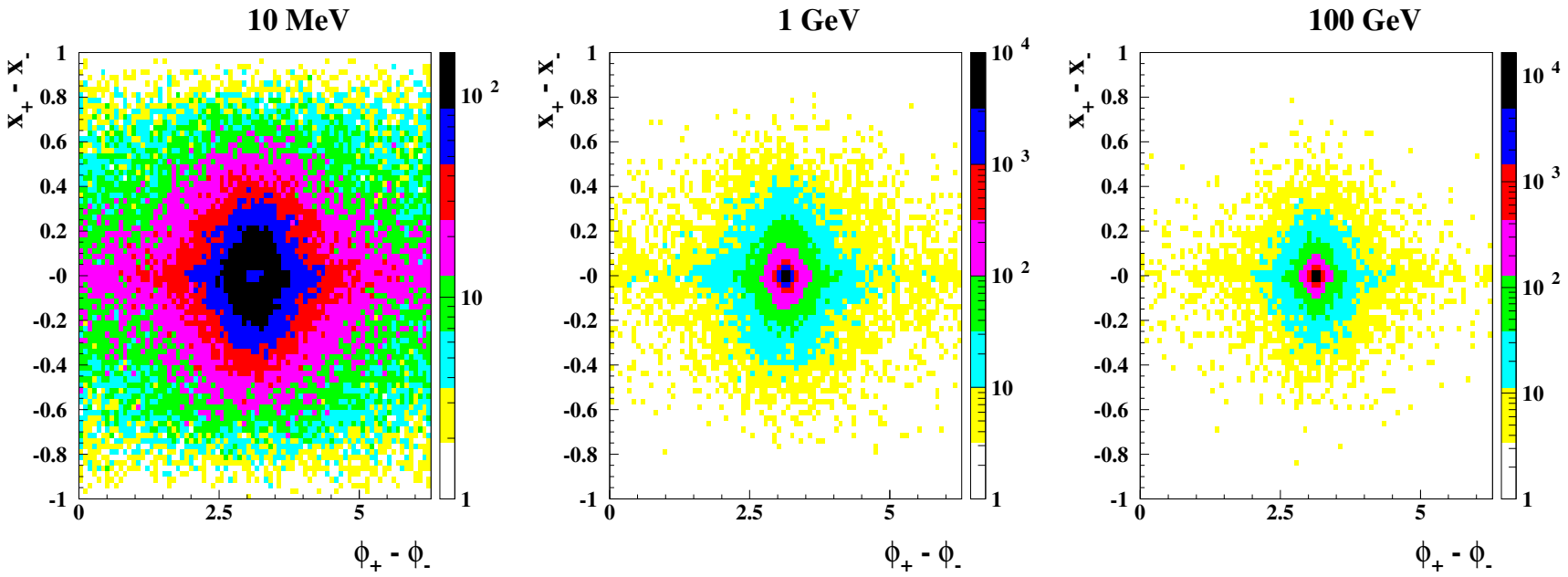
# $|x_+ - x_-|$ and $|\phi_+ - \phi_-|$ Distributions: Relation with $q$




