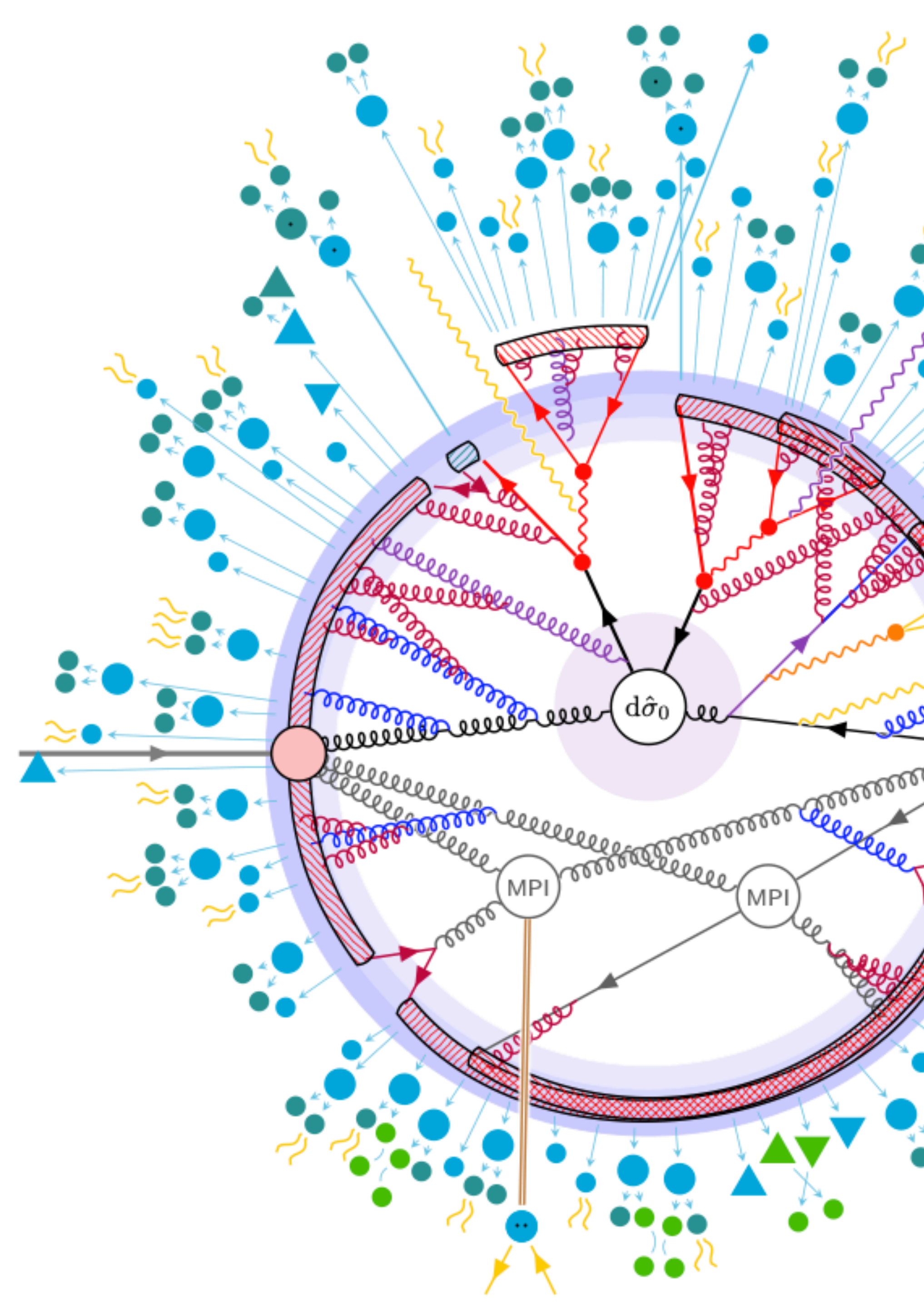


# The present and future of antenna showers

based on JHEP 07 (2024) 161
and PLB 836 (2023) 137614

Milan Joint Pheno Seminar
University of Milan Bicocca
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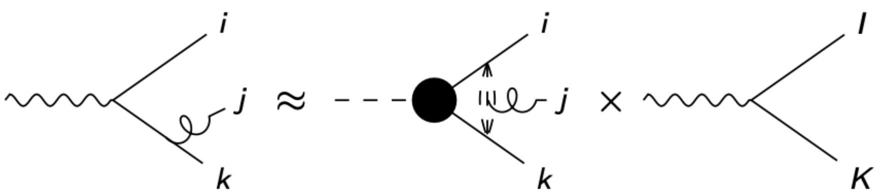
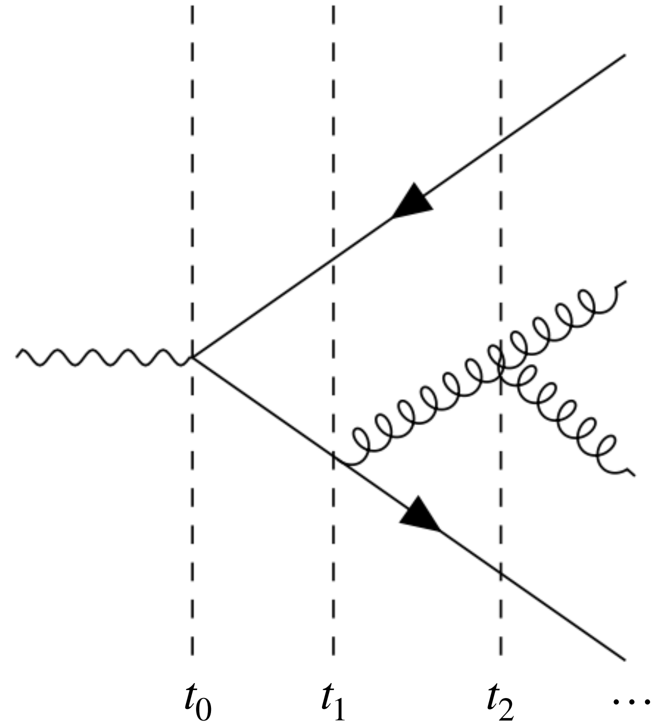
Parton showers dress a fixed-order calculation with radiation, describing the evolution from the **parton level** (quarks, gluons, …) to the **particle level** (hadrons).

- amplitudes factorise in limits where emissions are soft ($E_j \to 0$) or collinear ($\theta_{jk} \to 0$)



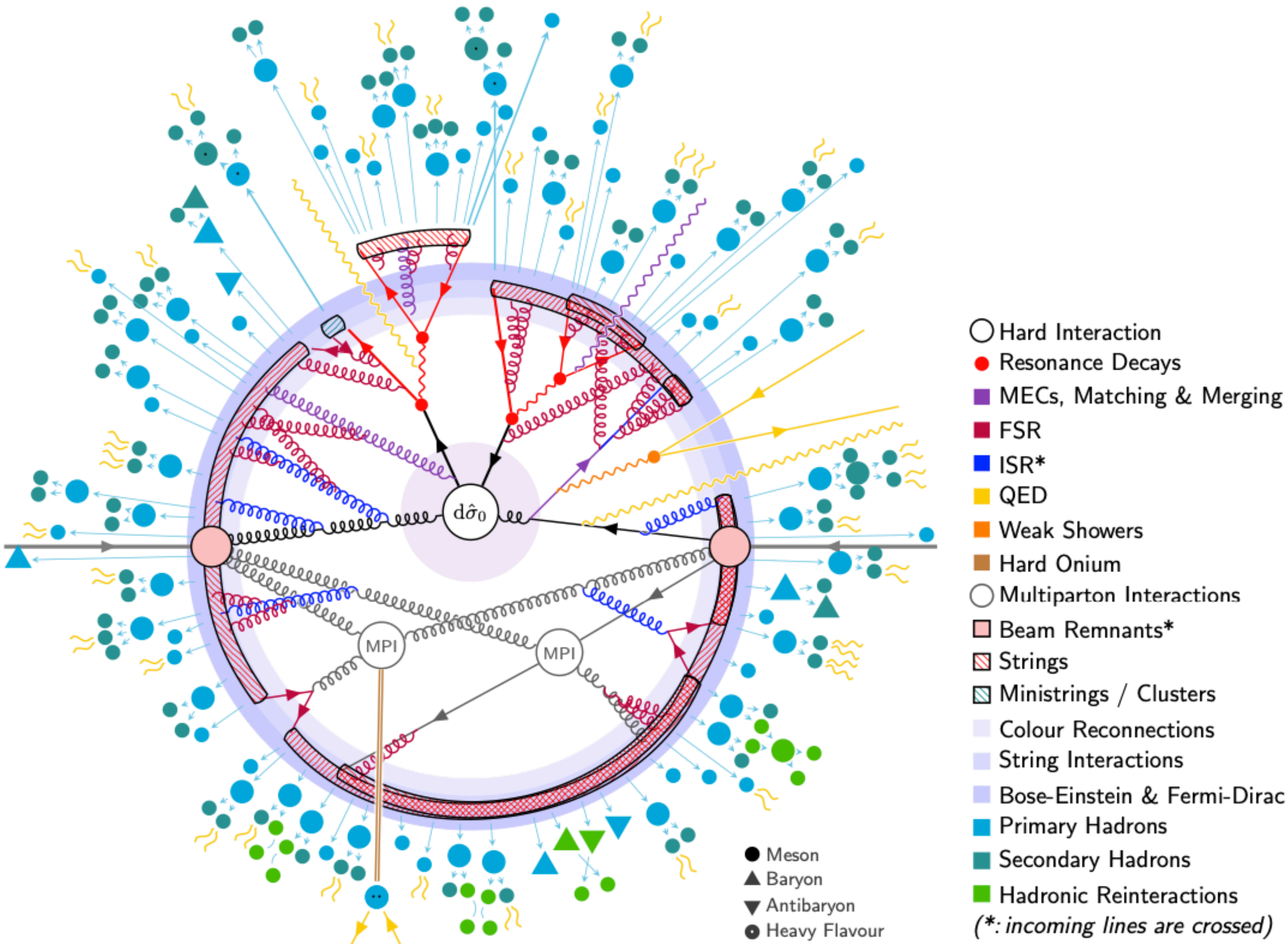

- starting from a large scale $t_0$, radiation is modelled under the assumption that it is soft/collinear and ordered $t_0 > t_1 > t_2 > \ldots > t_h$



