



How accurate are the partial N3LO fitting frameworks?

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Like an elaborate precision clock, a PDF fit relies on tight coordination of its multiple components to achieve the desired NxLO accuracy

To assess accuracy of a calculation,
Compare cross sections, not just PDFs

Parsimony/Occam's razor/information criteria:

Unnecessary free parameters constrained by the data tend to reduce, not increase, predictivity of a model

Does a combination of two PDF ensembles cover all possibilities arising in global fits?



Accurate (NNLO) PDFs

CT18 NNLO/CT18Z NNLO
MSHT20, NNPDF3.1/4.0 NNLO
ABMP, ATLAS, ... NNLO

multiple PDF solutions consistent with observations

Potentially more accurate (aN3LO) PDFs

MSHT20 aN3LO
NNPDF4.0 aN3LO

Different from NNLO?
More consistent?
Unique?

“NNLO+”

Mixed NNLO-N3LO calculations

E.g., a possible CT18 NNLO+ prescription (out of several)

1. Use CT18Z NNLO or CT18 NNLO error sets
2. Central prediction: take the average of predictions with $\hat{\sigma}_{NNLO}$ and $\hat{\sigma}_{N3LO}$
3. Scale uncertainty: compute using $\hat{\sigma}_{N3LO}$

How different from aN3LO predictions?

Necessary components of an N3LO PDF analysis

Component		Availability
Splitting functions		Partial N3LO
Hard cross sections	• DIS, light flavors	Full N3LO
	• NC DIS, heavy flavors	Full N3LO (Blümlein et al.), not yet in fitting codes
	• Vector boson production	Full N3LO for some processes, fixed N3LO/NLO K-factor tables
	• CC DIS, jet, $t\bar{t}$ production	N2LO
	• $pp \rightarrow W + c, pp \rightarrow Z + b, pp \rightarrow b$	NLO (massive); NNLO (ZM)

Looking forward to including all components **exactly and fully** to reduce the QCD scale uncertainty and guarantee the N3LO accuracy in the near future.

CTEQ-TEA and other groups include some N3LO contributions in their fitting codes: recent progress of MSHT and NNPDF in NNLO+ (aN3LO) fits

Probing parton luminosities with toy N3LO cross sections

Computations done by Max Ponce Chavez

Compute NNLO, N3LO cross sections with the **n3loxs** code (Baglio et al., [2209.06138](#))

1. Test gg luminosities via $gg \rightarrow \text{toy } H^0 X$; heavy top-quark limit with $N_f = 5$
2. Test $q\bar{q}$ luminosities via $q\bar{q} \rightarrow \text{toy } Z' X$

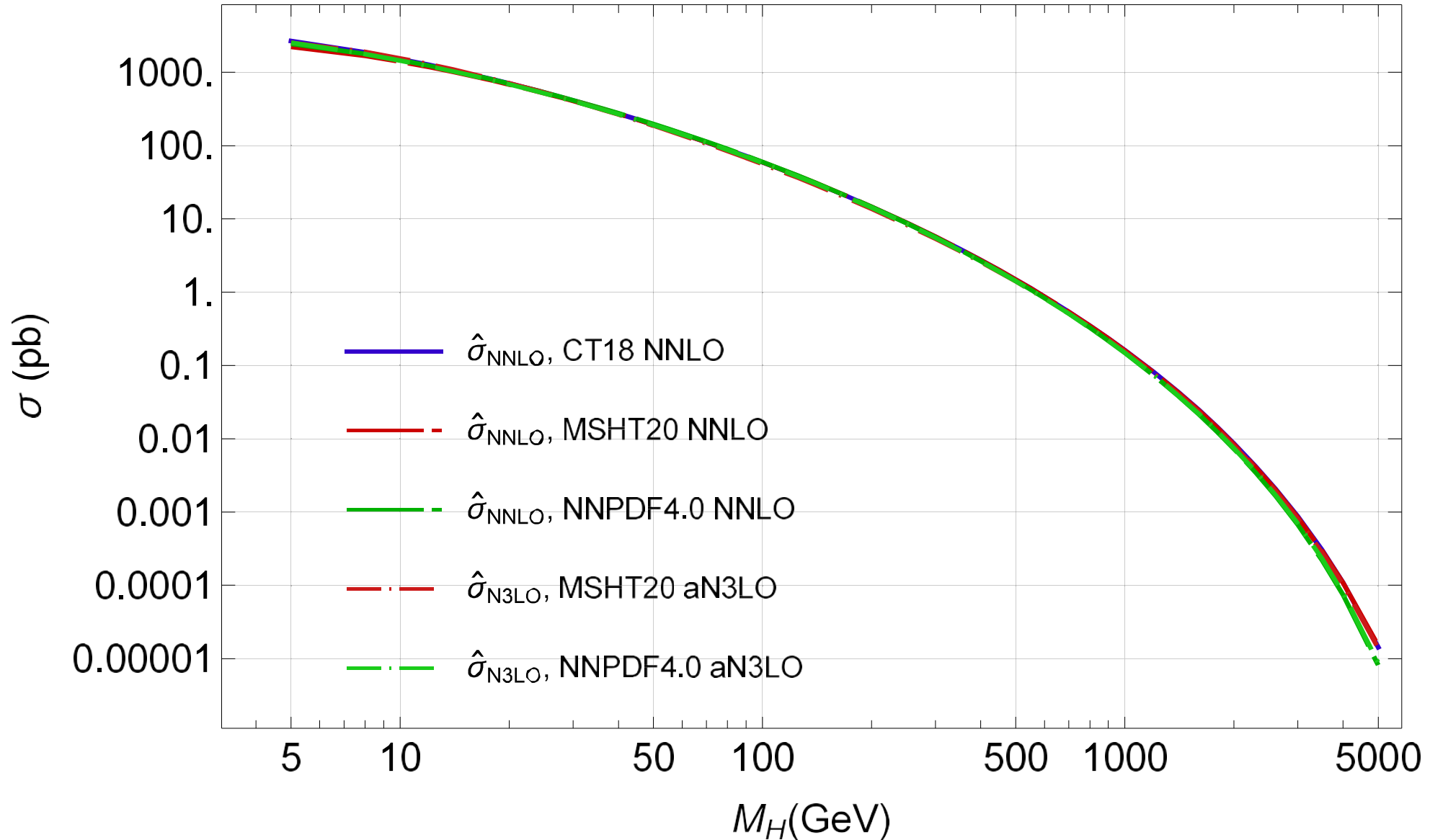
- Retain SM couplings; vary masses $M_{H,Z'}$ to test $f_a(x, \mu)$ at $x \sim \frac{M_{H,Z'}}{\sqrt{s}}$
- Estimate the 7-point QCD scale uncertainty around $\mu_{F,0} = \mu_{R,0} = M_H$ or $M_H/2$ for Higgs, $\mu_{F,0} = \mu_{R,0} = M_{Z'}$ for Z' .
- do not include the PDF uncertainty
 - Do the included N3LO contributions add up or cancel in the hadronic cross sections?

$pp \rightarrow \text{toy } H^0 X$, central scales $\mu_{F0} = \mu_{R0} = M_H$

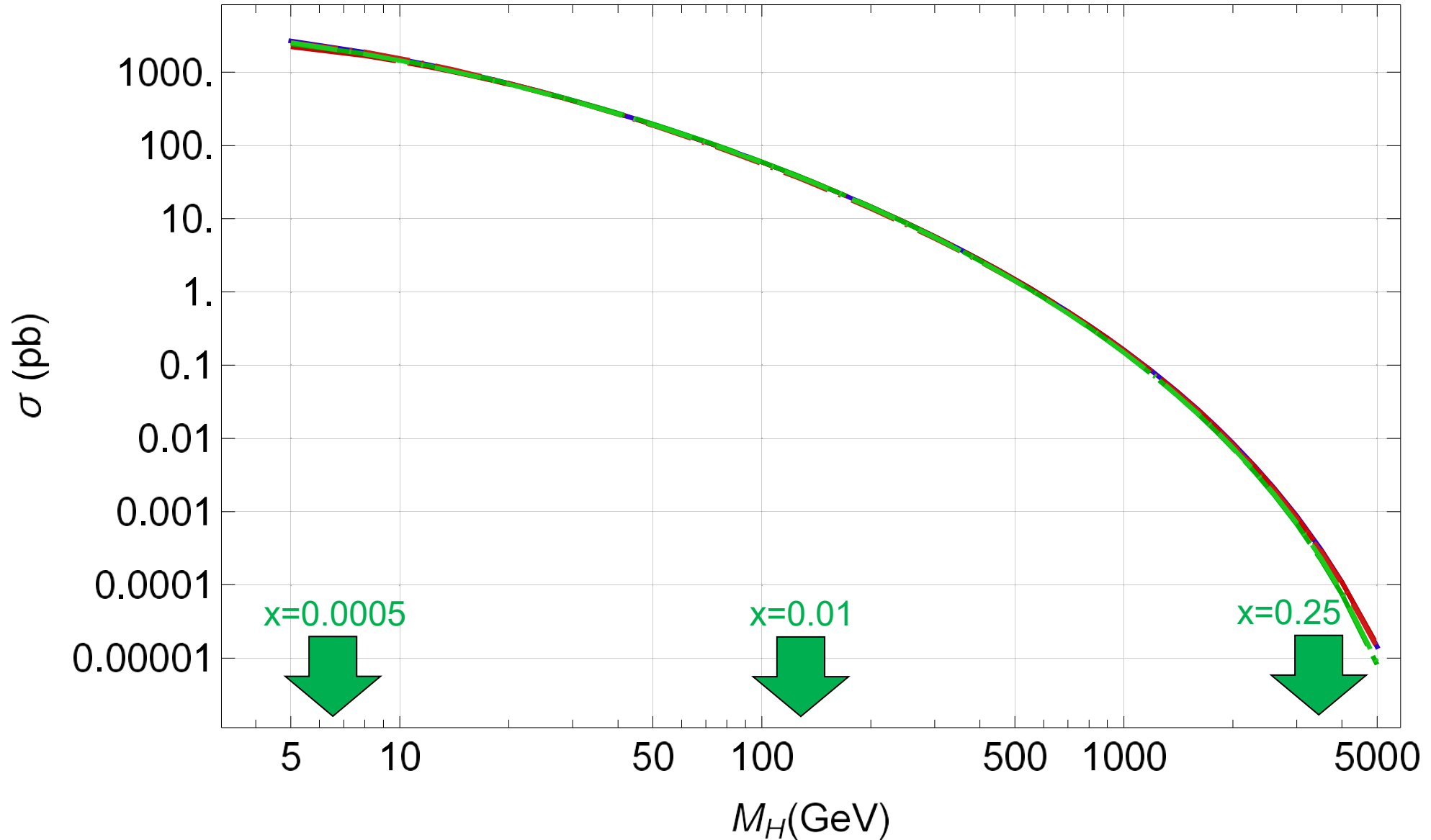
See $\mu_{F0} = \mu_{R0} = M_H/2$ in the supplemental .pdf file
(default choice in n3loxs, asymmetric 7pt errors)

All figures are preliminary!