$$q^2 =$$

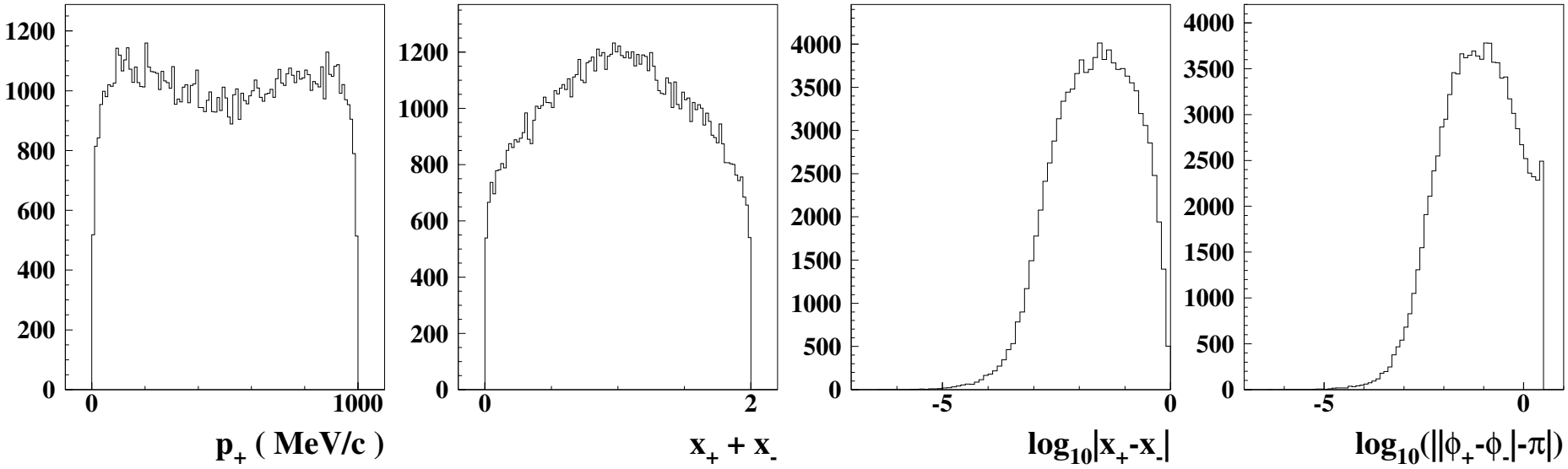
$$E^2 + p_+^2 + p_-^2 + 2p_+p_-(\cos\theta_+\cos\theta_- + \sin\theta_+\sin\theta_-\cos(\phi_+ - \phi_-)) - 2E(p_+\cos\theta_+ + p_-\cos\theta_-)$$

# $|x_+ - x_-|$ , $|\phi_+ - \phi_-|$ *Correlation*




Small correlation

# *Event Generation Mock-up*




Argon, 1 GeV

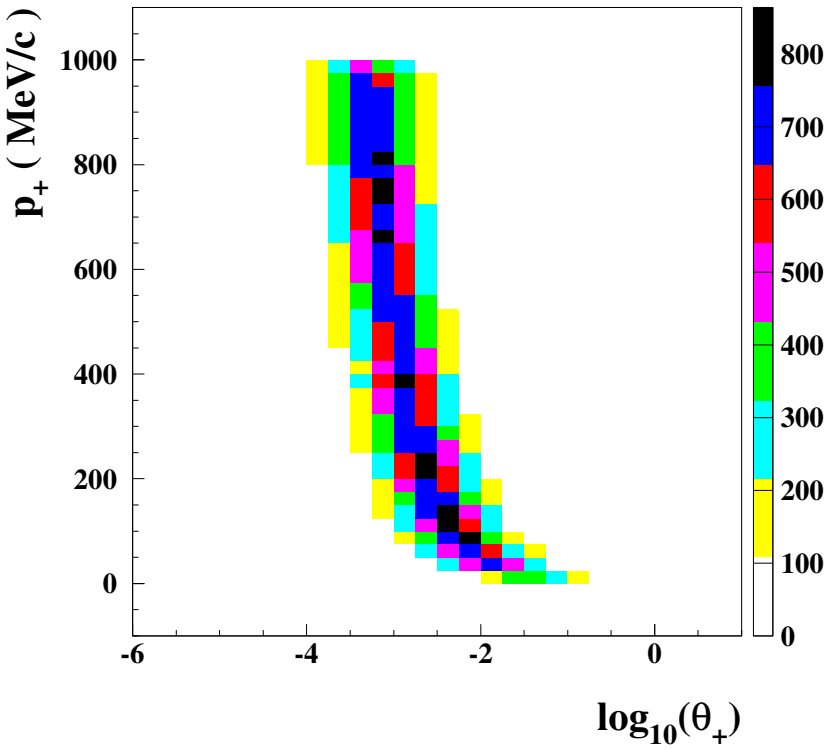
$p_+$, $x_+ + x_-$, $\log_{10}|x_+ - x_-|$, $\log_{10}||\phi_+ - \phi_-| - \pi|$, from histograms. $\phi_+ + \phi_-$, flat

