Signatures of gluon saturation from structure-function measurements arXiv:2203.05846

Nestor Armesto [1] Tuomas Lappi [2 3] Heikki Mäntysaari [2 3]
Hannu Paukkunen [2 3] Mirja Tevio [2 3]

[1]Instituto Galego de Física de Altas Enerxías IGFAE, Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Galicia-Spain

[2]Department of Physics, University of Jyväskylä, P.O. Box 35, 40014 University of Jyväskylä, Finland

[3]Helsinki Institute of Physics, P.O. Box 64, 00014 University of Helsinki, Finland

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JYVÄSKYLÄN YLIOPISTO
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- To see saturation effects on experimental data we have to distinguish the genuine difference between DGLAP and BK dynamics
- Both frameworks require input which are fitted to the same experimental data $\longrightarrow$ The results do not deviate dramatically and distinguishing DGLAP/BK dynamics is difficult

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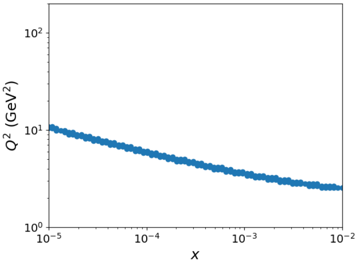

Matching line in (x, Q^2) plane

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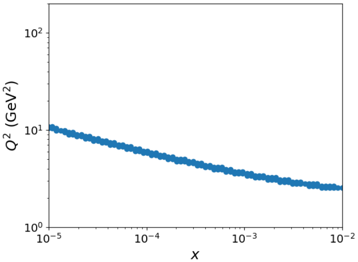

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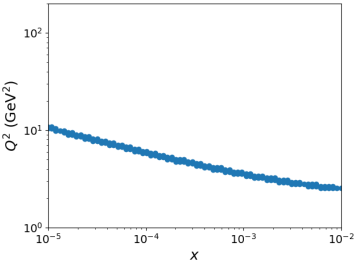
4. With differences we can quantify the precision needed at EIC and LHeC/FCC-he to distinguish saturation effects



Matching line in (x, Q^2) plane

$F_{2,\mathrm{L}}$ with collinear factorization vs. CGC

Collinear factorization:

- Collinear factorization $F_{2,\mathrm{L}}$ using APFEL [1] and LHAPDF [2] libraries
- `NNPDF31_nlo_as_0118_1000` as proton PDF set
- `nNNPDF20_nlo_as_0118_Au197` as nuclear PDF set
- Both PDF sets have 1000 Monte Carlo replicas

Color Glass Condensate (CGC):

- Dipole picture $F_{2,\mathrm{L}}$ fitted to HERA data
- Leading order total photon-nucleus cross sections
- Running coupling BK evolution [1]

- We match collinear factorization $F_{2,\mathrm{L}}$ to corresponding CGC structure functions in a line in (x, Q^2) plane

[1] T. Lappi and H. Mäntysaari. "Single inclusive particle production at high energy from HERA data to proton-nucleus collisions". In: *Phys. Rev. D* 88 (2013), p. 114020. arXiv: 1309.6963 [hep-ph]

PDF matching

Bayesian reweighting method [4, 5]:

For each PDF replica f_k we define

$$\chi_k^2 = \sum_{i=1}^{N_{\mathrm{data}}} \frac{(\mathcal{O}_i - \mathcal{O}_i[f_k])^2}{(\delta_{\mathrm{BK}}\mathcal{O}_i)^2}$$

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and so called **Giele-Keller** weights [6]

$$\omega_k = \frac{e^{-\frac{1}{2}\chi_k^2}}{\frac{1}{N_{\mathrm{rep}}}\sum_{k=1}^{N_{\mathrm{rep}}} e^{-\frac{1}{2}\chi_k^2}},$$

which always sum up to unity,

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Giele-Keller weights favor replicas with $\chi^2 \approx 0$.

