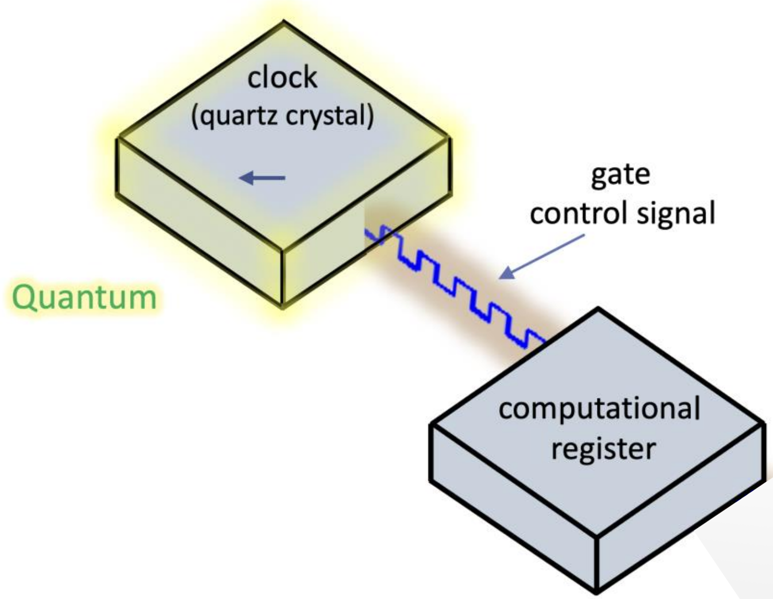
Quantum Frequential Computing:
a quadratic runtime advantage for computation

ICNF, Crete. 29/07/2025

ArXiv: 2403.02389

Mischa Woods

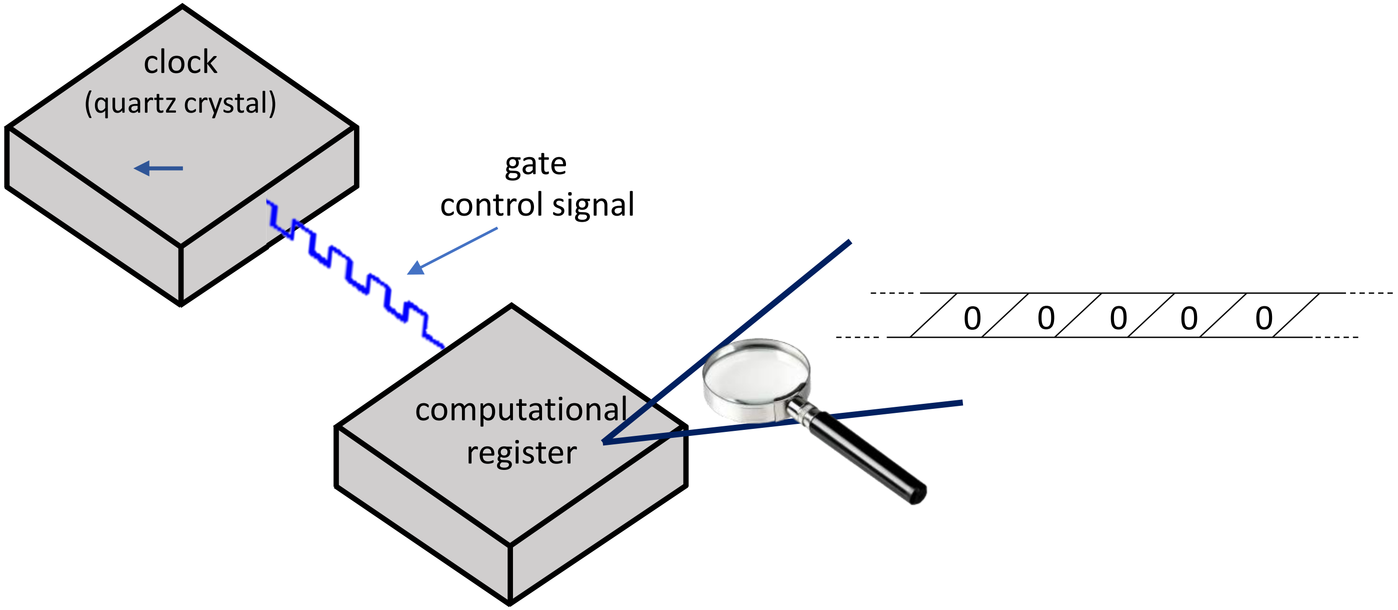
ENS Lyon, France



Inria

Motivation

Framework: classical & conventional quantum computers



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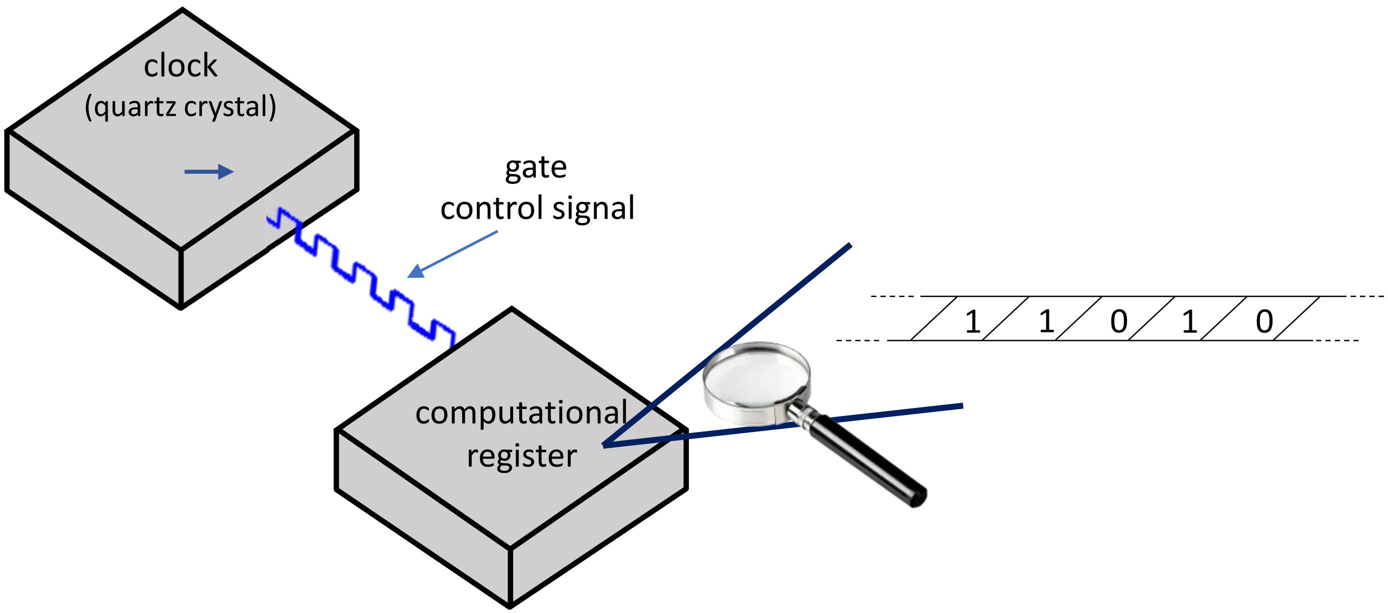

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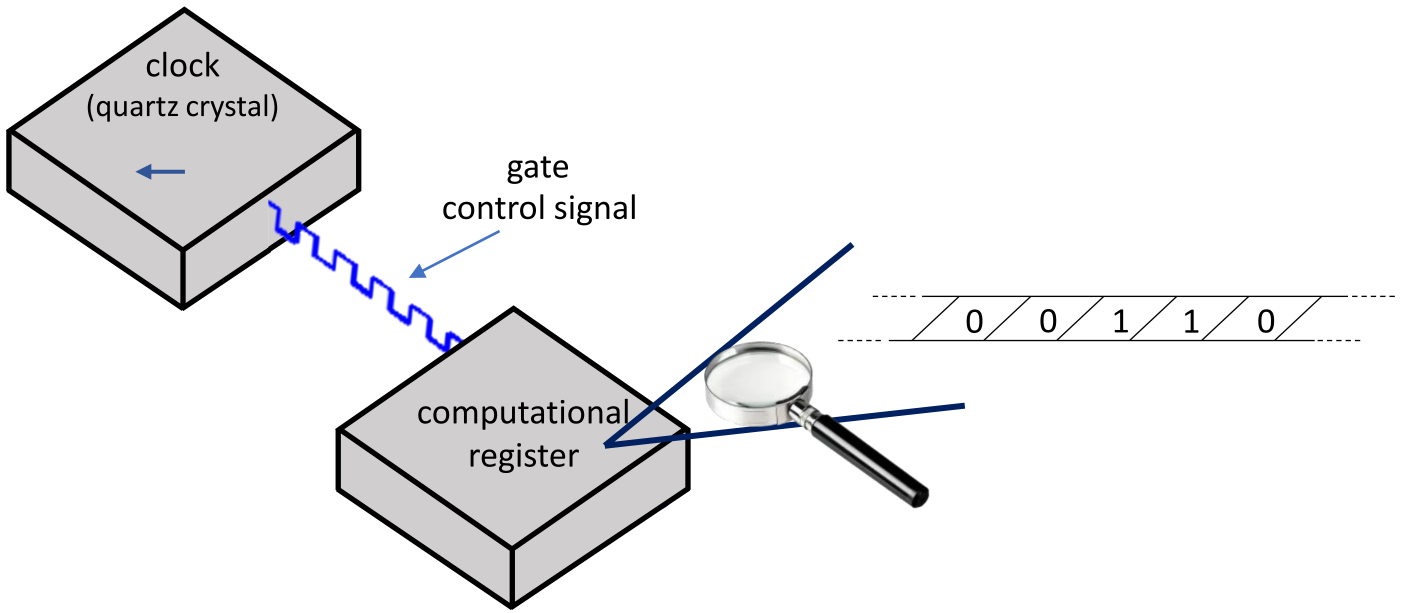

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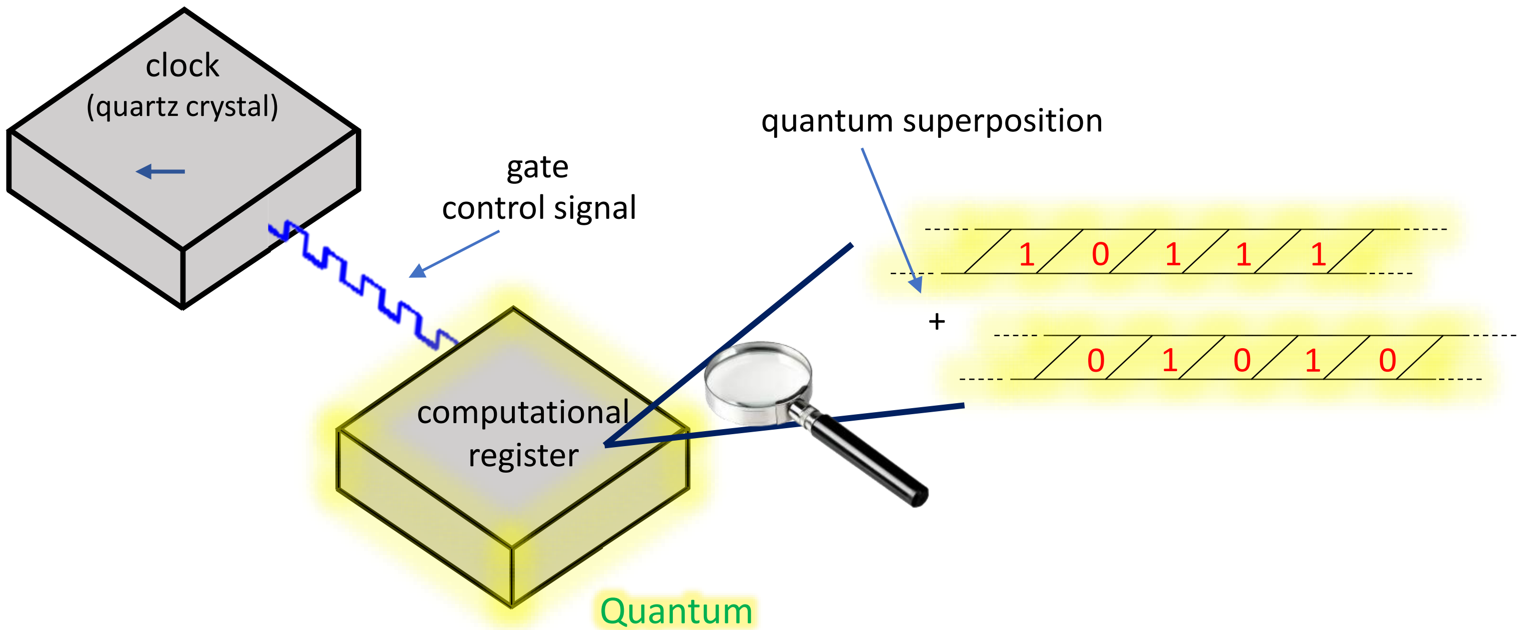

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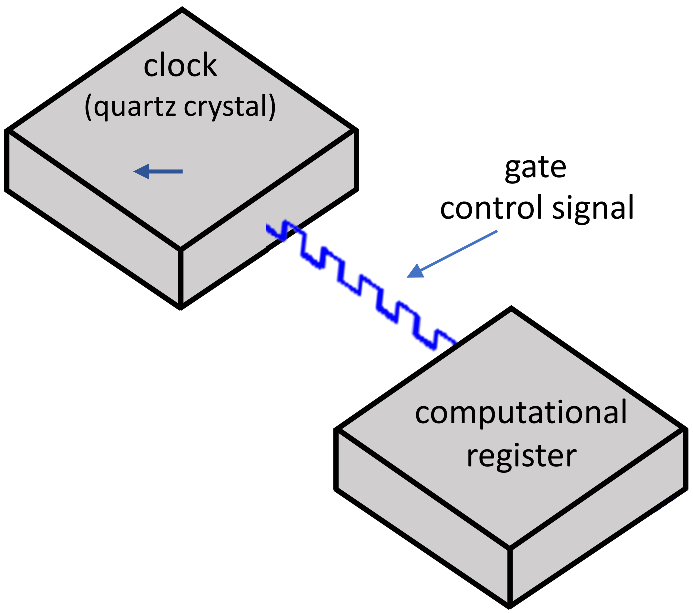

$$\text{run time} \sim \frac{\text{total } \# \text{ of gates}}{f}$$

gate frequency

Motivation

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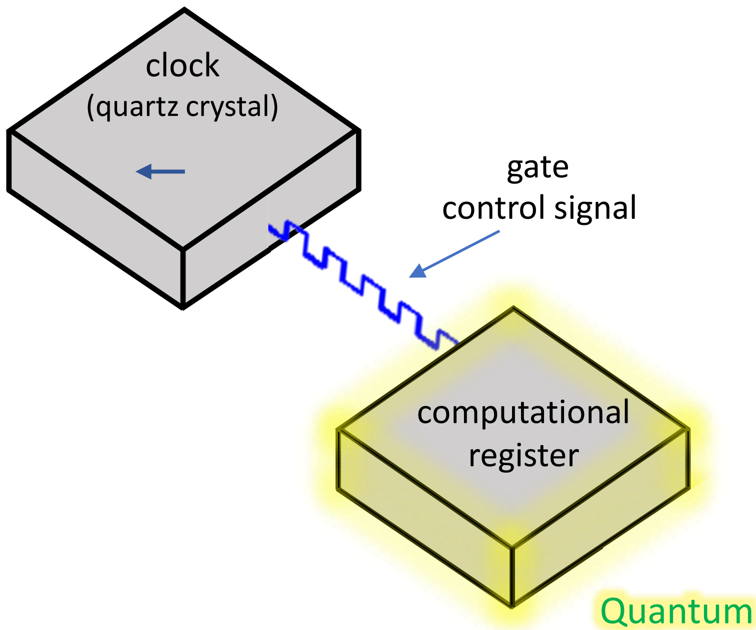
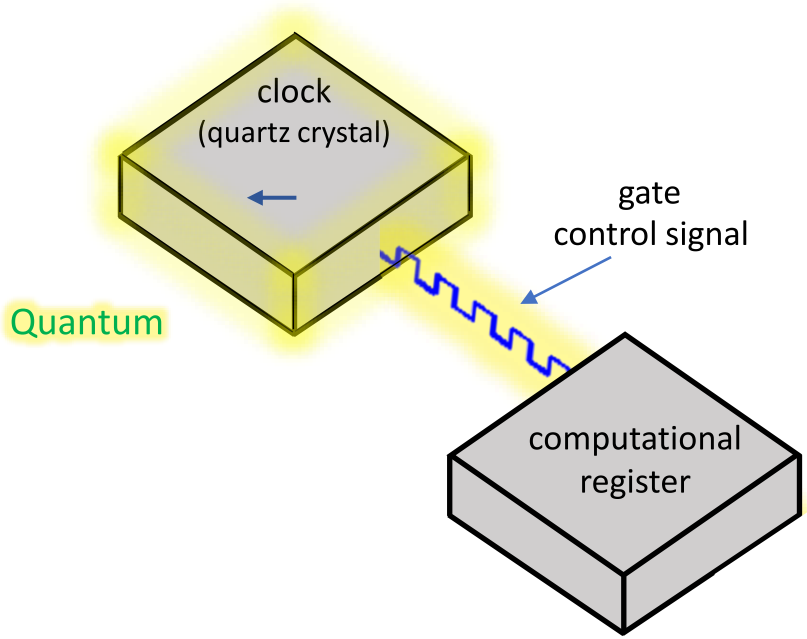


Conventional quantum advantage

$$\text{run time} \sim \frac{\text{total \# of gates}}{f}$$

Motivation

New Framework



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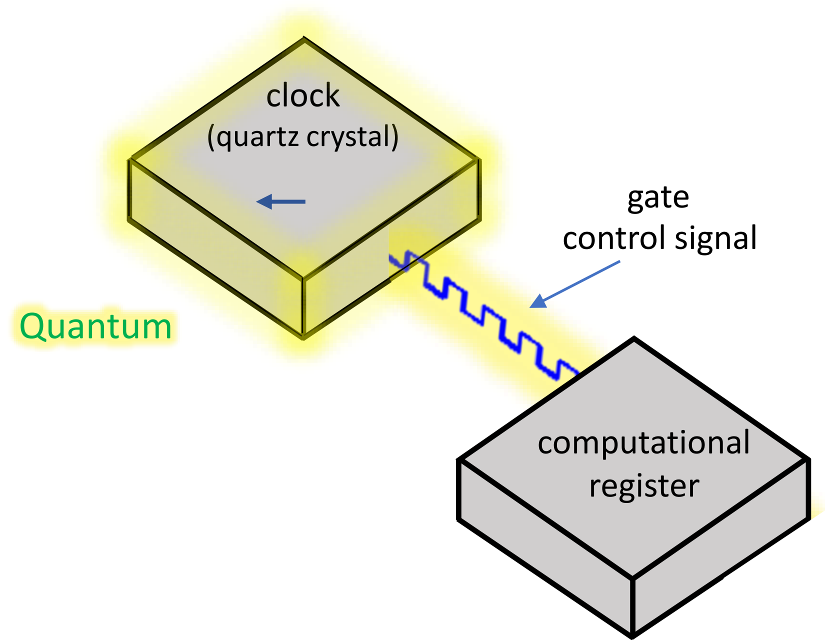
New quantum advantage?

Motivation

New Framework



Difficulties

The higher the clock frequency:

1: harder for gates to keep up

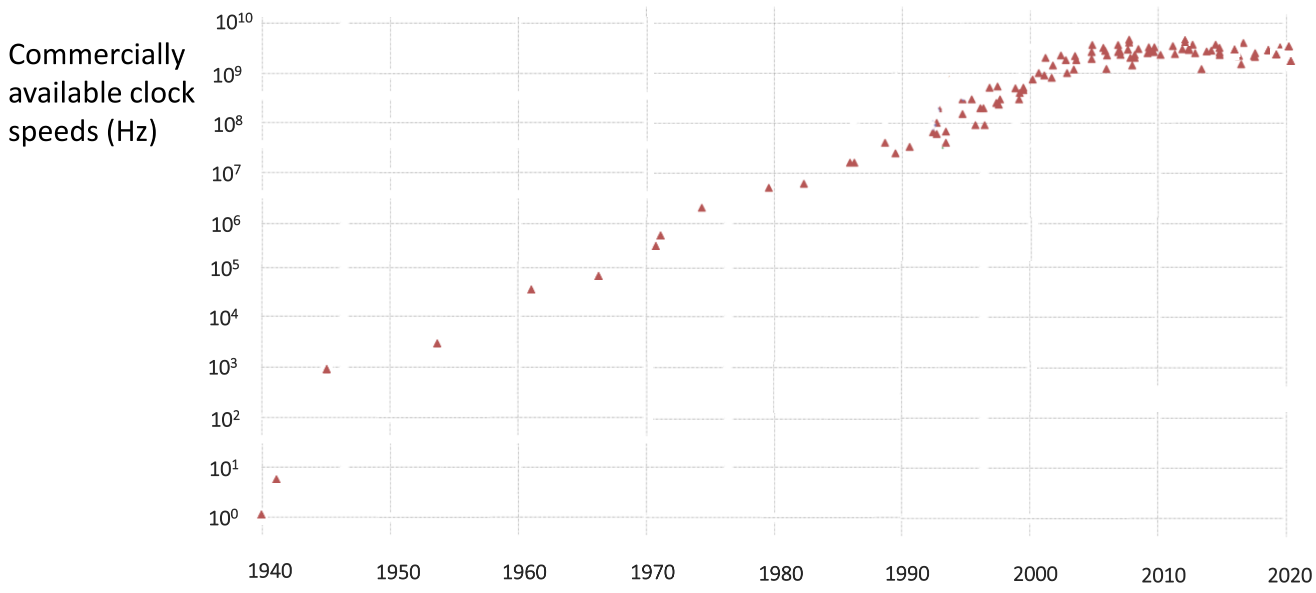
2: more heat generated

$$\text{run time} \sim \frac{\text{total \# of gates}}{f}$$

New quantum advantage?

Motivation

Why bother? Is frequency a bottleneck in the state of the art?



Source: The MosaicSim Simulator, O Matthews, *IEEE International Symposium on Performance Analysis of Systems and Software (2020)*
Disentangling Hype from Practicality: On Realistically Achieving Quantum Advantage, T. Hoefler, T. Häner, M. Troyer, Commun. ACM

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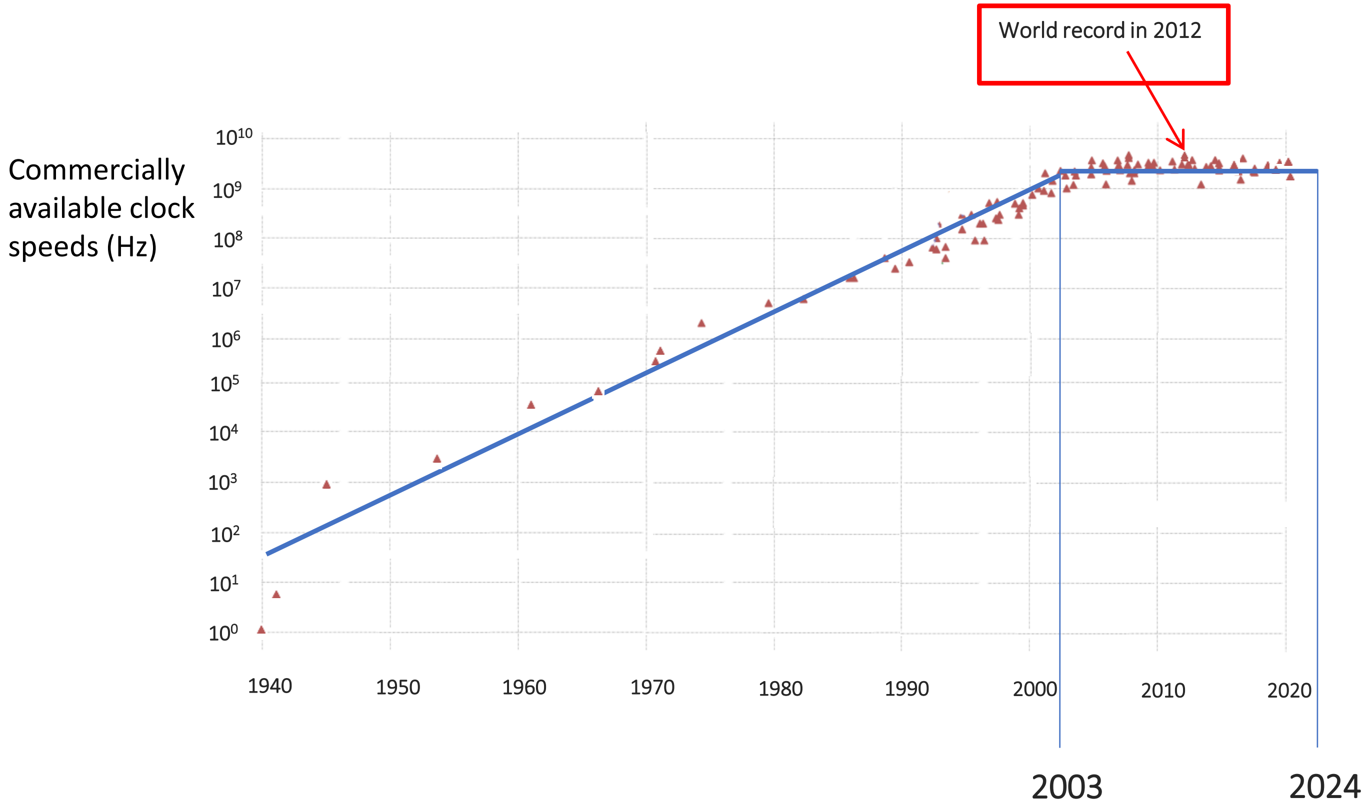
Why bother? Is frequency a bottleneck in the state of the art? —YES!



