Recent calculation of the magnetic moment of the muon

Z. Fodor

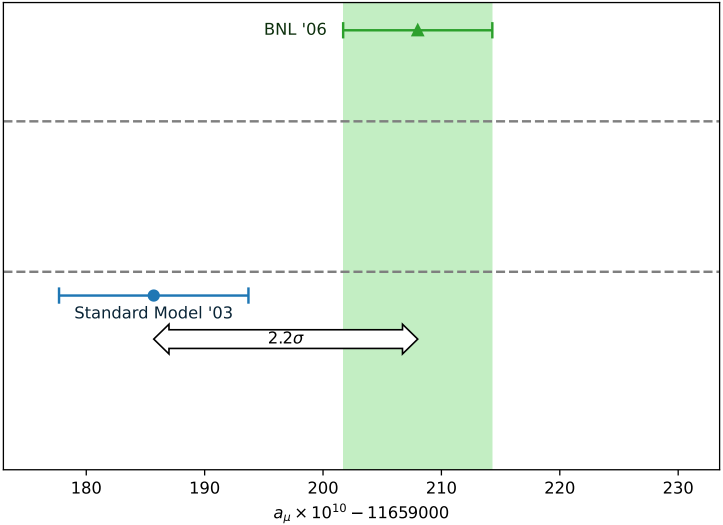
Penn State, Univ. Wuppertal, FZ Juelich, Univ. Budapest, UCSD

Nature 593 (7857), 51, 2021 (& arXiv: 2407.10913)

for the Budapest–Marseille–Wuppertal collaboration: BMW (& DMZ)

XIV ICNFP, Kolymbari, Greece, July 23, 2025

Take-home message: magnetic moment of the muon 2006



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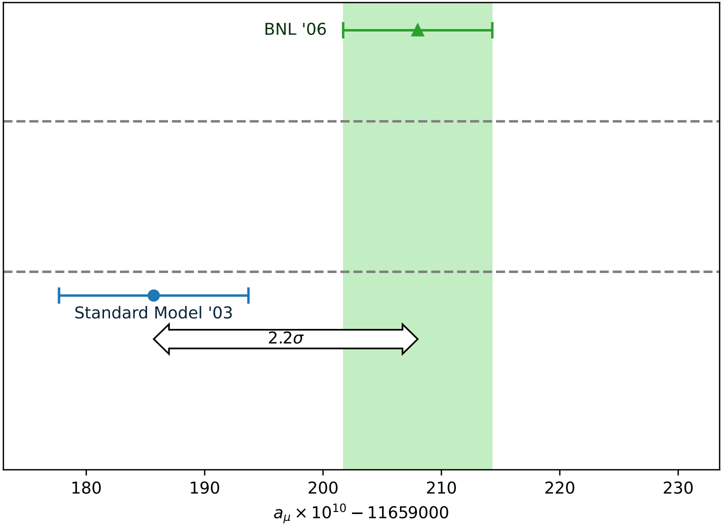

New physics needs $> 5\sigma$, new level of accuracies: both for theory/experiment

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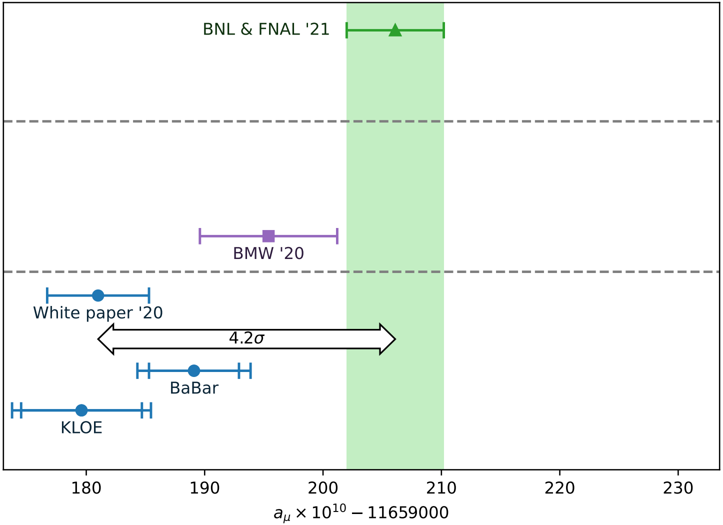

