Offset Tolerances for Run 2c

AWAKE Collaboration Meeting Nov 2024

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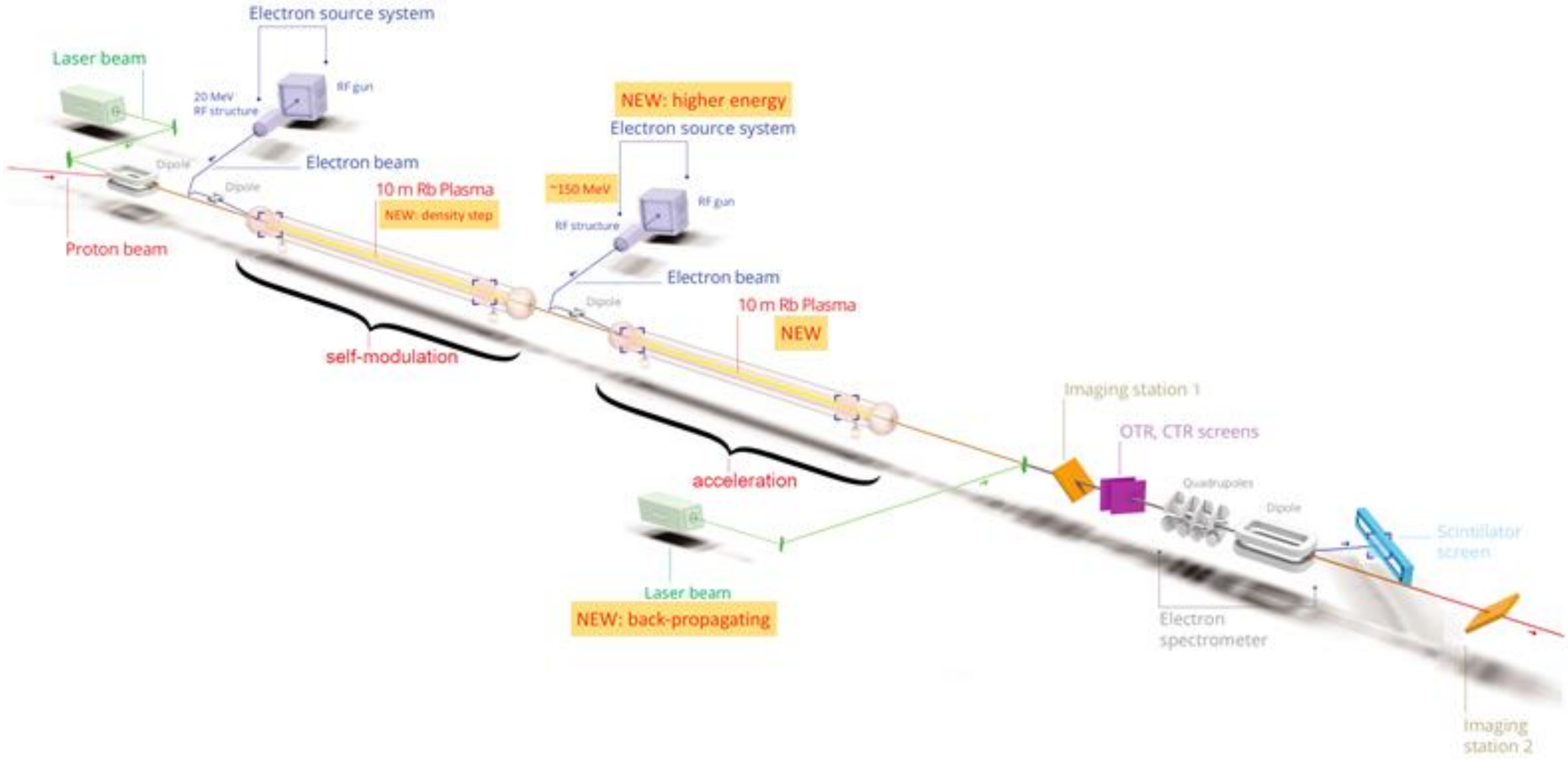
Introduction