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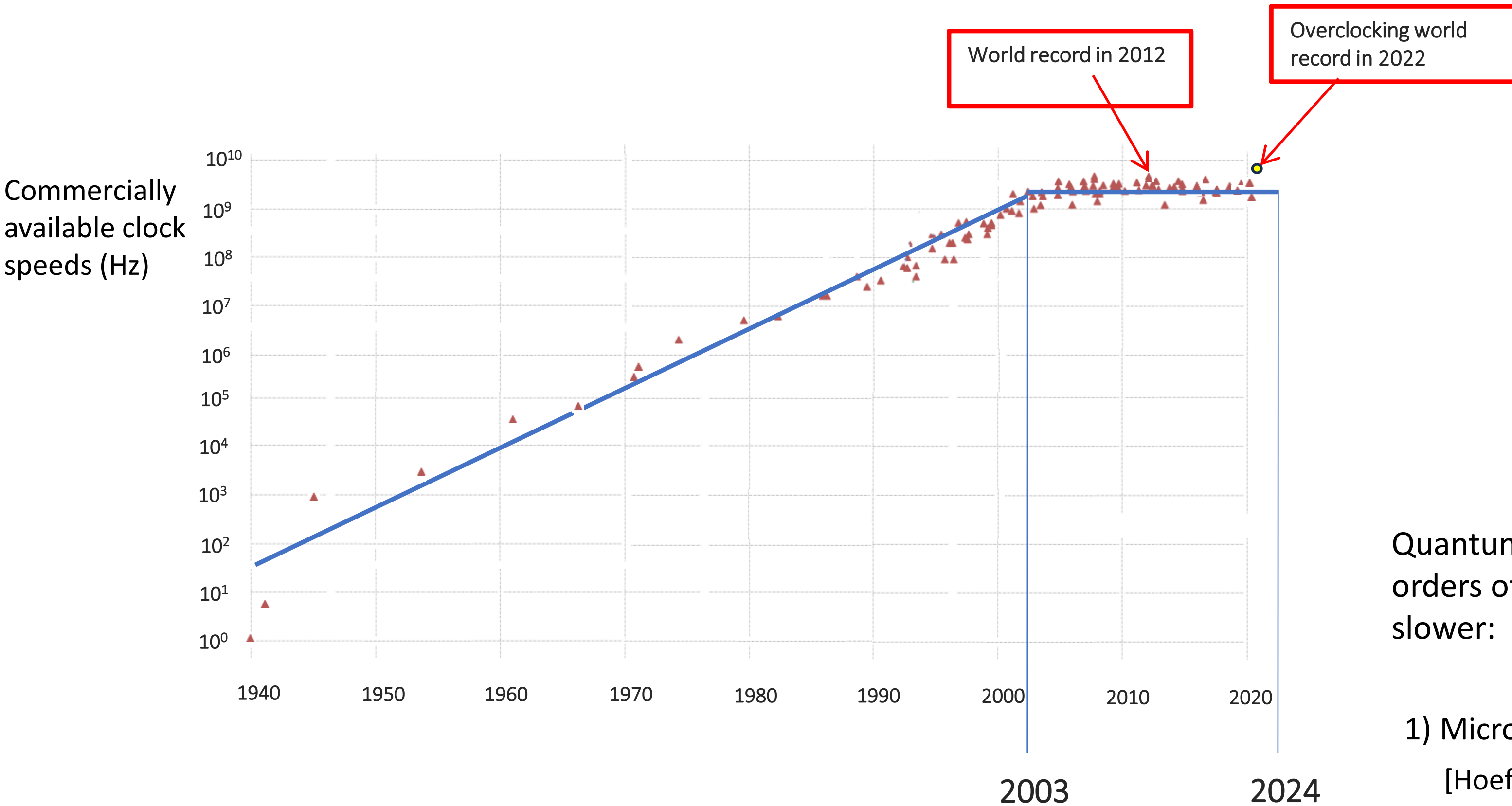

Quantum computers: orders of magnitude slower:

1) Microsoft Quantum

   [Hoefler et al, *Comm. ACM* (2023)]

2) Google Quantum

   [R. Babbush et al, PRX Quantum 2, 010103 (2021)]

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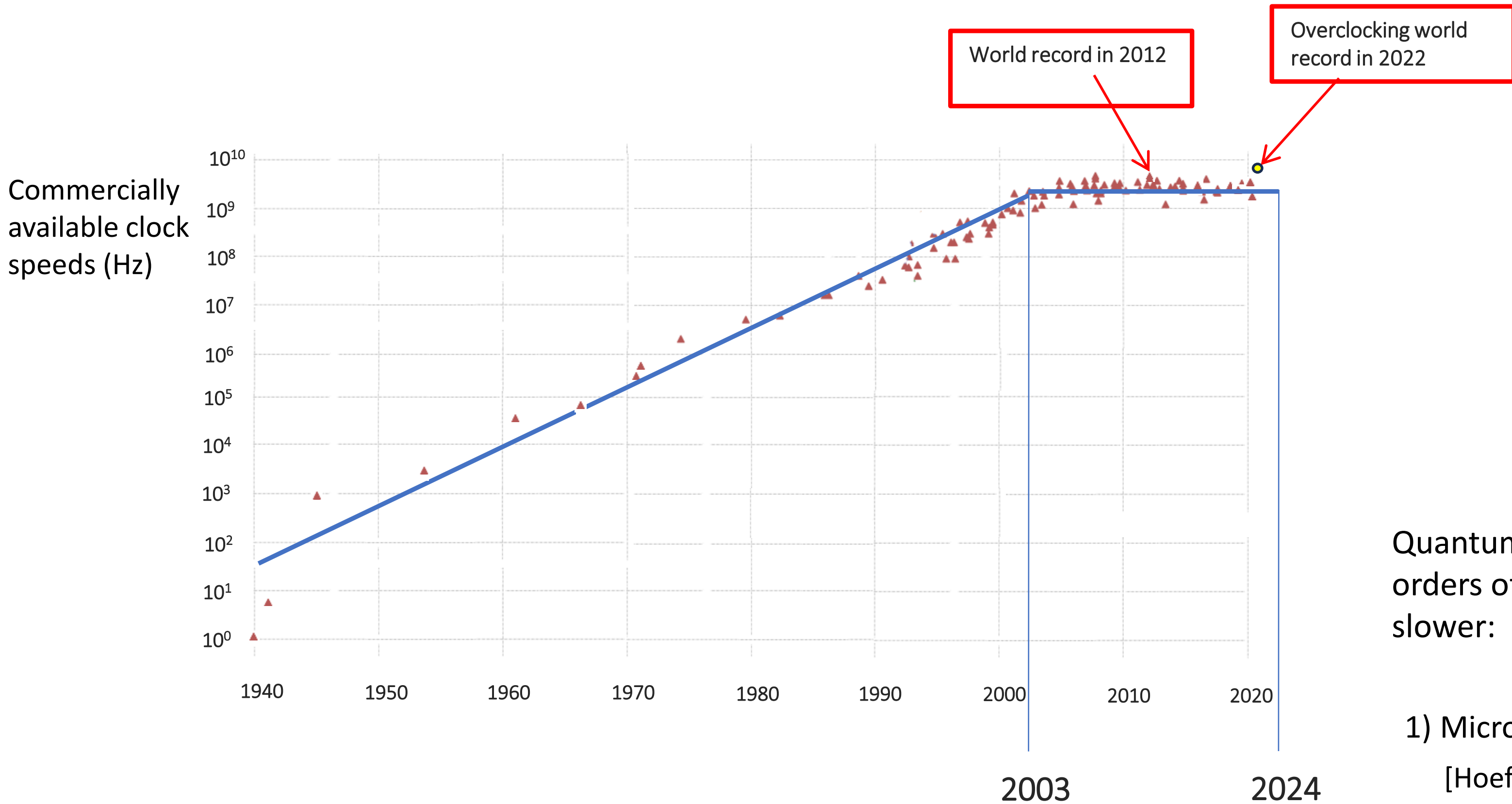

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Why bother? Is power consumption an issue? —YES!

— computations consumes 3% global power production. Set to balloon with AI.

[IEA.org/energy]

1) What are the fundamental relationships between energy, power, & gate frequency?

2) What (if any) are the quantum advantages if the control is quantum rather than classical?

Previous full-computer/themo modeling:

- The computer as a physical system, (Benioff) *Journal of Statistical Physics.*, 1980
- Quantum mechanical Hamiltonian models of Turing machines, (Benioff) *Journal of Statistical Physics*, 1982
- Tight binding Hamiltonians and Quantum Turing Machines", (Benioff) *Phys. Rev. Lett.*, 1997
- Feynman lectures on computation (Feynman) 1996
- Autonomous quantum machines and the finite sized Quasi-Ideal clock, (Woods et. al.) Annales Henri Poincaré, 2019
- Autonomous Quantum Devices (Woods et. al.) *Phys. Rev. X (2023)*
- Autonomous Quantum Processing Unit (Meier et. al.), *ArXiv:2402.00111*

Results

ArXiv: 2403.02389

- 1st part: Hamiltonian dynamics
    - Upper bounds on computational speed
    - Lower matching bounds
    - Only classical bus required
- 2nd part: Open quantum system model
    - Power & dissipation required for optimal speed in steady-state model

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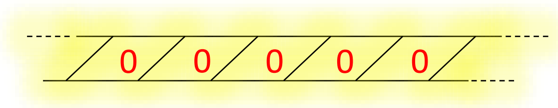
Upper bounds for classical and quantum computation

$e^{-itH_{\mathrm{Com}}} |0\rangle_{\mathrm{Com}}$



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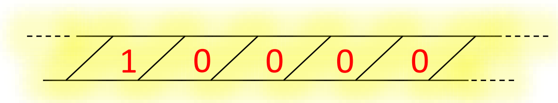

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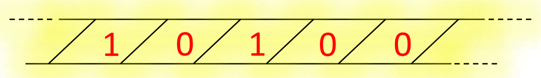

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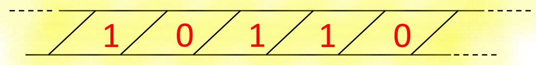

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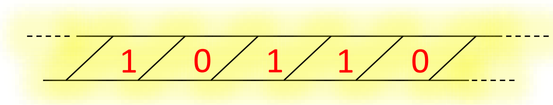

$$f \leq C_0 E$$

Quantum Speed limits:
Mandelstam–Tamm & Margolus, Levitin

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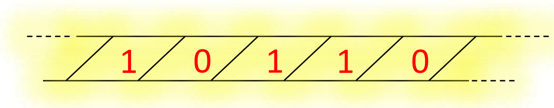

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Speed limits for semi-classical systems?

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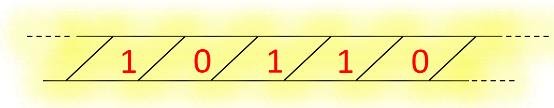

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Speed limits for semi-classical systems?

$f \leq C_0' \sqrt{E}$ If control not squeezed

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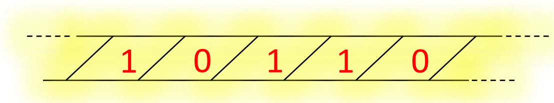

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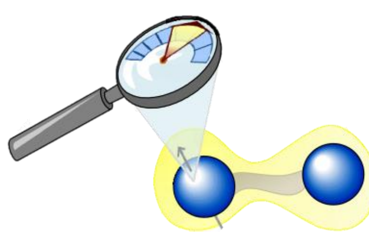
Proof uses tools from quantum metrology



V. Giovannetti, S. Lloyd, L. Maccone, doi: 10.1103/PhysRevLett.108.260405 (12)

L. Maccone, A. Riccardi
doi: 10.22331/q-2020-07-09-292 (20)

Upper bounds for classical and quantum computation

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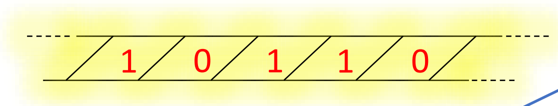

$f \leq C_0 E$

"Heisenberg Limit"

Quantum Speed limits:
Mandelstam–Tamm & Margolus, Levitin

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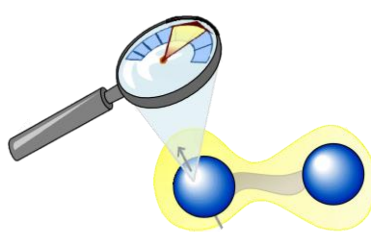
"Standard Quantum Limit"

Proof uses tools from quantum metrology



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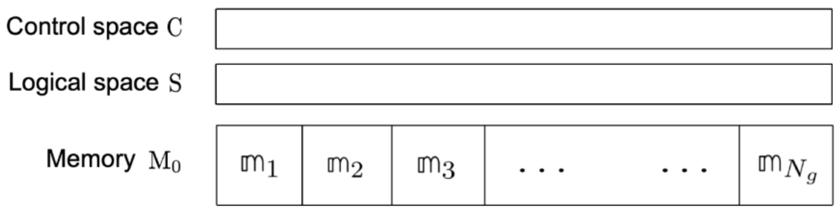
Existence of Optimal classical and Quantum limits

- Model:



- $$H_{\mathrm{M}_0\mathrm{SC}} = H_{\mathrm{C}} + \sum_{l=1}^{N_g} I^{(l)}_{\mathrm{M}_{0,l}\mathrm{S}} \otimes I^{(l)}_{\mathrm{C}}.$$

Existence of Optimal classical and Quantum limits

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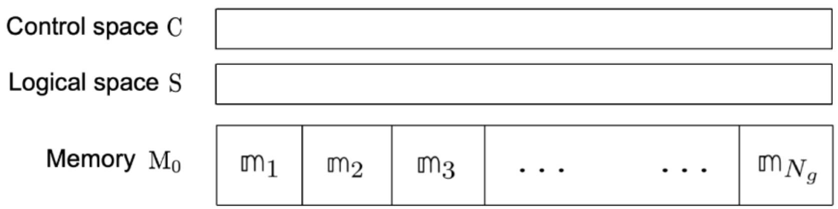

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$t_j = j t_1$

$|t_j\rangle_{\mathrm{S}} := U(\mathfrak{m}_j)U(\mathfrak{m}_{j-1})\ldots U(\mathfrak{m}_1)\,|0\rangle_{\mathrm{S}}$

- Want $\mathrm{e}^{-\mathrm{i}t_j H_{\mathrm{M_0SC}}} |0\rangle_{\mathrm{M}_0} |0\rangle_{\mathrm{S}} |0\rangle_{\mathrm{C}} \approx |0\rangle_{\mathrm{M}_0} |t_j\rangle_{\mathrm{S}} |t_j\rangle_{\mathrm{C}}$

- What is max $f := 1/t_1$?

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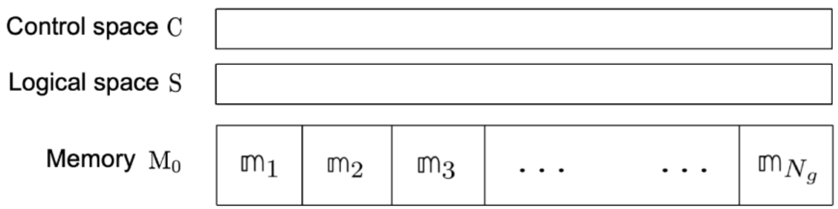

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- What is max $f := 1/t_1$?

- Answer: (theorem)

If $\{|t_j\rangle_{\mathrm{C}}\}_j$ semi-classical (not squeezed):

$$f = \frac{1}{T_0}(T_0 E)^{1/2} \qquad as\ E \to \infty$$

Otherwise:

$$f = E \qquad as\ E \to \infty$$

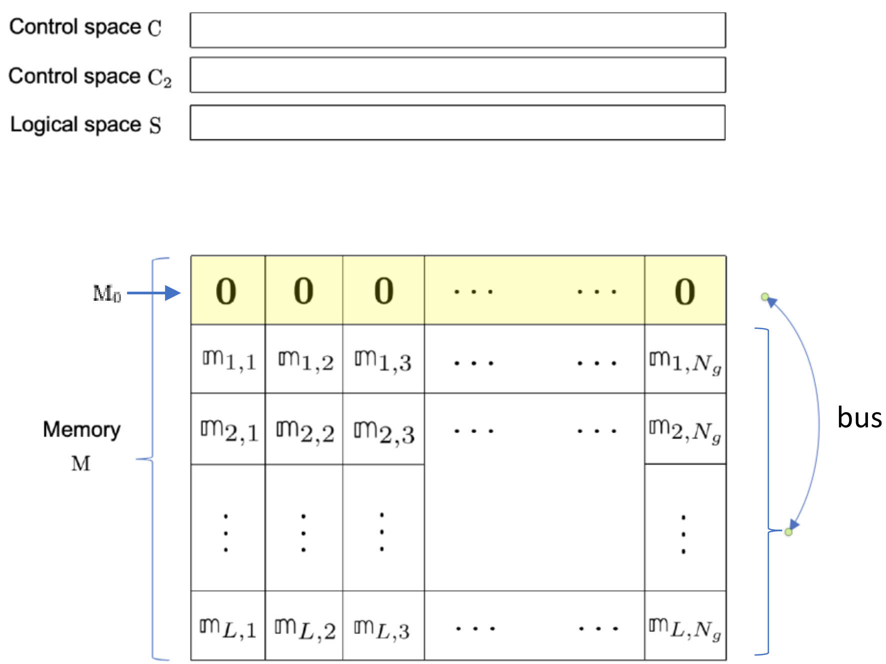
Bad bits: $H_{\mathrm{M_0SC}} = H_{\mathrm{C}} + \sum_{l=1}^{N_g} I^{(l)}_{\mathrm{M}_{0,l}\mathrm{S}} \otimes I^{(l)}_{\mathrm{C}}$

Optimal quantum limit only requires a classical internal bus

bus

$$H_{\mathrm{MSCC}_2} := H_{\mathrm{M}_0\mathrm{SC}} + H_{\mathrm{MC}_2}$$

Total number of interaction terms N_g. (double)



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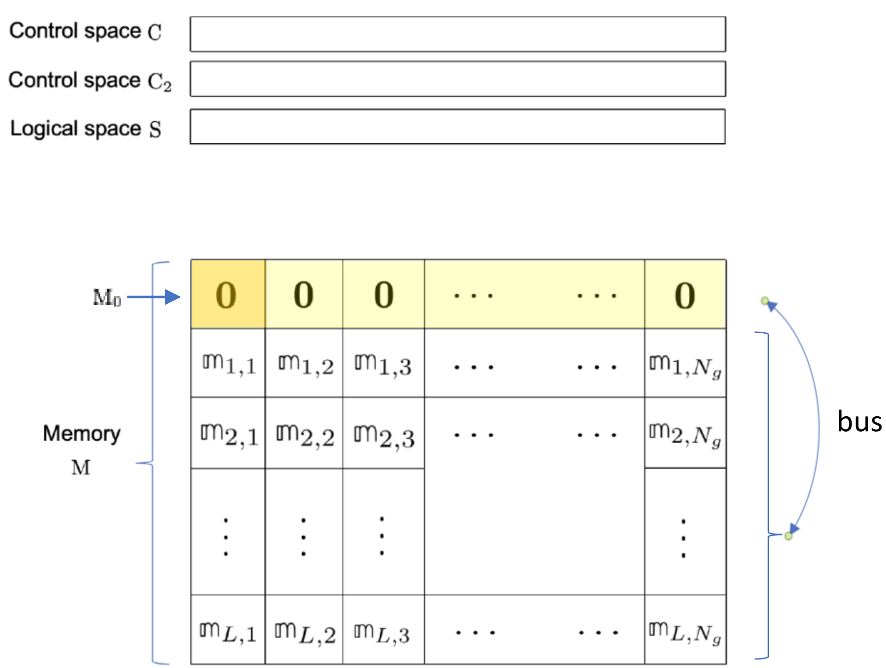

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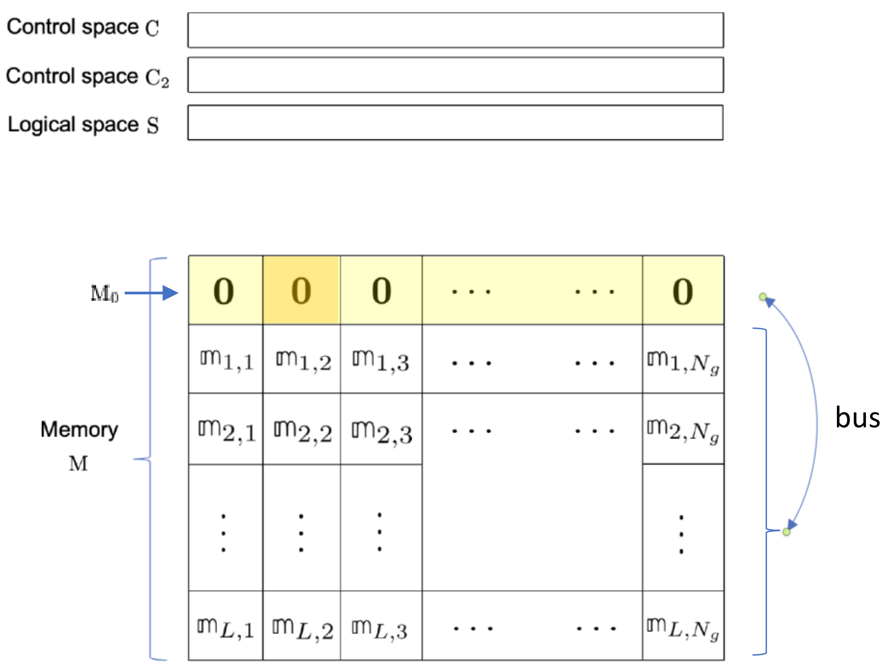

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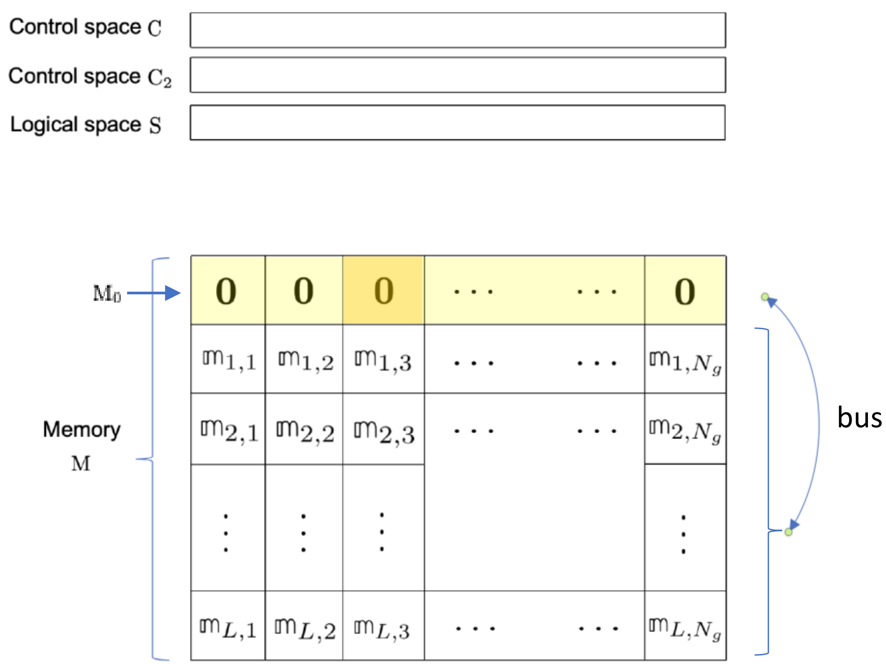