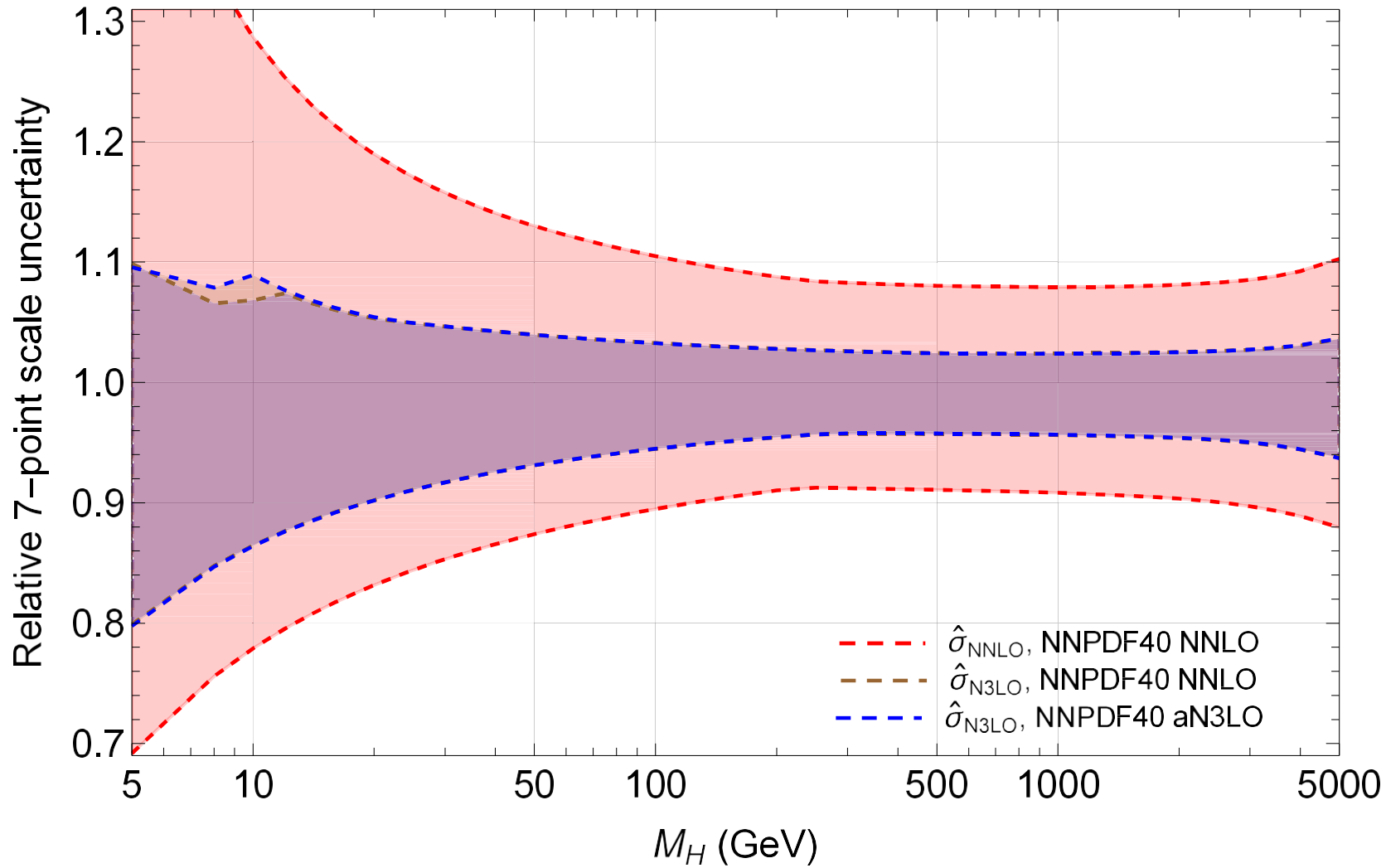
$gg \rightarrow \text{toy } H^0$, LHC 13 TeV, $m_t^{\overline{\text{MS}}} = 10 \text{ TeV}$, $\mu_0 = M_H$



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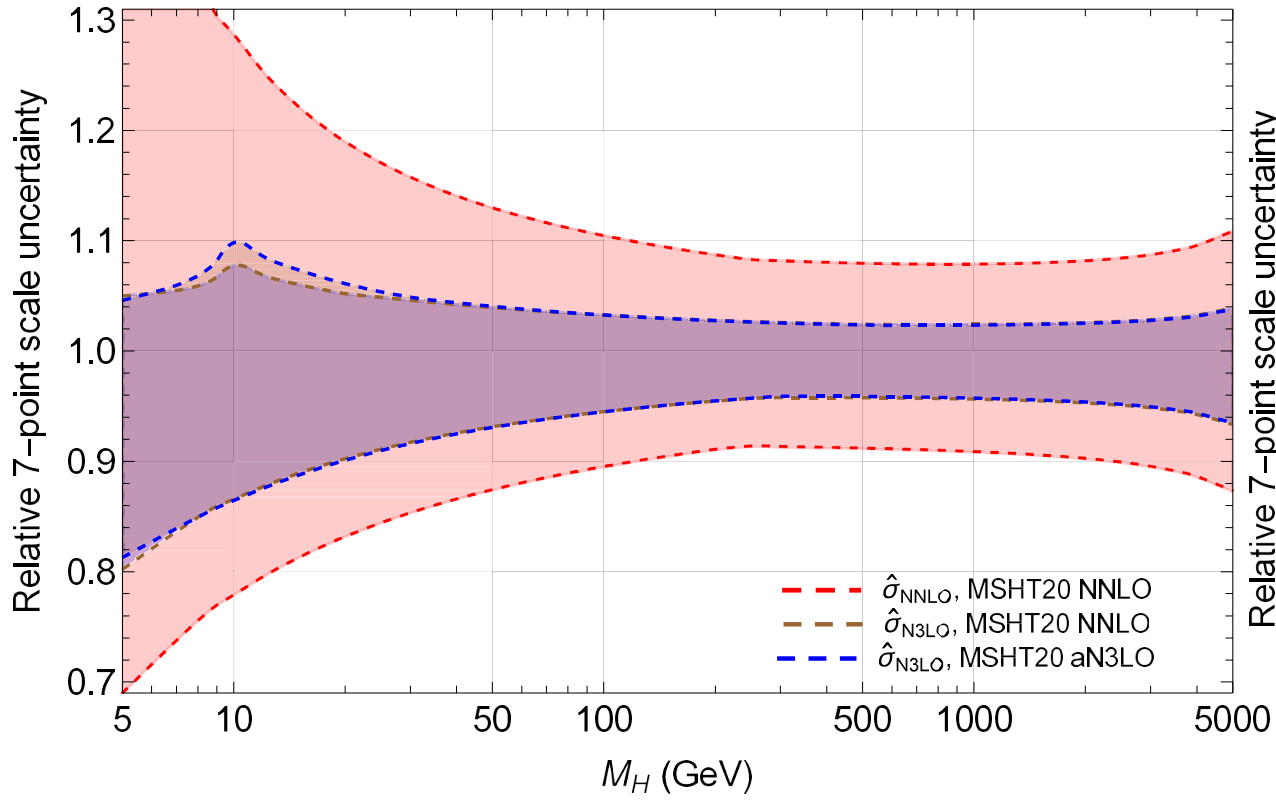
$gg \rightarrow \text{toy } H^0$, LHC 13 TeV, $m_t^{\overline{\text{MS}}} = 10 \text{ TeV}$, $\mu_0 = M_H$



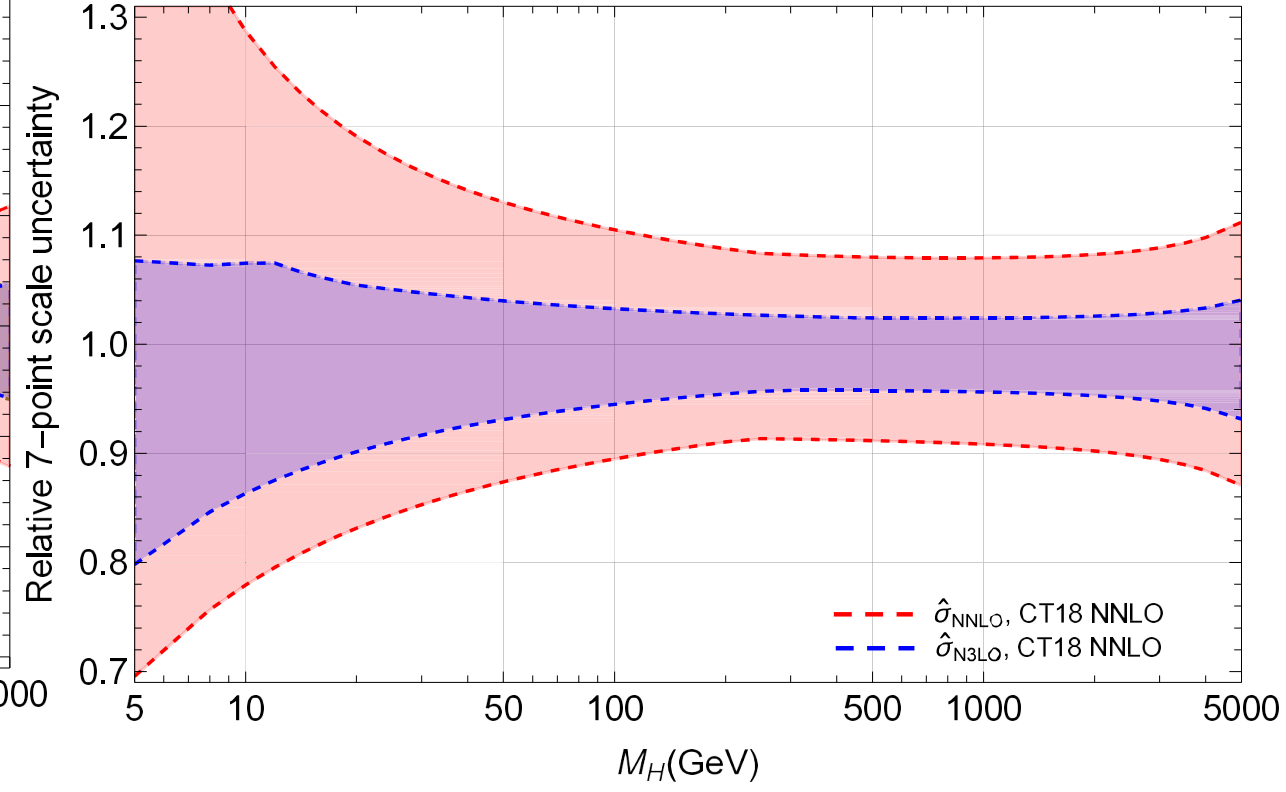
N3LO scale uncertainty is about the same with either NNLO or aN3LO PDFs

