Introduction to plasma wakefield acceleration

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plasma as an accelerator



a section of RF cavity

a plasma wave

- Conventional Accelerators are large (100 metres) and expensive 10-100M\$
- Conventional accelerators cannot achieve better than a few 10 MV/m or you get breakdown
- Plasma waves are a possible alternative providing a route to university scale accelerators and radiation sources

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~ 50 $\mu m;$ ~ 100 GV/m

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why do we want to use a laser-plasma accelerator?



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- Plasma based accelerators are a possible compact alternative
- in particular we are now quite good at accelerating electrons to ~ 1 GeV with ~ 100 TW lasers

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1 GeV electron beam ~ £3 M

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Wakefield acceleration



- when a boat travels through water it produces a wave behind it - a 'wake'
- the phase velocity of the wave is just the speed of the boat
- so we can use a laser pulse travelling at close to c in a plasma to drive a strong wave behind it.
- The wave in this case is an electron plasma oscillation

$$\omega_p = \left(\frac{n_0 e^2}{m_e \epsilon_0}\right)^{\frac{1}{2}}$$

Because these are high frequency oscillations the ions do not move and we can have very strong electric fields

Driving Force

For laser wakefield accelerators wake driven by ponderomotive force

$$\frac{d\mathbf{p}}{dt} = -\frac{e^2}{2m_e\omega_0^2}\nabla\langle E^2\rangle = -\frac{e^2}{2m_e}\nabla\langle A^2\rangle = -\frac{1}{2}m_ec^2\nabla\langle a^2\rangle$$

For particle beam drivers wake driven by space charge field of drive bunch

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = -e\mathbf{E}$$

$$\left(\frac{\partial^2}{\partial t^2} + \omega_p^2\right)\frac{n_1}{n_0} = -\frac{c^2}{2}\frac{\partial^2 a_{\text{laser}}^2}{\partial x^2} - \omega_p^2\frac{n_{\text{beam}}}{n_0}$$

Ponderomotive Force

- This simple derivation was for low intensity ($a_0 < 1$) also called non-relativistic intensities (I < 10¹⁸ Wcm⁻²).
- How do we extend to high intensities?
- method 1) just replace $m_e c^2$ with $\gamma m_e c^2$ but do it at the right stage

$$\mathbf{F}_{p} = -\frac{e^{2}}{2\langle\gamma\rangle m_{e}\omega_{0}^{2}}\nabla\langle E^{2}\rangle = -\frac{1}{2}m_{e}c^{2}\frac{1}{\langle\gamma\rangle}\nabla\langle a^{2}\rangle$$

 method 2) do it properly solving the equation of motion relativistically (see Quesnel + Mora Phys Rev E 1998)

$$\mathbf{F}_{p} = -\frac{1}{2}m_{e}c^{2}\frac{1}{\langle\gamma\rangle}\nabla\langle a^{2}\rangle$$



- The drive pulse of an intense laser pulse pushes away electrons just like a boat pushes away the water
- The much heavier ions are left behind this charge separation makes a very large electric field
- As the electrons rush back to their original position they overshoot forming a plasma wave
- Plasma wave amplitude is largest if the drive duration is less than the plasma wavelength $c\tau_L < \lambda_p$



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Driving Plasma waves

- The picture of wakefield I have shown so far is from a particle-in-cell numerical simulation
- But is it possible to "see" the plasma wave directly in experiments?

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- But is it possible to "see" the plasma wave directly in experiments?
 - Yes! This is using a technique called Fourier domain holography (Matlis Nature Physics 2006)



Driving Plasma waves

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- But is it possible to "see" the plasma wave directly in experiments?