$\Rightarrow \quad x_+, x_-, \theta_+, \theta_-, \phi_+, \phi_-,$

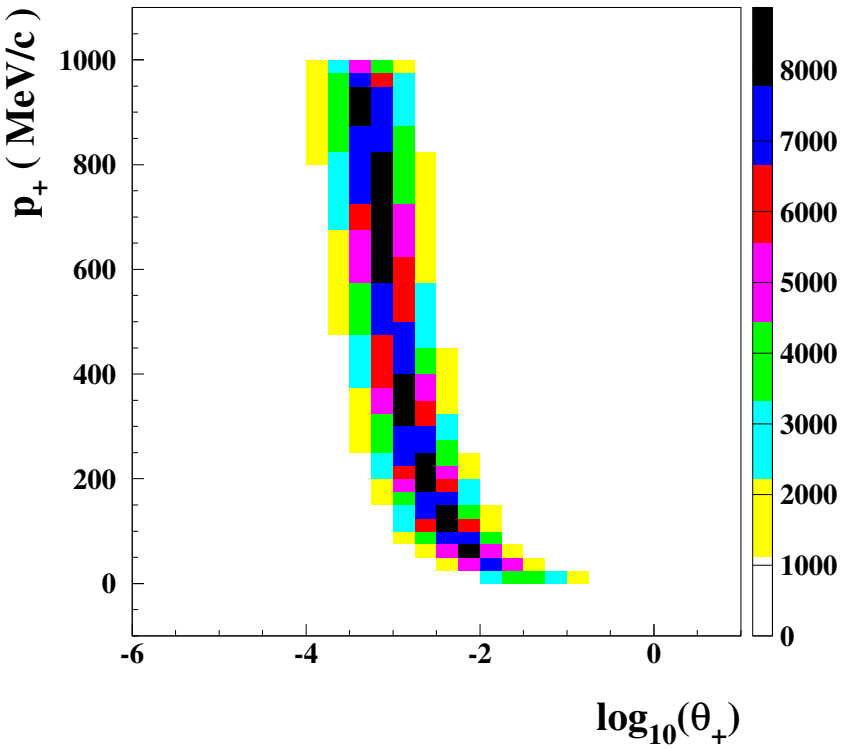
$\Rightarrow \quad E_- = k - E_+$

# *Checks (1) Angle / Momemtum*

5D

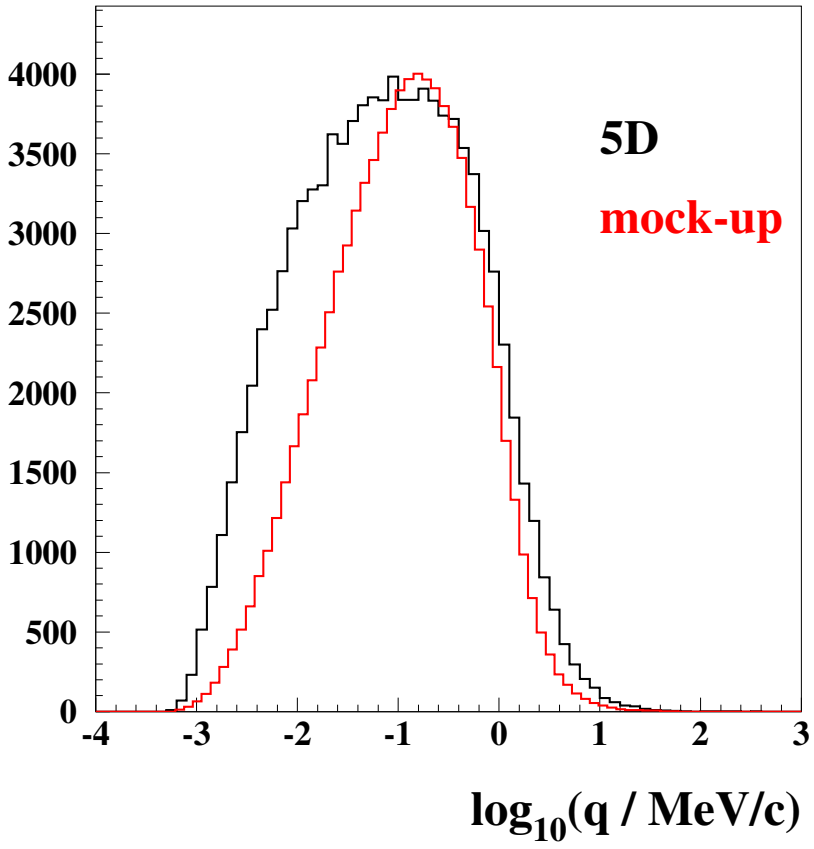



Mock-up




Argon, 1 GeV

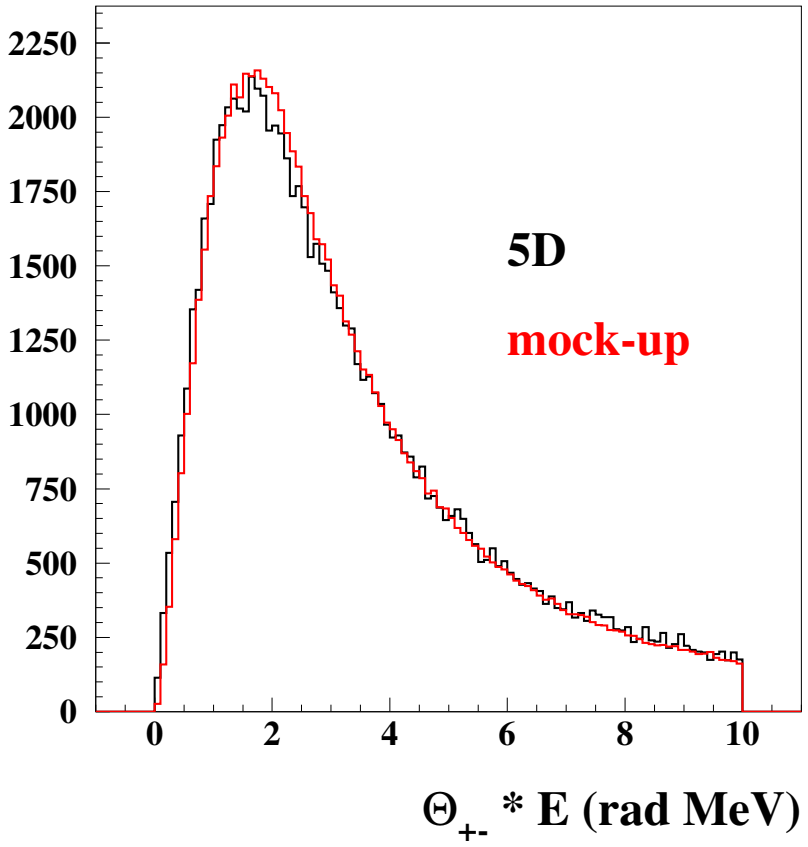
# *Checks (2) Recoil*




$\vec{q} = \vec{k} - (\vec{p}_+ + \vec{p}_-)$

Argon, 1 GeV

# *Checks (3) Pair Open Angle*




Argon, 1 GeV

# *Conclusion*

- G4BetheHeitler5DModel slow;

  Uneasy parameter space browsing for acceptance-rejection method / $1/q^4$ divergence

- Pre-existing models, direct generation (not acceptance-rejection ), incorrect $\vec{q}$ generation

  Issue in electron / positron angle(s) correlations

- $x_+ - x_-$ and distributions $\phi_+ - \phi_-$ distributions energy independant (asymptotically)

  $\Rightarrow$ New, direct, 5D event generation

  Some validation checks performed

- Polarization ?