Then we define reweighted observables as

$$\mathcal{O}^{\rm Rew} = \frac{1}{N_{\rm rep}}\sum_{k=1}^{N_{\rm rep}} \omega_k \mathcal{O}[f_k]$$

We also construct a PDF set matched to BK in (x, Q^2) line (Back up)

Choosing the matching line

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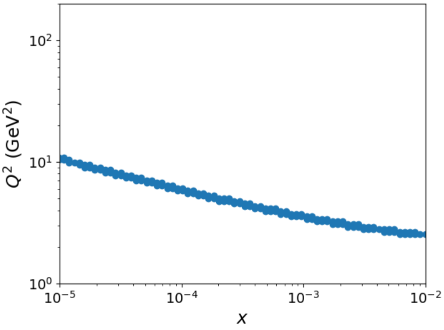
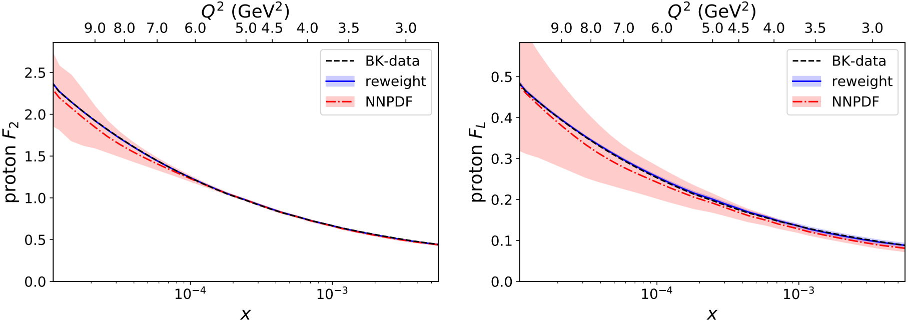
$\longrightarrow$ **We choose to do the matching on points** $Q^2(x) \approx 10 \times Q_s^2(x)$



Matching line in (x, Q^2) plane

Proton matching

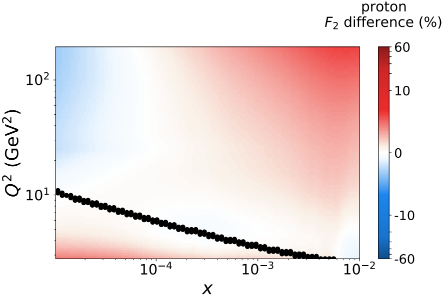


(a) F_2 (b) F_{L}

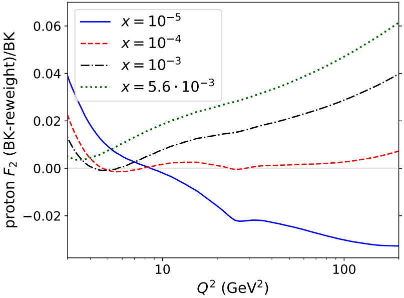
The structure functions for proton as a function of x at $Q^2 \approx 10 Q_s^2(x)$

- Separate matching for proton F_2 and F_{L} are both almost perfect

Relative difference of proton F_2^{Rew} to F_2^{BK}



(a) F_2

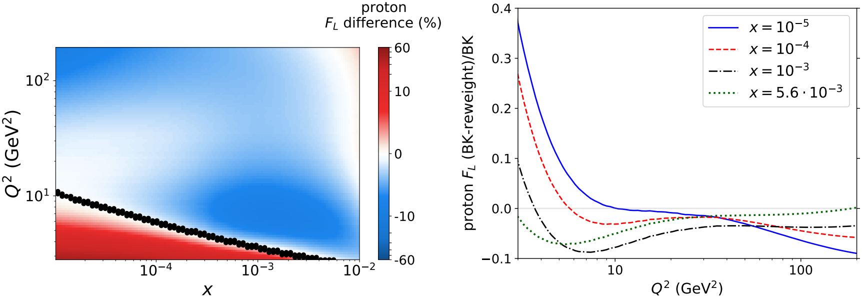


(b) F_2

Relative difference $(F_2^{\mathrm{BK}} - F_2^{\mathrm{Rew}})/F_2^{\mathrm{BK}}$

- For proton F_2 the relative difference is only a few percent
- Generically slower x dependence in BK evolution