Phase velocity and wavelength of plasma waves

- The laser pulse speed determines the wavelength and phase velocity.
- Think of each electron as a separate oscillator, that is set in motion by the laser when the laser gets to it.
- If the first electron (at z = 0) is set in motion at t = 0, the next electron (at $z = \Delta z$) will start oscillating at $t = \Delta t = \Delta z/v_g$ where is the velocity of the laser pulse in the plasma (group velocity)
- there will be a wave with a phase velocity of $v_p = \Delta z / \Delta t = v_g$
- The wavelength will therefore be

$$\lambda_p = \frac{2\pi v_g}{\omega_p} \simeq \frac{2\pi c}{\omega_p}$$

Phase velocity and wavelength of plasma waves



The wavelength of plasma waves is also experimentally verifiable

 $\lambda_p \simeq 10 \ \mu {
m m}$ at $n_e \simeq 10^{19} \ {
m cm}^{-3}$ (for $\lambda = 800 \ {
m nm}$ laser)

Dephasing



electrons travel slightly faster than the wave - eventually they stop being accelerated, this is called "dephasing"

Relativistic electrons ($v_e/c = \beta_e \rightarrow 1$) accelerating in the wave will move ahead of the wave which is moving at

$$\beta_p = \frac{v_g}{c} = \left(1 - \frac{n_e}{n_c}\right)^{\frac{1}{2}}$$

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$$t_d = \frac{\lambda_p}{2c\left(\beta_e - \beta_p\right)} \approx \frac{\lambda_p}{c} \frac{n_c}{n_e} \qquad L_{dp} = \lambda_p \frac{n_c}{n_e}$$

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- Dephasing length < 8 mm at $n_e = 4 \times 10^{18} \text{ cm}^{-3}$
- Dephasing is the fundamental limit to energy gain in LWFA

Limits to Acceleration: 2) pump depletion

Creating the plasma wave takes energy - this must come from the drive pulse.

 $U_{\text{plasma}} = \frac{1}{4} \epsilon_0 E_{z0}^2$ plasma wave electric field energy density $E_{z0} = \delta \frac{m_e c \omega_p}{e}$

 $W_{\text{plasma}} = U_{\text{plasma}}AL$ energy in plasma wave cross section A, length L

 $U_{\text{laser}} = \frac{1}{2} \epsilon_0 E_{L0}^2$ laser electric and magnetic field energy density $E_{L0} = a_0 \frac{m_e c \omega_0}{e}$

 $W_{\text{laser}} = U_{\text{laser}} A c \tau_L$ energy in laser pulse wave cross section *A*, duration τ $c \tau = \epsilon \lambda_p$

$$L_{pd} = 2\epsilon \left(\frac{a_0}{\delta}\right)^2 \frac{n_c}{n_e} \lambda_p$$

we can tailor parameters so pump depletion > dephasing

Limits to acceleration: 3) diffraction



- We need to keep the laser intense over the entire interaction
- Distance over which a laser diffracts in vacuum is the Rayleigh Range $z_R = \frac{\pi w_0^2}{\lambda_0}$
- For $z_R = 1$ cm we need focal spots ~ 50 µm difficult to make very intense focal spot this large
 - (e.g. you need P > 90 TW for $a_0 = 1$)

Limits to acceleration: 3) diffraction



- To overcome diffraction we need to guide the laser an optical fibre
- Can't use a normal optical fibre it will damage!
 - plasma waveguide plasma density minimum on axis
- Pre-formed plasma waveguides (Hooker group)
- Self-guiding pulse forms its own waveguide

The blow-out regime



If the drive beam is strong enough then it can completely expel all the electrons from near the laser pulse - we call this the blow-out or bubble regime

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$$c^{2}\frac{a_{0}}{\epsilon_{0}} - \frac{e^{2}n_{0}r_{b}}{\epsilon_{0}} = 0$$

$$m_e c^2 \frac{1}{w_0} - \frac{1}{\epsilon_0} = \frac{1}{\epsilon_0}$$

we can estimate r_b by balancing the ponderomotive force and space charge force of the ionic bubble

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$$m_e c^2 \frac{a_0}{w_0} - \frac{e^2 n_0 r_b}{\epsilon_0} = 0$$

$$r_b \approx \frac{a_0}{k_p^2 w_0}$$

it turns out that the situation is best if the laser spot size is matched to the bubble so we have:

$$r_b \approx 2\sqrt{a_0} \frac{c}{\omega_p}$$

- Using the equation for the electric field
- And the blow-out radius

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we can estimate the field strength of the bubble - it is:

$$E_{max} \approx \sqrt{a_0} \frac{m_e c \omega_p}{e}$$

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- And the blow-out radius

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- For $a_0 \approx 3$ and a plasma density of $n_0 = 4 \times 10^{18}$ cm⁻³ we get a maximum field of 330 GV/m !
- Combining this with the dephasing length we would get a maximum electron energy of 2.4 GeV
 - this is an overestimate as non-linear effects make the group velocity a bit slower