At $M_H \approx 10 \text{ GeV}$, more variability due to the $b\bar{b}$ mass threshold

$gg \rightarrow \text{toy } H^0$, LHC 13 TeV, $m_t^{\overline{\text{MS}}} = 10 \text{ TeV}$, $\mu_0 = M_H$



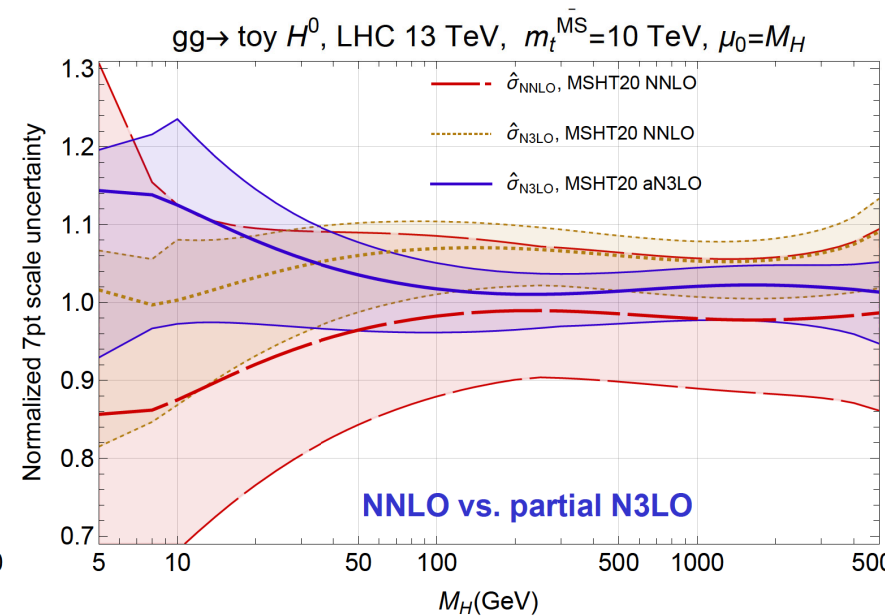
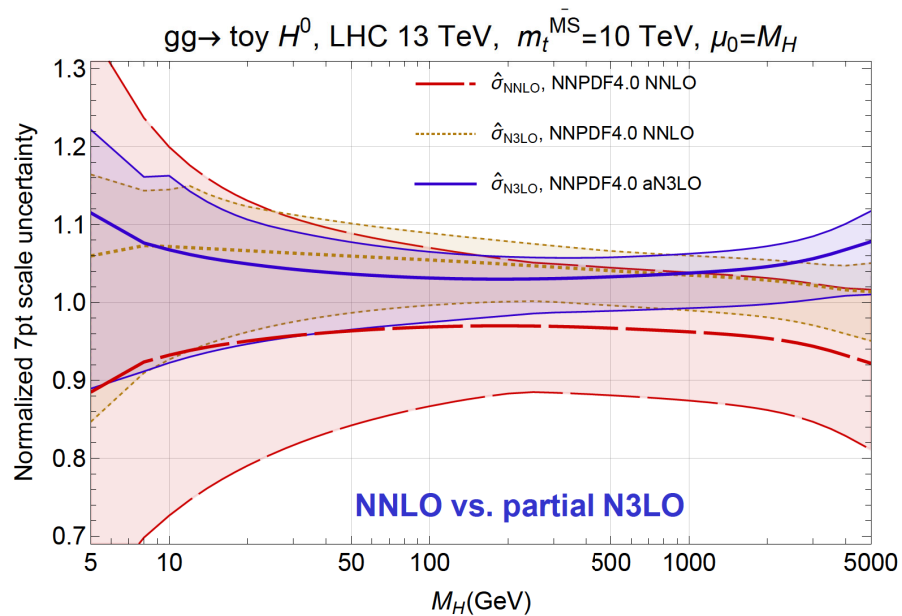
$gg \rightarrow \text{toy } H^0$, LHC 13 TeV, $m_t^{\overline{\text{MS}}} = 10 \text{ TeV}$, $\mu_0 = M_H$



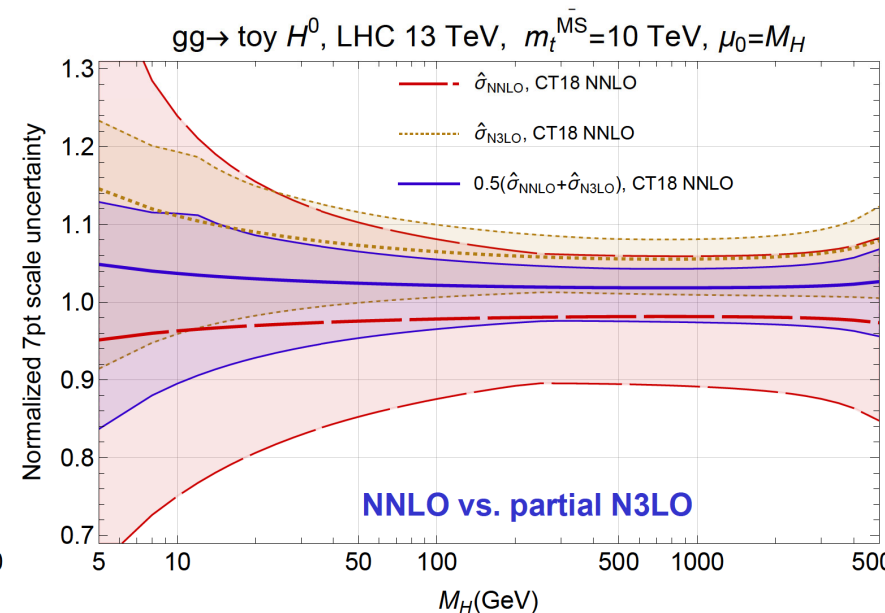
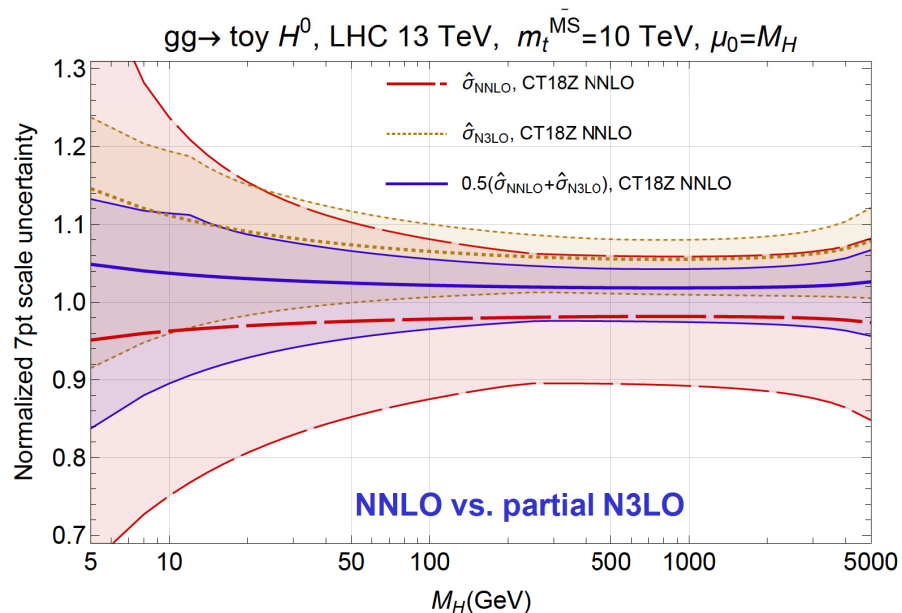
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NNLO+ vs NNLO

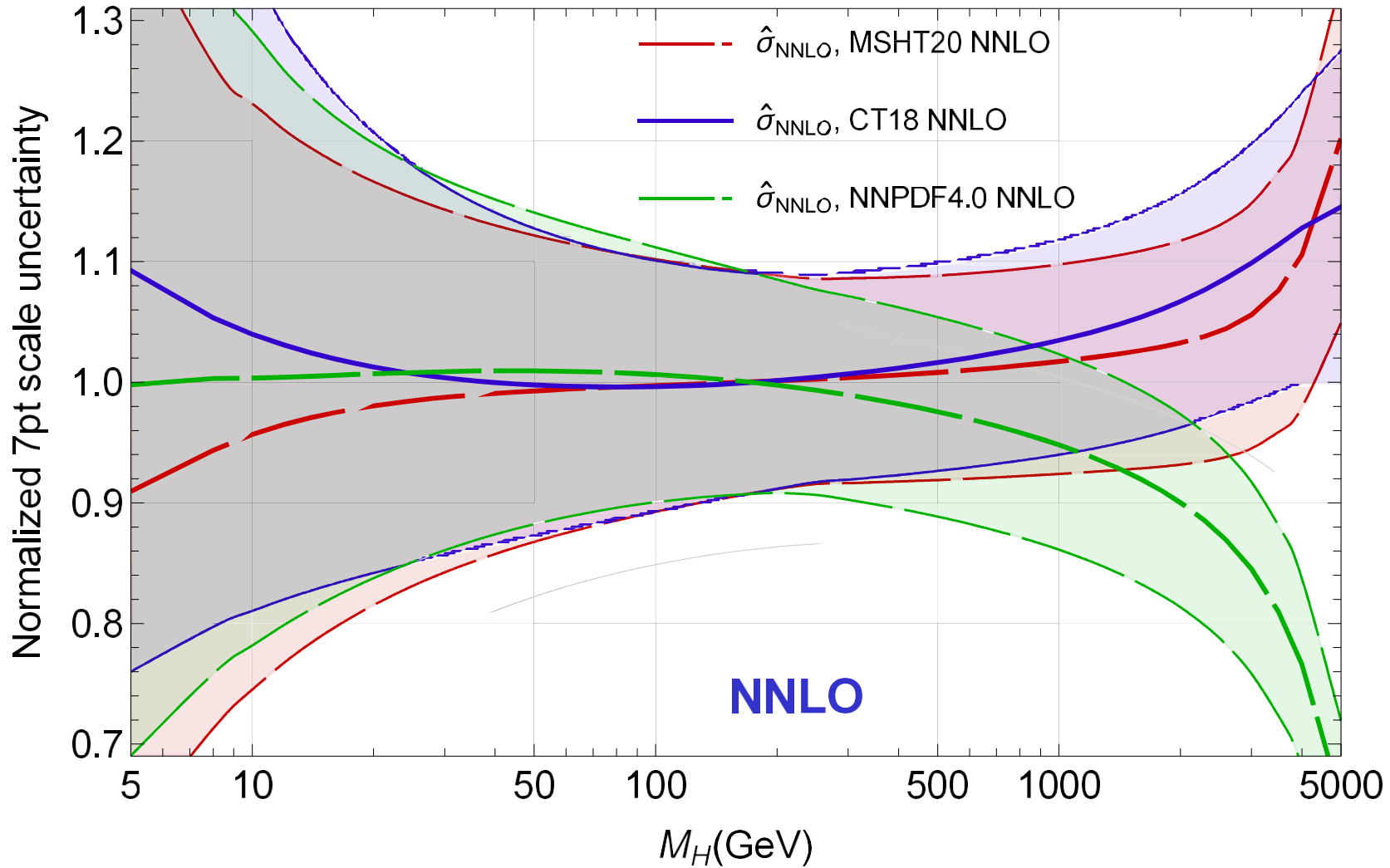
Normalized to the average of NNLO and NNLO+ cross sections