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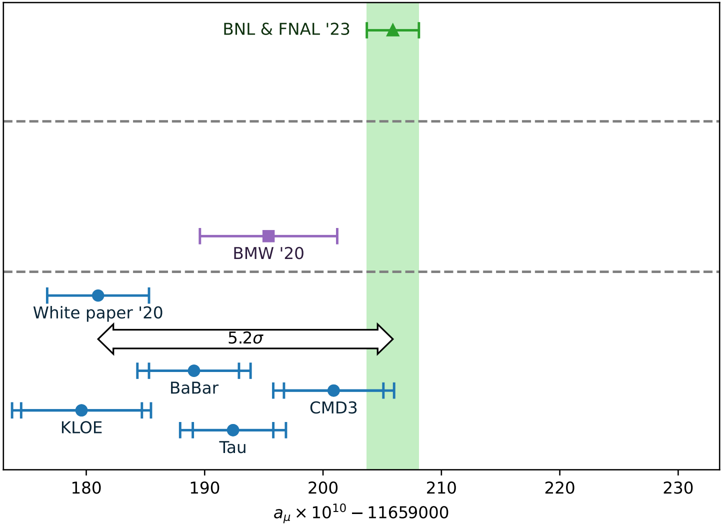
Take-home message: magnetic moment of the muon 2023



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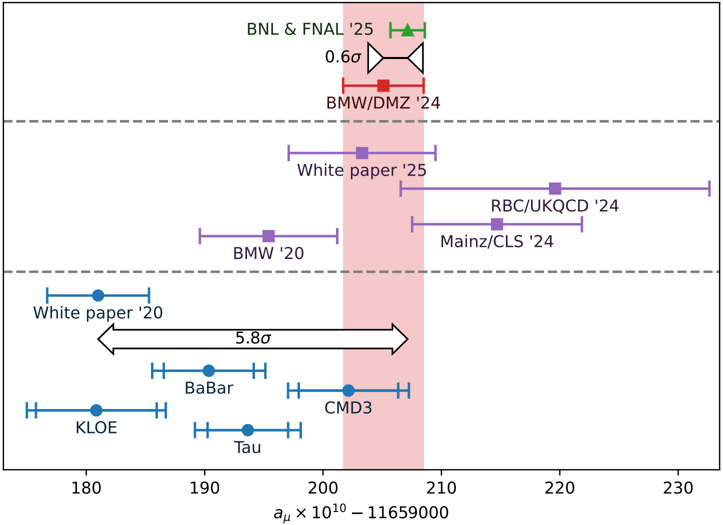
Take-home message: magnetic moment of the muon 2025 unpublished



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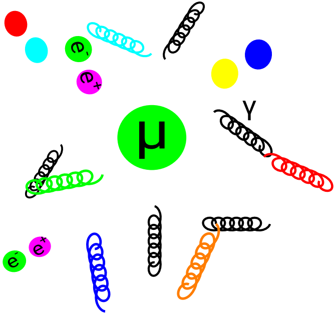

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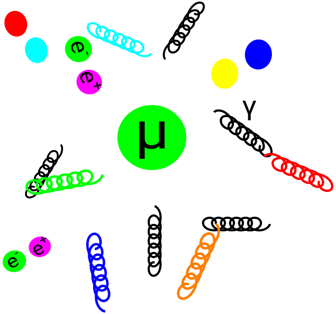

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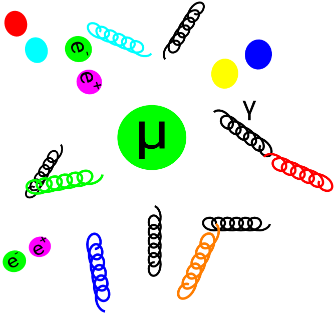

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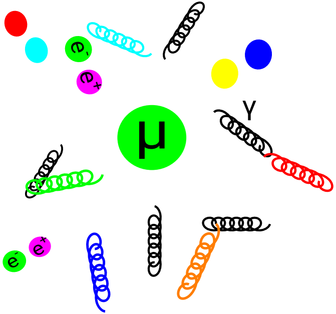

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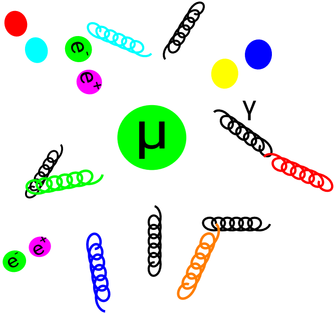

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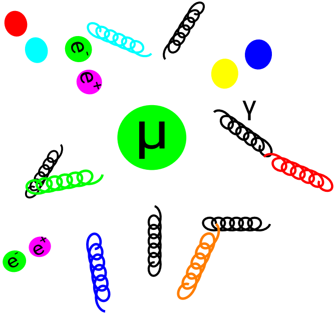

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	$a_\mu \times 10^{-10}$
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Total	11659181.0(4.3)

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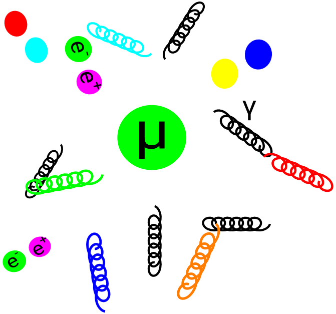

