Scalar and tensor charmonium resonances from lattice QCD

David Wilson

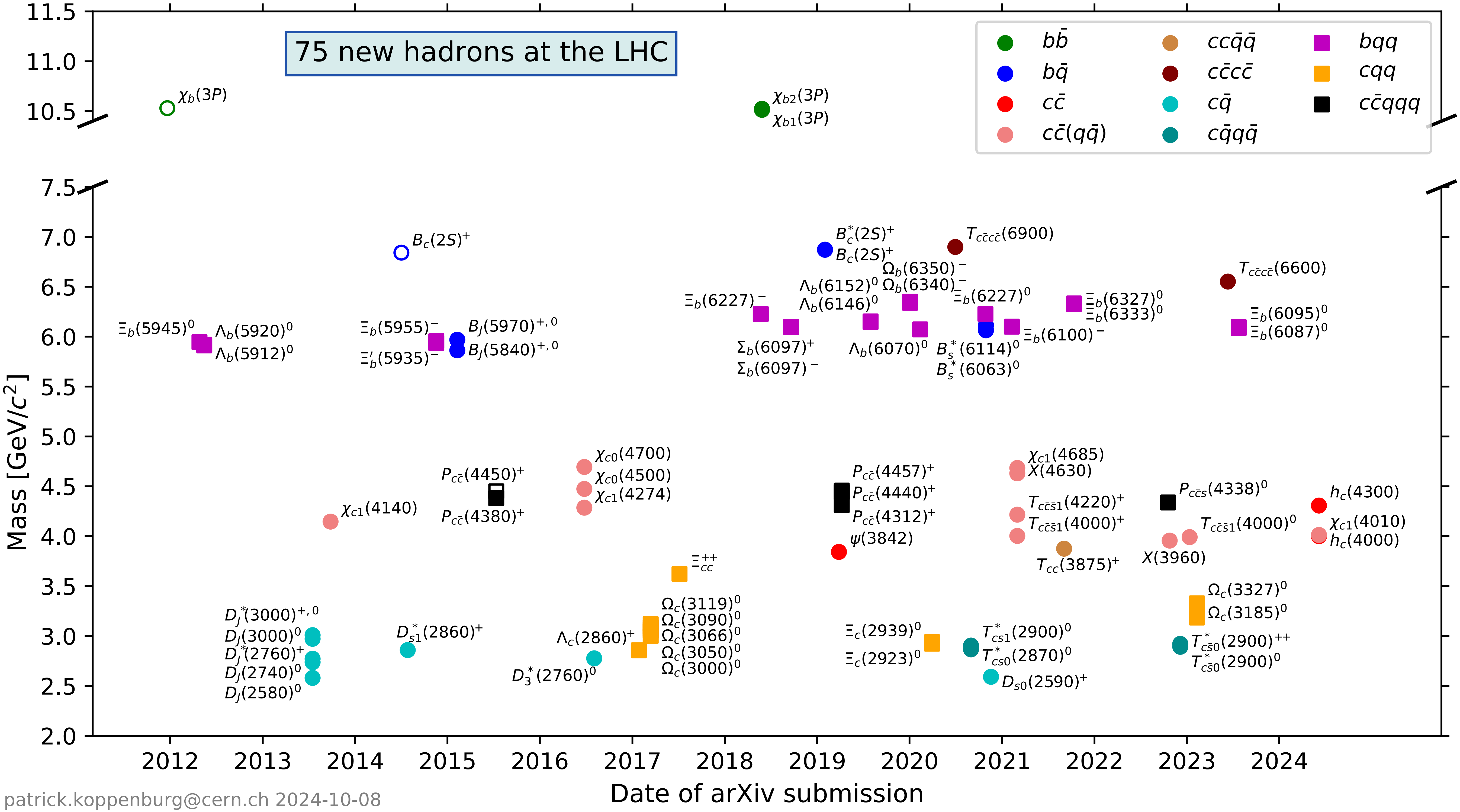
hadspec
hadspec.org

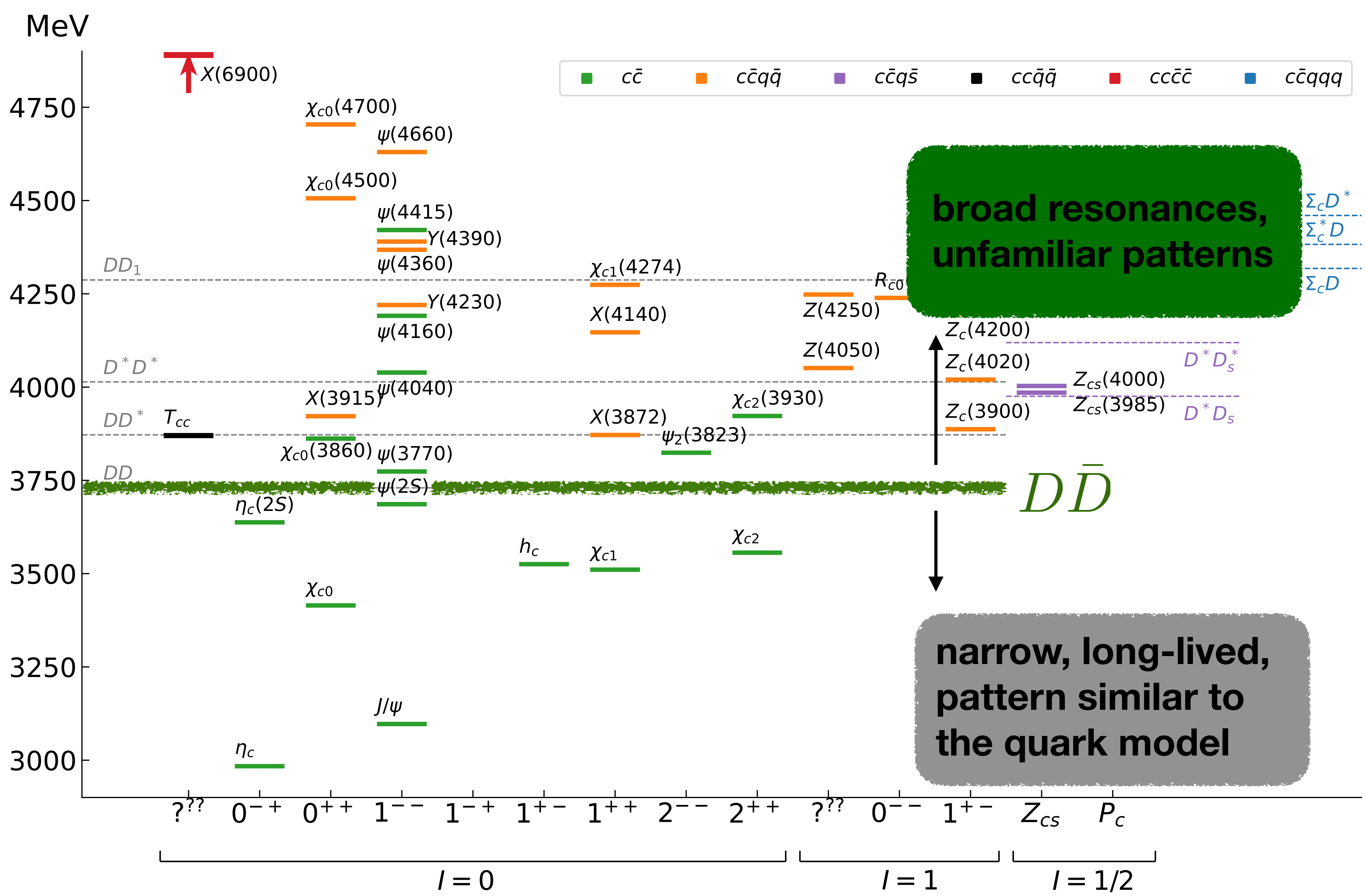
CERN lattice group coffee talk
22nd October 2024

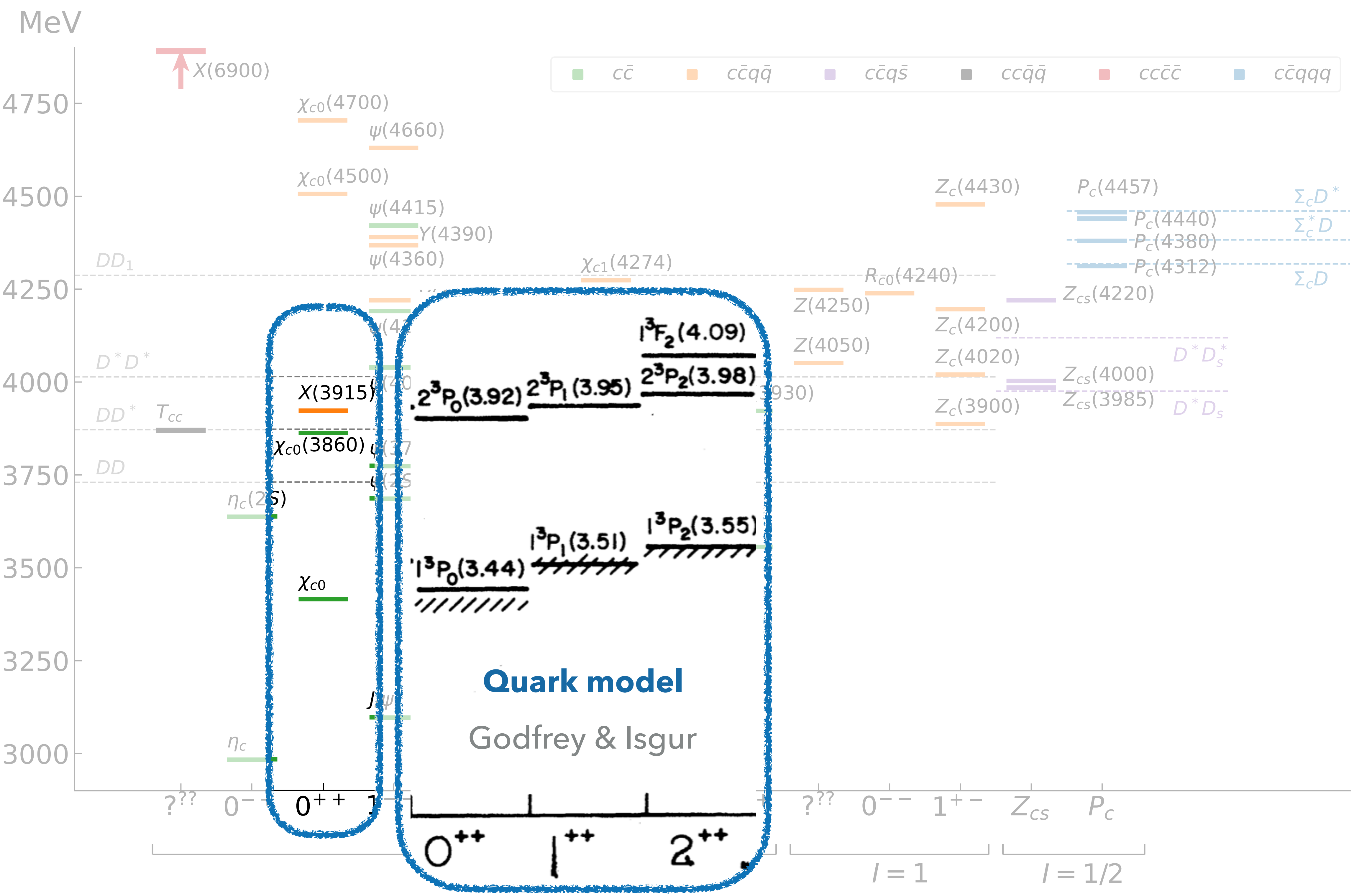
based on work:
PRL Editors' choice: arXiv: 2309.14070 (7 pages)
PRD Editors' choice: arXiv: 2309.14071 (55 pages)

UNIVERSITY OF CAMBRIDGE

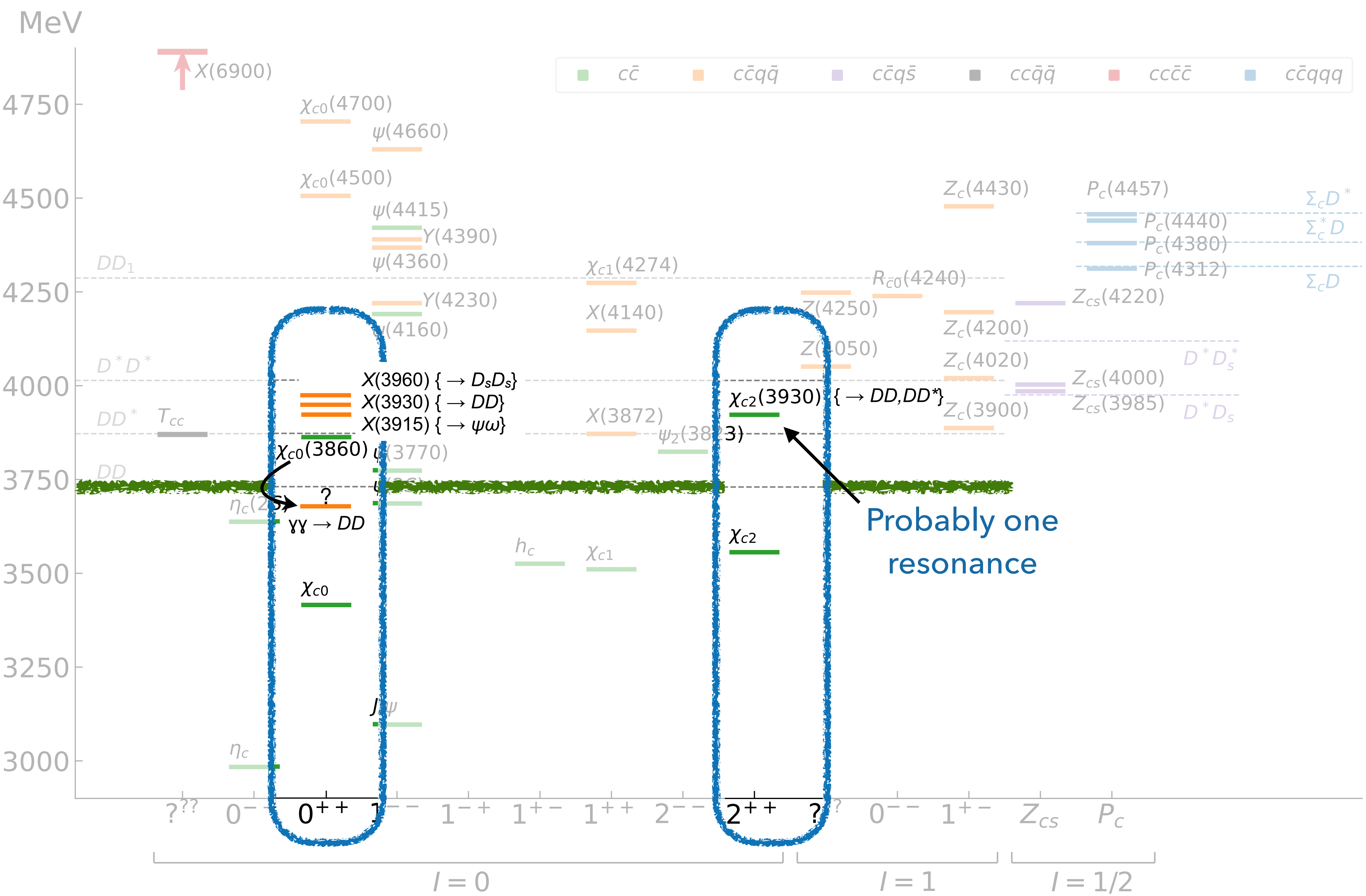
THE ROYAL SOCIETY


75 new hadrons at the LHC
Mass [GeV/c²]
Date of arXiv submission
bb̄
bq̄
cc̄
cc̄(qq̄)
ccq̄q̄
cc̄cc̄
cq̄
cq̄qq̄
bqq
cqq
cc̄qqq
χb(3P)
χb2(3P)
χb1(3P)
Bc(2S)+
Ξb(5945)0
Λb(5920)0
Λb(5912)0
Ξb(5955)−
Ξ′b(5935)−
BJ(5970)+,0
BJ(5840)+,0
Ξb(6227)−
Σb(6097)+
Σb(6097)−
Λb(6152)0
Λb(6146)0
Λb(6070)0
B*c(2S)+
Bc(2S)+
Ωb(6350)−
Ωb(6340)−
Ξb(6227)0
B*s(6114)0
B*s(6063)0
Ξb(6100)−
Tcc̄cc̄(6900)
Ξb(6327)0
Ξb(6333)0
Tcc̄cc̄(6600)
Ξb(6095)0
Ξb(6087)0
χc1(4140)
Pcc̄(4450)+
Pcc̄(4380)+
χc0(4700)
χc0(4500)
χc1(4274)
Pcc̄(4457)+
Pcc̄(4440)+
Pcc̄(4312)+
ψ(3842)
χc1(4685)
X(4630)
Tcc̄s̄1(4220)+
Tcc̄s̄1(4000)+
Tcc(3875)+
Pcc̄s(4338)0
Tcc̄s̄1(4000)0
X(3960)
hc(4300)
χc1(4010)
hc(4000)
Ξ++cc
D*J(3000)+,0
DJ(3000)0
D*J(2760)+
DJ(2740)0
DJ(2580)0
D*s1(2860)+
Λc(2860)+
D*3(2760)0
Ωc(3119)0
Ωc(3090)0
Ωc(3066)0
Ωc(3050)0
Ωc(3000)0
Ξc(2939)0
Ξc(2923)0
T*cs1(2900)0
T*cs0(2870)0
Ds0(2590)+
Ωc(3327)0
Ωc(3185)0
T*cs̄0(2900)++
T*cs̄0(2900)0
11.5
11.0
10.5
7.5
7.0
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
2012
2013
2014
2015
2016
2017
2018
2019
2020
2021
2022
2023
2024


MeV
4750
4500
4250
4000
3750
3500
3250
3000
X(6900)
χc0(4700)
ψ(4660)
χc0(4500)
ψ(4415)
Y(4390)
ψ(4360)
Y(4230)
ψ(4160)
ψ(4040)
X(3915)
χc0(3860)
ψ(3770)
ψ(2S)
ηc(2S)
χc0
J/ψ
ηc
Tcc
DD1
D*D*
DD*
DD
χc1(4274)
X(4140)
X(3872)
ψ2(3823)
χc2(3930)
hc
χc1
χc2
Z(4250)
Z(4050)
Rc0
Zc(4200)
Zc(4020)
Zc(3900)
Zcs(4000)
Zcs(3985)
D*Ds*
D*Ds
ΣcD*
Σc*D
ΣcD
DD̄
cc̄ cc̄qq̄ cc̄qs̄ ccq̄q̄ ccc̄c̄ cc̄qqq
broad resonances, unfamiliar patterns
narrow, long-lived, pattern similar to the quark model
??
0−+
0++
1−−
1−+
1+−
1++
2−−
2++
??
0−−
1+−
Zcs
Pc
I = 0
I = 1
I = 1/2



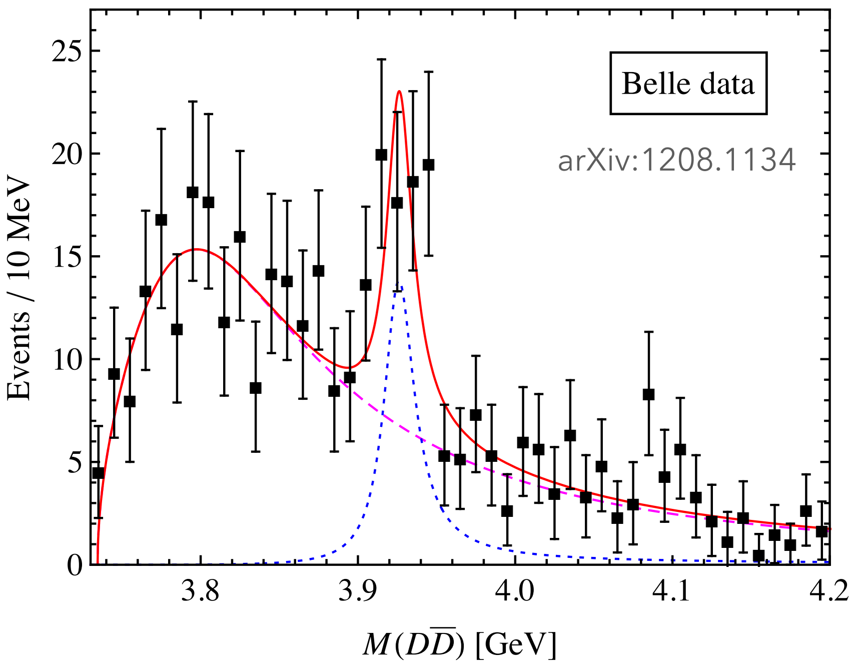
JPAC arXiv:2112.13436



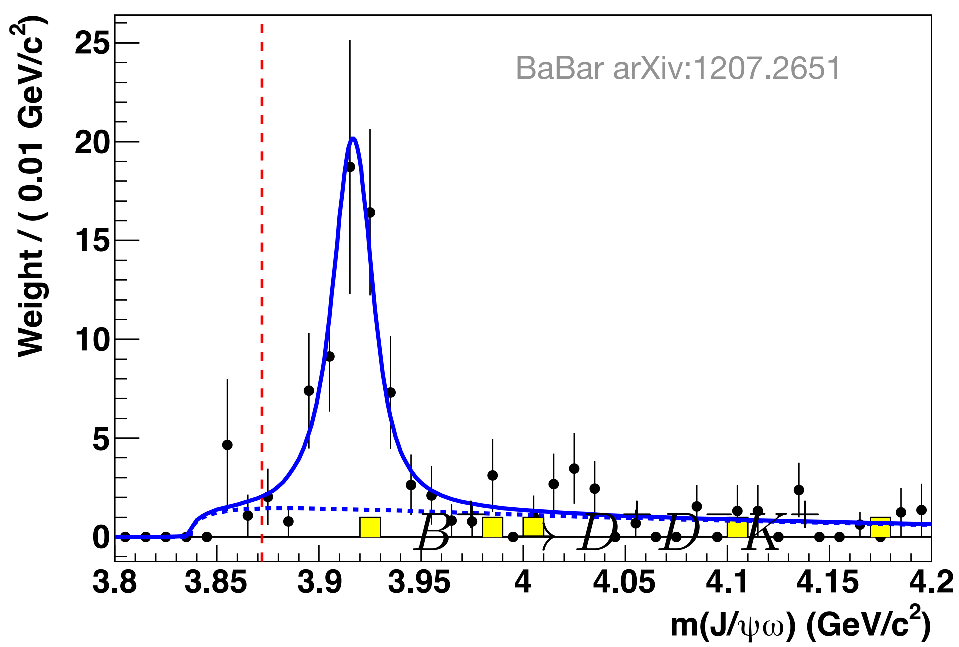
Level counting is completely unclear

- Near threshold behaviour?
- Multiple decoupled resonances?

$\gamma\gamma \to D\bar{D}$



$J/\psi\omega$



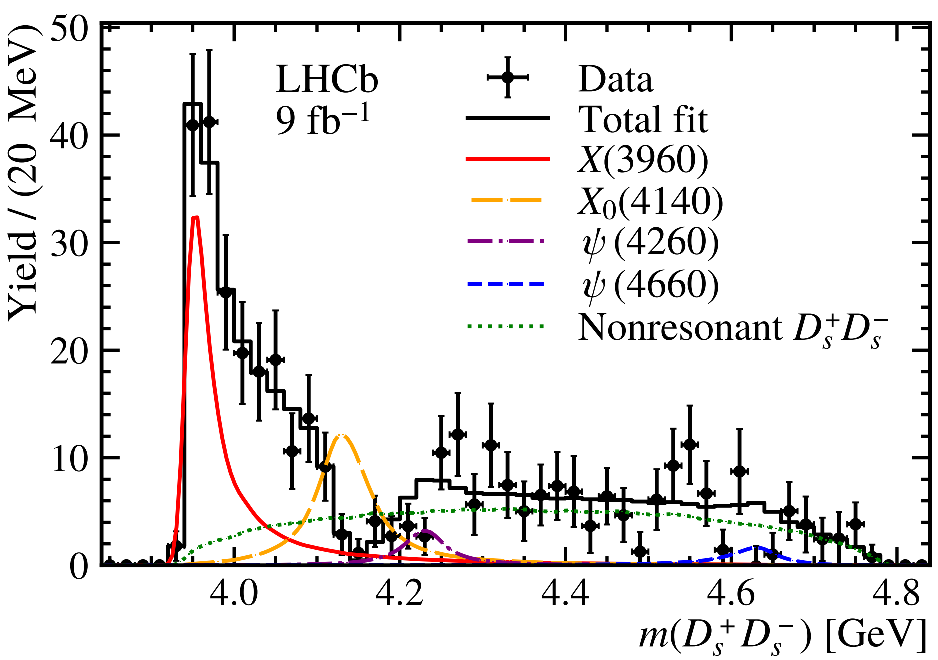
$m = (3919.4 \pm 2.2 \pm 1.6)$ MeV

$\Gamma = (13 \pm 6 \pm 3)$ MeV

$J^P = 0^+$

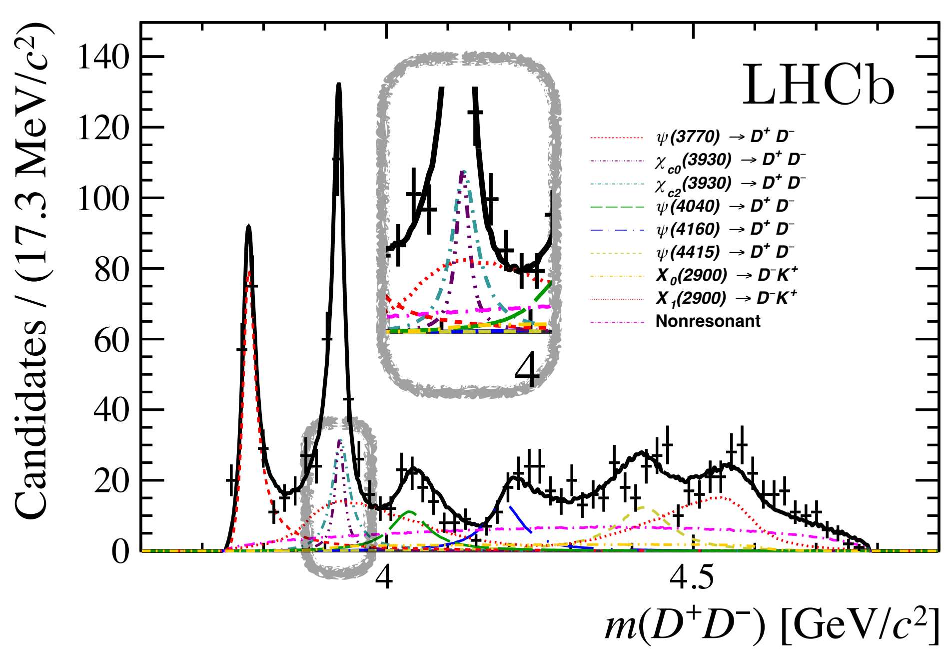
Just a few examples
Many many more
(References in the longer paper)

$B^+ \to D_s^+ D_s^- K^+$ arXiv:2210.15153 LHCb



near-threshold state at 3956 MeV

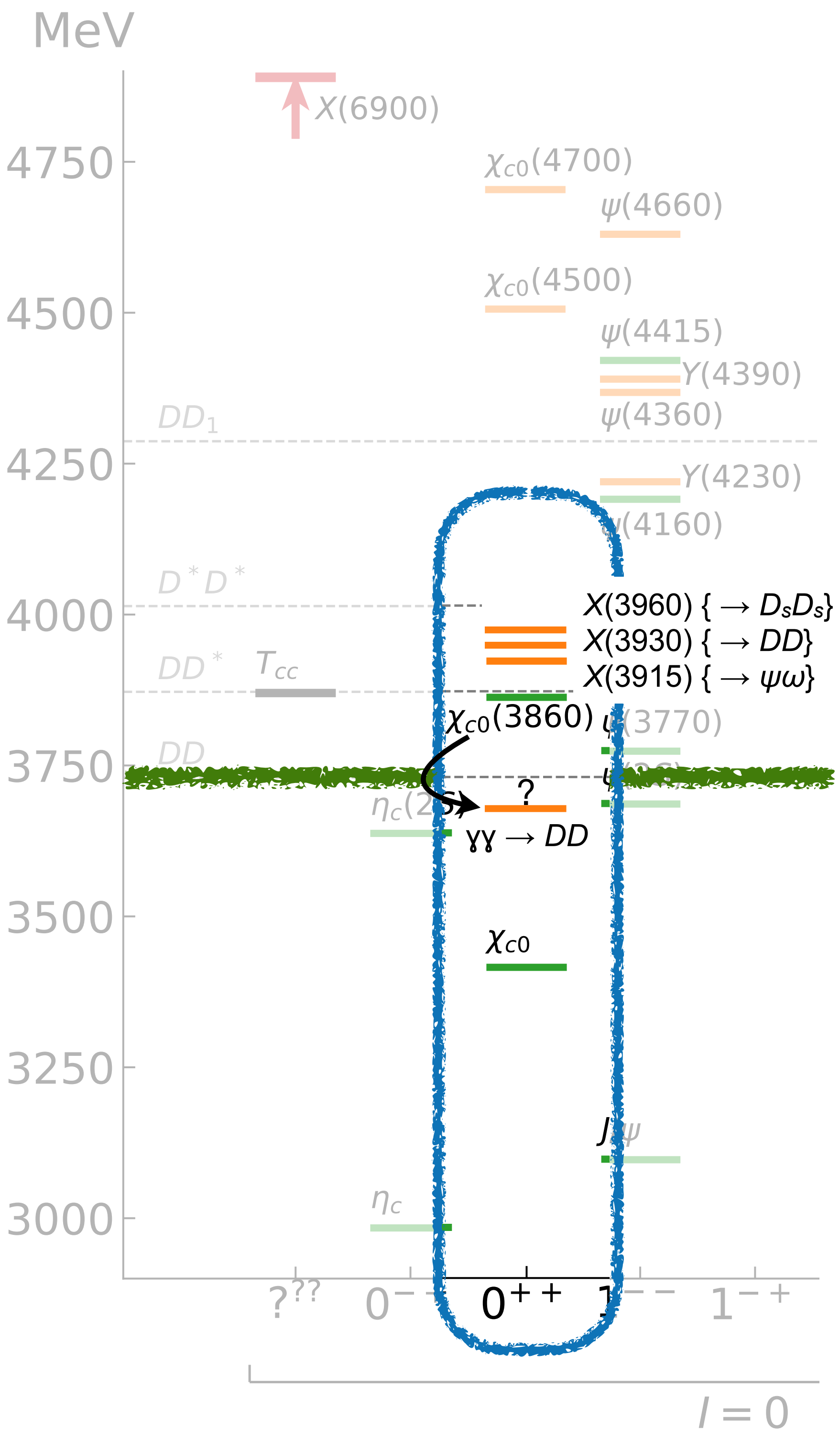
$B^+ \to D^+ D^- K^+$ arXiv:2009.00026



overlapping 0++ and 2++ resonances around 3925 MeV

Near threshold enhancement at $D_s\bar{D}_s$ threshold

no need for a low 0++ resonance



are all of these bumps resonances?

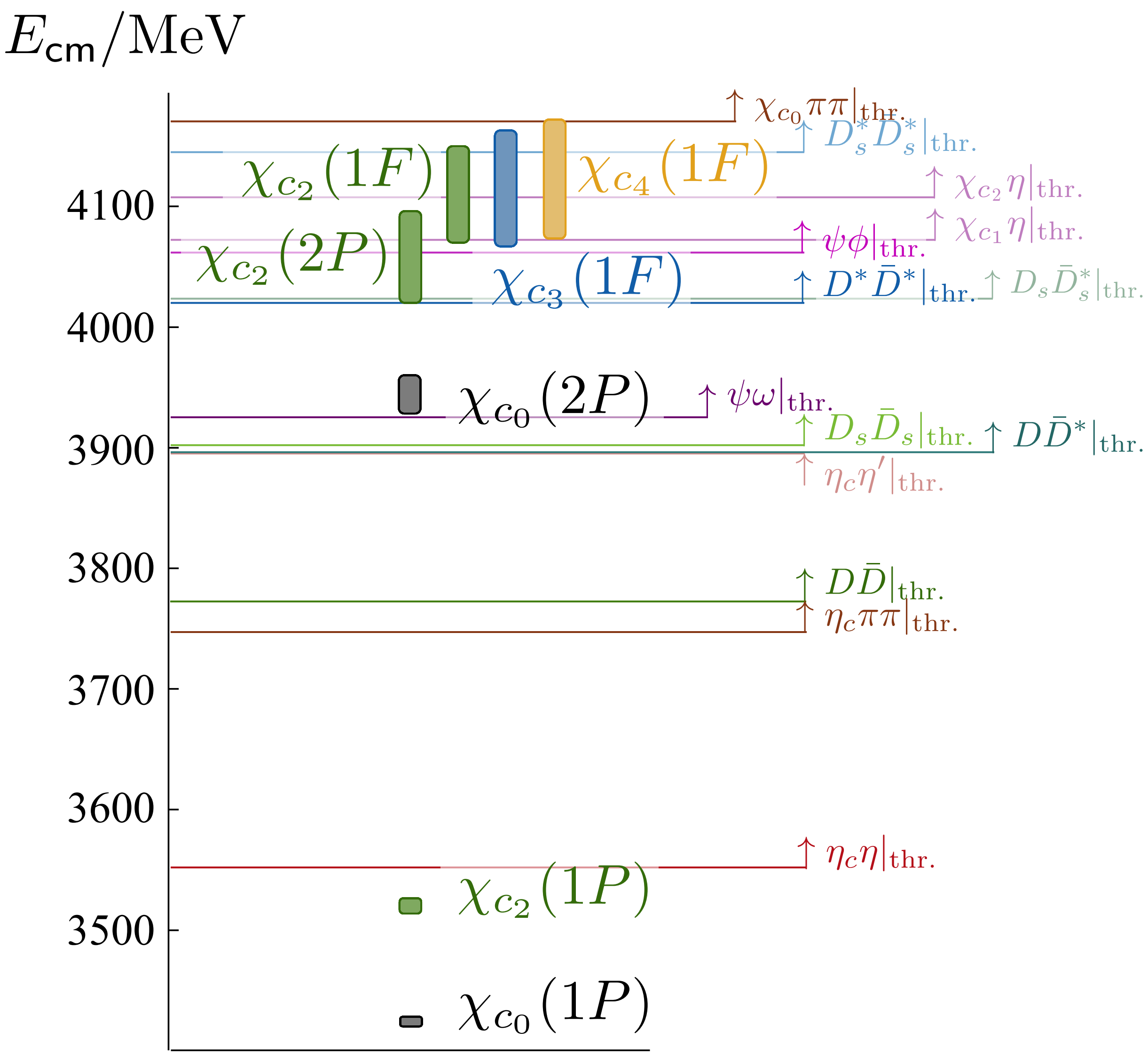
how are these experimental enhancements related to each other?

how many states are there in 0++ and 2++?

can we understand how the quark-model-like states and meson-meson like states contribute to the observed features?

first principles calculations are needed to start to understand this

Previously:



spectra from qqbar operators only,
Liu et al JHEP 1207 (2012) 126

"HadSpec" lattices

anisotropic (3.5 finer spacing in time)
Wilson-Clover

L/a_s=16, 20, 24
m_π = 391 MeV

rest and moving frames

N_f = 2+1 flavours
all light+strange annihilations included
no charm annihilation

using *distillation* (Peardon *et al* 2009)
many channels, many wick contractions

This study: Meson-meson + qqbar ops

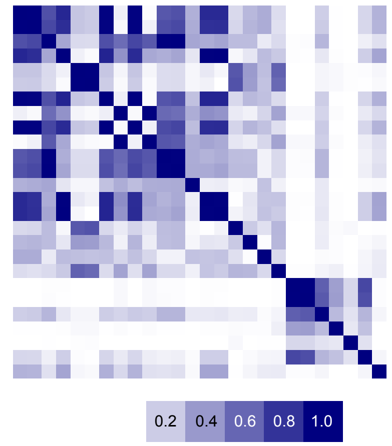
Derivative ops - good overlap upto J=4

Variationally-optimised single meson ops

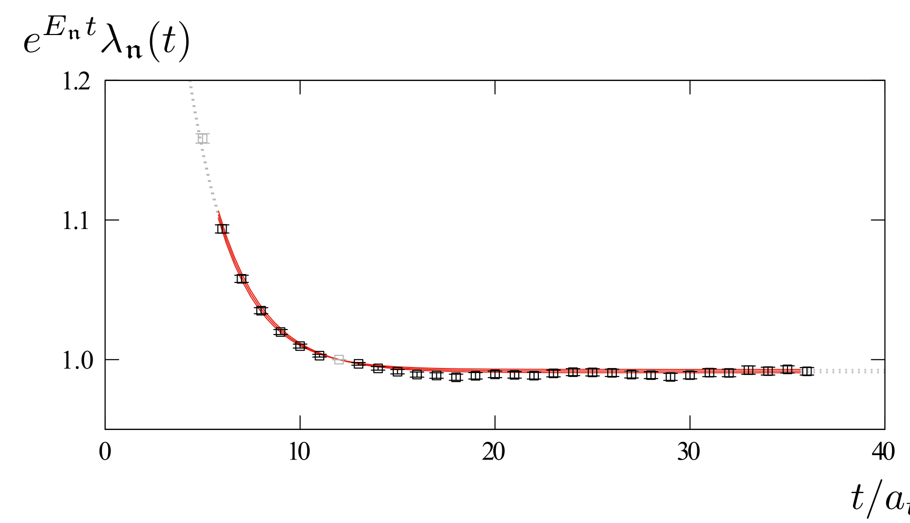
Well-established workflow

Lattice QCD

Compute Correlation Matrix



Generalised Eigenvalue Problem



Operators

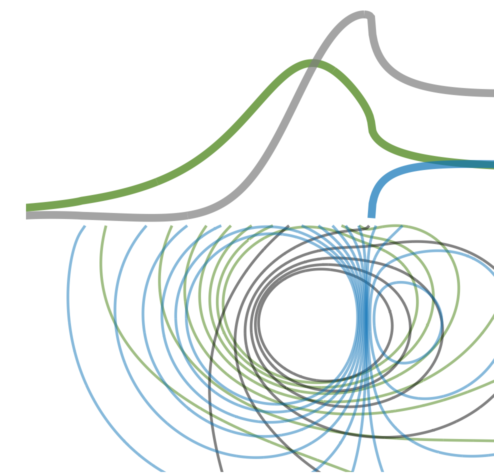
$\mathcal{O}^\dagger \sim \bar{q}\Gamma q$