CTEQ-TEA “NNLO+” correction is similar to the two aN3LO ones

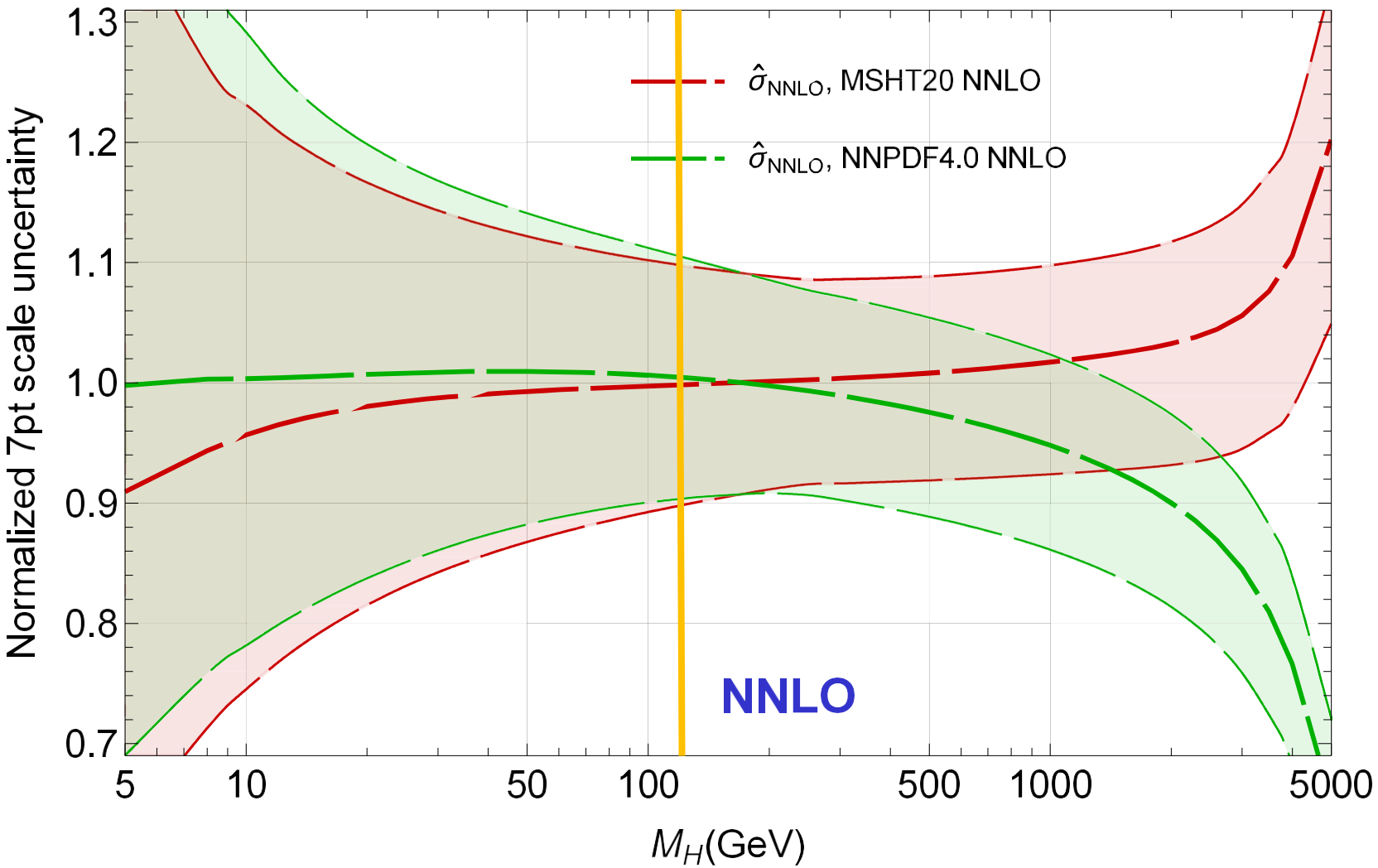


$gg \rightarrow \text{toy } H^0$, LHC 13 TeV, $m_t^{\text{MS}} = 10 \text{ TeV}$, $\mu_0 = M_H$

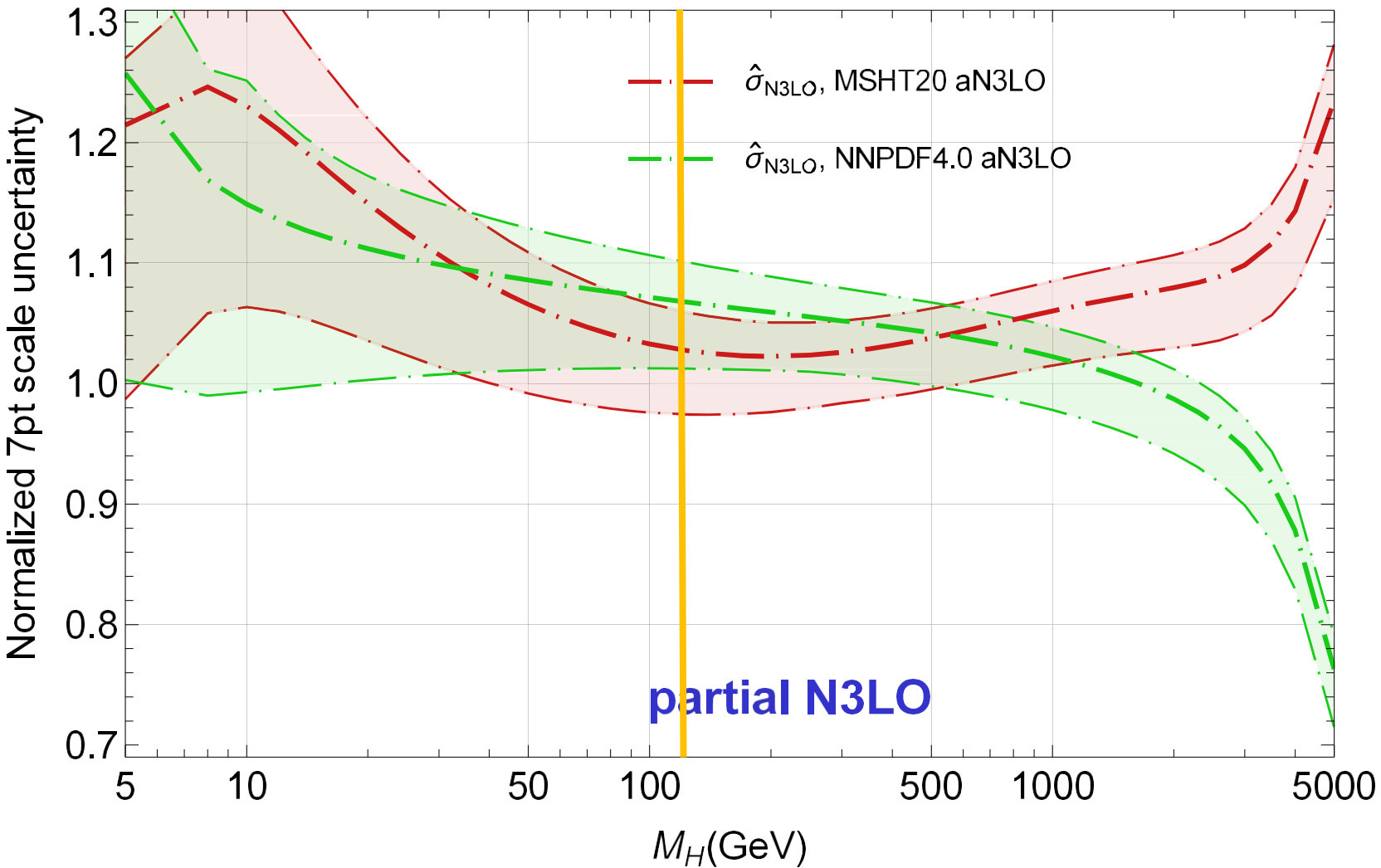


Scale uncertainty bands normalized by the average of central CT18, MSHT20, NNPDF4.0 cross sections

$gg \rightarrow \text{toy } H^0$, LHC 13 TeV, $m_t^{\overline{\text{MS}}} = 10 \text{ TeV}$, $\mu_0 = M_H$