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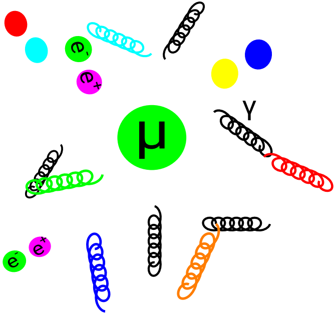

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QCD contribution: R-ratio method

R-Ratio: e^+e^- annihillation $\rightarrow$ photon $\rightarrow$ hadrons

Normalize the hadronic cross-section by the $\mu^+\mu^-$ cross section

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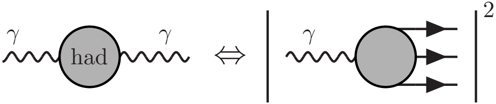
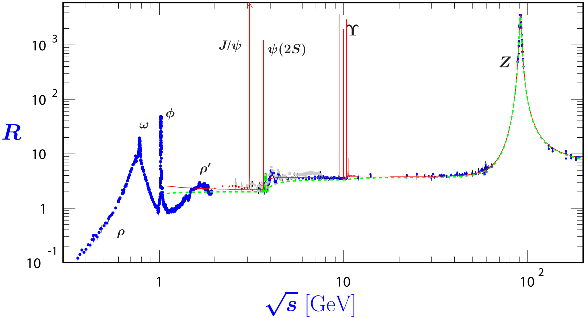


60 years of experiments: CMD, SND, BES, KLOE, BABAR, ...

$$\Rightarrow \left(\frac{\alpha}{\pi}\right)^2 \int \frac{ds}{s^2} K_\mu(s) R(s)$$



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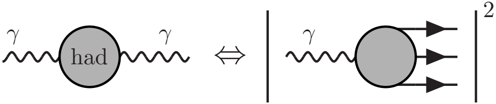

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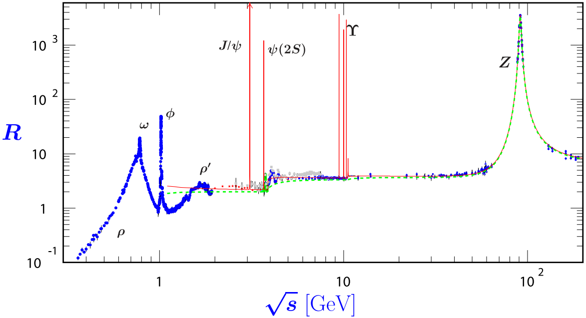

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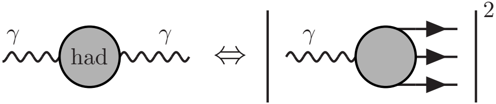

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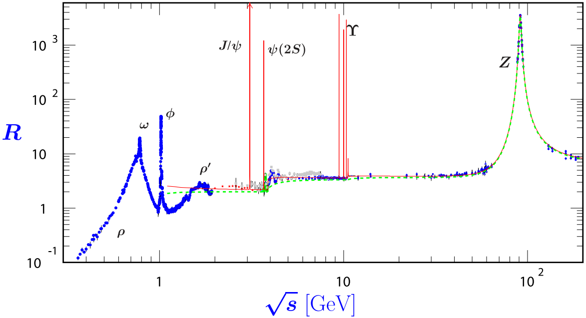

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2025 White Paper result: 713.2(6.1), or 0.86% error, lattice window averages

Tensions in the R-ratio method

CMD3 [2302.08834] $e^+e^- \to \pi^+\pi^-$ for $\sqrt{s}$: 0.60–0.88 GeV

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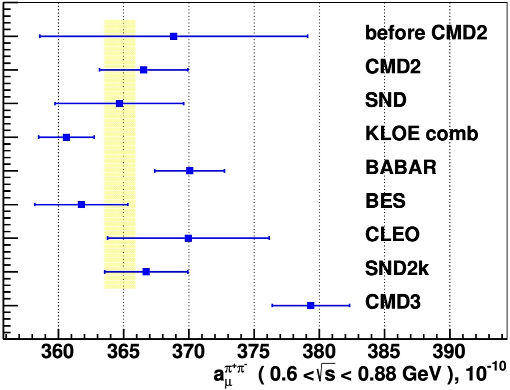

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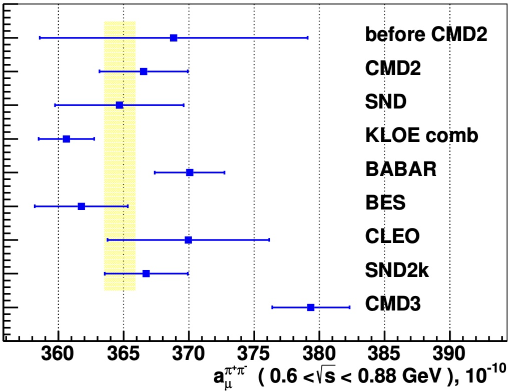