Relative difference of proton $F_{\mathrm{L}}^{\mathrm{Rew}}$ to $F_{\mathrm{L}}^{\mathrm{BK}}$

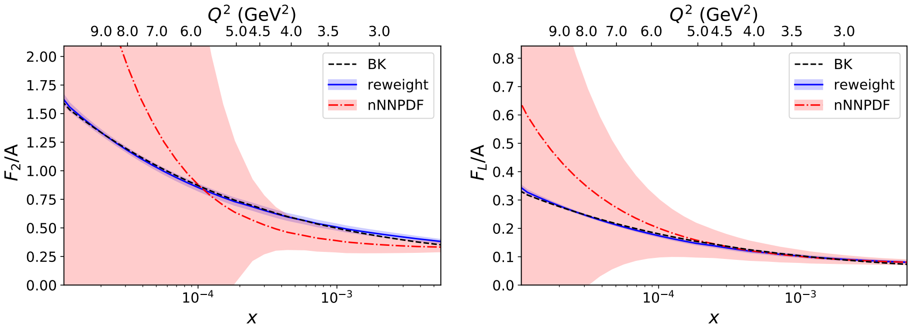


(a) F_{L} (b) F_{L}

The relative difference $(F_{\mathrm{L}}^{\mathrm{BK}} - F_{\mathrm{L}}^{\mathrm{Rew}})/F_{\mathrm{L}}^{\mathrm{BK}}$.

- For proton F_{L} the relative difference is:
  - $\lesssim 10\%$ for $x = 10^{-3} ... 5.6 \times 10^{-3}$ (EIC)
  - $\lesssim 40\%$ for $x = 10^{-5} ... 10^{-4}$ (LHeC/FCC-he)
- F_{L} is much more sensitive to saturation than F_2

Nuclear matching

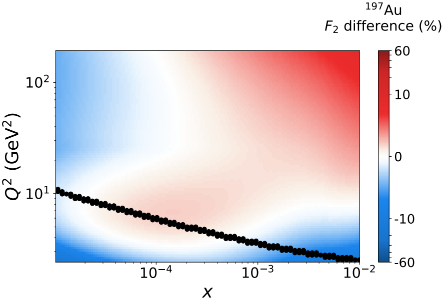


(a) F_2 (b) F_{L}

The structure functions for $^{197}\mathrm{Au}$ as a function of x at $Q^2 \approx 10Q_s^2(x)$.

- Nuclear reweight is not as successful as for proton since there are not enough Monte Carlo replicas to get a precise match

Relative difference of nuclear F_2 to $F_2^{\rm BK}$



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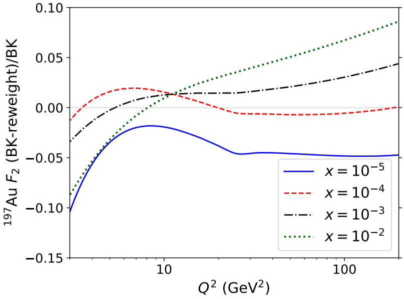
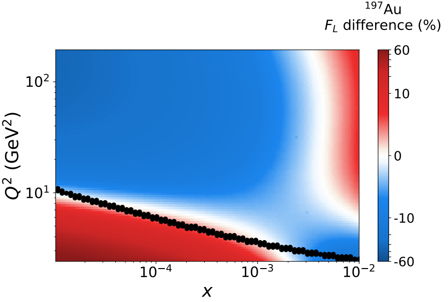


(b) F_2

Relative difference $(F_2^{\rm BK} - F_2^{\rm Rew})/F_2^{\rm BK}$.