$(\pi\pi)^\dagger \sim \sum_{\vec{p}} (\mathrm{CGs})\ \pi^\dagger\pi^\dagger$

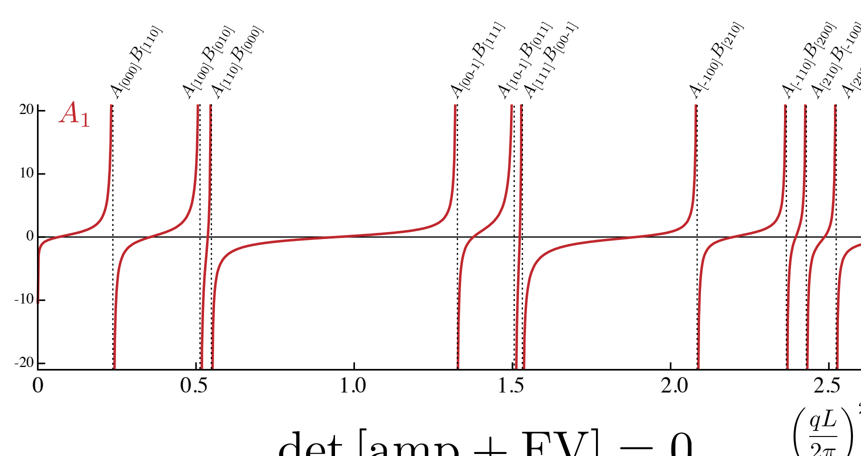
$\pi^\dagger \sim \sum_i v_i^{(\pi)}\mathcal{O}_i^\dagger$

$C_{ij}(t)v_j^{\mathfrak{n}} = \lambda_{\mathfrak{n}}(t)C_{ij}(t_0)v_j^{\mathfrak{n}}$

Obtain Finite Volume Spectrum



Lüscher Quantisation Condition

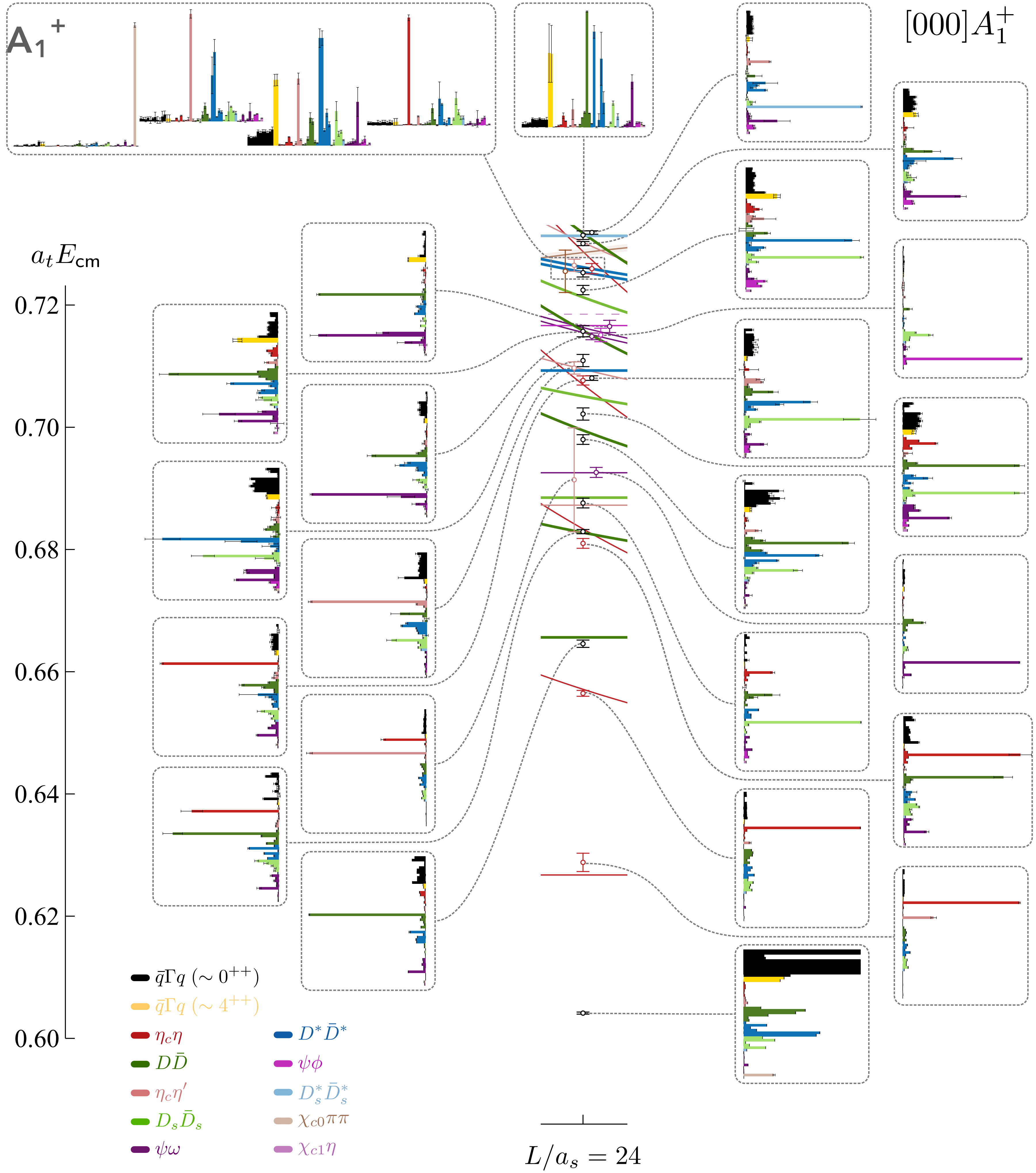


$\det\left[\mathrm{amp} + \mathrm{FV}\right] = 0$

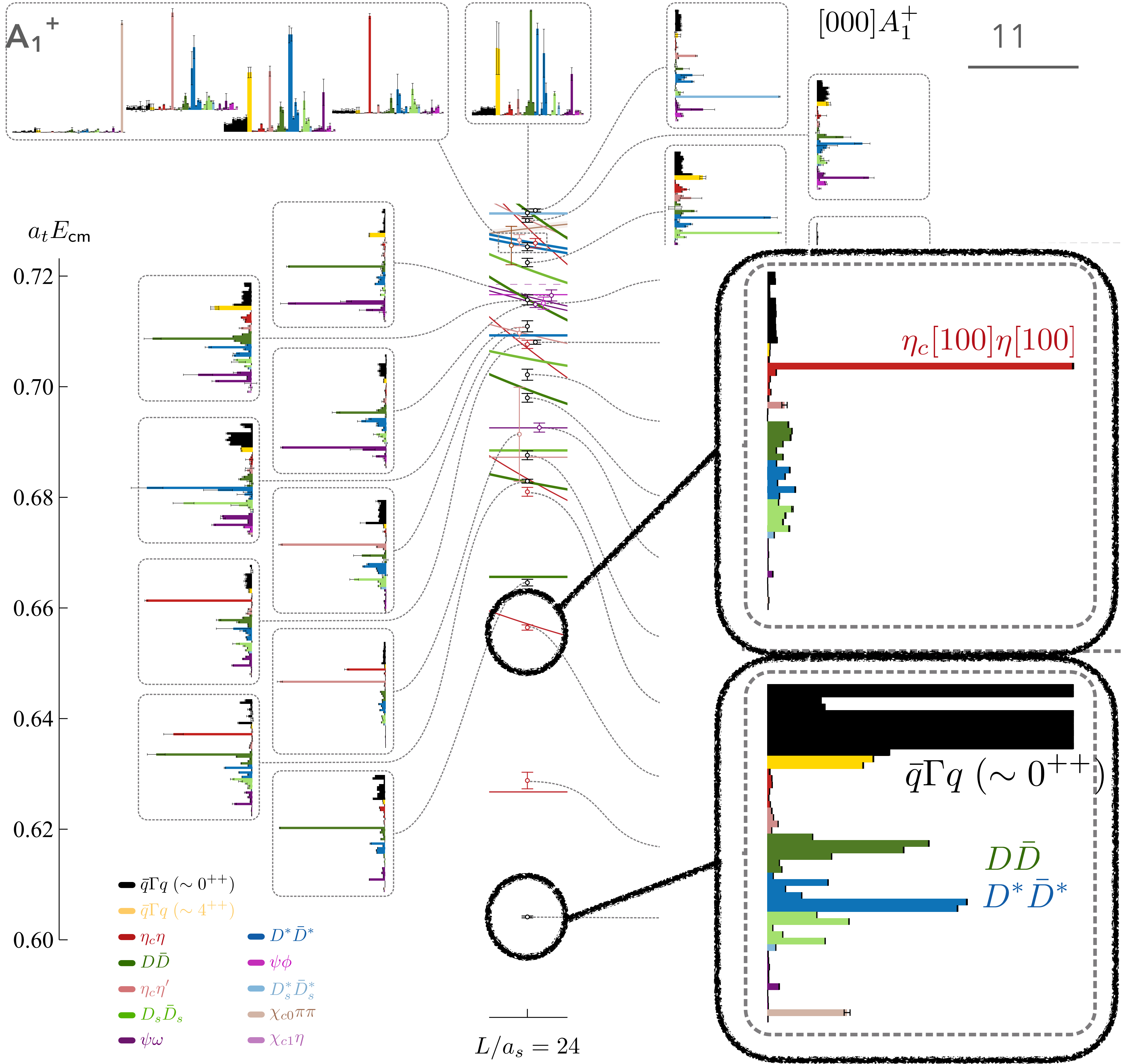
Determine Scattering Amplitudes

Poles, Couplings

Review: Briceño, Dudek, Young, https://doi.org/10.1103/RevModPhys.90.025001

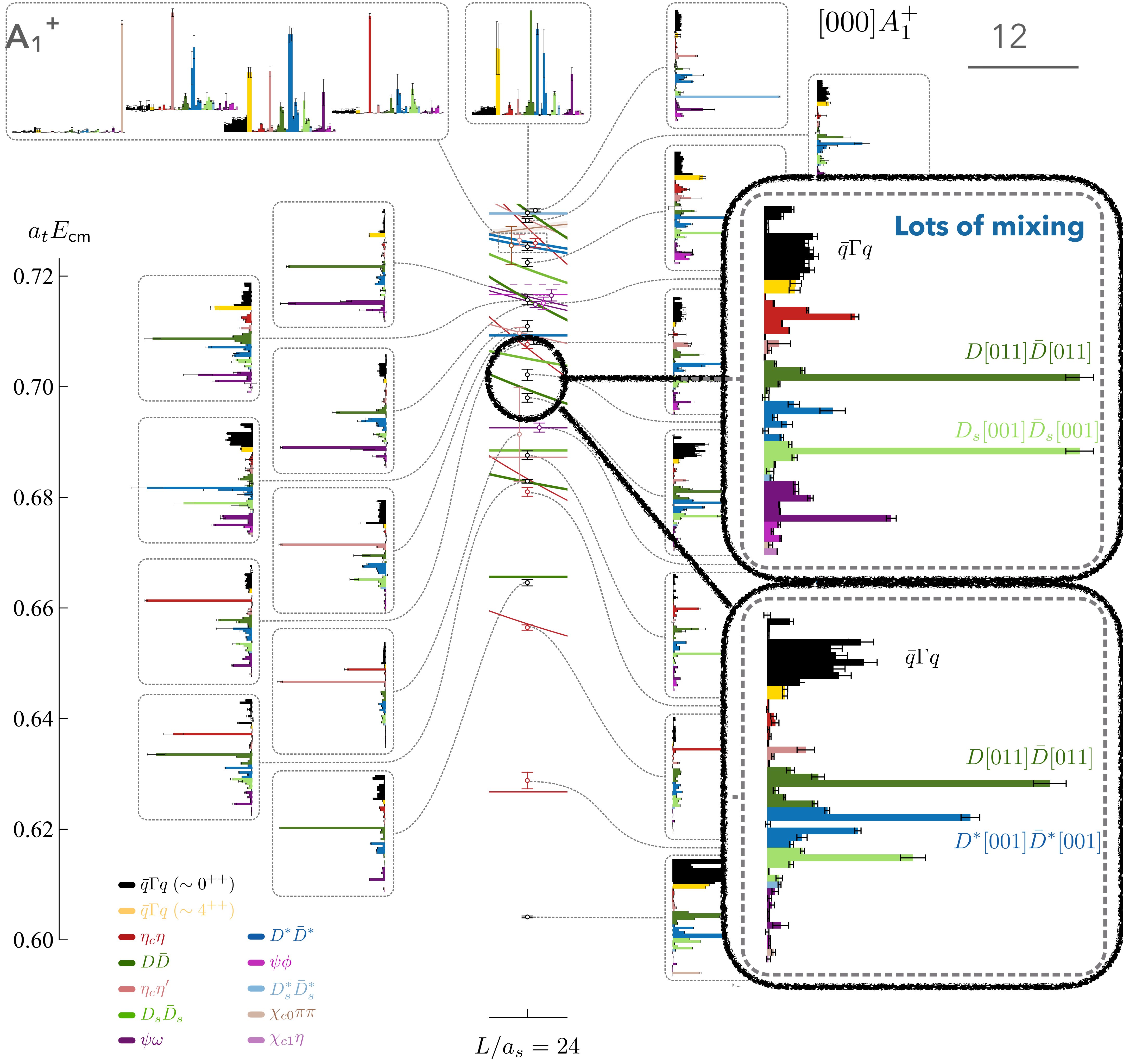

$[000]A_1^+$
$a_t E_{\mathsf{cm}}$
0.72
0.70
0.68
0.66
0.64
0.62
0.60
$\bar{q}\Gamma q$ ($\sim 0^{++}$)
$\bar{q}\Gamma q$ ($\sim 4^{++}$)
$\eta_c\eta$
$D\bar{D}$
$\eta_c\eta'$
$D_s\bar{D}_s$
$\psi\omega$
$D^*\bar{D}^*$
$\psi\phi$
$D_s^*\bar{D}_s^*$
$\chi_{c0}\pi\pi$
$\chi_{c1}\eta$
$L/a_s = 24$

S-wave - [000] A_1^+

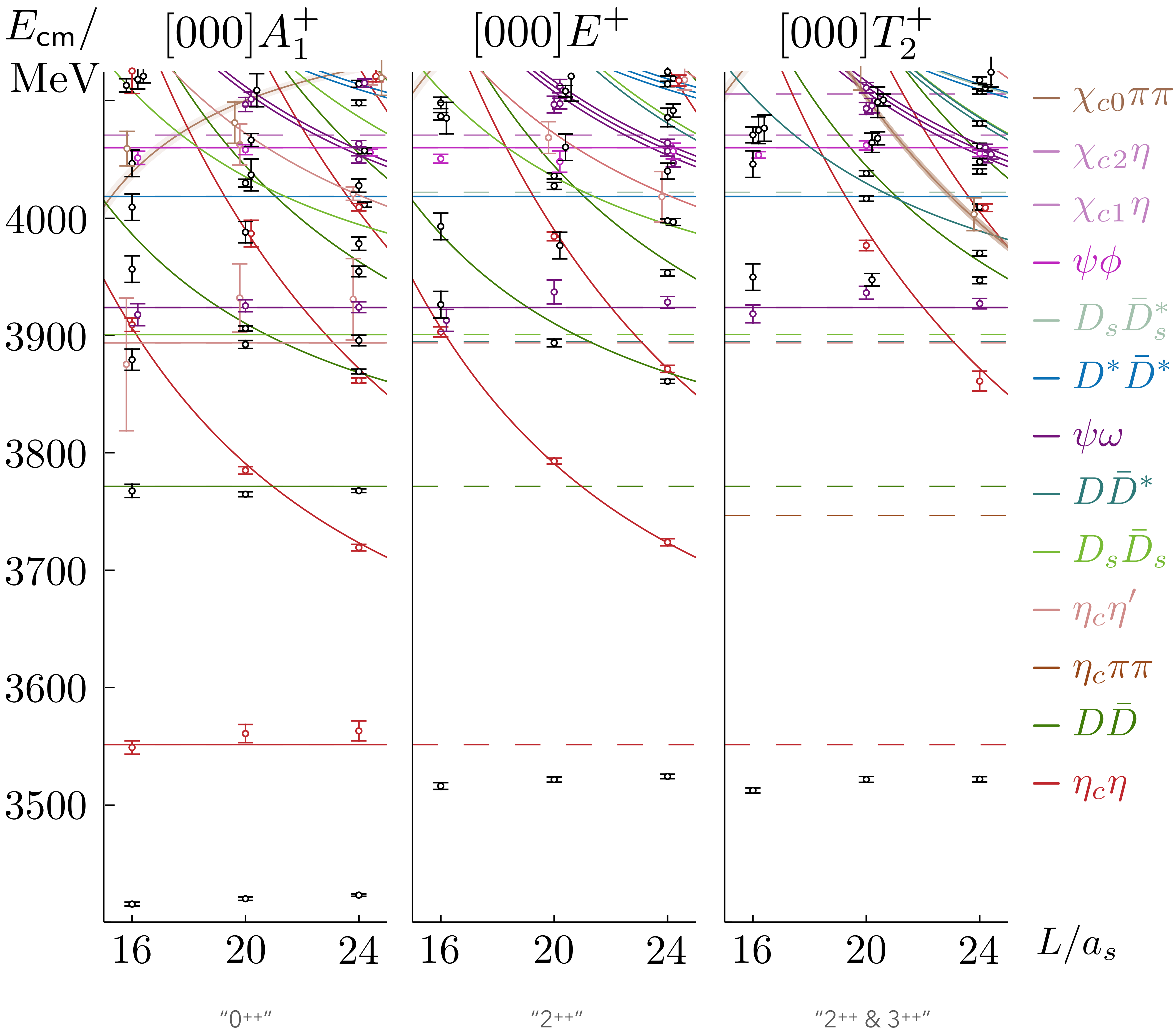


$[000]A_1^+$
$a_t E_{\mathrm{cm}}$
0.72
0.70
0.68
0.66
0.64
0.62
0.60
$\eta_c[100]\eta[100]$
$\bar{q}\Gamma q\ (\sim 0^{++})$
$D\bar{D}$
$D^*\bar{D}^*$
$\bar{q}\Gamma q\ (\sim 0^{++})$
$\bar{q}\Gamma q\ (\sim 4^{++})$
$\eta_c\eta$
$D\bar{D}$
$\eta_c\eta'$
$D_s\bar{D}_s$
$\psi\omega$
$D^*\bar{D}^*$
$\psi\phi$
$D_s^*\bar{D}_s^*$
$\chi_{c0}\pi\pi$
$\chi_{c1}\eta$
$L/a_s = 24$

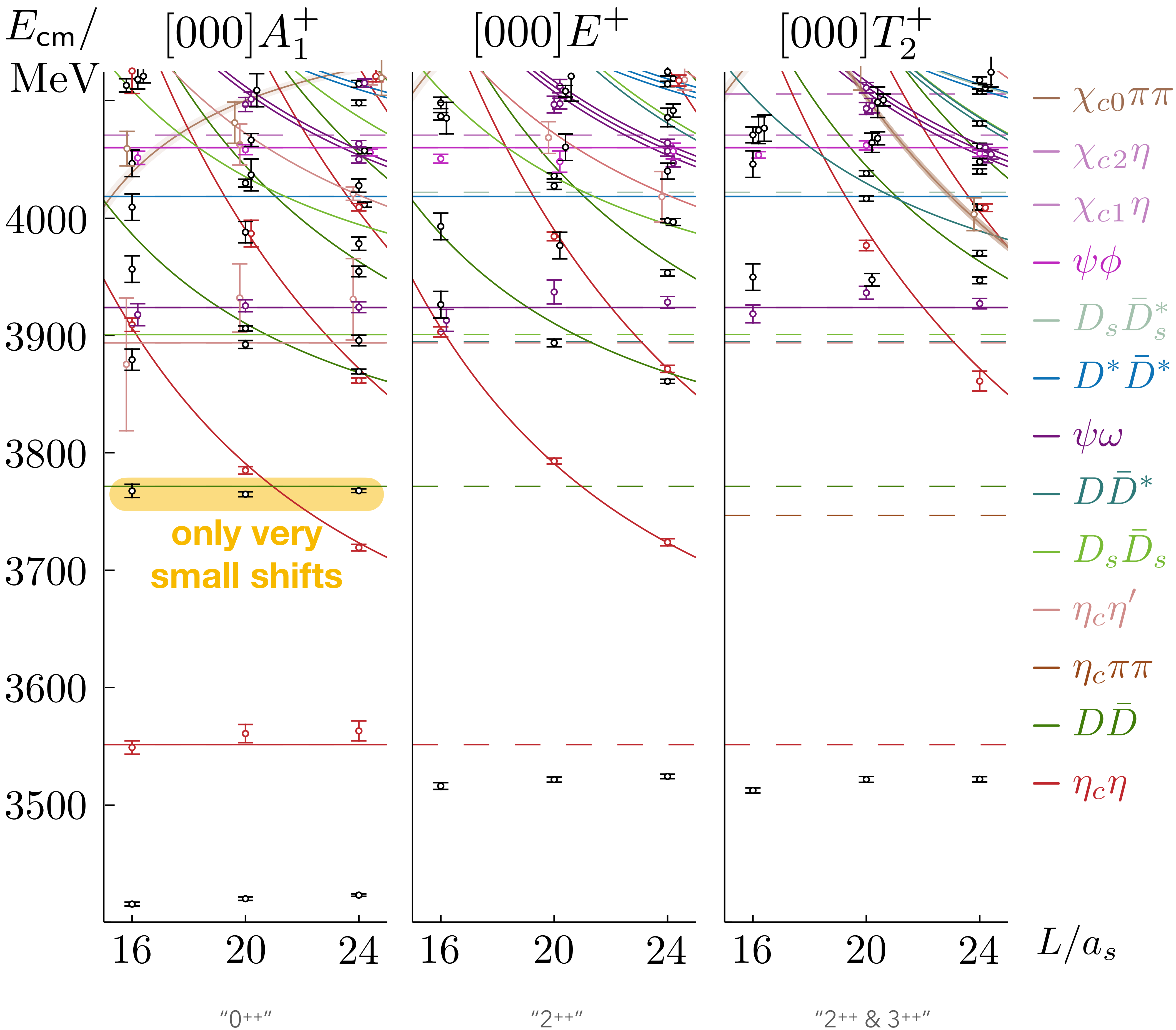
S-wave - [000] A_1^+

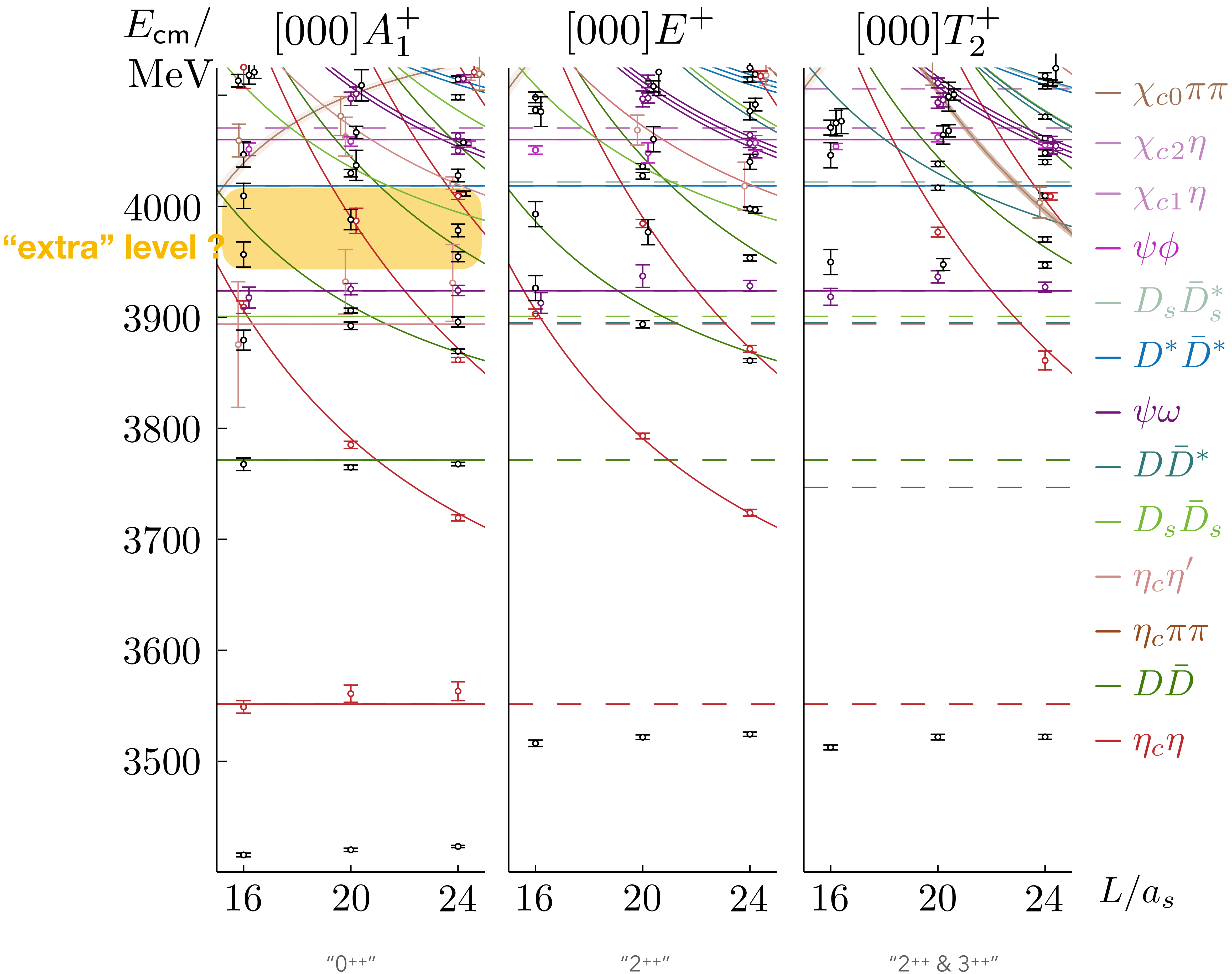


$[000]A_1^+$
$a_t E_{cm}$
0.72
0.70
0.68
0.66
0.64
0.62
0.60
$L/a_s = 24$
Lots of mixing
$\bar{q}\Gamma q$
$D[011]\bar{D}[011]$
$D_s[001]\bar{D}_s[001]$
$\bar{q}\Gamma q$
$D[011]\bar{D}[011]$
$D^*[001]\bar{D}^*[001]$
$\bar{q}\Gamma q\ (\sim 0^{++})$
$\bar{q}\Gamma q\ (\sim 4^{++})$
$\eta_c\eta$
$D\bar{D}$
$\eta_c\eta'$
$D_s\bar{D}_s$
$\psi\omega$
$D^*\bar{D}^*$
$\psi\phi$
$D_s^*\bar{D}_s^*$
$\chi_{c0}\pi\pi$
$\chi_{c1}\eta$

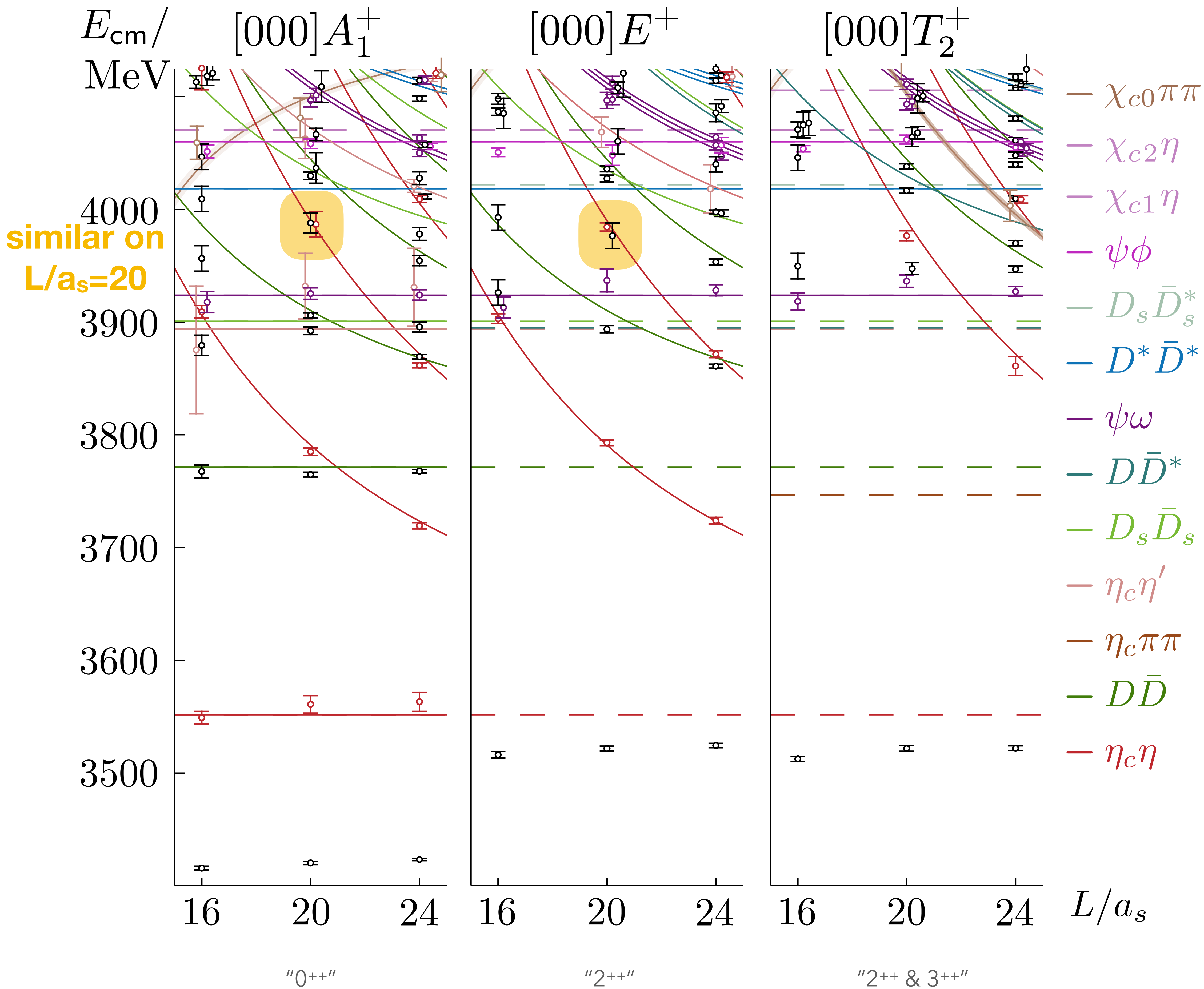


$E_{\mathrm{cm}}/$ MeV
$[000]A_1^+$
$[000]E^+$
$[000]T_2^+$
3500
3600
3700
3800
3900
4000
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
"0++"
"2++"
"2++ & 3++"

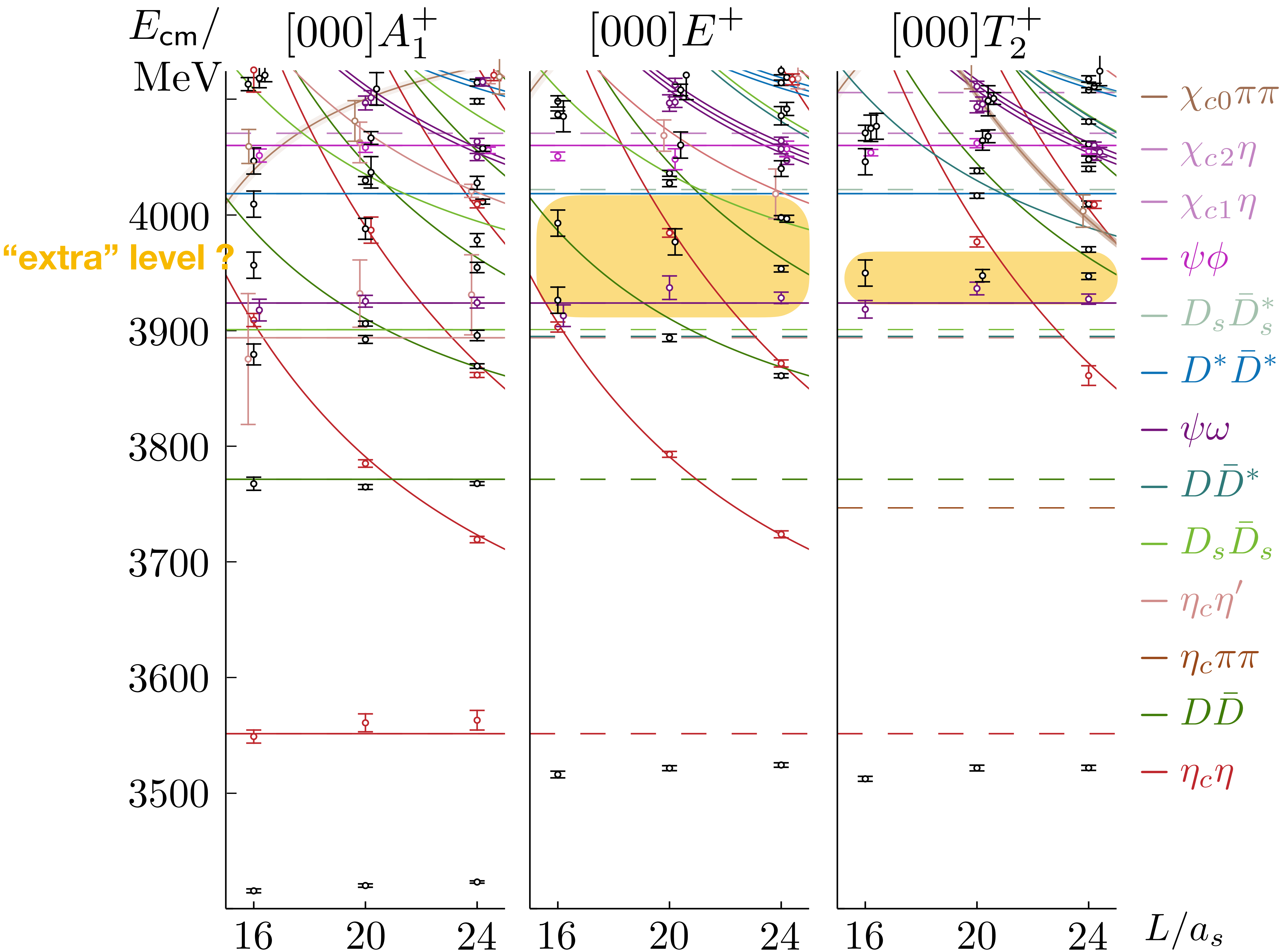

E_{cm}/ MeV
$[000]A_1^+$
$[000]E^+$
$[000]T_2^+$
4000
3900
3800
3700
3600
3500
16
20
24
L/a_s
only very small shifts
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
"0++"
"2++"
"2++ & 3++"



$E_{\rm cm}$/ MeV
$[000]A_1^+$
$[000]E^+$
$[000]T_2^+$
"extra" level ?
4000
3900
3800
3700
3600
3500
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
"0++"
"2++"
"2++ & 3++"

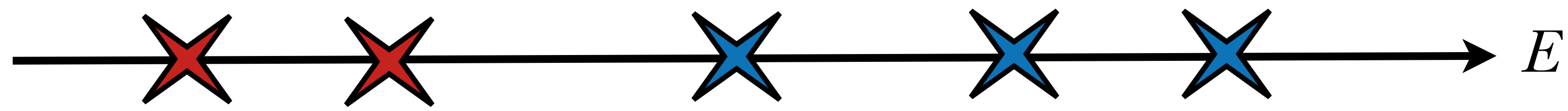


E_{cm}/ MeV
$[000]A_1^+$
$[000]E^+$
$[000]T_2^+$
similar on L/a_s=20
4000
3900
3800
3700
3600
3500
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
"0++"
"2++"
"2++ & 3++"



E_{cm}/ MeV
$[000]A_1^+$
$[000]E^+$
$[000]T_2^+$
"extra" level ?
4000
3900
3800
3700
3600
3500
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$

"0++" "2++" "2++ & 3++"

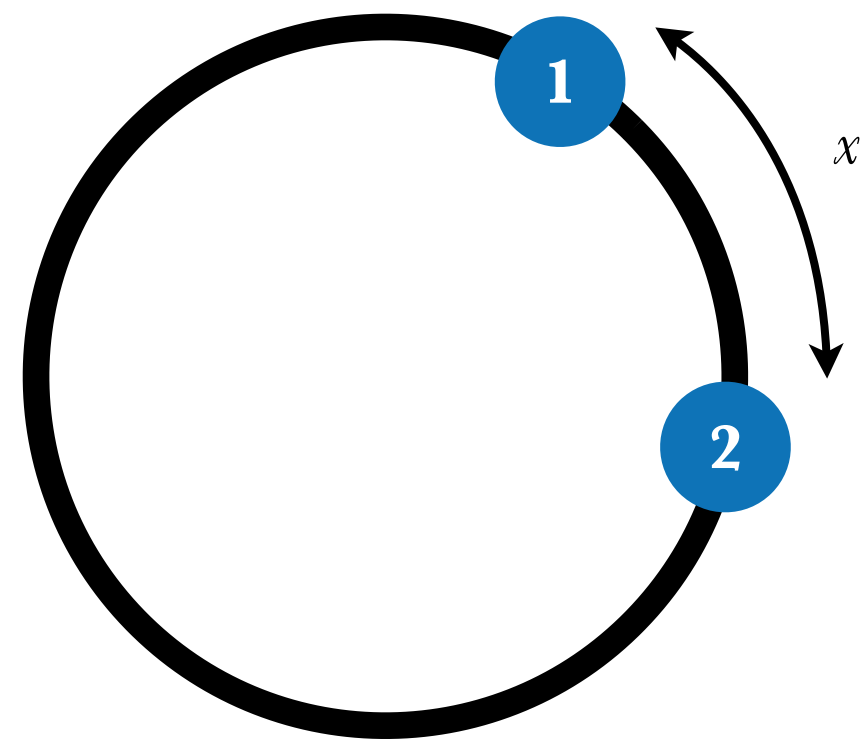


1-dimensional QM, periodic BC, two interacting particles: $V(x_1 - x_2) \neq 0$

$$\psi(0) = \psi(L), \quad \left.\frac{\partial \psi}{\partial x}\right|_{x=0} = \left.\frac{\partial \psi}{\partial x}\right|_{x=L}$$

$$\sin\left(\frac{pL}{2} + \delta(p)\right) = 0$$

$$p = \frac{2\pi n}{L} - \frac{2}{L}\delta(p)$$



Phase shifts via Lüscher's method:

$$\tan \delta_1 = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1; q^2)}$$

$$\mathcal{Z}_{00}(1; q^2) = \sum_{n \in \mathbb{Z}^3} \frac{1}{|\vec{n}|^2 - q^2}$$