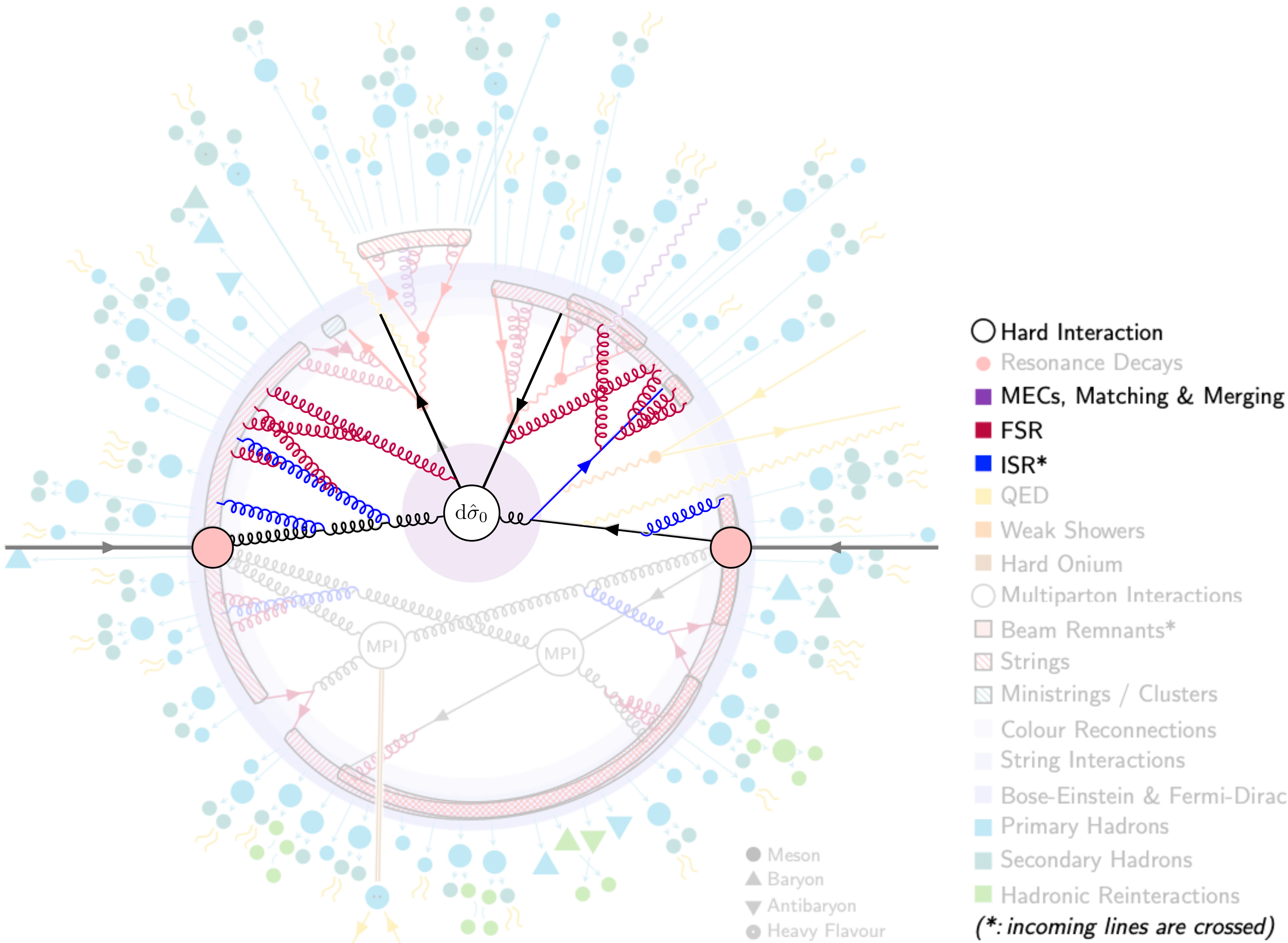

Evolves event from hard scale $t_0$ to soft scale $t_h$, but introduces logarithms

$$\alpha_s^n \to \alpha_s^n \log\left(\frac{t_0}{t}\right), \quad n \leq 2m, \ \textbf{large} \text{ if } t \ll t_0$$

# The full picture



# The full picture



# Overview

1. Constructing antenna showers

2. Logarithmic Accuracy
   based on [CTP JHEP07(2024)161]

3. NLO matching
   based on [CTP JHEP07(2024)161]

4. Towards NNLO matching
   based on [Campbell, Höche, Li, CTP, Skands PLB836(2023)137614]

# Constructing antenna showers

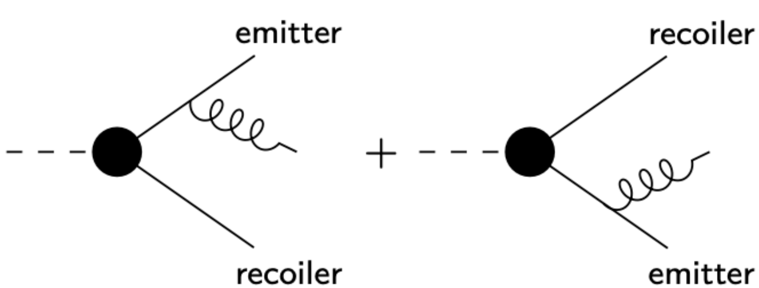
focussing on FSR

# More than one way to skin a cat…

**A specific parton-shower algorithm is defined by:**

- Branching kernels → *What is the probability for a parton to branch?*
- Recoil scheme → *How is the four-momentum of the emission generated?*
- Evolution variable → *In what measure does the event evolve?*

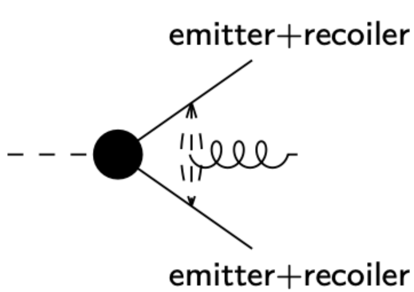
**Dipoles**




e.g. SHERPA CSS, HERWIG dipole, DIRE

- recoil taken by **opposite dipole end**
- **intrinsically** coherent

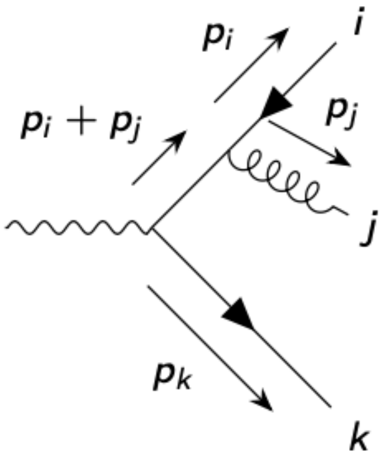
**Antennae**




e.g. ARIADNE, VINCIA

- **both parents** absorb transverse recoil
- **intrinsically** coherent

# Factorisation in unresolved limits



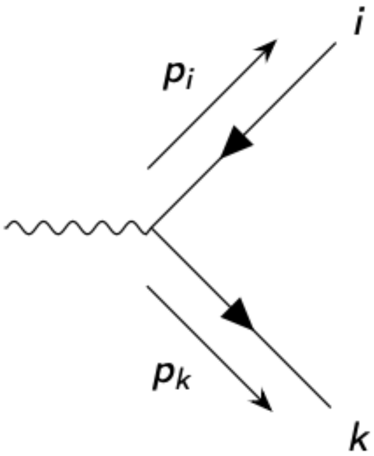

*ij*-coll:

$$g_s^2 \frac{1}{s_{ij}} \left( \frac{2z_i}{1-z_i} + (1-z_i) \right)$$

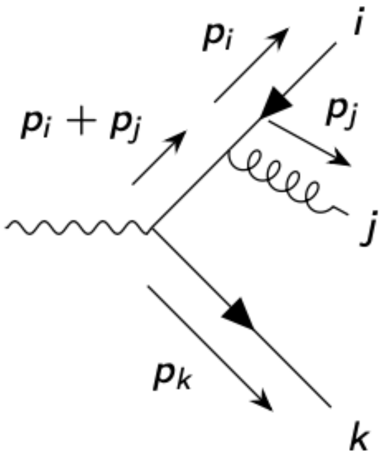
*j*-soft:

$$g_s^2 \frac{2 s_{ik}}{s_{ij} s_{jk}} \xrightarrow{i \| j} g_s^2 \frac{1}{s_{ij}} \frac{2z_i}{1-z_i}$$

$\times$



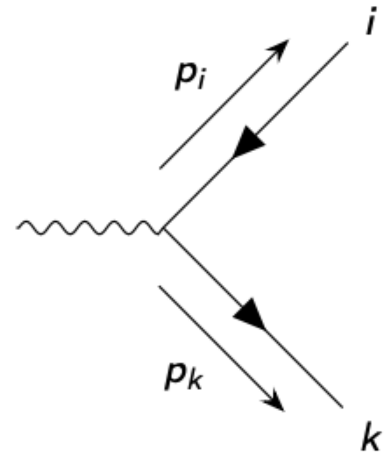
# Factorisation in unresolved limits

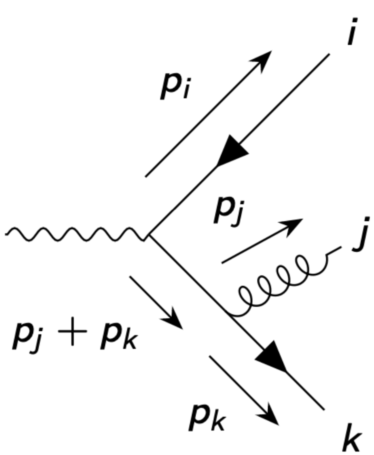



ij-coll: $g_s^2 \frac{1}{s_{ij}} \left( \frac{2z_i}{1-z_i} + (1-z_i) \right)$

j-soft: $g_s^2 \frac{2s_{ik}}{s_{ij}s_{jk}} \xrightarrow{i\|j} g_s^2 \frac{1}{s_{ij}} \frac{2z_i}{1-z_i}$

$\times$

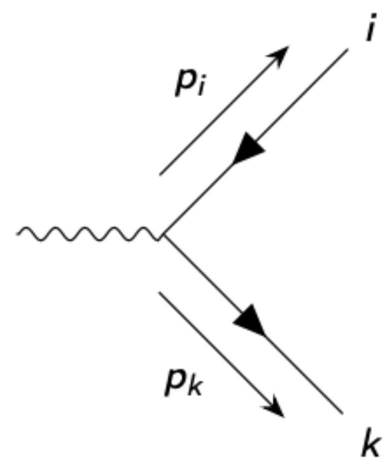



j-soft: $g_s^2 \frac{2s_{ik}}{s_{ij}s_{jk}} \xrightarrow{j\|k} g_s^2 \frac{1}{s_{ij}} \frac{2z_k}{1-z_k}$

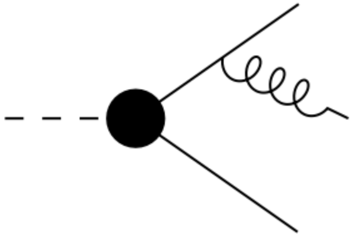
jk-coll: $g_s^2 \frac{1}{s_{ij}} \left( \frac{2z_k}{1-z_k} + (1-z_k) \right)$

$\times$




All about partitioning singular structures!