- For nuclear F_2 the relative difference is $\lesssim 10\%$
- The relative difference is much larger than in the proton case
  - It is expected since saturation effects are stronger in nuclei

Relative difference of nuclear $F_\mathrm{L}^\mathrm{Rew}$ to F_L^BK



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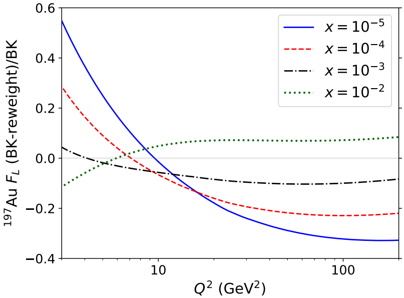

(b) F_L

The relative difference $(F_\mathrm{L}^\mathrm{BK} - F_\mathrm{L}^\mathrm{Rew})/F_\mathrm{L}^\mathrm{BK}$.

For nuclear F_L the relative difference is:

- $\lesssim 15\%$ for $x = 10^{-3}...10^{-2}$ (EIC)
- $\lesssim 60\%$ for $x = 10^{-5}...10^{-4}$ (LHeC/FCC-he)

Including small-x resummation (work in progress)

- At small x large logarithms $\log(1/x)$ in collinear framework
  $\longrightarrow$ need to resum
- Resummation should take DGLAP evolved F_2 and F_{L} closer to BK values
- With resummation can distinguish between BFKL and saturation effects
- How to implement small-x resummation:
  - Use "NNPDF31sx_nnlonllx_as_0118" PDF set for proton
  - Use APFEL+HELL implementation to enable resummation in F_2 and F_{L}

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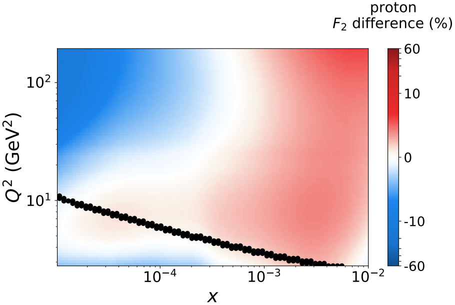

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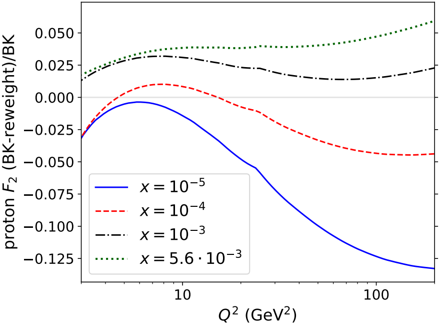

(b) F_2

- Differences are larger than w/o resummation
- Reweight not as successful due to limited number of replicas ($N_{\mathrm{rep}} = 100$) $\longrightarrow$ can not yet draw strong conclusions
- Have to improve reweighting (e.g. by scaling the replicas)

Summary

- With Bayesian reweighting we match proton/nuclear DGLAP structure functions to corresponding BK values in a line $Q^2 \approx 10 \times Q_s^2(x)$
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  - F_2 the measurements have to be $\mathcal{O}(1\%)$
- In LHeC/FCC-he the differences are a few times larger
- Saturation is stronger in heavy nuclei than in proton
- F_{L} is more sensitive to saturation than F_2
- Have to improve reweighting with small-x resummation

References

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Back up: Fixing matching parameters

- We want to match the reweighted values to BK values as closely as possible
  - Finite number of replicas (1000) prevent the absolute match
- Effective number of replicas [4, 7]

$$N_{\rm eff} = \exp \frac{1}{N_{\rm rep}} \sum_{k=1}^{N_{\rm rep}} \omega_k \ln\left(\frac{N_{\rm rep}}{\omega_k}\right)$$

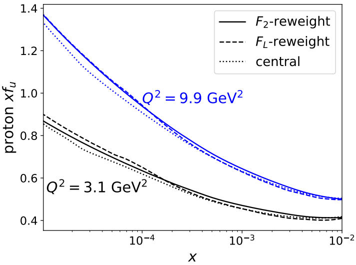
gives an approximation on how many PDF replicas have significant weight

