Higher-order QED radiative corrections in HEP: the status and prospects

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Future e^+e^- collider projects

Linear Colliders

- ILC, CLIC

E_{tot}

- ILC: 91; 250 GeV — 1 TeV
- CLIC: 500 GeV — 3 TeV

$\mathcal{L} \approx 2 \cdot 10^{34}$ cm^{-2}s^{-1}

Stat. uncertainty $\sim 10^{-4}$

Beam polarization:
e^-beam: $P = 80 - 90\%$
e^+beam: $P = 30 - 60\%$

Circular Colliders

- FCC-ee, CEPC
- Z-factory
- Super Charm-Tau Factory
- $\mu^+\mu^-$ collider (μTRISTAN)

E_{tot}

- 91; 160; 240; 350 GeV

$\mathcal{L} \approx 2 \cdot 10^{36}$ cm^{-2}s^{-1} (4 exp.)

Stat. uncertainty $\sim 10^{-6}$

Tera-Z mode!

Beam polarization: desirable

To-do list for QED

- Compute 2-loop QED radiative corrections to differential distributions of key processes: Bhabha scattering, muon decay, $e^+e^- \to \mu^+\mu^-$*, $e^+e^- \to \pi^+\pi^-$, $e^+e^- \to ZH$ etc.
- Estimate higher-order contributions within some approximations
- Account for interplay with QCD and electroweak effects
- Match with parton showers at N$^?$LO
- Construct reliable Monte Carlo codes

* See recent work by R. Lee et al., arXiv:2503.09245

Perturbative QED (I)

Fortunately, in our case the general perturbation theory can be applied:

$$\frac{\alpha}{2\pi} \approx 1.2 \cdot 10^{-3}, \quad \left(\frac{\alpha}{2\pi}\right)^2 \approx 1.4 \cdot 10^{-6}$$

Moreover, other effects: hadronic vacuum polarization, (electro)weak contributions, hadronic pair emission, etc. are small in, e.g., Bhabha scattering and can be treated one-by-one separately

Nevertheless, there are some enhancement factors:

1) First of all, the large logarithm $L \equiv \ln \frac{\Lambda^2}{m_e^2}$ where $\Lambda^2 \sim Q^2$ is the momentum transferred squared, e.g., $L(\Lambda = 1\,\text{GeV}) \approx 16$ and $L(\Lambda = M_Z) \approx 24$.

2) The energy region at the Z boson peak ($s \sim M_Z^2$) requires a special treatment since factor M_Z/Γ_Z appears in the annihilation channel

Perturbative QED (II)

Methods of resummation of higher-order QED corrections

- Resummation of vacuum polarization corrections (geometric series): $\alpha(0) \to \alpha(\mu_F^2)$
- Yennie–Frautschi–Suura (YFS) soft photon exponentiation and its extensions, see, e.g., PHOTOS
- Leading logarithms via QED structure functions or QED PDFs (E.Kuraev and V.Fadin 1985;
  A. De Rujula, R.Petronzio, A.Savoy-Navarro 1979)

N.B. Resummation of real photon radiation is good only for sufficiently inclusive observables...

Leading and next-to-leading logs in QED

The QED leading (LO) logarithmic corrections

$$\sim \left(\frac{\alpha}{2\pi}\right)^n \ln^n \frac{s}{m_e^2}$$

were relevant for LEP measurements of Bhabha, $e^+e^- \to \mu^+\mu^-$ etc. for $n \leq 3$ since $\ln(M_Z^2/m_e^2) \approx 24$

NLO contributions

$$\sim \left(\frac{\alpha}{2\pi}\right)^n \ln^{n-1} \frac{s}{m_e^2}$$

with at least $n = 3,\ 4$ are required for future e^+e^- colliders

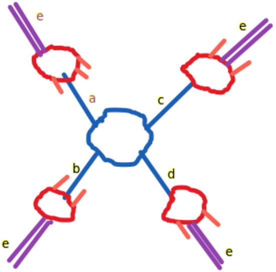
In the collinear approximation we can get them within the NLO QED structure function formalism

- F.A.Berends, W.L. van Neerven, G.J.Burgers, NPB'1988
- A.A., K.Melnikov, PRD'2002; A.A. JHEP'2003