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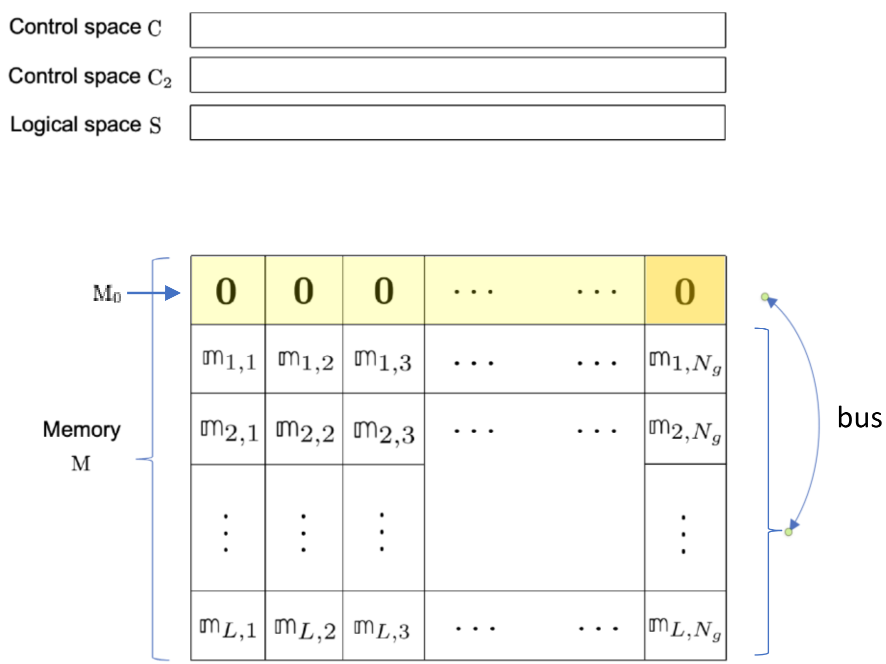

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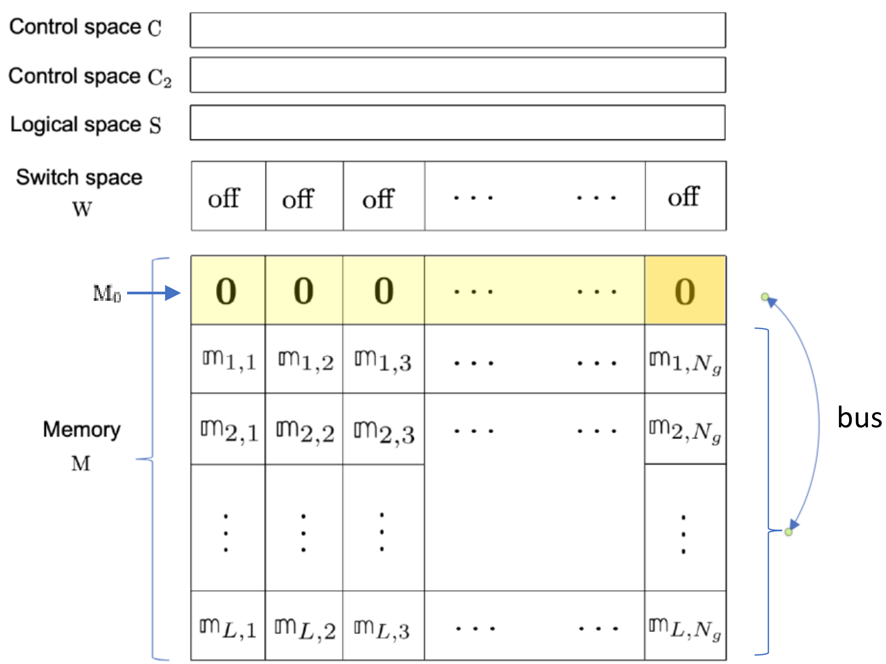

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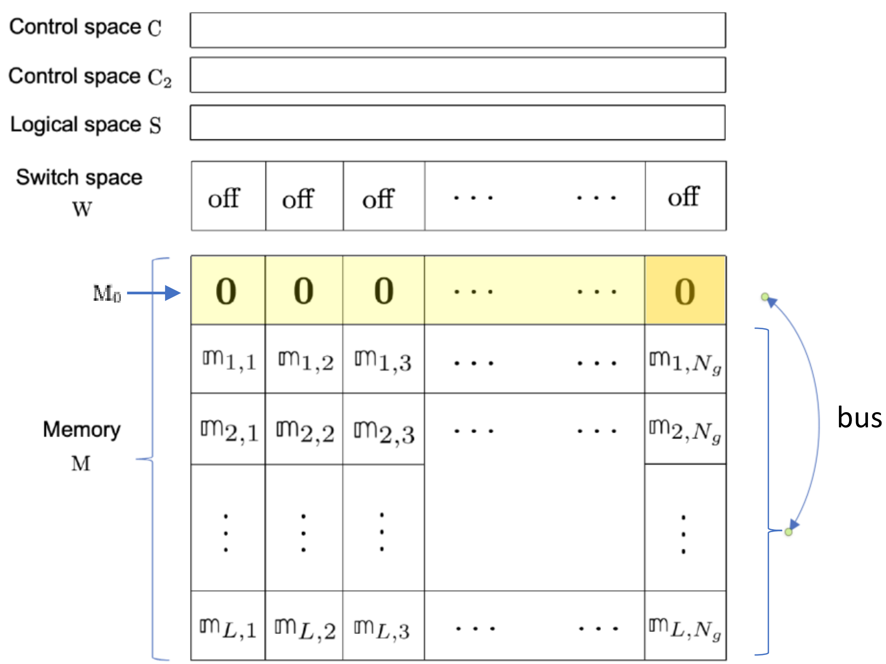


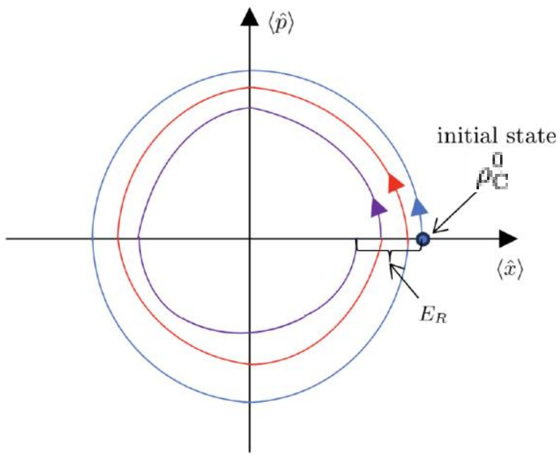
- (Theorem)

  $J = E$ as $E \to \infty$

  and ρ_{C_2} not squeezed

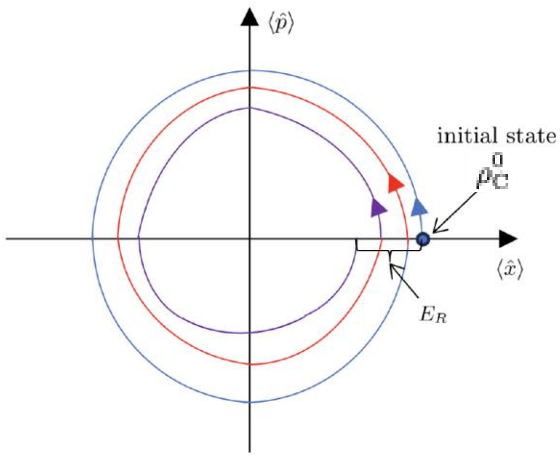
- Number of gates LN_g but only $2N_g$ interaction terms
- But still unstable to perturbations!

Nonequilibrium steady-state dynamics, power consumption and heat dissipation



Oscillators in Theorem 2

Nonequilibrium steady-state dynamics, power consumption and heat dissipation



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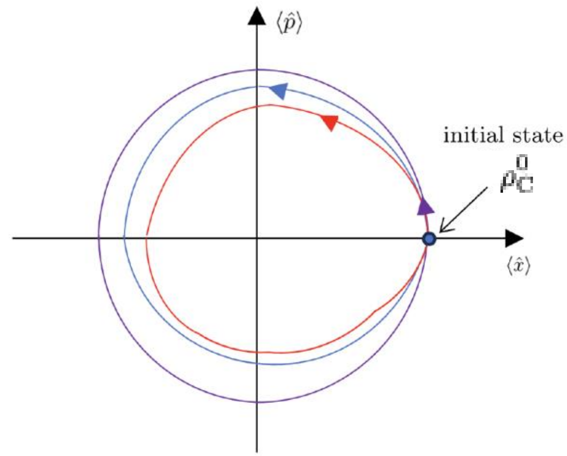


A self-Oscillator (e.g. from clock in an actual computer), see review:

[A. Jenkins, Physics Reports, (2013)]
[Batchtold et. Al, Rev. Mod. Phys. (2022)]

- self-oscillation: a form of error correction

- Well-studied for classical oscillators

- Relatively unknown in the quantum case

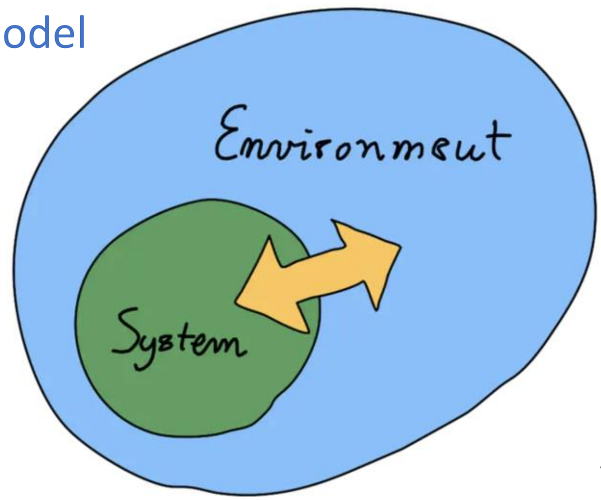
- Even if it works, will require power to run.

Maneki Neko

Cristal oscillator

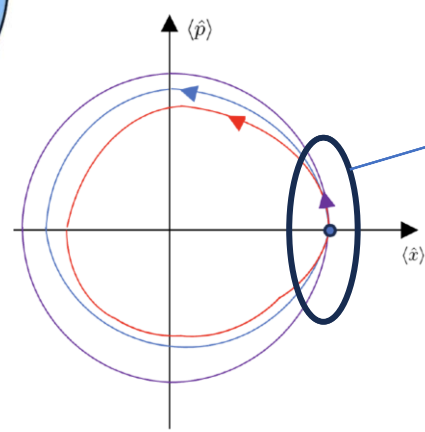
Nonequilibrium steady-state dynamics, power consumption and heat dissipation

- Model



$$\mathcal{L}_{\mathrm{M_0SWC}}(\cdot) = -\mathrm{i}\left[H_{\mathrm{M_0SWC}}, \cdot\,\right] + \mathcal{D}_{\mathrm{C}}(\cdot)$$

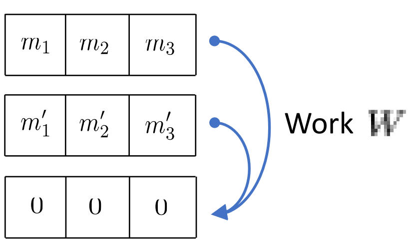
$$\mathcal{D}_{\mathrm{C}} = \mathcal{D}_{\mathrm{C}}^{\mathrm{re}} + \mathcal{D}_{\mathrm{C}}^{\mathrm{no\ re}} \qquad \mathcal{D}_{\mathrm{C}}^{\mathrm{re}}(\rho_{\mathrm{C}}) \propto \rho_{\mathrm{C}}^{0} \qquad \forall \rho_{\mathrm{C}}$$



Non isentropic regime

- Power consumption

Landauer Erasure/Charles Bennett (IBM)



Energy flow per cycle (two contributions)

- $P^{\mathrm{in}} = \dfrac{W}{T_0}$
- $P^{\mathrm{diss}} \approx -P^{\mathrm{in}}$

Nonequilibrium steady-state dynamics, power consumption and heat dissipation

- Upper bounds:

- (Theorem)

During isentropic regime:

$f \leq c_0 T_0 P^{\text{in}}$

$f \leq c_0 \sqrt{P^{\text{in}}}$ if oscillator on ρ_C semi-classical (not squeezed)

- Lower bounds:

- (Theorem)

$f = T_0 P^{\text{in}}$ as $P^{\text{in}} \to \infty$

Re-scaling Hamiltonian

$E \sim E_0/T_0$ (e.g. $\omega = 2\pi/T_0$)

Standard computer

$$f \sim \sqrt{\frac{E}{T_0}} = \frac{\sqrt{E_0}}{T_0}$$

Q. frequential computer

$$f \sim E = \frac{E_0}{T_0}$$

- Increasing $1/T_0$
  - Linearly increases $f \sim 1/T_0$ ☑
  - Requires linear increase in interaction strength ☒
  - Linear scaling with E implies more heat dissipation ☒

Re-scaling Hamiltonian

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Q. frequential computer

$$f \sim E = \frac{E_0}{T_0}$$

- Increasing $1/T_0$
  - Linearly increases $f \sim 1/T_0$ ✓
  - Requires linear increase in interaction strength ✗
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Often limiting factor!

Re-scaling Hamiltonian

$E \sim E_0/T_0$ (e.g. $\omega = 2\pi/T_0$)

Standard computer

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- Increasing $1/T_0$
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  - Requires linear increase in interaction strength ✗
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Often limiting factor!

- Quantum frequential computer
  - No need to increase interaction strength! ✓
  - Quadratic increase in frequency! ✓

Conclusion

ArXiv: 2403.02389 (Update coming soon!)

- Conventional computers have classical/semi-classical gate control
- Using squeezed states in control can result in a quadratic runtime advantage without increasing thermal dissipation or energy consumption
- Inherently less noisy gates

Next steps

- Experimental proposals to realize proof-of-principle quantum frequential computers (superconducting circuits, take inspiration from Heisenberg laser[1])
- Prove control-qubit interactions need to be non-linear

➢ Current/soon joining: Marek Winczewski (PostDoc), Pablo Alhambra (PhD)

➢ More Postdocs + PhDs available

[1] [T. Baker et. al, Nat. Phys. (2021)]

[Bonus] Optimal quantum limit only requires a classical internal bus

Theorem 2. *For all gate sets $\mathcal{U}_\mathcal{G}$, initial memory states $|0\rangle_\mathrm{M} \in \mathcal{C}_\mathrm{M}$ and initial logical states $|0\rangle_\mathrm{S} \in \mathcal{P}(\mathcal{H}_\mathrm{S})$, there exists $|0\rangle_\mathrm{C}$, $|0\rangle_{\mathrm{C}_2}$, $H_{\mathrm{M_0SC}}$ parametrised by the energy $E > 0$, such that for all $j = 1, 2, 3, \ldots, N_g$; $l = 0, 1, 2, \ldots, L$, the large-E scaling is as follows*

$$T\Big(\mathrm{e}^{-\mathrm{i}t_{j,l}H_{\mathrm{MWSCC_2}}}\,|0\rangle_\mathrm{M}\,|0\rangle_{\mathrm{C}_2}\,|0\rangle_\mathrm{W}\,|0\rangle_\mathrm{S}\,|0\rangle_\mathrm{C}\,,\ |t_{j,l}\rangle_{\mathrm{MC}_2}\,|t_{j,l}\rangle_\mathrm{W}\,|t_{j,l}\rangle_\mathrm{S}\,|t_{j,l}\rangle_\mathrm{C}\Big) \leq \frac{1}{SupPoly(E)} \quad as\ E \to \infty$$

where $|0\rangle\langle 0|_{\mathrm{C}_2}$, $\mathrm{tr}_\mathrm{M}\big[\,|t_{j,l}\rangle\langle t_{j,l}|_{\mathrm{MC}_2}\big] \in \mathcal{C}_{\mathrm{C}_2}$ *and*

$$f = \frac{1}{T_0}(T_0 E) + \frac{1}{SupPoly(E)} \quad as\ E \to \infty.$$

- Number of gates LN_g but only 2 N_g interaction terms
- Initial energy independent of total number of gates
- But still unstable to perturbations!

[bonus] Existence of Optimal classical and Quantum limits

Theorem 1. *For all gate sets $\mathcal{U}_\mathcal{G}$, initial memory states $|0\rangle_{\mathrm{M}_0} \in \mathcal{C}_{\mathrm{M}_0}$ and initial logical states $|0\rangle_\mathrm{S} \in \mathcal{P}(\mathcal{H}_\mathrm{S})$, there exists triplets $\{|t_j\rangle_\mathrm{C}\}_{j=0}^{N_g}$, N_g, $H_{\mathrm{M}_0\mathrm{SC}}$ parametrised by energy $E > 0$, such that for all $j = 1, 2, \ldots, N_g$ the large-E scaling is as follows*

$$T\left(\mathrm{e}^{-\mathrm{i}t_j H_{\mathrm{M}_0\mathrm{SC}}} |0\rangle_{\mathrm{M}_0} |0\rangle_\mathrm{S} |0\rangle_\mathrm{C}, \; |0\rangle_{\mathrm{M}_0} |t_j\rangle_\mathrm{S} |t_j\rangle_\mathrm{C}\right) \leq \frac{1}{SupPoly(E)} \qquad as\ E \to \infty$$

for the following two cases:
Case 1):

$$f = \frac{1}{T_0}(T_0 E)^{1/2} \quad + \quad \frac{1}{SupPoly(E)} \qquad as\ E \to \infty$$

and $|t_j\rangle_\mathrm{C} \in \mathcal{C}_\mathrm{C}$, $j = 0, 1, 2, \ldots, N_g$.
Case 2):

$$f = E \quad + \quad \frac{1}{SupPoly(E)} \qquad as\ E \to \infty$$

[Bonus] Optimal quantum limit only requires a classical internal bus

Proof outline:

Lemma C.1 (Quantum-control-and-bus error decoupling). *For $j = 1,2,3,\dots,N_g$ and $l = 0,1,2,\dots,L$ we have*

$$\left\| \mathrm{e}^{-\mathrm{i}t_{j,l}H_{\mathrm{MWSCC_2}}} |0\rangle_{\mathrm{M}} |0\rangle_{\mathrm{W}} |0\rangle_{\mathrm{S}} |0\rangle_{\mathrm{C}} |0\rangle_{\mathrm{C_2}} - |t_{j,l}\rangle_{\mathrm{MC_2}} |t_{j,l}\rangle_{\mathrm{W}} |t_{j,l}\rangle_{\mathrm{S}} |t_{j,l}\rangle_{\mathrm{C}} \right\|_2 \tag{C.7}$$

$$\leq \sum_{r=0}^{l} \sum_{k=1}^{j} \Bigg(\left\| |t_{k,r}\rangle_{\mathrm{W}_k} |t_{k,r}\rangle_{\mathrm{S}} |t_{k,r}\rangle_{\mathrm{C}} - \mathrm{e}^{-\mathrm{i}t_1 H_{\mathrm{WSC}}^{(k,\mathfrak{m}_{r,k})}} |t_{k-1,r}\rangle_{\mathrm{W}_k} |t_{k-1,r}\rangle_{\mathrm{S}} |t_{k-1,r}\rangle_{\mathrm{C}} \right\|_2 \tag{C.8}$$

$$+ \left\| |t_{k,r}\rangle_{\bar{\mathrm{M}}_{0,k}\mathrm{C_2}} |t_{k,r}\rangle_{\bar{\mathrm{W}}_k} - \mathrm{e}^{-\mathrm{i}t_1 \bar{H}_{\mathrm{MWC_2}}^{(k)}} |t_{k-1,r}\rangle_{\bar{\mathrm{M}}_{0,k}\mathrm{C_2}} |t_{k-1,r}\rangle_{\bar{\mathrm{W}}_k} \right\|_2 \tag{C.9}$$

$$+ t_1 \max_{x\in[0,t_1]} \max_{\{\mathfrak{m}_l \in \mathcal{G}\cup\{\mathbf{0}\}\}_{l=1}^{N_g}} \left\| \bar{H}_{\mathrm{WSC}}^{(k,\vec{\mathfrak{m}})} \mathrm{e}^{-\mathrm{i}xH_{\mathrm{WSC}}^{(k,\mathfrak{m}_k)}} |t_{k-1,r}\rangle_{\mathrm{W}} |t_{k-1,r}\rangle_{\mathrm{S}} |t_{k-1,r}\rangle_{\mathrm{C}} \right\|_2 \tag{C.10}$$

$$+ t_1 \max_{y\in[0,t_1]} \left\| \left(I_{\mathrm{MW}}^{(k)} \otimes I_{\mathrm{C_2}}^{(k)} \right) \mathrm{e}^{-\mathrm{i}y\bar{H}_{\mathrm{MWC_2}}^{(k)}} |t_{k-1,r}\rangle_{\mathrm{MC_2}} |t_{k-1,r}\rangle_{\mathrm{W}} \right\|_2 \Bigg) \tag{C.11}$$

where

$$H_{\mathrm{WSC}}^{(k,\mathfrak{m}_k)} := {}_{\mathrm{M}_{0,\#}}\langle\vec{\mathfrak{m}}|H_{\mathrm{M_0WSC}}^{(k)}|\vec{\mathfrak{m}}\rangle_{\mathrm{M}_{0,\#}} = H_{\mathrm{C}} + I_{\mathrm{S}}^{(k,\mathfrak{m}_k)} \otimes I_{\mathrm{C}}^{(k)} + I_{\mathrm{W}}^{(k,\mathfrak{m}_k)} \otimes I_{\mathrm{C}}^{(k)} \tag{C.12}$$

$$H_{\mathrm{WSC}}^{(k,\mathfrak{m}_{r,k})} := H_{\mathrm{WSC}}^{(k,\mathfrak{m}_k)}\Big|_{\mathfrak{m}_k \mapsto \mathfrak{m}_{r,k}} \tag{C.13}$$

$$\bar{H}_{\mathrm{WSC}}^{(k,\vec{\mathfrak{m}})} := {}_{\mathrm{M}_{0,\#}}\langle\vec{\mathfrak{m}}| \left(H_{\mathrm{M_0WSC}} - H_{\mathrm{M_0WSC}}^{(k)} \right) |\vec{\mathfrak{m}}\rangle_{\mathrm{M}_{0,\#}} = \sum_{\substack{q=1\\q\neq k}}^{N_g} \left(I_{\mathrm{S}}^{(q,\mathfrak{m}_q)} \otimes I_{\mathrm{C}}^{(q)} + I_{\mathrm{W}}^{(q,\mathfrak{m}_q)} \otimes I_{\mathrm{C}}^{(q)} \right), \tag{C.14}$$

where $\vec{\mathfrak{m}} = (\mathfrak{m}_1, \mathfrak{m}_2, \dots, \mathfrak{m}_{N_g})$, $|\vec{\mathfrak{m}}\rangle_{\mathrm{M}_{0,\#}} := |\mathfrak{m}_1\rangle_{\mathrm{M}_{0,1}} |\mathfrak{m}_2\rangle_{\mathrm{M}_{0,2}} \cdots |\mathfrak{m}_{N_g}\rangle_{\mathrm{M}_{0,N_g}}$ *with* $\mathfrak{m}_l \in \mathcal{G}\cup\{\mathbf{0}\}$ *and recall* $I_{\mathrm{S}}^{(q,\mathfrak{m}_q)}$ *is defined in eq.* (B.100) *for* $\mathfrak{m}_l \in \mathcal{G}$*. For* $\mathfrak{m}_l = \mathbf{0}$ *we define* $I_{\mathrm{S}}^{(k,\mathbf{0})} := \hat{0}$ *(with* $\hat{0}$ *the zero operator) and* $I_{\mathrm{W}}^{(k,\mathbf{0})} := I_{\mathrm{W}_k}^{(k)}$ *and* $I_{\mathrm{W}_k}^{(k,\mathfrak{m}_k)} := \hat{0}$ *for all* $\mathfrak{m}_l \in \mathcal{G}$.

[Bonus] Nonequilibrium steady-state dynamics, power consumption and heat dissipation

- Upper bounds:

Theorem 5 (Upper bounds on frequency in the nonequilibrium steady-state regime). *For all states in the lth isentropic regime, $\tilde{\rho}_{\mathrm{AC}}(t+\tau_l|\tau_l) \in \mathcal{S}_l^{ISE}$ the gate frequency f_{ε_0} is upper bounded for all $\varepsilon_0 \in [0, 1/2]$ as follows:*

$$f_{\varepsilon_0} \leq \begin{cases} \dfrac{\sqrt{P_l^{\mathrm{in}} + \varepsilon_{\mathrm{H}}^0}}{(\lambda+1)c_0}, & \text{if } \rho_{\mathrm{AC}}(\tau_l|\tau_l) \in \mathcal{C}_{\mathrm{AC}}^{clas.,\,l}, \\ \dfrac{P_l^{\mathrm{in}} + \varepsilon_{\mathrm{H}}^0}{T_0(\lambda+1)\kappa}, & \text{otherwise.} \end{cases} \tag{VI.1}$$

Here $\lambda > 0, \kappa > 0, c_0 > 0$ are numerical constants, $\varepsilon_{\mathrm{H}}^0 \geq 0$ is a small (or zero) quantity related to the quality of the self-oscillator and defined in appendix E 1.