gg→ toy H^0 , LHC 13 TeV, $m_t^{\overline{\text{MS}}}=10$ TeV, $\mu_0=M_H$

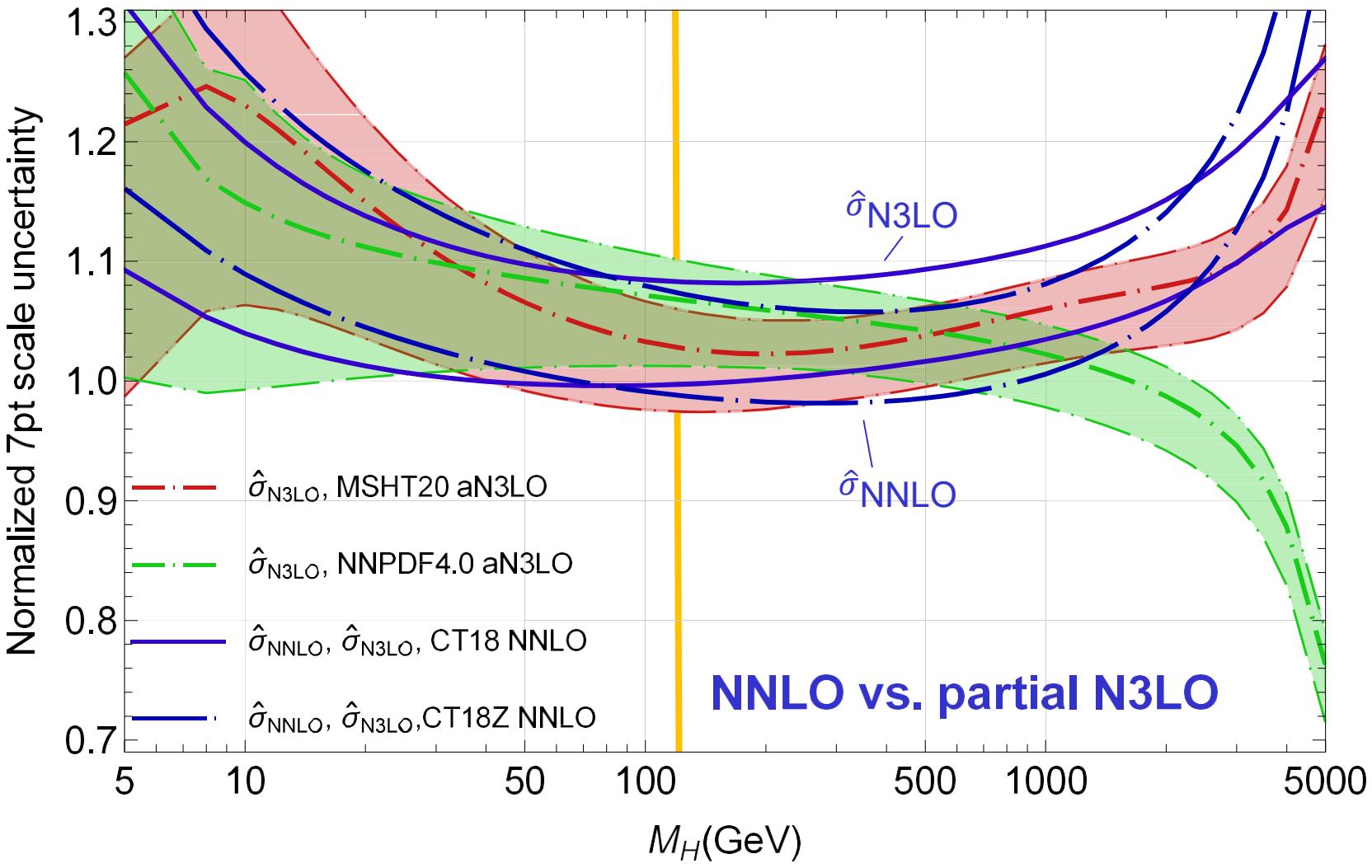


Enlarged $g(x)$ at $x < 10^{-3}$

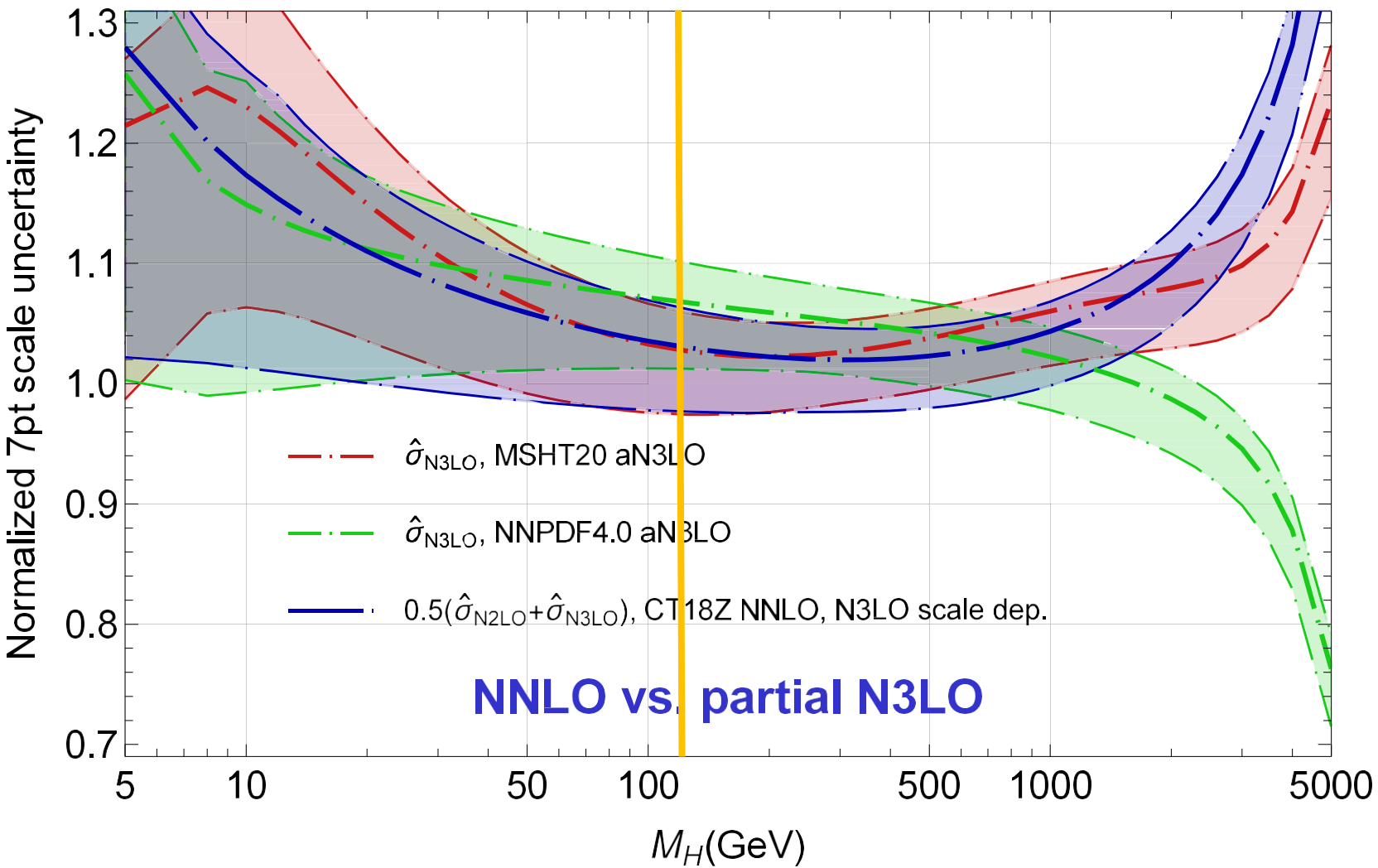
Weaker agreement at $M_H = 125$ GeV than at NNLO

Persistent differences at $x > 0.1$ reflect tensions in fitted data

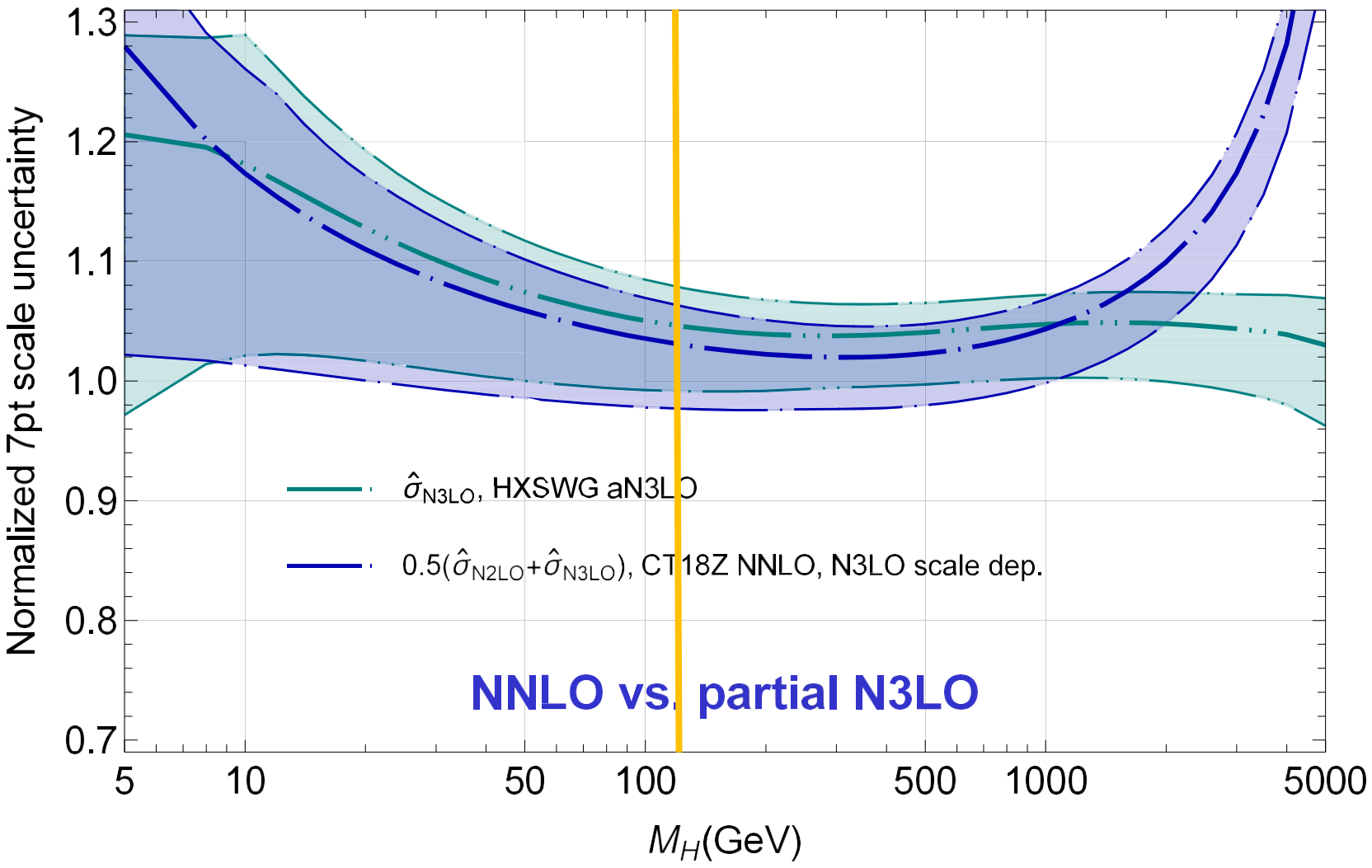
gg→ toy H^0 , LHC 13 TeV, $m_t^{\overline{\text{MS}}}=10$ TeV, $\mu_0=M_H$