- We adjust $\delta_{\rm BK}$ in χ^2_k in order to fix $N_{\rm eff} \approx 10$

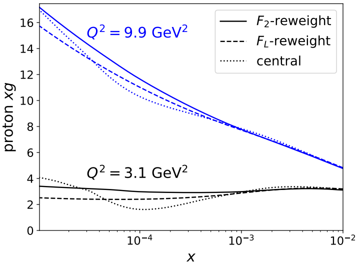
$$\chi^2_k = \sum_{i=1}^{N_{\rm data}} \frac{(y_i - y_i[f_k])^2}{(\delta_{\rm BK} y_i)^2}$$

$$\omega_k = \frac{e^{-\frac{1}{2}\chi^2_k}}{\frac{1}{N_{\rm rep}} \sum_{k=1}^{N_{\rm rep}} e^{-\frac{1}{2}\chi^2_k}}$$

$$\mathcal{O}^{\rm Rew} = \frac{1}{N_{\rm rep}} \sum_{k=1}^{N_{\rm rep}} \omega_k \mathcal{O}[f_k]$$



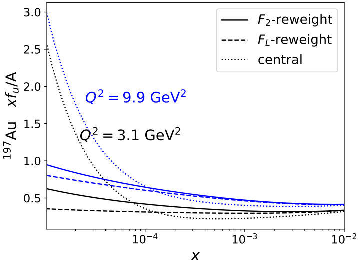
(a) Proton up quark



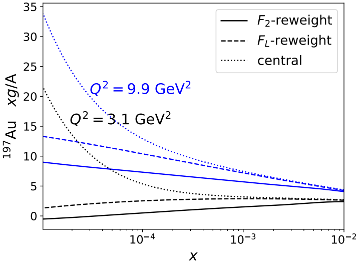
(b) Proton gluon

- Reweighting has slightly stronger effect on gluon distribution than on up quark
- Moderate effects expected since NNPDF3.1 PDFs are fitted to same HERA data as BK boundary conditions

Back up: Weighted nuclear PDFs

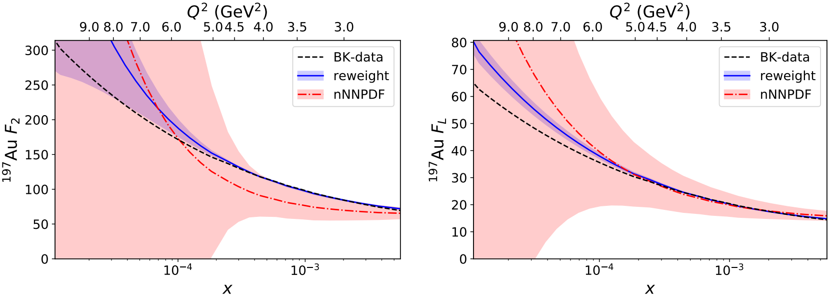


(c) Nuclear up quark



(d) Nuclear gluon

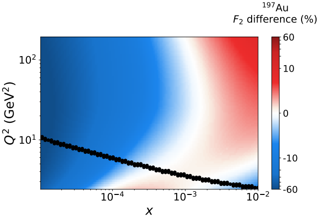
- Nuclear PDFs are affecter more than proton PDFs
- Reweighting has stronger effect on gluon distribution than on up quark



(a) F_2 (b) F_{L}

Nuclear reweight in region $x = 10^{-4} \ldots 10^{-2}$

Back up: Reweight with smaller x region



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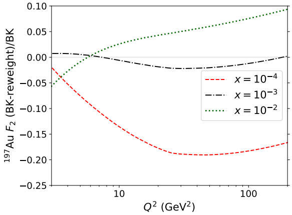
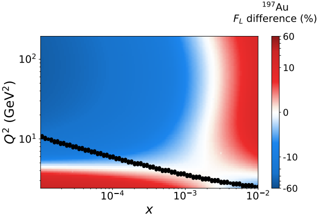


(b) F_2

The relative difference $(F_2^{\mathrm{BK}} - F_2^{\mathrm{Rew}})/F_2^{\mathrm{BK}}$ with nuclear reweight in region $x = 10^{-4}...10^{-2}$.