- Lower bounds:

Theorem 3 (Nonequilibrium steady-state optimal quantum frequential computers exist). *For all gate sets $\mathcal{U}_{\mathcal{G}}$, initial gate sequences $(\mathfrak{m}_{l,k})_{l,k}$ with elements in $\mathcal{G}$, and initial logical states $|0\rangle_{\mathrm{S}} \in \mathcal{P}(\mathcal{H}_{\mathrm{S}})$, there exists states and $\mathcal{L}_{\mathrm{M_0SWC}}$ parametrised by the power $P^{\mathrm{in}} > 0$, such that for all $j = 1, 2, 3, \ldots, N_g$; $l \in \mathbb{N}_{\geq 0}$, the following large-$P^{\mathrm{in}}$ scaling holds simultaneously*

1) The lth renewal event is well localised in time:

$$\int_{\tau_l+T_0-t_1}^{\tau_l+T_0} \mathrm{d}t\, P(t,+1|\tau_l) = 1-\varepsilon_r, \qquad 0 < \varepsilon_r \leq \frac{1}{SupPoly(P^{\mathrm{in}})}$$

2) The deviations in the state between renewals are small: For $j = 1, 2, \ldots, N_g$,

$$T\Big(\rho_{\mathrm{M_0SC}}(t_j|\tau_l),\ |[t_j|\tau_l]\rangle_{\mathrm{M_0SWA}}\, |[t_j|\tau_l]\rangle_{\mathrm{C}}\Big) \leq \frac{1}{SupPoly(P^{\mathrm{in}})}$$

3) The gate frequency has the asymptomatically optimal scaling in terms of power:

$$f = T_0 P^{\mathrm{in}} + \frac{1}{SupPoly(P^{\mathrm{in}})} \quad \text{as } P^{\mathrm{in}} \to \infty.$$

Summary of results

f = frequency
p = power
C_r = cooling rate

ArXiv: 2403.02389
M. Woods

1) Classical & semi-classical gate control:
All classical & conventional quantum computers satisfy
$$f \lesssim p^{1/2} \sim C_r{}^{1/2}$$

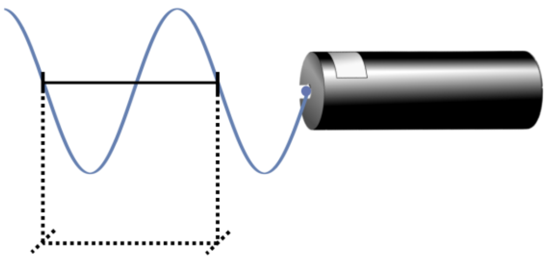
2) Optimal quantum control:
When control is fully quantum and optimal
$$f \sim p \sim C_r$$

3) Quantum frequential computers only need a classical bus:
$$f \sim p \sim C_r$$
$$f_{\mathrm{bus}} \sim \sqrt{p} \sim \sqrt{C_r}$$

Quantum Frequential Computers have a quadratic runtime advantage

No bandwidth issues

Proof of principle implementation: waveguides & quantum circuits?



- Lasers are self-oscillators

Q1: Can conventional lasers achieve $f \sim \sqrt{p}$ with control-qubit conventional coupling ?

- conventional laser = Schawlow-Townes limited: $\mathfrak{C} \sim \mu^2$
- conventional coupling = $a^\dagger \otimes \sigma + h.c.$

Q2: Can non-conventional lasers achieve $f \sim p$ with control-qubits non-conventional coupling?

- non-conventional laser = Heisenberg limited: $\mathfrak{C} \sim \mu^4$
- non-conventional coupling: $e^\dagger \otimes \sigma + h.c.$

↑ Non-linear in $a,\ a^\dagger$

[T. Baker et. al, Nat. Phys. (2021)]

Benjamin Huard
&
Mazyar Mirrahimi
(Inria)

Bonus slide: Re-scaling time

- Consider Hamiltonians $H(\omega) = \omega H,\ \omega \in \mathbb{R}^+$ e.g. $H(\omega) = \omega a^\dagger a$

- Increasing ω leads to "trivial" squeezing. E.g.

$$(\Delta x)^2 = \frac{\hbar}{2m\omega}\mathrm{e}^{-2\zeta} \qquad (\Delta p)^2 = \frac{m\hbar\omega}{2}\mathrm{e}^{2\zeta}$$

- Consider two states :
  - Case 1: $|\text{not sqz. w.r.t. } \omega = 1\rangle$, $f \leq C_1\sqrt{E(\omega=1)}$
  - Case 2: $|\text{sqz. w.r.t. } \omega = 1\rangle$, $f \leq C_2\, E(\omega=1)$

- Now apply "trivial" squeezing (i.e. evolution under $H(\omega),\ \omega > 1$)
  - Case 1: $f \leq \omega\, C_1\sqrt{E(\omega=1)}$
  - Case 2: $f \leq \omega\, C_2\, E(\omega=1)$

  Energy is $E(\omega) = \omega E(\omega=1)$

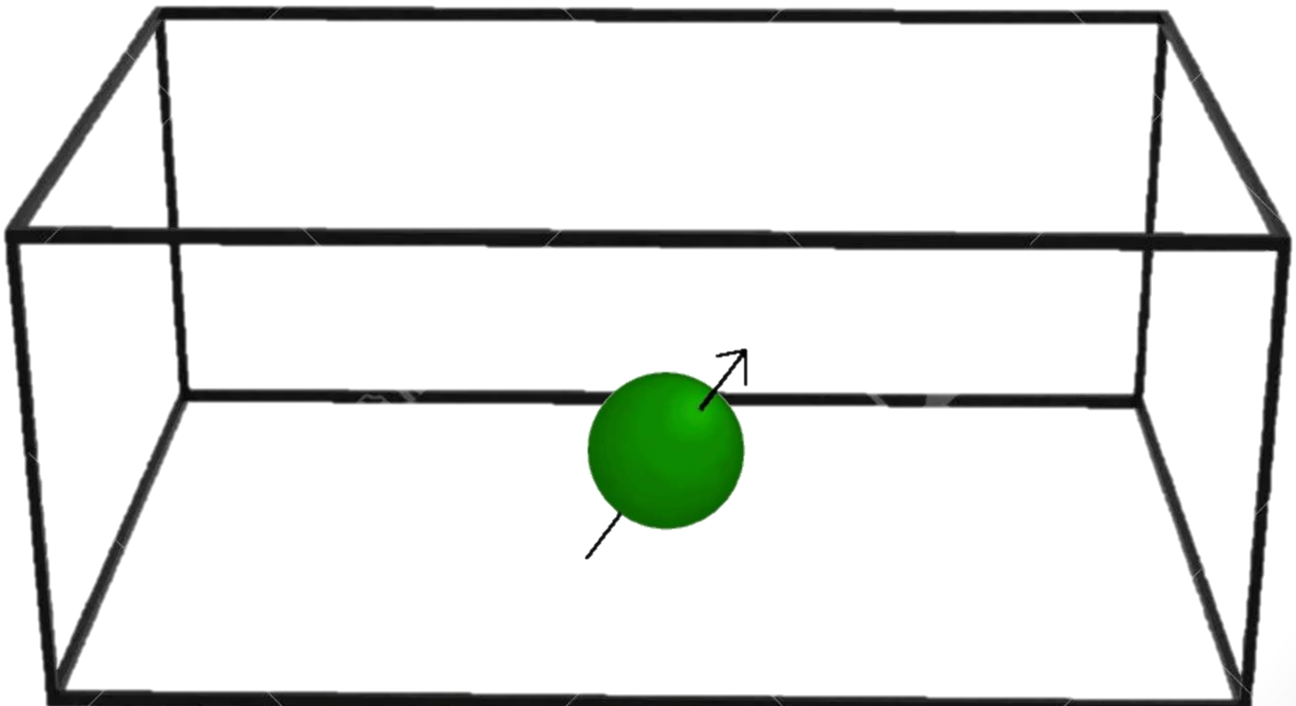
- Note:
  1. Increasing ω requires stronger interaction terms ✗
  2. For fixed ω, increasing $E(\omega)$ *doesn't* require stronger interaction terms. ✓

To do

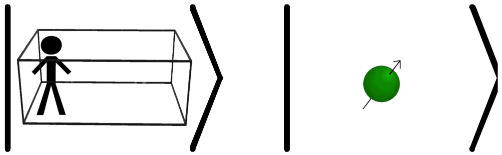
- 1. backup for non constant Ham term
- 2. Have future slides: show connection to laser: self-oscillator. Embedding? Special coupling required?

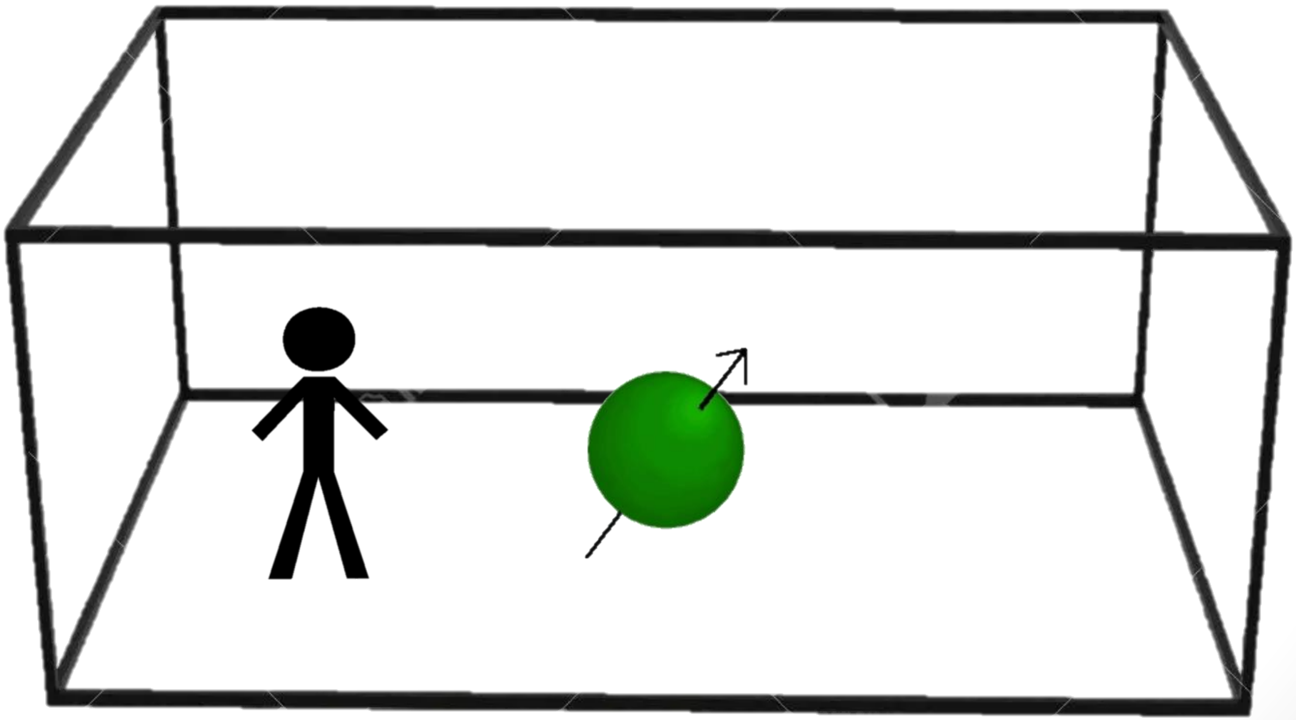
- ---Draw area under slide. Intuition about why squeezed states help.

Wigner's Friend



Wigner's Friend





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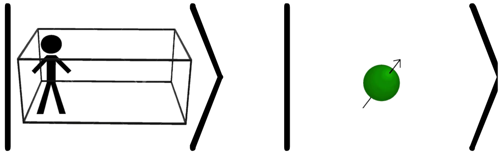

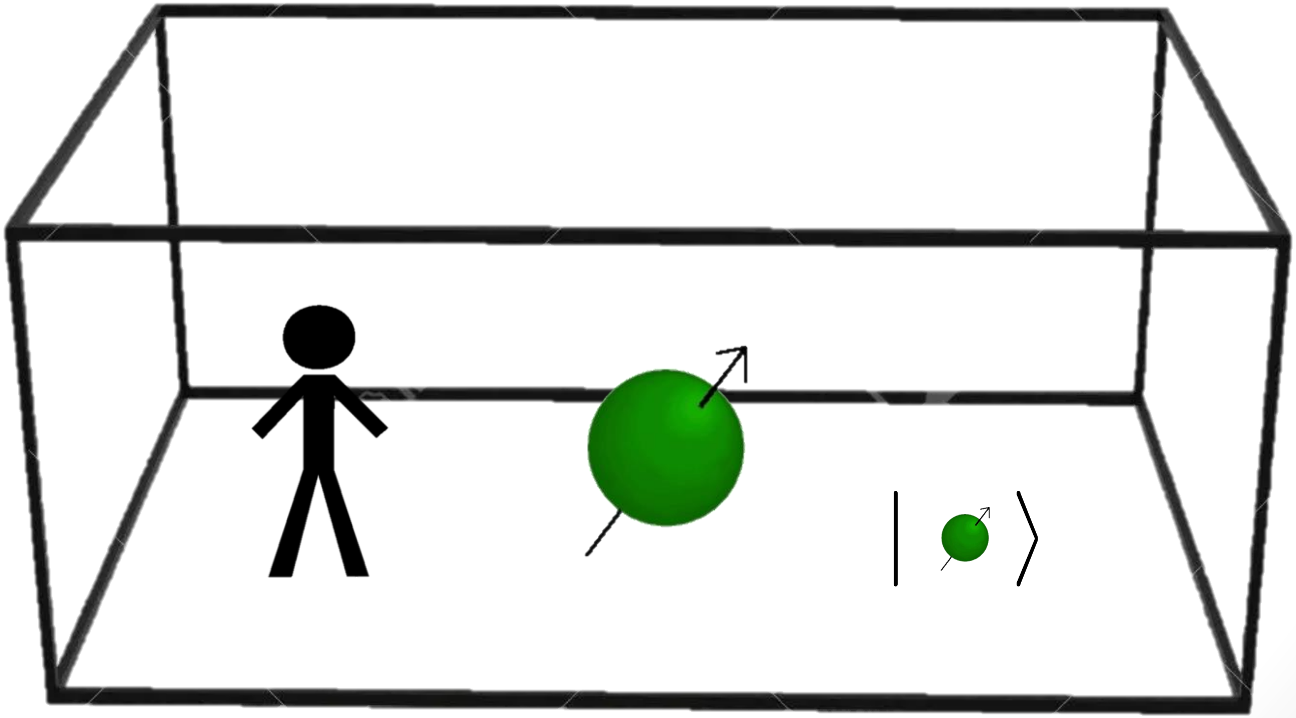

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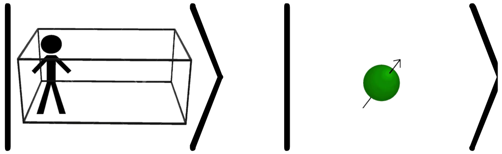

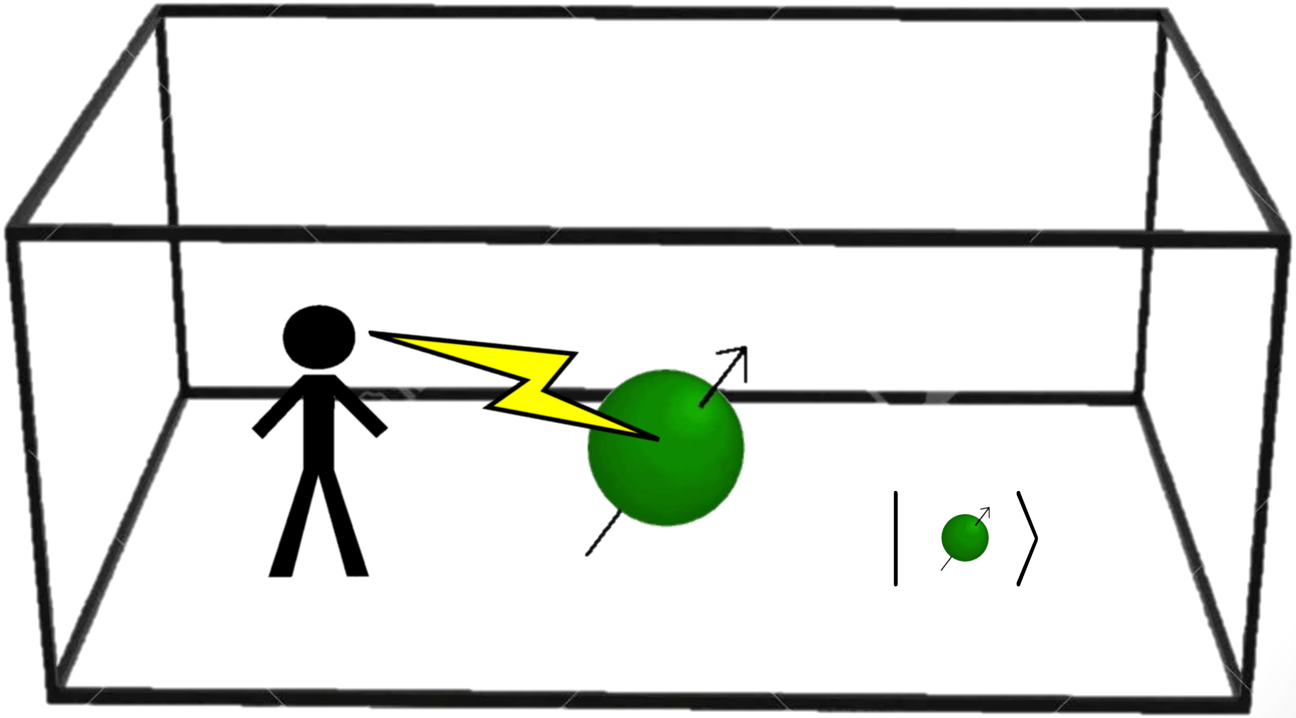

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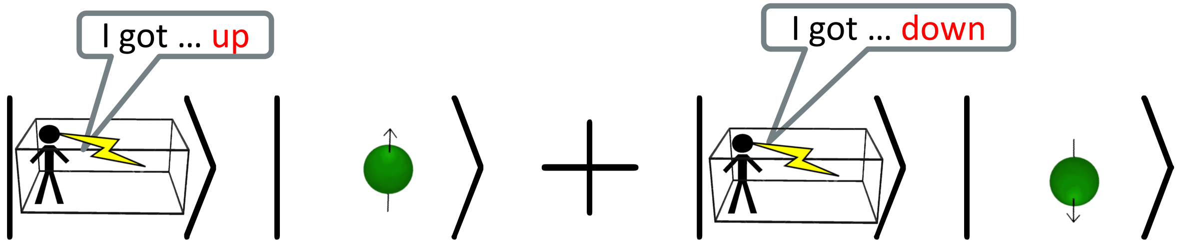

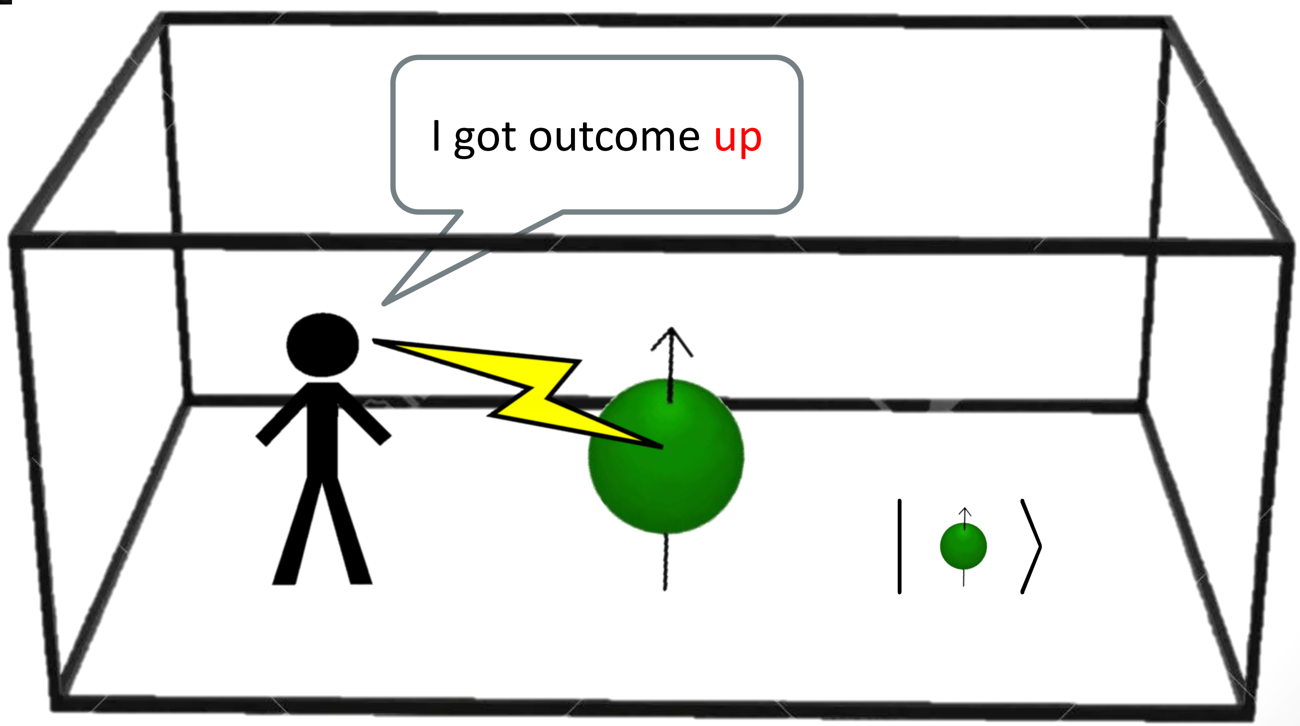

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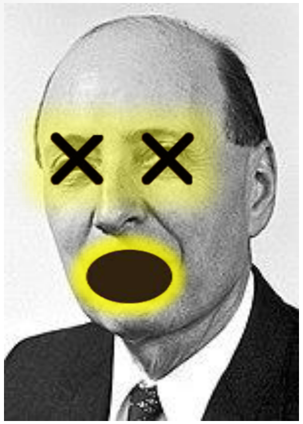