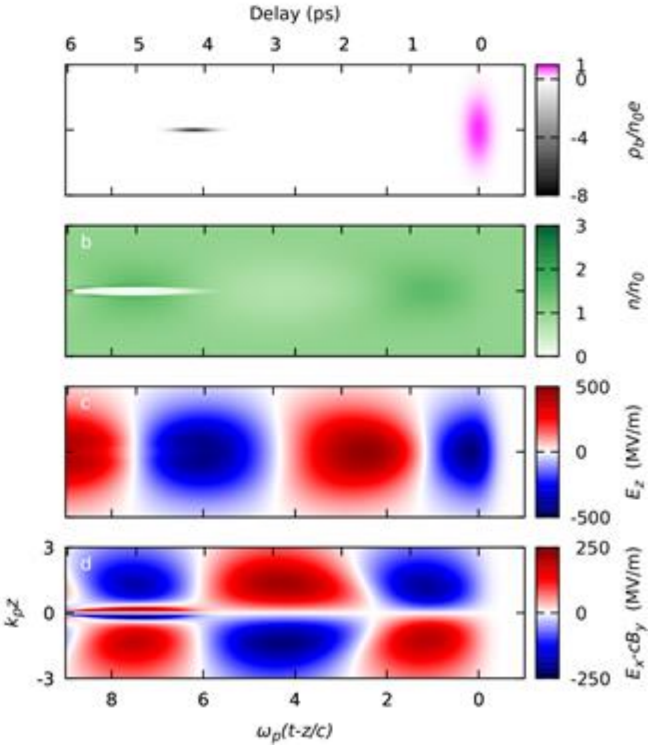


Motivation & setup

An injection offset introduces transverse momentum to the witness bunch. This will inevitably degrade emittance and lower the final beam quality. Injection at an angle will produce a similar effect.

What degree of misalignment can be tolerated, and can we quantify the effect on the final bunch?

Once again, we are using the very practical toy model to scan different electron beam configurations



Electron beam properties

We will consider two emittance cases

- 2 µm & 16 µm

and four different total charges

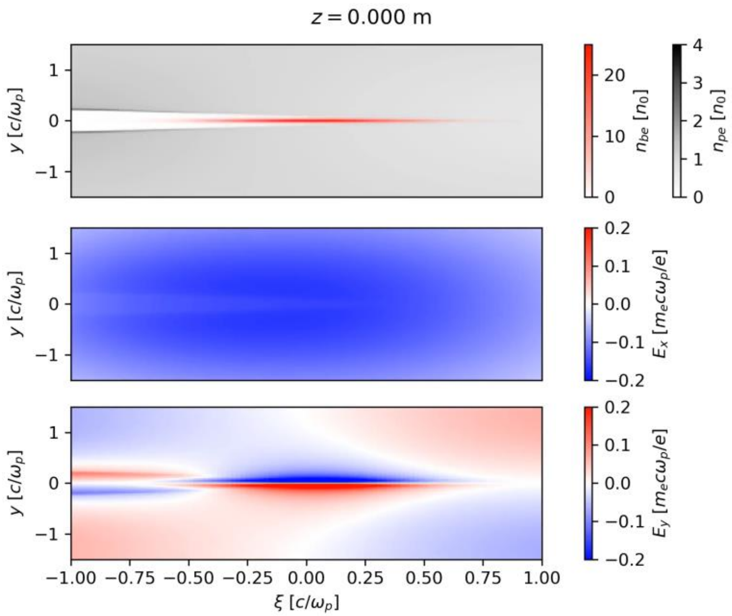
- 100 pC, 200 pC, 300 pC & 400 pC

with two offset distances

- 0 µm or 20 µm.

The remaining properties are taken as

- 150 MeV initial energy
- 0.1% energy spread
- 60 µm length
- 5.76 µm / 16.3 µm spot size
- 7×10^{14} cm^{-3} background plasma

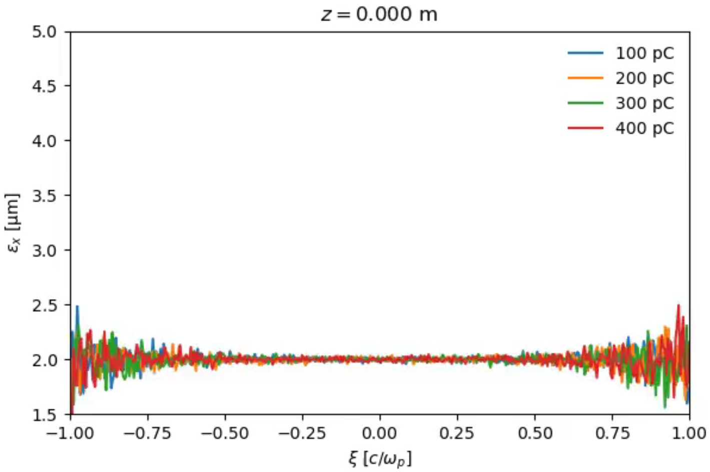


100 pC, 2 µm emittance, no offset:

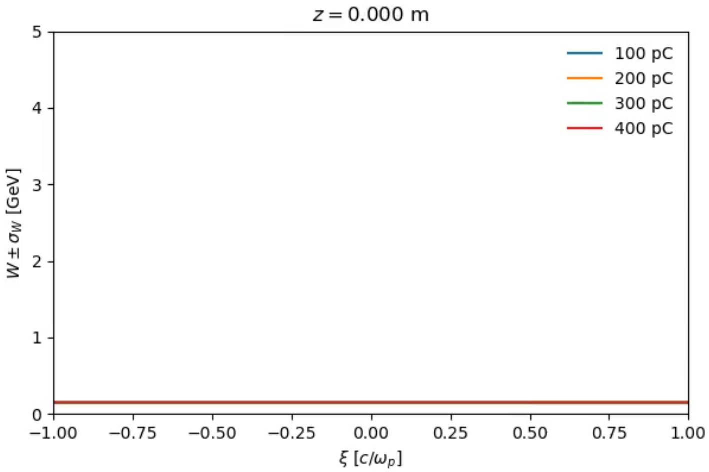
- 3.93 GeV ± 5.4% energy
- 2.5 µm projected emittance

Slice analysis

Next we can look at the properties along the bunch. With many snapshots this allows us to build up a picture of the bunch properties resolved in both time and space.