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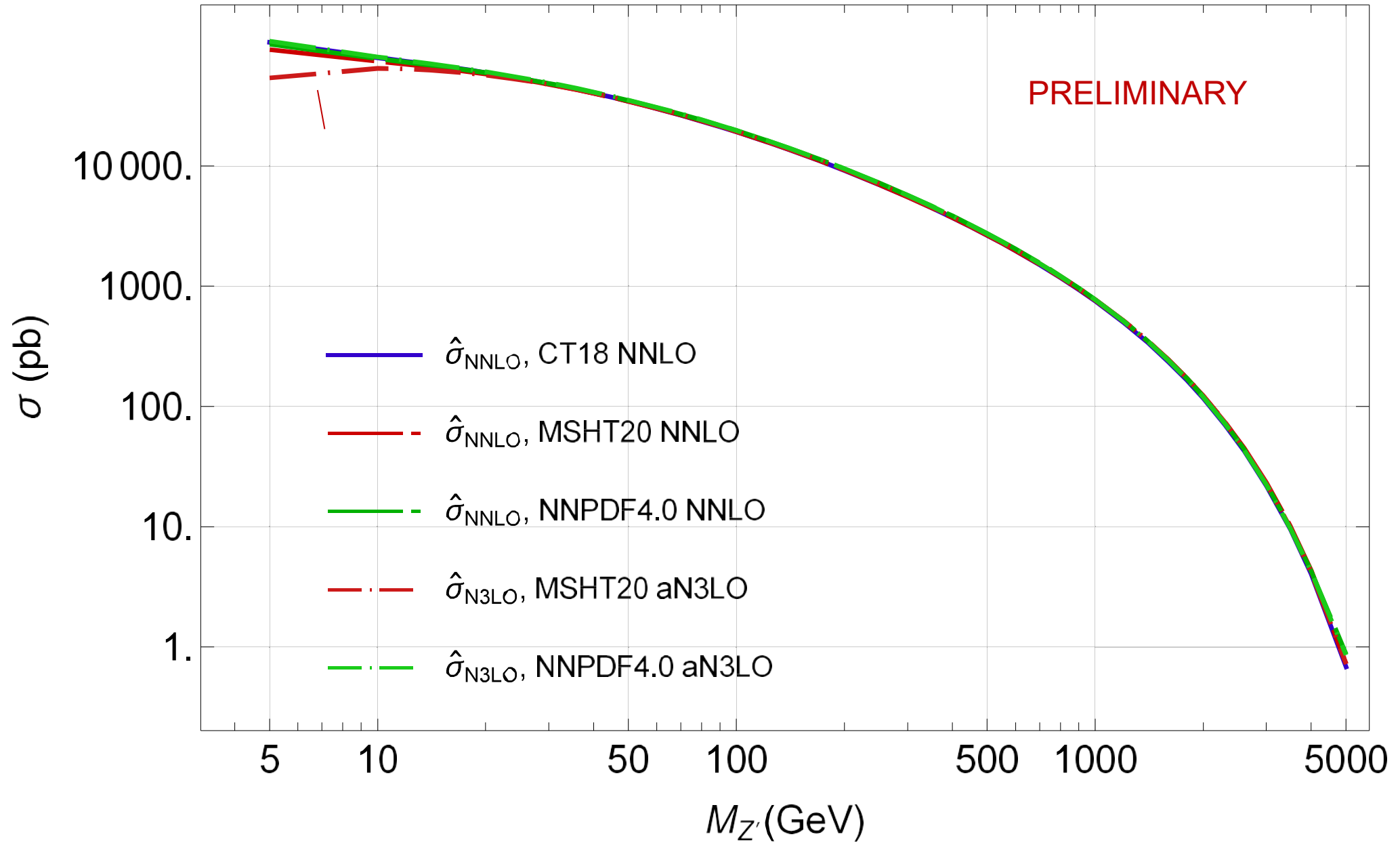
The aN3LO corrections reproduce well the shape of the CT18Z gluon PDF obtained at NNLO with a saturation-inspired factorization scale in DIS, also consistently with BFKL-resumed NNPDF and xFitter NNLO PDFs

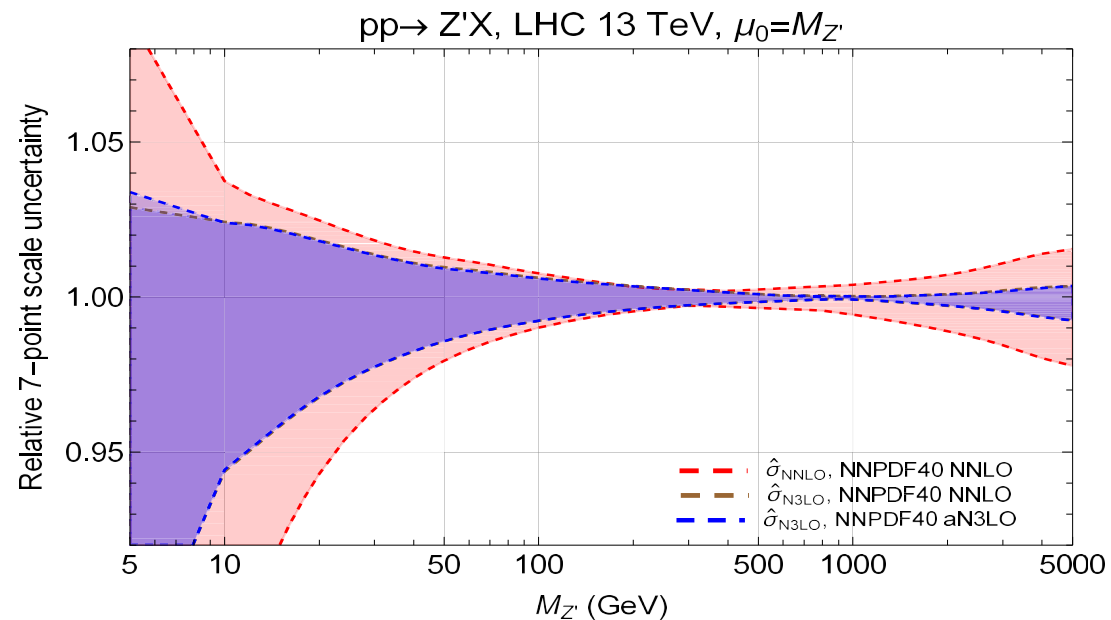
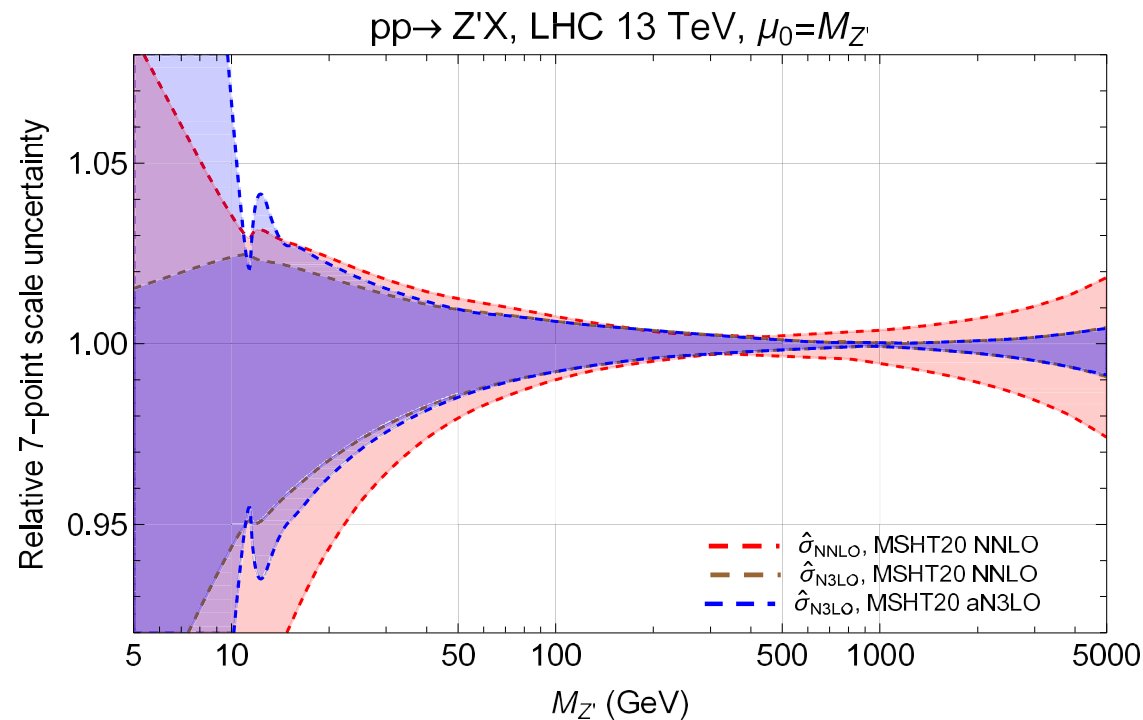
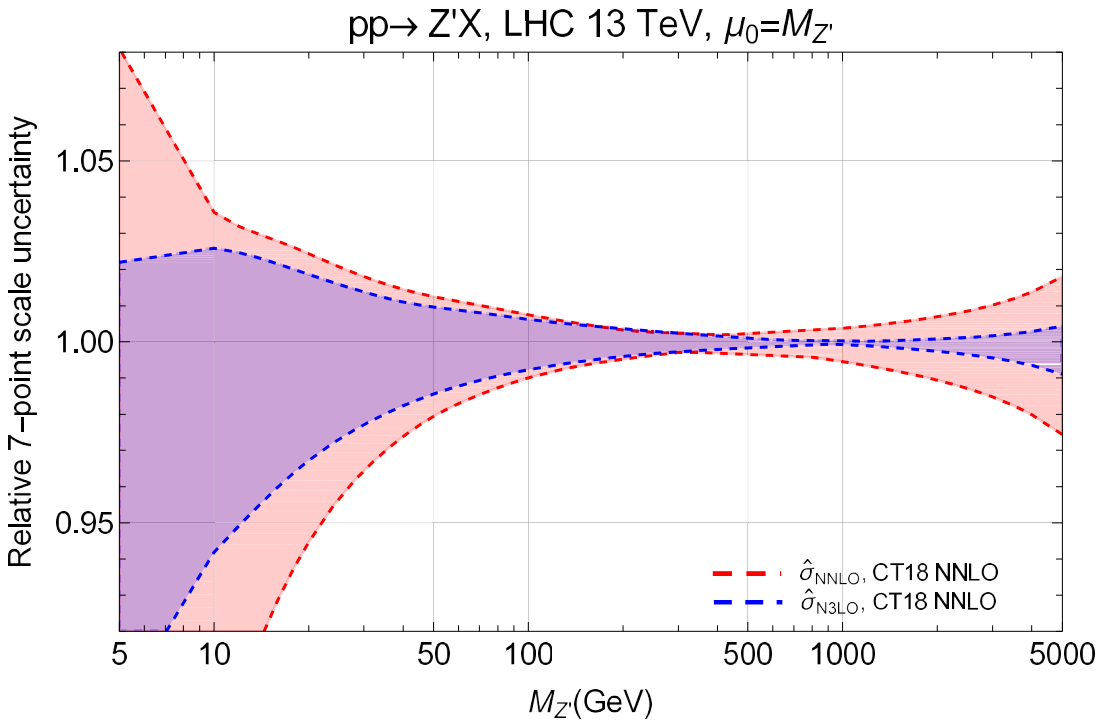
The CT18 NNLO+ prescription agrees with the combined HXSWG prescription everywhere except at $M_H > 1.5 \text{ TeV}$ ($x > 0.1$), where tensions between the earlier and newer data sets introduce some differences; added LHC jet/ $t\bar{t}$ data reduce this difference in the CT25 fit ([2408.04020](#))