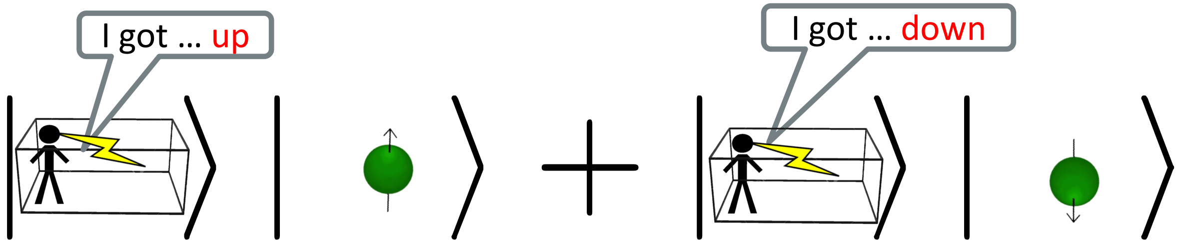

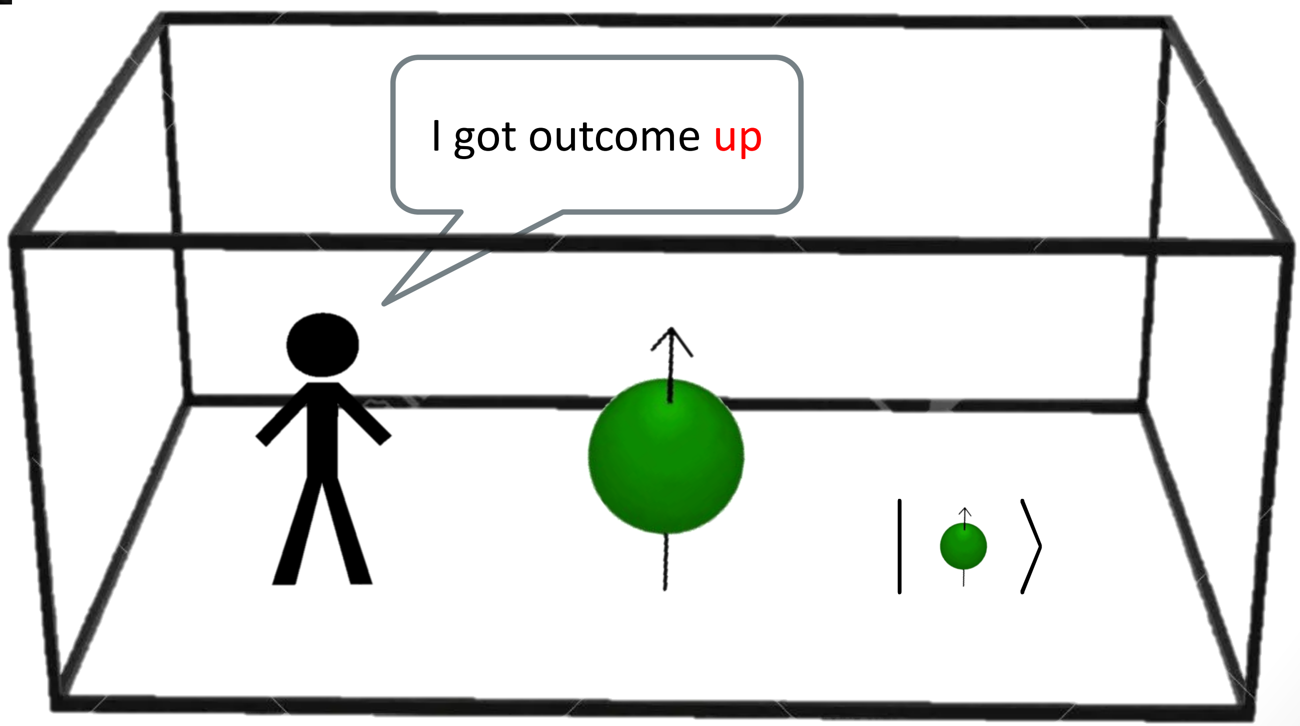

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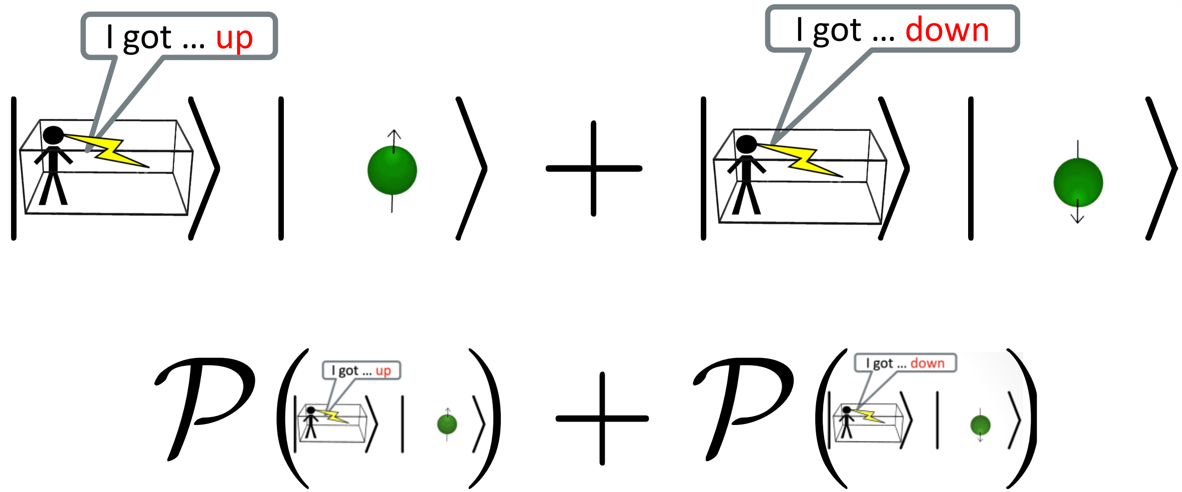

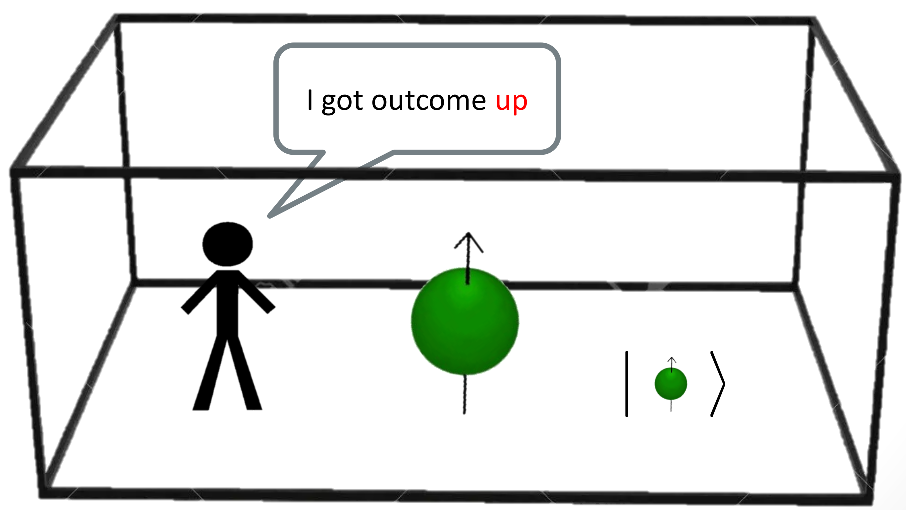

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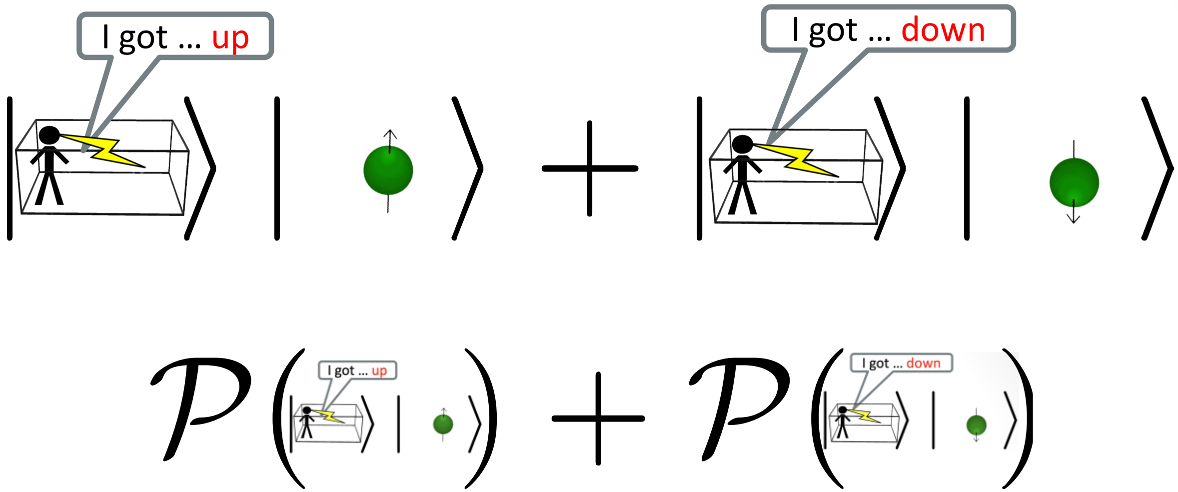
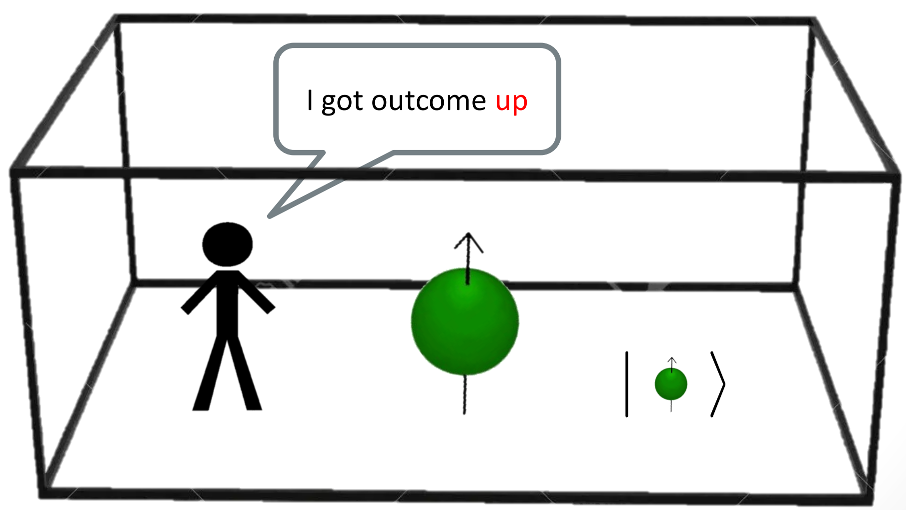


Takeaway:

Confusing? --- Yes!

Logical contradiction? --- No!



FR Experiment

Renato Renner & Daniela Frauchiger
Nat. Comms **volume 9**, Article number: 3711 (2018)

In a Nutshell:

Can we get a logical contradiction under reasonable assumptions?

Their claim --- yes!

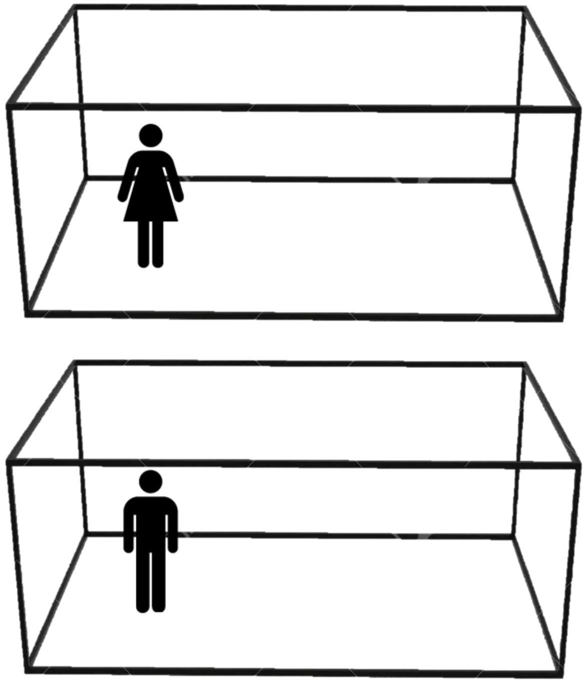
FR Experiment: as viewed from the super observers

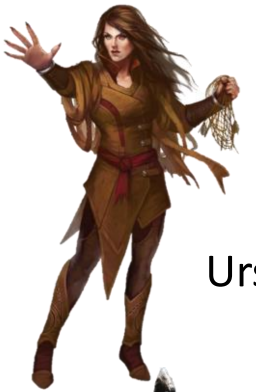
Lab A

$|0\rangle_{\mathrm{A}}$

Lab B

$|0\rangle_{\mathrm{B}}$





Ursula

Wigner

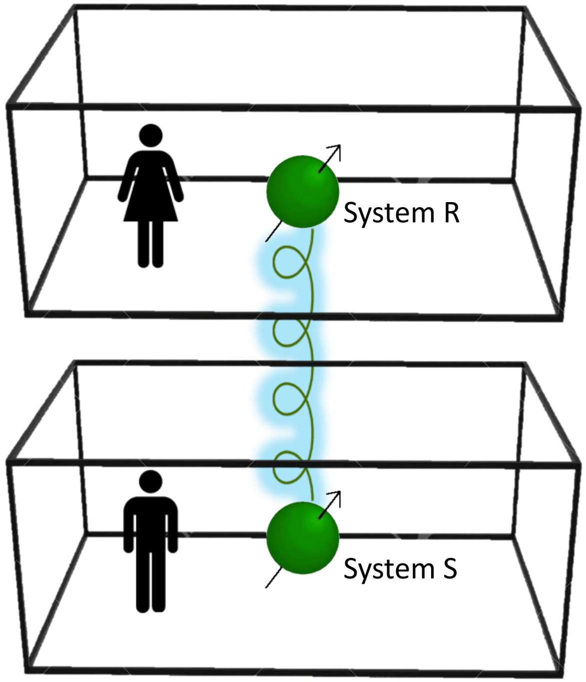
FR Experiment: as viewed from the super observers

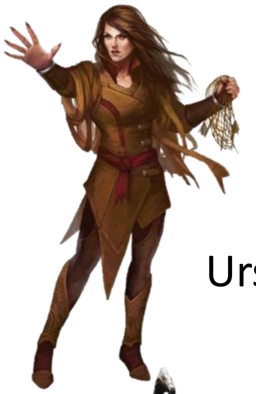
Lab A

$|0\rangle_{\mathrm{A}}$

Lab B

$|0\rangle_{\mathrm{B}}$

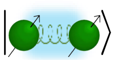




Ursula

Wigner

At t=1

$|$$\rangle_{\mathrm{RS}} :=$

$$\frac{1}{\sqrt{3}}(|00\rangle_{\mathrm{RS}} + |10\rangle_{\mathrm{RS}} + |11\rangle_{\mathrm{RS}})$$

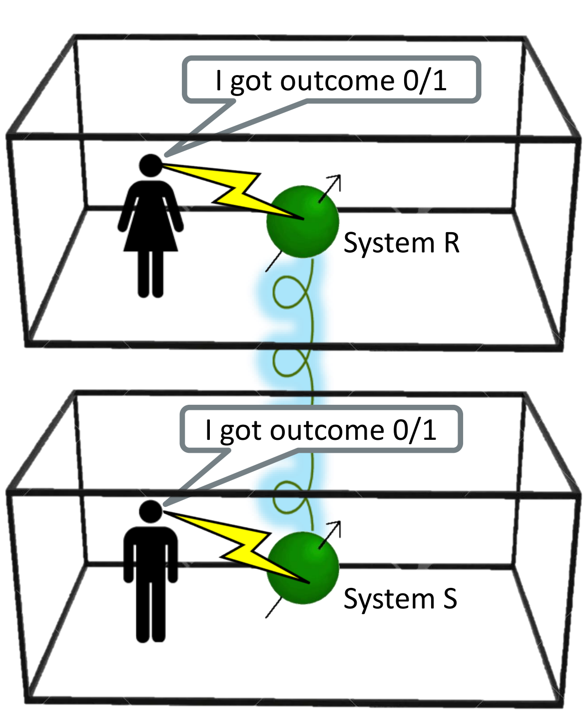
FR Experiment: as viewed from the super observers

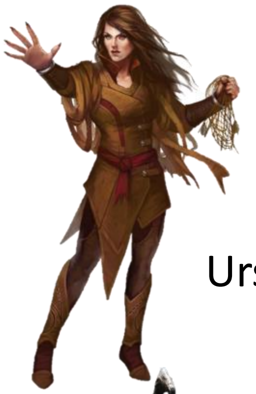
Lab A

$|0\rangle_{\mathrm{A}}$

Lab B

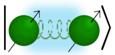
$|0\rangle_{\mathrm{B}}$





Ursula

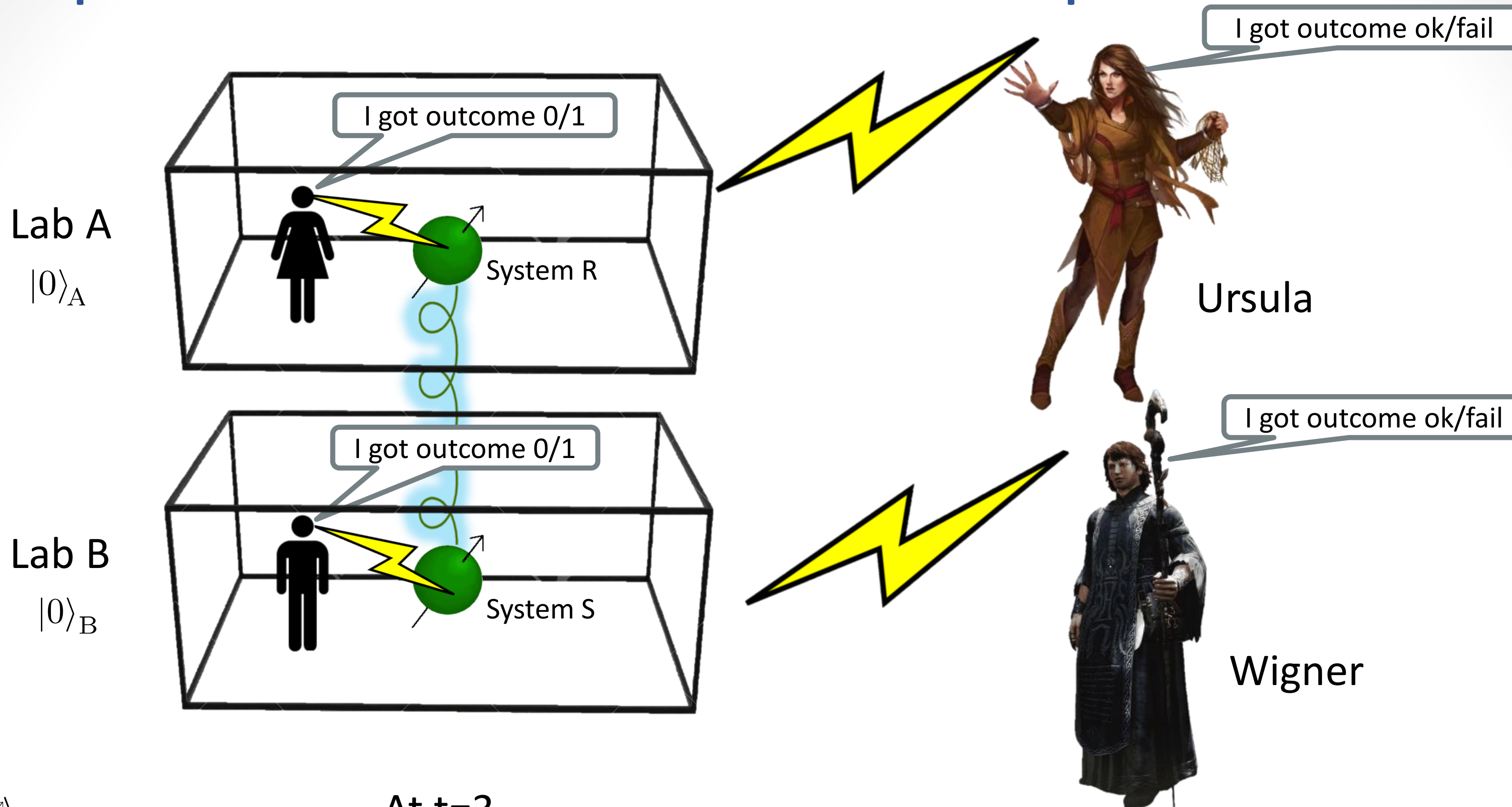
Wigner

$|$$\rangle_{\mathrm{RS}} :=$

$$\frac{1}{\sqrt{3}}(|00\rangle_{\mathrm{RS}} + |10\rangle_{\mathrm{RS}} + |11\rangle_{\mathrm{RS}})$$

At t=2

FR Experiment: as viewed from the super observers



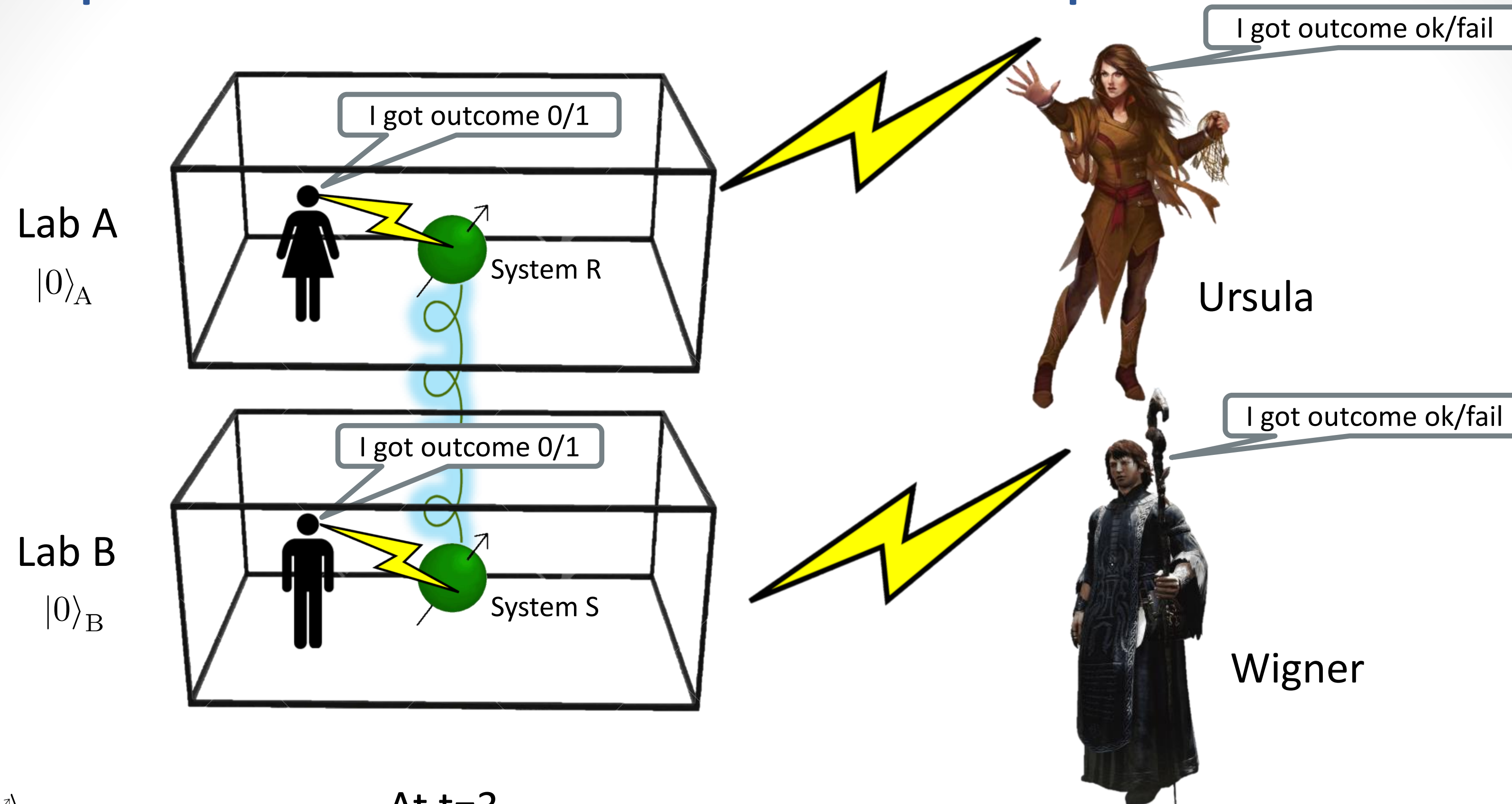
$|\text{(entangled pair)}\rangle_{RS} :=$
$$\frac{1}{\sqrt{3}}(|00\rangle_{RS} + |10\rangle_{RS} + |11\rangle_{RS})$$

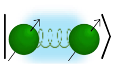
At t=3

Ursula
$$|ok\rangle_{AS} := (|00\rangle_{AS} - |11\rangle_{AS})/\sqrt{2}$$
$$|fail\rangle_{AS} := (|00\rangle_{AS} + |11\rangle_{AS})/\sqrt{2}$$

Wigner
$$|ok\rangle_{BR} := \ldots$$
$$|fail\rangle_{BR} := \ldots$$

FR Experiment: as viewed from the super observers





$$|\,\cdot\,\rangle_{RS} := \frac{1}{\sqrt{3}}(|00\rangle_{RS} + |10\rangle_{RS} + |11\rangle_{RS})$$