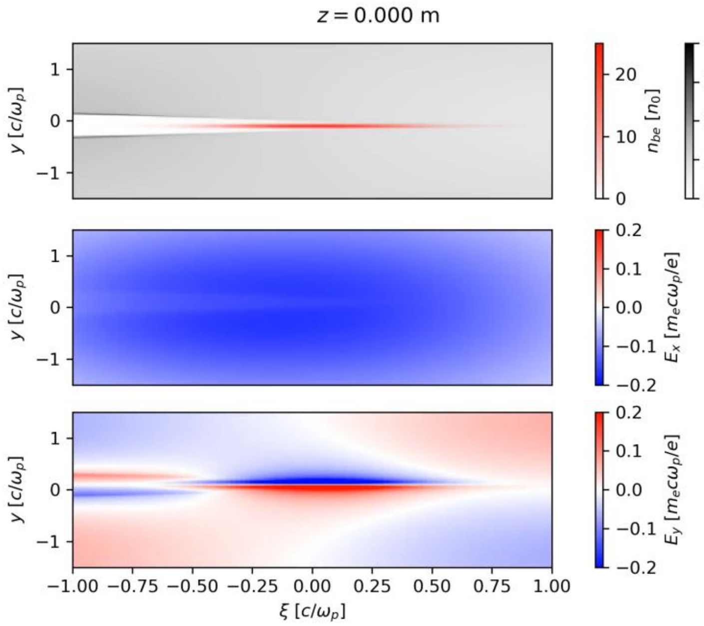
Emittance



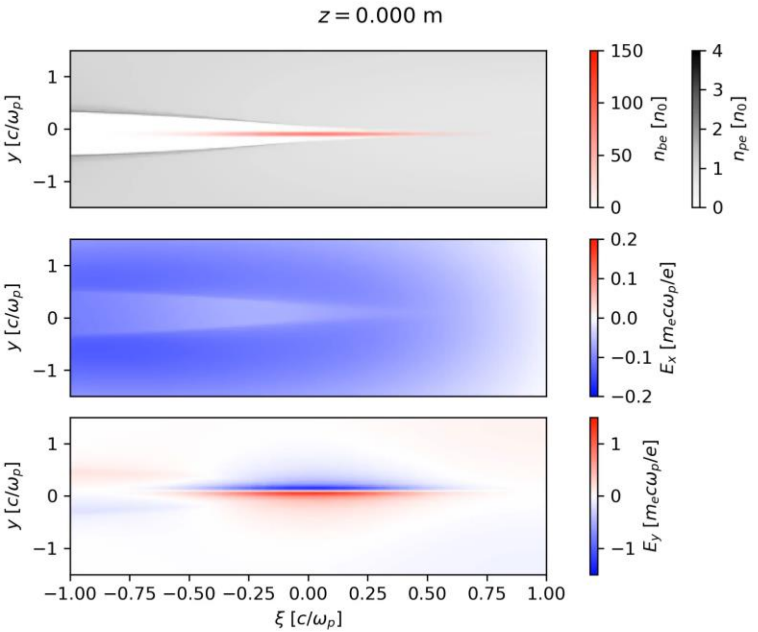
Energy & energy spread

Qualatative effects of an offset

When there is an offset, the behaviour changes immediately



100 pC

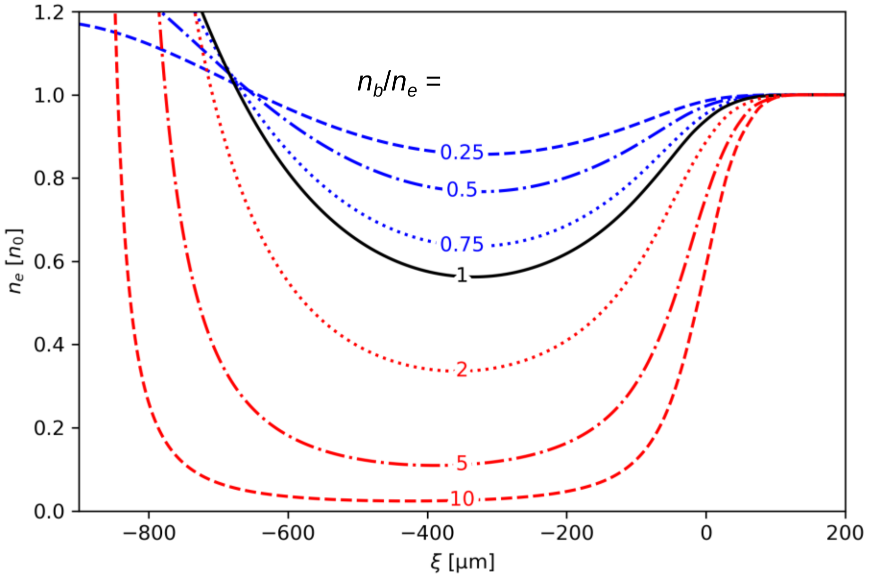


400 pC

What are blowout conditions?

- Often we say when $n_b > n_e$
- In reality, not so easy. Depends on density and spot size, but also duration and energy (a little)
- And on top of this, the emittance preservation effect is not an on/off situation either

Example on-axis density plots for witness beams with different peak density ratios



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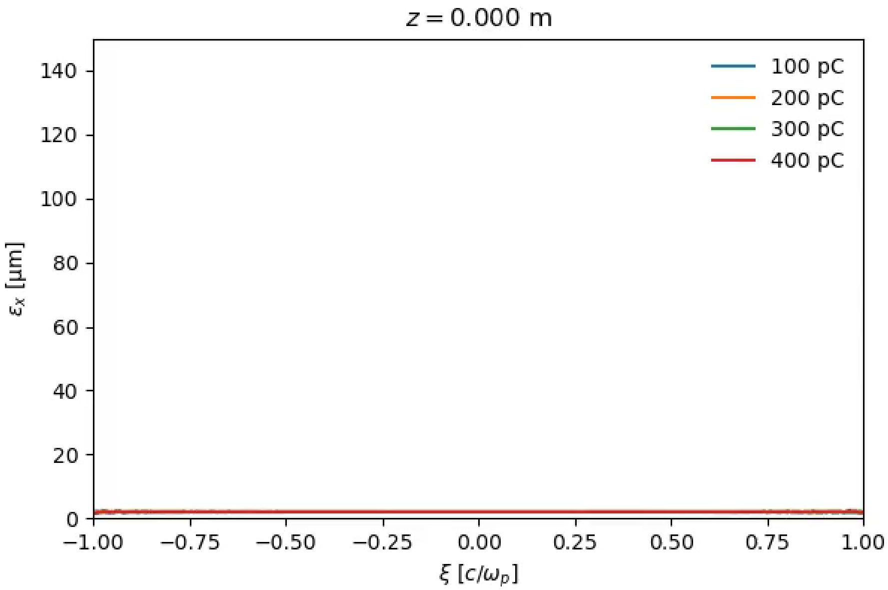
Emittance growth with an offset