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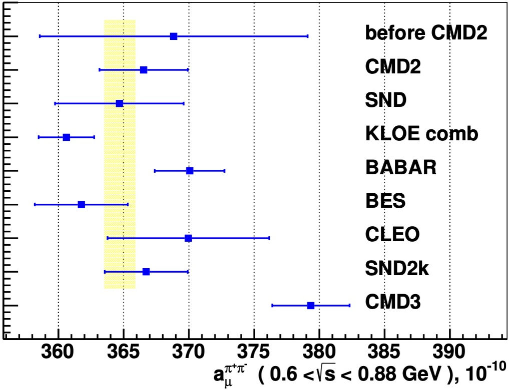

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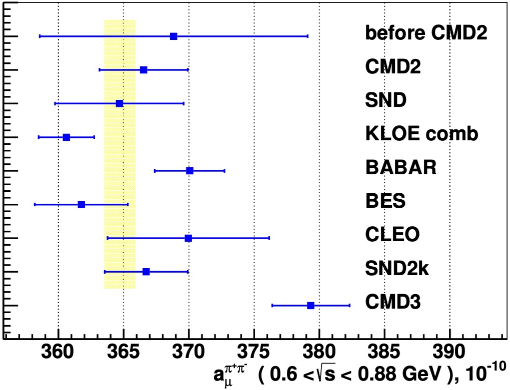

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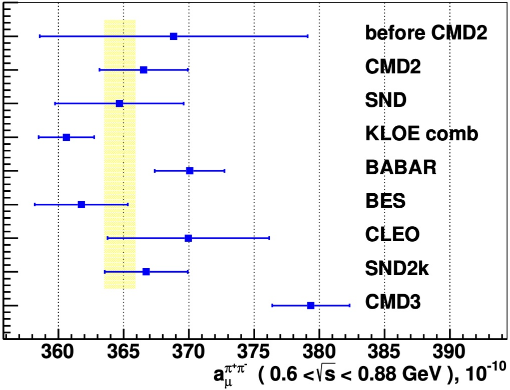

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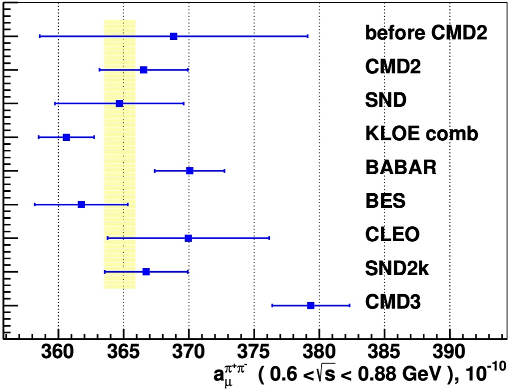

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central value (remember) 15(0.88 GeV) shift

Magnetic moment on the lattice Nature 593 (2021) 7857, 51

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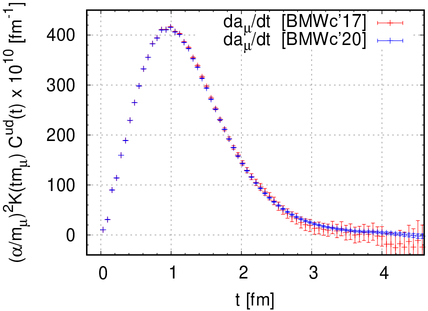
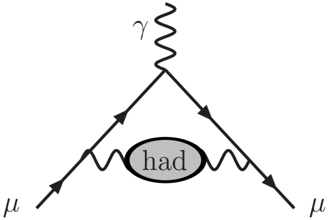
K(t) describes the muon-line:





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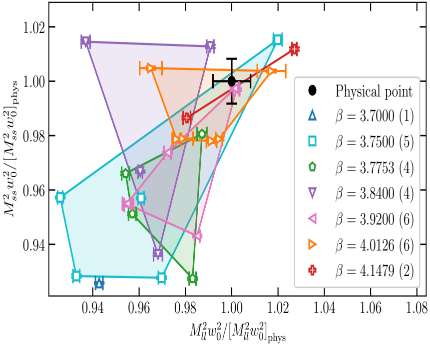