QED NLO master formula

The NLO Bhabha cross section reads

$$d\sigma = \sum_{a,b,c,d=e,\bar{e},\gamma} \int_{\bar{z}_1}^{1} dz_1 \int_{\bar{z}_2}^{1} dz_2 \mathcal{D}^{\rm str}_{ae}(z_1)\mathcal{D}^{\rm str}_{b\bar{e}}(z_2)$$
$$\times \left[d\sigma^{(0)}_{ab\to cd}(z_1,z_2) + d\bar{\sigma}^{(1)}_{ab\to cd}(z_1,z_2) \right]$$
$$\times \int_{\bar{y}_1}^{1} \frac{dy_1}{Y_1} \int_{\bar{y}_2}^{1} \frac{dy_2}{Y_2} \mathcal{D}^{\rm frg}_{ec}\left(\frac{y_1}{Y_1}\right) \mathcal{D}^{\rm frg}_{\bar{e}d}\left(\frac{y_2}{Y_2}\right)$$
$$+\mathcal{O}\left(\alpha^n L^{n-2}, \frac{m_e^2}{s}\right)$$



High-order ISR in e^+e^- annihilation

$$\frac{d\sigma_{e^+e^-\to\gamma^*}}{ds'} = \frac{1}{s}\sigma^{(0)}(s') \sum_{a,b=e^-,\gamma,e^+} D_{ae^-} \otimes \tilde{\sigma}_{ab\to\gamma^*} \otimes D_{be^+}$$

$a\backslash b$	e^+	γ	e^-
e^-	$D_{e^-e^-}D_{e^+e^+}\sigma_{e^-e^+}$ LO (1)	$D_{\gamma e^-}D_{e^-e^-}\sigma_{e^-\gamma}$ NLO $(\alpha^2 L)$	$D_{e^-e^-}D_{e^-e^+}\sigma_{e^-e^-}$ NNLO $(\alpha^4 L^2)$
γ	$D_{\gamma e^-}D_{e^+e^+}\sigma_{e^+\gamma}$ NLO $(\alpha^2 L)$	$D_{\gamma e^-}D_{\gamma e^+}\sigma_{\gamma\gamma}$ NNLO $(\alpha^4 L^2)$	$D_{\gamma e^-}D_{e^-e^+}\sigma_{e^-\gamma}$ NLO $(\alpha^4 L^3)$
e^+	$D_{e^+e^-}D_{e^+e^+}\sigma_{e^+e^+}$ NNLO $(\alpha^4 L^2)$	$D_{e^+e^-}D_{\gamma e^+}\sigma_{e^+\gamma}$ NLO $(\alpha^4 L^3)$	$D_{e^+e^-}D_{e^-e^+}\sigma_{e^+e^-}$ LO $(\alpha^4 L^4)$

Contributions from $D_{e^-e^+}$ and $D_{e^+e^-}$ are missed in [J. Ablinger, J. Blümlein, A. De Freitas and K. Schönwald, "Subleading Logarithmic QED Initial State Corrections to $e^+e^- \to \gamma^*/Z^{0^*}$ to $O(\alpha^6 L^5)$," NPB 955 (2020) 115045]

QED NLO DGLAP evolution equations

$$\mathcal{D}_{ba}\left(x, \frac{\mu_R^2}{\mu_F^2}\right) = \delta_{ab}\delta(1-x) + \sum_{c=e,\gamma,\bar{e}} \int\limits_{\mu_R^2}^{\mu_F^2} \frac{dt}{t} \int\limits_x^1 \frac{dy}{y} P_{bc}(y,t) \mathcal{D}_{ca}\left(\frac{x}{y}, \frac{\mu_R^2}{t}\right)$$

a, b, c are massless partons ($\sim e^{\pm}, \gamma$)

μ_F is a factorization (energy) scale

μ_R is a renormalization (energy) scale

D_{ba} is a parton density function (PDF)

P_{bc} is a splitting function or kernel of the DGLAP equation

N.B. In QED $\mu_R = m_e \approx 0$ is the natural choice well motivated by known analytic results

Iterative solution

The NLO "electron in electron" PDF reads [A.A., U.Voznaya, JPG 2023]