At t=3

Ursula

$$|ok\rangle_{AS} := (|00\rangle_{AS} - |11\rangle_{AS})/\sqrt{2}$$

$$|fail\rangle_{AS} := (|00\rangle_{AS} + |11\rangle_{AS})/\sqrt{2}$$

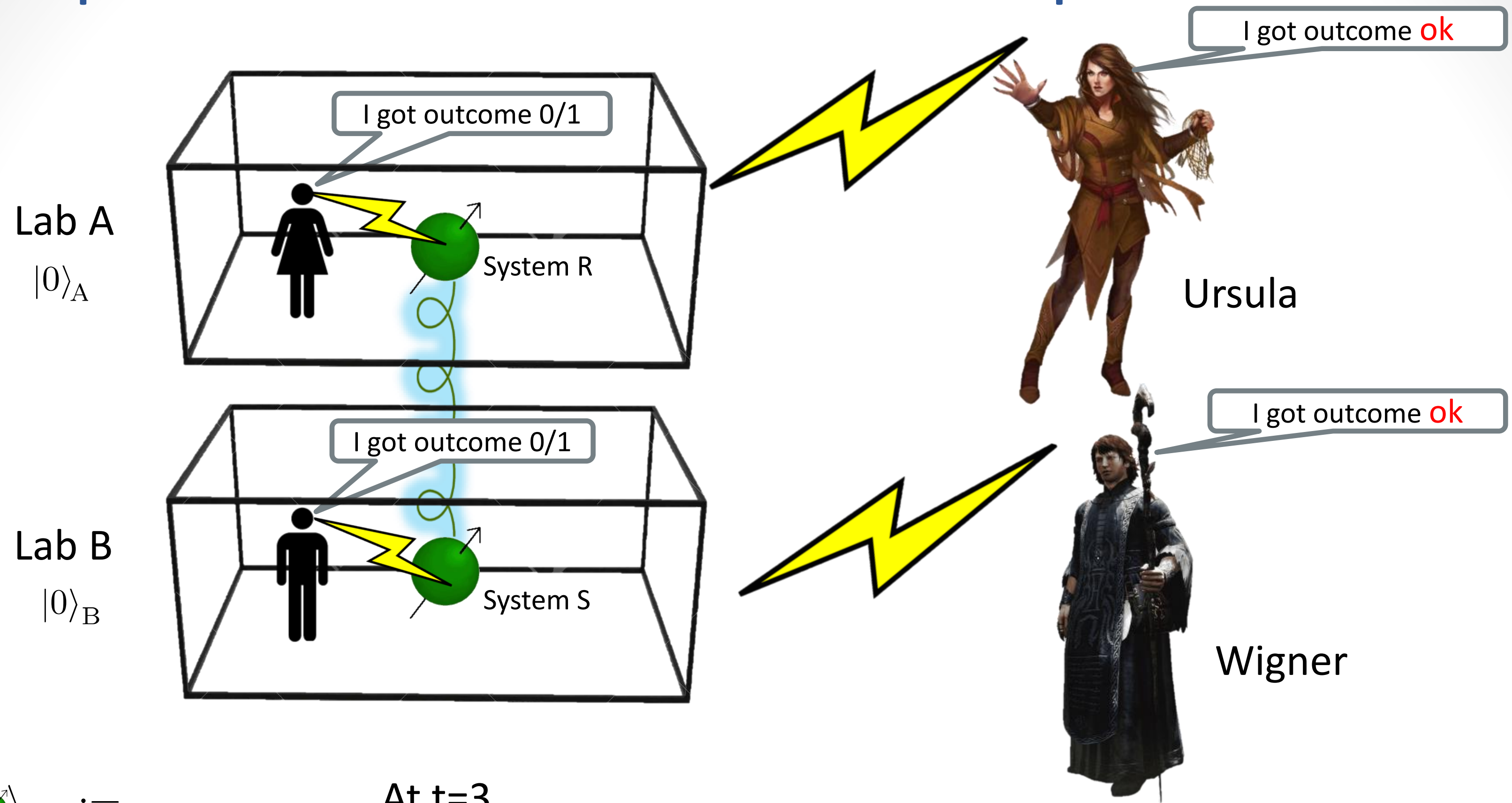
Repeat many times until …

Wigner

$$|ok\rangle_{BR} := \ldots$$

$$|fail\rangle_{BR} := \ldots$$

FR Experiment: as viewed from the super observers



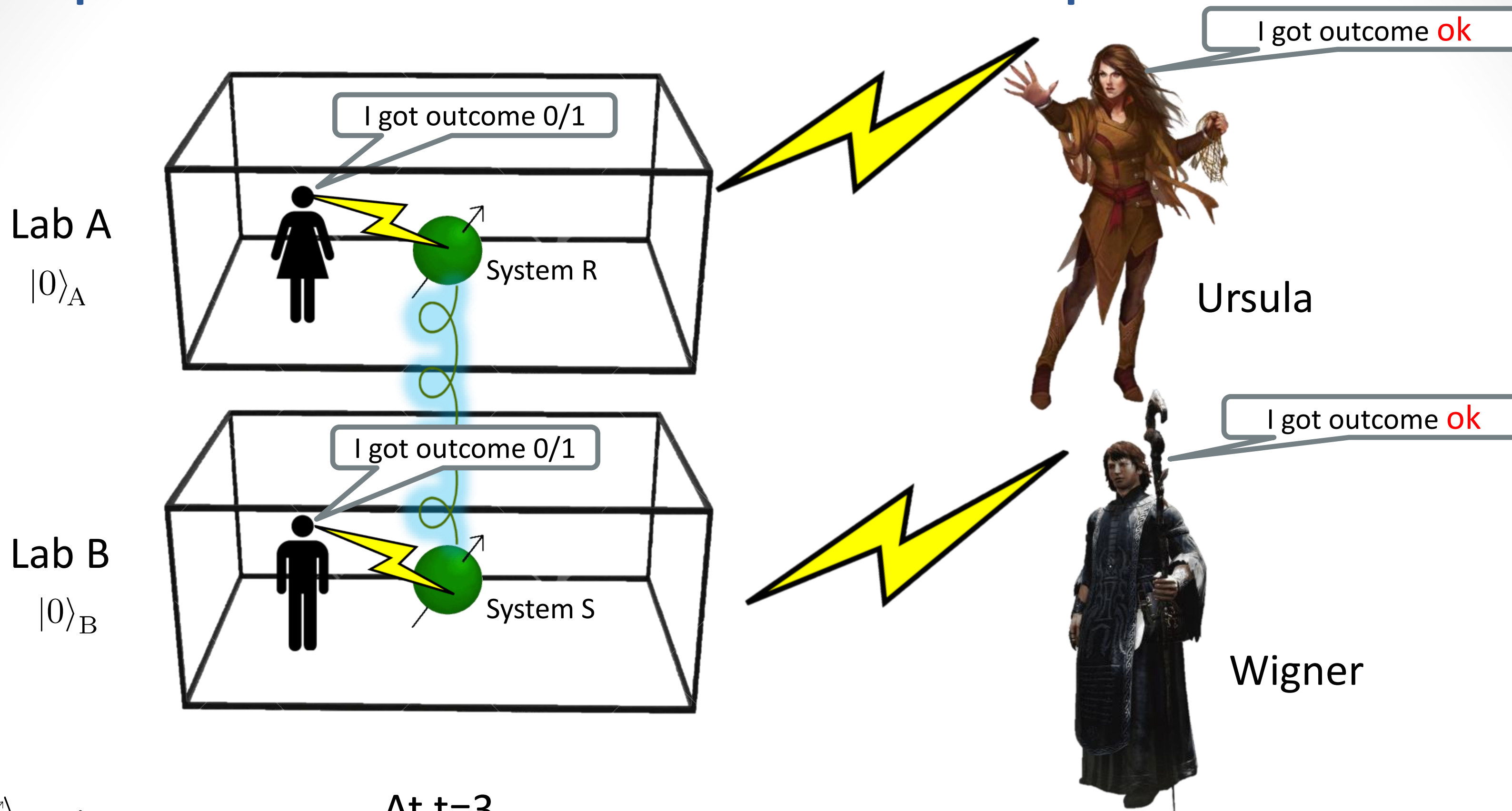
$|\bullet\!\!\sim\!\!\bullet\rangle_{RS} := \frac{1}{\sqrt{3}}(|00\rangle_{RS} + |10\rangle_{RS} + |11\rangle_{RS})$

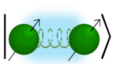
At t=3

Ursula $\begin{cases} |\mathrm{ok}\rangle_{AS} := (|00\rangle_{AS} - |11\rangle_{AS})/\sqrt{2} \\ |\mathrm{fail}\rangle_{AS} := (|00\rangle_{AS} + |11\rangle_{AS})/\sqrt{2} \end{cases}$

Wigner $\begin{cases} |\mathrm{ok}\rangle_{BR} := \ldots \\ |\mathrm{fail}\rangle_{BR} := \ldots \end{cases}$

FR Experiment: as viewed from the super observers



$|$$\rangle_{RS} :=$

$\frac{1}{\sqrt{3}}(|00\rangle_{RS} + |10\rangle_{RS} + |11\rangle_{RS})$

At t=3

Ursula $\begin{cases} |ok\rangle_{AS} := (|00\rangle_{AS} - |11\rangle_{AS})/\sqrt{2} \\ |fail\rangle_{AS} := (|00\rangle_{AS} + |11\rangle_{AS})/\sqrt{2} \end{cases}$

Wigner $\begin{cases} |ok\rangle_{BR} := \ldots \\ |fail\rangle_{BR} := \ldots \end{cases}$

- All parties  know this protocol.

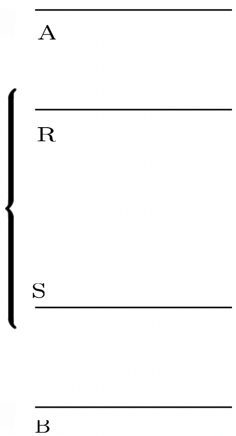
What can they deduce about each other's outcomes?

FR Experiment: as viewed from super observers

$|0\rangle$

$|\psi^{t=1}\rangle_{\mathrm{RS}} :=$

$\frac{1}{\sqrt{3}}(|00\rangle_{\mathrm{RS}} + |10\rangle_{\mathrm{RS}} + |11\rangle_{\mathrm{RS}})$



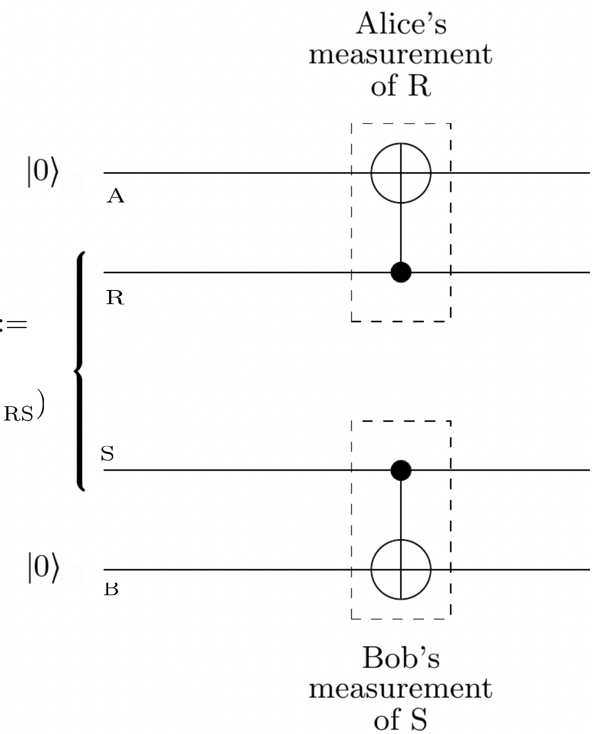
$|0\rangle$

$t = 1$

$$|\psi^{t=1}\rangle_{\mathrm{RASB}} := \frac{1}{\sqrt{3}}(|0000\rangle + |1000\rangle + |1010\rangle)_{\mathrm{RASB}}$$

FR Experiment: as viewed from super observers


$$|\psi^{t=1}\rangle_{\mathrm{RS}} := \frac{1}{\sqrt{3}}(|00\rangle_{\mathrm{RS}} + |10\rangle_{\mathrm{RS}} + |11\rangle_{\mathrm{RS}})$$

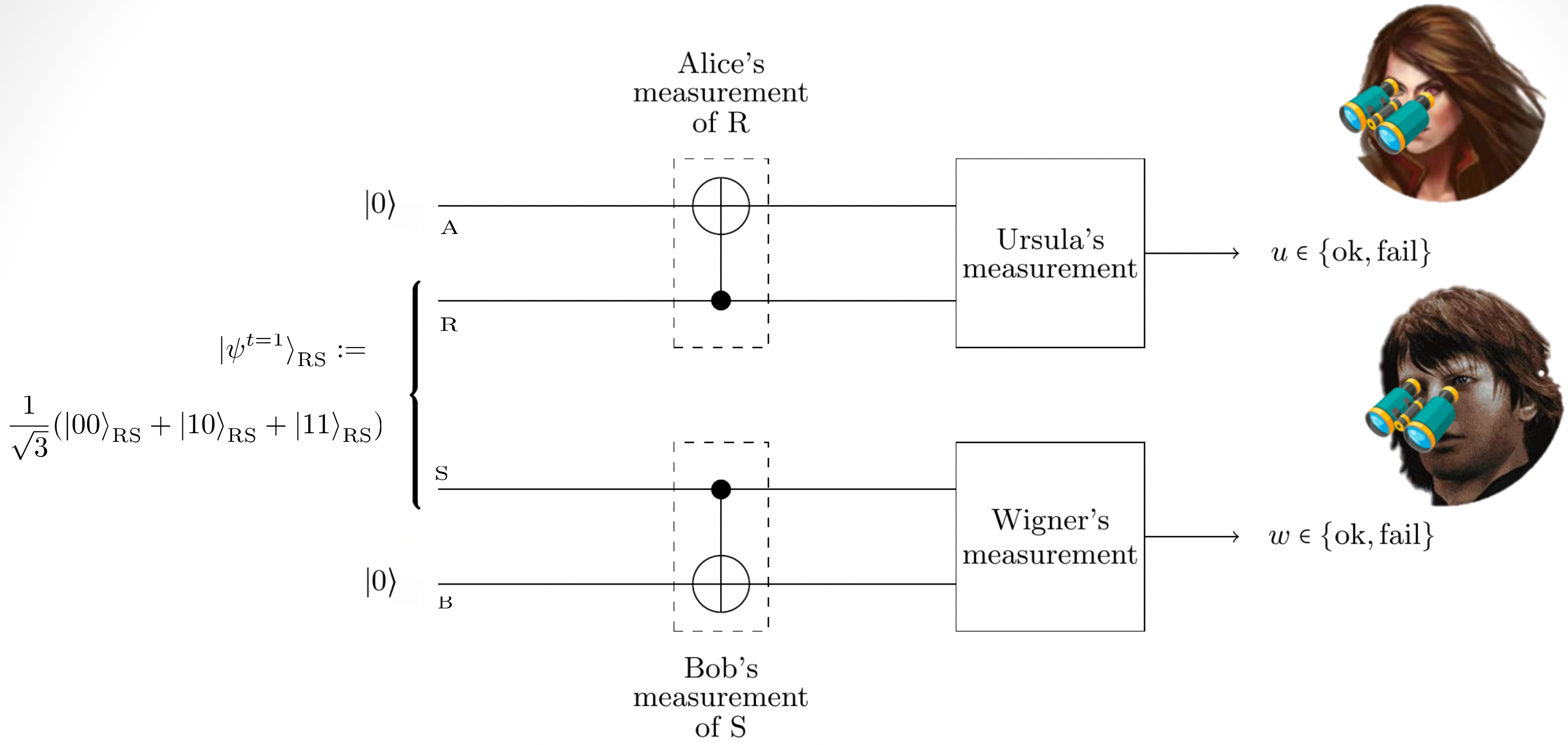


$t = 1$ $t = 2$

$$|\psi^{t=1}\rangle_{\mathrm{RASB}} := \frac{1}{\sqrt{3}}(|0000\rangle + |1000\rangle + |1010\rangle)_{\mathrm{RASB}}$$

$$|\psi^{t=2}\rangle_{\mathrm{RASB}} = \frac{1}{\sqrt{3}}(|0000\rangle + |1100\rangle + |1111\rangle)_{\mathrm{RASB}}$$

FR Experiment: as viewed from super observers



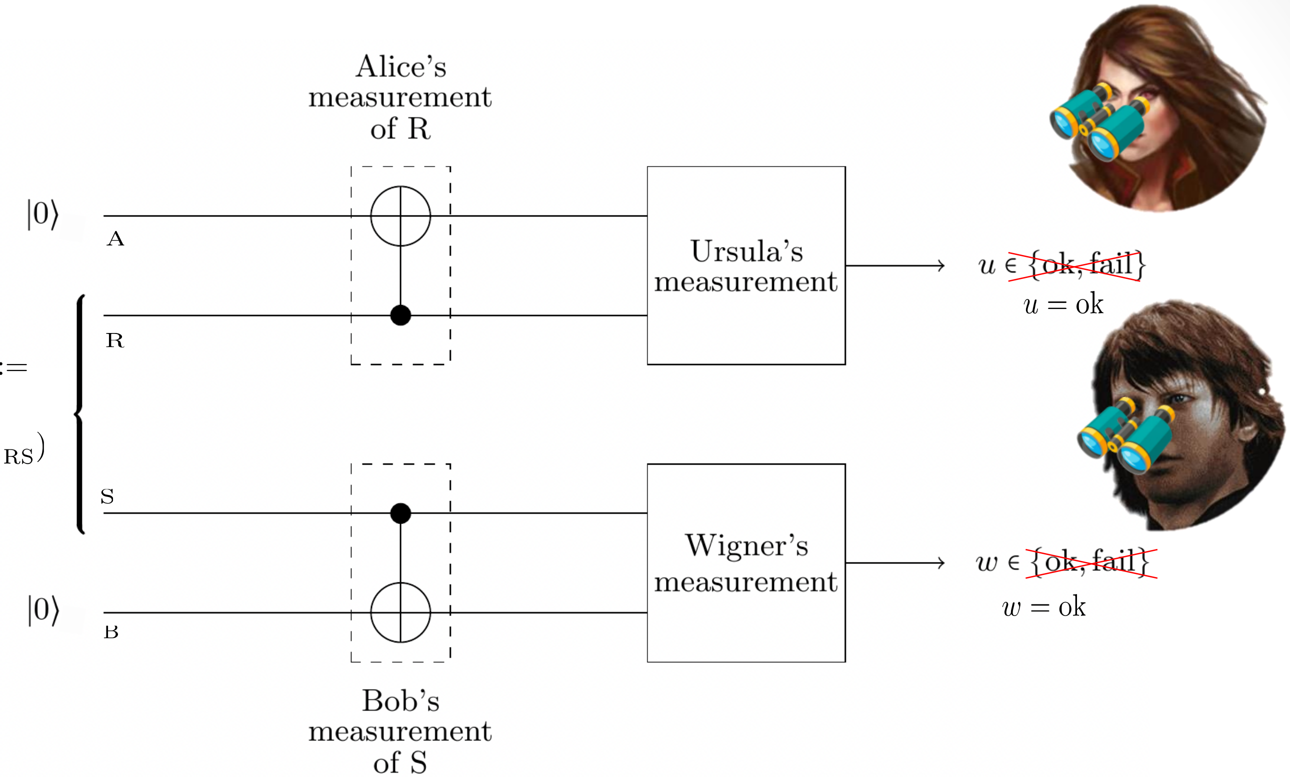
$t = 1$ $t = 2$ $t = 3$

$$|\psi^{t=1}\rangle_{RASB} := \frac{1}{\sqrt{3}}(|0000\rangle + |1000\rangle + |1010\rangle)_{RASB}$$

$$|\psi^{t=2}\rangle_{RASB} = \frac{1}{\sqrt{3}}(|0000\rangle + |1100\rangle + |1111\rangle)_{RASB}$$

FR Experiment: as viewed from super observers

$$|\psi^{t=1}\rangle_{RS} := \frac{1}{\sqrt{3}}(|00\rangle_{RS} + |10\rangle_{RS} + |11\rangle_{RS})$$



$t = 1$

$$|\psi^{t=1}\rangle_{RASB} := \frac{1}{\sqrt{3}}(|0000\rangle + |1000\rangle + |1010\rangle)_{RASB}$$

$t = 2$

$$|\psi^{t=2}\rangle_{RASB} = \frac{1}{\sqrt{3}}(|0000\rangle + |1100\rangle + |1111\rangle)_{RASB}$$

$t = 3$

$$|\langle ok|\langle ok||\psi^{t=2}\rangle|^2 = P(u = w = \text{ok}|\psi^{t=2})$$

FR Reasoning assumptions

Q: Born rule:

Any agent A: I know I can apply the quantum mechanical Born rule

C: Knowledge inheritance

$$K_{\mathrm{A}_i} K_{\mathrm{A}_j}(s) \Rightarrow K_{\mathrm{A}_i}(s)$$

U: Unitarity

Any agent A: I can model other Agent's labs unitarily

D: Distributive Axiom

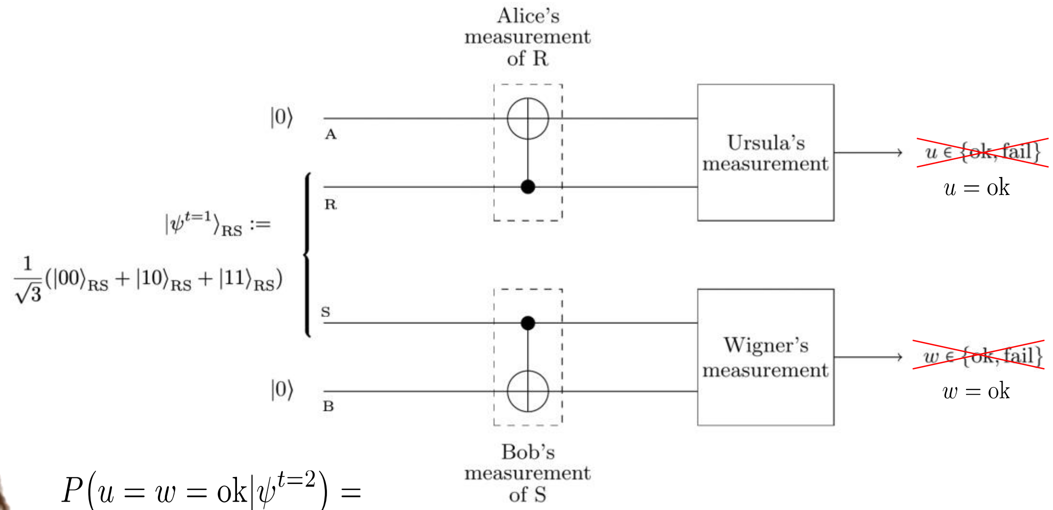
$$K_{\mathrm{A}_i}(s_1 \wedge (s_1 \Rightarrow s_2))$$

$$\Rightarrow K_{\mathrm{A}_i}(s_2)$$

S: Consistency

Any agent A: I know that a measurement outcome cannot take two distinct values

FR Experiment: Reasoning



$$P(u = w = \text{ok}|\psi^{t=2}) =$$

$$|\langle ok| \langle ok| |\psi^{t=2}\rangle|^2 = 1/12$$

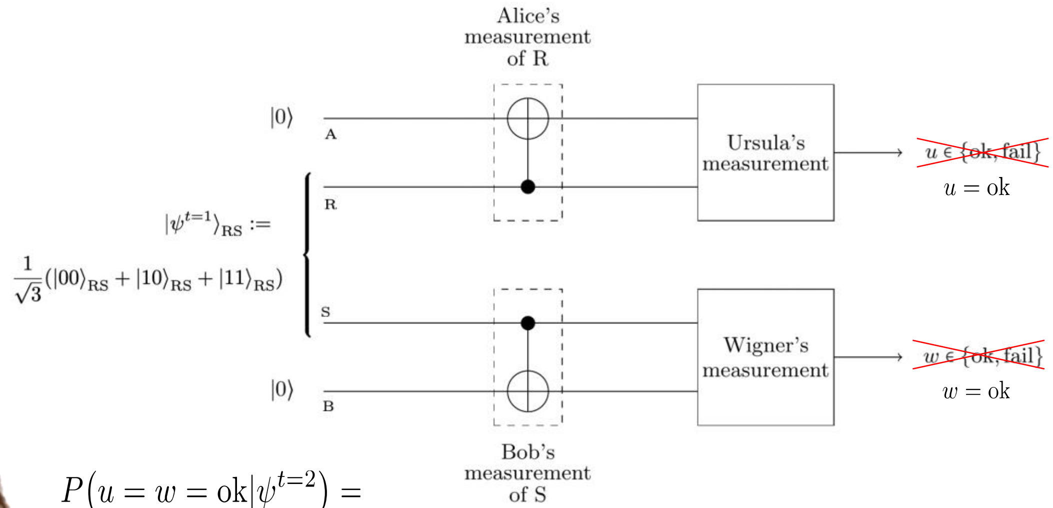
$$\therefore \quad K_U(u = w = \text{ok})$$

$$\langle 00|_{SB} \langle \text{ok}|_{RA} |\psi^{t=2}\rangle_{RASB} = 0$$

$$\implies K_U(u = \text{ok} \Rightarrow b = 1)$$

$$\therefore \quad K_U(b = 1)$$

FR Experiment: Reasoning



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$$\implies K_U(u = \text{ok} \Rightarrow b = 1)$$
$$\therefore \quad K_U(b = 1)$$