$pp \rightarrow Z'X$, central scales $\mu_{F0} = \mu_{R0} = M_{Z'}$

[More figures in the supplemental .pdf file]

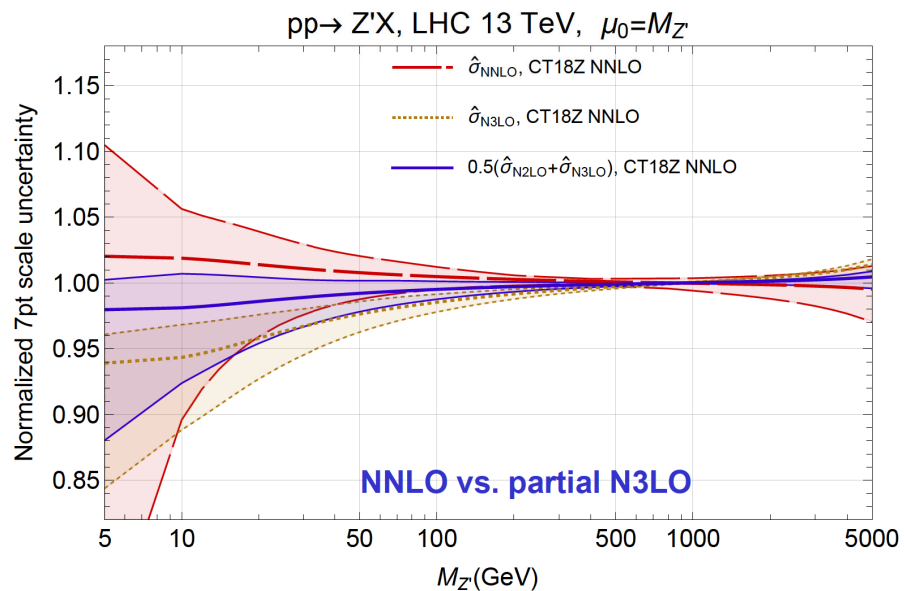
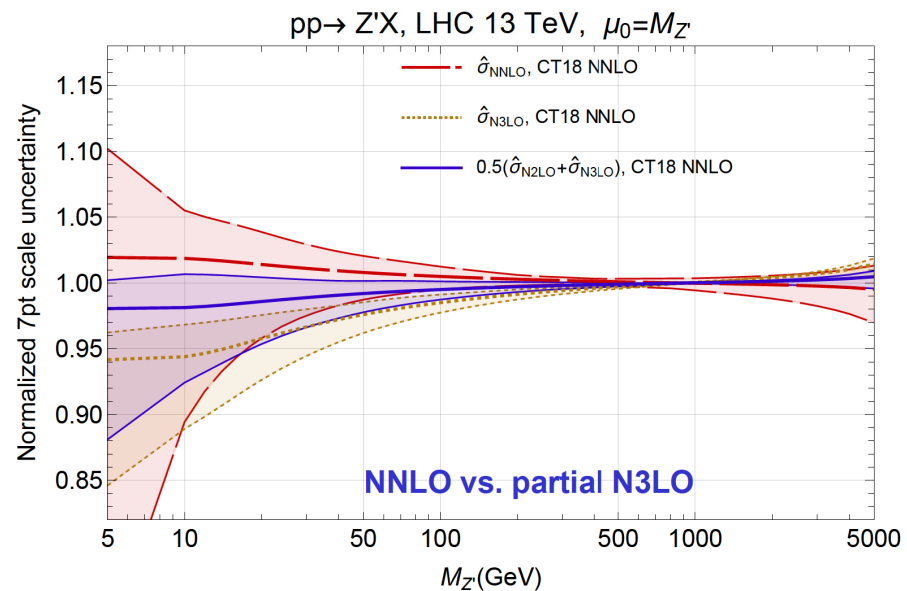
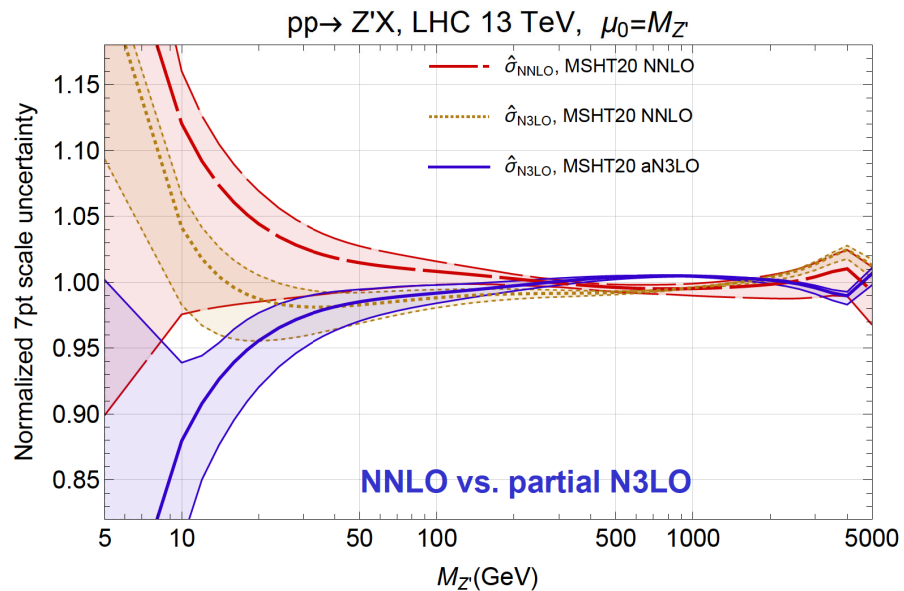
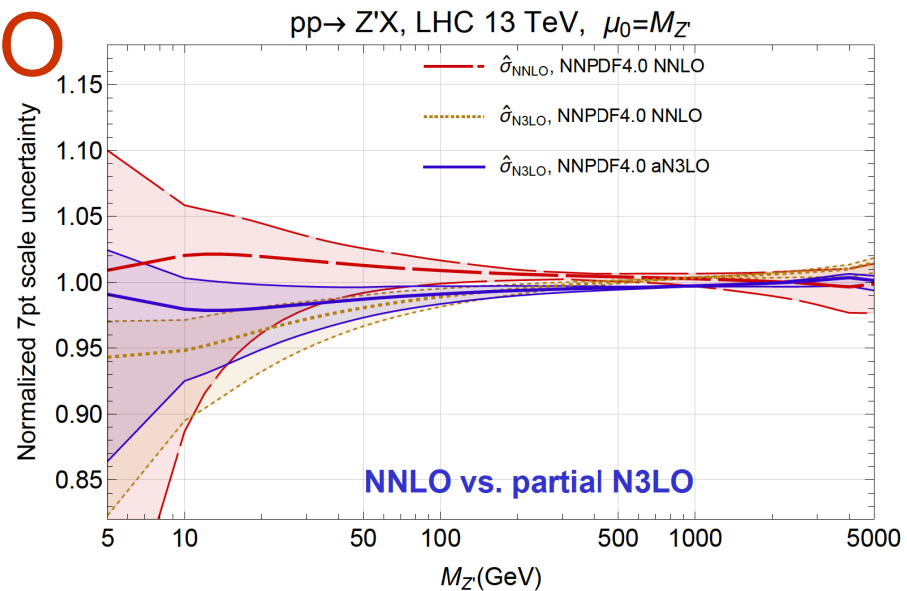
pp \rightarrow Z' X, LHC 13 TeV, $\mu_0=M_{Z'}$