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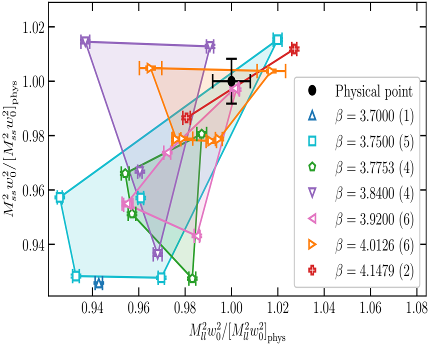
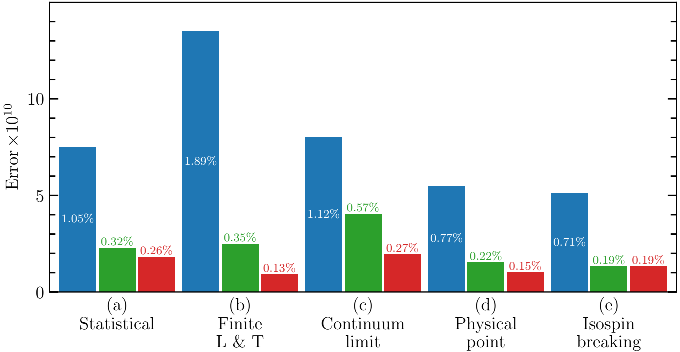


β	a[fm]	$L \times T$	#conf
3.7000	0.1315	48 × 64	904
3.7500	0.1191	56 × 96	2072
3.7753	0.1116	56 × 84	1907
3.8400	0.0952	64 × 96	3139
3.9200	0.0787	80 × 128	4296
4.0126	0.0640	96 × 144	6980
4.1479	0.0480	128 × 192	5017

CPU demand scales as $\approx a^{-8}$:
very careful planning needed

Error reduction 2017 (blue) – 2020 (green) - 2024 (red)



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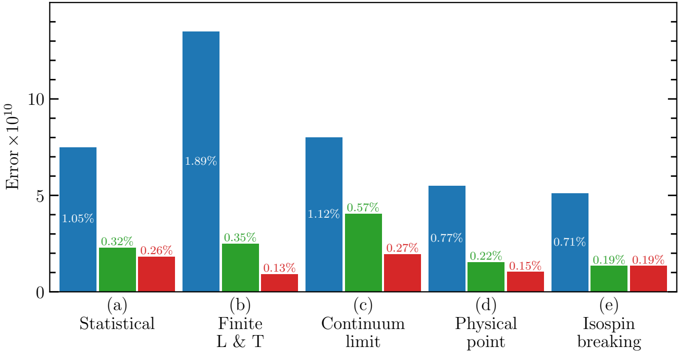

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$\Longrightarrow$ in 2020 dedicated large volume simulation

earlier only 6 fm box, increased to 11 fm

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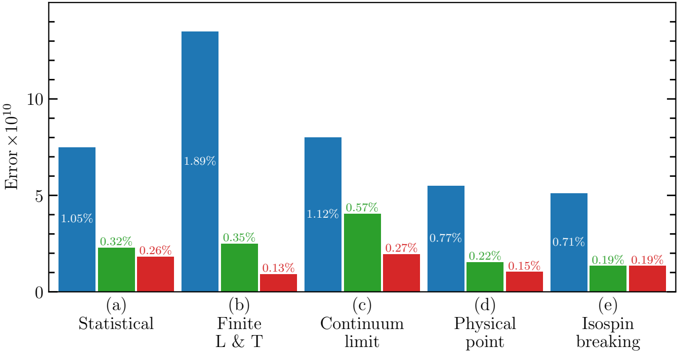

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$\Longrightarrow$ new, even finer lattice with 0.048 fm lattice spacing (alltogether seven)

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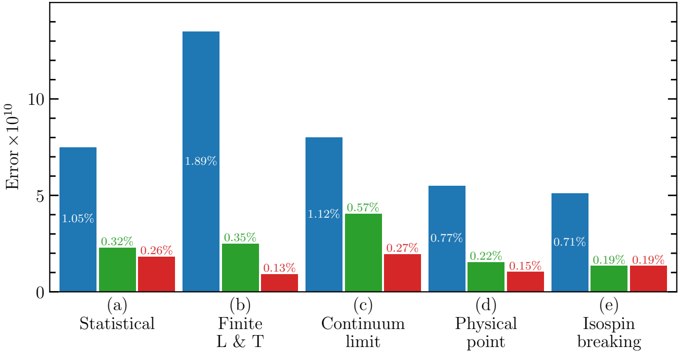

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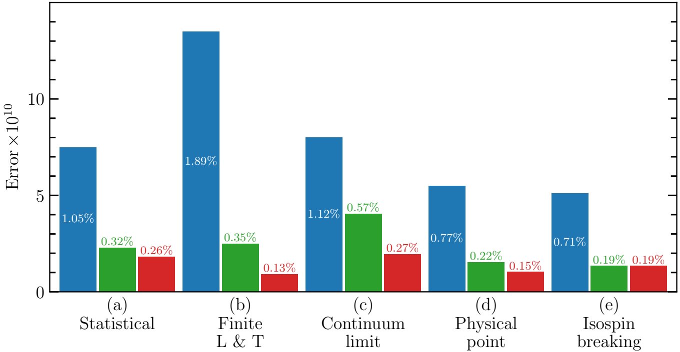