**Dipole showers** describe radiation from (leading-colour) dipoles with **local** on-shell kinematics:




$$p_i^\mu = a p_I^\mu - p_\perp^\mu$$
$$p_j^\mu = (1-a)p_I^\mu + (1-b)p_K^\mu + p_\perp^\mu$$
$$p_k^\mu = b p_K^\mu$$

transverse recoil taken by $p_I$

**Branching kernels** contain collinear limit and partial fraction soft eikonal:

$$P_{q\to qg}(p_i, p_j, p_k) = \frac{1}{s_{ij}} \left[ \frac{2 s_{ik}}{s_{ij} + s_{jk}} + (1 - z_i) \right]$$

soft $\qquad$ $j \parallel i$

Note: "rest" of soft limit reproduced by neighbouring dipole with $i \leftrightarrow k$

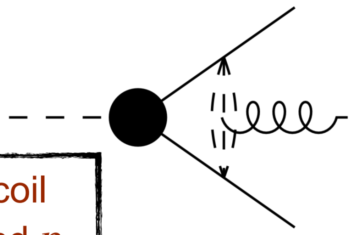
**Ordering variable** typically chosen as some notion of transverse momentum:

$$t \equiv p_{\mathrm{T}}^2 = z(1-z) s_{IK}$$

$$z = \frac{s_{ik}}{s_{ik} + s_{jk}}$$

$$s_{IK} \equiv s_{ijk} = s_{ij} + s_{jk} + s_{ik}$$

**Antenna showers** describe radiation from dipoles with **local emitter-recoiler-agnostic** on-shell kinematics:




$$p_i^\mu = a_i p_I^\mu + b_i p_K^\mu - c p_\perp^\mu$$
$$p_j^\mu = (1 - a_i - a_k) p_I^\mu + (1 - b_i - b_k) p_K^\mu + p_\perp^\mu$$
$$p_k^\mu = a_k p_I^\mu + b_k p_K^\mu - (1 - c) p_\perp^\mu$$

transverse recoil taken by $p_I$ and $p_K$

**Antenna functions** (= branching kernels) contain soft + (part of) collinear limits:

$$A_{g/qg}(p_i, p_j, p_k) = \frac{2 s_{ik}}{s_{ij} s_{jk}} + \frac{s_{jk}}{s_{ij} s_{ijk}} + \frac{s_{ij} s_{ik}}{s_{jk} s_{ijk}^2}$$

soft | $j \parallel i$ | $j \parallel k$

Note: "rest" of $j \parallel k$ limit reproduced by neighbouring antenna with $z \leftrightarrow 1 - z$

**Ordering variable** typically chosen as symmetric "ARIADNE-$p_\perp$":

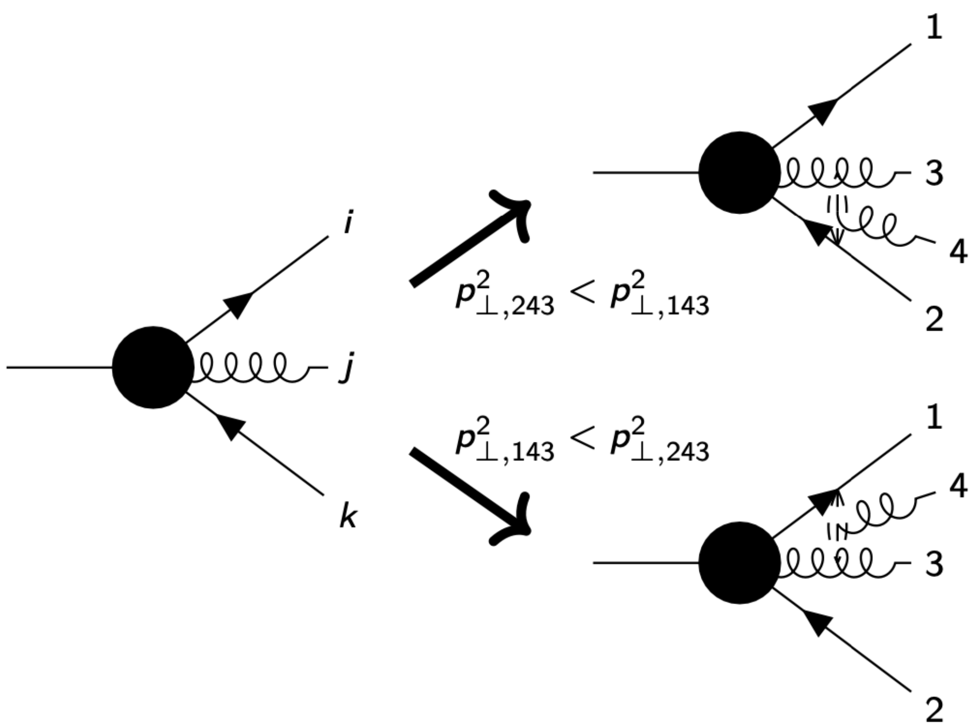
$$t \equiv p_\perp^2 = \frac{s_{ij} s_{jk}}{s_{ijk}} \to \begin{cases} (1 - z_i) s_{ij} & j \parallel i \\ (1 - z_j) s_{jk} & j \parallel k \end{cases}$$

$$z_i = \frac{s_{ik}}{s_{ik} + s_{jk}} \approx \frac{s_{ik}}{s_{ijk}}$$

## Aside: sector antenna showers [Brooks, CTP, Skands 2003.00702]

**Idea:** combine antenna showers with deterministic jet-clustering algorithm

- shower only generates branchings that would be clustered by a $3 \mapsto 2$ clustering algorithm ~ ARCLUS [Lönnblad Z. Phys. C 58 (1993)]




$$p^2_{\perp,ijk} = \frac{s_{ij}s_{jk}}{s_{IK}}$$

⇒ softest gluon always regarded as emitted one

⇒ only one (most singular) antenna contributes at each phase-space point

Since PYTHIA 8.304: sector showers default option in VINCIA

# Parton showers in PYTHIA

**Simple shower** `PartonShowers:model = 1`

- PYTHIA's default $p_\mathrm{T}$-ordered shower algorithm
- based on DGLAP splitting functions + dipole-like kinematics

**VINCIA** `PartonShowers:model = 2`

- antenna shower ordered in $p_\perp$
- based on antenna functions + local antenna kinematics

**DIRE** `PartonShowers:model = 3`

legacy support only

- dipole shower ordered in $p_\perp$
- based on dipole splitting functions + local dipole kinematics

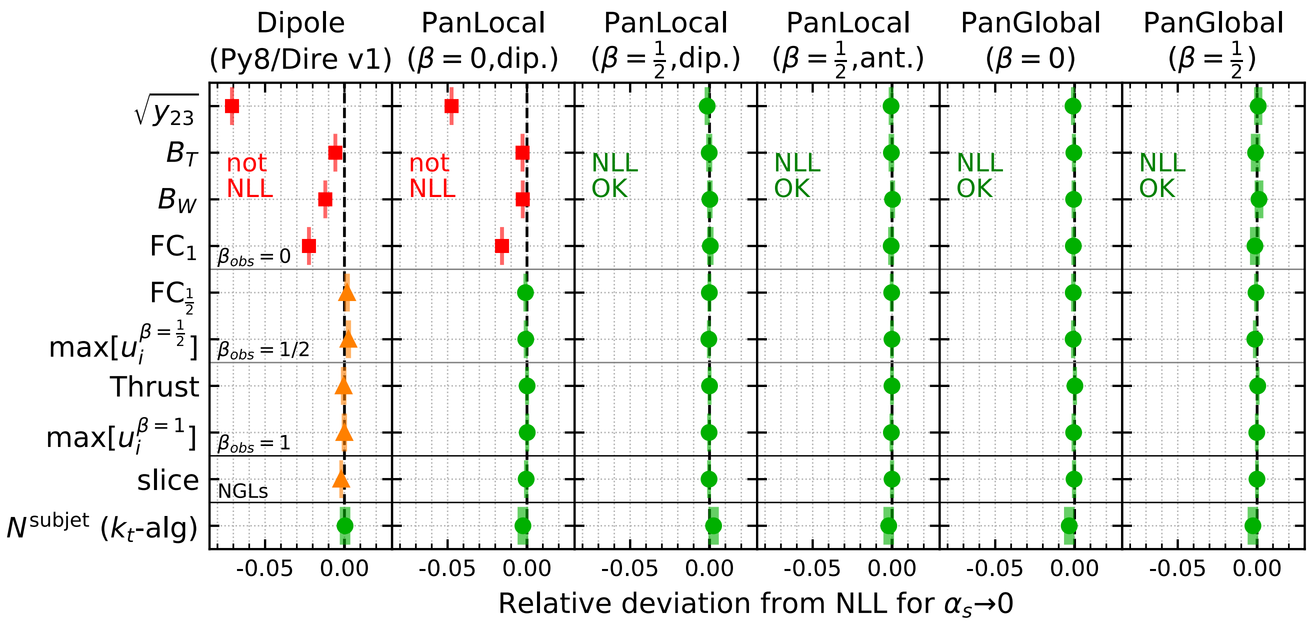
**APOLLO** `to be released?`

NEW

- dipole-antenna shower ordered in notion of $p_\perp$
- based on partitioned antenna functions + global dipole kinematics

# Logarithmic accuracy

## Conventional dipole(-like) showers are **inconsistent** with NLL resummation




[PanScales PRL125(2020)5,052002]

## Why care?

### Impossible to ignore recent progress on logarithmically accurate showers

(PanScales, ALARIC, HERWIG, Deductor, …)

### Improved formal control over shower and matching precision

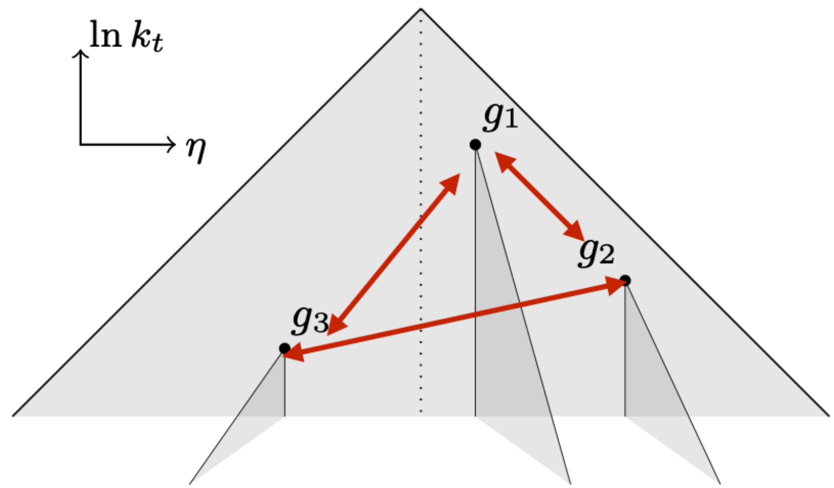
(consistency with resummation, higher-order matching, theoretical uncertainties, …)

## The problem with dipole/antenna showers

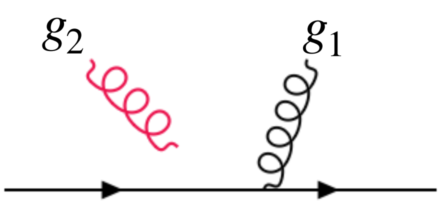
**Main issue**: local dipole recoil schemes are not **NLL safe**

(leaving problems with the correct assignment of $C_\mathrm{F}$ and $C_\mathrm{A}$ aside — noted already in [Gustafson NPB 392(1993)251])

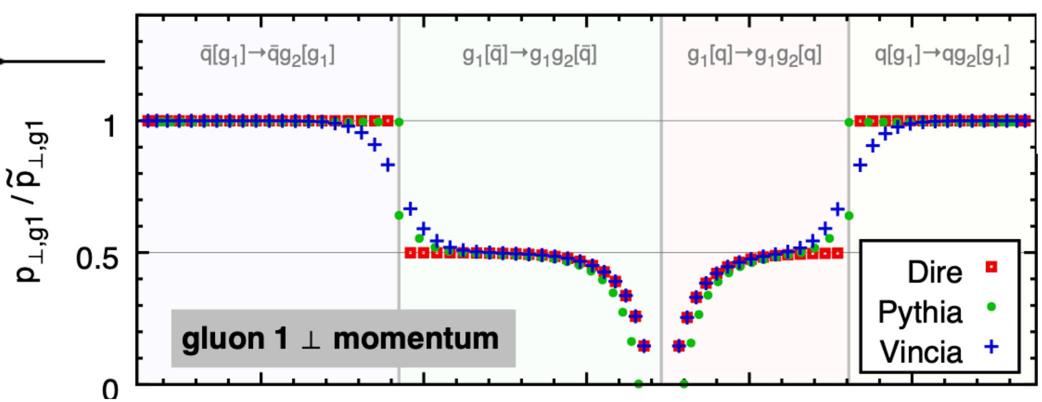
**NLL safety**: [PanScales PRL125(2020)5,052002]
subsequent emissions must not affect previous ones if they are well separated in at least one direction in the Lund plane