Lüscher 1986, 1991

generalisation to a 3-dimensional strongly-coupled QFT

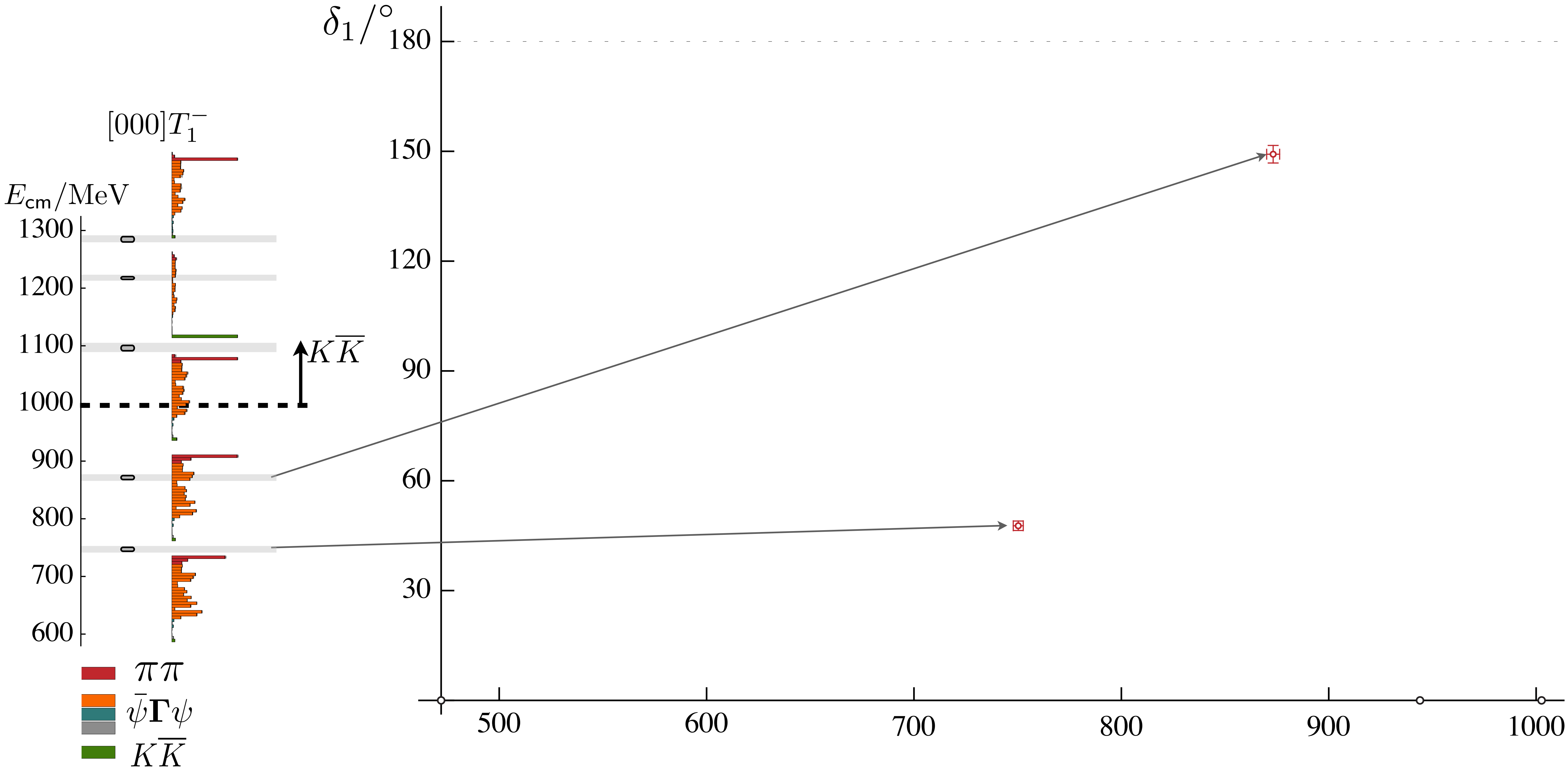
→ powerful non-trivial mapping from finite vol spectrum to infinite volume phase

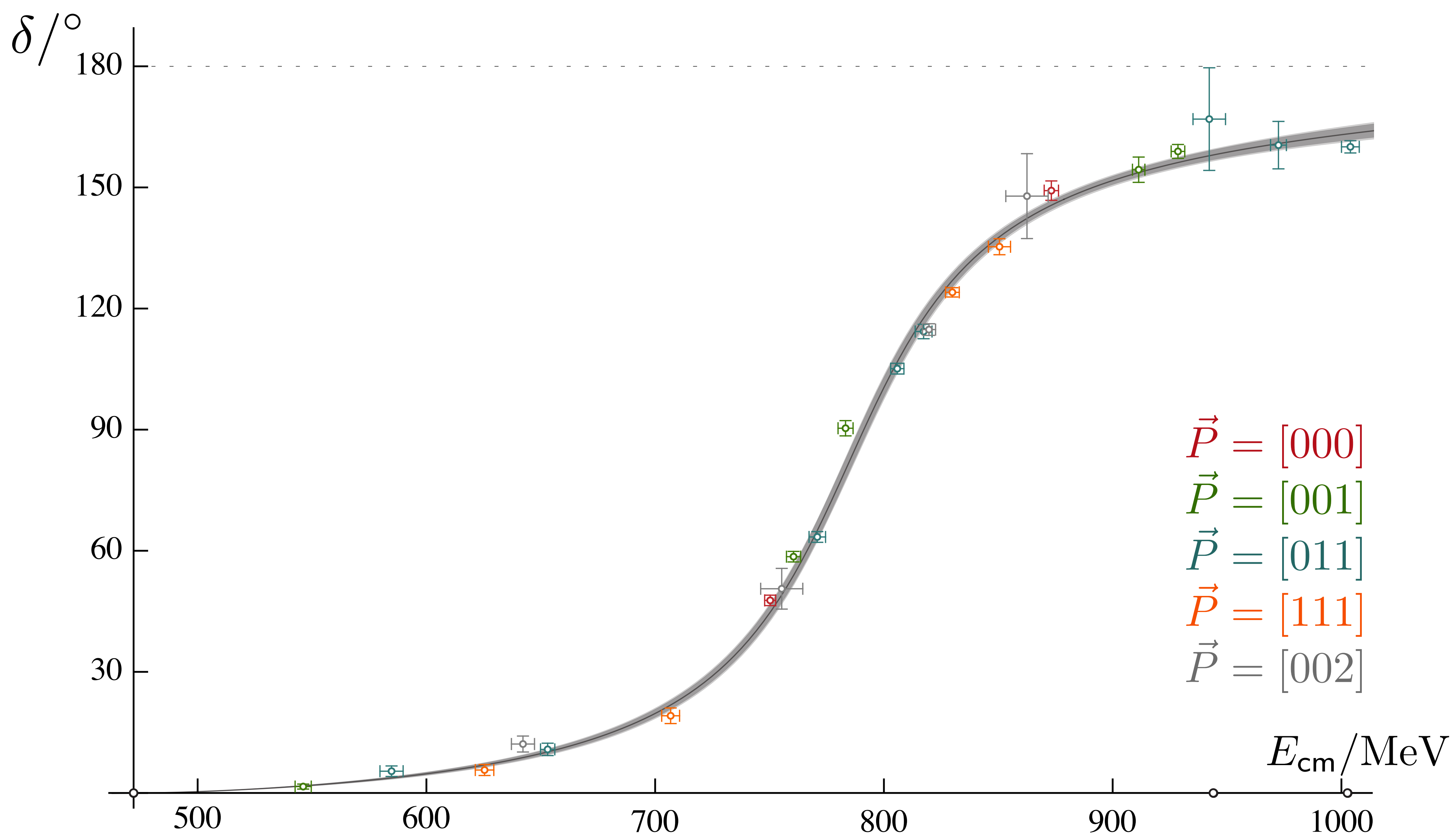
See also Kim, Sachrajda, Sharpe: Nucl. Phys. B727 (2005) (arXiv:hep-lat/0507006)

Review by Briceno, Dudek, Young: Rev. Mod. Phys. 90, 025001 (arXiv:1706.06223)

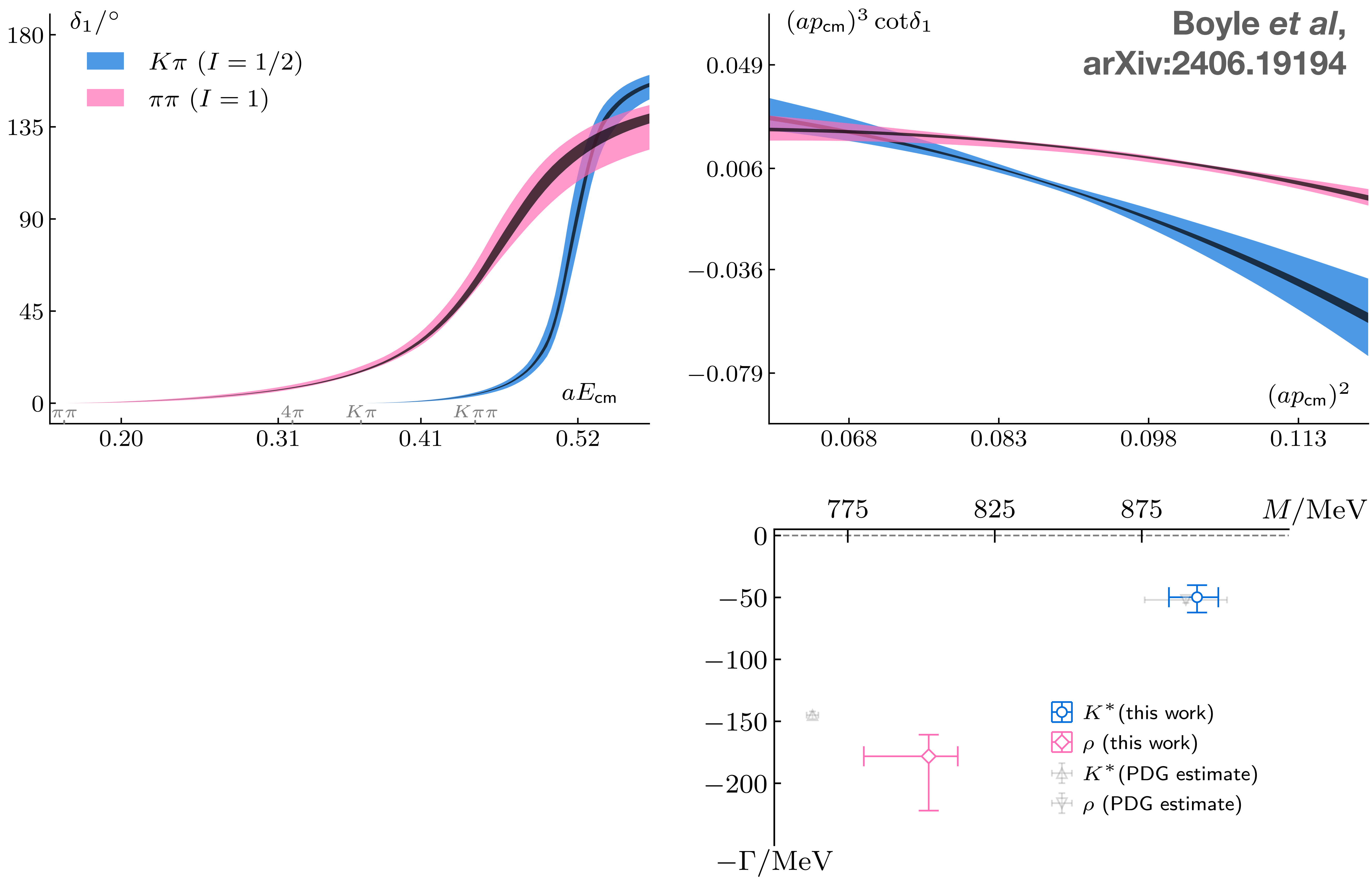
Phase shifts via the Lüscher method: $\tan\delta_1 = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1;q^2)}$

$$\mathcal{Z}_{00}(1;q^2) = \sum_{n\in\mathbb{Z}^3} \frac{1}{|\vec{n}|^2 - q^2}$$




δ/°
180
150
120
90
60
30
500
600
700
800
900
1000
E_cm/MeV
P = [000]
P = [001]
P = [011]
P = [111]
P = [002]

Boyle *et al*, arXiv:2406.19194



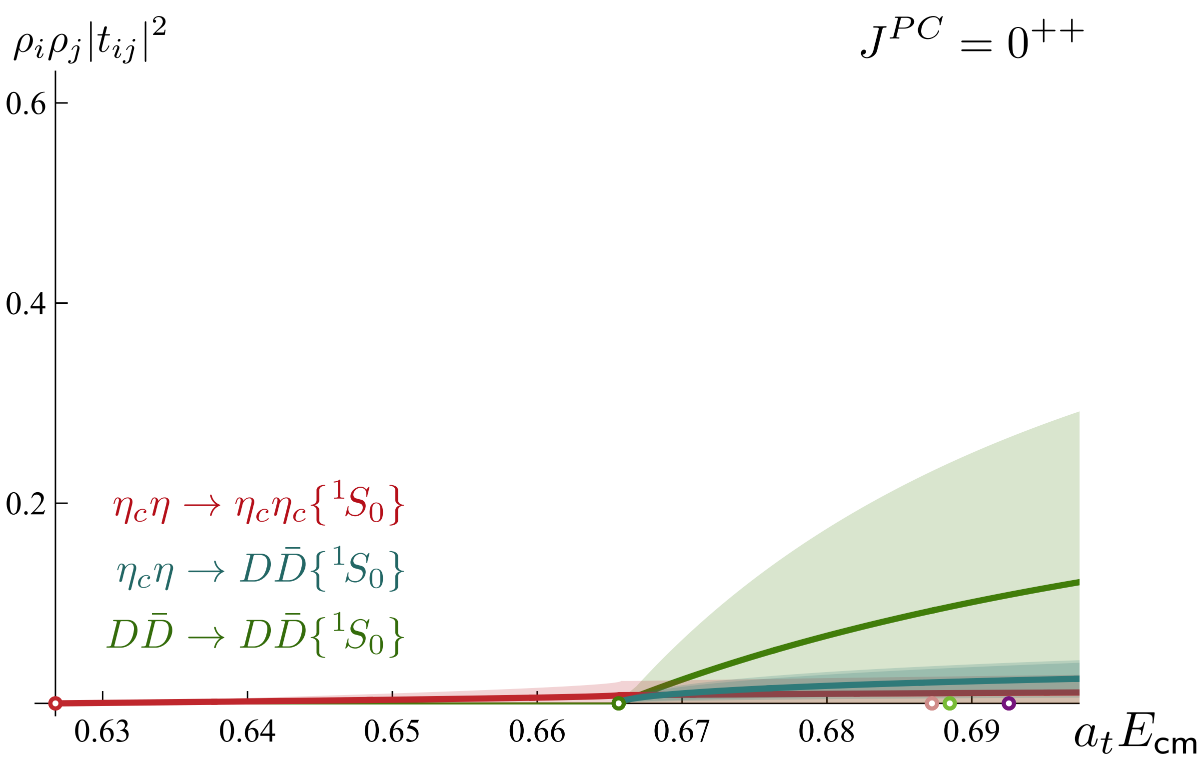
$$\boldsymbol{S} = \boldsymbol{1} + 2i\boldsymbol{\rho}^{\frac{1}{2}} \cdot \boldsymbol{t} \cdot \boldsymbol{\rho}^{\frac{1}{2}}$$

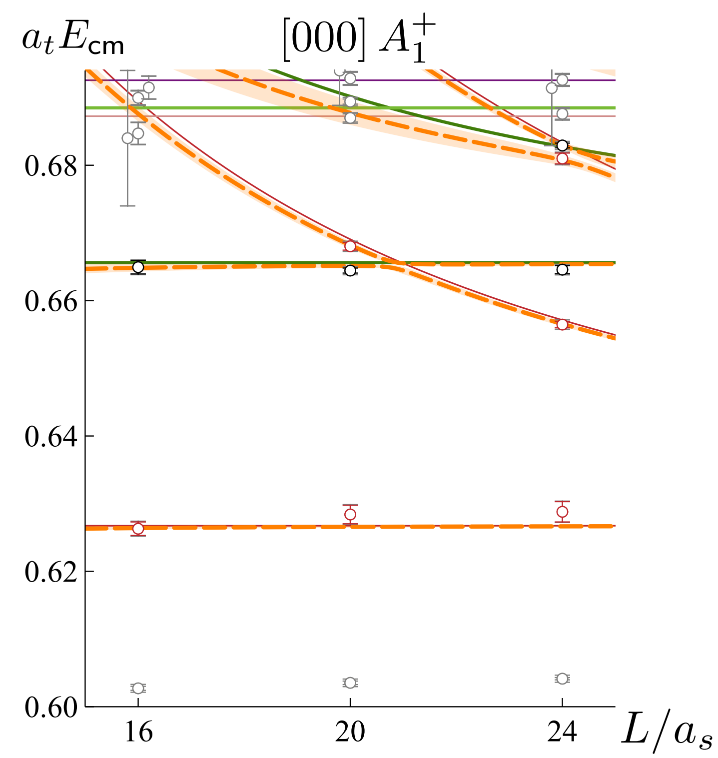
$$\boldsymbol{t}^{-1} = \boldsymbol{K}^{-1} + \boldsymbol{I}$$

$$\mathrm{Im} I_{ij} = -\rho_i = 2k_i/\sqrt{s}$$

$$\det[\boldsymbol{1} + i\boldsymbol{\rho}\cdot\boldsymbol{t}\,(\boldsymbol{1} + i\boldsymbol{\mathcal{M}}(L))] = 0$$

$$\boldsymbol{K} = \begin{bmatrix} \gamma_{\eta_c\eta\to\eta_c\eta} & \gamma_{\eta_c\eta\to D\bar{D}} \\ \gamma_{\eta_c\eta\to D\bar{D}} & \gamma_{D\bar{D}\to D\bar{D}} \end{bmatrix}$$



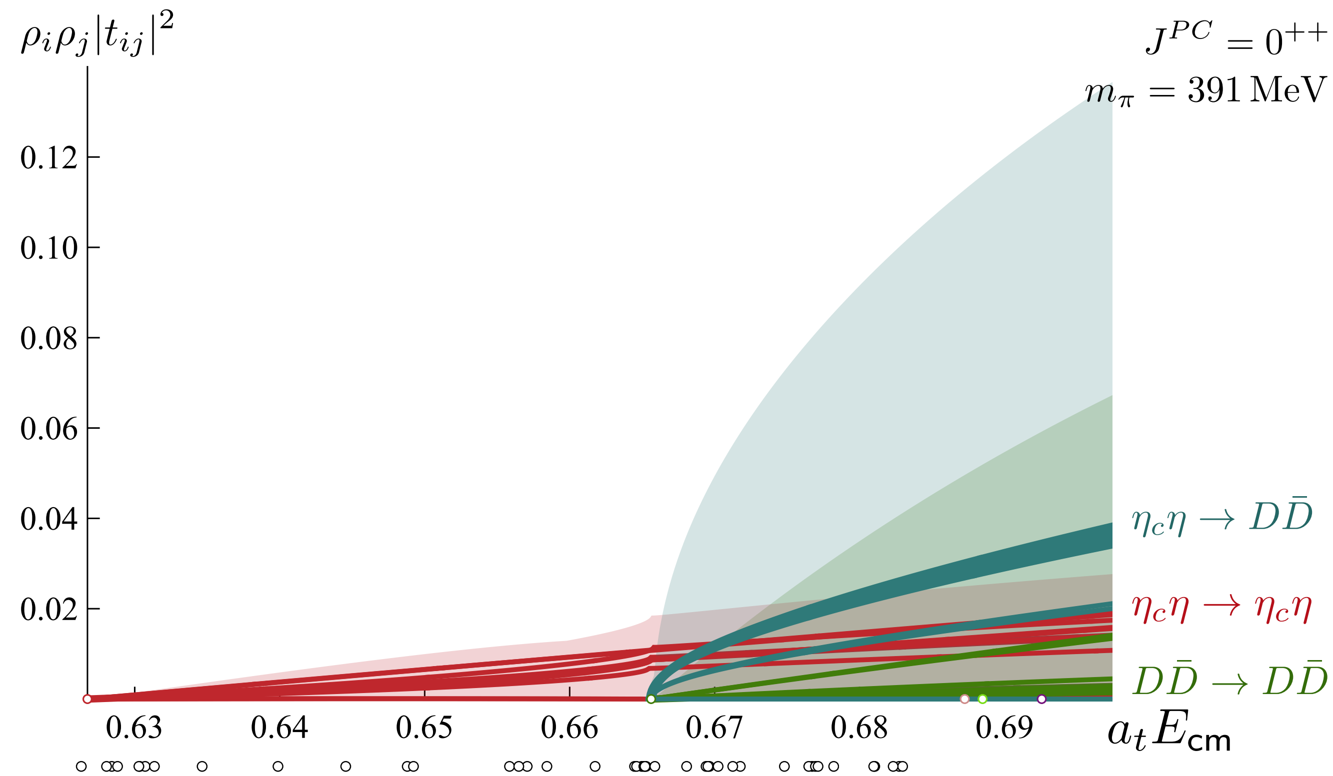


using rest-frame only

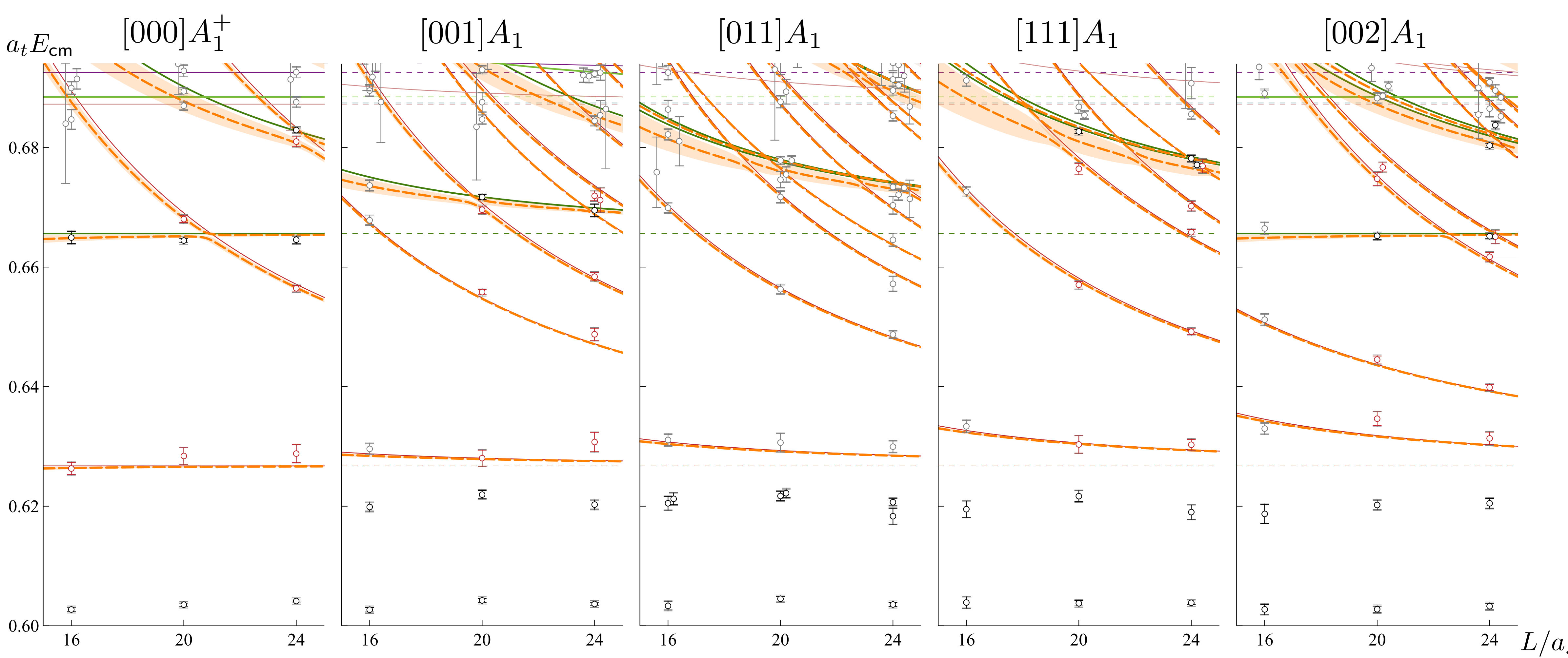
$$\begin{aligned} \gamma_{\eta_c\eta\to\eta_c\eta} &= (0.34 \pm 0.23 \pm 0.09) \\ \gamma_{\eta_c\eta\to D\bar{D}} &= (0.58 \pm 0.29 \pm 0.05) \\ \gamma_{D\bar{D}\to D\bar{D}} &= (1.39 \pm 1.19 \pm 0.24) \end{aligned} \quad \begin{bmatrix} 1.00 & 0.77 & -0.24 \\ & 1.00 & -0.22 \\ & & 1.00 \end{bmatrix}$$

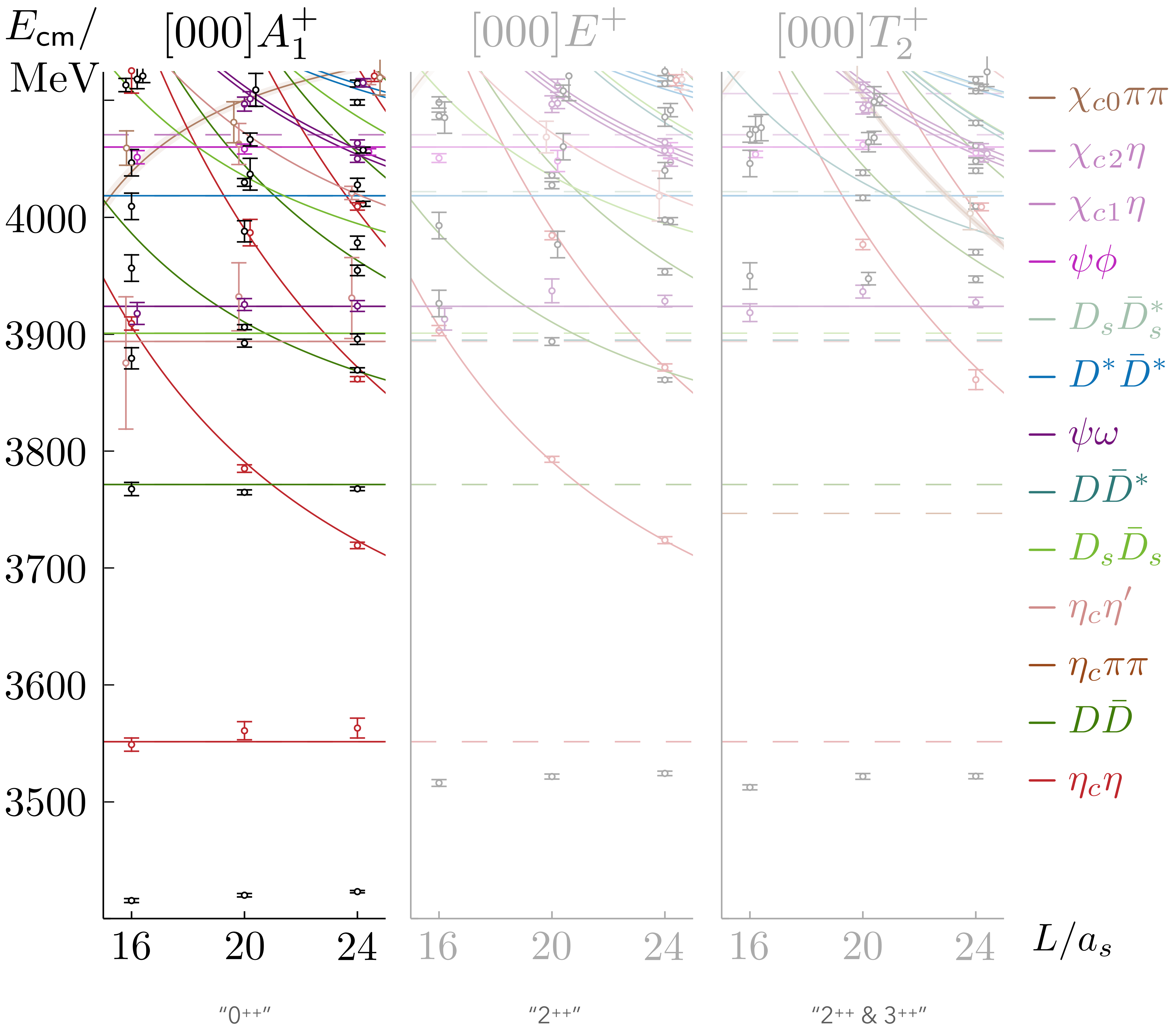
$$\chi^2/N_{\mathrm{dof}} = \frac{5.65}{10-3} = 0.81$$

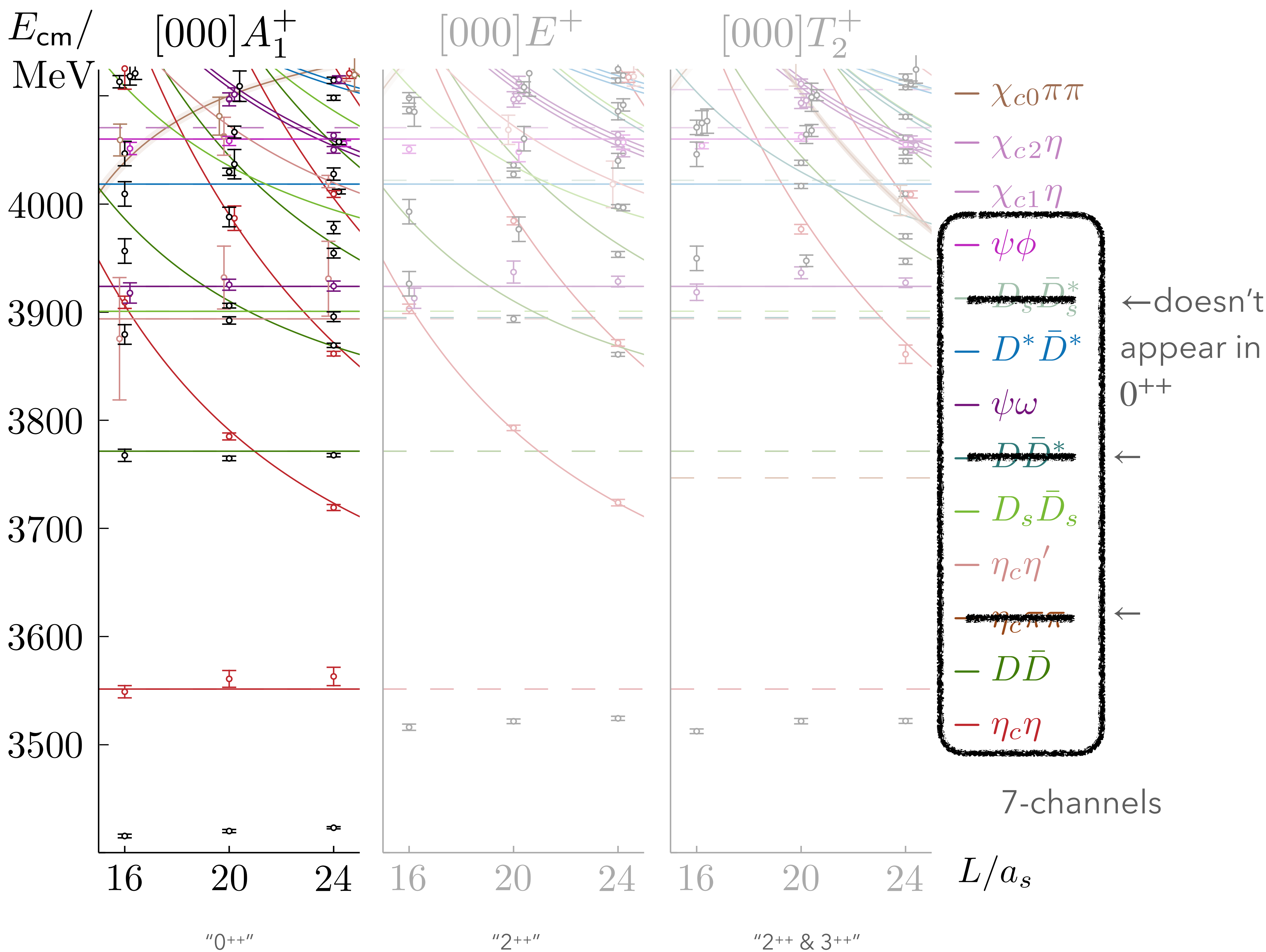
(smaller range)



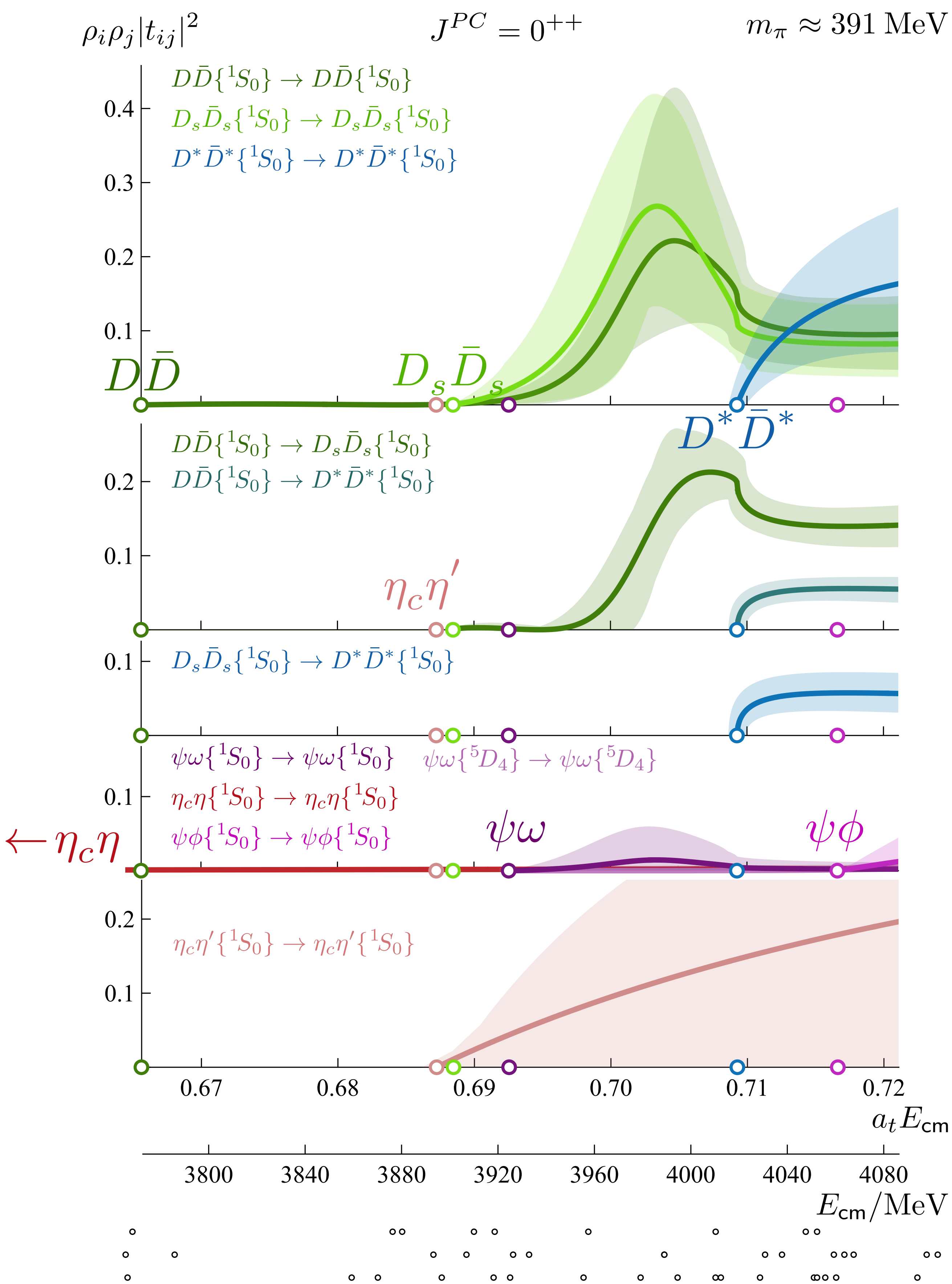
using zero and non-zero total momentum




E_{cm}/ MeV
$[000]A_1^+$
$[000]E^+$
$[000]T_2^+$
4000
3900
3800
3700
3600
3500
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
"0++"
"2++"
"2++ & 3++"



E_{cm}/ MeV
[000] A_1^+
[000] E^+
[000] T_2^+
4000
3900
3800
3700
3600
3500
16 20 24
16 20 24
16 20 24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
←doesn't appear in 0^{++}
←
←
7-channels
"0++"
"2++"
"2++ & 3++"



three channels open close together:
$\eta_c\eta'$, $D_s\bar{D}_s$, $\psi\omega$

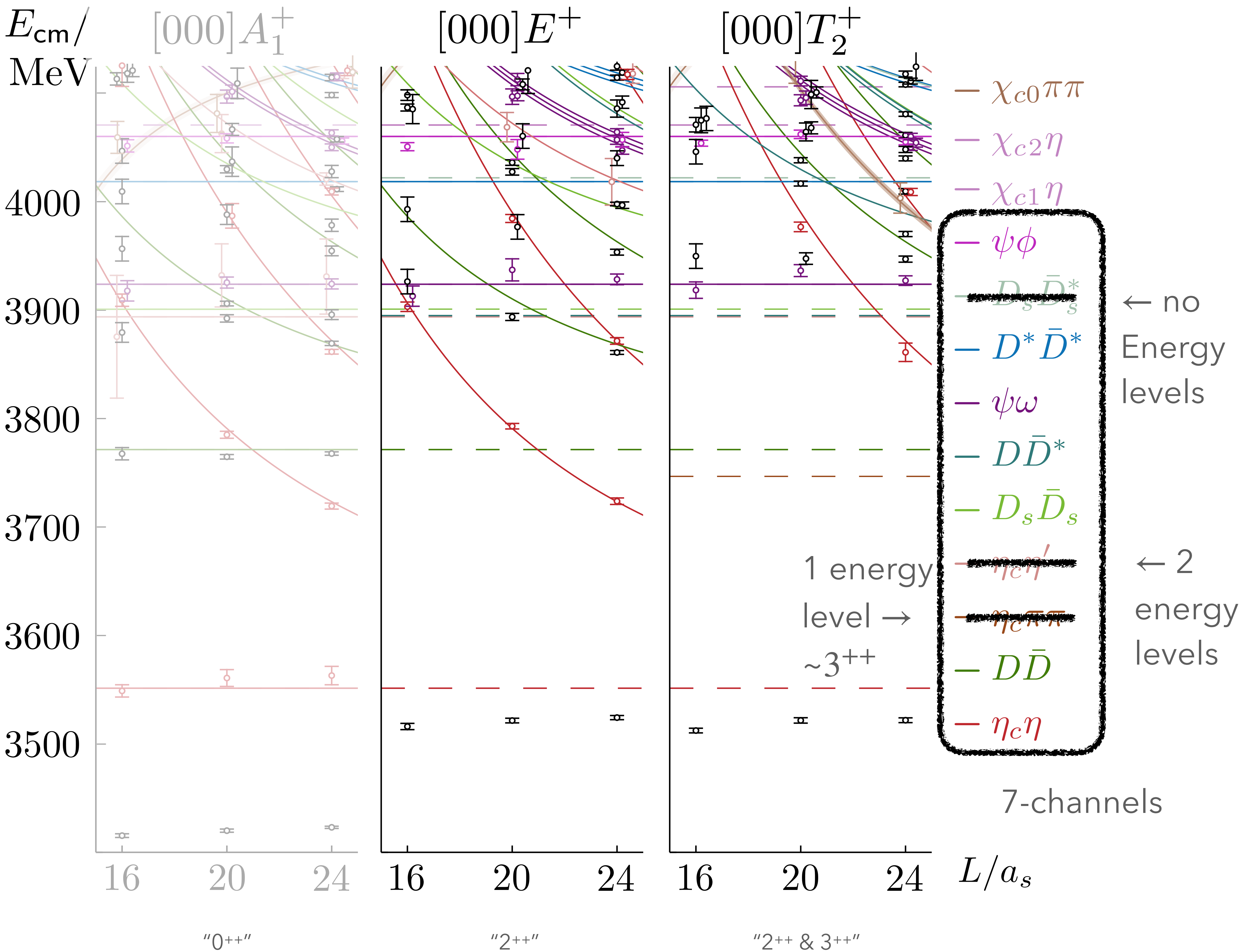
operator overlaps suggest $D^*\bar{D}^*$ is important