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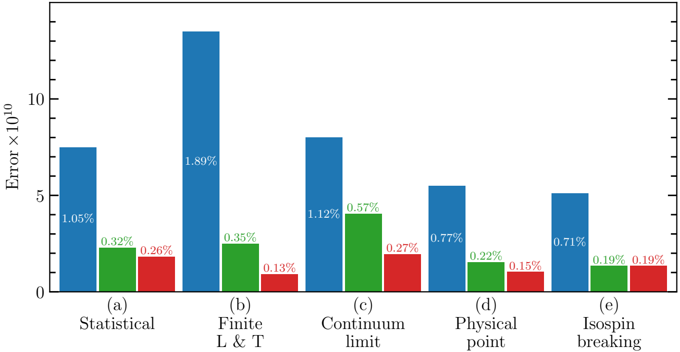

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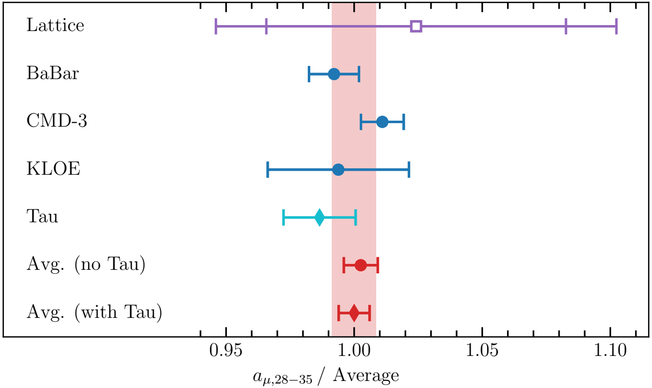
- Scale for physical point & dedicated isospin breaking analysis

Tail (long distance) contribution: $t > 2.8$ fm



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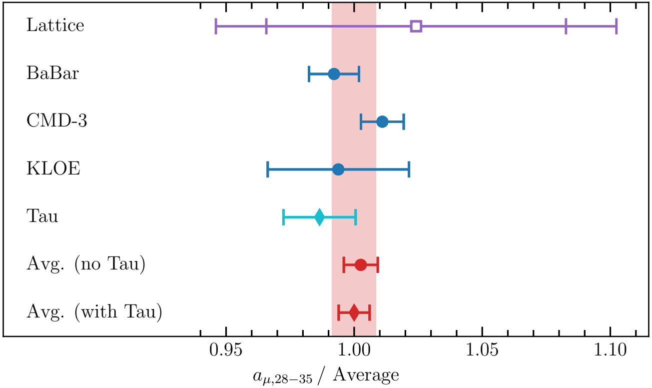
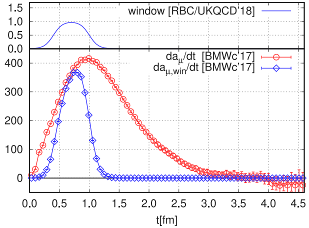


- $t > 2.8$ fm contributes only less than 5%
- data driven uncertainty: an order of magnitude better than lattice
- low energy part (below ρ) all agree
- we could be generous with errors: even 5 times larger wouldn't change the final error

Window observable

- Restrict correlator to window between $t_1 = 0.4\,\mathrm{fm}$ and $t_2 = 1.0\,\mathrm{fm}$

[RBC/UKQCD'18]



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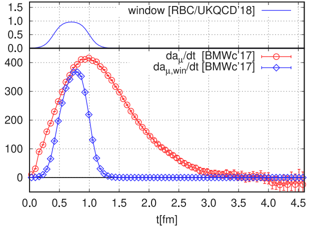

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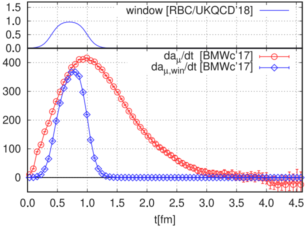

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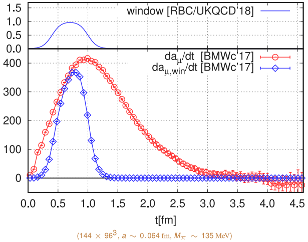

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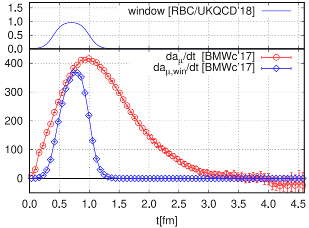
easier (CPU and manpower)

Window observable

- Restrict correlator to window between $t_1 = 0.4\,\text{fm}$ and $t_2 = 1.0\,\text{fm}$



