Beam Diagnostics & Instrumentation

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• Recap: Transverse dynamics

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- Example: Transverse beam profile

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- Extra: Data Acquisition and Sampling

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What properties might we want to know?





Diagnostic Tools

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- Electric/ magnetic field sensors
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We process their signals with dedicated electronics to measure the property we're interested in. We often filter out the information we want at this stage.



Transverse Profile

Key point: Diagnostics in the context of phase space.





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Distribution of particles with amplitude from:

- Current intercepting scraper
- Number of particles lost/ remaining





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This method is considered reliable, but what issues can you see?







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Questions?

Key point: A beam's EM fields provide a wealth of information.

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Understanding the distribution of the image charges or fields, allows us to measure beam position and more, without intercepting it.





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Figure 1.2. Electromagnetic field carried by an ultrarelativistic point charge: (a), (b) in free space; (c) in a perfectly conducting smooth pipe.



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Beam position monitors are useful for studying longitudinal, and transverse dynamics.



Consider a pencil beam ($v \approx c$, current $I_b(t)$) at position (r, ϕ_0) , inside a perfectly conducting, circular chamber of radius b.





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$$i_w(r,\phi,t) = \frac{-I_b(t)}{2\pi b} \left(1 + 2\sum_{n=1}^{\infty} \left(\frac{r}{b}\right)^n \cos(n(\phi - \phi_0)) \right)$$





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$$I_{R} = \int_{\phi=-\phi_{a}}^{\phi_{a}} i_{w}(r,\phi,t) b \,\mathrm{d}\phi = \frac{-I_{b}(t)}{2\pi} \left(\phi_{a} + 4\sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{r}{b}\right)^{n} \sin\left(\frac{n\phi_{a}}{2}\right) \cos(n\phi_{0})\right)$$

$$I_{L} = \frac{-I_{b}(t)}{2\pi} \left(\phi_{a} + 4\sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{r}{b}\right)^{n} (-1)^{n} \sin\left(\frac{n\phi_{a}}{2}\right) \cos(n\phi_{0})\right)$$
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Technology
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Taking the difference between these currents,

$$I_R - I_L = \frac{-2I_b(t)}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{r}{b}\right)^n \cos(n\phi_0) \sin\left(\frac{n\phi_a}{2}\right) (1 - (-1)^n) \\ = \frac{-4I_b(t)}{\pi} \frac{r}{b} \cos(\phi_0) \sin\left(\frac{\phi_a}{2}\right) + A_3 \left(\frac{r}{b}\right)^3 + A_5 \left(\frac{r}{b}\right)^5 + \dots$$



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Provided $r \ll b$, the difference is approximately proportional to $I_b(t)r\cos(\phi_0)$. With only a vertical offset, $\phi_0 = \frac{\pi}{2}$, so $I_R - I_L = 0$. This is a horizontal BPM. For vertical position, use two more electrodes centred on $\phi = \pm \frac{\pi}{2}$.



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For now assume $\phi_0 = 0$ so we are only discussing horizontal position.

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In theory, we could use this difference signal

$$I_R - I_L \approx \frac{-4I_b(t)}{\pi} \frac{r}{b} \sin\left(\frac{\phi_a}{2}\right),$$

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It would be preferable to remove this dependence. Consider the sum signal

$$I_R + I_L = \frac{-I_b(t)}{2\pi} \left(2\phi_a + 4\sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{r}{b}\right)^n \sin\left(\frac{n\phi_a}{2}\right) (1 + (-1)^n) \right) \approx -I_b(t) \frac{\phi_a}{\pi}$$



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We often use the ratio, because this is independent of $I_b(t)$

$$\frac{I_R - I_L}{I_R + I_L} \approx \frac{4}{\phi_a} \frac{r}{b} \sin\left(\frac{\phi_a}{2}\right)$$



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Device suitability depends on bunch structure. Split-plate BPMs popular for large bunches, and buttons are common at light sources.



Questions?
Key point 1: A beam can be made to reveal its properties. Key point 2: Fourier analysis is very useful.

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Oscillation frequency gives non-integer tune.





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To understand how to obtain the tune, let's write down an ideal BPM difference signal.