Injecting electrons into the wave



- For a surfer to "catch a wave" he must swim to get up to speed before the wave arrives
- ▶ if he is too slow the wave will just pass over him
- we must find a way of accelerating electrons up to the correct speed for them to be trapped by the wave and accelerated

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- Nature is kind to us when the wakefield has a large enough amplitude some electrons can be trapped
- They are all injected at the back of the bubble so can be accelerated to the same energy - quasi-monoenergetic electron beams



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- this a plot of the longitudinal position ($\xi = z ct$) in the wave against the longitudinal momentum p_z (called the $p_z \xi$ phase space)
 - The black arrows show electron trajectories
 - Trapped electrons follow closed orbits
- self-injection in the bubble only happens over a small range of ξ at the back of the bubble
- phase space rotation exchanges initial spread in p_z for spread in ξ



This animation demonstrates how phase space rotation changes the electron spectrum



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what sort of electron beams can we get?

Back in 2004 the Imperial College group, a group in the US (LBNL) and a group in France (LOA) were the first to report narrow energy spread beams from a laser wakefield accelerator





Faure et al Nature 2004

Mangles et al Nature 2004

What sort of electron beams can we get?



• There has been steady progress in LWFAs

» e.g. beam energy doubles every 2.5 years (roughly!)

*to add data to this database please go to: <u>https://forms.gle/D3zR2uHpjos9RQXt6</u>

What sort of electron beams can we get?



- Increase in Beam energy has has been achieved by using higher power lasers.
 - why do we need higher power lasers to get higher energy electrons?

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What sort of electron beams can we get?



• Current record is 8 GeV in 20 cm with a 0.85 PW laser

Beam energy is not the only important parameter!

Laser wakefield accelerators can produce ultra short duration electron beams

 LWFA bunches measured to be less than 2 fs



Lundh et al Nat. Phys. 7, 219–222 (2011)

Laser wakefield accelerators can operate consistently



Dedicated LWFA facility running at 10 Hz for more than 24 hours

 Stable, consistent operation

Laser wakefield accelerators can produce narrow energy spread beams



LT Ke et al PRL **126**, 214801 (2021)

 New injection methods can produce very narrow energy spread beams at the per mille level

Laser wakefield accelerators can be controlled and optimised



Shalloo et al, Nat Comms, 11, 6355 (2020)

 Machine learning methods can be used to optimize and control laser wakefield accelerators
 »6D optimization of LWFA using Bayesian Optimization

When a high power laser enters a plasma, something amazing happens...



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• Laser enters plasma, driving a plasma wave in its wake



• Wake amplitude grows and electrons from background plasma are swept up and accelerated by the wake



• Electrons undergo betatron oscillations in the wake, generating X-rays

Laser wakefield accelerators are a source of interesting X-ray beams



Co-location with other laser pulses makes laser wakefield accelerators tools for diverse experiments

Laser Wakefield Accelerators for Dynamic Imaging





J Wood et al Sci Rep 8, 11010 (2018) J Wood PhD Thesis, Imperial 2017

• Small $(1 \ \mu m)$ source size enables use to make high resolution imaging of laser driven shocks

Co-location with other laser pulses makes laser wakefield accelerator tools for diverse experiments



Mahieu et al, Nat Comms 9, 3276 (2018)

- X-rays generated by laser wakefield beams as ultrafast probe of dense matter
 - Non-equilibrium dynamics of matter in extreme conditions

Co-location with other laser pulses makes laser wakefield accelerator tools for diverse experiments



Cole et al, PRX 8, 011020 (2018)

- Collisions between high intensity lasers and electrons can probe electrodynamics in extreme fields
 - experimental evidence for "radiation reaction"
 - Signatures of quantum nature of this in strong fields



Poder et al, PRX **8**, 031004 (2018)

Summary

This lecture has covered:

- introduction to laser wakefield acceleration
 - driving plasma waves with lasers
 - Limits to acceleration with LWFA
 - Trends and status of L
- Any questions?

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