- Some degree of emittance preservation occurs in high-charge bunches even with an offset

Charge (pC)	Final projected emittance (μm)
100	79
200	48
300	32
400	25



Bunch oscillation theory

Our goal is to calculate the potential, and thus focusing fields in a quasilinear wake, the longitudinal part is sinusoidal and can be eliminated, and the problem takes the form of a screened Poisson equation

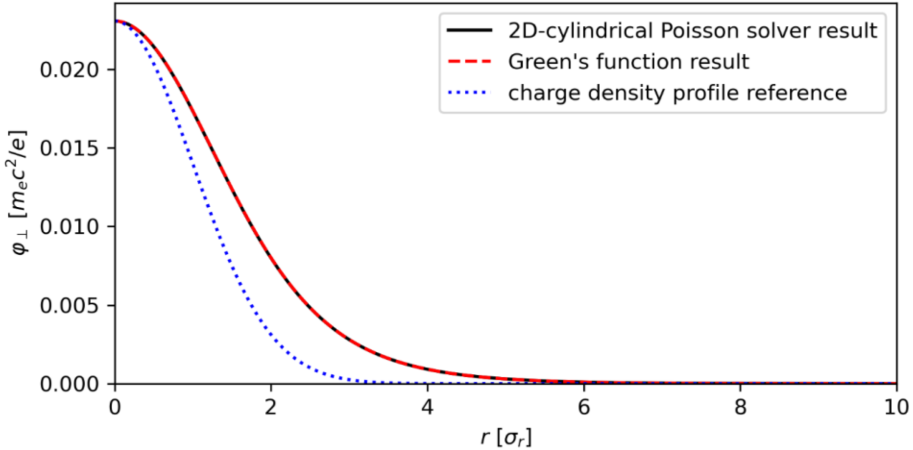
$$\nabla^2\varphi = \frac{\rho}{\epsilon_0} \quad \rightarrow \quad \frac{\rho}{\epsilon_0}\left(\nabla_\perp^2 - k_p^2\right)\varphi_\perp = \frac{\rho_\perp}{\epsilon_0}$$

This can then be treated via the Green's function method, or spectral methods etc. as you prefer. Analytical solutions present a challenge.

Numerical results agree well, if the wake bucket density can be predicted.

$$\varphi_0 = -\frac{\rho_0}{\epsilon_0}\int_0^\infty K_0(k_p r)\exp\left(-\frac{r^2}{2\sigma_r^2}\right) r\,\mathrm{d}r$$

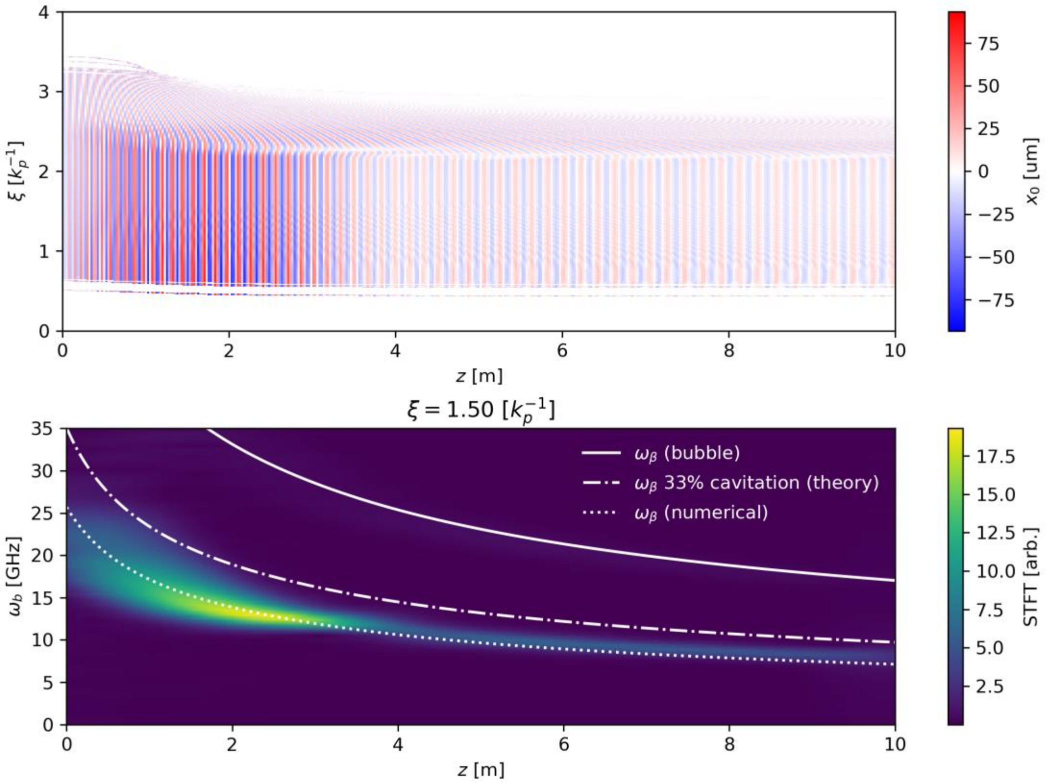
$$\varphi_\perp = -\frac{1}{2\pi}K_0(k_p r) ** \frac{\rho_0}{\epsilon_0}\exp\left(-\frac{r^2}{2\sigma_r^2}\right)$$