$$\begin{aligned}
\mathcal{D}_{ee}(x,\mu_F,m_e) &= \delta(1-x) + \frac{\alpha}{2\pi} L P^{(0)}_{ee}(x) + \frac{\alpha}{2\pi} d^{(1)}_{ee}(x,m_e,m_e) \\
&+ \left(\frac{\alpha}{2\pi}\right)^2 L^2 \left(\frac{1}{2} P^{(0)}_{ee} \otimes P^{(0)}_{ee}(x) + \frac{1}{3} P^{(0)}_{ee}(x) + \frac{1}{2} P^{(0)}_{e\gamma} \otimes P^{(0)}_{\gamma e}(x)\right) \\
&+ \left(\frac{\alpha}{2\pi}\right)^2 L \left(P^{(0)}_{e\gamma} \otimes d^{(1)}_{\gamma e}(x,m_e,m_e) + P^{(0)}_{ee} \otimes d^{(1)}_{ee}(x,m_e,m_e) - \frac{10}{9} P^{(0)}_{ee}(x) + P^{(1)}_{ee}(x)\right) \\
&+ \left(\frac{\alpha}{2\pi}\right)^3 L^3 \left(\frac{1}{6} P^{(0)}_{ee} \otimes P^{(0)}_{ee} \otimes P^{(0)}_{ee}(x) + \frac{1}{6} P^{(0)}_{e\gamma} \otimes P^{(0)}_{\gamma\gamma} \otimes P^{(0)}_{\gamma e}(x) + \ldots\right) \\
&+ \left(\frac{\alpha}{2\pi}\right)^3 L^2 \left(P^{(0)}_{ee} \otimes P^{(1)}_{ee}(x) + P^{(0)}_{ee} \otimes P^{(0)}_{ee} \otimes d^{(1)}_{ee}(x,m_e,m_e) + \frac{1}{3} P^{(1)}_{ee}(x) - \frac{10}{9} P^{(0)}_{ee} \otimes P^{(0)}_{ee}(x) + \ldots\right) \\
&+ \mathcal{O}(\alpha^2 L^0, \alpha^3 L^1) + \ldots
\end{aligned}$$

The large logarithm $L \equiv \ln \frac{\mu_F^2}{\mu_R^2}$ with factorization scale $\mu_F^2 \sim s$ or $\sim -t$; and renormalization scale $\mu_R = m_e$. Here $\alpha \equiv \alpha(0)$.

A deviation from [M.Skrzypek 1992] is found in singlet-channel contribution in $\mathcal{O}(\alpha^3 L^3)$

$\mathcal{O}(\alpha)$ matching

The expansion of the master formula for ISR gives

$$d\sigma^{(1)}_{e\bar{e}\to\gamma^*} = \frac{\alpha}{2\pi}\left\{2LP^{(0)} \otimes d\sigma^{(0)}_{e\bar{e}\to\gamma^*} + 2d^{(1)}_{ee} \otimes d\sigma^{(0)}_{e\bar{e}\to\gamma^*}\right\} + d\,\bar{\sigma}^{(1)}_{e\bar{e}\to\gamma^*}$$

We know the massive $d\sigma^{(1)}$ and massless $d\,\bar{\sigma}^{(1)}$ ($m_e \equiv 0$ with $\overline{\mathrm{MS}}$ subtraction) results in $\mathcal{O}(\alpha)$ $\Rightarrow$

$$d^{(1)}_{ee} = \left[\frac{1+z^2}{1-z}\left(\ln\frac{\mu_R^2}{m_e^2} - 1 - \ln(1-z)\right)\right]_+, \quad P^{(0)}_{ee}(z) = \left[\frac{1+z^2}{1-z}\right]_+, \quad L = \ln\frac{\mu_F^2}{\mu_R^2}$$

Scheme dependence comes from here

Factorization and renormalization scale dependence is also from here

N.B. "Massification procedure” [McMule Coll.]

Factorization scale choice

We apply the BLM-like prescription, i.e., hide the bulk of one-loop corrections into the scale choice

For e^+e^- annihilation

$$\frac{d\sigma^{(1)}_{e\bar{e}\to\gamma^*}}{d\sigma^{(0)}_{e\bar{e}\to\gamma^*}} = \frac{\alpha}{\pi}\left[\frac{1+z^2}{1-z}\right]_+ \left(\ln\frac{s}{m_e^2} - 1\right) + \delta(1-z)(\ldots) \Rightarrow \mu_F^2 = s \quad \text{or } \mu_F^2 = \frac{s}{e}$$

Remind Drell-Yan processes where we usually take $\mu_F^2 = s' \equiv zs$, i.e., the enegry scale of the hard subprocess

Factorization at NLO

Remind the one-loop correction $\Rightarrow \mu_F^2 = s$

But Berends et al. and Blümlein et al. used $\mu_F = zs$ in PDFs and $\mu_F^2 = ys \neq zs$ in

$$\frac{d\sigma^{(1)}_{e\bar{e}\to\gamma^*}}{d\sigma^{(0)}_{e\bar{e}\to\gamma^*}} = \frac{\alpha}{\pi}\left[\frac{1+y^2}{1-y}\right]_+ \left(\ln\frac{ys}{m_e^2} - 1\right) + \delta(1-y)(\ldots)$$

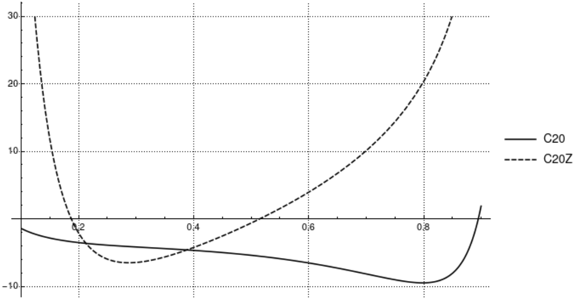
where y is an integration variable in convolution $\overline{\sigma}^{(1)}_{e\bar{e}} \otimes P^{(0)}_{ee}$, sic!