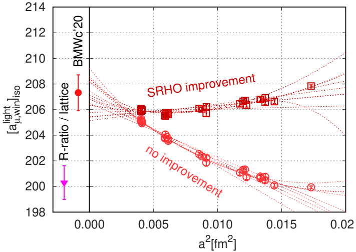
$(144 \times 96^3,\ a \sim 0.064\,\text{fm},\ M_\pi \sim 135\,\text{MeV})$

[RBC/UKQCD'18]

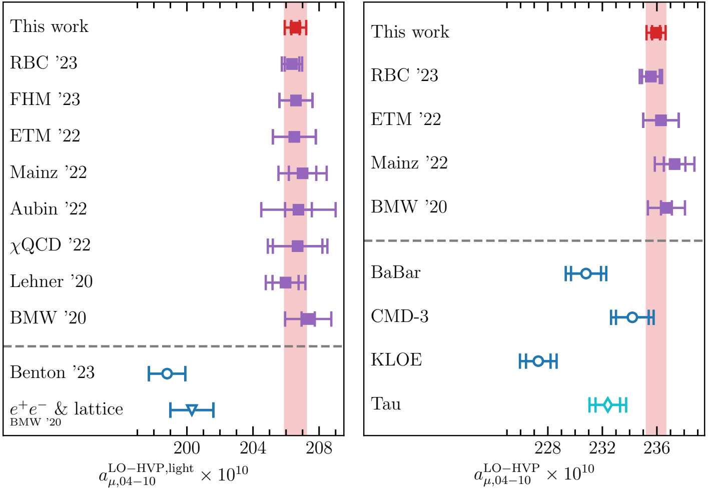
- Less challenging than full a_μ
  - signal/noise
  - lattice artefacts (short & long)
  - finite size effects
  - R-ratio possible: other kernel

about two orders of magnitude easier (CPU and manpower)

histogram of 250,000 fits
with and without improvements
3.7 σ tension (already in 2020)

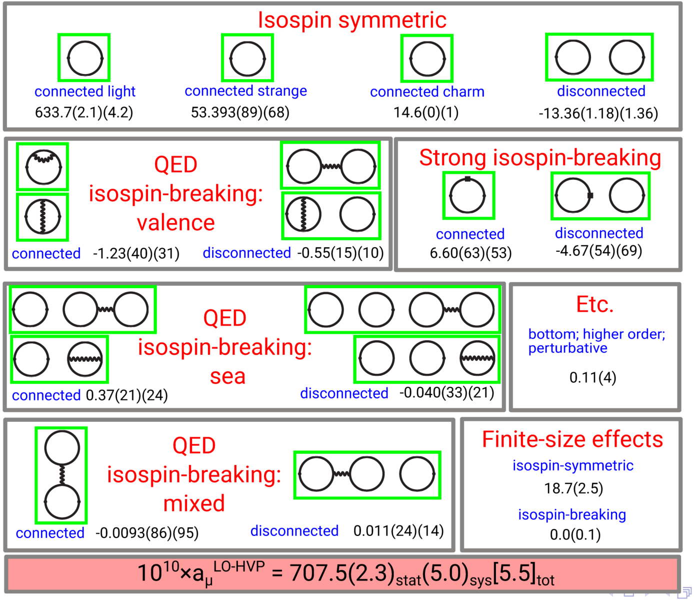


Window observable tension: other groups



Huge tension with our result and with the average even more

Final result: 2020



Isospin symmetric

connected light 633.7(2.1)(4.2)

connected strange 53.393(89)(68)

connected charm 14.6(0)(1)

disconnected -13.36(1.18)(1.36)

QED isospin-breaking: valence

connected -1.23(40)(31)

disconnected -0.55(15)(10)

Strong isospin-breaking

connected 6.60(63)(53)

disconnected -4.67(54)(69)

QED isospin-breaking: sea

connected 0.37(21)(24)

disconnected -0.040(33)(21)

Etc.

bottom; higher order; perturbative

0.11(4)

QED isospin-breaking: mixed

connected -0.0093(86)(95)

disconnected 0.011(24)(14)

Finite-size effects

isospin-symmetric 18.7(2.5)

isospin-breaking 0.0(0.1)

