Longitudinal Beam Dynamics Problem Set

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Problem 1.1. The ISIS Rapid Cycling Synchrotron is a 26 m radius ring comprising 10 sections (also known as **superperiods**). Each superperiod consists of:

- A 4.4 m long dipole
- A quadrupole doublet
- An insertion (varies between superperiods)
- A singlet quadrupole

It accelerates protons from a kinetic energy of 70 MeV to 800 MeV along the rising edge of a sinusoidally varying dipole field at a repetition rate of 50 Hz.

. Given these details, please calculate the following.

a) The bending radius of the ISIS dipoles

b) The minimum and maximum value of the dipole magnetic field

c) The maximal rate-of-change of the magnetic field

Problem 1.2. Write an expression describing the variation of the ISIS dipole field as a function of time ($B(t)$).

Problem 1.3. The vacuum chamber inside the AC-driven dipole and quadrupole magnets is made of ceramic, whereas elsewhere in the ring it is made of steel. Given what you have calculated so far, can you explain why this is?

Problem 1.4. Calculate the revolution frequency of particles in the ISIS RCS at the start and end of the acceleration cycle.

Problem 1.5. What is the minimum RF voltage required as a function of time (0 - 10 ms) to accelerate a proton at ISIS? Why do we need more?

Problem 1.6. The average dispersion in the ISIS dipoles is ~1 m. Calculate the Lorentz factor at transition, γ_t, and the associated kinetic energy. Calculate the value of the phase-slip factor , η, at 0, 5, and 10 ms.

We can derive a one-turn mapping for ΔE and ϕ from the Hamiltonian given on Slide 36:

$$\Delta E_{n+1} = \Delta E_n + V_1(\sin\phi_n - \sin\phi_s)$$
$$\Delta\phi_{n+1} = \phi_n - \frac{2\pi h\eta}{E_0\beta^2\gamma}\Delta E_{n+1}$$

where V_1 is the maximal gap voltage of our fundamental RF system and E_0 is the rest energy of our particles.

Problem 2.1. Write a simulation program in the programming language of your choice that assumes several initial particle co-ordinates $(\phi, \Delta E)$, with $0 < \phi < \pi$, and calculates and applies the mapping equations over n turns. Assume a constant energy of 70 MeV (**no acceleration**) and use your previously calculated value for η when $t = 0$ ms. Assume a harmonic number $h = 2$, and total peak gap voltage of $V_1 = 19$ kV. Plot the trajectory of 5 particles evenly spaced in ϕ for the range specified previously, assuming they all have $\Delta E = 0$. Ensure that a sufficient of turns are iterated over to produce a full revolution in the Longitudinal Phase Space (LPS). Please show all trajectories on a single plot.

Problem 2.2. Now additionally track the trajectory of 4 more particles with initial coordinates $(0, +0.5\text{ MeV})$, $(0, +1\text{ MeV})$, $(\pi, +0.5\text{ MeV})$, $(\pi, +1\text{ MeV})$.

Problem 2.3. Plot the separatrix for this RF setup on top of your LPS.

Now let us investigate the LPS at $t = 5$ ms (peak acceleration). Assume now that $V_1 = 150$ kV.

Problem 2.4. Calculate the synchronous phase in this case.

Problem 2.5. Track several particles with $-\pi < \phi < \pi$ over sufficient turns to resolve trajectories fully. Assume all RF parameters are constant during this simulation. Once again, plot the separatrix in this case, and confirm the shape of your RF bucket by tracking a particle just inside the separatrix. Include all trajectories on a single plot.

Problem 2.6. Calculate the RF bucket height and RF bucket area at peak acceleration. You may use numerical integration for the latter.

Hand in a copy of your program(s) with its (their) output. Marks are available for the code itself. Do not hesitate to get in touch if you have any questions or are having any issues with any of these problems. I am not trying to catch you out!

Problem 3.1. OPTIONAL Consider now an RF setup with 2 harmonics, $h = 2$ and $h = 4$. The energy gain per turn for the synchronous particle in this case becomes:

$$\Delta E_s = q(V_1 \sin\phi_s + V_2 \sin(2\phi_s + \theta)),$$

where V_2 is the total peak voltage of the $h = 2$ system, and θ is the relative phase offset between the two harmonics. Have a go at calculating the synchronous phase at $t = 5$ ms when $V_1 = 120$ keV, $V_2 = 60$ keV and $\theta = 0$. **N.B. This cannot be solved analytically!**

Problem 3.2. OPTIONAL Adapt your program to track particle trajectories for the dual harmonic system, this time looking at $t = 0$ ms, with:

a) $\theta = 0$

b) $\theta = \pi$