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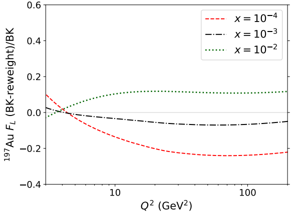
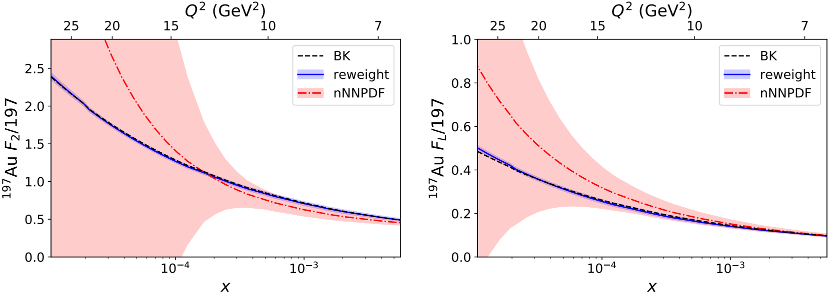


(b) F_{L}

The relative difference $(F_{\mathrm{L}}^{\mathrm{BK}} - F_{\mathrm{L}}^{\mathrm{Rew}})/F_{\mathrm{L}}^{\mathrm{BK}}$ with nuclear reweight in region $x = 10^{-4}...10^{-2}$.

Back up: Reweight in line $Q^2(x) \approx 27 \times Q_s^2(x)$



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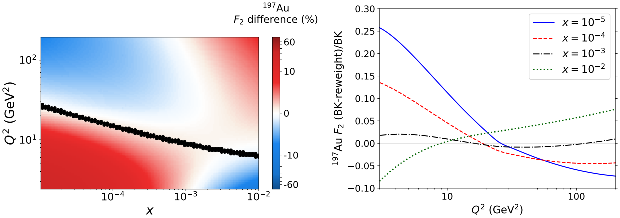

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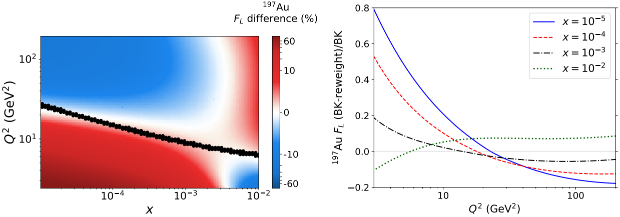

(a) F_{L} (b) F_{L}

The relative difference $(F_{\mathrm{L}}^{\mathrm{BK}} - F_{\mathrm{L}}^{\mathrm{Rew}})/F_{\mathrm{L}}^{\mathrm{BK}}$ with nuclear reweight in line $Q^2(x) \approx 27 \times Q_s^2(x)$.

Back up: Weights

Giele-Keller weights which favor replicas with $\chi^2/N_{\rm data} \approx 0$

$$\omega_k = \frac{e^{-\frac{1}{2}\chi_k^2}}{\frac{1}{N_{\rm rep}}\sum_{k=1}^{N_{\rm rep}} e^{-\frac{1}{2}\chi_k^2}}$$

Weights used with experimental data favor replicas with $\chi^2/N_{\rm data} \approx 1$

$$\omega_k = \frac{(\chi_k^2)^{(N_{\rm data}-1)/2} e^{-\frac{1}{2}\chi_k^2}}{\frac{1}{N_{\rm rep}}\sum_{k=1}^{N_{\rm rep}} (\chi_k^2)^{(N_{\rm data}-1)/2} e^{-\frac{1}{2}\chi_k^2}}$$