## Beam Diagnostics & Instrumentation

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• Recap: Transverse dynamics

- Recap: Transverse dynamics
- Example: Transverse beam profile

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- Example: Average beam position

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- Example: Transverse beam profile
- **•** Example: Average beam position
- Example: Betatron tune
- Other Diagnostic Systems
- Conclusion
- Extra: Data Acquisition and Sampling

### Decoupled

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Evolution of a position and momentum pair can be expressed by linear equations. E.g. for a drift

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x_1 = x_0 + Lp_{x0} + O(2)
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### What properties might we want to know?





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- Interceptive devices (wires, scrapers, screens, Faraday cups etc)
- Secondary particle detectors (ion chambers, scintillators etc)
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We process their signals with dedicated electronics to measure the property we're interested in. We often filter out the information we want at this stage.



# **Transverse Profile**

Key point: Diagnostics in the context of phase space.





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Distribution of particles with amplitude from:

- Current intercepting scraper
- Number of particles lost/remaining



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Particles remaining after many turns

 $p_x$ 

 $\ddot{\phantom{0}}$ 

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This method is considered reliable, but what issues can you see?



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# **Questions?**

Key point: A beam's EM fields provide a wealth of information.

Profile monitors can measure position, but it's more common to use the beam's EM fields [1].





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Understanding the distribution of the image charges or fields, allows us to measure beam position and more, without intercepting it.



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Figure 1.2. Electromagnetic field carried by an ultrarelativistic point charge: (a), (b) in free space; (c) in a perfectly conducting smooth pipe.



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Beam position monitors are useful for studying longitudinal, and transverse dynamics.



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i_w(r, \phi, t) = \frac{-I_b(t)}{2\pi b} \left( 1 + 2 \sum_{n=1}^{\infty} \left( \frac{r}{b} \right)^n \cos(n(\phi - \phi_0)) \right)
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The total current flowing <u>in an angular regions  $\pm \frac{\phi_a}{2}$ </u>  $\frac{b_a}{2}$  and  $\pi \pm \frac{a}{2}$  $\frac{a}{2}$  are  $I_R$  and  $I_L$ 





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I_R = \int_{\phi = -\phi_a}^{\phi_a} i_w(r, \phi, t) b \,d\phi = \frac{-I_b(t)}{2\pi} \left( \phi_a + 4 \sum_{n=1}^{\infty} \frac{1}{n} \left( \frac{r}{b} \right)^n \sin\left(\frac{n\phi_a}{2}\right) \cos(n\phi_0) \right)
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$$
I_L = \frac{-I_b(t)}{\sum_{\text{recoine and} 2\pi} \sin\left(\phi_a + 4 \sum_{n=1}^{\infty} \frac{1}{n} \left( \frac{r}{b} \right)^n (-1)^n \sin\left(\frac{n\phi_a}{2}\right) \cos(n\phi_0) \right)
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Taking the difference between these currents,

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I_R - I_L = \frac{-2I_b(t)}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{r}{b}\right)^n \cos(n\phi_0) \sin\left(\frac{n\phi_a}{2}\right) (1 - (-1)^n)
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With only a vertical offset,  $\phi_0 = \frac{\pi}{2}$  $\frac{\pi}{2}$ , so *I*<sub>*R*</sub> − *I*<sub>*L*</sub> = 0. This is a horizontal BPM. For vertical position, use two more electrodes centred on  $\phi = \pm \frac{\pi}{2}$  $\frac{\pi}{2}$ .



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For now assume  $\phi_0 = 0$  so we are only discussing horizontal position.



In theory, we could use this difference signal

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I_R - I_L \approx \frac{-4I_b(t)}{\pi} \frac{r}{b} \sin\left(\frac{\phi_a}{2}\right),\,
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It would be preferable to remove this dependence. Consider the sum signal

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I_R + I_L = \frac{-I_b(t)}{2\pi} \left( 2\phi_a + 4\sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{r}{b}\right)^n \sin\left(\frac{n\phi_a}{2}\right) (1 + (-1)^n) \right) \approx -I_b(t)\frac{\phi_a}{\pi}
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We often use the ratio, because this is independent of  $I_b(t)$ 

$$
\frac{I_R - I_L}{I_R + I_L} \approx \frac{4}{\phi_a} \frac{r}{b} \sin\left(\frac{\phi_a}{2}\right)
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Examples of beam position monitors include [4]: - capacitive pickups



- capacitive pickups
	- button pickups





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	- split-plate pickups





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Device suitability depends on bunch structure. Split-plate BPMs popular for large bunches, and buttons are common at light sources.



# **Questions?**
Key point 1: A beam can be made to reveal its properties. Key point 2: Fourier analysis is very useful.

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A common technique to measure tune is to use a kicker to increase *p<sup>x</sup>* on a single turn, which will cause the beam's average position to oscillate.

Oscillation frequency gives non-integer tune.<br>
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To understand how to obtain the tune, let's write down an ideal BPM difference signal.



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= A \sum_{n = -\infty}^{\infty} \delta(t - nT_p) * y(t)
$$



where *∗* is a convolution



To include the transverse oscillations, we write

$$
V(t) = y(t) * A \cos(\omega_{\beta} t) \sum_{n=-\infty}^{\infty} \delta(t - nT_p),
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where  $\omega_{\beta}=2\pi\frac{Q}{T_x}$  $\frac{Q}{T_p}$  is the angular betatron frequency.