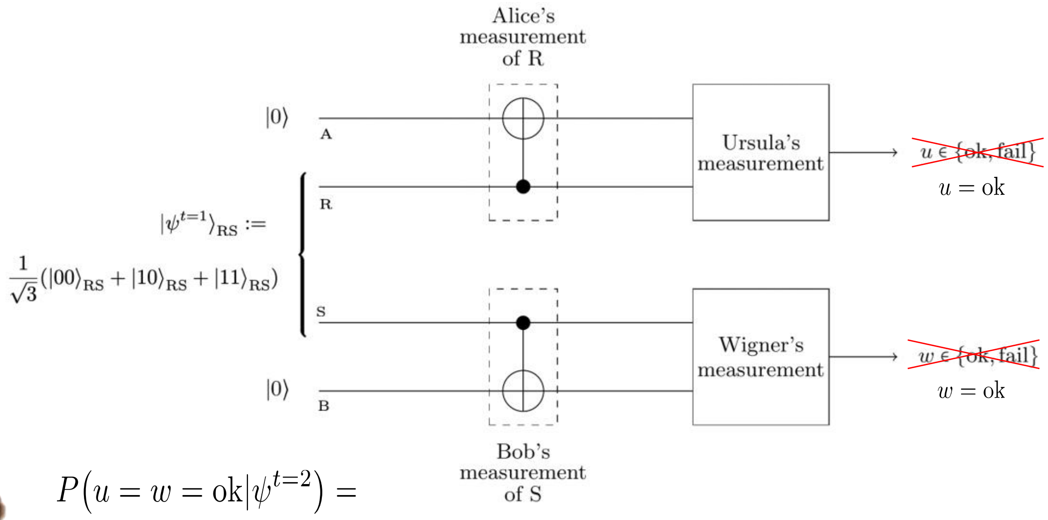
What Bob must think if b=1:

$$\underbrace{\langle 00|_{RA}\langle 11|_{SB}|\psi^{t=2}\rangle_{RASB}}_{\text{Prob. Alice sees a=1 given b=1}} = 0$$

Prob. Alice sees a=1 given b=1

$$K_U K_B(b = 1 \Rightarrow a = 1)$$
$$\therefore K_U(b = 1 \Rightarrow a = 1)$$
$$\therefore \; K_U(a = 1)$$

FR Experiment: Reasoning



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What Bob must think if b=1:

$$\underbrace{\langle 00|_{RA} \langle 11|_{SB} |\psi^{t=2}\rangle_{RASB}}_{\text{Prob. Alice sees a=1 given b=1}} = 0$$

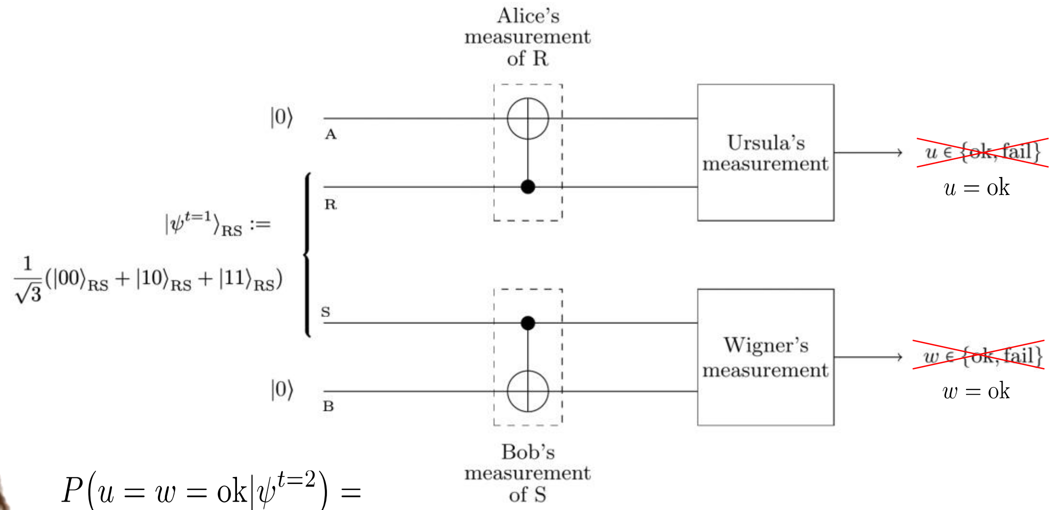
$$K_U K_B(b = 1 \Rightarrow a = 1)$$
$$\therefore K_U(b = 1 \Rightarrow a = 1)$$
$$\therefore K_U(a = 1)$$

What Bob thinks Alice must think if a=1:

$$\underbrace{\langle \text{ok}|_{SB} \langle 11|_{RA} |\psi^{t=2}\rangle_{RASB}}_{\text{Prob. w=ok given a=1}} = 0$$

$$K_U K_B K_A(a = 1 \Rightarrow w = \text{fail})$$
$$\therefore K_U(a = 1 \Rightarrow w = \text{fail})$$
$$\therefore K_U(w = \text{fail})$$

FR Experiment: Reasoning



$$P(u = w = \text{ok}|\psi^{t=2}) =$$
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$$\langle 00|_{SB}\langle \text{ok}|_{RA}|\psi^{t=2}\rangle_{RASB} = 0$$
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$$\therefore K_U(b = 1)$$

What Bob must think if b=1:

$$\langle 00|_{RA}\langle 11|_{SB}|\psi^{t=2}\rangle_{RASB} = 0$$

Prob. Alice sees a=1 given b=1

$$K_U K_B(b = 1 \Rightarrow a = 1)$$
$$\therefore K_U(b = 1 \Rightarrow a = 1)$$
$$\therefore K_U(a = 1)$$

What Bob thinks Alice must think if a=1:

$$\langle \text{ok}|_{SB}\langle 11|_{RA}|\psi^{t=2}\rangle_{RASB} = 0$$

Prob. w=ok given a=1

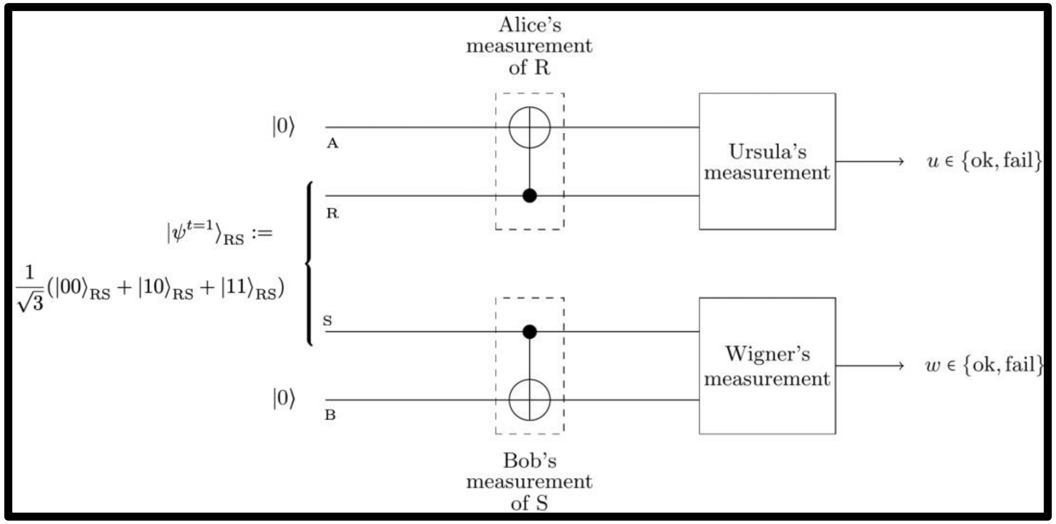
$$K_U K_B K_A(a = 1 \Rightarrow w = \text{fail})$$
$$\therefore K_U(a = 1 \Rightarrow w = \text{fail})$$
$$\therefore K_U(w = \text{fail})$$

~~S~~

D. Frauchiger & R. Renner:

Thm: Q, U, C, D & S cannot simultaneously hold!

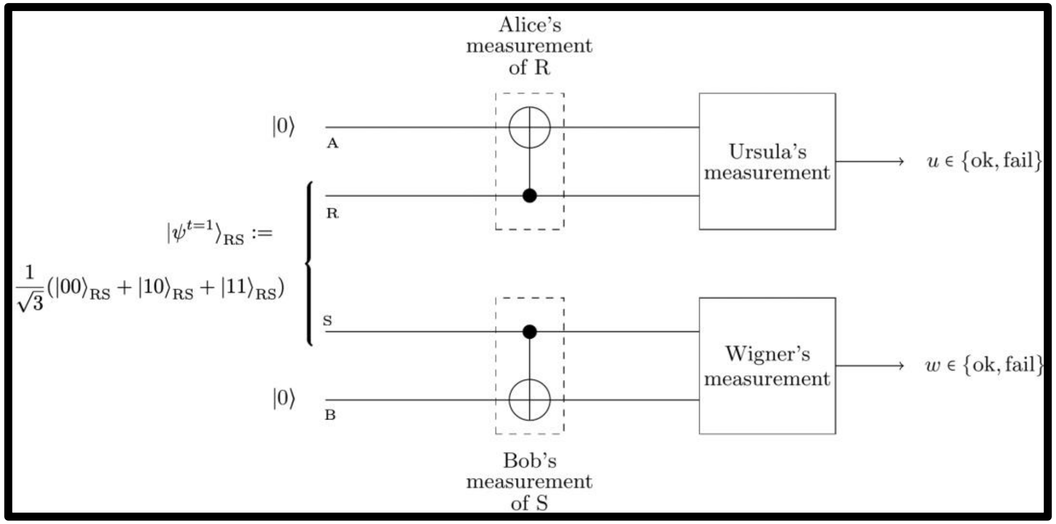
Our Resolution



V. Vilasini & M Woods:

Thm: Q, U, C, D & S can~~not~~ simultaneously hold!

Our Resolution

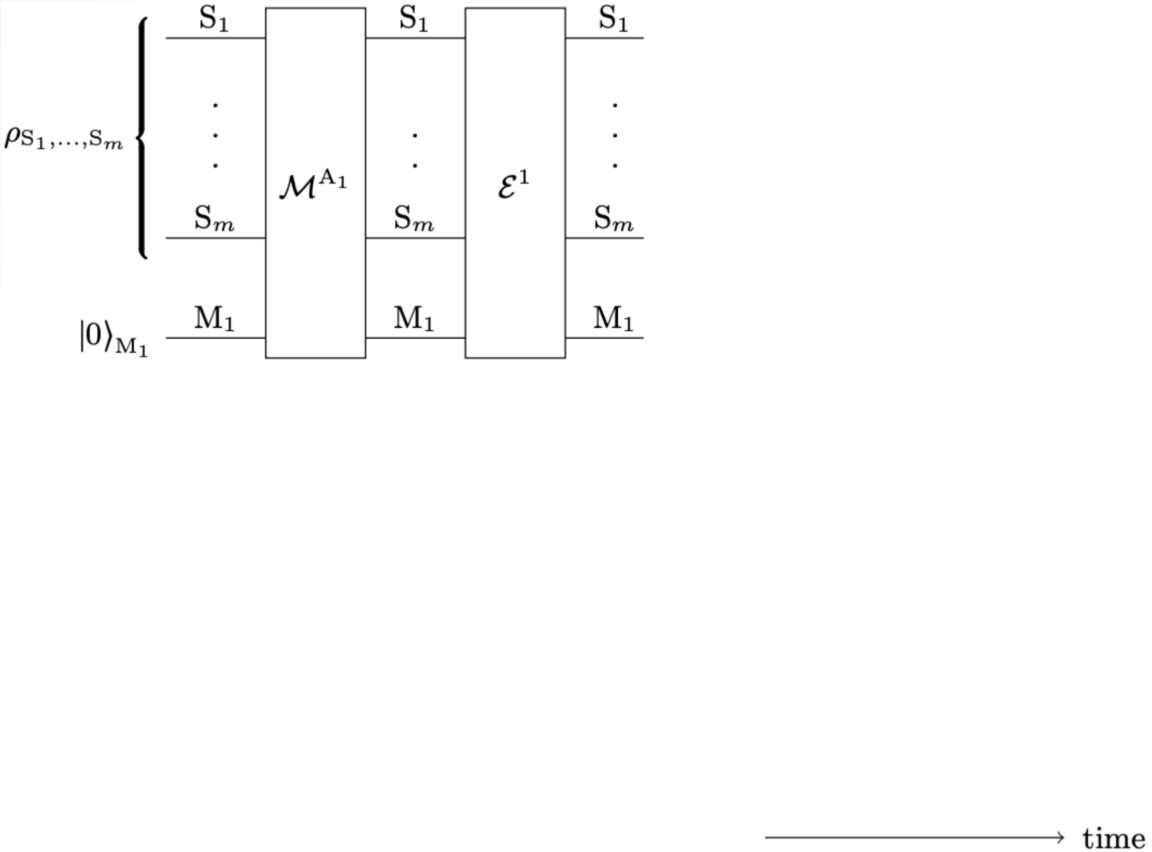


V. Vilasini & M Woods:

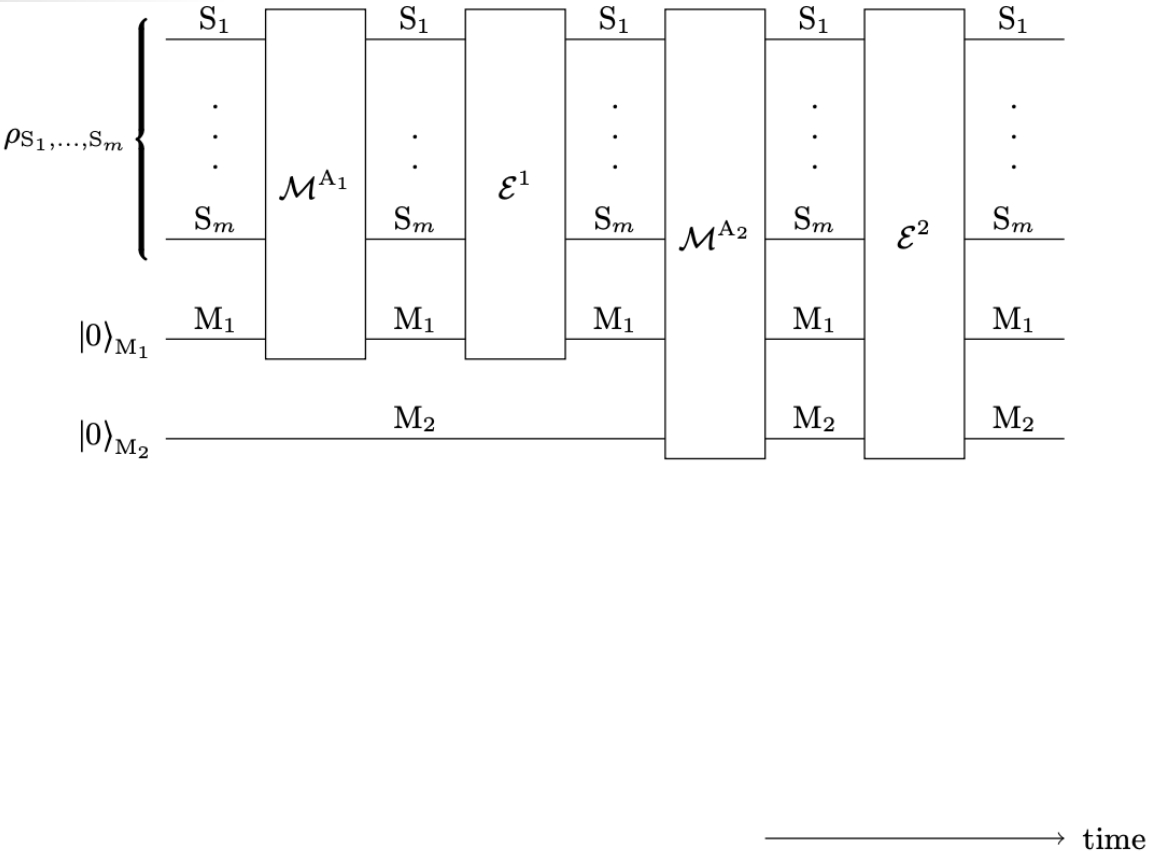
Thm: **Q**, **U**, **C**, **D** & **S** can~~not~~ simultaneously hold!

Be **more precise** in how they are applied: keep track of *who* is applying **Q** to *which* system

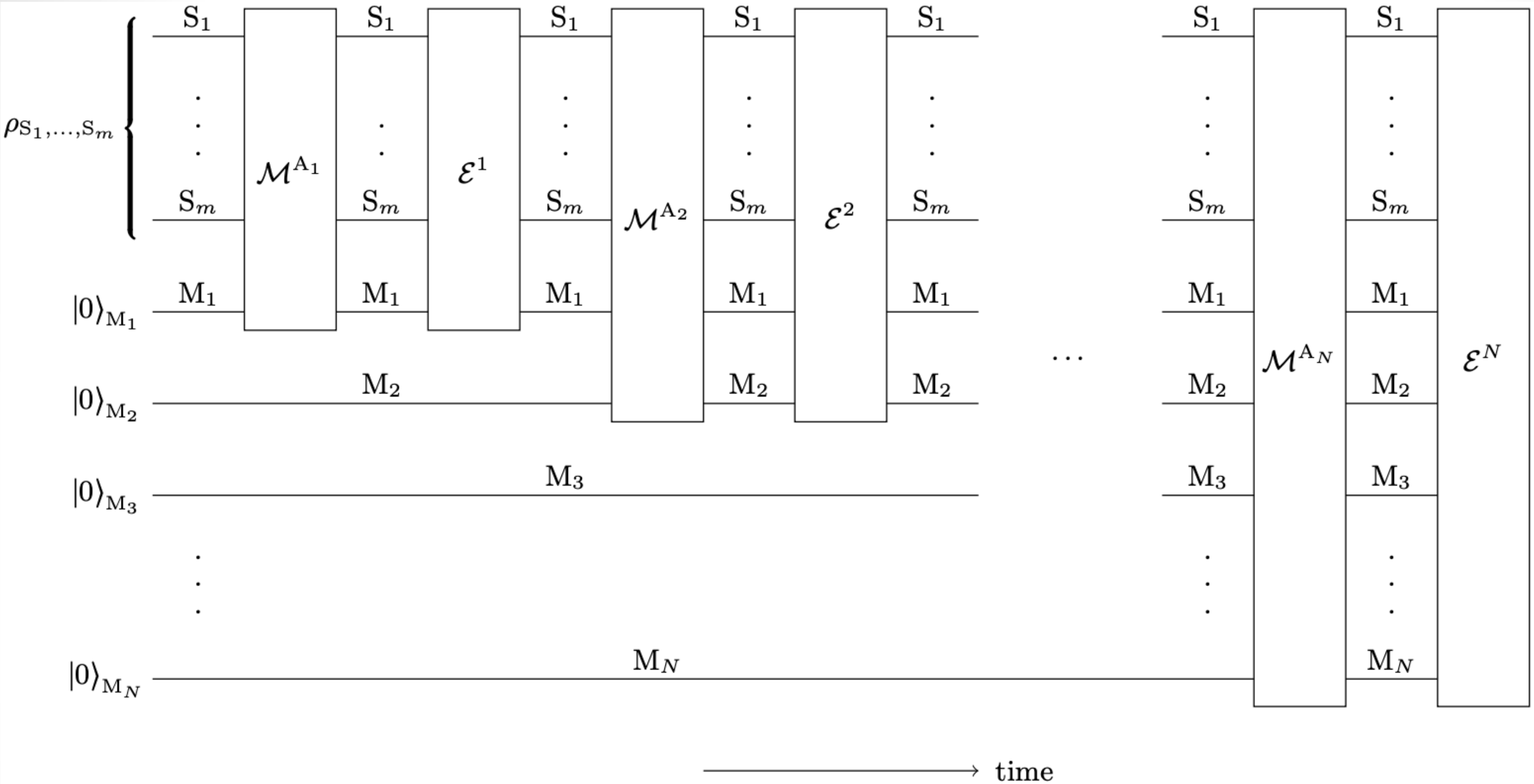
Our Resolution: LWFS general model



Our Resolution: LWFS general model

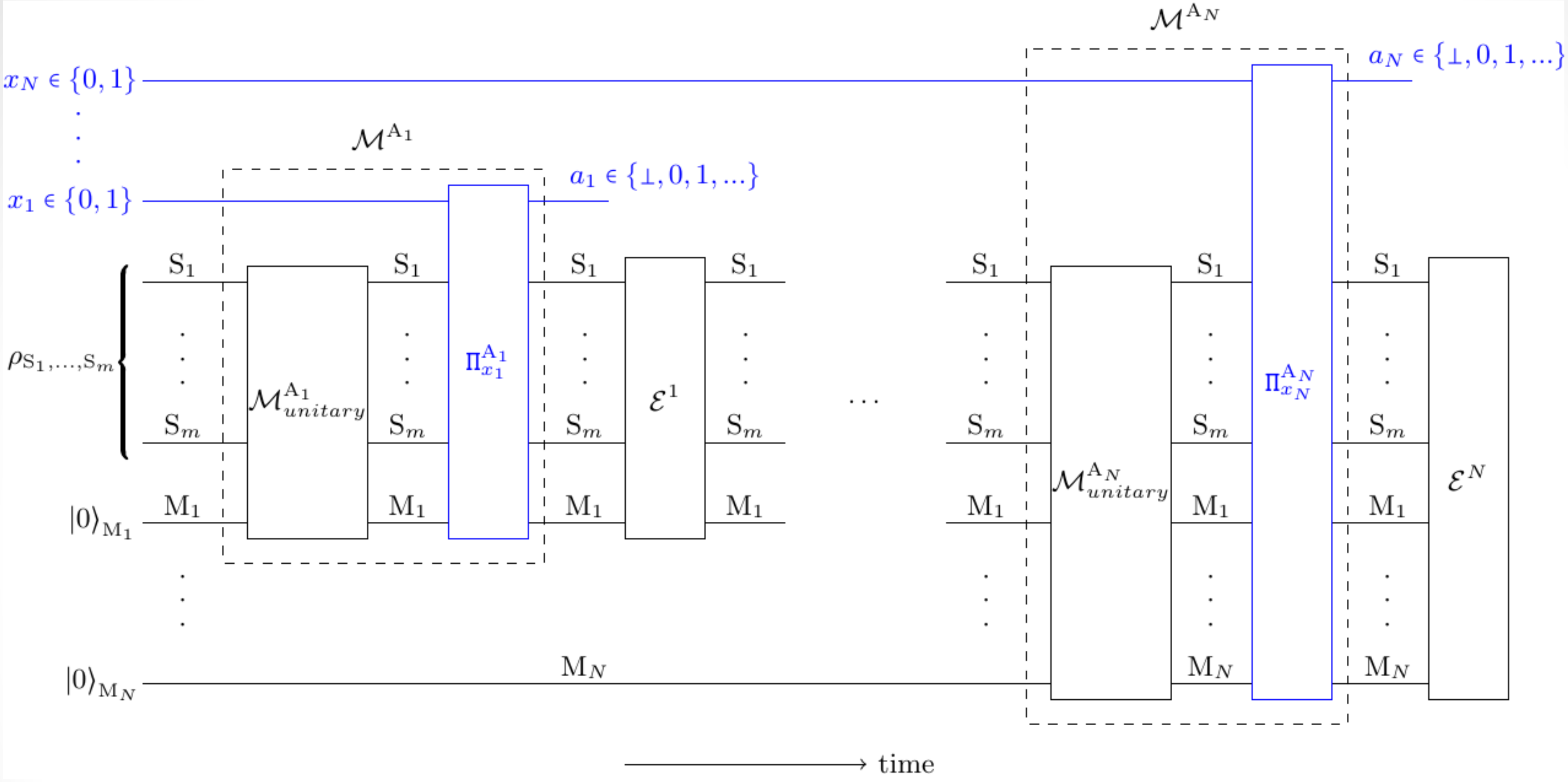


Our Resolution: LWFS general model



- Agents A_j will want to make *predictions.* Need *augmented* circuit.