Accidentally their "choice" leads to the proper result in $\mathcal{O}(\alpha^2 L^1)$. But it breaks in higher orders.

Factorization scale choice: numerics (I)

$$d\sigma^{\mathrm{NLO}}_{e\bar{e}\to\gamma^*} = d\sigma^{(0)}_{e\bar{e}\to\gamma^*}\left\{1+\sum_{k=1}^{\infty}\left(\frac{\alpha}{2\pi}\right)^k\sum_{l=k-1}^{k} c_{kl}L^l + \mathcal{O}(\alpha^k L^{k-2})\right\}$$

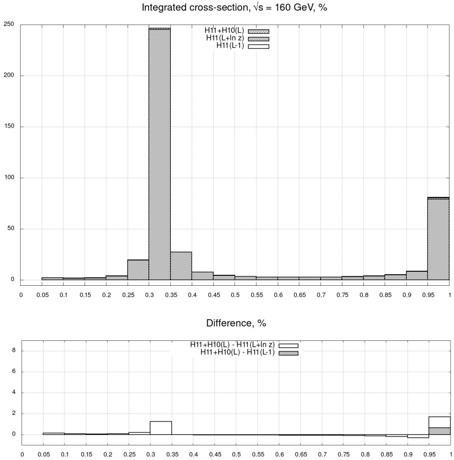
[A.A., U.Voznaya, PRD'2024]



Values of $c_{20}(z)\times 10^6$ are plotted for $\mu_F^2 = s$ (solid line) and for $\mu_F^2 = zs$ (dashed line) [Berends; Blümlein]

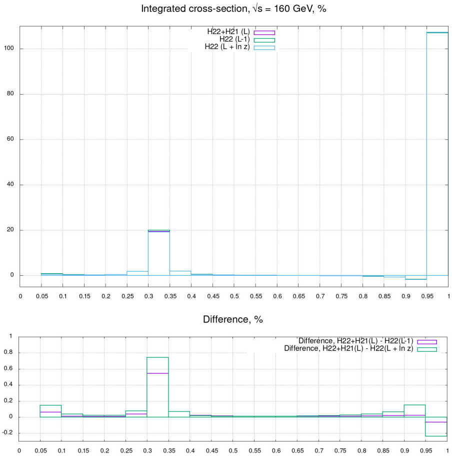
Factorization scale choice: numerics (II)

$\sqrt{s} = 160$ GeV, $\mathcal{O}(\alpha^1)$



Factorization scale choice: numerics (III)

$\sqrt{s} = 160$ GeV, $\mathcal{O}(\alpha^2)$



ISR corrections to $e^+e^- \to Z(\gamma^*)$ ($\sqrt{s} = M_Z$)

LO $\mathcal{O}(\alpha^n L^n)$ and NLO $\mathcal{O}(\alpha^n L^{n-1})$ ISR corrections in % at the Z-peak for $z_{\min} = 0.1$

Type / n	1	2	3	4	5
LO γ	−32.7365	4.8843	−0.3776	0.0034	0.0032
NLO γ	2.0017	−0.5952	0.0710	−0.0019	
LO pair	—	−0.3057	0.0875	0.0016	−0.0001
NLO pair	—	0.1585	−0.0460	0.0038	
Σ	−30.7348	4.1419	−0.2651	0.0069	0.0031

Even higher orders seem to be relevant numerically $\Longrightarrow$ exponentiantion

Exponentiation of the leading logs is straghtforward and known [Gribov-Lipatov, Kuraev-Fadin, ...]

NLO exponentiation in the MSbar scheme is ambigous

Outlook

- A new high-energy e^+e^- collider is well motivated by the necessity to study SM (its Higgs sector in particular) in more detail
- New calculations of two-loop and higher-order corrections within QED and full SM are required
- We have a progress in NLO QED PDFs and fragmentation functions
- QED provides explicit results and can serve as a toy model for cross checks of QCD
- Optimisation of factorization scale and scheme choices is important
- NLO exponentiation is in progress