$\psi\phi$ has been seen to be important in some places

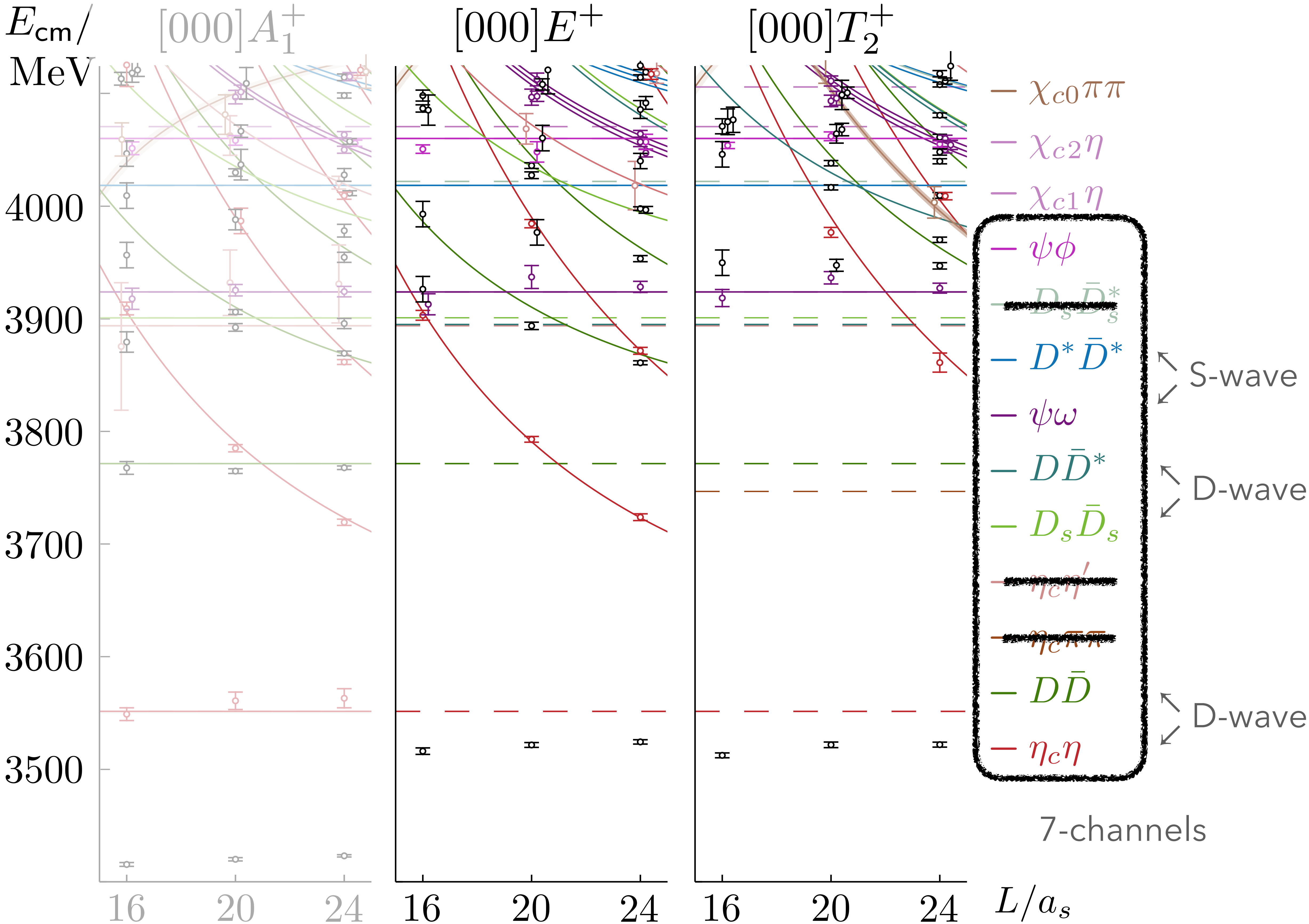
consider 7-channel system

$$K_{ij} = \frac{g_i g_j}{m^2 - s} + \gamma_{ij}$$

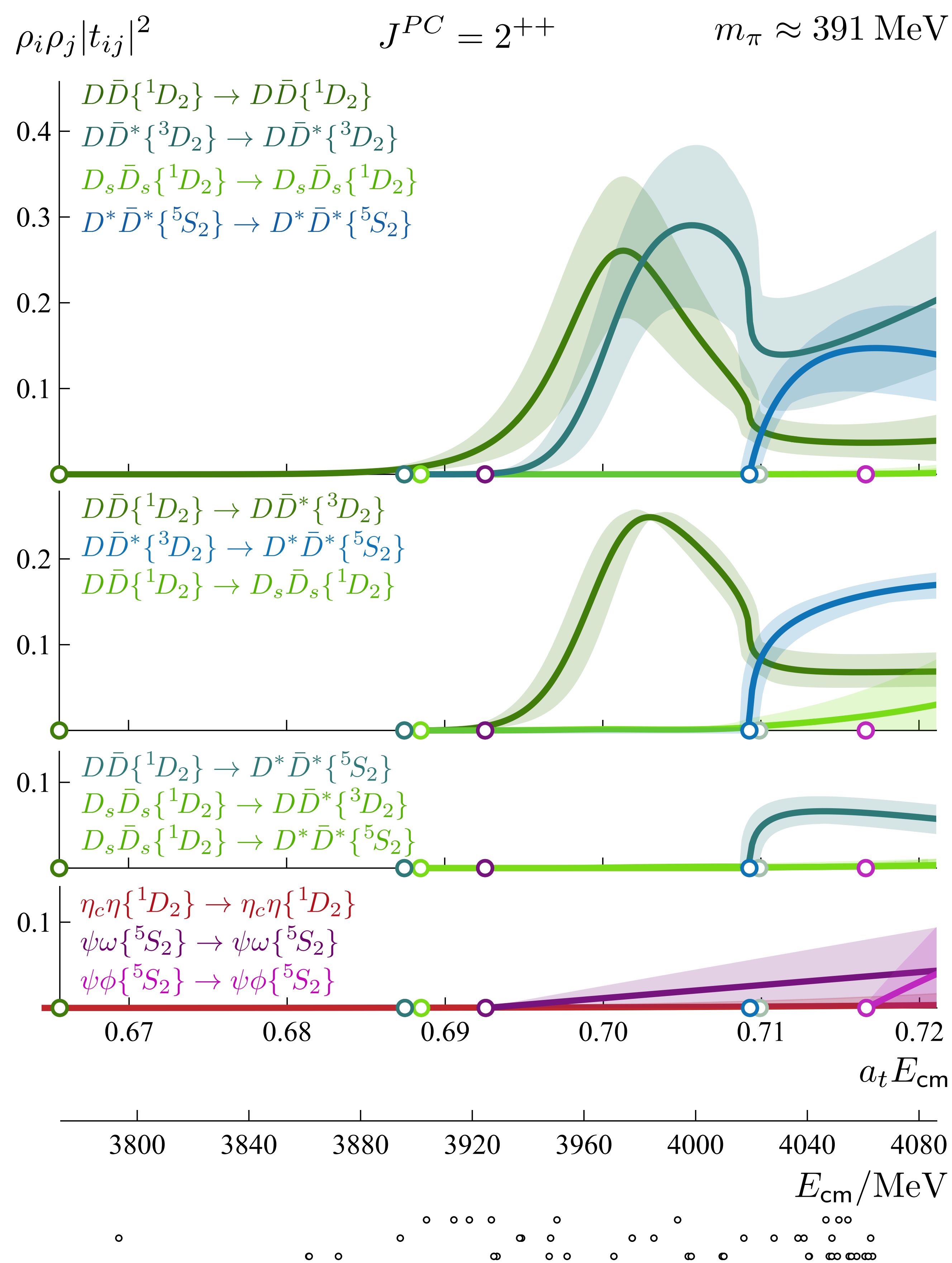
K-matrix pole terms are necessary to obtain a good quality of fit



E_{cm}/ MeV
[000]A_1^+
[000]E^+
[000]T_2^+
4000
3900
3800
3700
3600
3500
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
← no Energy levels
← 2 energy levels
1 energy level → ~3^{++}
7-channels
“0^{++}”
“2^{++}”
“2^{++} & 3^{++}”


E_{cm}/ MeV
$[000]A_1^+$
$[000]E^+$
$[000]T_2^+$
4000
3900
3800
3700
3600
3500
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
S-wave
D-wave
D-wave
7-channels

"0++" "2++" "2++ & 3++"



7-channels, mixture of *S* and *D*

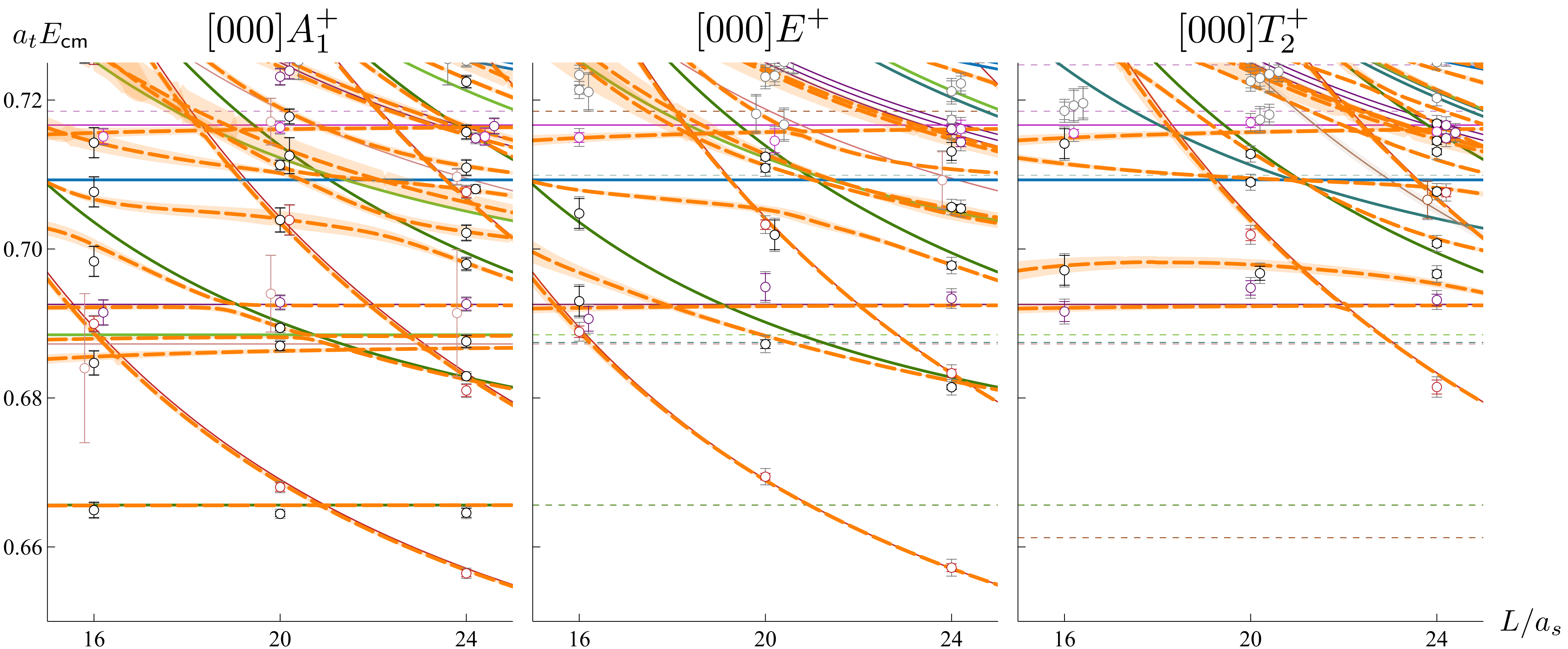
$D\bar{D}, D_s\bar{D}_s\{^1D_2\}$ $D\bar{D}^*\{^3D_2\}$ $D^*\bar{D}^*\{^5S_2\}$

$\eta_c\eta\{^1D_2\}$ $\psi\omega, \psi\phi\{^5S_2\}$

peaks at a similar energy

very small DsDs amplitudes -
some phase space suppression

DD* is large -
similar phase space to DsDs



$$\det[\mathbf{1} + i\boldsymbol{\rho}\cdot\boldsymbol{t}\,(\mathbf{1} + i\boldsymbol{\mathcal{M}}(L))] = 0$$

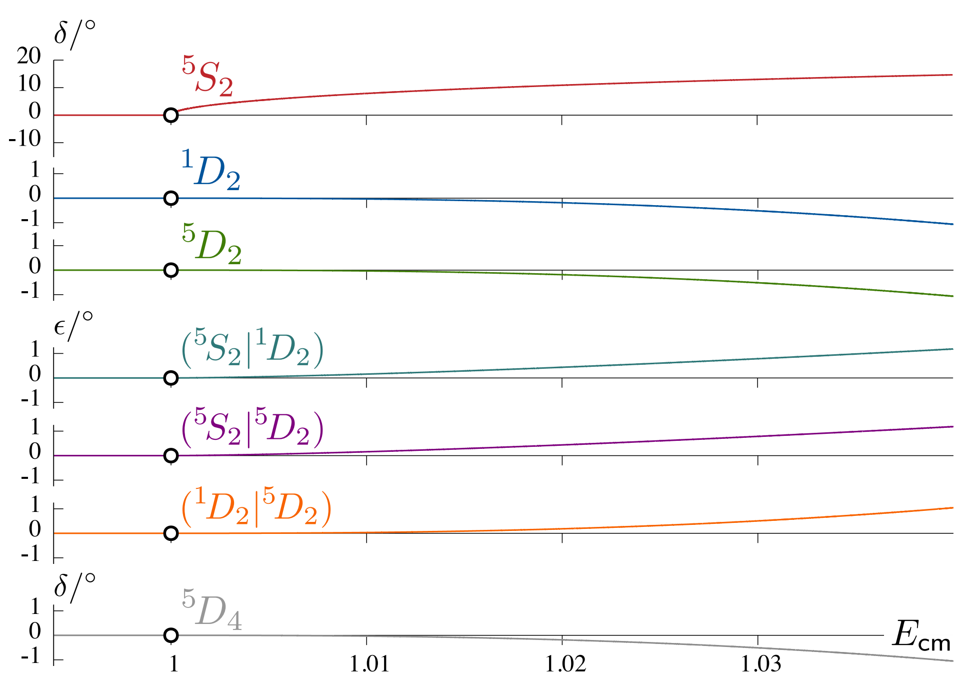
$$\det\left[\boldsymbol{D}(E_{\mathsf{cm}})\right] = 0$$

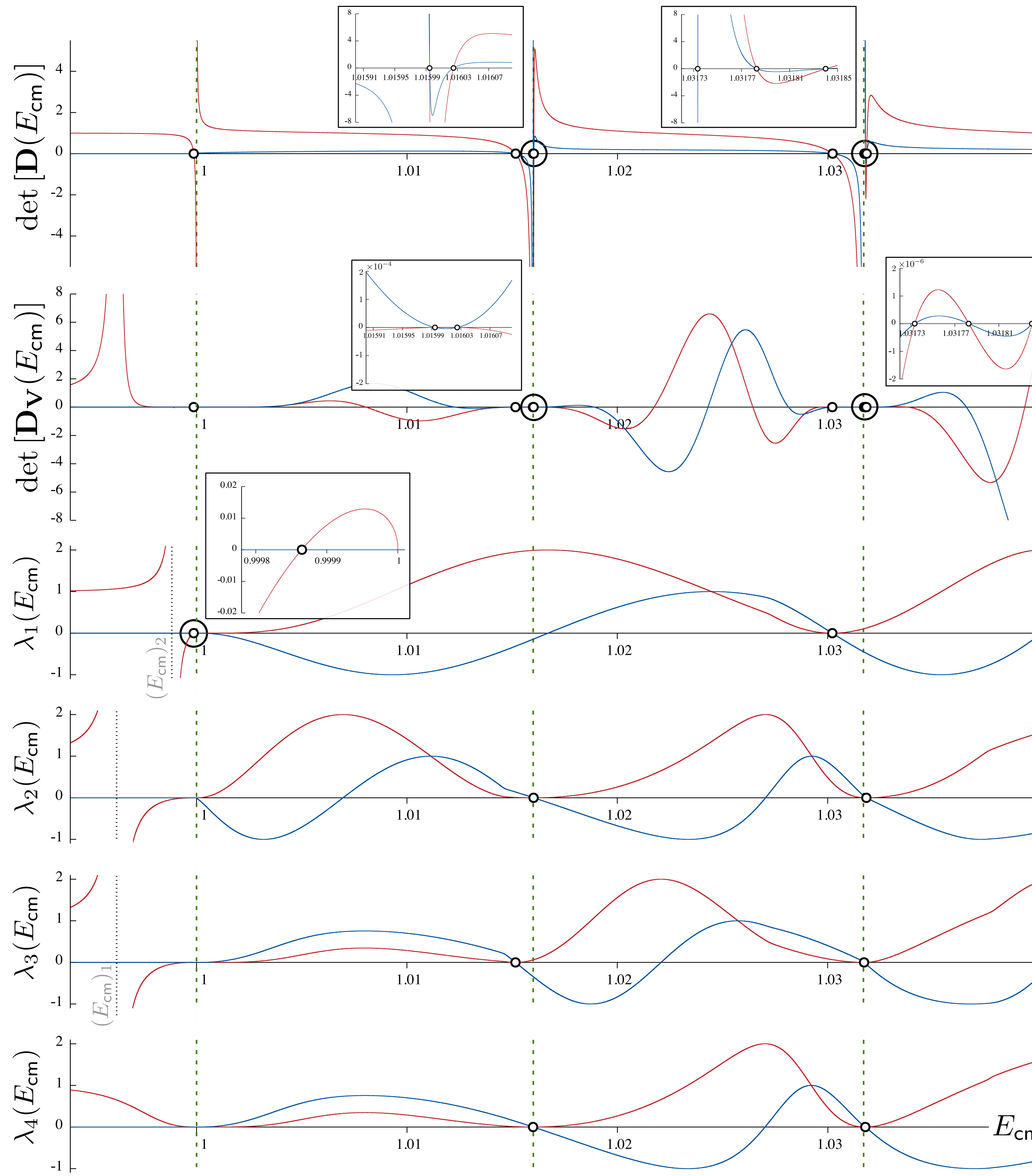
$$\boldsymbol{D}(E_{\mathsf{cm}}) = \boldsymbol{1} + i\boldsymbol{\rho}(E_{\mathsf{cm}})\cdot\boldsymbol{t}(E_{\mathsf{cm}})\cdot\big(\boldsymbol{1}+i\boldsymbol{\mathcal{M}}(E_{\mathsf{cm}},L)\big)$$

$$\boldsymbol{D}_V(E_{\mathsf{cm}}) = \boldsymbol{1} + \boldsymbol{S}(E_{\mathsf{cm}})\cdot\boldsymbol{V}(E_{\mathsf{cm}},L)$$

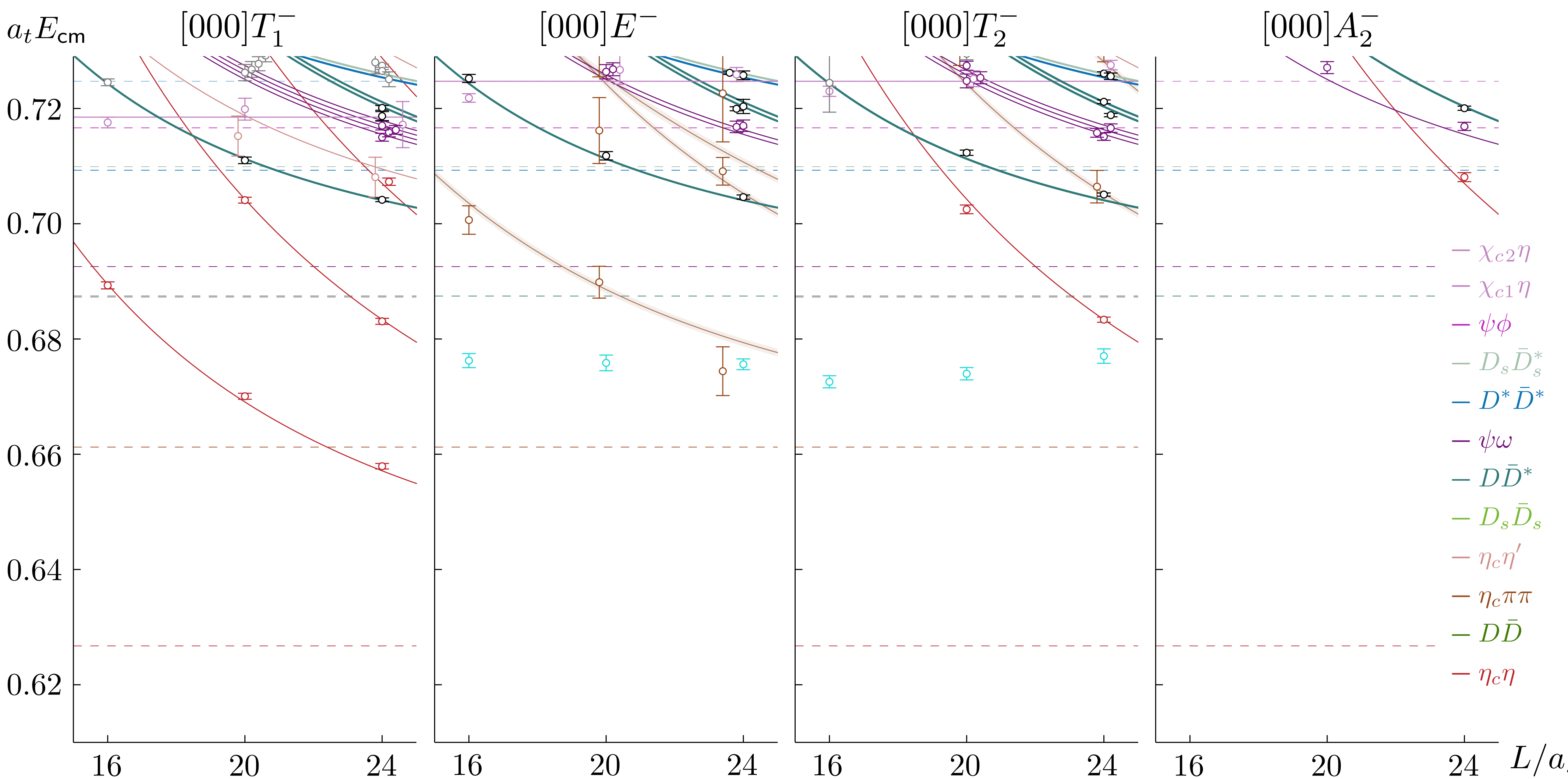
$$\det\left[\boldsymbol{D}_V(E_{\mathsf{cm}})\right] = \prod_{p=1}^{n}\lambda_p(E_{\mathsf{cm}})$$

$$\boldsymbol{D}_V(E_{\mathsf{cm}})\,\boldsymbol{v}^{(p)}(E_{\mathsf{cm}}) = \lambda_p(E_{\mathsf{cm}})\,\boldsymbol{v}^{(p)}(E_{\mathsf{cm}})$$





Woss, DW, Dudek, arXiv:2001.08474, PRD 101, 114505 (2020)

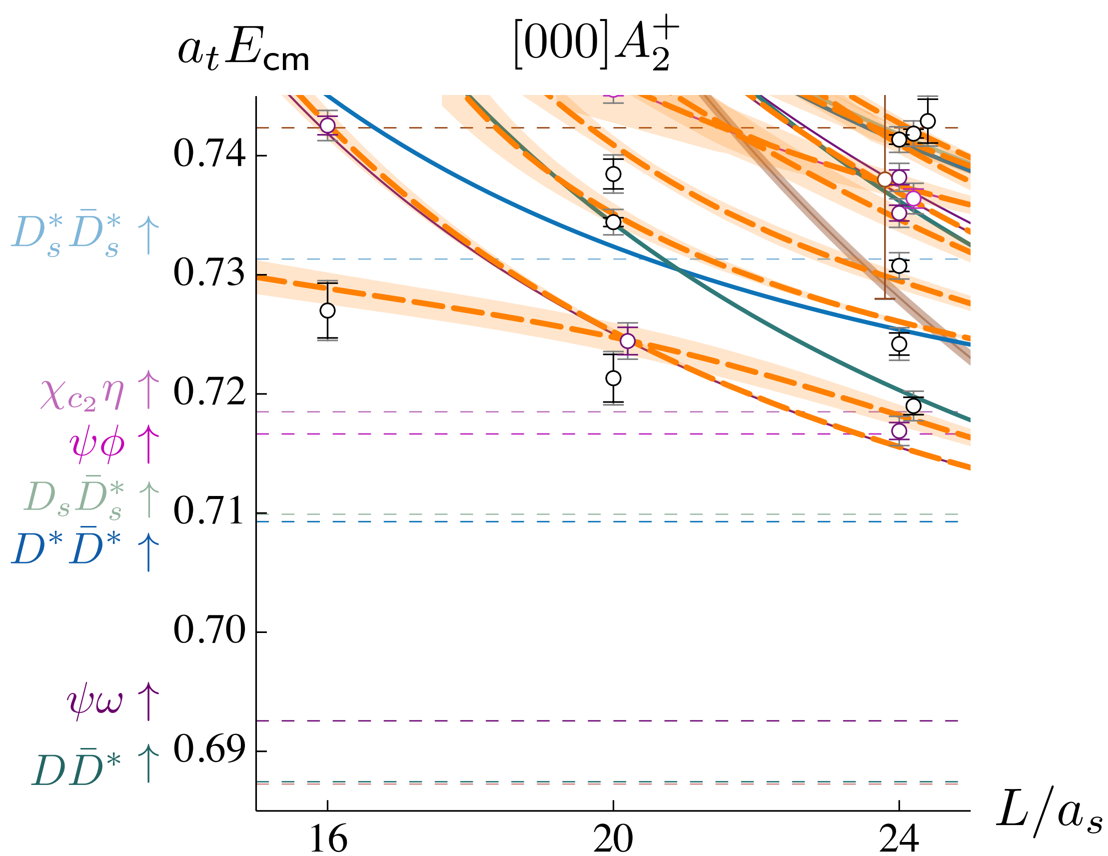


(we also computed lattice irreps with non-zero total momentum)

P=- partial waves can then contribute

very little going on here

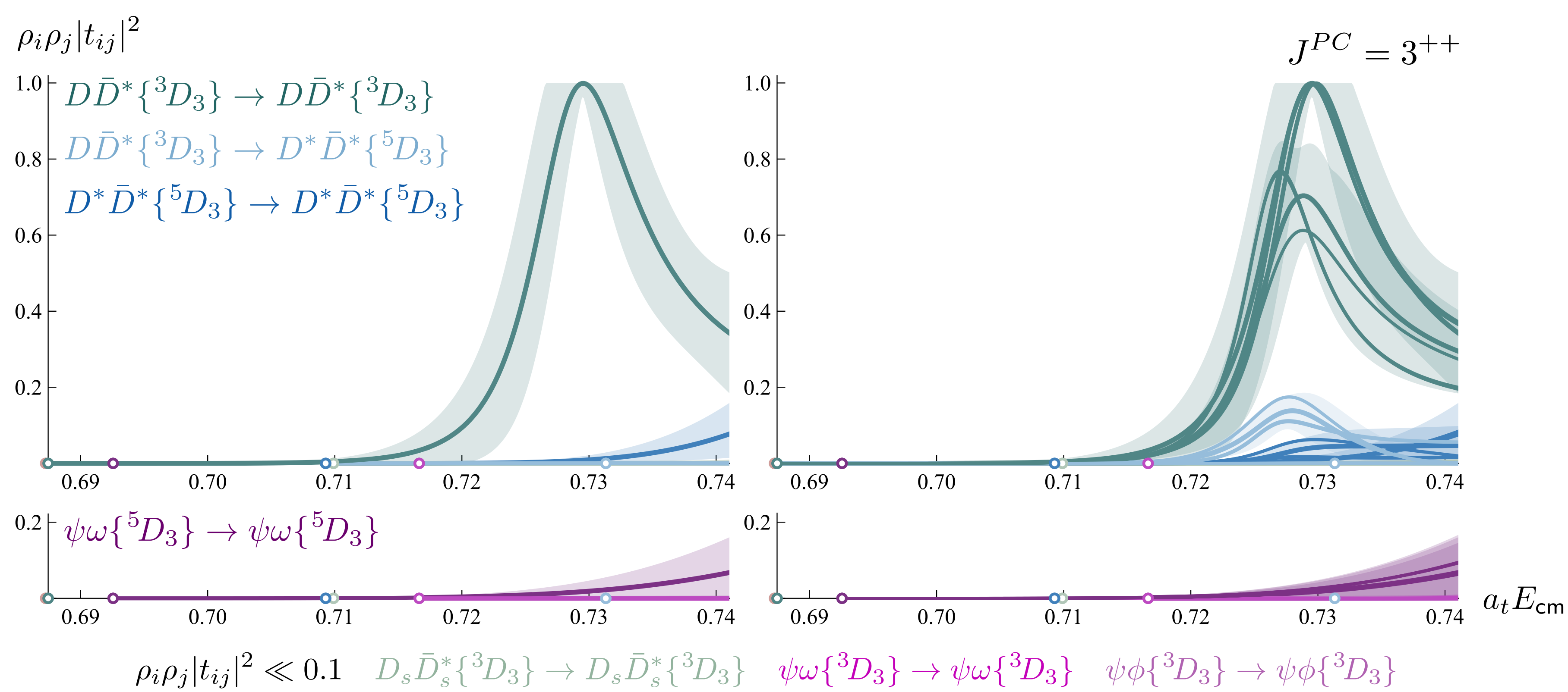
an η_{c2} 2^{-+} state arises below DD*



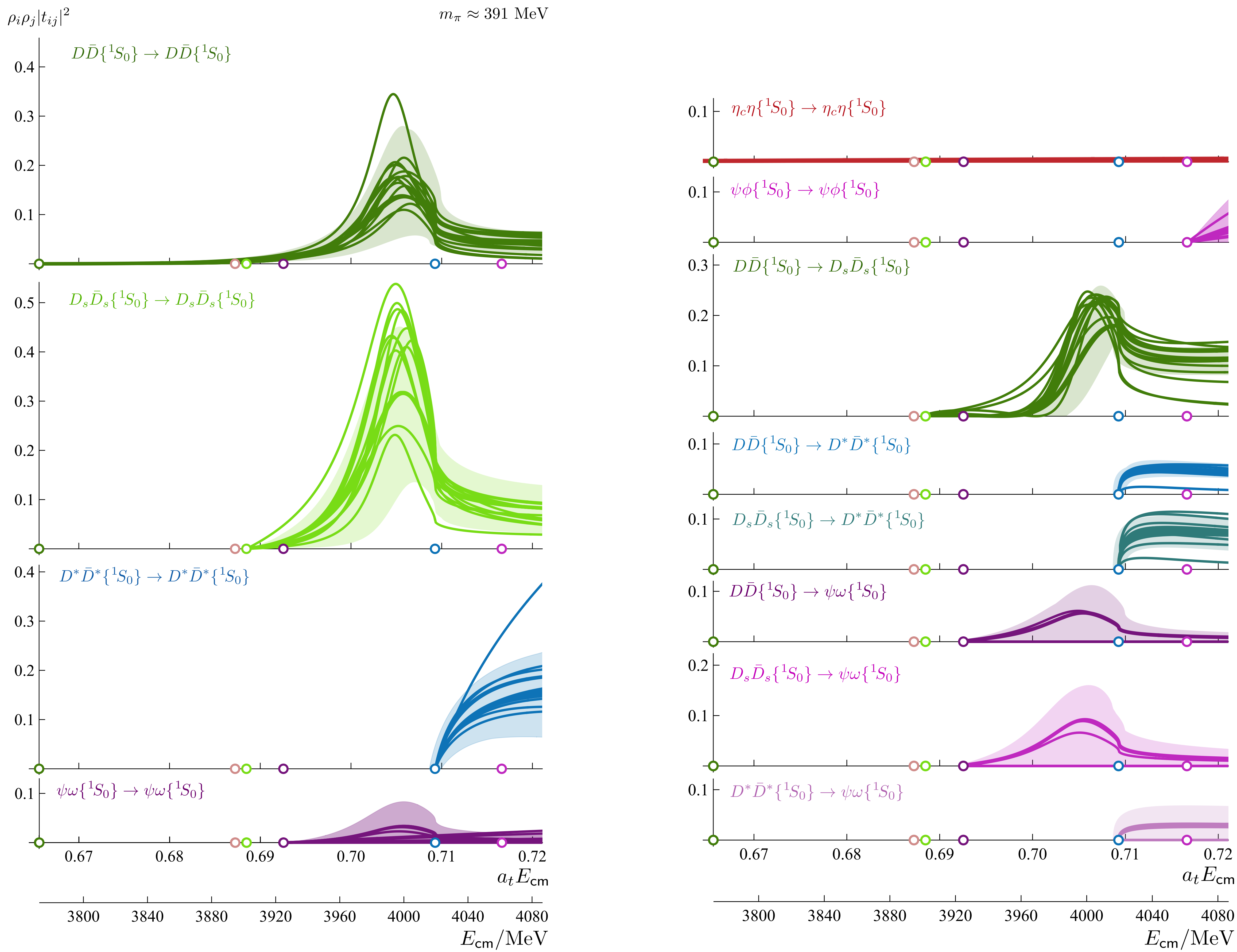
extra level and resonance higher up

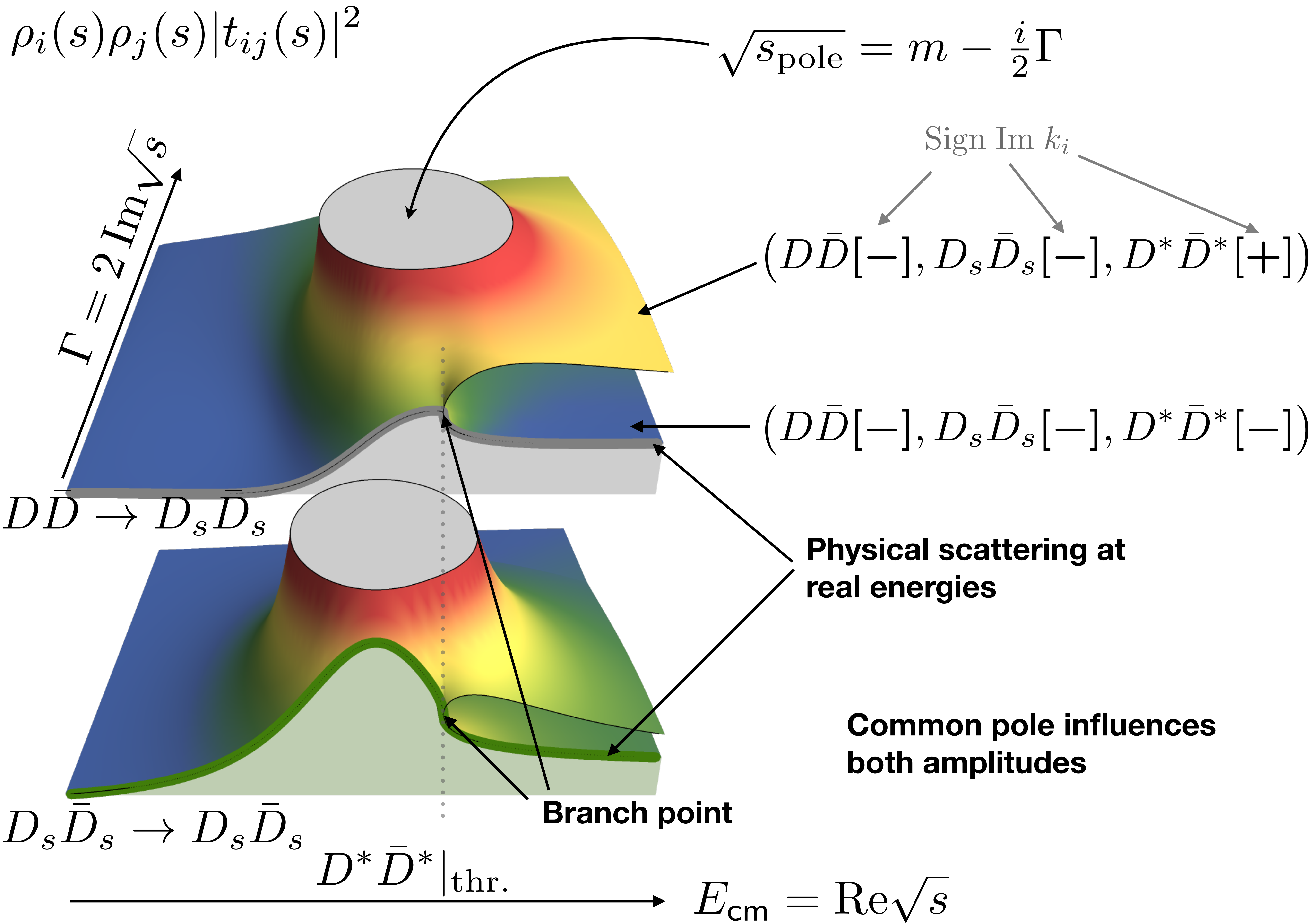
two classes of amplitudes were found:

- zero D*D* coupling
- finite D*D* coupling
- all had significant DD* coupling
- amps very small below 4050 MeV ($a_t E_{cm}$=0.715)



amplitude variations - scalar



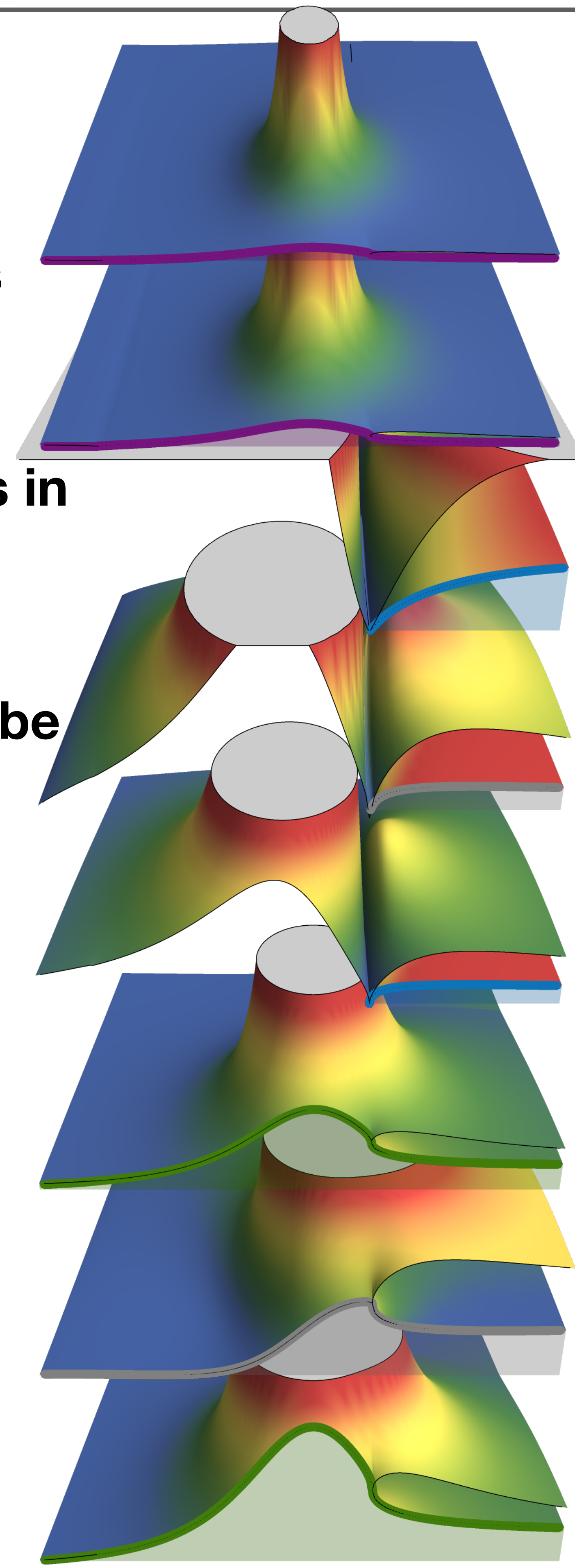

$\rho_i(s)\rho_j(s)|t_{ij}(s)|^2$
$\sqrt{s_{\mathrm{pole}}} = m - \frac{i}{2}\Gamma$
$\Gamma = 2\,\mathrm{Im}\sqrt{s}$
Sign Im k_i
$(D\bar{D}[-], D_s\bar{D}_s[-], D^*\bar{D}^*[+])$
$(D\bar{D}[-], D_s\bar{D}_s[-], D^*\bar{D}^*[-])$
$D\bar{D} \to D_s\bar{D}_s$
Physical scattering at real energies
Common pole influences both amplitudes
$D_s\bar{D}_s \to D_s\bar{D}_s$
Branch point
$D^*\bar{D}^*|_{\mathrm{thr.}}$
$E_{\mathrm{cm}} = \mathrm{Re}\sqrt{s}$

$$\rho_i(s)\rho_j(s)|t_{ij}(s)|^2$$

one resonance pole
— many different amplitudes

We don't need different poles in different coupled amplitudes

A single resonance pole can be responsible for many bumps and features



$J/\psi\omega \to J/\psi\omega$

$D\bar{D} \to J/\psi\omega$

$D^*\bar{D}^* \to D^*\bar{D}^*$

$D_s\bar{D}_s \to D^*\bar{D}^*$

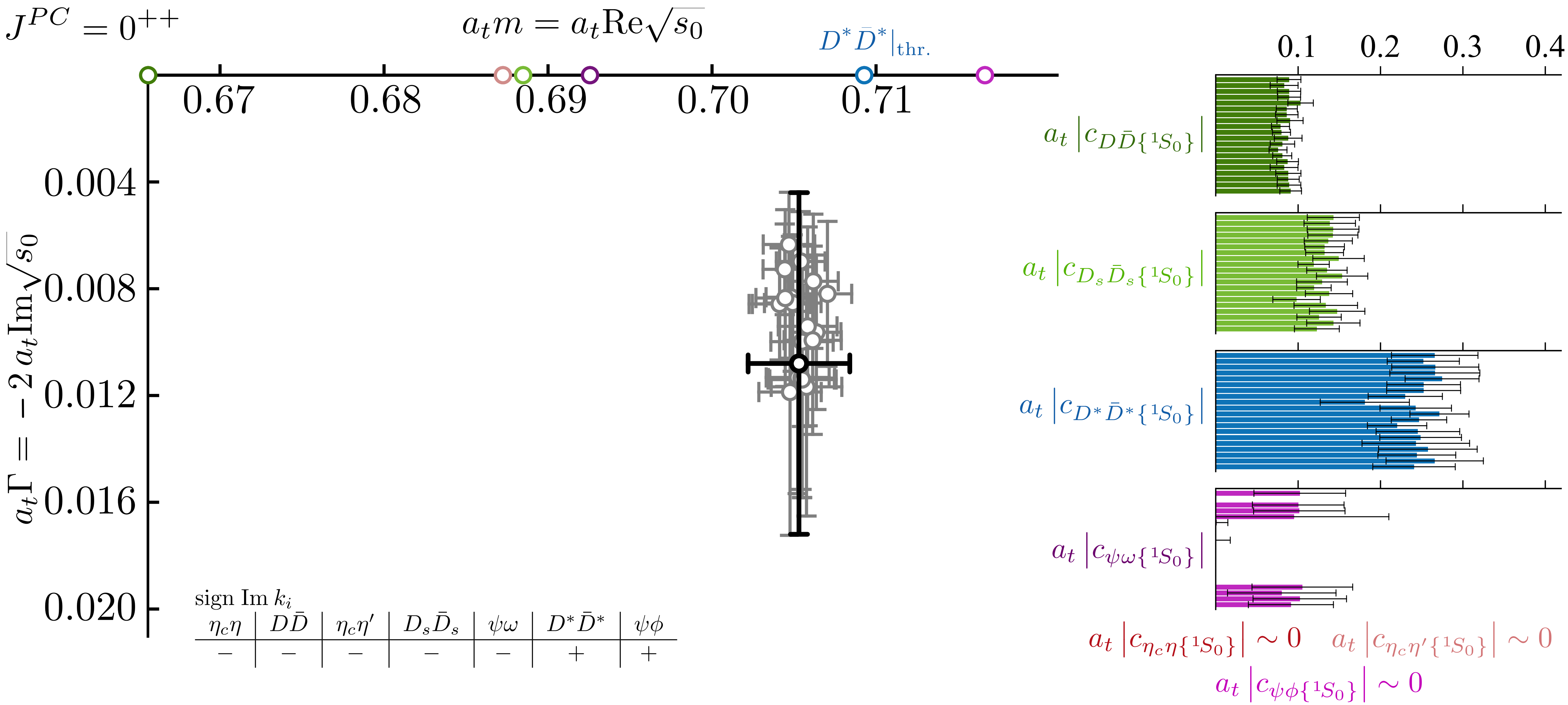
$D\bar{D} \to D^*\bar{D}^*$

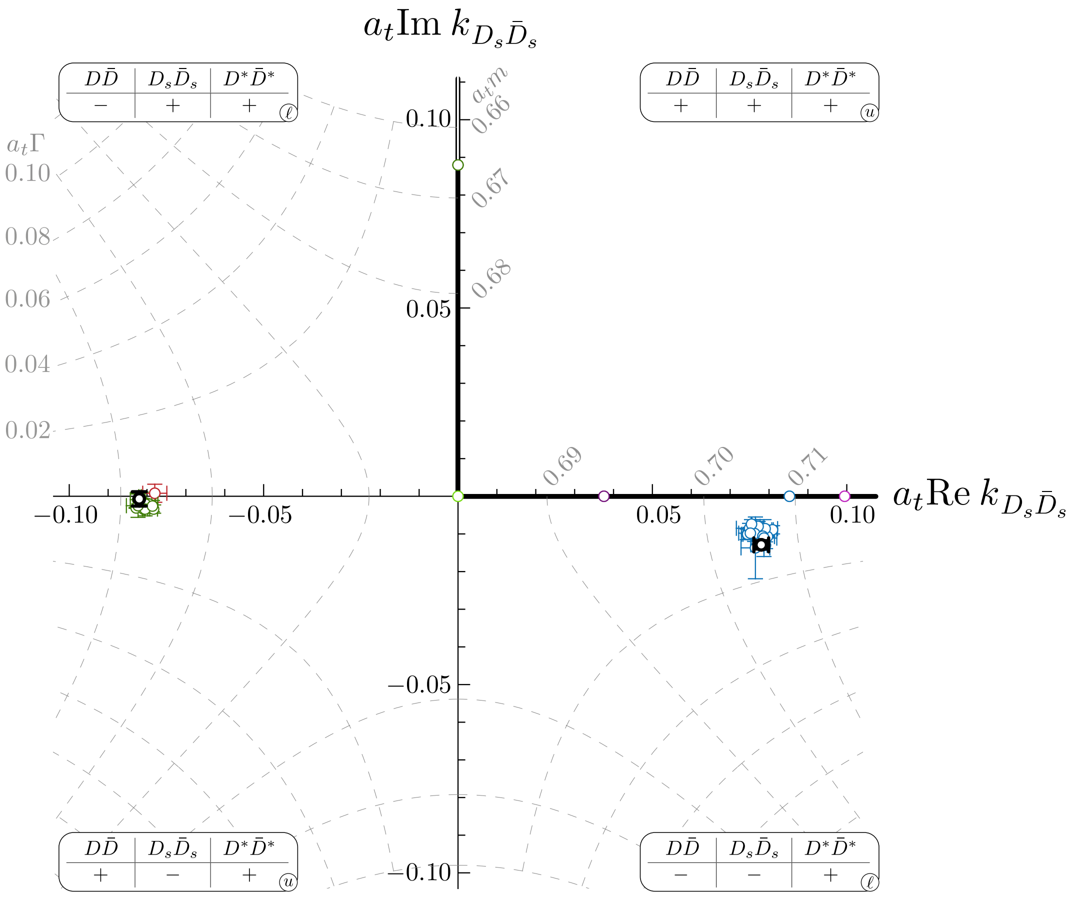
$D\bar{D} \to D\bar{D}$

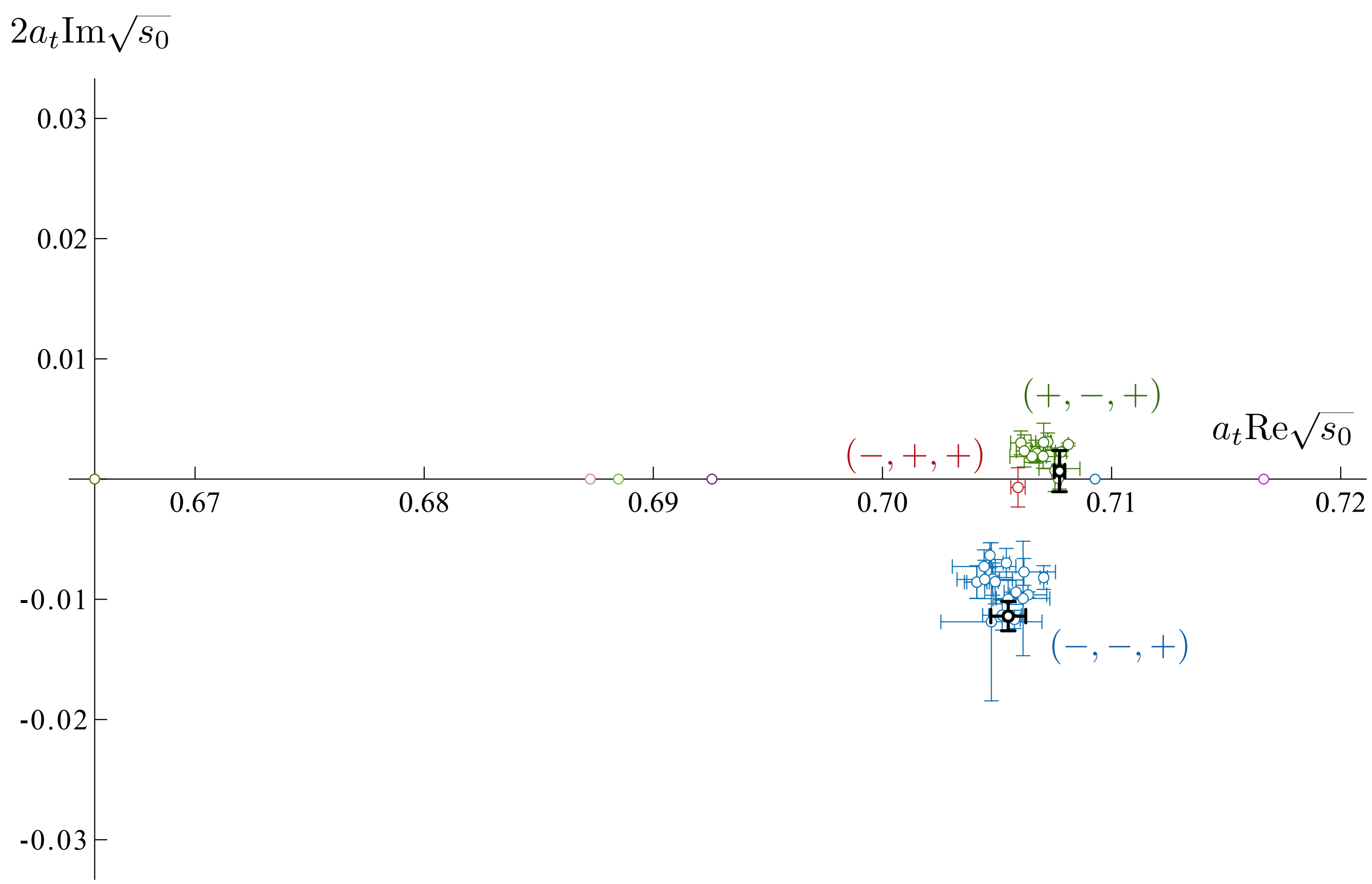
$D\bar{D} \to D_s\bar{D}_s$

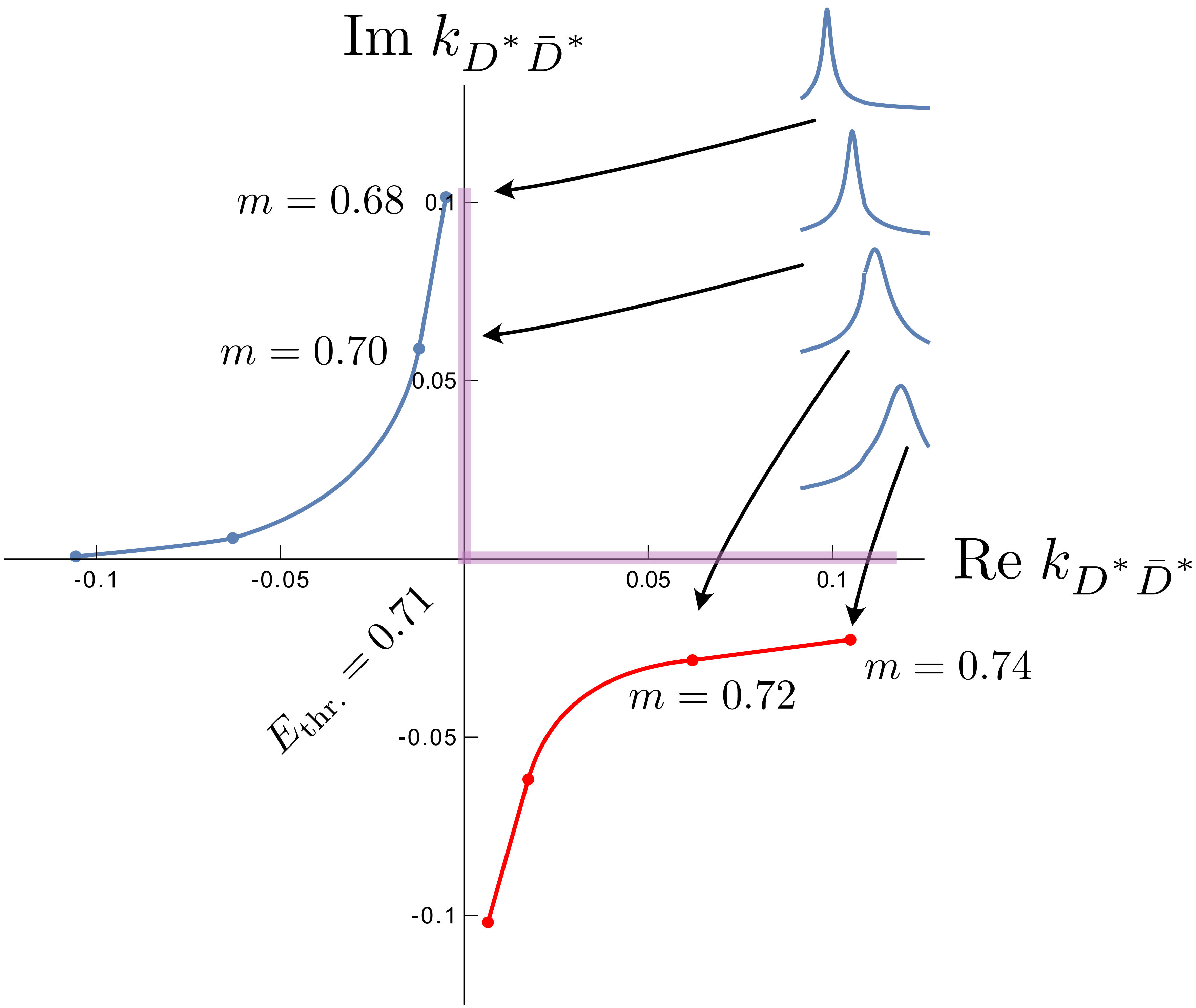
$D_s\bar{D}_s \to D_s\bar{D}_s$

Similar story for 2^{++}

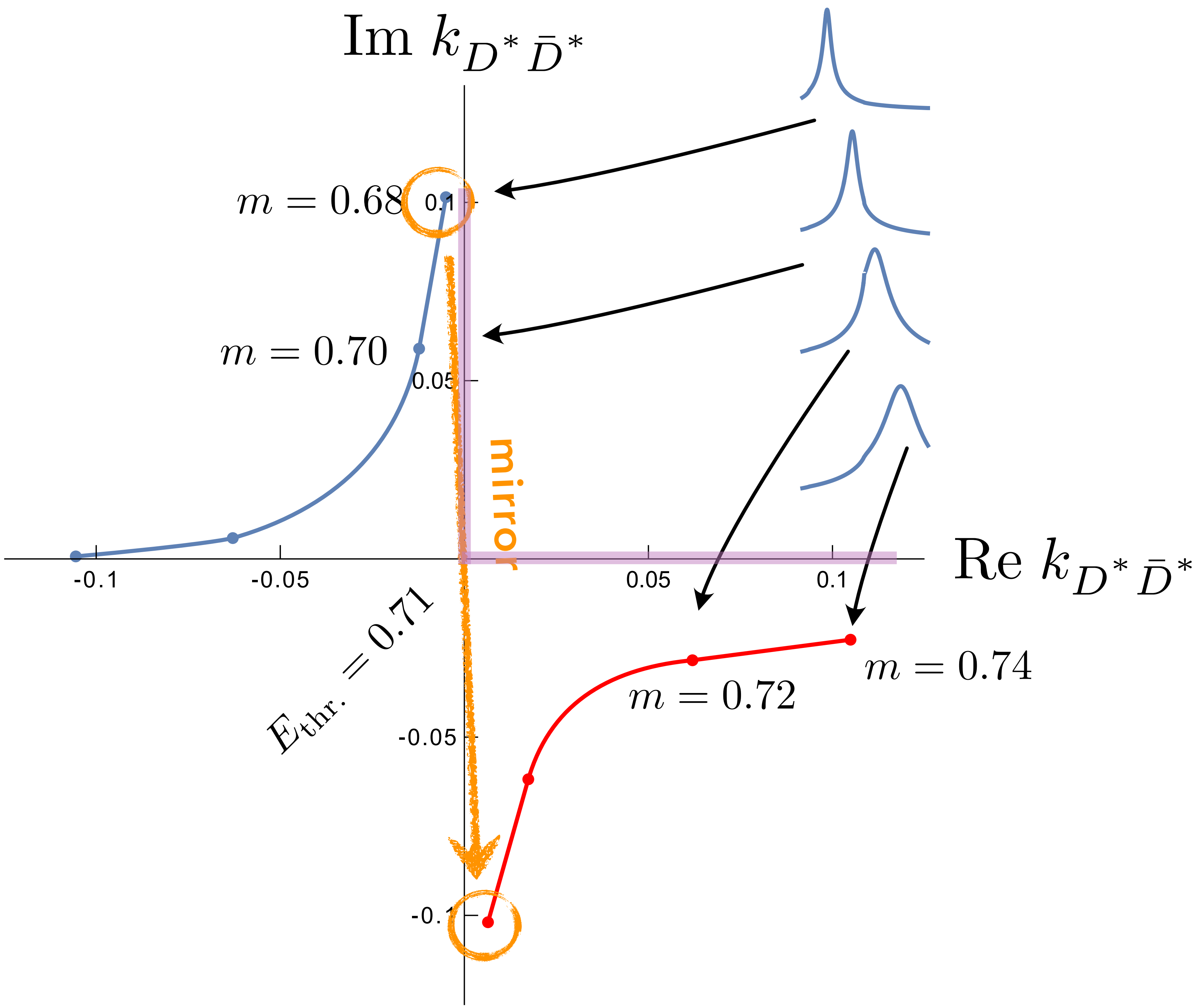

$J^{PC} = 0^{++}$
$a_t m = a_t \mathrm{Re}\sqrt{s_0}$
$D^* \bar{D}^*|_{\mathrm{thr.}}$
0.67
0.68
0.69
0.70
0.71
$a_t\Gamma = -2\, a_t \mathrm{Im}\sqrt{s_0}$
0.004
0.008
0.012
0.016
0.020
sign Im k_i
$\eta_c\eta$ | $D\bar{D}$ | $\eta_c\eta'$ | $D_s\bar{D}_s$ | $\psi\omega$ | $D^*\bar{D}^*$ | $\psi\phi$
$-$ | $-$ | $-$ | $-$ | $-$ | $+$ | $+$
0.1
0.2
0.3
0.4
$a_t \left|c_{D\bar{D}\{^1S_0\}}\right|$
$a_t \left|c_{D_s\bar{D}_s\{^1S_0\}}\right|$
$a_t \left|c_{D^*\bar{D}^*\{^1S_0\}}\right|$
$a_t \left|c_{\psi\omega\{^1S_0\}}\right|$
$a_t \left|c_{\eta_c\eta\{^1S_0\}}\right| \sim 0$
$a_t \left|c_{\eta_c\eta'\{^1S_0\}}\right| \sim 0$
$a_t \left|c_{\psi\phi\{^1S_0\}}\right| \sim 0$


$a_t \mathrm{Im}\, k_{D_s\bar{D}_s}$
$a_t \mathrm{Re}\, k_{D_s\bar{D}_s}$
$a_t\Gamma$
$a_t m$
$D\bar{D}$ | $D_s\bar{D}_s$ | $D^*\bar{D}^*$
−0.10
−0.05
0.05
0.10

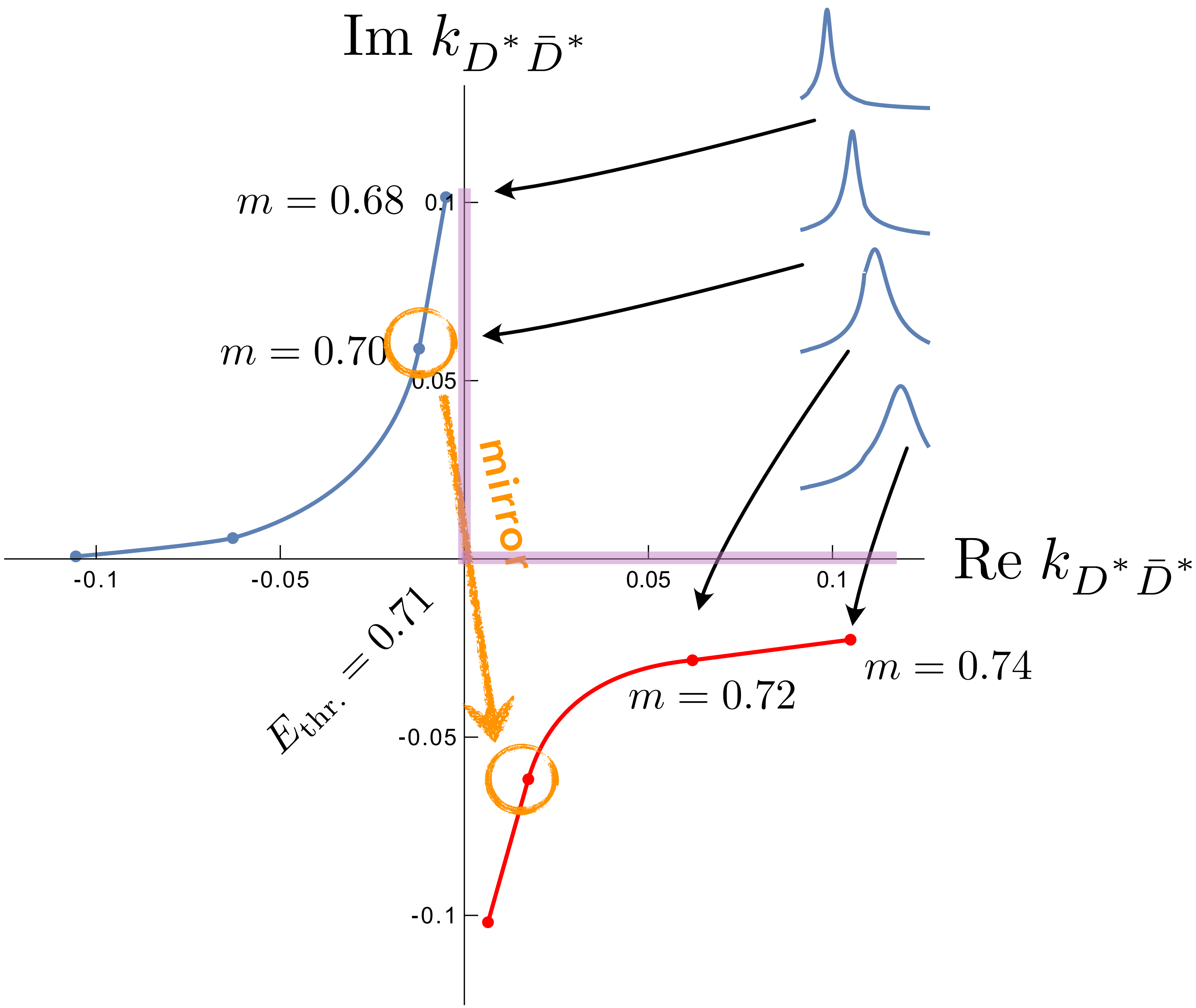

$2a_t \mathrm{Im}\sqrt{s_0}$
$a_t \mathrm{Re}\sqrt{s_0}$
$(+,-,+)$
$(-,+,+)$
$(-,-,+)$
0.67
0.68
0.69
0.70
0.71
0.72



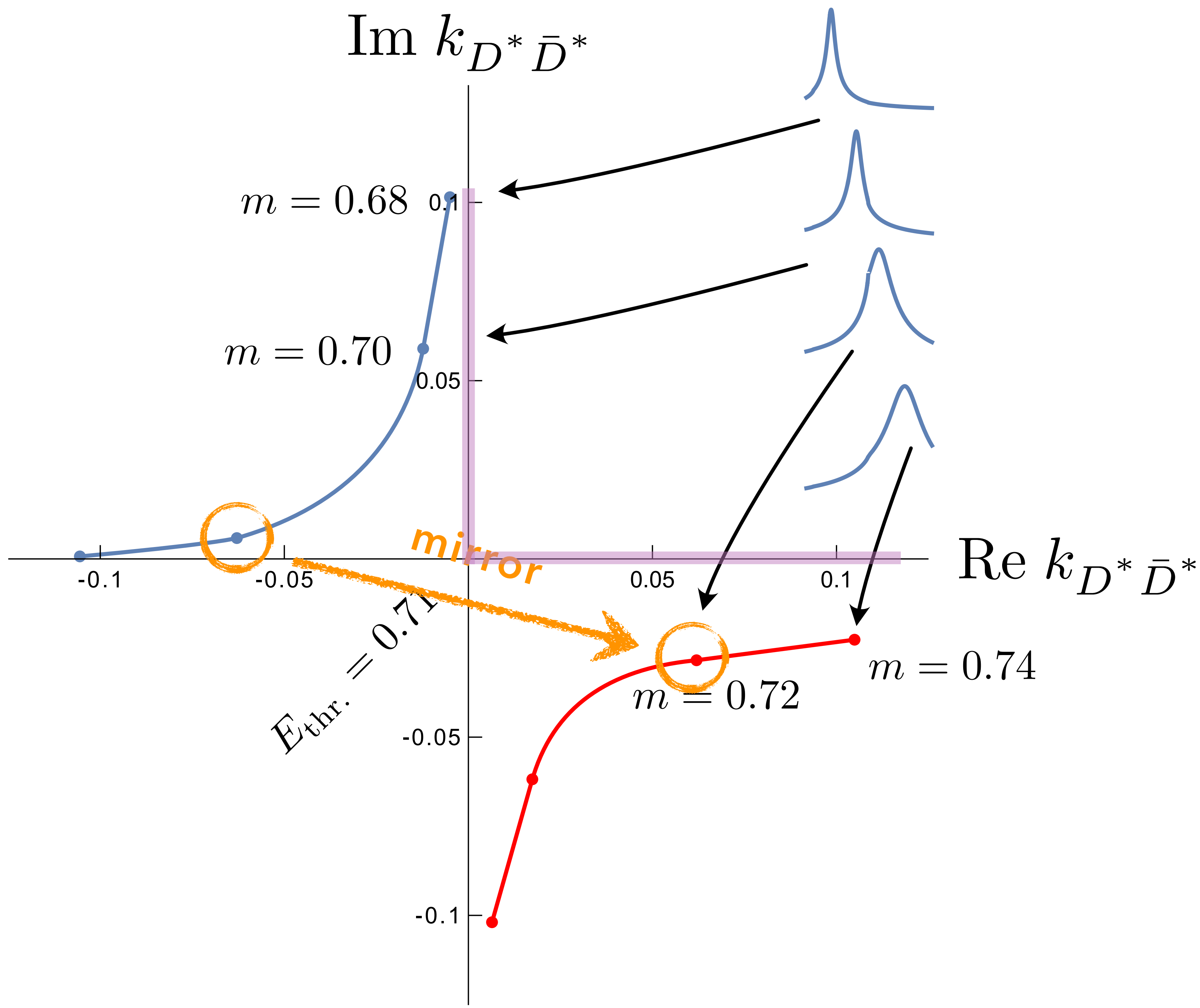
$$t_{ij} = \frac{g_i g_j}{m_0^2 - s - i g_{D\bar{D}}^2 \rho_{D\bar{D}} - i g_{D^*\bar{D}^*}^2 \rho_{D^*\bar{D}^*}}$$



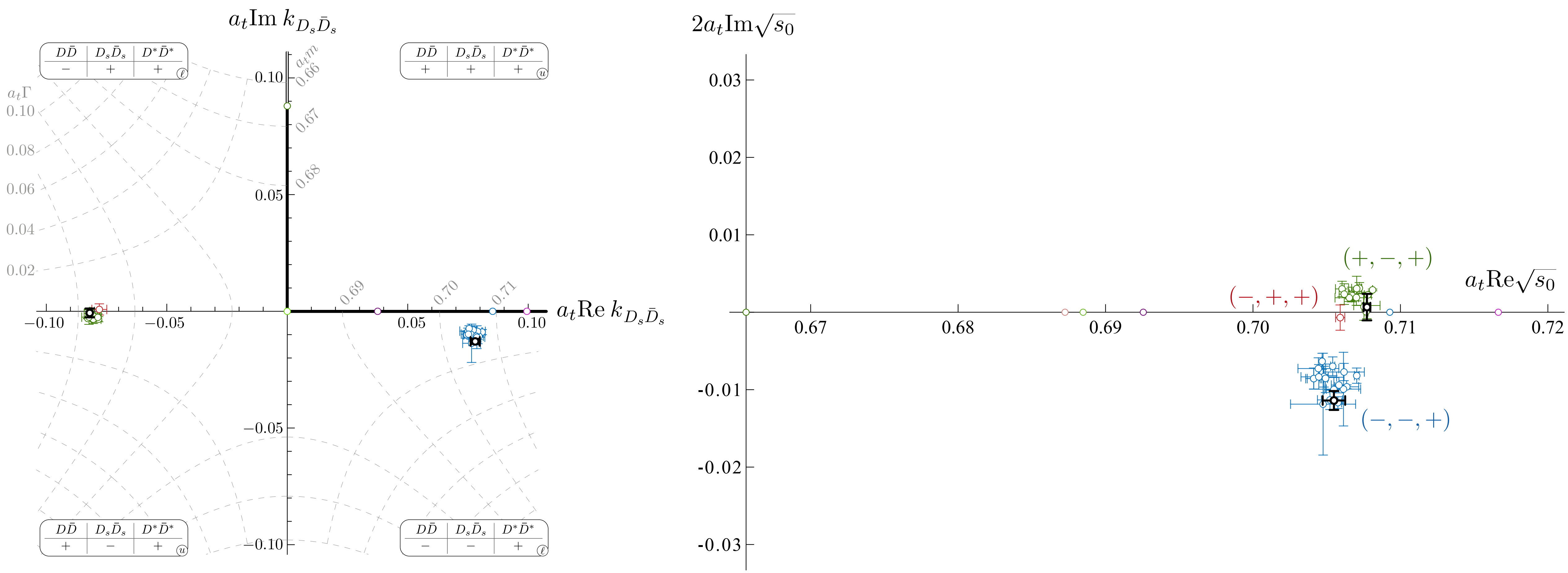
$$t_{ij} = \frac{g_i g_j}{m_0^2 - s - i g_{D\bar{D}}^2 \rho_{D\bar{D}} - i g_{D^*\bar{D}^*}^2 \rho_{D^*\bar{D}^*}}$$



$$t_{ij} = \frac{g_i g_j}{m_0^2 - s - i g_{D\bar{D}}^2 \rho_{D\bar{D}} - i g_{D^*\bar{D}^*}^2 \rho_{D^*\bar{D}^*}}$$

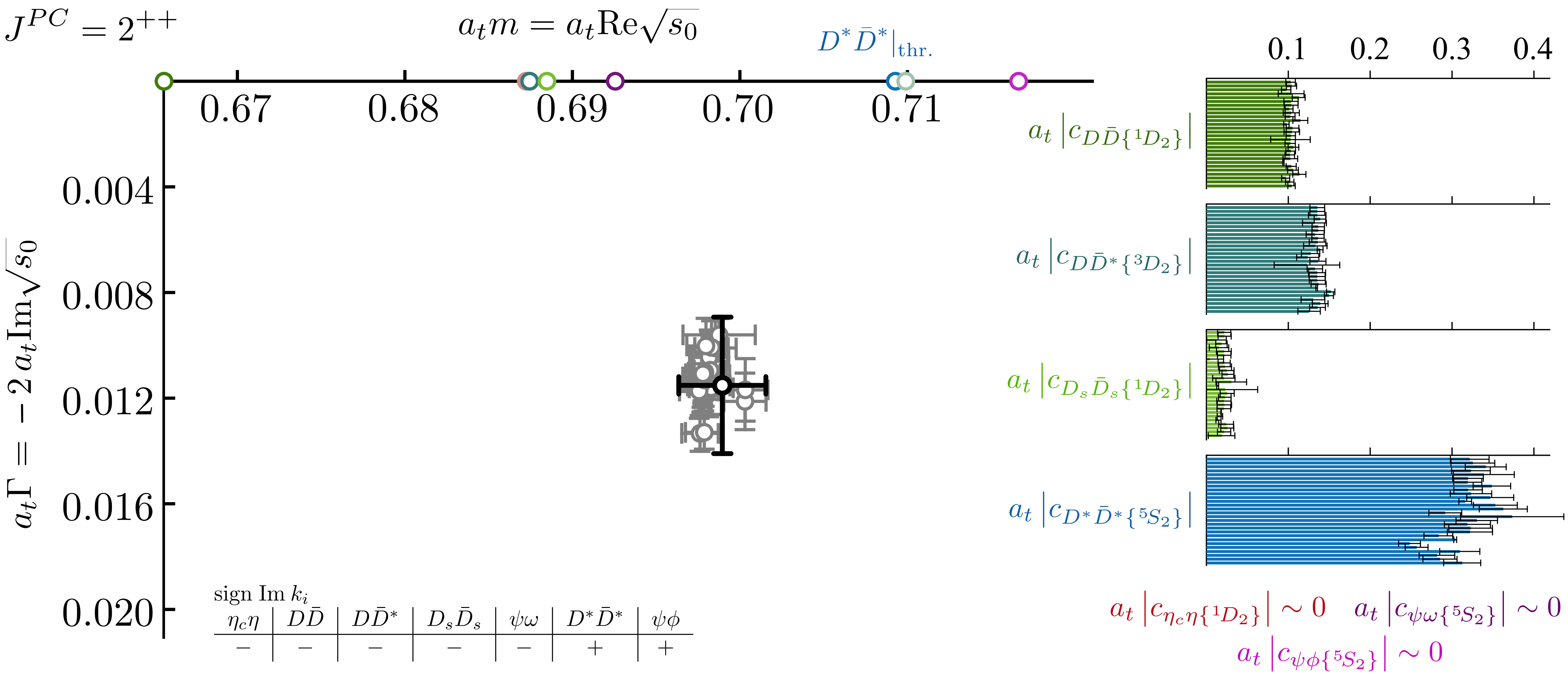


$$t_{ij} = \frac{g_i g_j}{m_0^2 - s - i g_{D\bar{D}}^2 \rho_{D\bar{D}} - i g_{D^*\bar{D}^*}^2 \rho_{D^*\bar{D}^*}}$$



the green/red cluster of poles are just mirror poles

- amplitude is **dominated by a single resonance pole** in this energy region

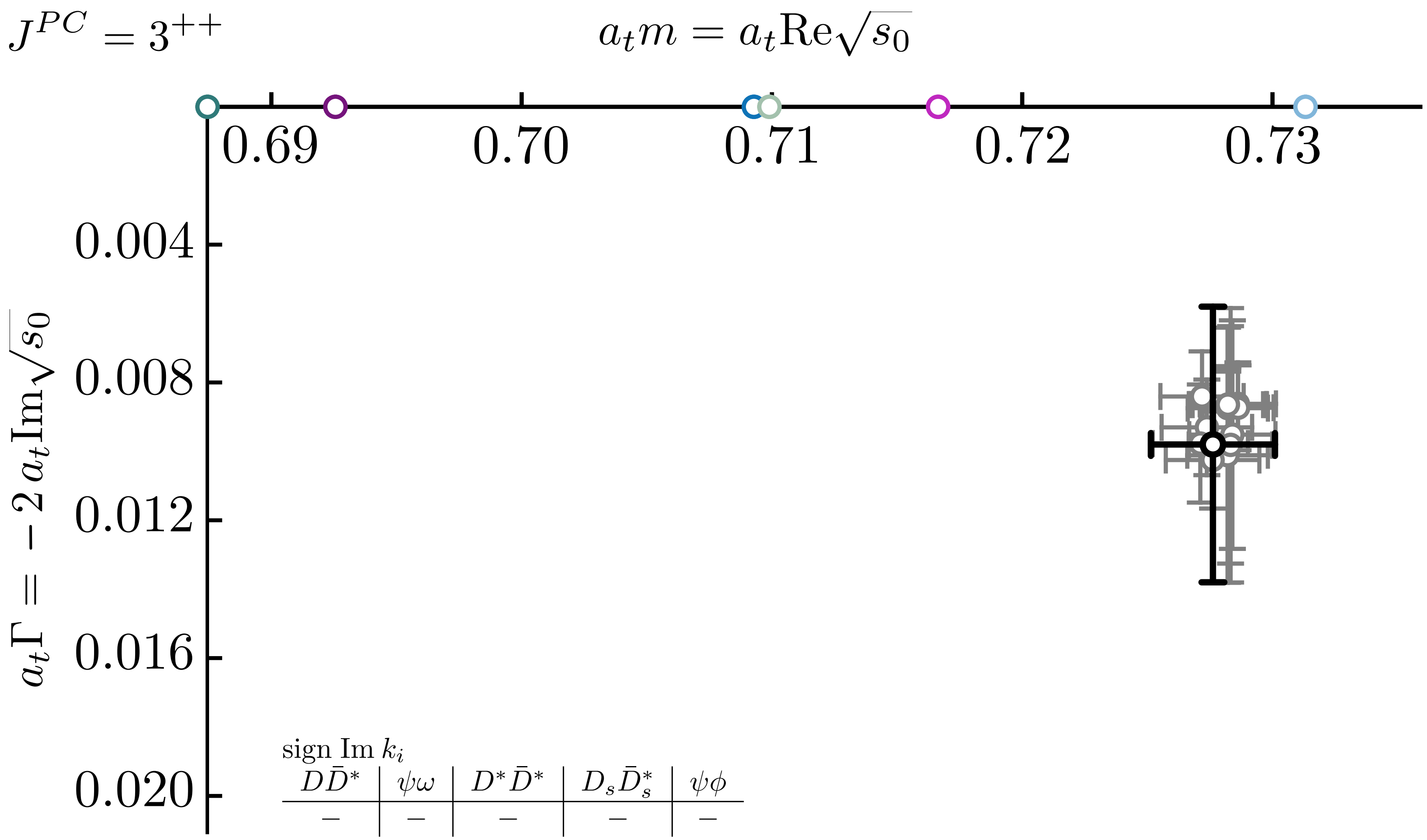


additional poles were found

- don't appear to be important

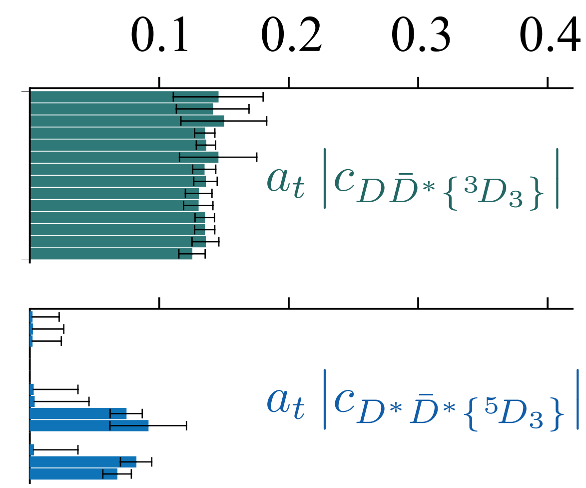
"coupling-ratio" phenomena seen in K-matrix pole parameters