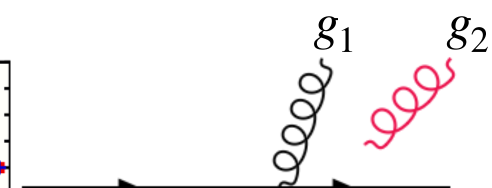



adapted from S. Ferrario-Ravasio




impact of gluon-2 emission on gluon-1 momentum



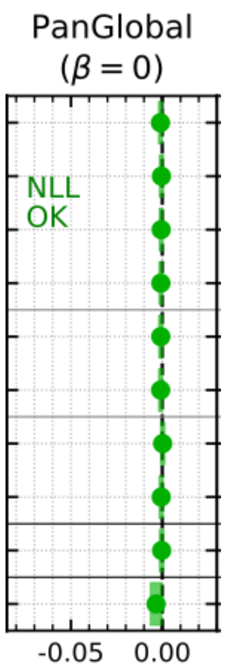




[Dasgupta et al. JHEP09(2018)033]

Recoil issues can be fixed with **global transverse recoil**
(→ PanGlobal [PanScales PRL125(2020)5,052002])

$$p_i^\mu = r a_i p_I^\mu$$
$$p_j^\mu = r(1-a_i)p_I^\mu + r(1-b_k)p_K^\mu + r p_\perp^\mu$$
$$p_k^\mu = r b_k p_K^\mu$$
$$p_r^\mu = p_R^\mu - p_\perp^\mu$$

transverse recoil taken by entire event $p_R = \sum_{k\in \mathrm{FS}} p_k^\mu$




Requires **rescaling** $r = \sqrt{\frac{p_R^2}{p_r^2}}$ to conserve invariant mass of the event

**But**: **very difficult** to deal with rescaling when keeping phase space **exact**!
Note: PanScales uses soft/collinear phase-space approximation

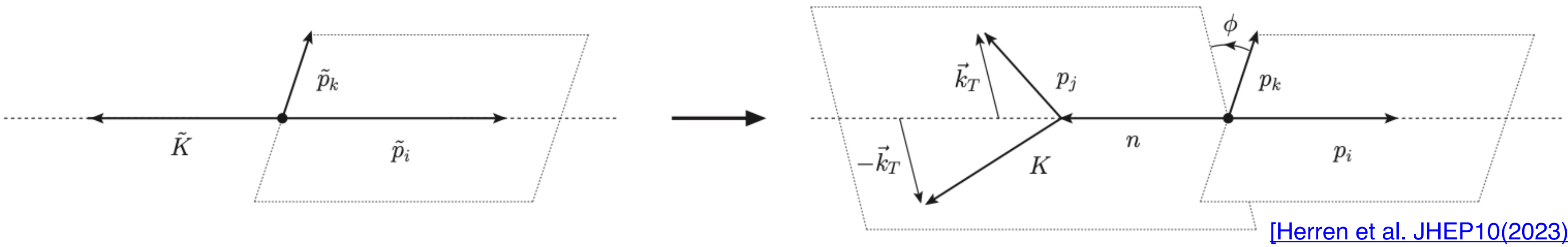
# Global dipole kinematics

**Alternative solution**: give up on emitter-recoiler-agnostic kinematics
(→ ALARIC [Herren et al. JHEP10(2023)091])

$$p_i^\mu = z\tilde{p}_{ij}^\mu$$
$$p_j^\mu = a\tilde{p}_{ij}^\mu + bK^\mu + p_\perp^\mu$$
$$K^\mu = (1-z-a)\tilde{p}_{ij}^\mu + (1-b)\tilde{K}^\mu - p_\perp^\mu$$

transverse recoil taken by initial state $\tilde{K}^\mu = \sum_{k\in \mathrm{IS}} p_k^\mu$

subsequent boost distributes transverse recoil among **all final-state particles**
Note: analogous to dipole subtraction with identified hadrons [Catani, Seymour NPB485(1997)291]



[Herren et al. JHEP10(2023)091]

Requires **partitioning** of antenna functions into $j \parallel i$ and $j \parallel k$ terms

⇒ APOLLO: **A**ntenna **P**artitioning **O**vercoming **L**ogarithmically **L**imiting **O**bstacles

Introduce auxiliary vector $n_j^\mu$ to partition soft term

$p_i n_j \neq 0,\ p_k nj \neq 0$

$$\frac{2s_{ik}}{s_{ij}s_{jk}} = \frac{1}{s_{ij}}\frac{2s_{ik}(p_i n_j)}{s_{jk}(p_i n_j) + s_{ij}(p_k n_j)} + \frac{1}{s_{jk}}\frac{2s_{ik}(p_k n_j)}{s_{jk}(p_i n_j) + s_{ij}(p_k n_j)}$$

In practice: $n_j^\mu = K^\mu + p_j^\mu$

Collinear terms straightforward ($1/s_{ij}$ vs $1/s_{jk}$)

Two (partitioned) antenna functions / branching kernels

$$P_{\mathrm{qg}}(p_i, p_j, p_k; n_j) = \frac{1}{s_{ij}}\left[\frac{2s_{ik}(p_i n_j)}{s_{jk}(p_i n_j) + s_{ij}(p_k n_j)} + \frac{s_{jk}}{s_{ijk}}\right]$$

$$P_{\mathrm{gg}}(p_i, p_j, p_k; n_j) = \frac{1}{s_{ij}}\left[\frac{2s_{ik}(p_i n_j)}{s_{jk}(p_i n_j) + s_{ij}(p_k n_j)} + \frac{s_{jk}s_{ik}}{s_{ijk}^2}\right]$$

(Note: branching kernels implicitly depend on branching angle $\phi$)

Evolution variable chosen in analogy to collinear limit of ARIADNE-$p_\perp$

$$t = (1-z)s_{ij} = z(1-z)2\tilde{p}_{ij}\tilde{K}$$

$$z = \frac{p_i n_j}{(p_i + p_j)n_j}$$

Gluon splittings **irrelevant** for NLL consistency, because **purely collinear**

Note: in principle all purely-collinear branchings irrelevant at NLL

Use kinematics with intrinsic $i \leftrightarrow j$ symmetry (i.e., $\mathrm{q} \leftrightarrow \bar{\mathrm{q}}$ symmetry)

$$p_i^\mu = a_i \tilde{p}_{ij}^\mu + b_i \tilde{K}^\mu - p_\perp^\mu$$
$$p_j^\mu = a_j \tilde{p}_{ij}^\mu + b_j K^\mu + p_\perp^\mu$$
$$K^\mu = (1 - a_i - a_j)\tilde{p}_{ij}^\mu + (1 - b_i - b_j)\tilde{K}^\mu$$

transverse recoil taken by $\tilde{p}_{ij}$ (not NLL safe)

Branching kernel constructed as

$$P_{\mathrm{q}\bar{\mathrm{q}}}(p_i, p_j, p_k) = \frac{1}{s_{ij}}\left[1 - \frac{2 s_{jk} s_{ik}}{s_{ijk}^2}\right]$$

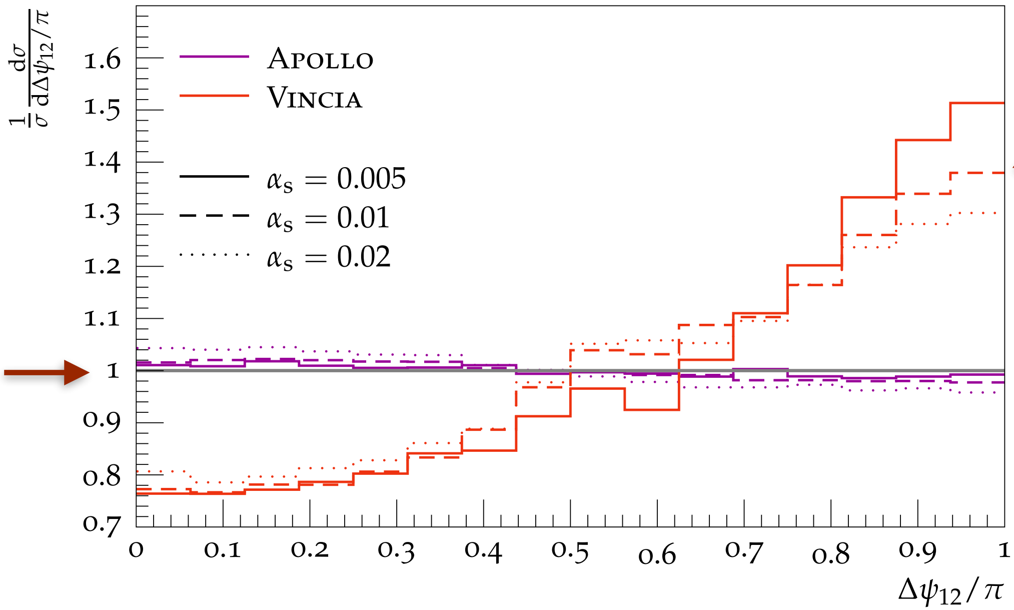
Evolution variable $t = (1 - z)s_{ij}$ identical to gluon-emission case

(not necessary, but simplifies multiplicative NLO matching)

Take limit $\alpha_s \to 0$ with $\lambda = \alpha_s \log(v) =$ const to **separate NLL terms** in ratio $\frac{\Sigma^{\mathrm{PS}}}{\Sigma^{\mathrm{NLL}}}$

Numerically achieved by extrapolating results for small $\alpha_s$ in the limit $t_c \to 0$
(numerically quite challenging, done in dedicated python code)