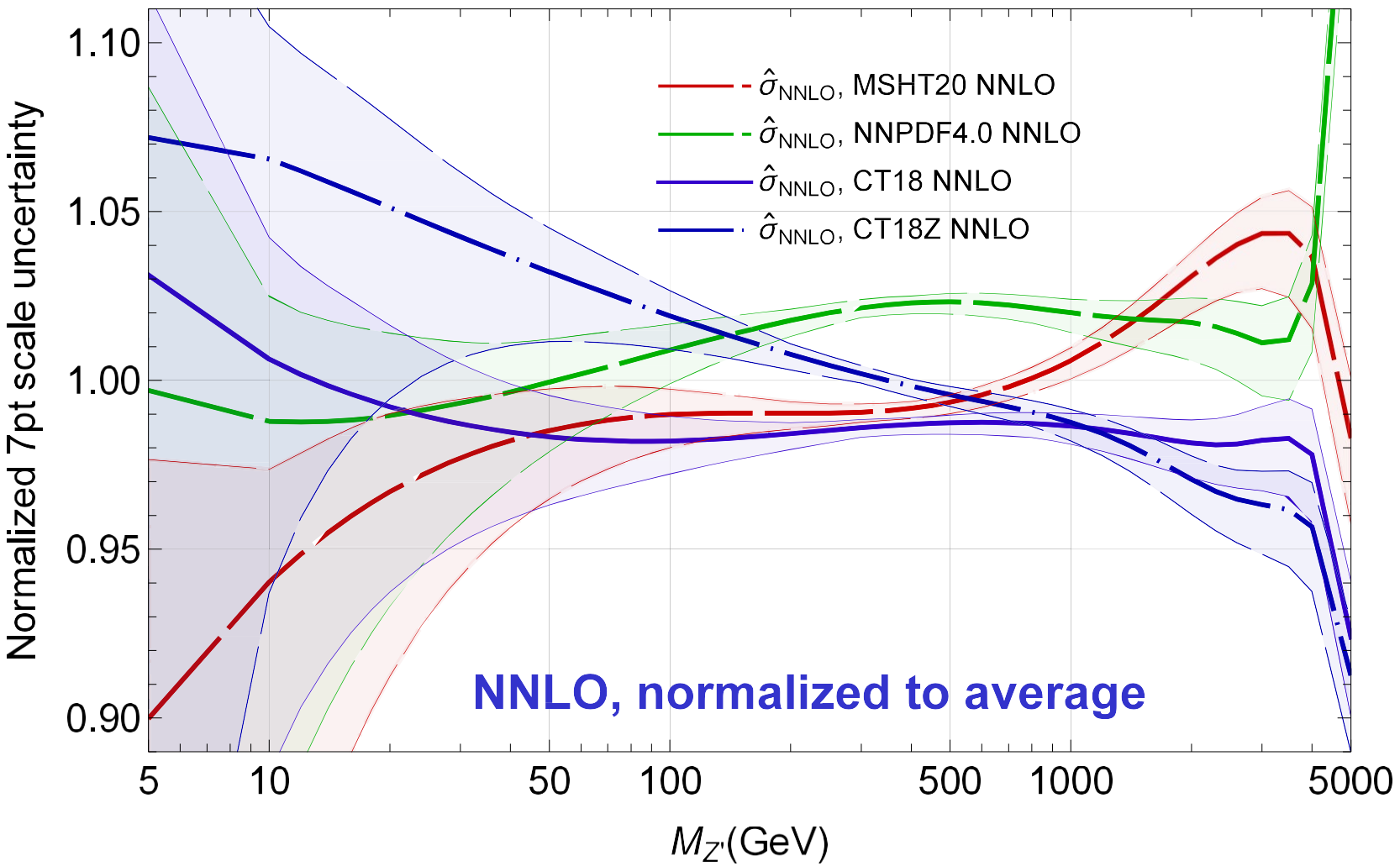


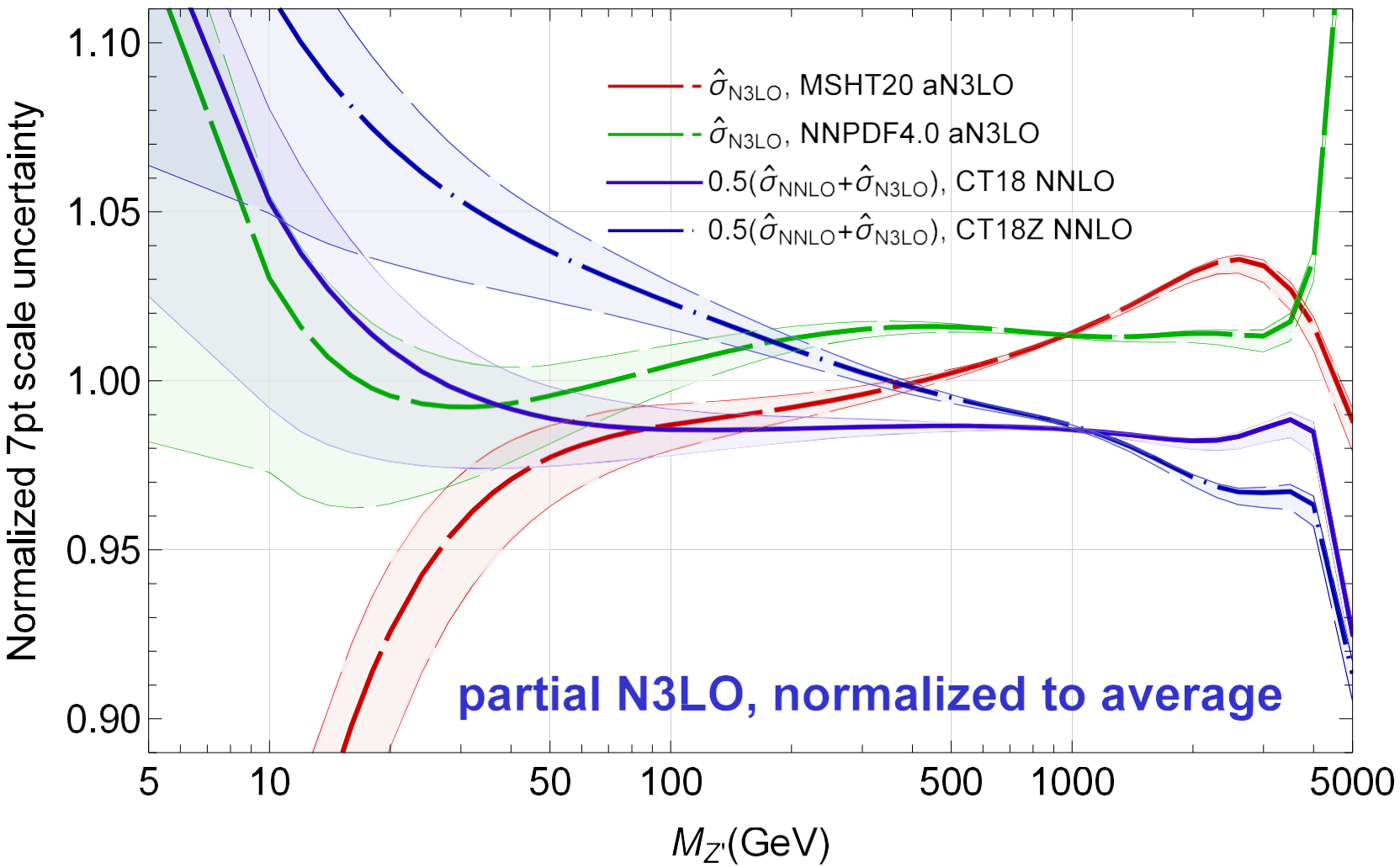
Notice the scale on the y-axis: uncertainties are smaller than in the $gg \rightarrow HX$ case
 N3LO scale uncertainty is about the same with either NNLO or aN3LO PDFs
 b -quark threshold effects at 10 GeV

NNLO+ vs NNLO



CTEQ-TEA “NNLO+”
correction is similar to
the two aN3LO ones



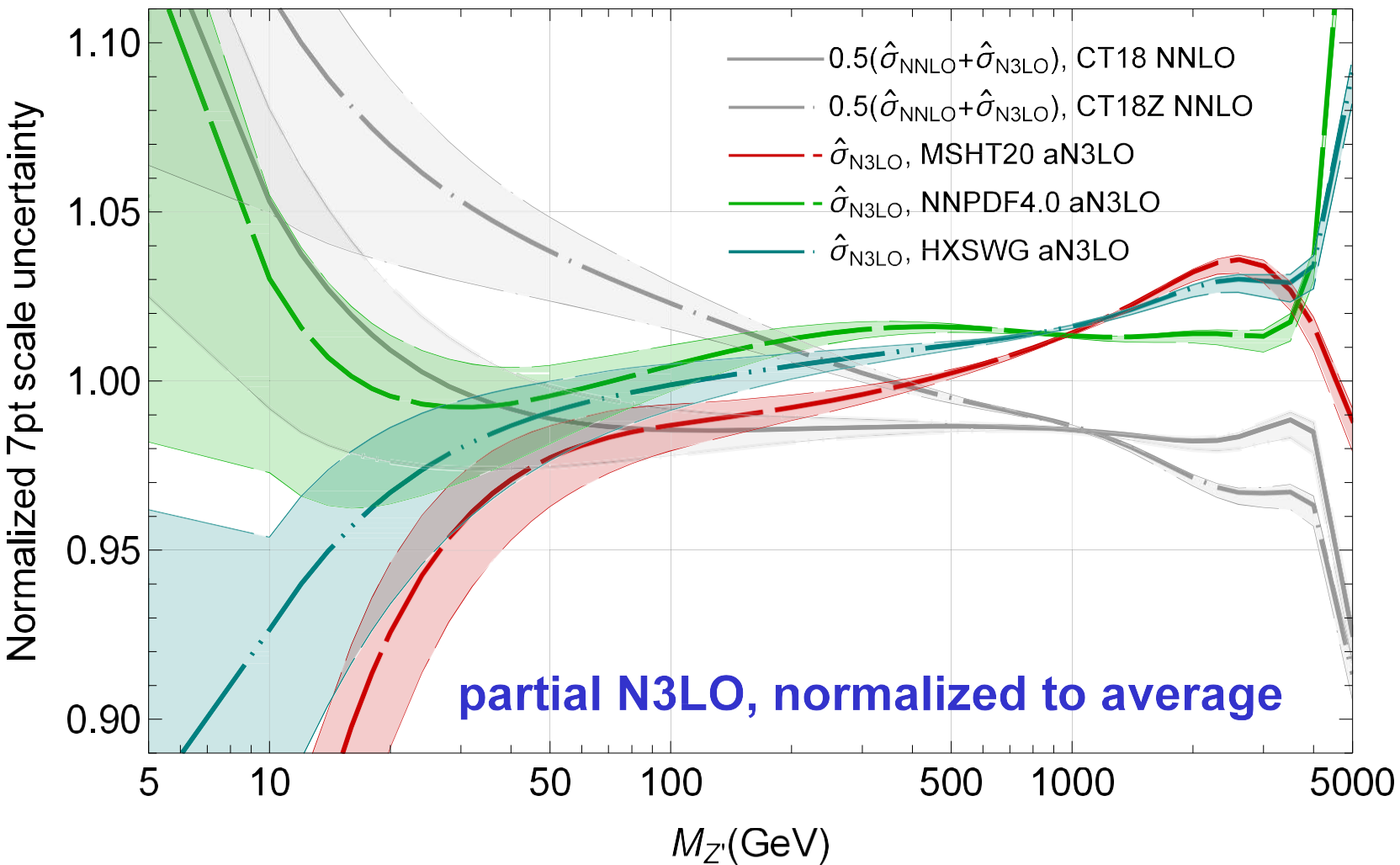


Increased MSHT-NNPDF
aN3LO mismatch at
 $M_{Z'} < 50 \text{ GeV}$

Good agreement
among the groups at
50-500 GeV

Persistent differences at
 $M_{Z'} > 1 \text{ TeV}$, possibly to
be reduced with new data

$pp \rightarrow Z'X$, LHC 13 TeV, $\mu_0 = M_{Z'}$



The HXSWG aN3LO combination...

- performs best for 50-500 GeV;
- in agreement with CT18Z NNLO+ at these masses;
- at other $M_{Z'}$, does not capture the full range of NNLO and NNLO+ predictions

NNLO or NNLO+?

Sustained progress by all groups in including N3LO contributions in the PDF fits.

N3LO contributions are still incomplete. Cross section comparisons are not conclusive about superiority of any single NNLO+ technique. The described CT18 NNLO+ and HXSWG aN3LO prescriptions perform similarly for $gg \rightarrow H^0$ total cross sections, while these prescriptions capture a part of the variability in the $q\bar{q} \rightarrow Z'$ cross sections.

None of the NNLO+ techniques is reliable before thorough benchmarking is performed. A suitable time for such benchmarking is after the imminent implementation of N3LO HQ DIS cross sections in all global fits around 2025.

Many other PDF uncertainties are larger than the N3LO-NNLO differences. Among these, the uncertainties due to the choice of the PDF priors and modeling of systematics affect all global fits and do not automatically decrease at NNLO+. Accounting for them is central for replicability. See, e.g., Courtoy et al., [2205.10444](#).

Does the aN3LO combination manuscript acknowledge these uncertainties?

Human aspects

- Interactions within the PDF4LHC group
- Impact on the broad community