- possible to rescale K-matrix g_i factors and obtain similar amplitudes
- t-matrix couplings are found to be well-determined



sign Im k_i

$D\bar{D}^*$	$\psi\omega$	$D^*\bar{D}^*$	$D_s\bar{D}_s^*$	$\psi\phi$
–	–	–	–	–

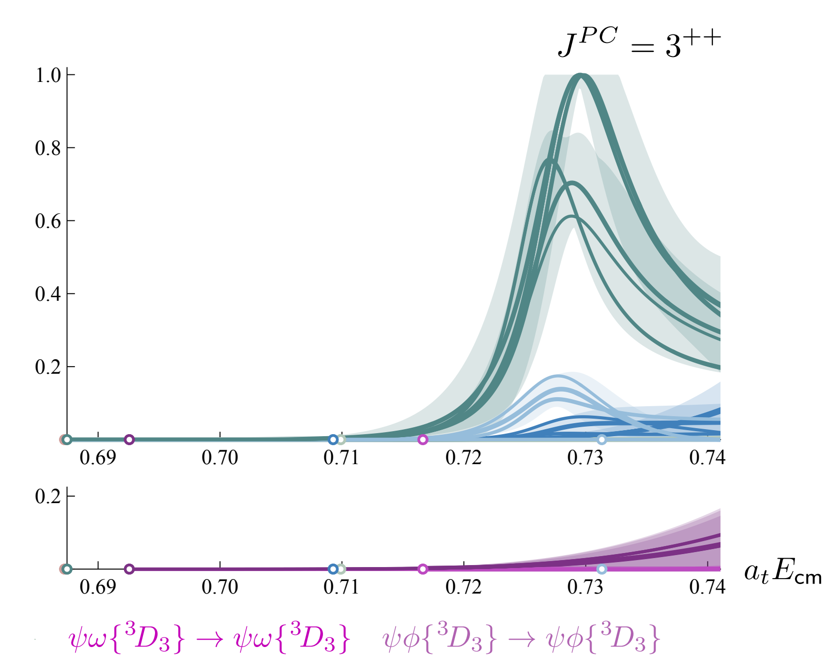


$a_t \left|c_{D_s\bar{D}_s^*\{^3D_3\}}\right| \sim 0$

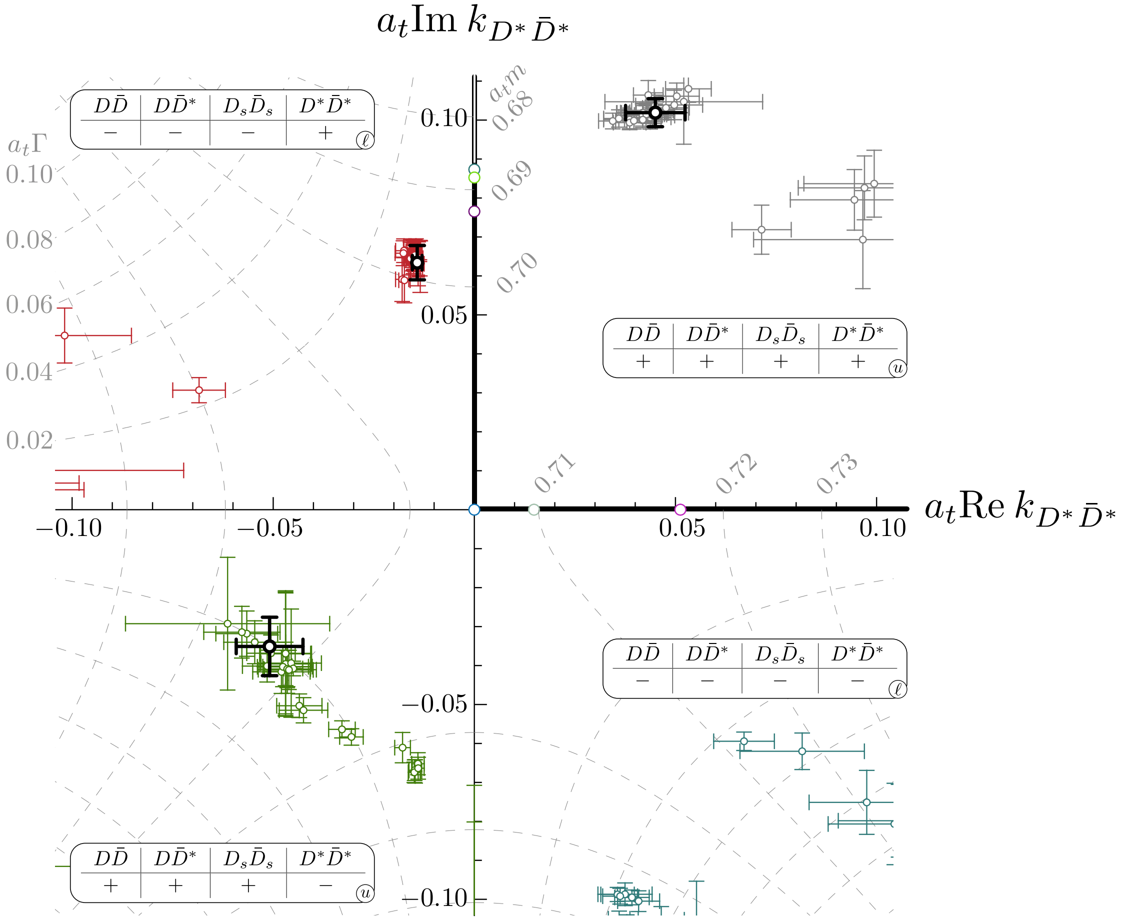
$a_t \left|c_{\psi\omega\{^3D_3\}}\right| \sim 0$

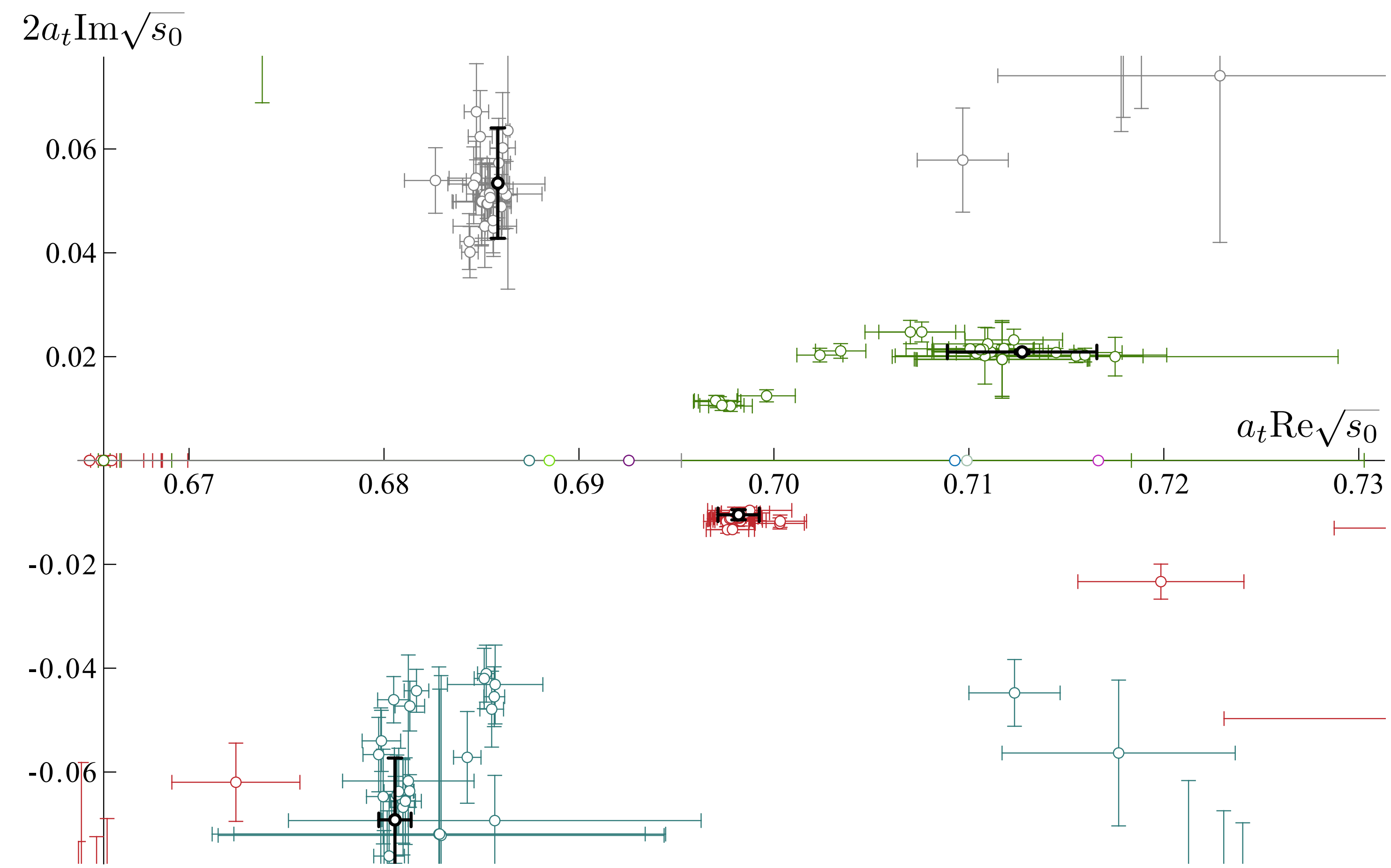
$a_t \left|c_{\psi\omega\{^5D_3\}}\right| \sim 0$

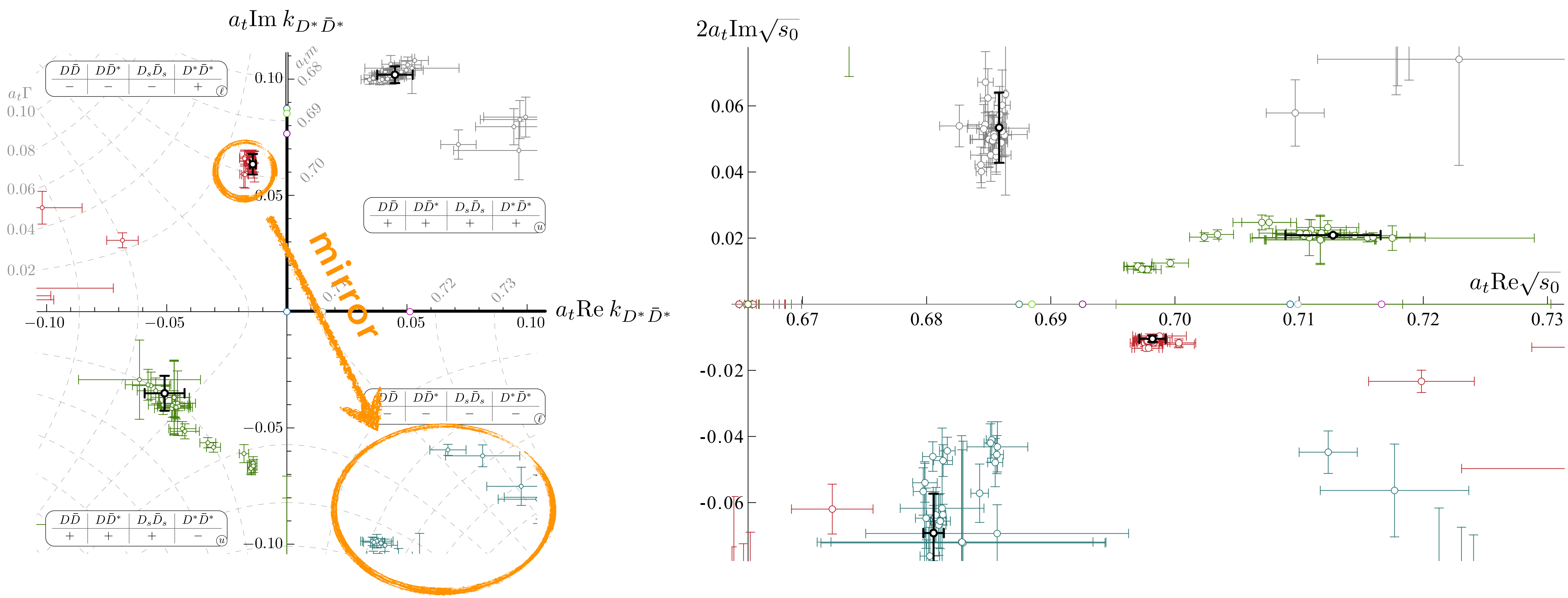
$a_t \left|c_{\psi\phi\{^3D_3\}}\right| \sim 0$



poles - tensor - more going on here, but mostly not physics

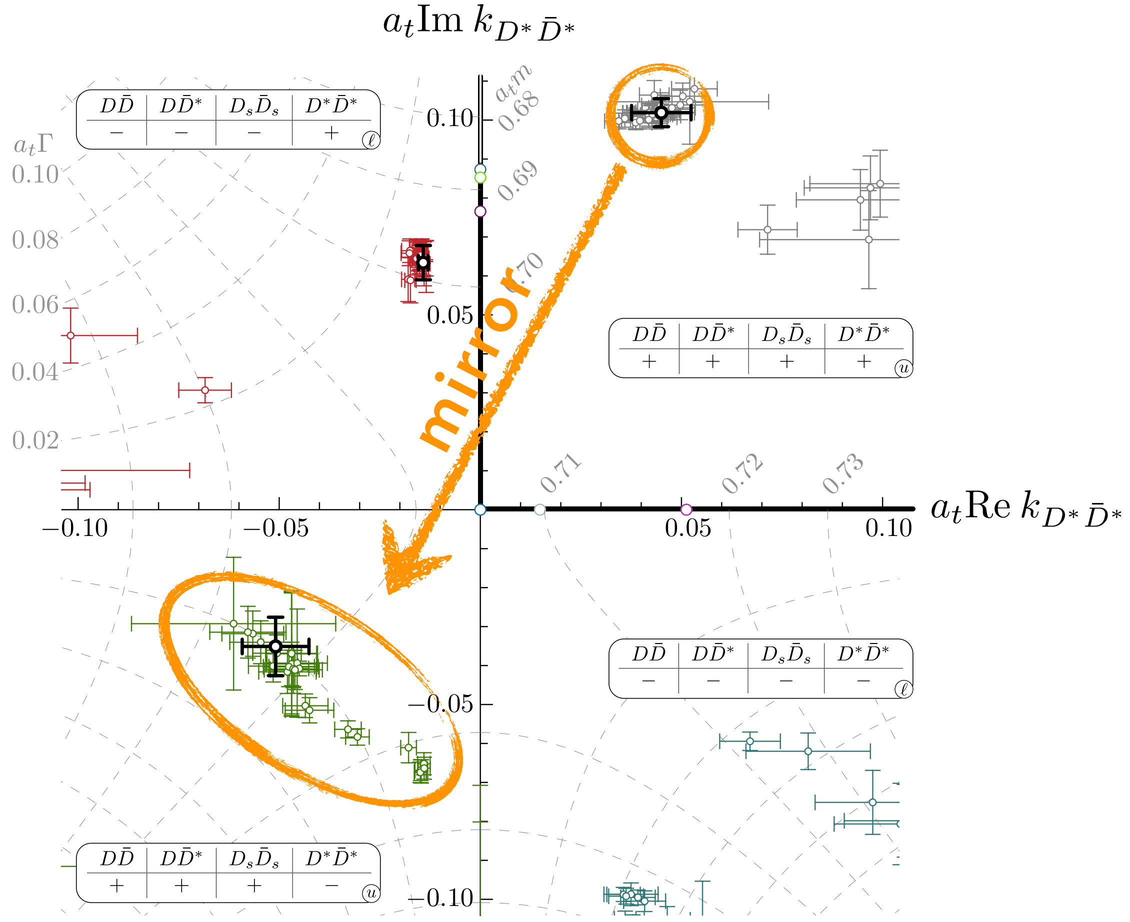


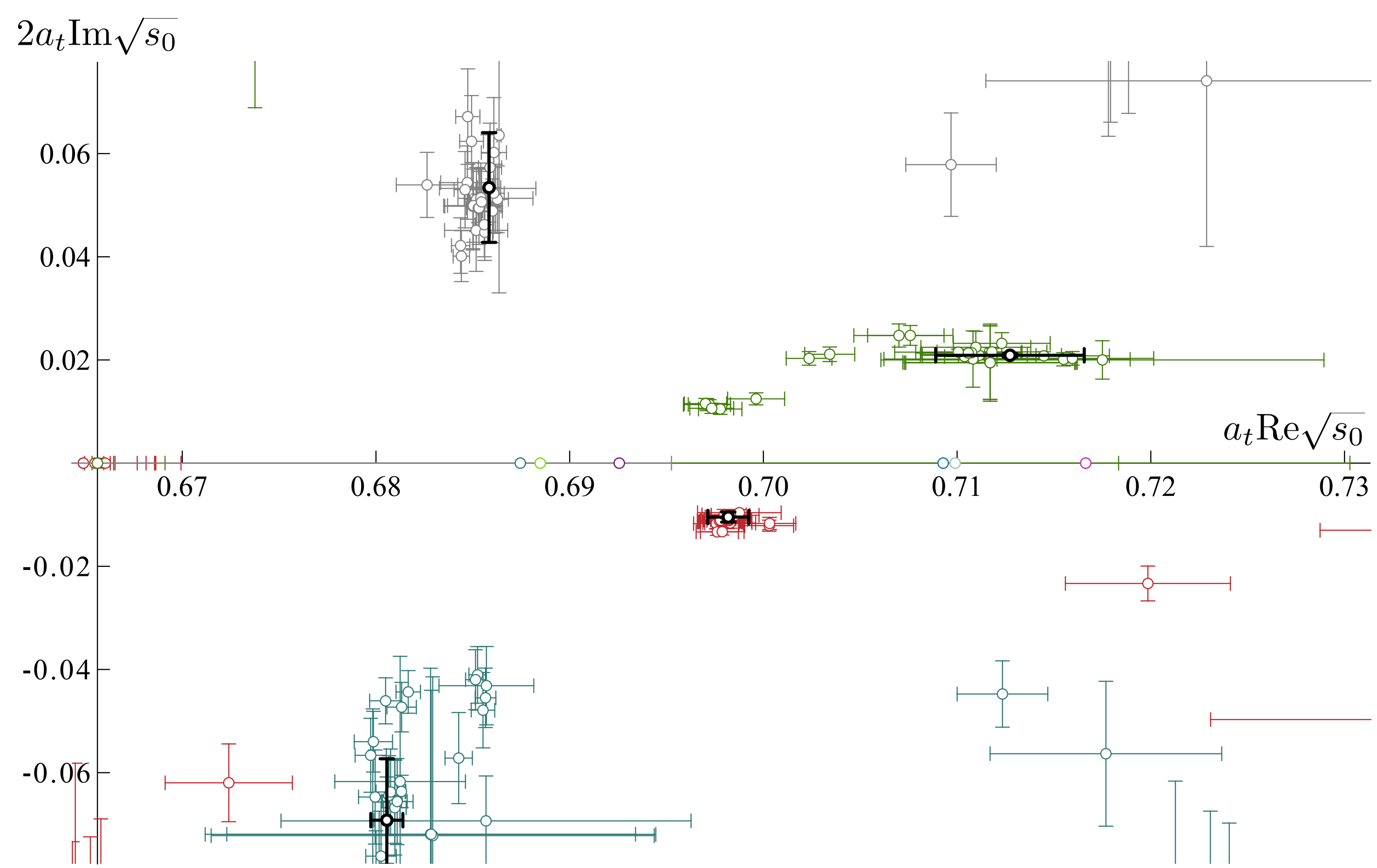




mirror pole - similar to a Flatté

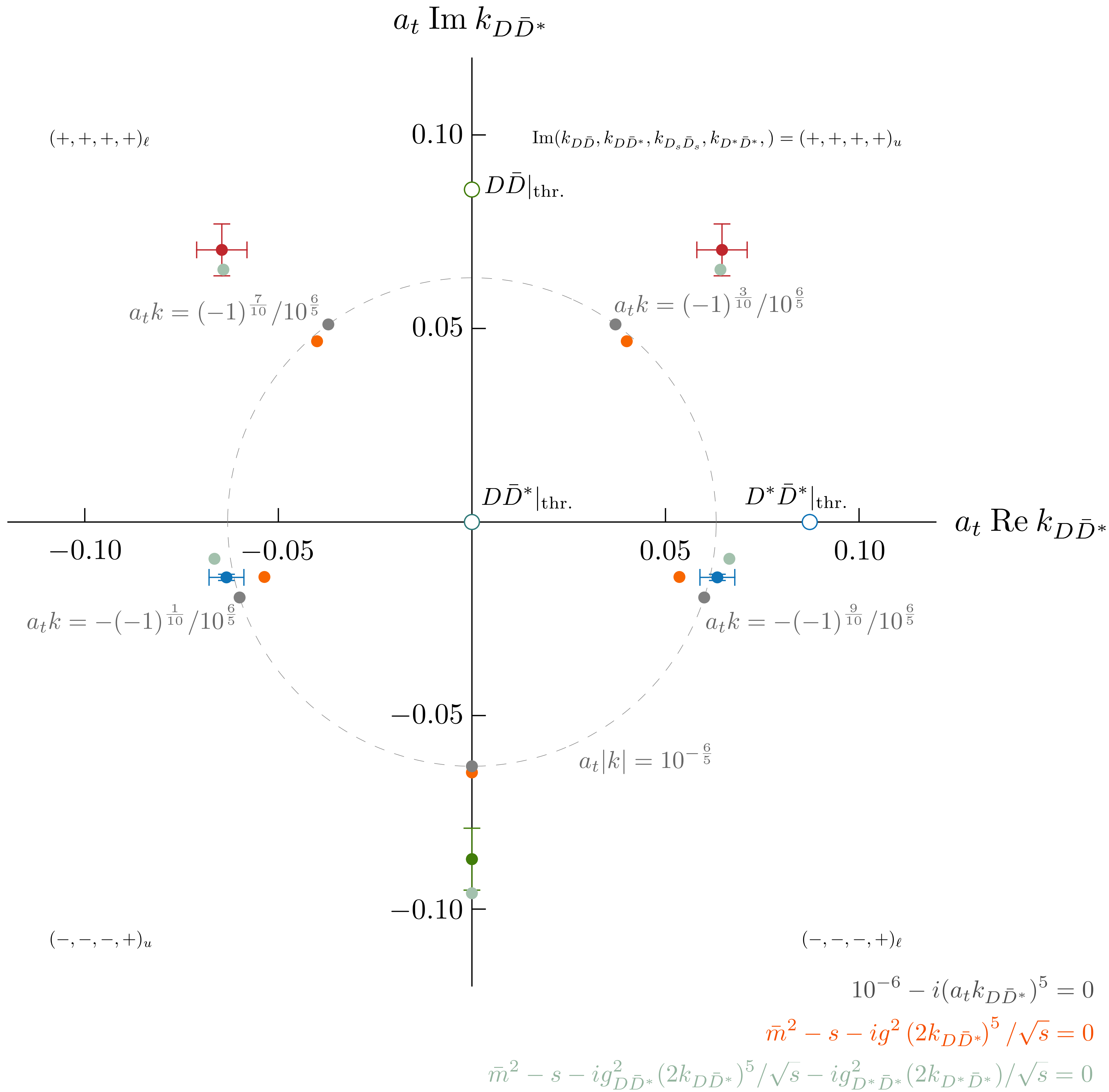
poles - tensor - more going on here, but mostly not physics



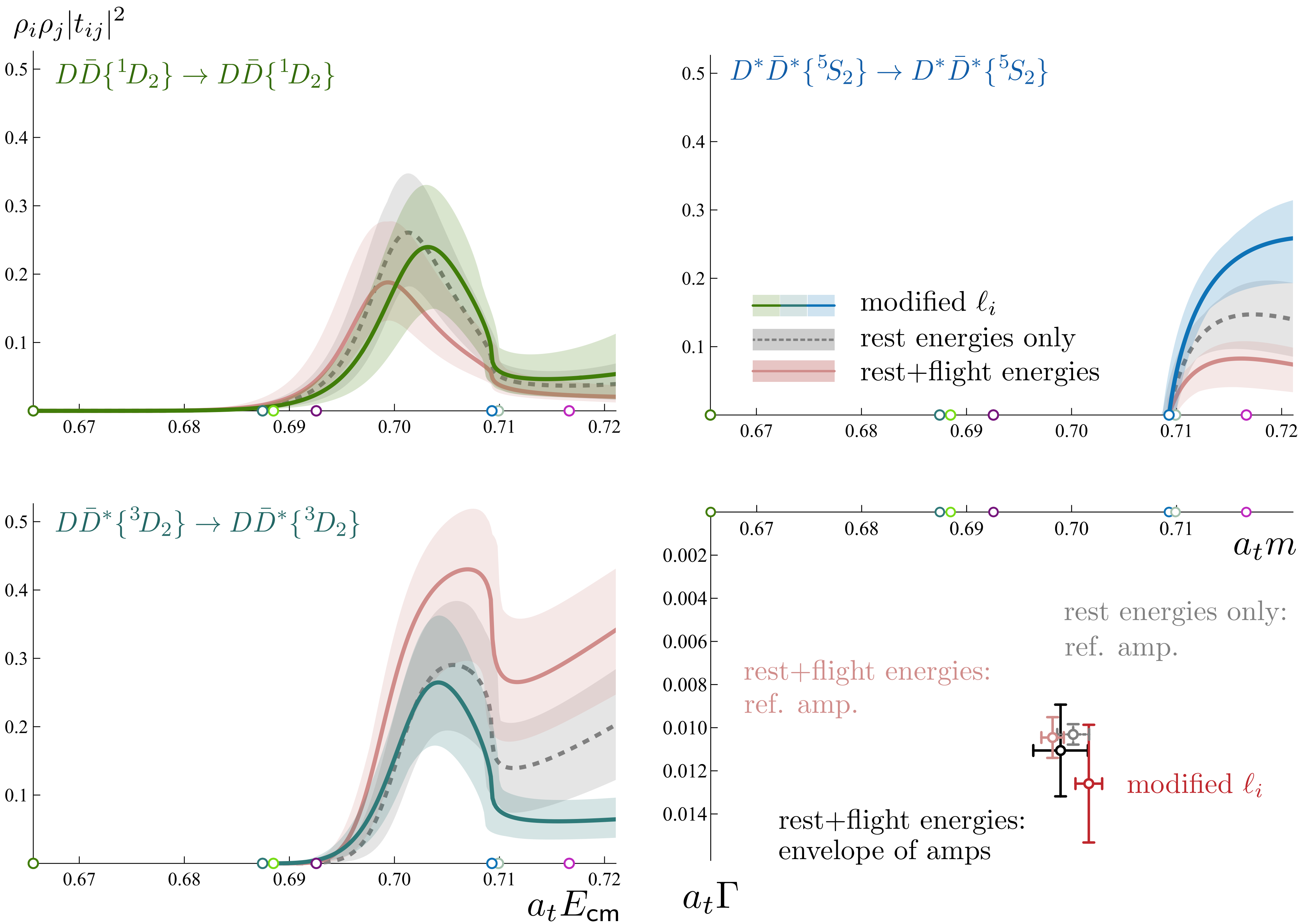


"green" pole is a mirror of the physical sheet pole

physical sheet pole arises because of the large g_{DD^*}



$a_t \,\mathrm{Im}\, k_{D\bar{D}^*}$
$a_t \,\mathrm{Re}\, k_{D\bar{D}^*}$
0.10
0.05
−0.05
−0.10
−0.10
−0.05
0.05
0.10
$(+,+,+,+)_\ell$
$\mathrm{Im}(k_{D\bar{D}}, k_{D\bar{D}^*}, k_{D_s\bar{D}_s}, k_{D^*\bar{D}^*},) = (+,+,+,+)_u$
$D\bar{D}|_{\mathrm{thr.}}$
$D\bar{D}^*|_{\mathrm{thr.}}$
$D^*\bar{D}^*|_{\mathrm{thr.}}$
$a_t k = (-1)^{\frac{7}{10}}/10^{\frac{6}{5}}$
$a_t k = (-1)^{\frac{3}{10}}/10^{\frac{6}{5}}$
$a_t k = -(-1)^{\frac{1}{10}}/10^{\frac{6}{5}}$
$a_t k = -(-1)^{\frac{9}{10}}/10^{\frac{6}{5}}$
$a_t |k| = 10^{-\frac{6}{5}}$
$(-,-,-,+)_u$
$(-,-,-,+)_\ell$
$10^{-6} - i(a_t k_{D\bar{D}^*})^5 = 0$
$\bar{m}^2 - s - ig^2\,(2k_{D\bar{D}^*})^5/\sqrt{s} = 0$
$\bar{m}^2 - s - ig^2_{D\bar{D}^*}(2k_{D\bar{D}^*})^5/\sqrt{s} - ig^2_{D^*\bar{D}^*}(2k_{D^*\bar{D}^*})/\sqrt{s} = 0$

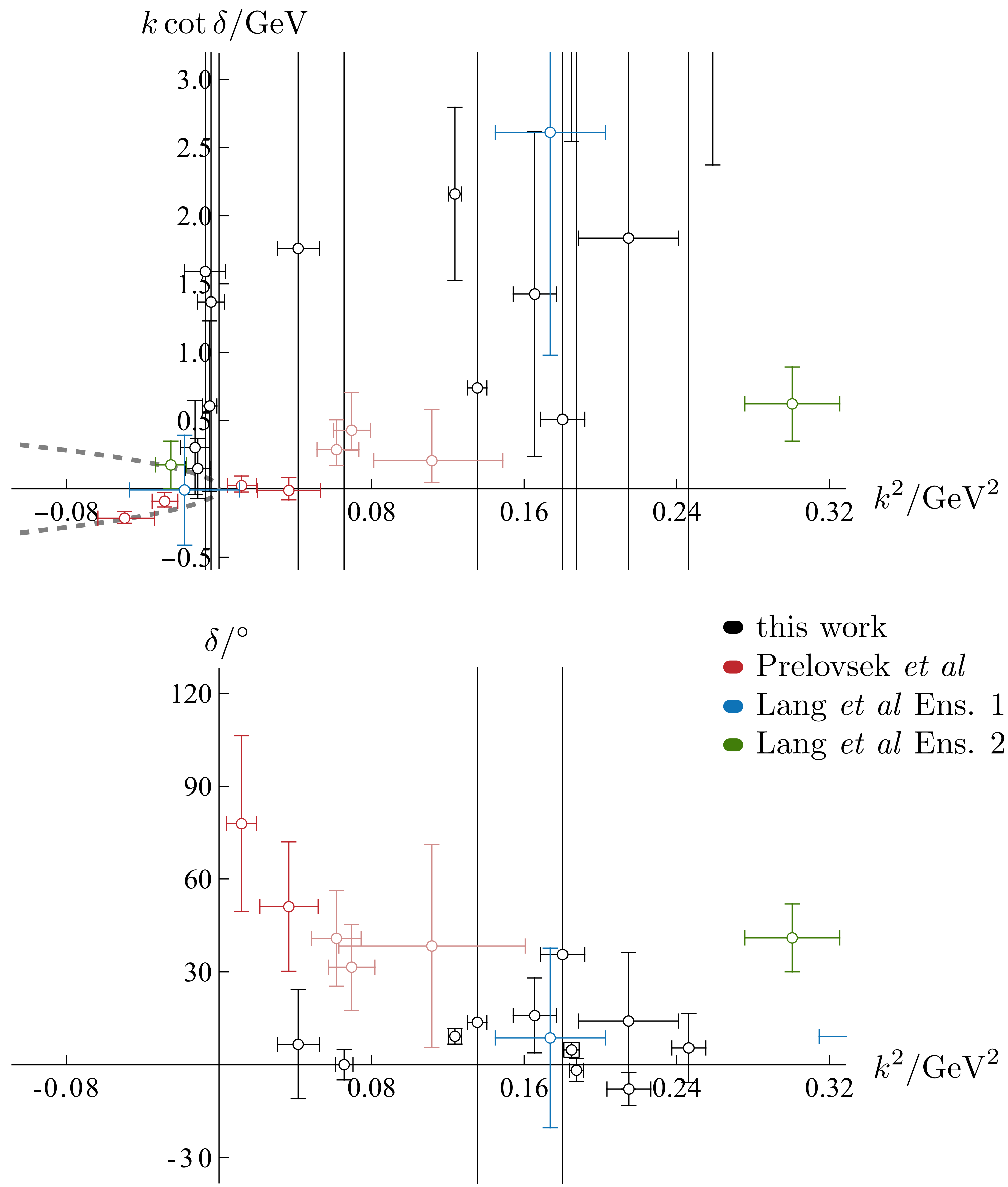


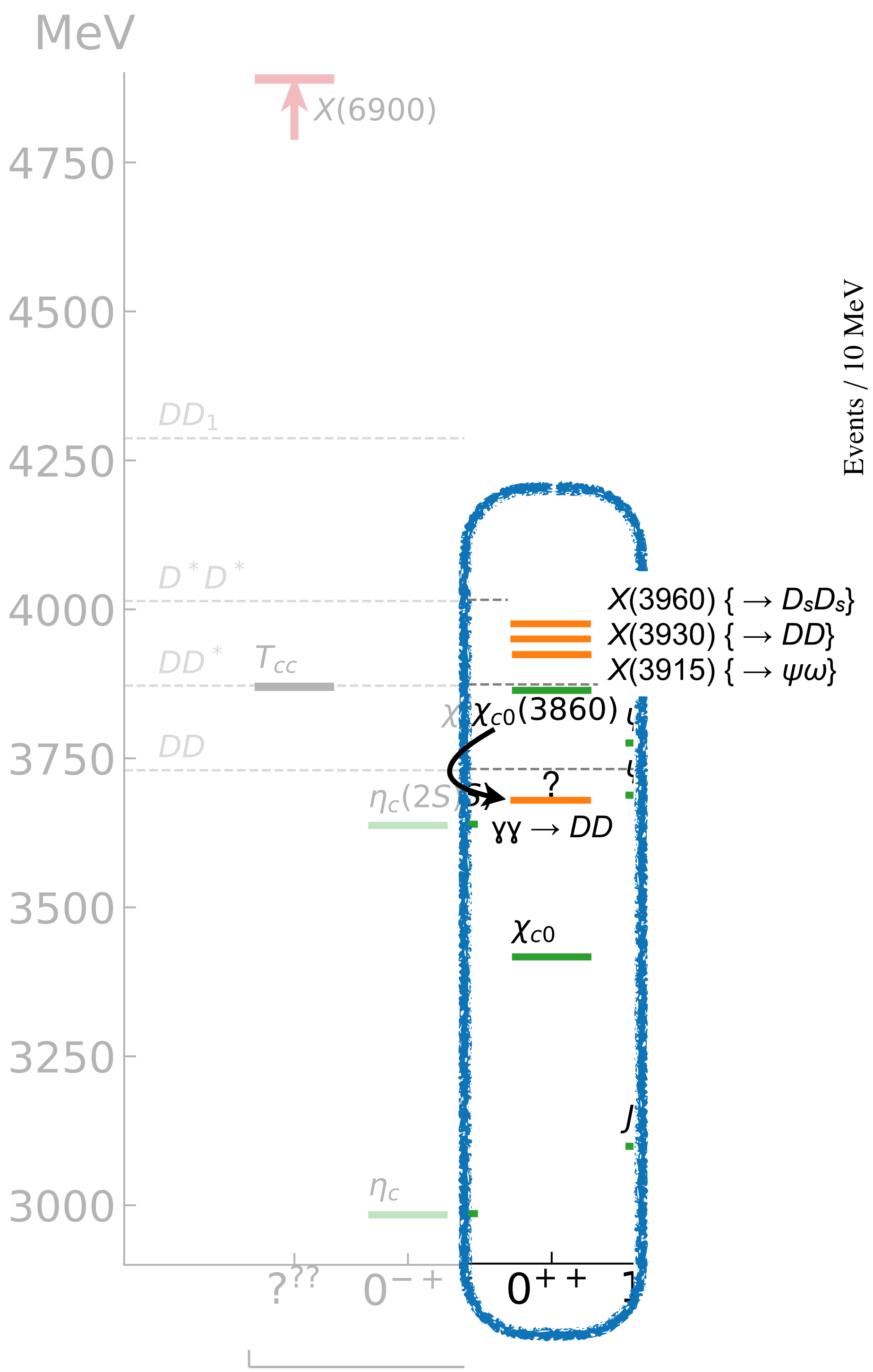
- different physical sheet pole
- no obvious nearby (+,+,+,-) sheet pole (there are some with a_t E >0.74)

Results from Prelovsek, Padmanath et al, suggest effects at DDbar and DsDsbar thresholds