APOLLO is NLL

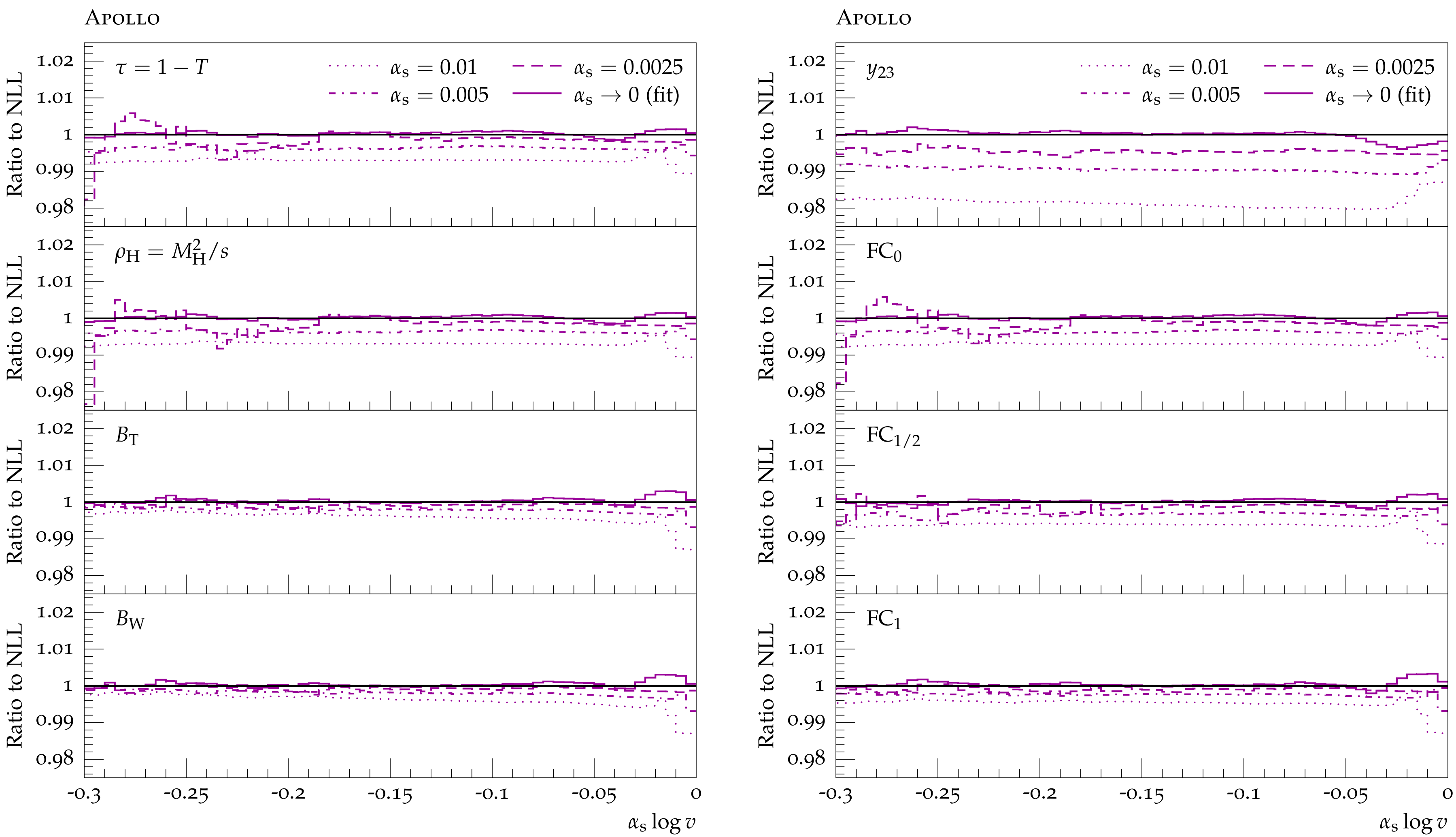



VINCIA not NLL

$\Delta\psi_{12}$ = angle between first and second emission

# NLL tests

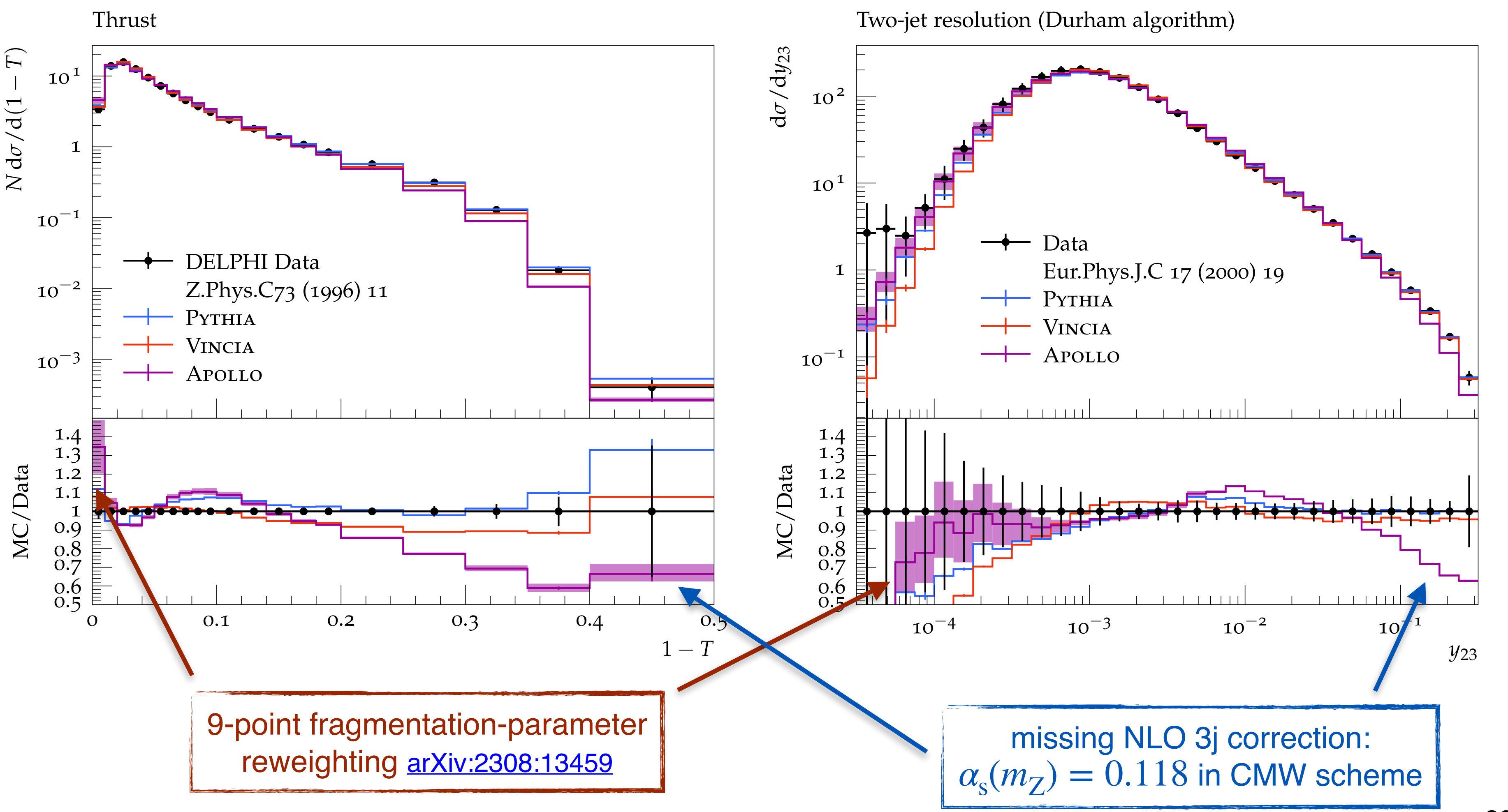
[CTP JHEP07(2024)161]




APOLLO is consistent with NLL for wide range of global event shapes

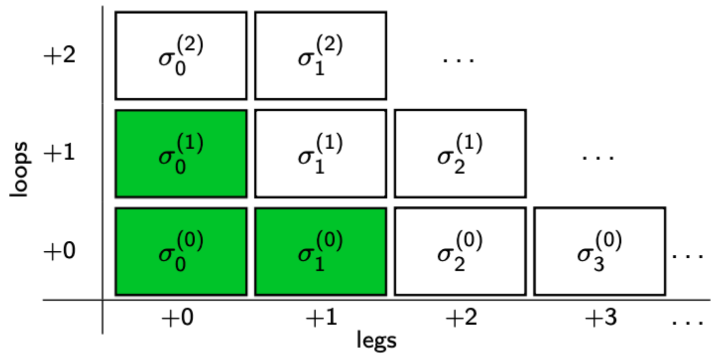
# Preliminary comparison to LEP data and default PYTHIA and VINCIA

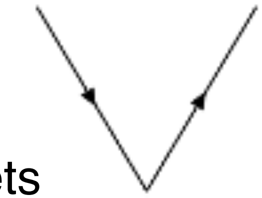
No dedicated tune, fix $a_{\mathrm{Lund}} = 0.44$ and $b_{\mathrm{Lund}} = 0.55$ based on “first bins” of event shapes @ LEP



# NLO matching

**NLO**

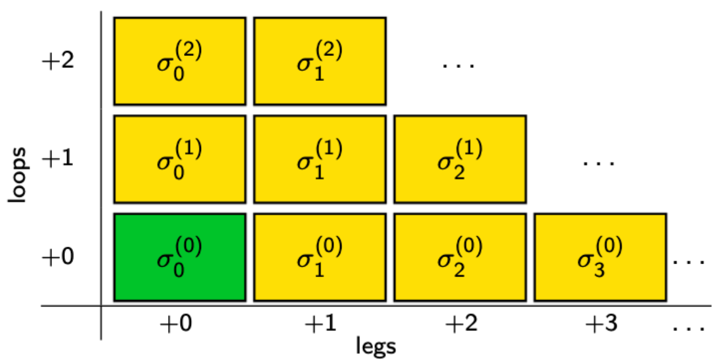


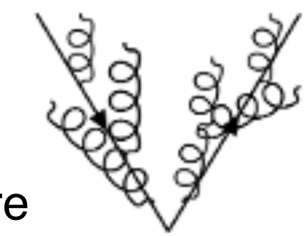



**Fixed-order calculations** → hard jets

- reliable at high scales, without scale hierarchies
- accurate for limited number of legs (+ loops)
- perturbative accuracy (LO, NLO, NNLO, …)

**LO+PS**



**Parton showers** → jet substructure

- reliable at small scales, with scale hierarchies
- approximate predictions for many particles
- logarithmic accuracy (LL, NLL, NNLL, …)

⇒ large complementarity, so ideally combine them!

The simplest way to combine parton showers with fixed-order calculations:
Matrix-element corrections (MECs)

Replace splitting functions (for first branching) by full matrix element.

Schematically:

$$\Delta(t_0, t_1) = \exp\left\{ -\sum_i \sum_{j \neq i} \int_{t_1}^{t_0} \mathrm{d}\Phi_{ij}\, C_{ij}\, P_{ij}(p_i, p_j, p_k; n_j)\; w_{\mathrm{MEC}}(\Phi_{n+1}) \right\}$$

with MEC factor:

$$w_{\mathrm{MEC}}(\Phi_{n+1}) = \frac{R(p_1, \ldots, p_{n+1})}{\sum_i \sum_{j \neq i} C_{ij} P_{ij}(p_i, p_j, p_k; n_j) B(\tilde{p}_1, \ldots, \tilde{p}_n)}$$

For simple processes (colour-singlet decays/production), the MEC can be absorbed into the definition of the branching kernel.

E.g. $\mathrm{H} \to \mathrm{qg\bar{q}}$ (with non-vanishing Yukawa but kinematically massless quarks)

$$|\mathcal{M}_{\mathrm{H}\to\mathrm{qg\bar{q}}}|^2 = \left( \frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{s_{jk}}{s_{ij}s_{ijk}} + \frac{s_{ij}}{s_{jk}s_{ijk}} + \frac{2}{s_{ijk}} \right) |\mathcal{M}_{\mathrm{H}\to\mathrm{q\bar{q}}}|^2$$

$$P_{\mathrm{H}\to\mathrm{qg\bar{q}}}(p_i, p_j; p_k, n_j) = \frac{2s_{ik}(p_i n_j)}{s_{ij}(s_{jk}(p_i n_j) + s_{ij}(p_k n_j))} + \frac{s_{jk}}{s_{ij}s_{ijk}} + \frac{1}{s_{ijk}}$$

$$P_{\mathrm{H}\to\mathrm{qg\bar{q}}}(p_k, p_j; p_i, n_j) = \frac{2s_{ik}(p_k n_j)}{s_{ij}(s_{jk}(p_i n_j) + s_{ij}(p_k n_j))} + \frac{s_{ij}}{s_{jk}s_{ijk}} + \frac{1}{s_{ijk}}$$

# Full-colour MECs

**In general:**
Need to account for subleading-colour pieces with varying signs $(-1/N_\mathrm{C})^\ell$.

Kinematics explicitly depend only on $p_i^\mu$, $p_j^\mu$, and global reference $K^\mu$.