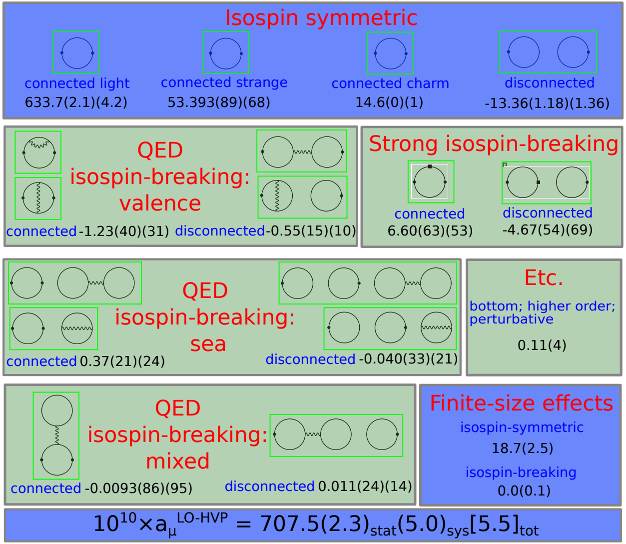
$$10^{10}\times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$$

Final result: 2020⇒2024

Final result: 2020⇒2024

Improved in new work

Some checks in new work



Isospin symmetric

connected light	connected strange	connected charm	disconnected
633.7(2.1)(4.2)	53.393(89)(68)	14.6(0)(1)	-13.36(1.18)(1.36)

QED isospin-breaking: valence

connected -1.23(40)(31) disconnected -0.55(15)(10)

Strong isospin-breaking

connected 6.60(63)(53)

disconnected -4.67(54)(69)

QED isospin-breaking: sea

connected 0.37(21)(24) disconnected -0.040(33)(21)

Etc.

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0.11(4)

QED isospin-breaking: mixed

connected -0.0093(86)(95) disconnected 0.011(24)(14)

Finite-size effects

isospin-symmetric

18.7(2.5)

isospin-breaking

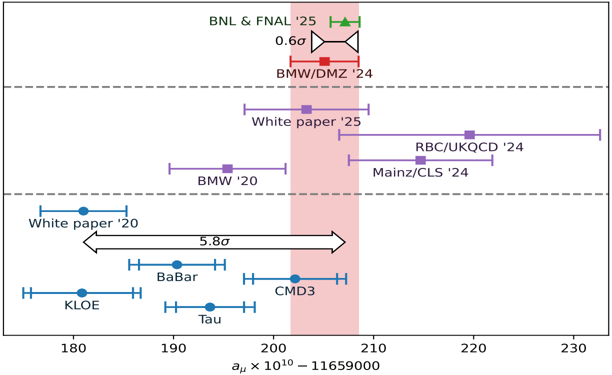
0.0(0.1)

$$10^{10}\times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$$

↓ 2024

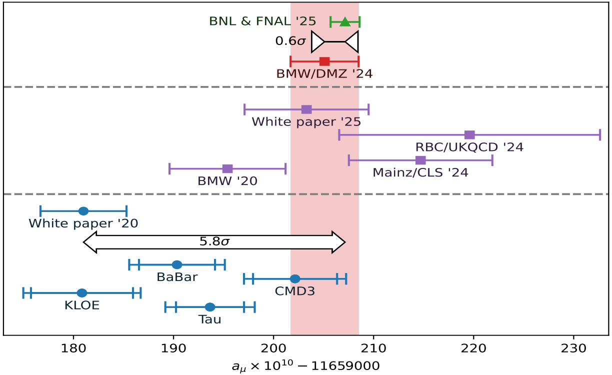
$$10^{10}\times a_\mu^{\text{LO-HVP}} = 714.1(2.2)_{\text{stat}}(2.5)_{\text{sys}}[3.3]_{\text{tot}}$$

Take-home message: success of Quantum Field Theory up to 11 digits



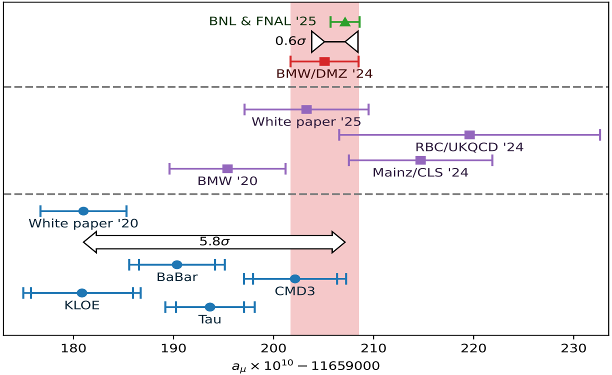
- Very different theories/techniques and we are sensitive to them:

Take-home message: success of Quantum Field Theory up to 11 digits



- Very different theories/techniques and we are sensitive to them:
  QED: many loops/diagrams with renormalization
  weak theory: renormalization and spontaneous symmetry breaking
  QCD: lattice formulation and non-perturbative renormalization

Take-home message: success of Quantum Field Theory up to 11 digits



- Very different theories/techniques and we are sensitive to them:
  QED: many loops/diagrams with renormalization
  weak theory: renormalization and spontaneous symmetry breaking
  QCD: lattice formulation and non-perturbative renormalization
- BMW is just one result: other groups should confirm or refute