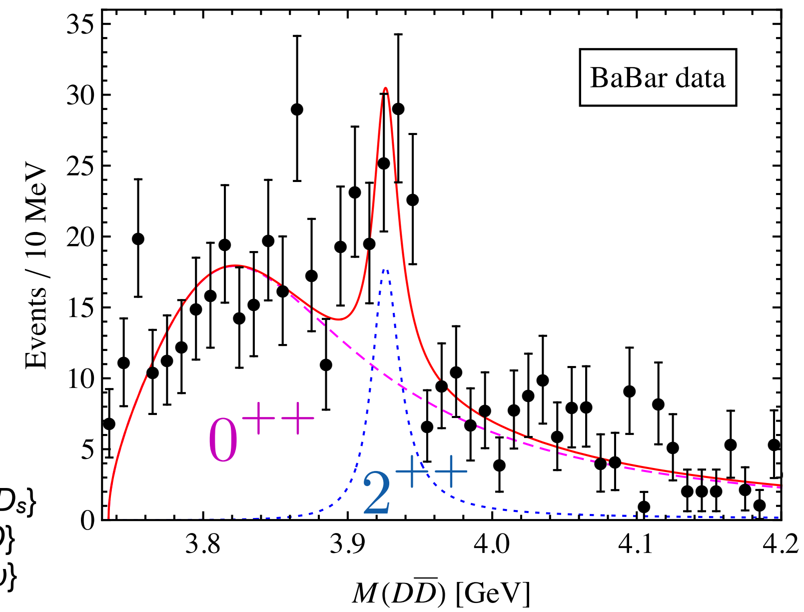
- pion mass ~ 280 MeV
- light quark heavier than physical, strange quark lighter than physical

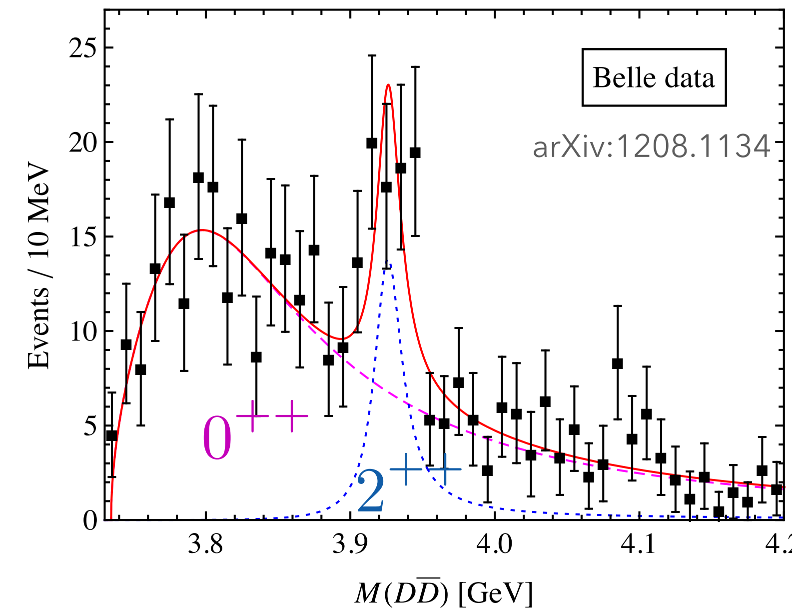
hard to justify such a large change due to the light quark mass (no one-pion-exchange term)





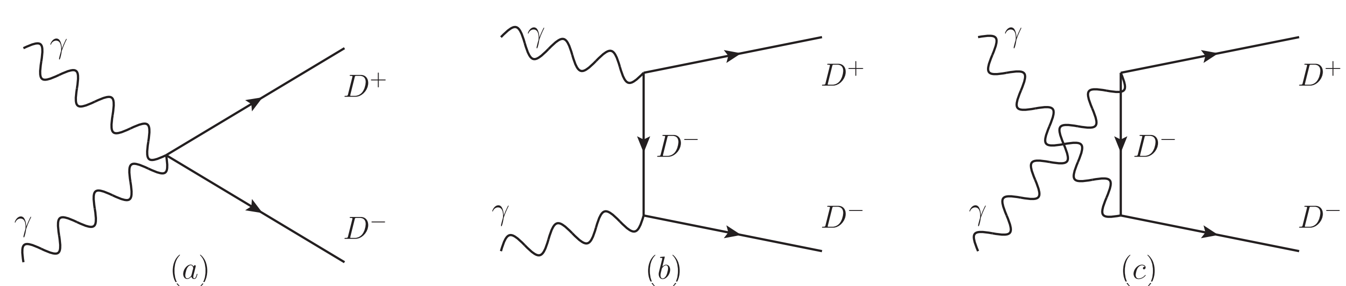
- BaBar, Belle - resonance around 3860 MeV $\gamma\gamma \to D\bar{D}$





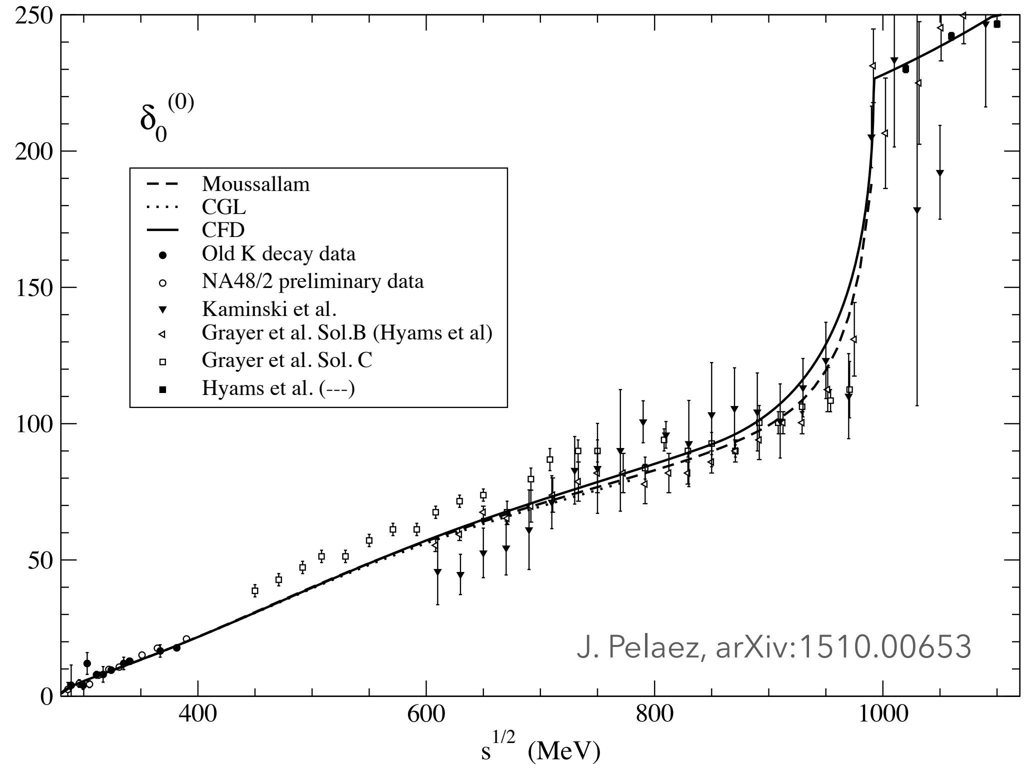
- Guo & Meissner (2012)
  m = 3840 MeV, Γ = 220 MeV

- Wang et al (2021), Daneika et al (2022):
  Complications from Born exchanges lead to a lower state around 3700 MeV

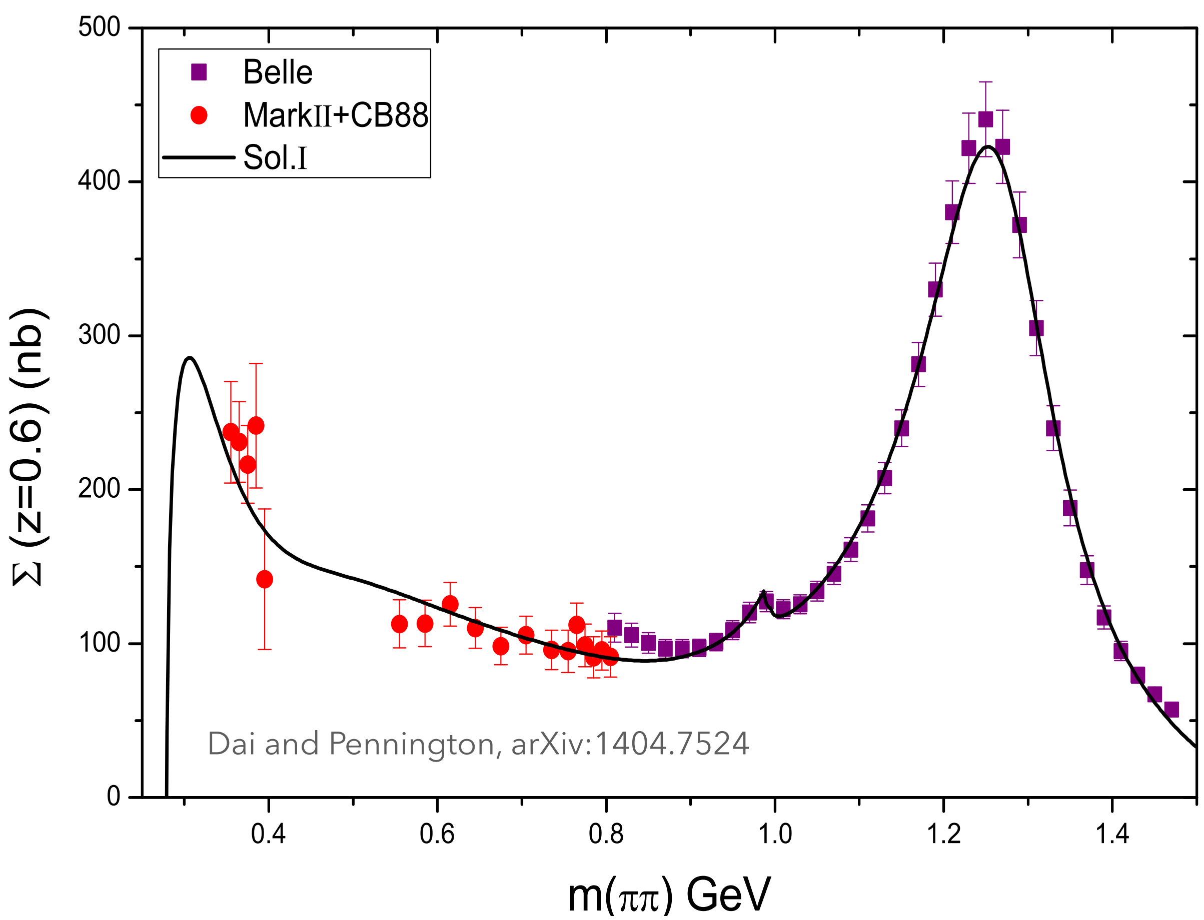


arXiv:2010.15431

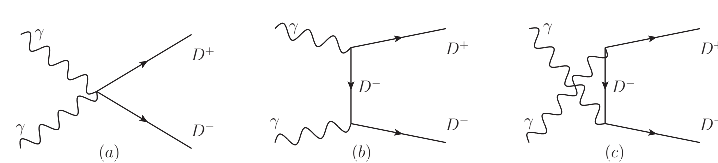
no state around 3840-3860 MeV (?)



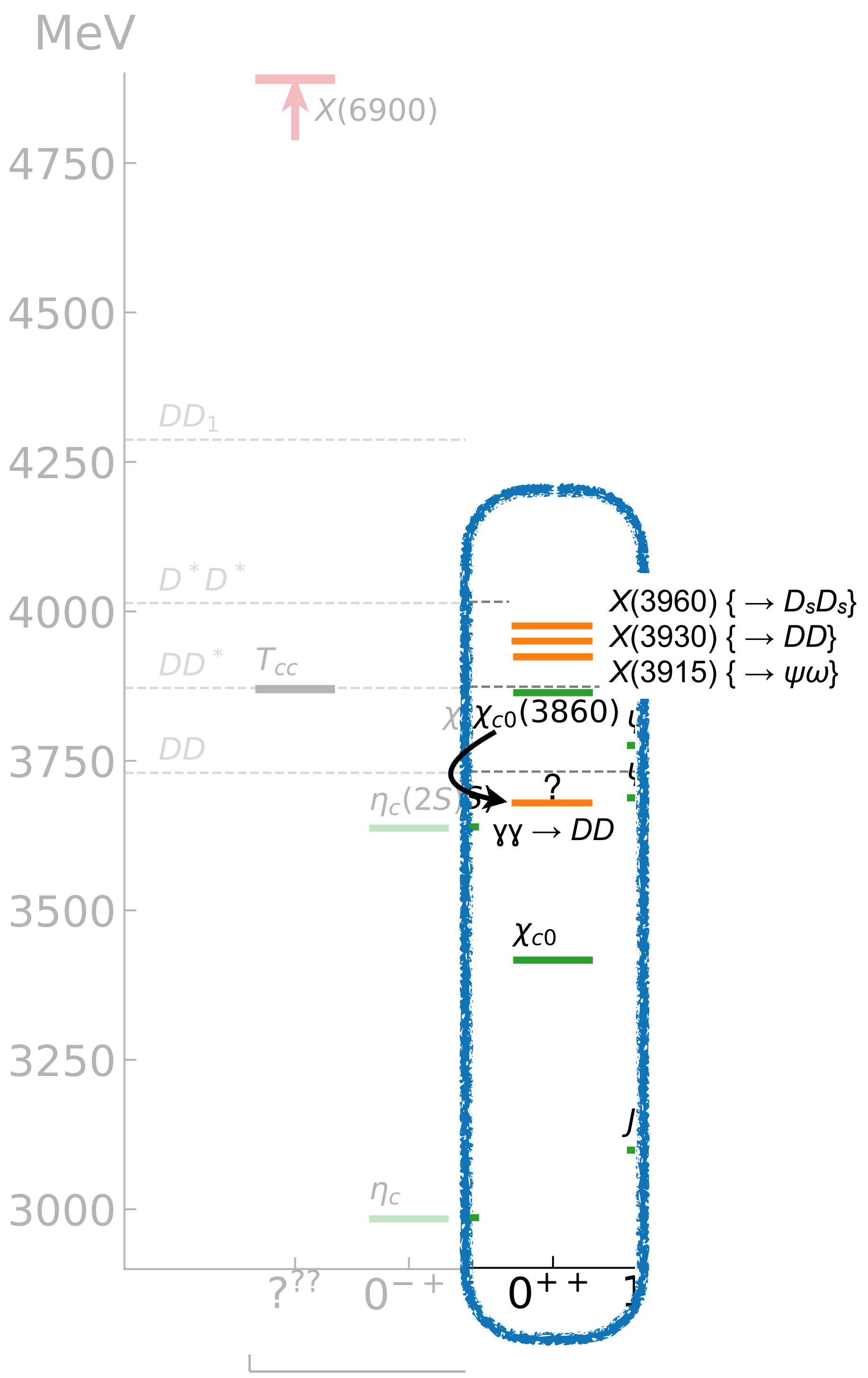
$$\pi\pi \to \pi\pi \quad (S-\text{wave})$$



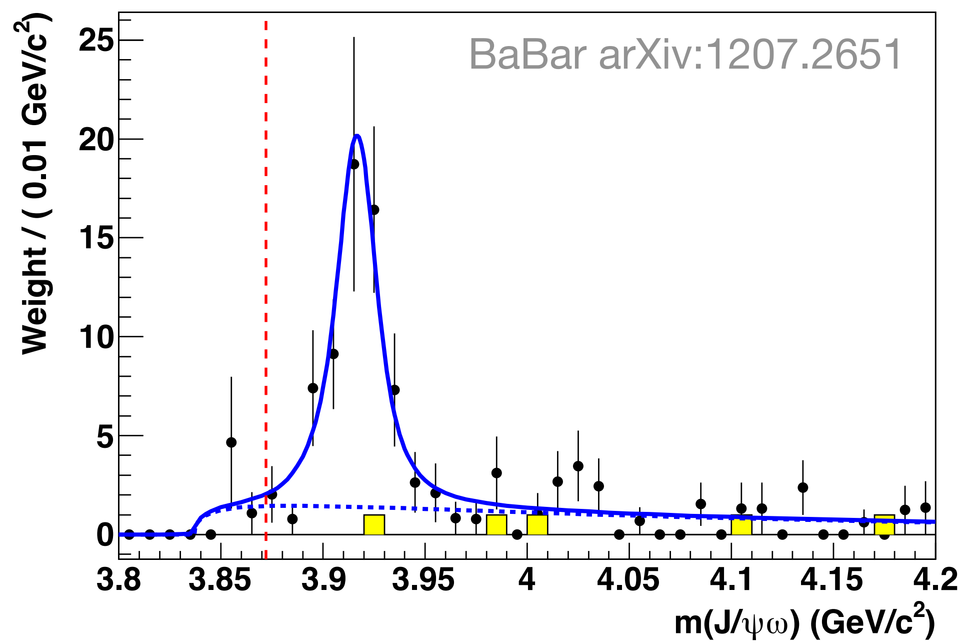
$$\gamma\gamma \to \pi\pi$$



extra structure at threshold, not linked to a resonance or bound state



- BaBar, Belle - resonance around 3915 MeV in $J/\psi\omega$



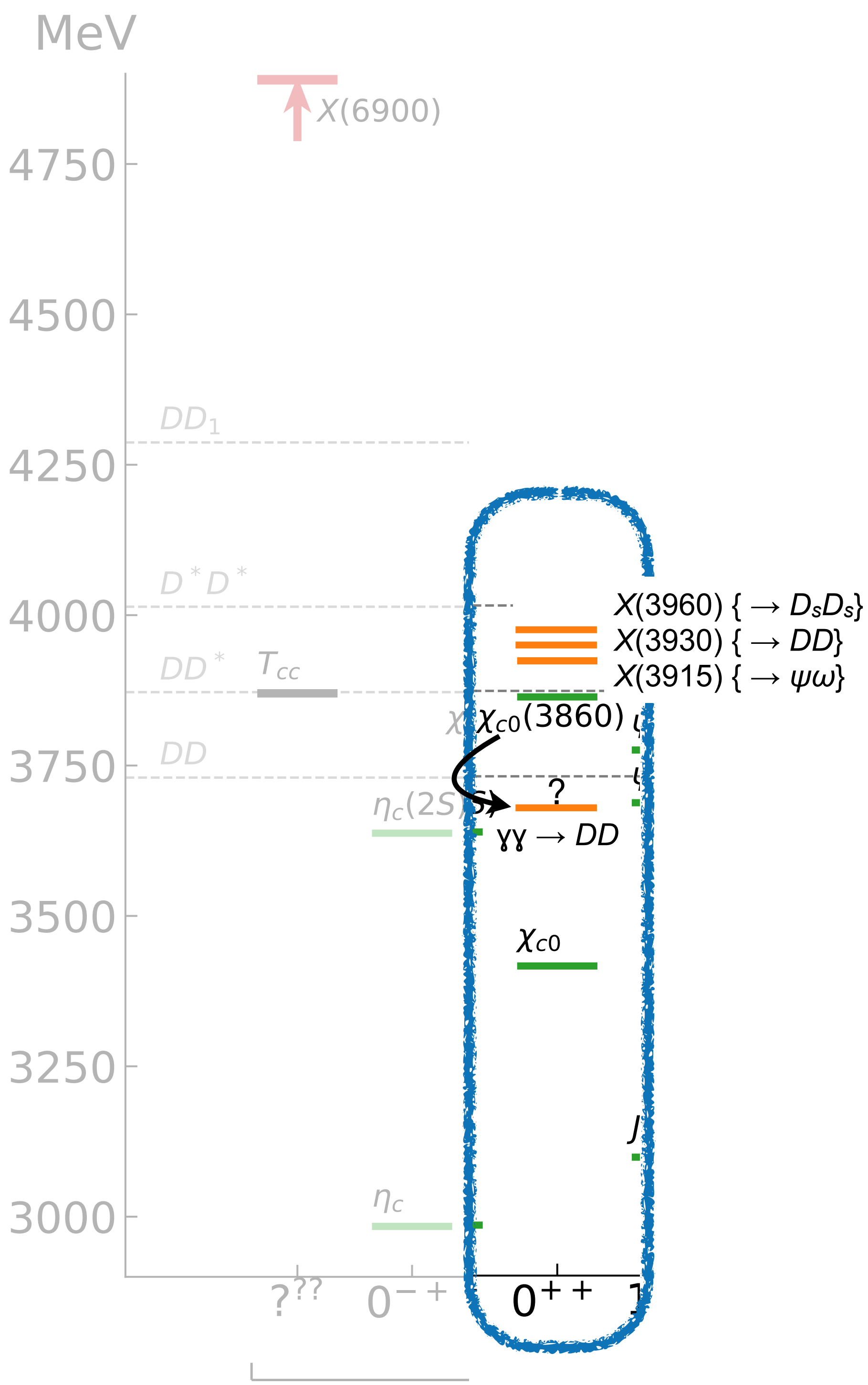
$m = (3919.4 \pm 2.2 \pm 1.6)$ MeV

$\Gamma = (13 \pm 6 \pm 3)$ MeV

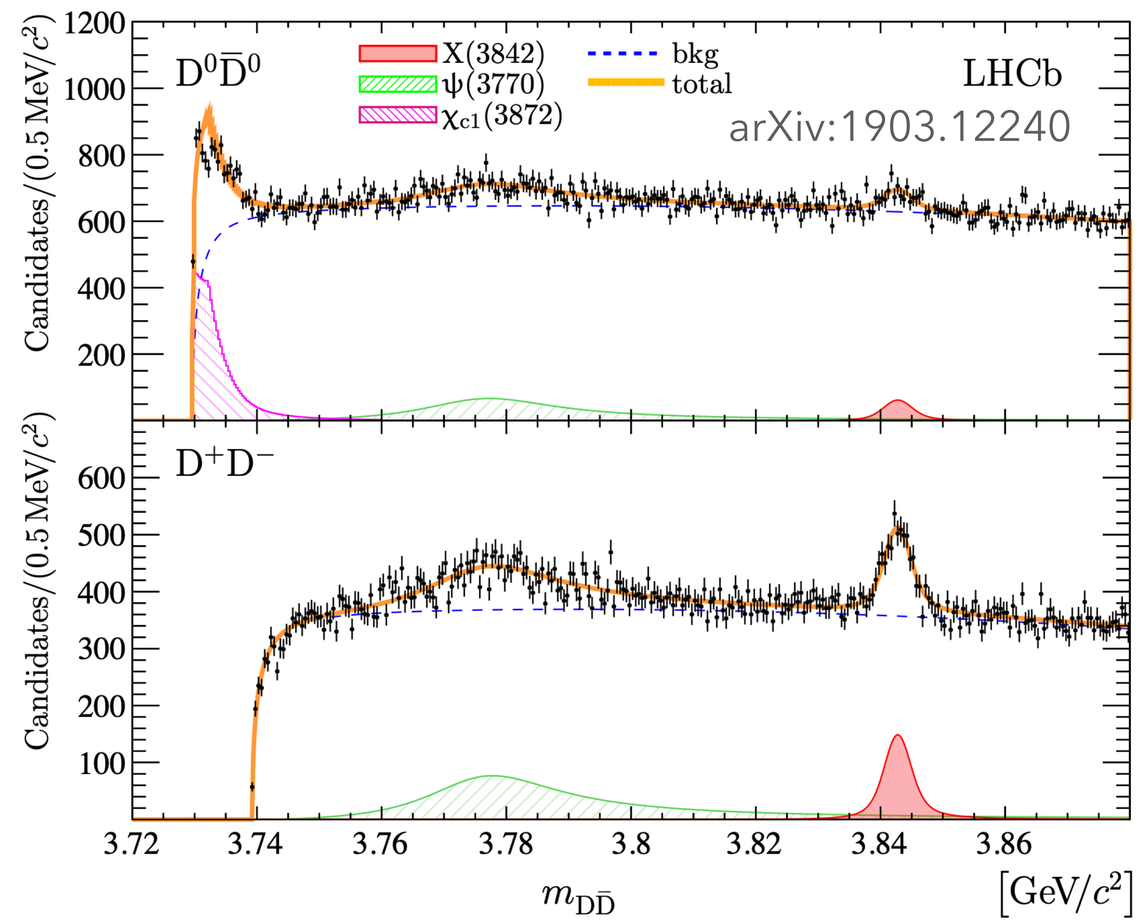
$J^P = 0^+$

(several other studies of this)

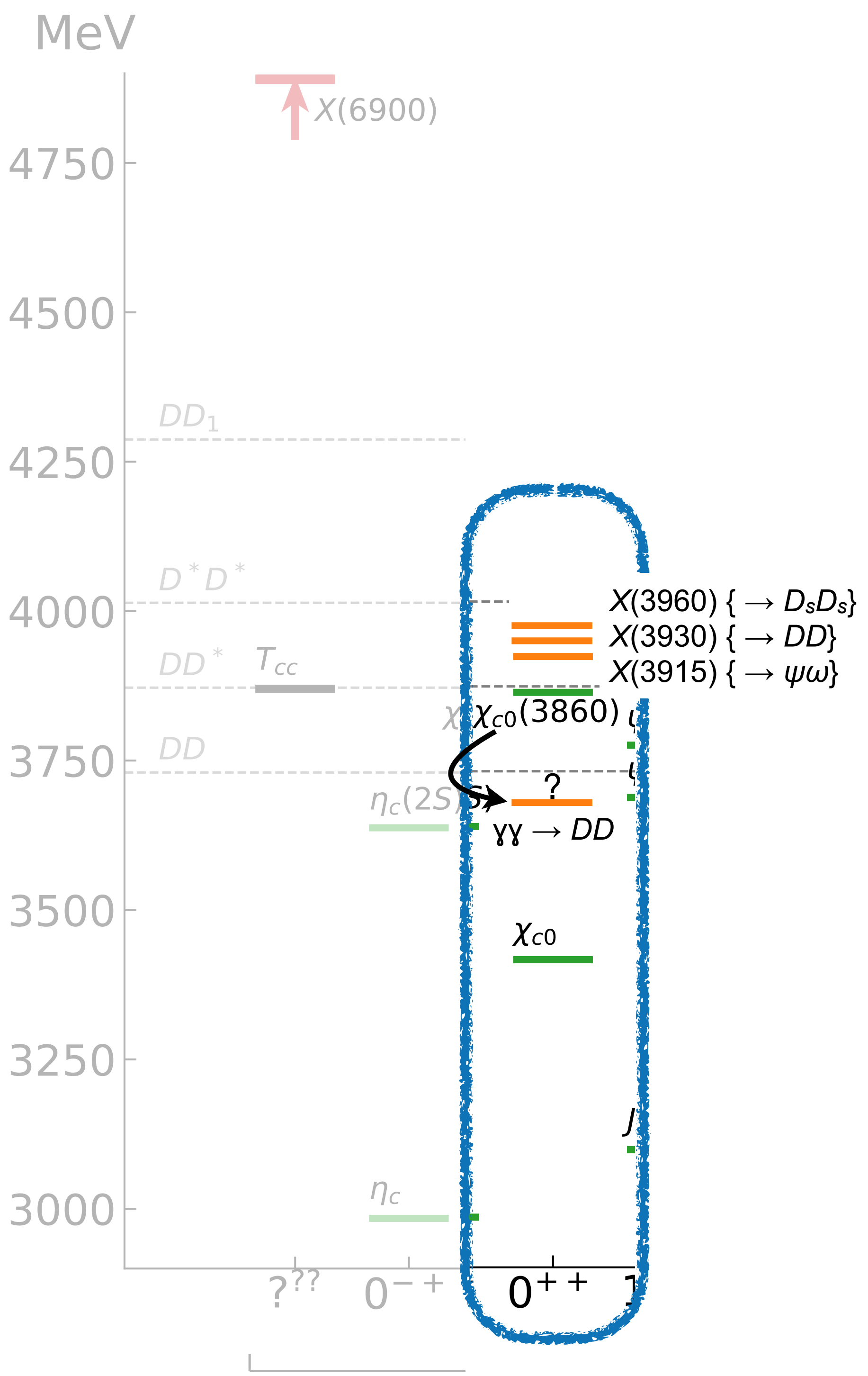
experimental results - 3700 to 3860 MeV



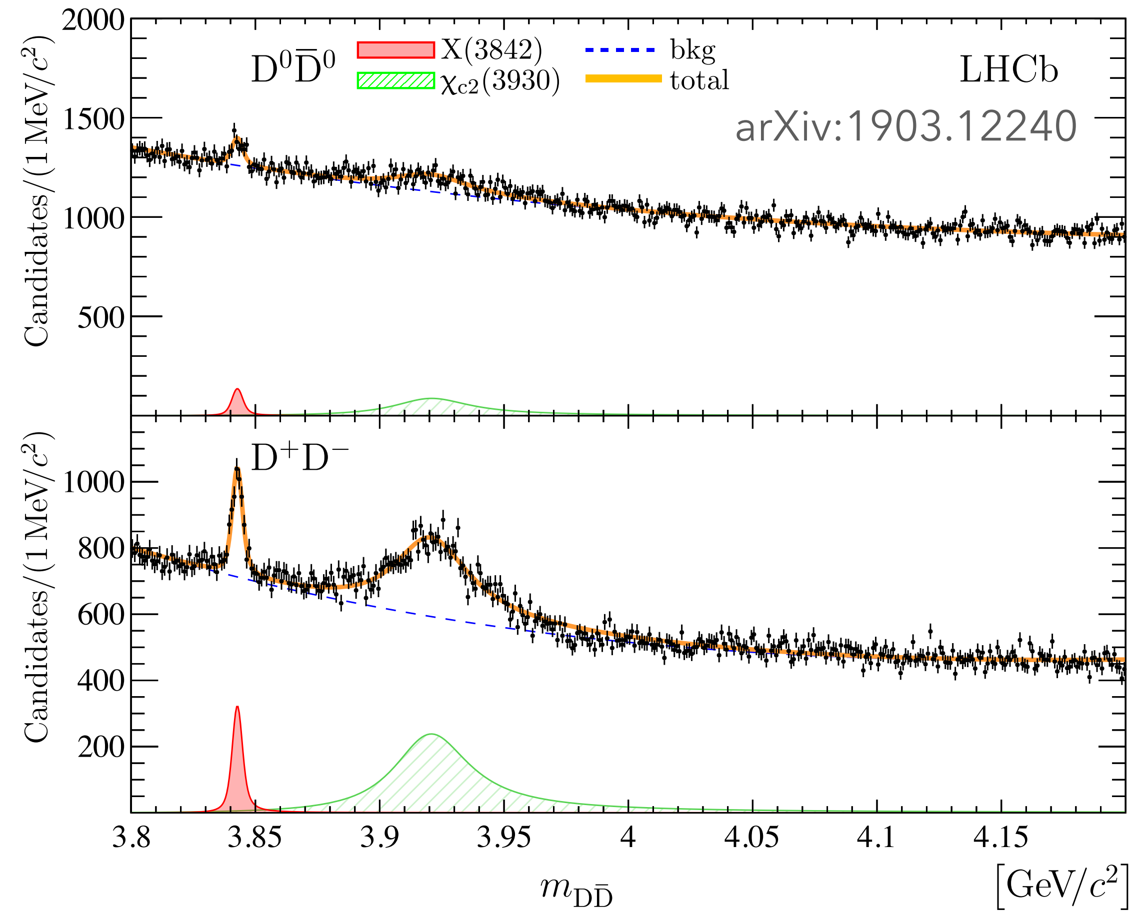
DDbar at LHCb
hadronic production process



Peak at DDbar threshold attributed to "feed-down" from X(3872) decays



Same study from LHCb, higher energies

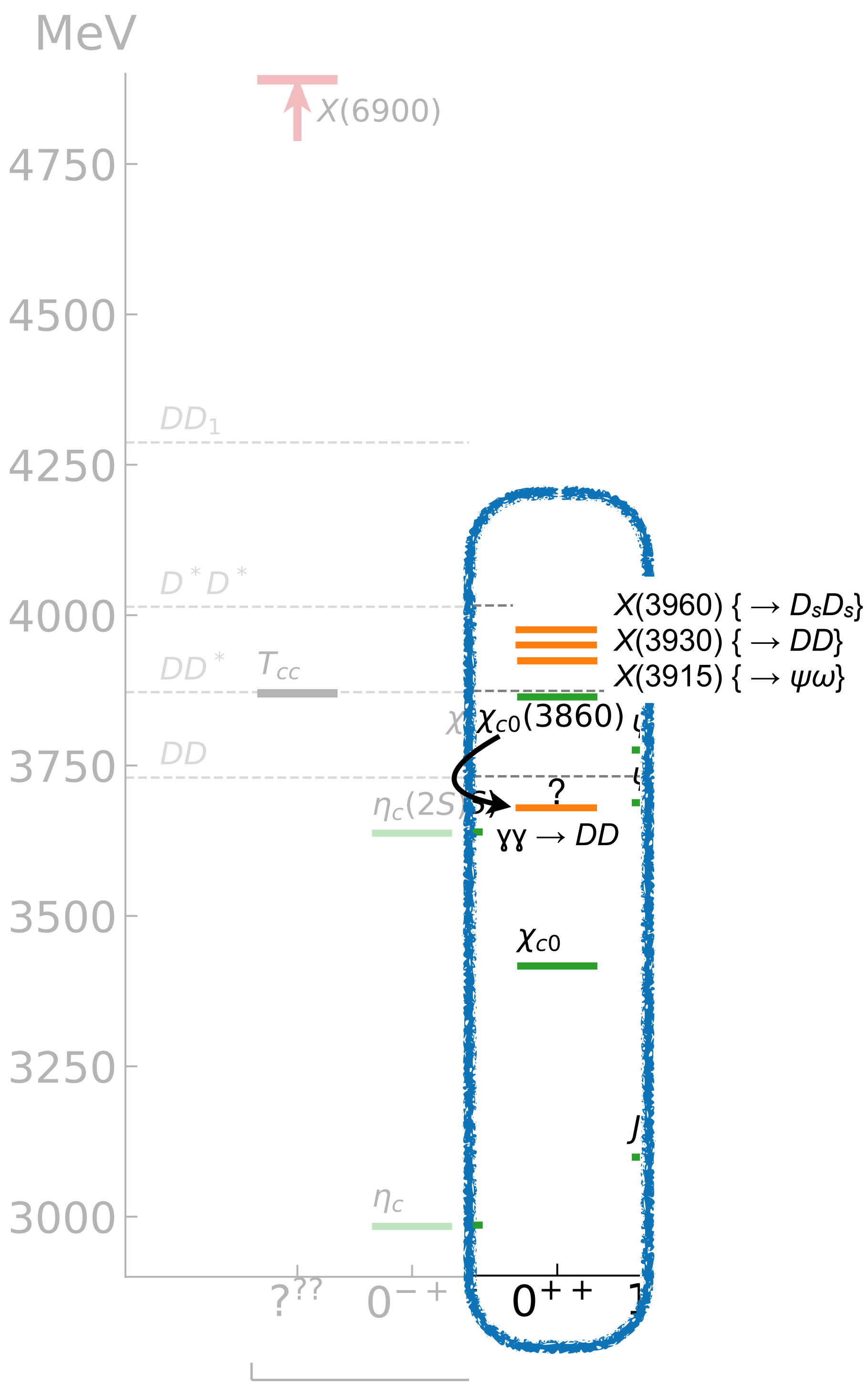


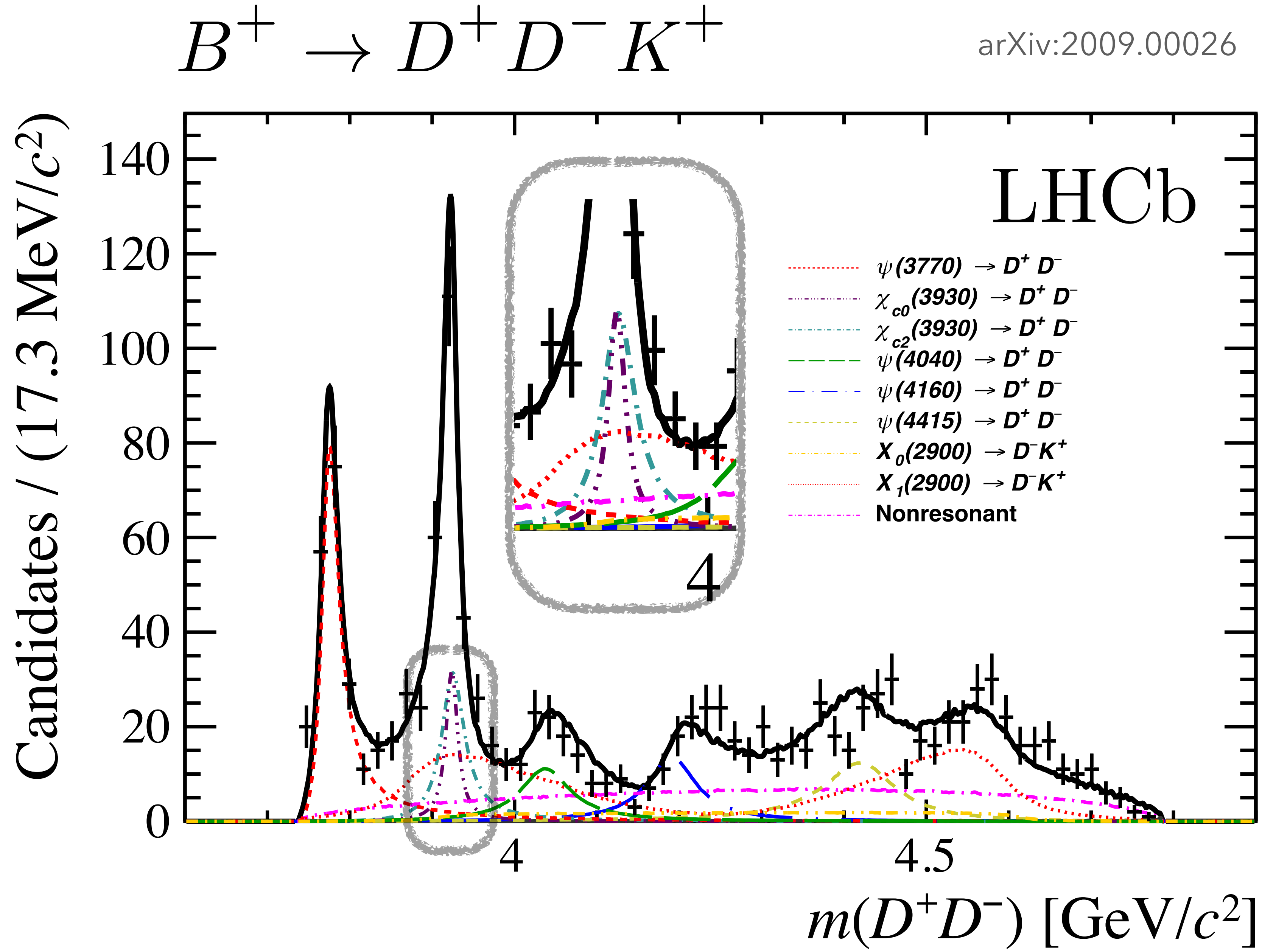
$\chi_{c2}(3930)$

$m \approx 3922$ MeV

$\Gamma \approx 37$ MeV

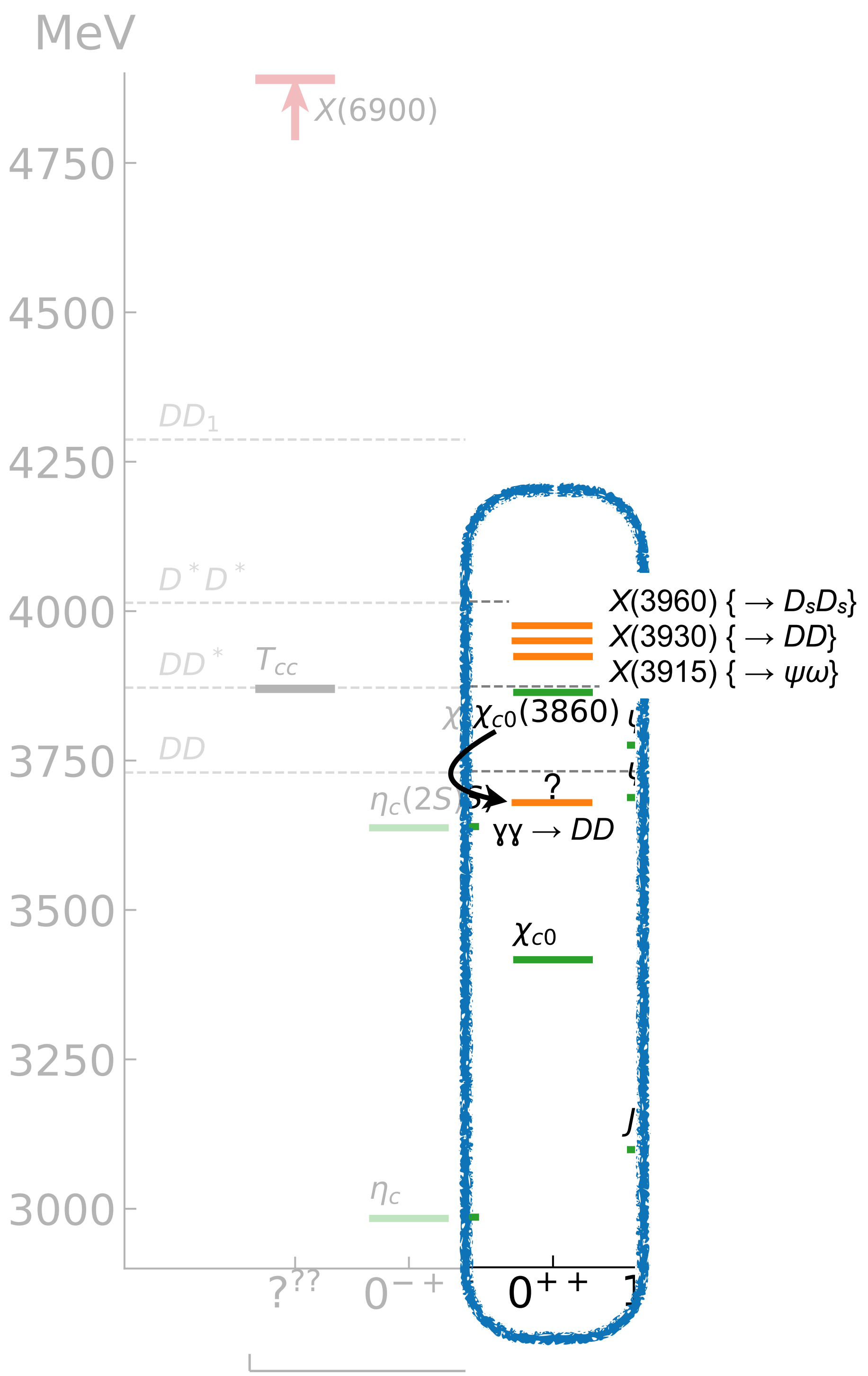
not obviously inconsistent with earlier Belle & BaBar results