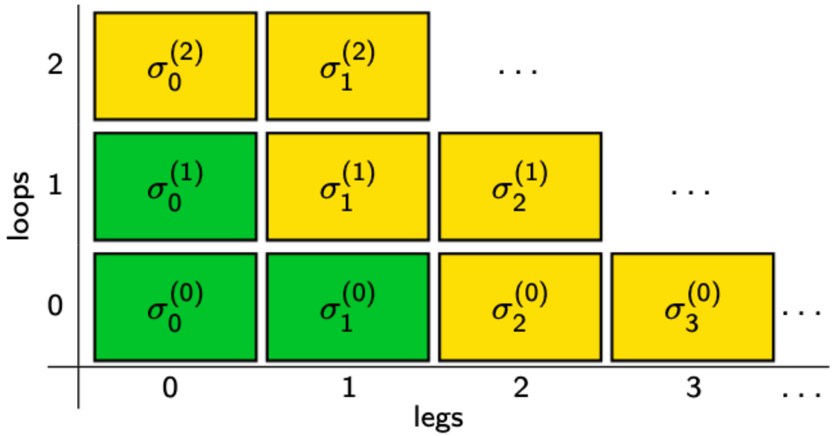
Define colour-corrected branching kernel as

$$P_{ij}^{(\mathrm{cc})}(p_i, p_j; K) = \sum_{k \neq i,j} C_{ij,k} P_{ij}(p_i, p_j, p_k; K)$$

Since $t$ and $z$ are Lorentz invariant, the full-colour ME can be reproduced by

$$w_{\mathrm{MEC}}^{(\mathrm{FC})}(\Phi_{n+1}) = \frac{R(p_1, \ldots, p_{n+1})}{\sum_i \sum_{j \neq i} \sum_{k \neq i,j} C_{ij,k} P_{ij}^{\mathrm{trial}}(t, z) B(\tilde{p}_1, \ldots, \tilde{p}_n)} \geq 0$$

$$t = (1-z)s_{ij}\,, \quad z = \frac{p_i n_j}{(p_i + p_j) n_j}$$



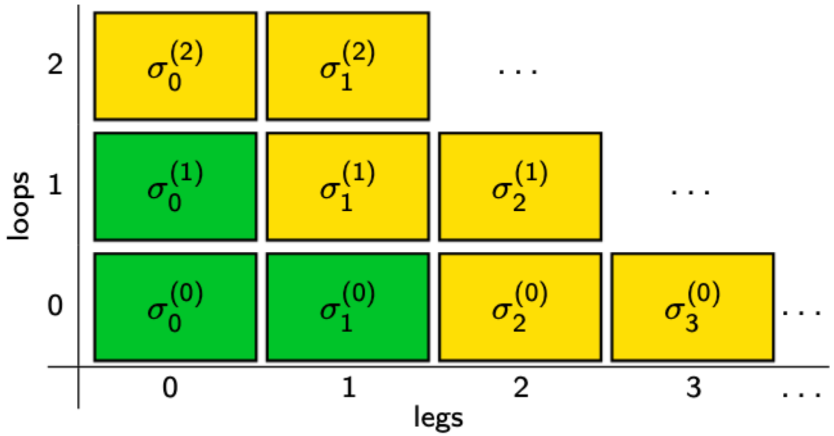

Strategy developed > 20 years ago
[Norrbin, Sjöstrand NPB603(2001)297-342]

Nowadays known as POWHEG matching
[Nason JHEP11(2004)040]

**Idea:** first-order expansion of matrix-element-corrected shower reproduces NLO calculation if Born-level event is weighted by NLO $K$-factor:

$$\langle O \rangle^{\mathrm{POWHEG}}_{\mathrm{NLO+PS}} = \int \mathrm{d}\Phi_2\, \mathrm{B}(\Phi_2)\, K_{\mathrm{NLO}}(\Phi_2)\, \mathcal{S}_2(t_0, O; \Phi_2)$$

First-order MECs can easily be implemented via **finite terms** in branching kernels
$\Rightarrow$ for colour-singlet decays NLO matching automatic in PYTHIA

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First-order MECs can easily be implemented via **finite terms** in branching kernels
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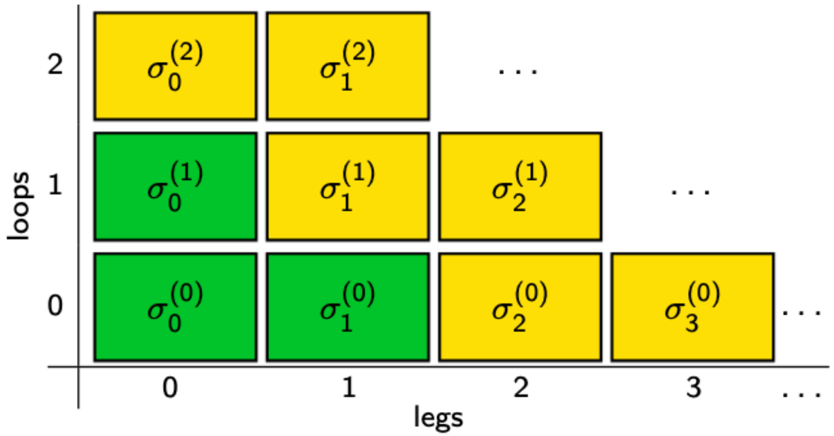
**In general need:**
(1) Born-local NLO weight
(2) Matrix-element correction in first branching

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**In general need:**
(1) Born-local NLO weight
(2) Matrix-element correction in first branching ✓

# Born-local NLO weights

Two general and NLL-safe schemes for multiplicative NLO matching:

**Colour-ordered projectors**
- same kinematics as in shower
- requires dedicated knowledge of colour structure

**Born-local subtraction**
- splitting kinematics (NLL unsafe!)
- NLL safety restored up to ME-corrected order

In both cases:

POWHEG parton shower

**identical** evolution variable in first branching and subsequent branchings

Note: this means no mismatch, no need for PowhegHooks, no vetoed shower!

Impose colour ordering on real matrix element

$$R(p_1, \ldots, p_{n+1}) = \sum_{\ell=1}^{m} C^{(\ell)} R^{(\ell)}(p_1, \ldots, p_{n+1})$$

and decompose each colour layer according to branching kernels

$$R^{(\ell)}(p_1, \ldots, p_{n+1}) = \sum_i \sum_{j \neq i} W_{ij}^{(\ell)} R_{ij}^{(\ell)}(p_1, \ldots, p_{n+1})$$

with $W_{ij}^{(\ell)} = \dfrac{P_{ij}(p_i, p_j; K) B(\tilde{p}_1, \ldots, \tilde{p}_n)}{\sum_i \sum_{i \neq j} P_{ij}(p_i, p_j; K) B(\tilde{p}_1, \ldots, \tilde{p}_n)}$ ← well defined as $P_{ij} > 0$

The real correction can then be integrated locally using shower kinematics

$$\Rightarrow \sum_i \sum_{j \neq i} \sum_{\ell} C^{(\ell)} \int \mathrm{d}\Phi_{ij} \left[ W_{ij}^{(\ell)} R^{(\ell)}(p_1, \ldots, p_{n+1}) - P_{ij}(p_i, p_j; K) B(\tilde{p}_1, \ldots, \tilde{p}_n) \right]$$

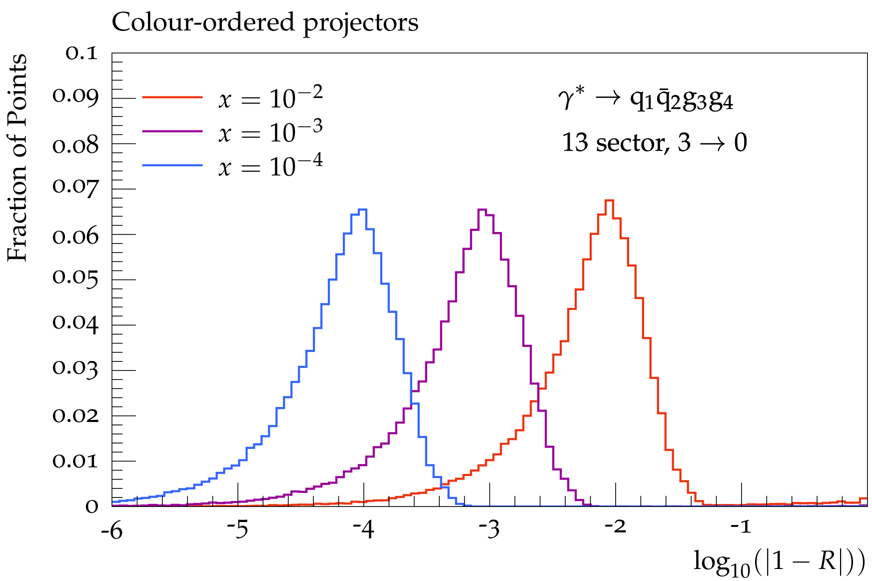
# Colour-ordered projectors — convergence tests

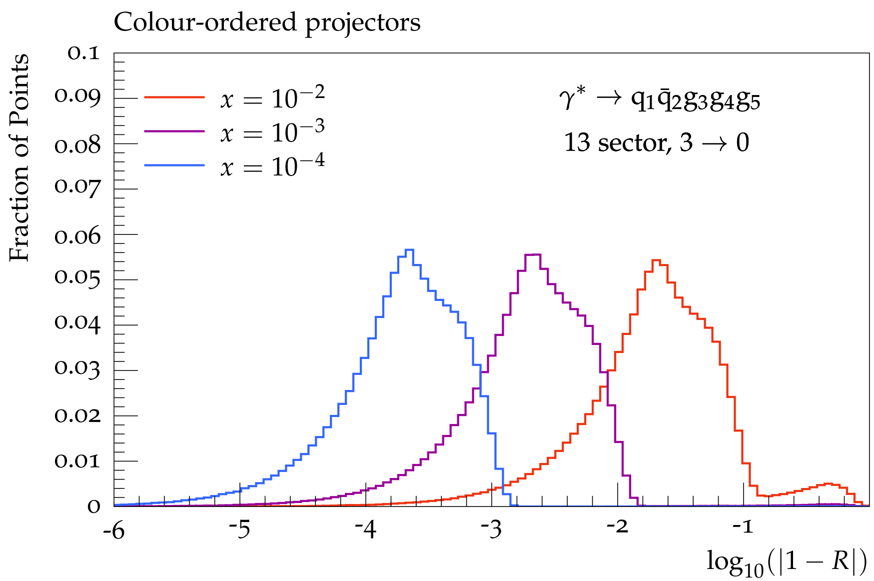
Apply rescaling in singular limits:

soft: $E_j \to xE_j$ , collinear: $\sqrt{s_{ij}} \to x\sqrt{s_{ij}}$

and express convergence in number of agreeing digits

$$\log_{10}(|1-R|) \text{ with } R = \frac{\sum_\ell C^{(\ell)} P_{ij}(p_i, p_j; K) B(\tilde{p}_1, \ldots, \tilde{p}_n)}{\sum_\ell C^{(\ell)} W_{ij}^{(\ell)} R^{(\ell)}(p_1, \ldots, p_{n+1})}$$



# Born-local subtraction

In analogy to local analytic sector subtraction introduce sector functions
[Magnea et al. JHEP12(2018)107]

$$R(p_1, \ldots, p_{n+1}) = \sum_i \sum_{j \neq i} W_{ij}^{\mathrm{sct}} R(p_1, \ldots, p_{n+1}) \text{ with } \sum_i \sum_{j \neq i} W_{ij}^{\mathrm{sct}} = 1$$

and construct subtraction term as

$$K_{ij}(p_1, \ldots, p_{n+1}) := \left( \boldsymbol{S}_j^{\downarrow} + \boldsymbol{C}_{ij}^{\downarrow} - \boldsymbol{S}_j^{\downarrow} \boldsymbol{C}_{ij}^{\downarrow} \right) R(p_1, \ldots, p_{n+1}) W_{ij}^{\mathrm{sct}}$$

$\boldsymbol{S}_i^{\downarrow}$: projector into $i$-soft limit
$\boldsymbol{C}_{ij}^{\downarrow}$: projector into $i \parallel j$ limit

The real correction can then be integrated locally

$$\Rightarrow \sum_i \sum_{j \neq i} \int \mathrm{d}\Phi_{ij} \left[ W_{ij}^{\mathrm{sct}} R^{(\ell)}(p_1, \ldots, p_{n+1}) - K_{ij}(p_1, \ldots, p_{n+1}) \right]$$

**But** requires NLL unsafe splitting kinematics for $i \leftrightarrow j$ symmetry!

# Born-local subtraction — kinematics

The first ("POWHEG") branching is generated with NLL unsafe splitting kinematics:

$$p_i^\mu = a_i \tilde{p}_{ij}^\mu + b_i \tilde{K}^\mu - p_\perp^\mu$$
$$p_j^\mu = a_j \tilde{p}_{ij}^\mu + b_j K^\mu + p_\perp^\mu$$
$$K^\mu = (1 - a_i - a_j)\tilde{p}_{ij}^\mu + (1 - b_i - b_j)\tilde{K}^\mu$$

Subsequent branchings are generated with NLL safe radiation kinematics:

$$p_i^\mu = z\tilde{p}_{ij}^\mu$$
$$p_j^\mu = a\tilde{p}_{ij}^\mu + bK^\mu + p_\perp^\mu$$
$$K^\mu = (1 - z - a)\tilde{p}_{ij}^\mu + (1 - b)\tilde{K}^\mu - p_\perp^\mu$$

**No mismatches** due to evolution in single measure $t = (1 - z)s_{ij}$

The shower expansion reproduces the correct strongly-ordered matrix element in all configurations where subsequent emissions are well separated in at least one direction in the Lund plane!