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$$= A \sum_{n=-\infty}^{\infty} \delta(t - nT_p) * y(t)$$



where * is a convolution



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$$V(t) = y(t) * A\cos(\omega_{\beta}t) \sum_{n=-\infty}^{\infty} \delta(t - nT_p),$$

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I define the Fourier transform and its inverse as

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Find the Fourier transform (FT) of V(t) with the convolution theorem of FT's

$$\mathcal{F}(V(t)) = \tilde{V}(\omega) = \mathcal{F}(y(t) * g(t)) = \mathcal{F}(y(t)) \cdot \mathcal{F}(g(t))$$
$$= \mathcal{F}[y(t)] \mathcal{F}\left[A\cos(\omega_{\beta}t)\sum_{n=-\infty}^{\infty}\delta(t-nT_p)\right]$$

and its inverse $\mathcal{F}(h(t)k(t)) = \frac{1}{2\pi}\mathcal{F}(h(t)) * \mathcal{F}(k(t))$, plus the important result

$$F\left(\sum_{n=-\infty}^{\infty}\delta(t-nT_p)\right) = \frac{2\pi}{T_p}\sum_{n=-\infty}^{\infty}\delta\left(\omega-n\frac{2\pi}{T_p}\right).$$



$$\tilde{V}(\omega) = \frac{A}{2} \mathcal{F}[y(t)] \,\omega_0 \sum_{n=-\infty}^{\infty} \left[\delta(\omega - n\omega_0 - \omega_\beta) + \delta(\omega - n\omega_0 + \omega_\beta)\right]$$



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We can't distinguish the integer part of the tune from a single monitor, because $\omega_{\beta} = \omega_0(m+q)$ where m is an integer. The delta-functions become $\delta(\omega - (n \pm m)\omega_0 \pm q\omega_0)$. Since this appears in an infinite sum, we can't distinguish between n and $n \pm m$.



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The quantity of interest, q can then be found by measuring the difference in frequency between the revolution harmonics and the sidebands and dividing by $f_0 = \frac{1}{T_r}$



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We can also get an intuitive picture of why it is that we can only measure up to the half-integer. Plot $\tilde{V}(\omega)$ for increasing values of fractional tune:



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We can also get an intuitive picture of why it is that we can only measure up to the half-integer. Plot $\tilde{V}(\omega)$ for increasing values of fractional tune: q = 0.2.





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We can also get an intuitive picture of why it is that we can only measure up to the half-integer. Plot $\tilde{V}(\omega)$ for increasing values of fractional tune: q = 0.3.





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We can also get an intuitive picture of why it is that we can only measure up to the half-integer. Plot $\tilde{V}(\omega)$ for increasing values of fractional tune: q = 0.4.





$$\tilde{V}(\omega) = \frac{A}{2} \mathcal{F}[y(t)] \,\omega_0 \sum_{n=-\infty}^{\infty} \left[\delta(\omega - n\omega_0 - \omega_\beta) + \delta(\omega - n\omega_0 + \omega_\beta)\right]$$

We can also get an intuitive picture of why it is that we can only measure up to the half-integer. Plot $\tilde{V}(\omega)$ for increasing values of fractional tune: q = 0.5.





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We can also get an intuitive picture of why it is that we can only measure up to the half-integer. Plot $\tilde{V}(\omega)$ for increasing values of fractional tune: q = 0.6.





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Sidebands have swapped places! We can't know if q = 0.4 or 1 - 0.4 = 0.6.



Tune Measurement Example

This is an example BPM difference signal during a tune measurement on the ISIS synchrotron. A kick was applied at around $200 \,\mu$ s.




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Fractional tune is either 0.19 or 0.81.

Questions?

Other Diagnostic Systems

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But there are two systems I haven't mentioned which are very important, and I would like to mention them, even without much detail.

Like some of the more detailed examples, they can be used in combination with other equipment and techniques to measure a range of beam and machine parameters.



Measure the number of accelerated particles. [5, 6]



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Wall current monitors generate a voltage from the image current flowing in the vacuum vessel.





Beam Loss Monitors

Ideally we would accelerate all the injected particles, but some beam loss is inevitable, and we must plan for equipment failures [5, 7]



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In a scintillator BLM, light is generated when an incoming particle passes through its sensitive volume. The light signal is converted into an electrical signal.





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- Predicting an idealised version of a signal gives us a method for understanding a problem without knowing all the background physics.
- Sometimes the quantity of interest can be obtained directly from the spectra.
- Please review some of the excellent references for more detail.



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Extra: Data Acquisition and Sampling

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Whatever diagnostics you use, you will be digitising analogue signals with an ADC [8]. This procedure, whilst very common, is a critical step in the measurement process. There are a few things that it is good to be aware of.



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After a signal has been sampled, there is no general correction method. Must be considered in advance!





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If we didn't know what signal to expect, then we might choose the maximum sample rate and bandwidth. If the signal was known to have a low frequency, then we could reduce input bandwidth and sample rate.