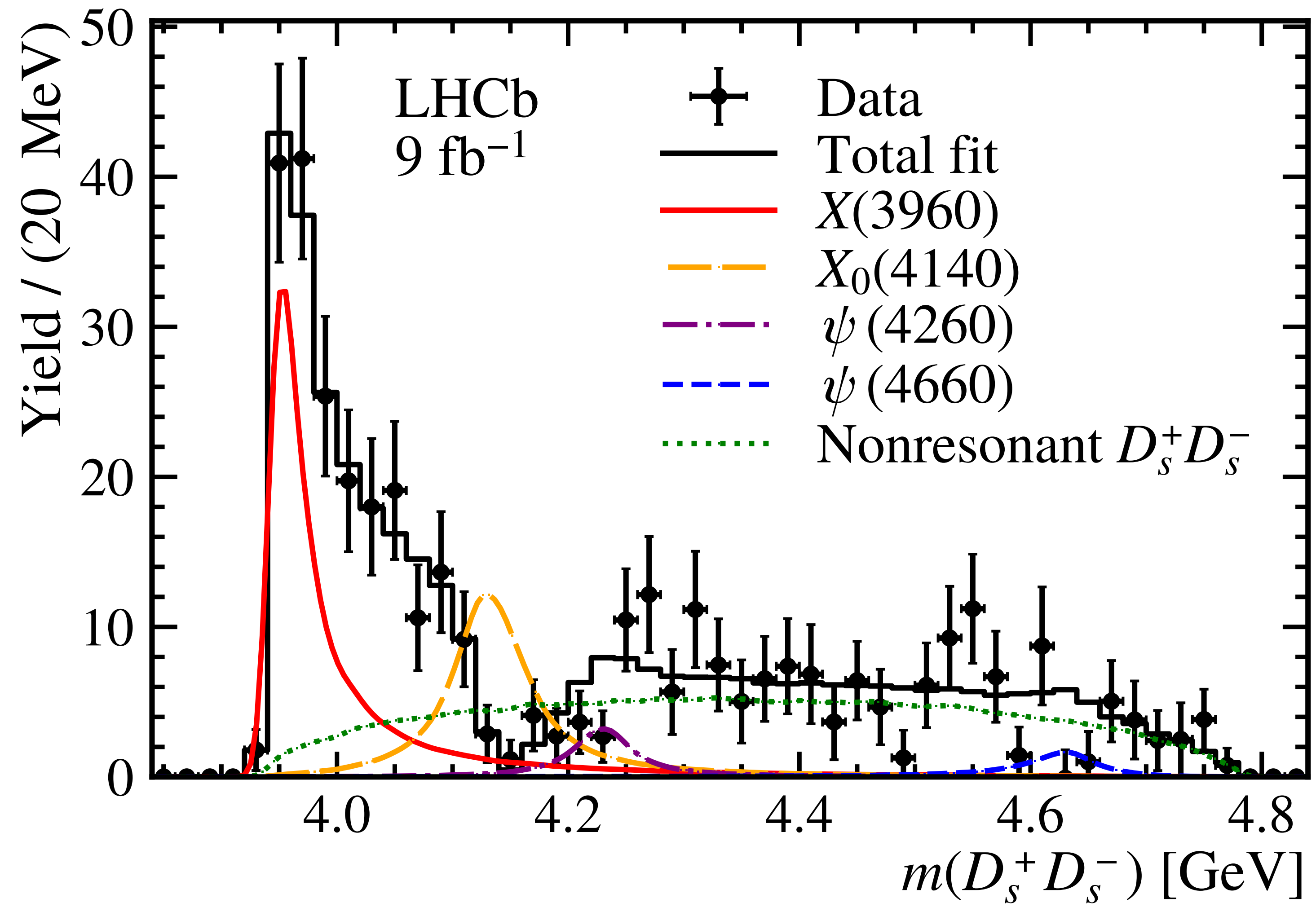
overlapping 0++ and 2++ resonances around 3925 MeV

no need for a low 0++ resonance



$B^+ \to D_s^+ D_s^- K^+$

arXiv:2210.15153
LHCb



enhancement in DsDs at threshold "X(3960)"

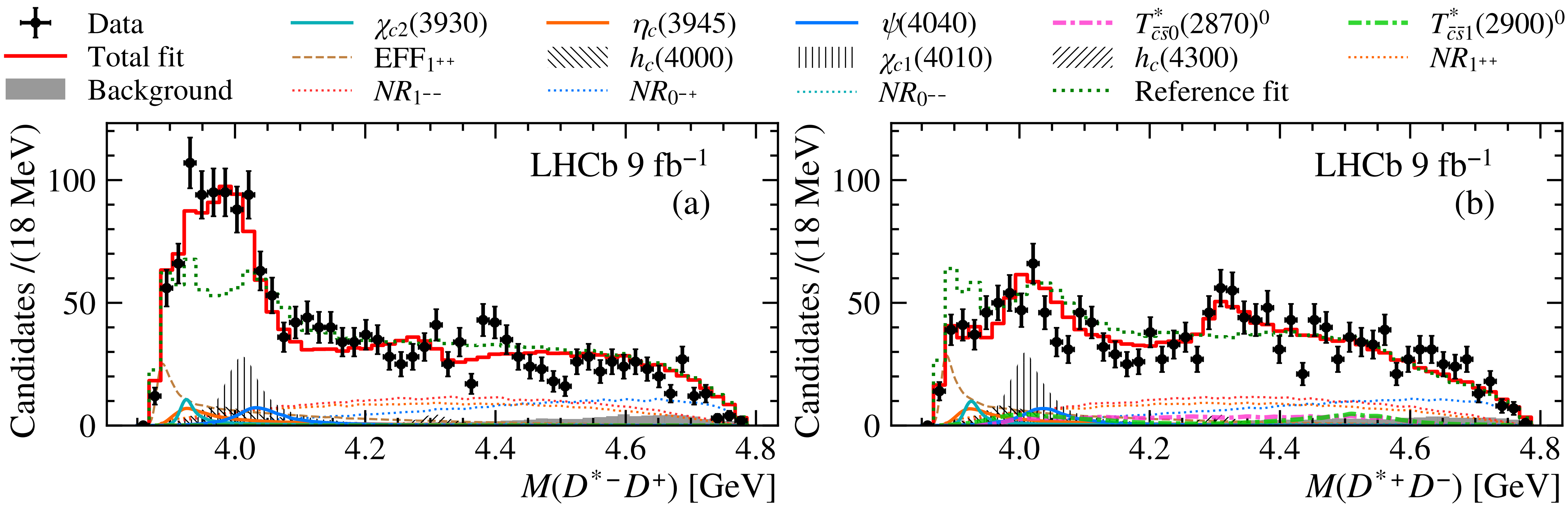
$m \approx 3956$ MeV

$\Gamma \approx 43$ MeV

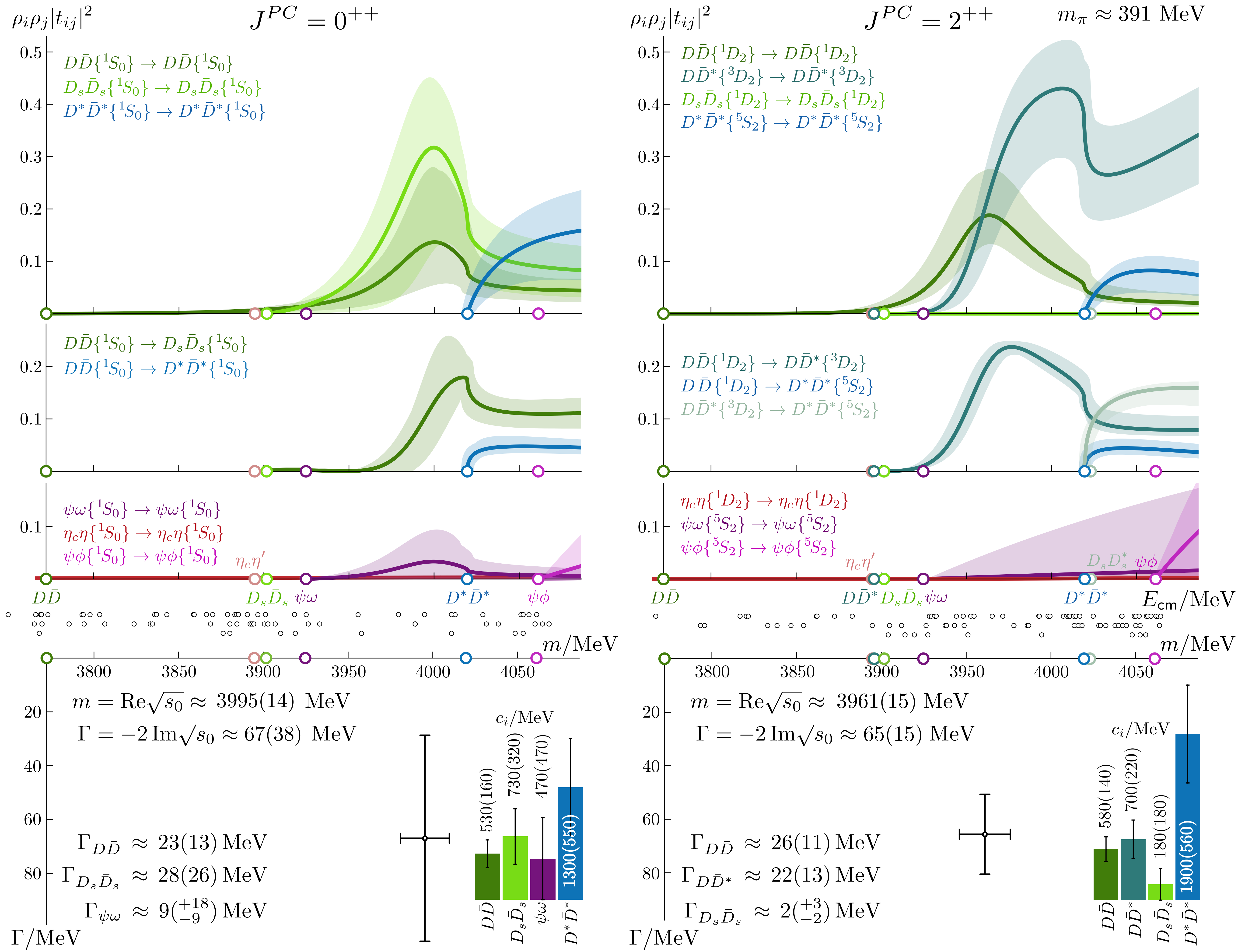
$J^{PC} = 0^{++}$

$B^+ \to D^{*\pm} D^{\mp} K^+$

LHCb arXiv:2406.03156



Very complicated
Many amplitudes contribute with similar strength
New resonances proposed around 4000 MeV



$\rho_i\rho_j|t_{ij}|^2$
$J^{PC} = 0^{++}$
$J^{PC} = 2^{++}$
$m_\pi \approx 391$ MeV
$D\bar{D}\{^1S_0\} \to D\bar{D}\{^1S_0\}$
$D_s\bar{D}_s\{^1S_0\} \to D_s\bar{D}_s\{^1S_0\}$
$D^*\bar{D}^*\{^1S_0\} \to D^*\bar{D}^*\{^1S_0\}$
$D\bar{D}\{^1S_0\} \to D_s\bar{D}_s\{^1S_0\}$
$D\bar{D}\{^1S_0\} \to D^*\bar{D}^*\{^1S_0\}$
$\psi\omega\{^1S_0\} \to \psi\omega\{^1S_0\}$
$\eta_c\eta\{^1S_0\} \to \eta_c\eta\{^1S_0\}$
$\psi\phi\{^1S_0\} \to \psi\phi\{^1S_0\}$
$D\bar{D}\{^1D_2\} \to D\bar{D}\{^1D_2\}$
$D\bar{D}^*\{^3D_2\} \to D\bar{D}^*\{^3D_2\}$
$D_s\bar{D}_s\{^1D_2\} \to D_s\bar{D}_s\{^1D_2\}$
$D^*\bar{D}^*\{^5S_2\} \to D^*\bar{D}^*\{^5S_2\}$
$D\bar{D}\{^1D_2\} \to D\bar{D}^*\{^3D_2\}$
$D\bar{D}\{^1D_2\} \to D^*\bar{D}^*\{^5S_2\}$
$D\bar{D}^*\{^3D_2\} \to D^*\bar{D}^*\{^5S_2\}$
$\eta_c\eta\{^1D_2\} \to \eta_c\eta\{^1D_2\}$
$\psi\omega\{^5S_2\} \to \psi\omega\{^5S_2\}$
$\psi\phi\{^5S_2\} \to \psi\phi\{^5S_2\}$
$\eta_c\eta'$
$D_sD_s^*$
$D\bar{D}$ $D_s\bar{D}_s$ $\psi\omega$ $D^*\bar{D}^*$ $\psi\phi$
$D\bar{D}$ $D\bar{D}^*$ $D_s\bar{D}_s$ $\psi\omega$ $D^*\bar{D}^*$
E_{cm}/MeV
m/MeV
3800 3850 3900 3950 4000 4050
Γ/MeV
$m = \mathrm{Re}\sqrt{s_0} \approx 3995(14)$ MeV
$\Gamma = -2\,\mathrm{Im}\sqrt{s_0} \approx 67(38)$ MeV
$\Gamma_{D\bar{D}} \approx 23(13)$ MeV
$\Gamma_{D_s\bar{D}_s} \approx 28(26)$ MeV
$\Gamma_{\psi\omega} \approx 9(^{+18}_{-9})$ MeV
c_i/MeV
530(160) 730(320) 470(470) 1300(550)
$m = \mathrm{Re}\sqrt{s_0} \approx 3961(15)$ MeV
$\Gamma = -2\,\mathrm{Im}\sqrt{s_0} \approx 65(15)$ MeV
$\Gamma_{D\bar{D}} \approx 26(11)$ MeV
$\Gamma_{D\bar{D}^*} \approx 22(13)$ MeV
$\Gamma_{D_s\bar{D}_s} \approx 2(^{+3}_{-2})$ MeV
580(140) 700(220) 180(180) 1900(560)

Scalar and tensor charmonium

- at m_π=391 MeV, one scalar and one tensor pole is found.
- The **level counting** is not obviously different from the **quark model**

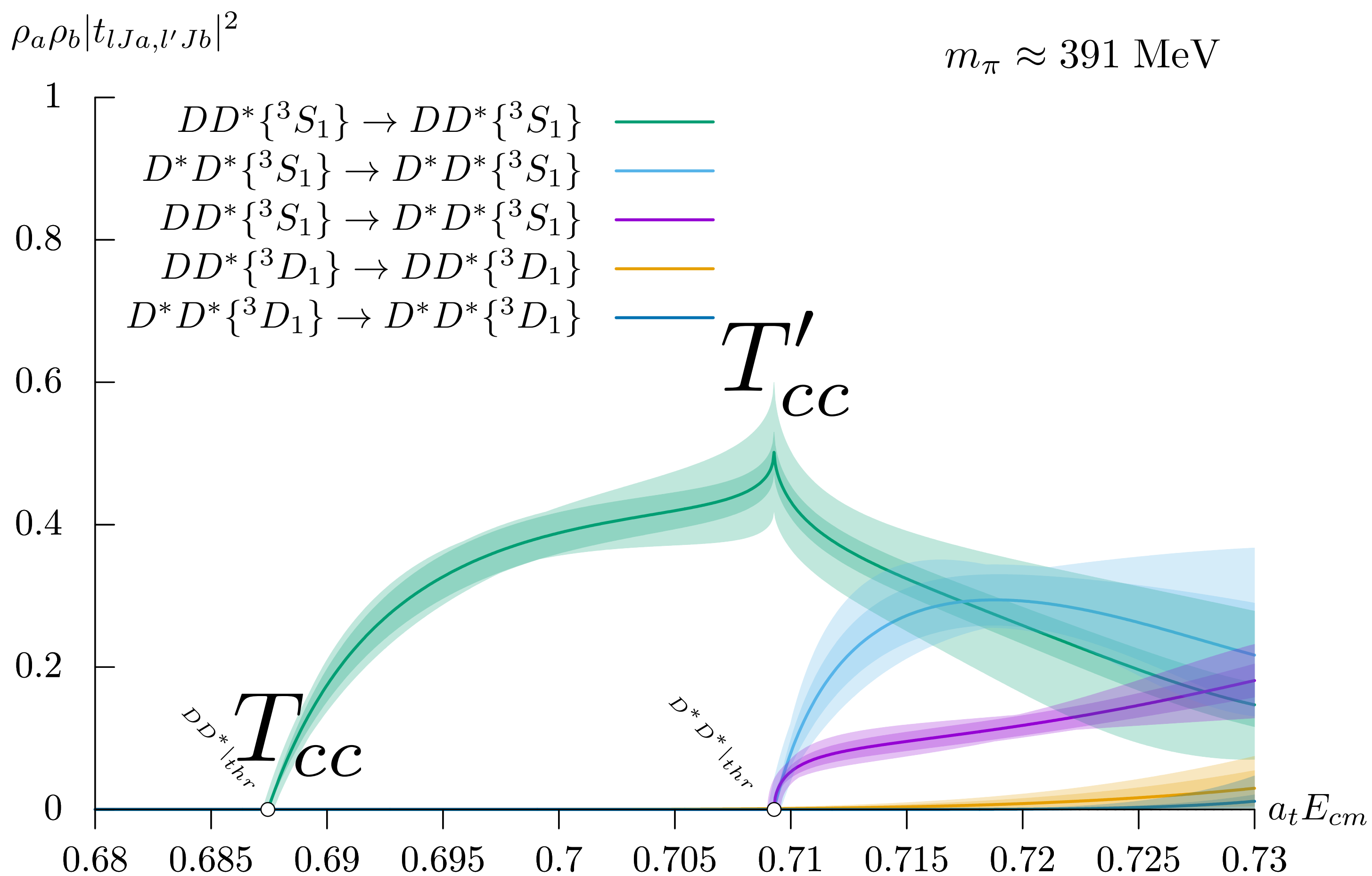
- large **coupled-channel** effects in OZI **connected D-meson channels**
- OZI **disconnected** channels look **small everywhere**

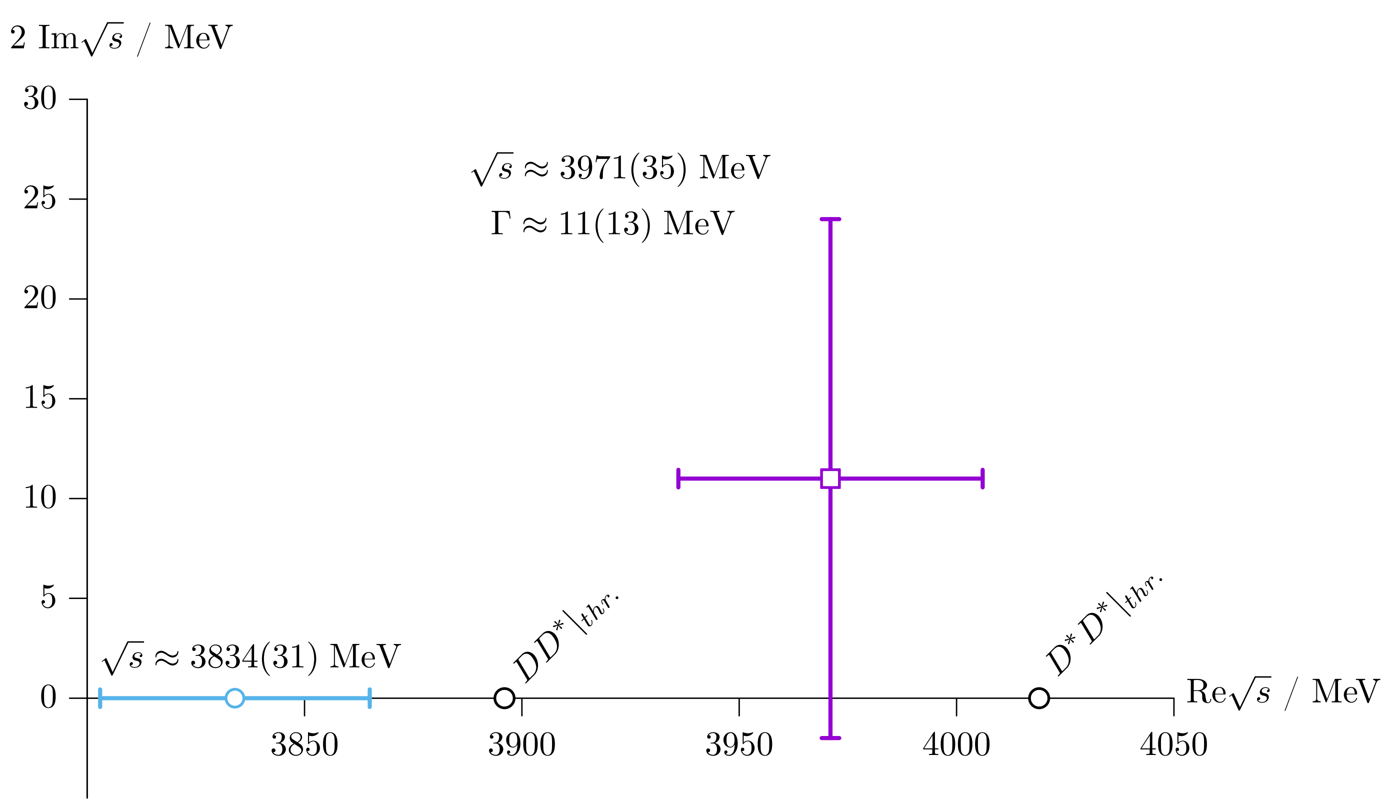
- we have extracted a **complete** unitary **S-matrix** and this naturally **connects** features seen in **different channels** and simplifies the overall picture
- some amplitudes are **very different** to the simple **Breit-Wigners** often used in experimental analyses

- a clear, as yet unobserved, 3++ resonance is present in DDbar* & a bound state in 2-+
- we **do not find** a **near-threshold DDbar** state (between 3700 and 3860 MeV)

- these methods can also be applied to the X(3872) 1++ channel

Full details and references:
arXiv: 2309.14070 (7 pages)
arXiv: 2309.14071 (55 pages)

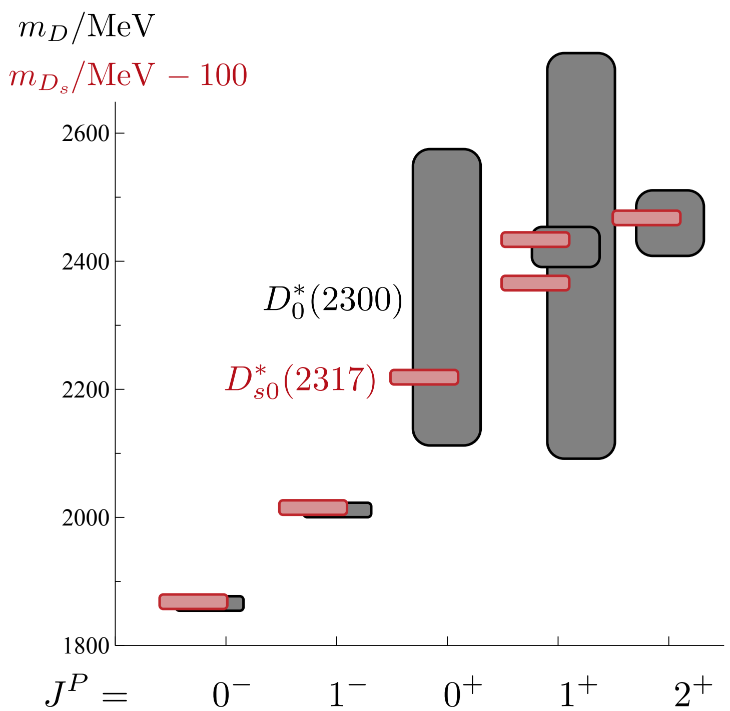


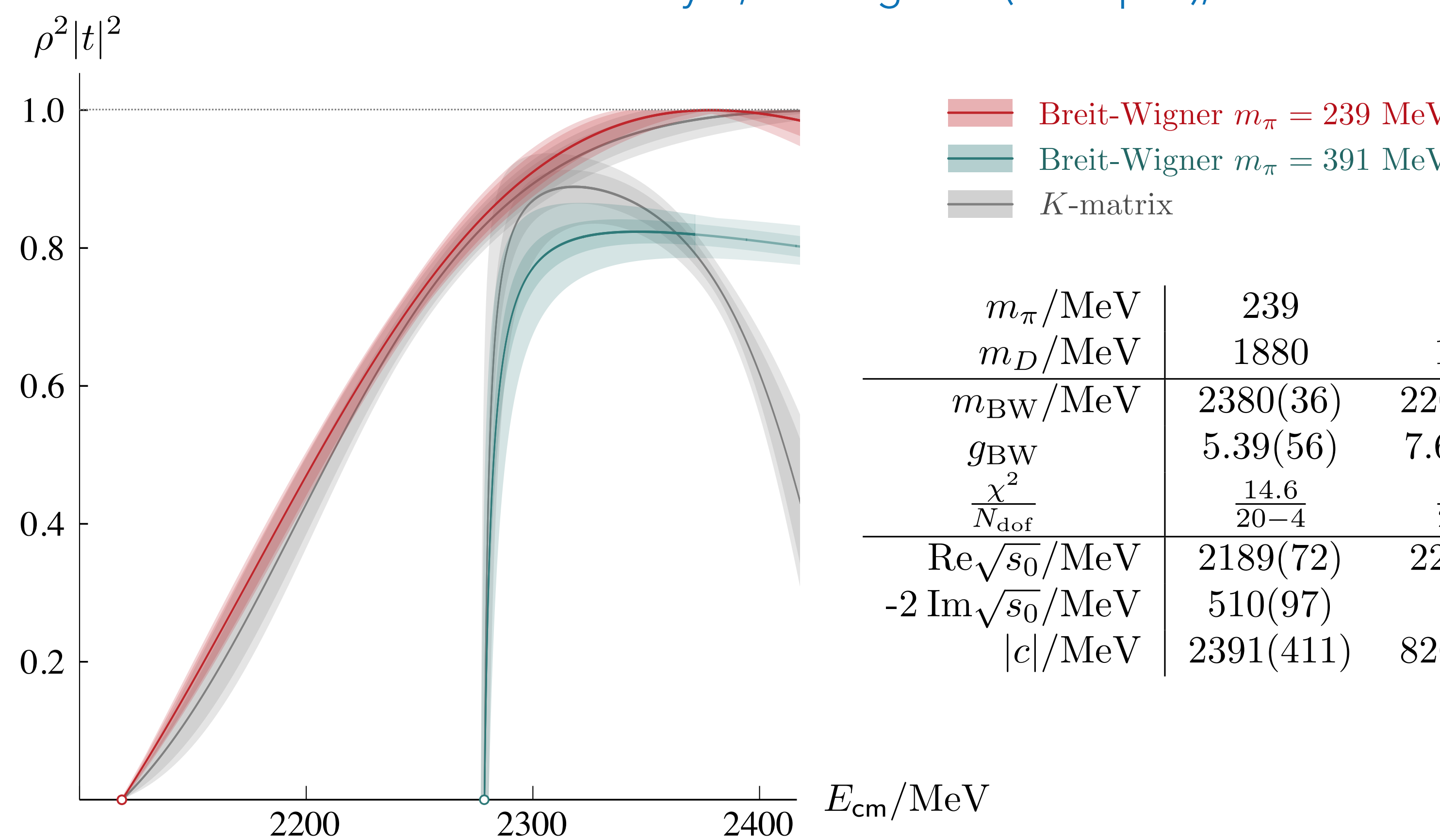


DD*-D*D* coupled channel
Whyte, Wilson, Thomas
[arXiv:2405.15741]

- *S* and *D*-wave in $J^P=1^+$
- virtual bound state below DD* and resonance below D*D*
- (neglecting left cuts)

L. Gayer, N. Lang et al (HadSpec), arXiv:2102.04973

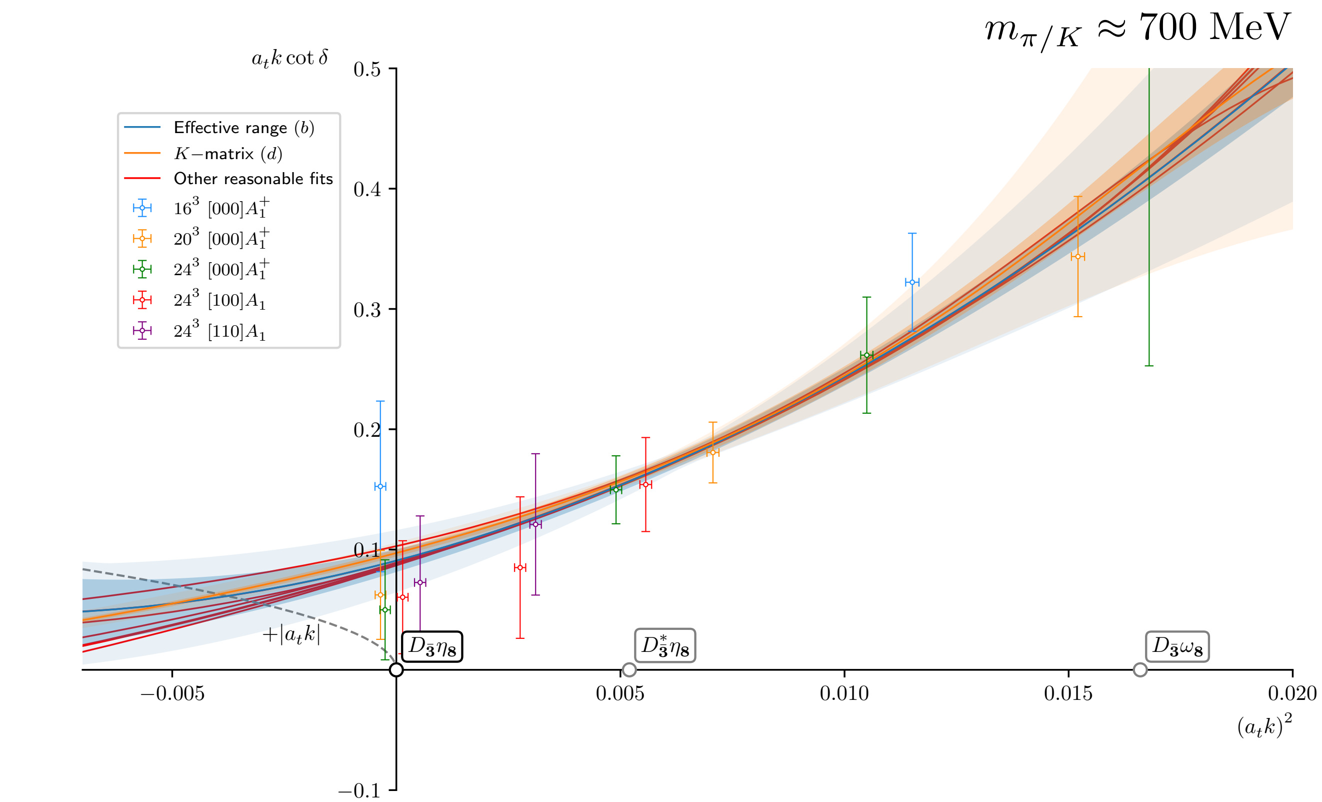




m_π/MeV	239	391
m_D/MeV	1880	1887
m_{BW}/MeV	2380(36)	2206(32)
g_{BW}	5.39(56)	7.62(75)
$\frac{\chi^2}{N_{dof}}$	$\frac{14.6}{20-4}$	$\frac{36.0}{29-5}$
Re$\sqrt{s_0}$/MeV	2189(72)	2275(1)
-2 Im$\sqrt{s_0}$/MeV	510(97)	-
$\|c\|$/MeV	2391(411)	826(133)

$D_0^*(2300)$ & $D_{s0}^*(2317)$

what is the mass ordering?
why are the masses so close?
why are the widths so different?

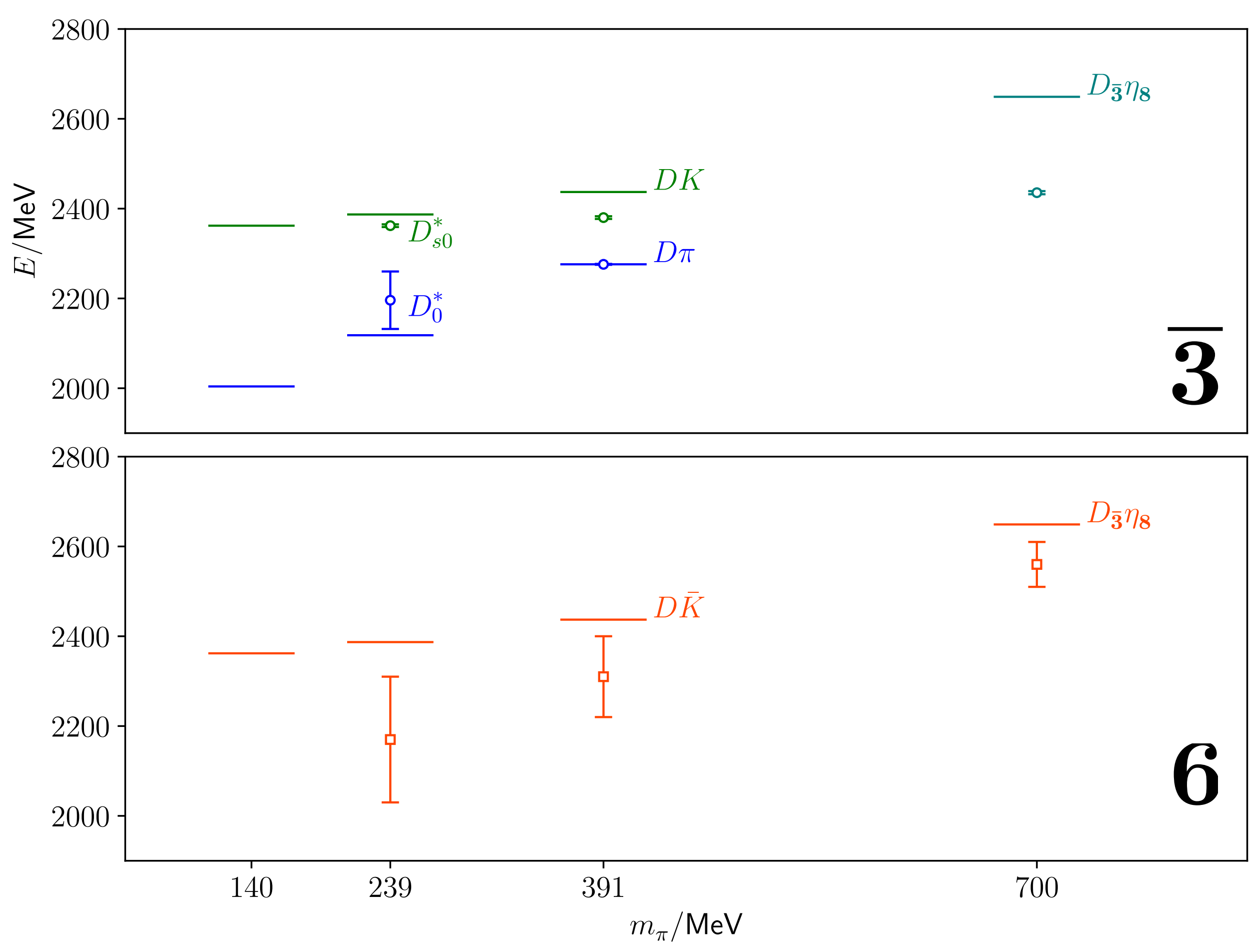


Dπ/DK scattering with SU(3) flavour symmetry

Yeo, Thomas, Wilson

arXiv:2403.10498

- S-wave interactions in flavour SU(3) 3bar, 6, 15bar
- Virtual bound state sextet pole
- Also deeply bound 3bar state, similar to Ds0(2317), much greater binding



SU(3) flavour:
D-meson and light meson

$$\bar{\mathbf{3}} \otimes \mathbf{8} \to \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}$$

$$\bar{\mathbf{3}} \otimes \mathbf{1} \to \bar{\mathbf{3}}$$