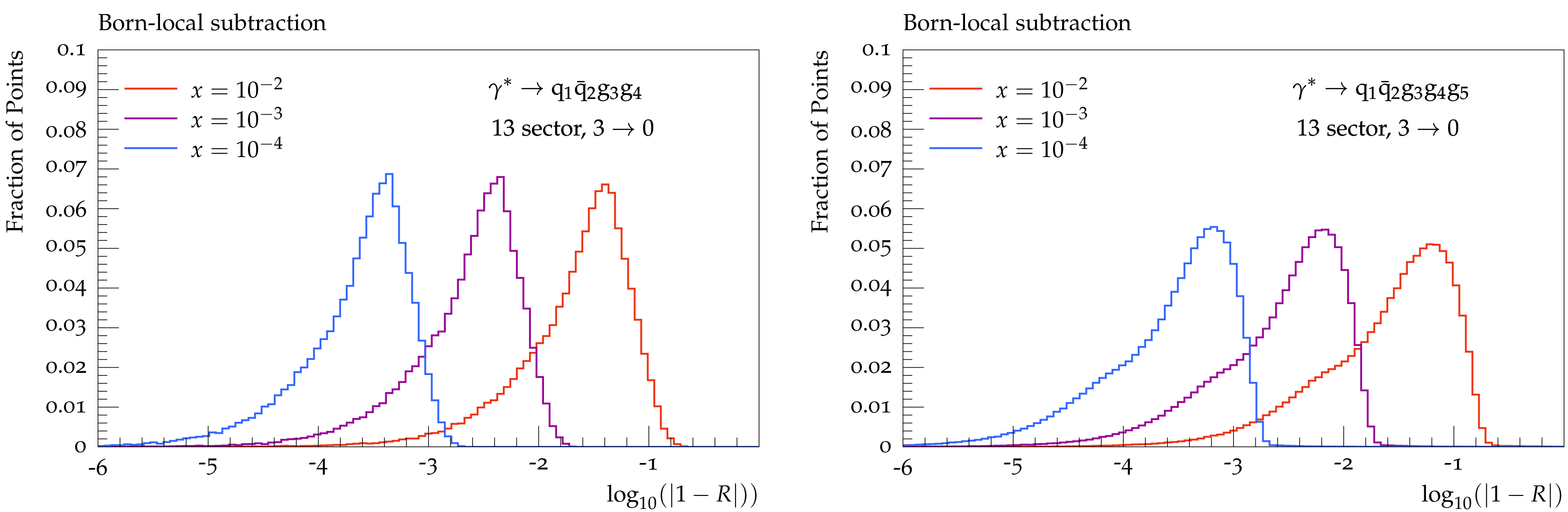
# Born-local subtraction — convergence tests

Apply rescaling in singular limits:

soft: $E_j \to xE_j$ ,  collinear: $\sqrt{s_{ij}} \to x\sqrt{s_{ij}}$

and express convergence in number of agreeing digits

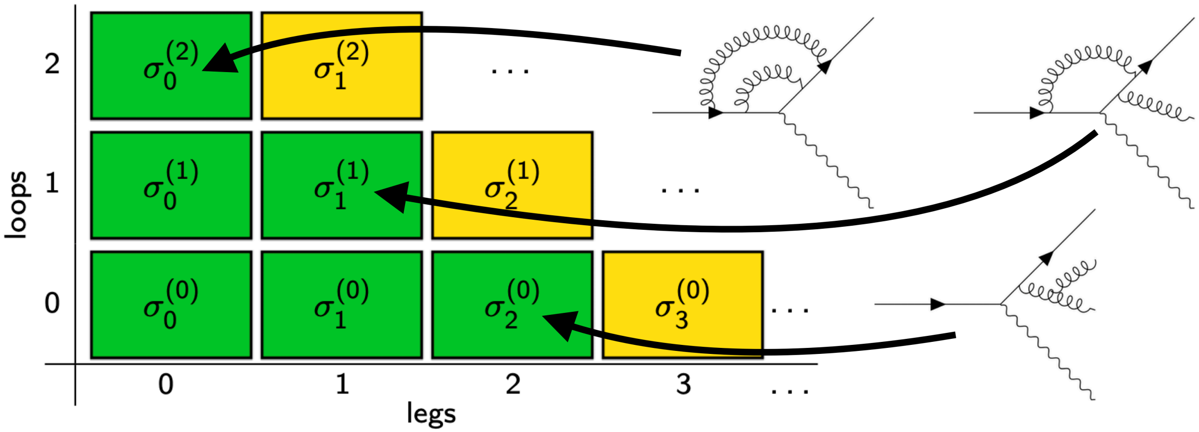
$$\log_{10}(|1 - R|) \text{ with } R = \frac{K_{ij}(p_1, \ldots, p_{n+1})}{W_{ij}^{\text{sct}} R(p_1, \ldots, p_{n+1})}$$



# Towards NNLO matching

# VINCIANNLO

[Campbell, Höche, Li, CTP, Skands PLB836(2023)137614]




VINCIA NNLO

**Idea:** fully-differential multiplicative NNLO matching scheme (“POWHEG at NNLO”)

$$\langle O \rangle^{\mathrm{VINCIA}}_{\mathrm{NNLO+PS}} = \int \mathrm{d}\Phi_2\, \mathrm{B}(\Phi_2)\, K_{\mathrm{NNLO}}(\Phi_2)\, \mathcal{S}_2(t_0, O; \Phi_2)$$

**Need:**

(1) Born-local NNLO $K$-factor

(2) NLO Matrix-element correction in first branching

(3) LO Matrix-element correction in second iterated branching

(4) Direct $2 \mapsto 4$ branching with Matrix-element correction

## NNLO K-factors [Campbell, Höche, Li, CTP, Skands PLB836(2023)137614]

(1) Born-local NNLO $K$-factor

$$k_{\mathrm{NNLO}}(\Phi_2) = 1 + \frac{\mathrm{V}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{I}_{\mathrm{S}}^{\mathrm{NLO}}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{VV}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{I}_{\mathrm{T}}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{I}_{\mathrm{S}}(\Phi_2)}{\mathrm{B}(\Phi_2)}$$
$$+ \int \mathrm{d}\Phi_{+1} \left[ \frac{\mathrm{R}(\Phi_2,\Phi_{+1})}{\mathrm{B}(\Phi_2)} - \frac{\mathrm{S}^{\mathrm{NLO}}(\Phi_2,\Phi_{+1})}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{RV}(\Phi_2,\Phi_{+1})}{\mathrm{B}(\Phi_2)} - \frac{\mathrm{T}(\Phi_2,\Phi_{+1})}{\mathrm{B}(\Phi_2)} \right]$$
$$+ \int \mathrm{d}\Phi_{+2} \left[ \frac{\mathrm{RR}(\Phi_2,\Phi_{+2})}{\mathrm{B}(\Phi_2)} - \frac{\mathrm{S}(\Phi_2,\Phi_{+2})}{\mathrm{B}(\Phi_2)} \right]$$

**Fixed-Order Coefficients:**

| Loops \ Legs | 0 | 1 | 2 |
|---|---|---|---|
| 0 | B | R | RR |
| 1 | V | RV | |
| 2 | VV | | |

**Subtraction Terms (not tied to shower formalism):**

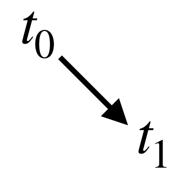
| Loops \ Legs | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | $S^{NLO}$ | S |
| 1 | $I_S^{NLO}$ | T | |
| 2 | $I_S$, $I_T$ | | |

**Note:** requires Born-local subtraction.
In the antenna formalism only given for simplest cases!

**Key aspect:**
up to matched order, include process-specific NLO corrections into shower evolution

(2) correct first branching to exclusive NLO rate

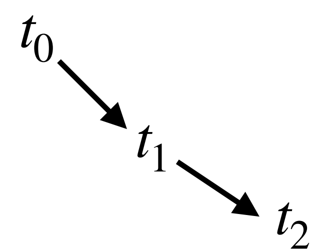
$$\Delta^{\mathrm{NLO}}_{2\mapsto 3}(t_0, t_1) = \exp\left\{ -\int_{t_1}^{t_0} \mathrm{d}\Phi_{+1} \frac{\mathrm{R}(\Phi_3)}{\mathrm{B}(\Phi_2)} \left(1 + w_{\mathrm{RV}}(\Phi_3)\right) \right\}$$




RV

(3) correct second branching to LO matrix element

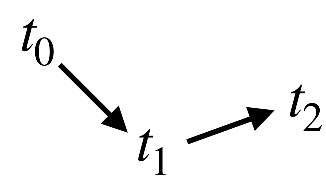
$$\Delta^{\mathrm{LO}}_{3\mapsto 4}(t_1, t_2) = \exp\left\{ -\int_{t_2}^{t_1} \mathrm{d}\Phi_{+1} \frac{\mathrm{RR}(\Phi_4)}{\mathrm{R}(\Phi_3)} \right\}$$