Turns since kick



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I define the Fourier transform and its inverse as

$$
\mathcal{F}(f(t)) = \tilde{f}(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t} dt, \qquad \mathcal{F}^{-1}(\tilde{f}(\omega)) = f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{f}(\omega)e^{i\omega t} d\omega
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Find the Fourier transform  $(FT)$  of  $V(t)$  with the convolution theorem of FT's

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*F* ( ∑*<sup>∞</sup>*

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$$

 $=\frac{2\pi}{\pi}$ *Tp*

∑*<sup>∞</sup>*

*δ*  $\sqrt{2}$ 

*ω* − *n* $\frac{2π}{T_2}$ *Tp*

 $\setminus$ *.*

*n*=*−∞*

and its inverse  $\mathcal{F}(h(t)k(t)) = \frac{1}{2\pi}\mathcal{F}(h(t)) * \mathcal{F}(k(t))$ , plus the important result  $\setminus$ 

 $\delta(t - nT_p)$ 



$$
\tilde{V}(\omega) = \frac{A}{2} \mathcal{F}[y(t)] \omega_0 \sum_{n=-\infty}^{\infty} [\delta(\omega - n\omega_0 - \omega_\beta) + \delta(\omega - n\omega_0 + \omega_\beta)]
$$



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We can't distinguish the integer part of the tune from a single monitor, because  $\omega_{\beta} = \omega_0(m + q)$  where *m* is an integer. The delta-functions become  $\delta(\omega - (n \pm m)\omega_0 \pm q\omega_0)$ . Since this appears in an infinite sum, we can't distinguish between *n* and  $n \pm m$ .



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The quantity of interest, *q* can then be found by measuring the difference in frequency between the revolution harmonics and the sidebands and dividing by  $f_0 = \frac{1}{T_c}$ *Tp*



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We can also get an intuitive picture of why it is that we can only measure up to the half-integer. Plot  $\tilde{V}(\omega)$  for increasing values of fractional tune:



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We can also get an intuitive picture of why it is that we can only measure up to the half-integer. Plot  $\tilde{V}(\omega)$  for increasing values of fractional tune:  $q = 0.1$ .





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$$
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We can also get an intuitive picture of why it is that we can only measure up to the half-integer. Plot  $\tilde{V}(\omega)$  for increasing values of fractional tune:  $q = 0.3$ .





$$
\tilde{V}(\omega) = \frac{A}{2} \mathcal{F}[y(t)] \omega_0 \sum_{n=-\infty}^{\infty} [\delta(\omega - n\omega_0 - \omega_\beta) + \delta(\omega - n\omega_0 + \omega_\beta)]
$$

We can also get an intuitive picture of why it is that we can only measure up to the half-integer. Plot  $\tilde{V}(\omega)$  for increasing values of fractional tune:  $q = 0.4$ .





$$
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$$

We can also get an intuitive picture of why it is that we can only measure up to the half-integer. Plot  $\tilde{V}(\omega)$  for increasing values of fractional tune:  $q = 0.5$ .





$$
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We can also get an intuitive picture of why it is that we can only measure up to the half-integer. Plot  $\tilde{V}(\omega)$  for increasing values of fractional tune:  $q = 0.6$ .





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Sidebands have swapped places! We can't know if  $q = 0.4$  or  $1 - 0.4 = 0.6$ .



# Tune Measurement Example

This is an example BPM difference signal during a tune measurement on the ISIS synchrotron. A kick was applied at around 200 µs.




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Use an FFT to obtain a spectra of this signal.





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Since ISIS has two bunches,

Muon Source

$$
\underbrace{2\frac{1.35-1.225}{\text{35}}}_{\text{Facilities Council}} = 0.
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**LK** 

**ISIS Neutron and** Muon Source

$$
2\frac{2\frac{1.35 - 1.225}{1.35}}{1.35} = 0.19,
$$



Fractional tune is either 0.19 or 0.81.

# **Questions?**

# **Other Diagnostic Systems**

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In such a short time, it isn't possible to review even a few systems in any detail.

But there are two systems I haven't mentioned which are very important, and I would like to mention them, even without much detail.

Like some of the more detailed examples, they can be used in combination with other equipment and techniques to measure a range of beam and machine parameters.



Measure the number of accelerated particles. [5, 6]



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Wall current monitors generate a voltage from the image current flowing in the vacuum vessel.





## Beam Loss Monitors

Ideally we would accelerate all the injected particles, but some beam loss is inevitable, and we must plan for equipment failures [5, 7]



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In an ion chamber, electron-ion pairs are generated when an ionising particle passes through its gas. The pair is separated with a high-voltage and the resulting current is measured.





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In a scintillator BLM, light is generated when an incoming particle passes through its sensitive volume. The light signal is converted into an electrical signal.





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- Predicting an idealised version of a signal gives us a method for understanding a problem without knowing all the background physics.
- Sometimes the quantity of interest can be obtained directly from the spectra.
- Please review some of the excellent references for more detail.



#### **References**

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# **Extra: Data Acquisition and Sampling**

## Sampling

Whatever diagnostics you use, you will be digitising analogue signals with an ADC [8].



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Whatever diagnostics you use, you will be digitising analogue signals with an ADC [8]. This procedure, whilst very common, is a critical step in the measurement process. There are a few things that it is good to be aware of.



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If the sample rate is less than twice the highest frequency component, high-frequencies will be mapped to lower frequencies. This is called aliasing.

After a signal has been sampled, there is no general correction method. Must be considered in advance!





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The signal we want to measure is this sinusoid,





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If we didn't know what signal to expect, then we might choose the maximum sample rate and bandwidth. If the signal was known to have a low frequency, then we could reduce input bandwidth and sample rate.