Centroid motion analysis

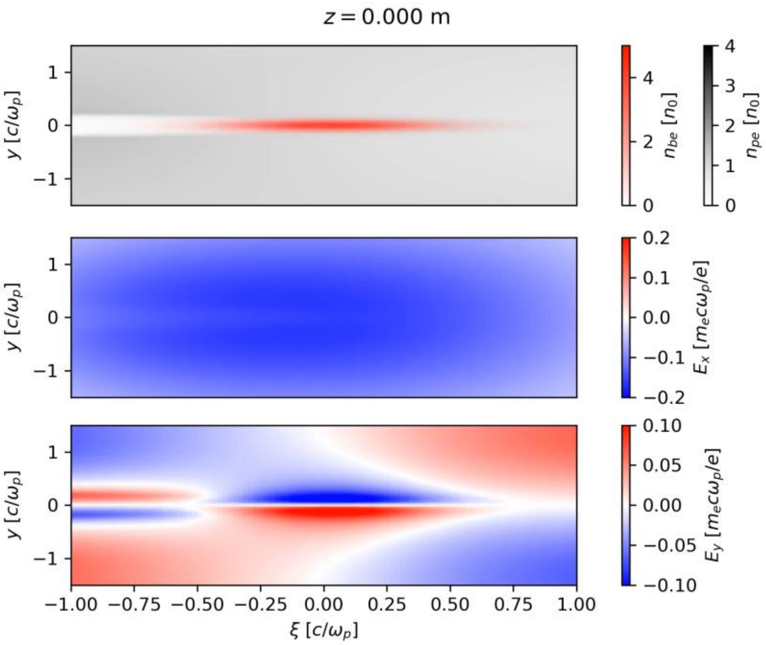
- Using the slice information, we can look at the centroid position over time for a slice and compute a STFT to obtain the time-resolved betatron frequency
- If we know the local plasma density (~2/3n_0) in the absence of a witness, we can calculate the transverse E-field
- An oscillation rate can be derived, à la the well known betatron frequency