RR

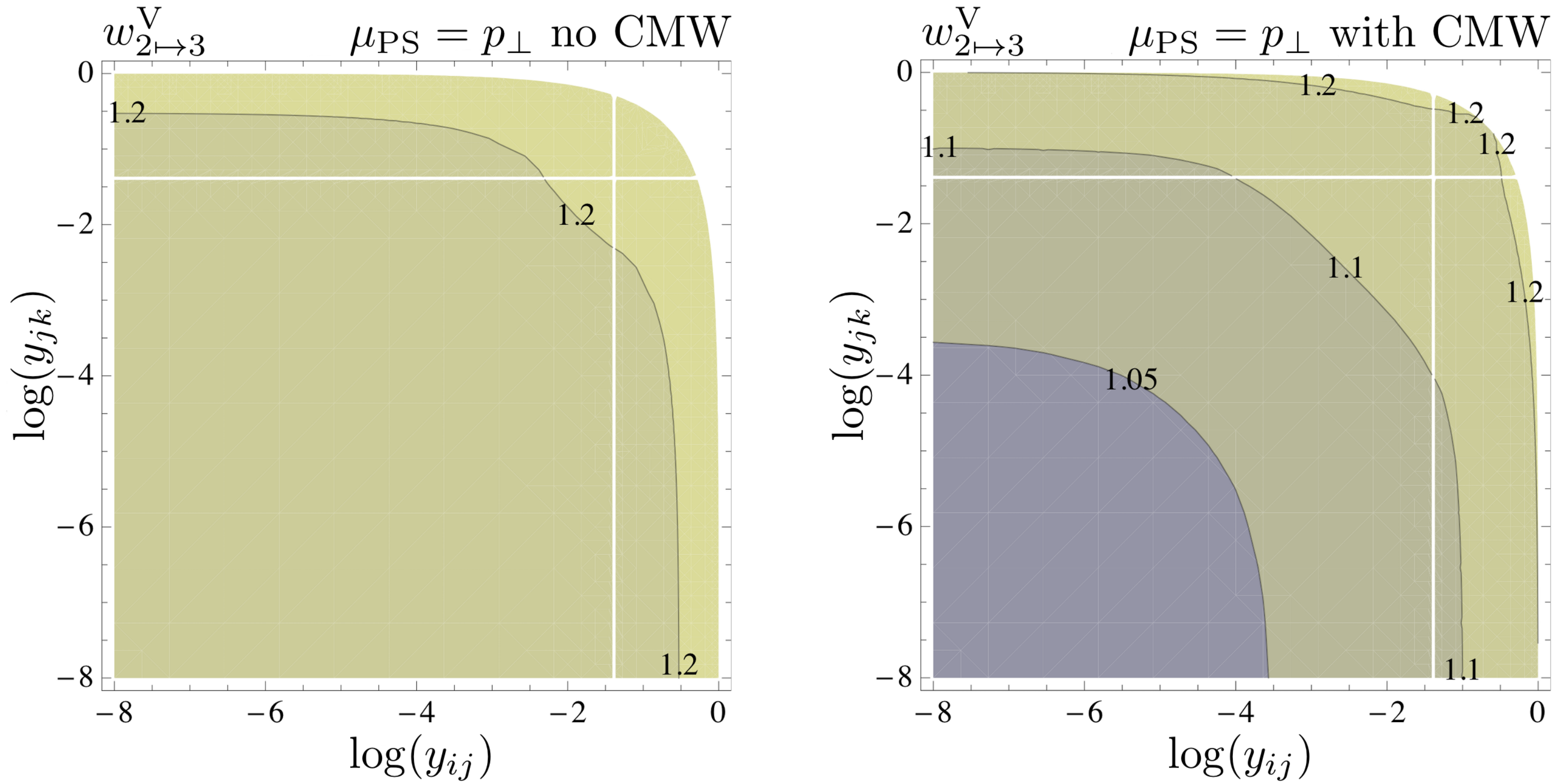
(4) add $2 \mapsto 4$ branching and correct it to LO matrix element

$$\Delta^{\mathrm{LO}}_{2\mapsto 4}(t_0, t_1) = \exp\left\{ -\int_{t_2}^{t_0} \mathrm{d}\Phi_{+2} \frac{\mathrm{RR}(\Phi_4)}{\mathrm{B}(\Phi_2)} \Theta(t_2 - t_1) \right\}$$




RR

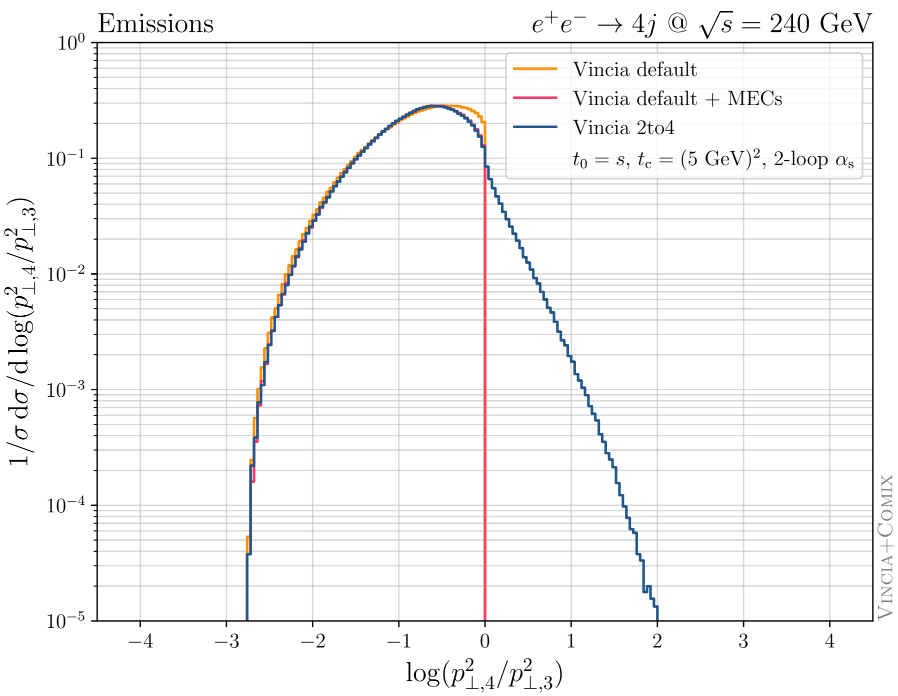
Real-virtual correction factor $w_{\mathrm{RV}}$ ("POWHEG in the exponent") studied analytically

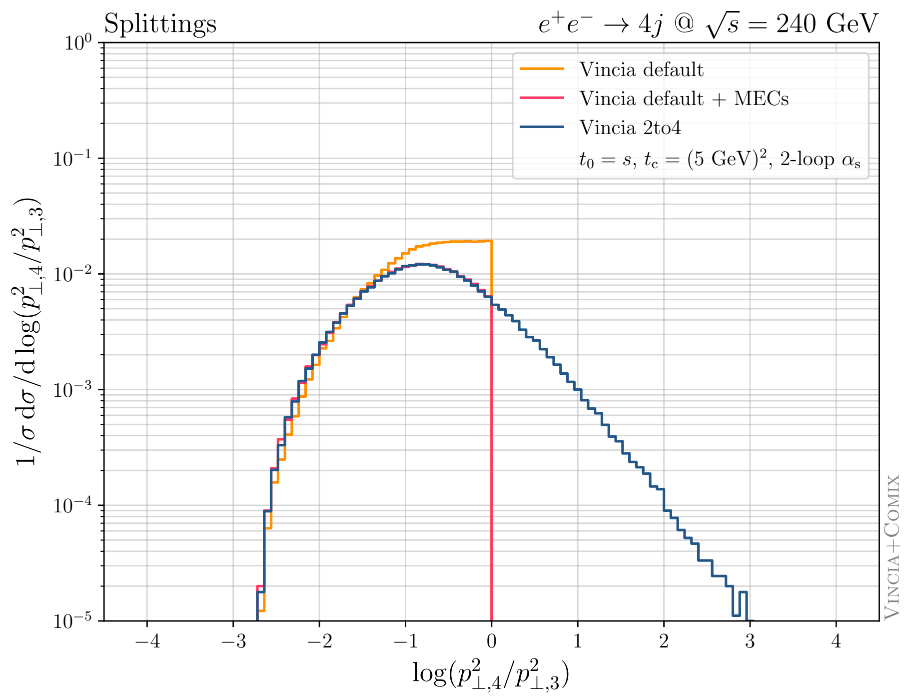



**Now** generalisation & semi-automation in form of NLO MECs

# Double-real corrections [Campbell, Höche, Li, CTP, Skands PLB836(2023)137614]

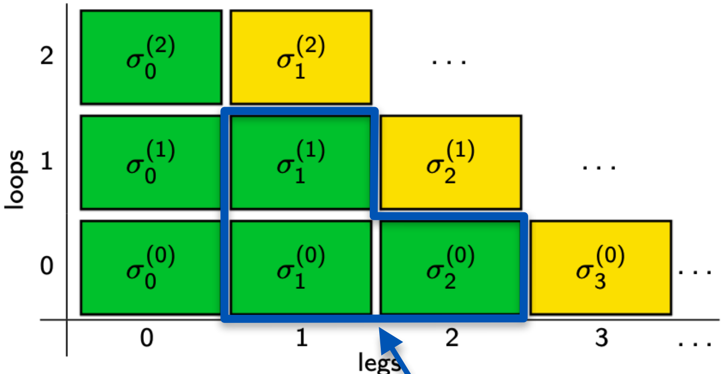
Direct $2 \mapsto 4$ shower fills unordered region of phase space with $p_{\perp,4}^2 > p_{\perp,3}^2$




**Sectorisation** enforces a strict cutoff at $p_{\perp,4}^2 = p_{\perp,3}^2$. No recoil effects!

# VINCIANNLO in resonance decays

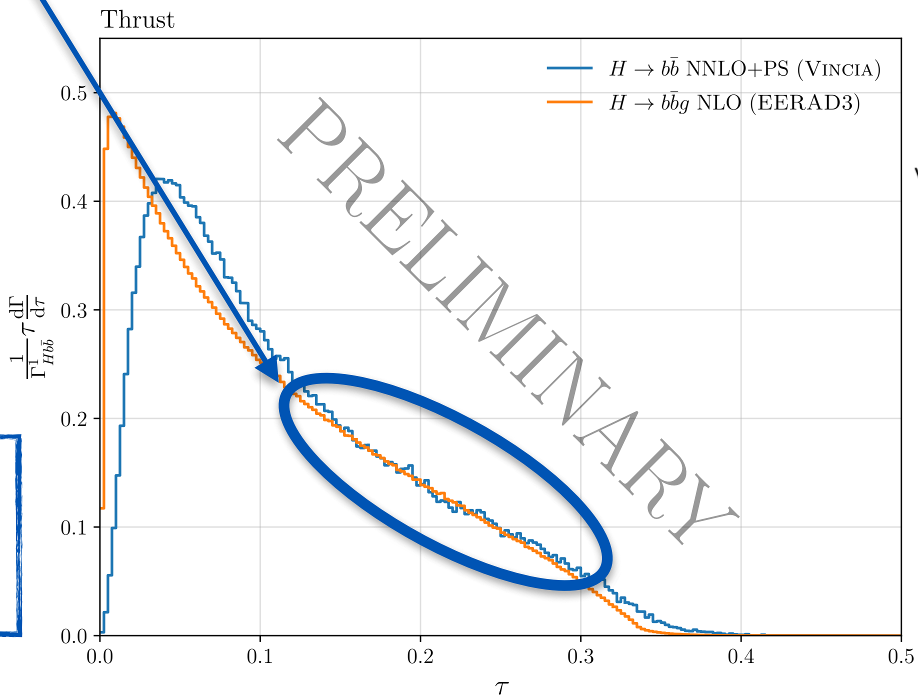



By **construction** decay width is NNLO accurate.

NNLO Born accuracy also implies
**NLO accuracy in first branching** and
**LO accuracy in second branching**.

E.g. $\mathrm{H} \to \mathrm{b\bar{b}j}$ at parton level (massless b-quarks with non-vanishing Yukawa)

vs. NLO from [Coloretti, Gehrmann-De Ridder, CTP JHEP06(2022)009]




Generalisation underway in collaboration with B. El-Menoufi, H.T. Li, L. Scyboz, P. Skands

# Conclusions

# Conclusions

Antenna showers are rooted in the soft limit.

Emitter-recoiler-agnostic antenna kinematics interpolate between dipole kinematics.

When aiming for formal NLL accuracy, the emitter-recoiler-agnostic picture may not be kept alive at the same time as keeping the phase space exact.

**Even with global “dipole-like” kinematics, a close connection to physical matrix elements/antenna functions can be kept intact.**

**Antenna showers offer a promising candidate for general NNLO matching.**