Our Resolution: LWFS general model



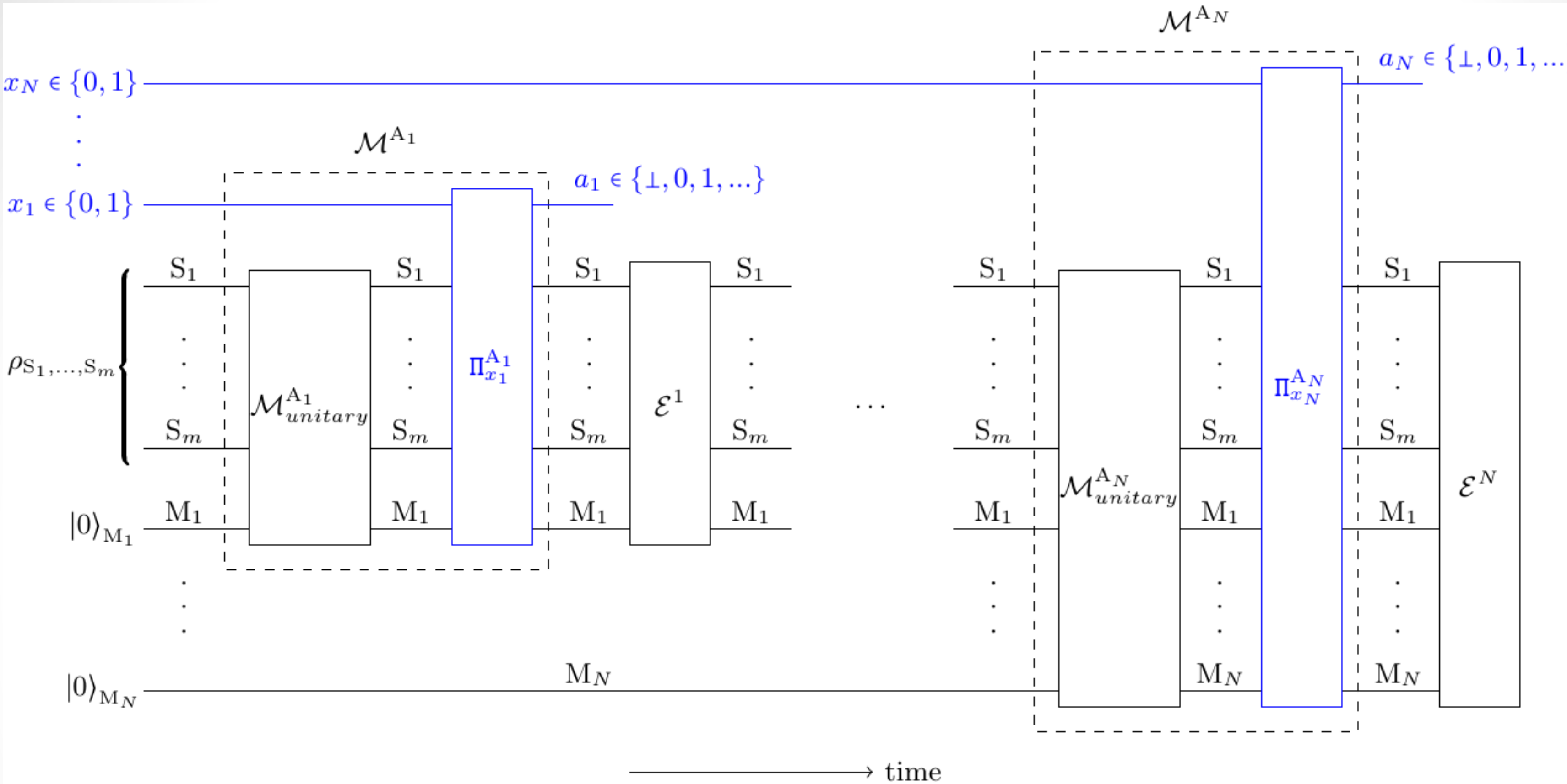
Sample space: $\Omega = \{a_1, x_1, a_2, x_2, \ldots, a_N, x_N\}_{\{a_j, x_j\}_j}$

Set of all events: power set of Ω

Predictions: $P(a_{j_1}, a_{j_2}, \ldots, a_{j_p}, \vec{x}) := P(a_{j_1}, a_{j_2}, \ldots, a_{j_p} | \vec{x}) P(\vec{x}), \ \vec{x} = (x_1, x_2, \ldots, x_N)$

$P\left(a_{j_1}, a_{j_2}, \ldots, a_{j_p} \,\middle|\, a_{l_1}, \ldots, a_{l_q}, \vec{x}\right)$

Our Resolution: LWFS general model



Sample space: $\Omega = \{a_1, x_1, a_2, x_2, \ldots, a_N, x_N\}_{\{a_j, x_j\}_j}$

Set of all events: power set of Ω

Defined via Q (Born rule)

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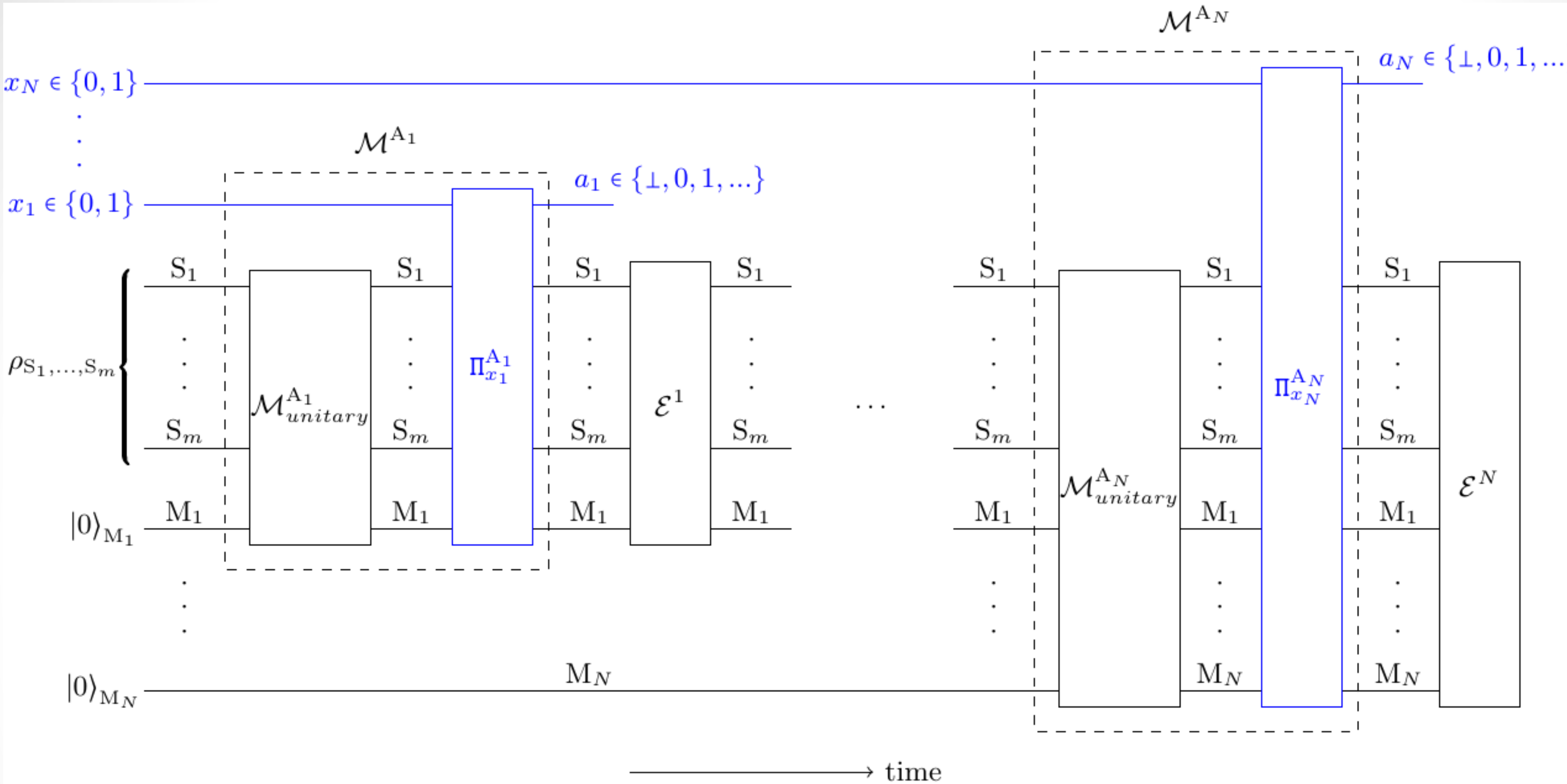
Our Resolution: LWFS general model



Sample space: $\Omega = \{a_1, x_1, a_2, x_2, \ldots, a_N, x_N\}_{\{a_j, x_j\}_j}$

Set of all events: power set of Ω

Defined via Q
(Born rule)

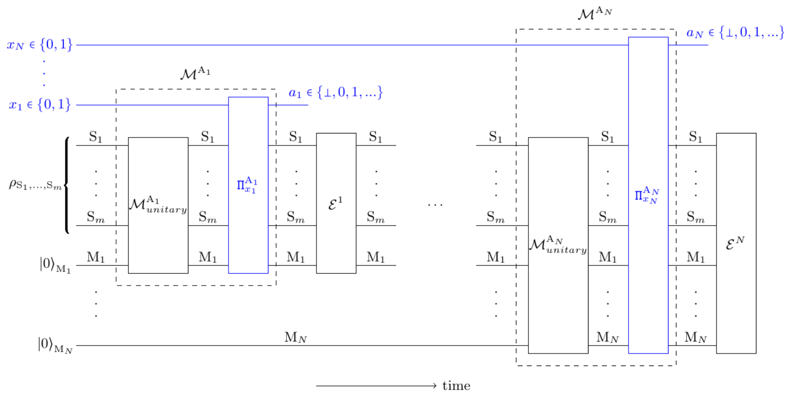
Predictions: $P(a_{j_1}, a_{j_2}, \ldots, a_{j_p}, \vec{x}) := P(a_{j_1}, a_{j_2}, \ldots, a_{j_p} | \vec{x}) P(\vec{x}), \;\; \vec{x} = (x_1, x_2, \ldots, x_N)$

$P\left(a_{j_1}, a_{j_2}, \ldots, a_{j_p} \,\middle|\, a_{l_1}, \ldots, a_{l_q}, \vec{x}\right)$

$x_{j_1} = x_{j_2} = \ldots = x_{j_p} = x_{l_1} = \ldots = x_{l_q} = 1$

Other settings free to choose

Our Resolution: LWFS general model



In our model:

✓ Thm: **Q**, **U**, **C**, **D** and **S** are all self-consistent.

✓ Temporally consistency

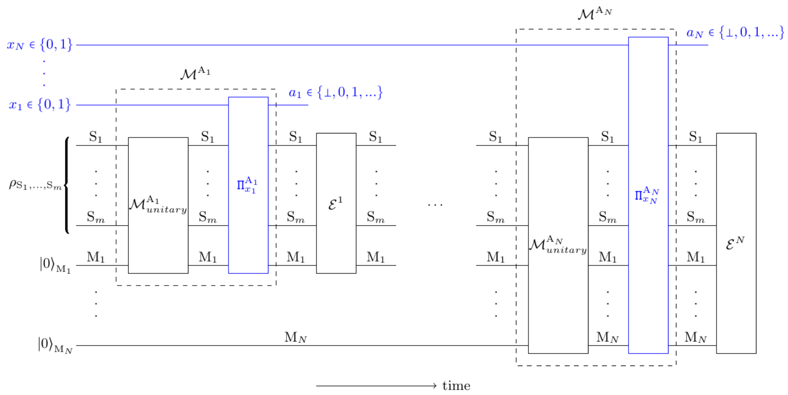
- Difference with FR experiment:

$$P\left(a_{j_1}, a_{j_2}, \ldots, a_{j_p} \,\middle|\, a_{l_1}, \ldots a_{l_q}, \vec{x}\right)$$

$$x_{j_1} = x_{j_2} = \ldots = x_{j_p} = x_{l_1} = \ldots = x_{l_q} = 1$$

Other settings ~~free to choose~~ chosen, but not stated

Our Resolution: LWFS general model



In our model:

✓ Thm: **Q**, **U**, **C**, **D** and **S** are all self-consistent.

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- Difference with FR experiment:

$$P\left(a_{j_1}, a_{j_2}, \ldots, a_{j_p} \,\middle|\, a_{l_1}, \ldots a_{l_q}, \vec{x}\right)$$

$$x_{j_1} = x_{j_2} = \ldots = x_{j_p} = x_{l_1} = \ldots = x_{l_q} = 1$$

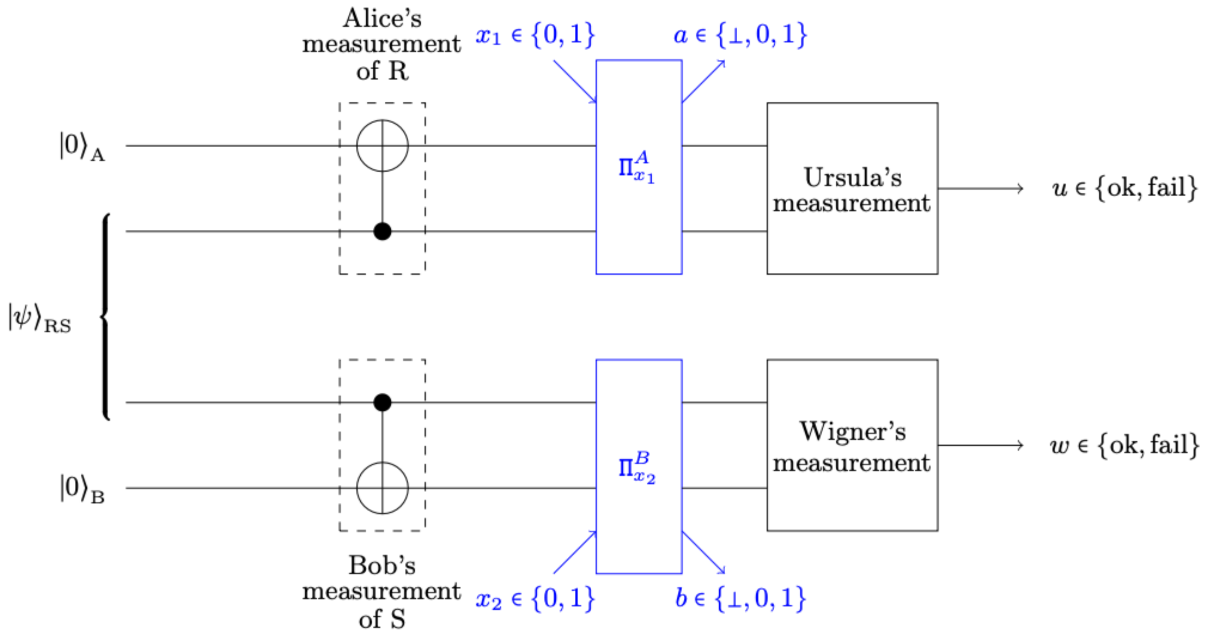
Other settings ~~free to choose~~ chosen, but not stated

➢ Formalize this difference:

Assumption **I**: if $P\left(a_{j_1}, a_{j_2}, \ldots, a_{j_p} \,\middle|\, a_{l_1}, \ldots a_{l_q}, \vec{x}\right)$ is $\vec{x}$ independent, it is said to be *setting independent*

➢ Thm: assumption **I** is *necessary* for at least one prediction in order to recover the FR paradox

Our Resolution: augmented circuit



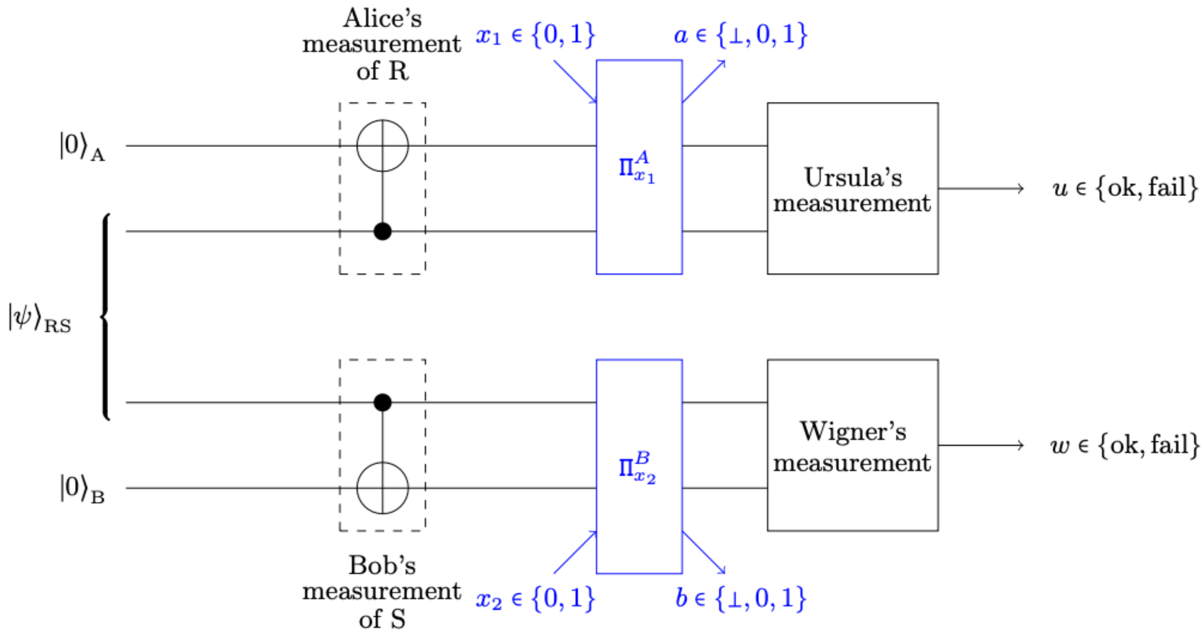
original formulation

$$u = \text{ok} \Rightarrow b = 1$$
$$b = 1 \Rightarrow a = 1$$
$$a = 1 \Rightarrow w = \text{fail}$$
$$\therefore \; u = \text{ok} \implies w = \text{fail}$$

$$\begin{cases} P\big(b=1 | w=u=\text{ok}, (x_1,x_2)=(0,1)\big) = |\langle 11|_{RA} \langle 11|_{SB} |\psi^{t=2}\rangle_{RASB}|^2 = 1 \\ P\big(b=1 | w=u=\text{ok}, (x_1,x_2)=(1,1)\big) = 1/3 \quad \therefore \; \sout{(x_1,x_2)=(1,1)} \end{cases}$$

$$P\big(a=1, b=1 | w=u=\text{ok}, (x_1,x_2)=(1,1)\big) = 1/3$$

Our Resolution: augmented circuit



original formulation	our explicit formulation
$u = \text{ok} \Rightarrow b = 1$	$(x_1, x_2) = (0,1) \wedge u = \text{ok} \Rightarrow b = 1$
$b = 1 \Rightarrow a = 1$	$(x_1, x_2) = (1,1) \wedge b = 1 \Rightarrow a = 1$
$a = 1 \Rightarrow w = \text{fail}$	$(x_1, x_2) = (1,0) \wedge a = 1 \Rightarrow w = \text{fail}$

$\therefore \; u = \text{ok} \implies w = \text{fail}$ $\qquad\qquad$ $\therefore \; u = \text{ok} \not\Longrightarrow w = \text{fail}$

$$\begin{cases} P\big(b=1|w=u=\text{ok},(x_1,x_2)=(0,1)\big) = |\,\langle 11|_{\text{RA}}\,\langle 11|_{\text{SB}}\,|\psi^{t=2}\rangle_{\text{RASB}}\,|^2 = 1 \\ P\big(b=1|w=u=\text{ok},(x_1,x_2)=(1,1)\big) = 1/3 \quad \therefore \; ~~(x_1,x_2)=(1,1)~~ \end{cases}$$

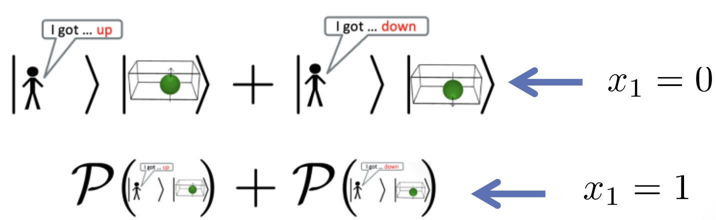
$$P\big(a=1,b=1|w=u=\text{ok},(x_1,x_2)=(1,1)\big) = 1/3$$

Renato's challenge:

- The set of **rules must be unambiguous**. Using them should thus correspond to executing an algorithm, which could in principle be run on a machine.
- The set of rules should **not lead to contradictions**, e.g., when they are applied to the FR thought experiment.
- The rules should **be applicable to any experiment** that is realistic in principle (whereby building a universal quantum computer is considered realistic in principle), including multi-agent setups **where one agent may acquire information from another agent**. For such experiments, the predictions of standard quantum theory (without any additions such as hidden variables or a collapse mechanism) are unambiguous and should agree with those of the proposed set of rules.
- The set of rules should be **compatible with basic physical principles** (such as non-signalling to the past).

Our work: resolves the challenge!

Wigner's original mystery:



- Still wide open
- But doesn't lead to a logical contradiction
- Need solution to the measurement "problem" to solve Wigner's mystery

Related work and

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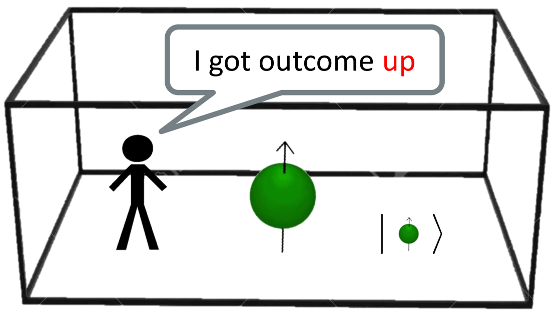
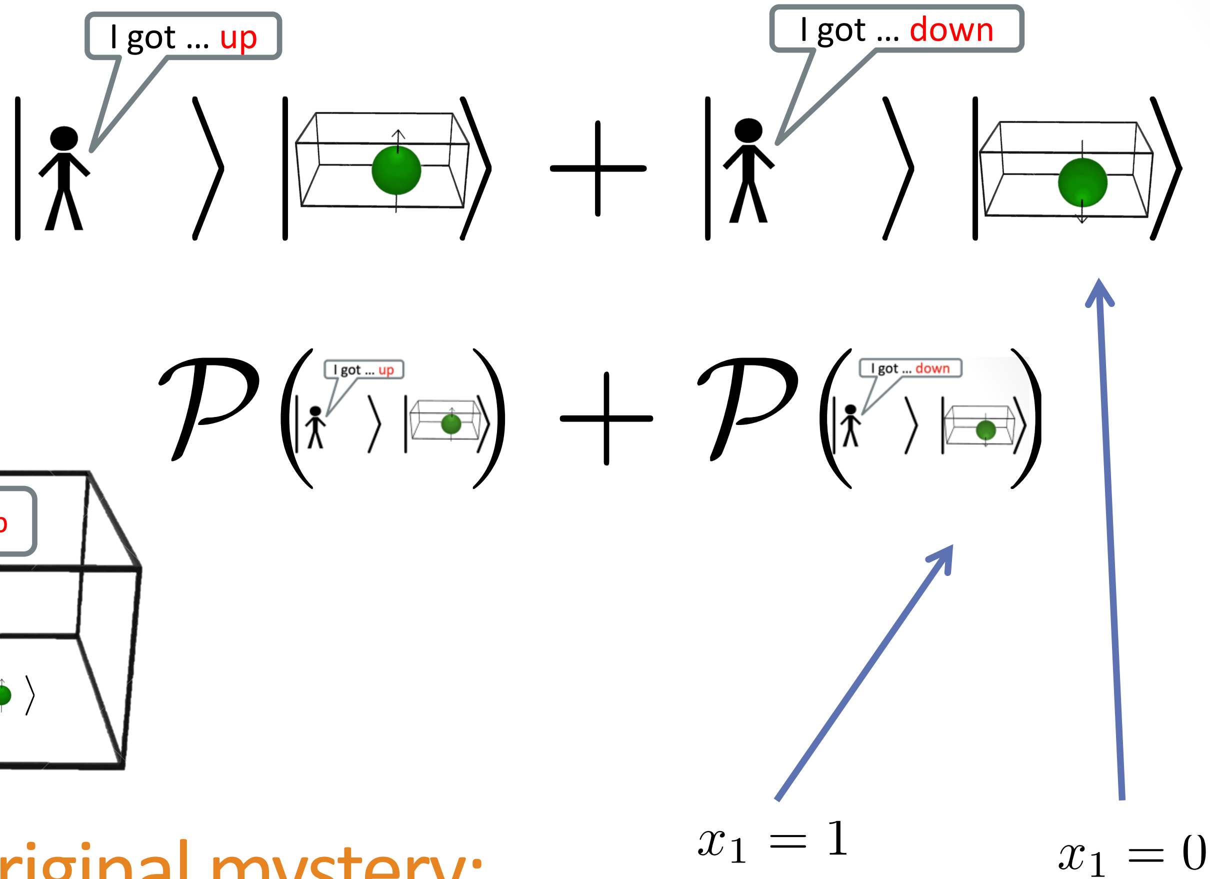
Wigner's original mystery





Wigner's original mystery:

1) still wide open
2) Impossible to make it a *logical* contradiction