$$\omega_\beta / \omega_p = (2\gamma_e)^{-1/2}$$

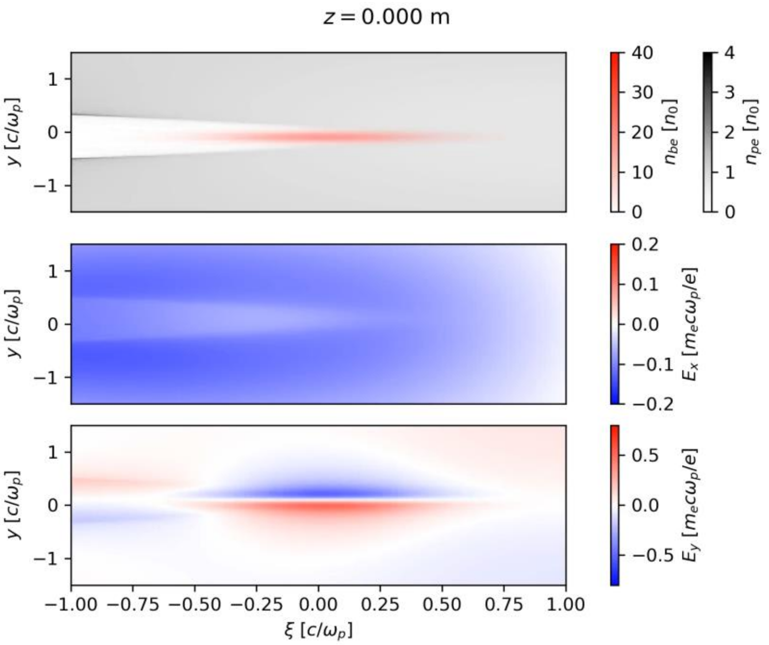


High emittance case

The 16 µm emittance case is less well behaved, but qualitatively similar



100 pC, no offset

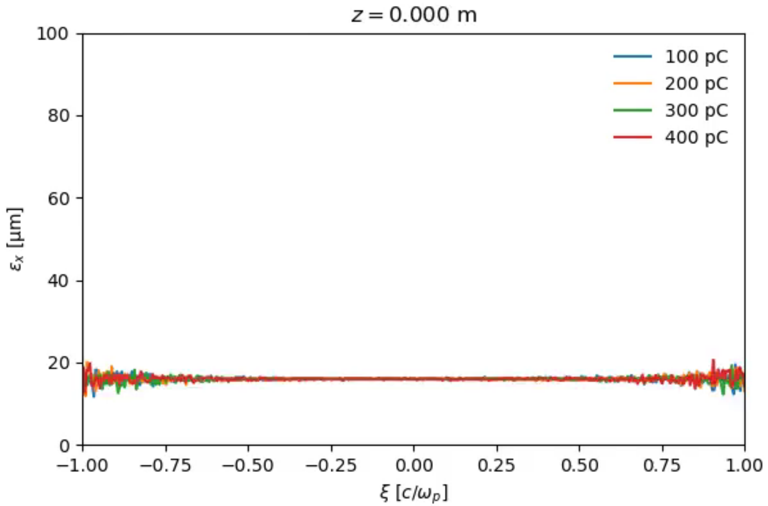


400 pC, 20 µm offset

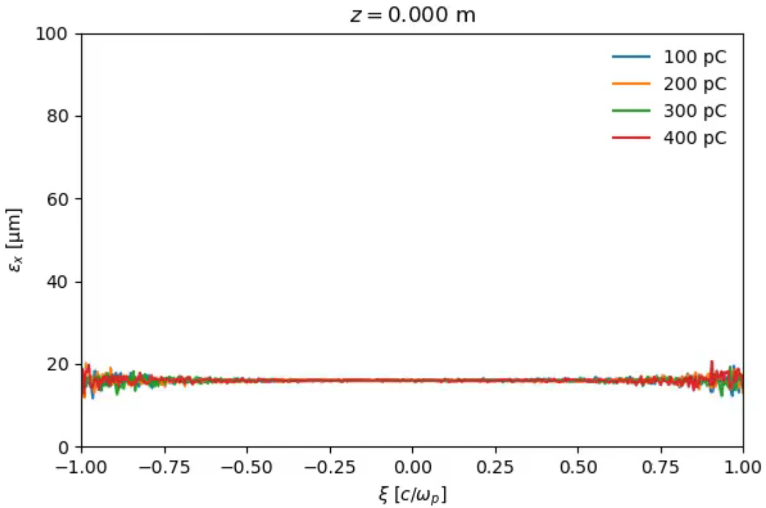
Emittance preservation with higher initial emittance

Emittance preservation sets in later, due to the lower current (larger spot size)

Notably it seems to have a relatively higher tolerance for an offset



20 µm offset
100pC: 16 µm → 32 µm



20 µm offset
100pC: 16 µm → 43 µm

Summary (so far)

- Offsets degrade beam quality
  - High charge can mitigate this effect, slice emittance remains good at the rear of the bunch
- Higher initial emittance has proportionally more emittance growth with no offset
  - But proportionally less growth with an offset
- The collective motion of an offset bunch can be somewhat modeled as a single particle oscillating in a screened potential
  - There are probably partial analytical solutions for this
- As always, plenty more work that could be done. Depends what's most pressing or most interesting

Thank you for your attention

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