Formation of dark matter primordial black holes

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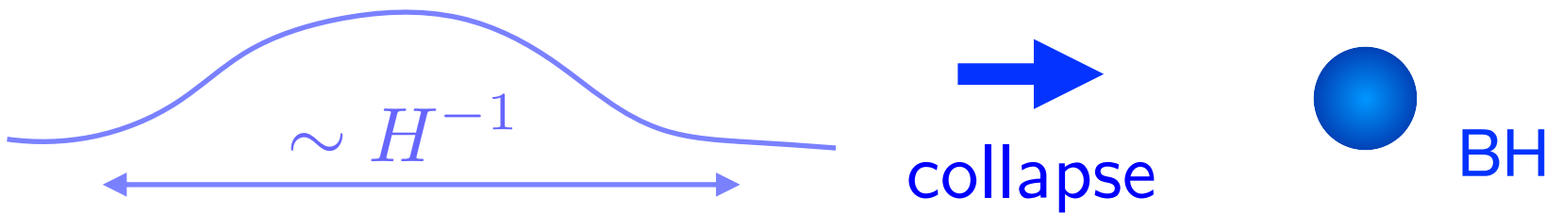
Refs. Ando, MK, Nakatsuka Phys. Rev. D 98, 083508 (2018)
Inomata, MK, Mukaida, Yanagida Phys. Rev. D 109, 043508 (2024)
Kasai, MK, Murai, Neda in preparation
Kasuya, MK, Kusenko, Neda in preparation

1. Introduction

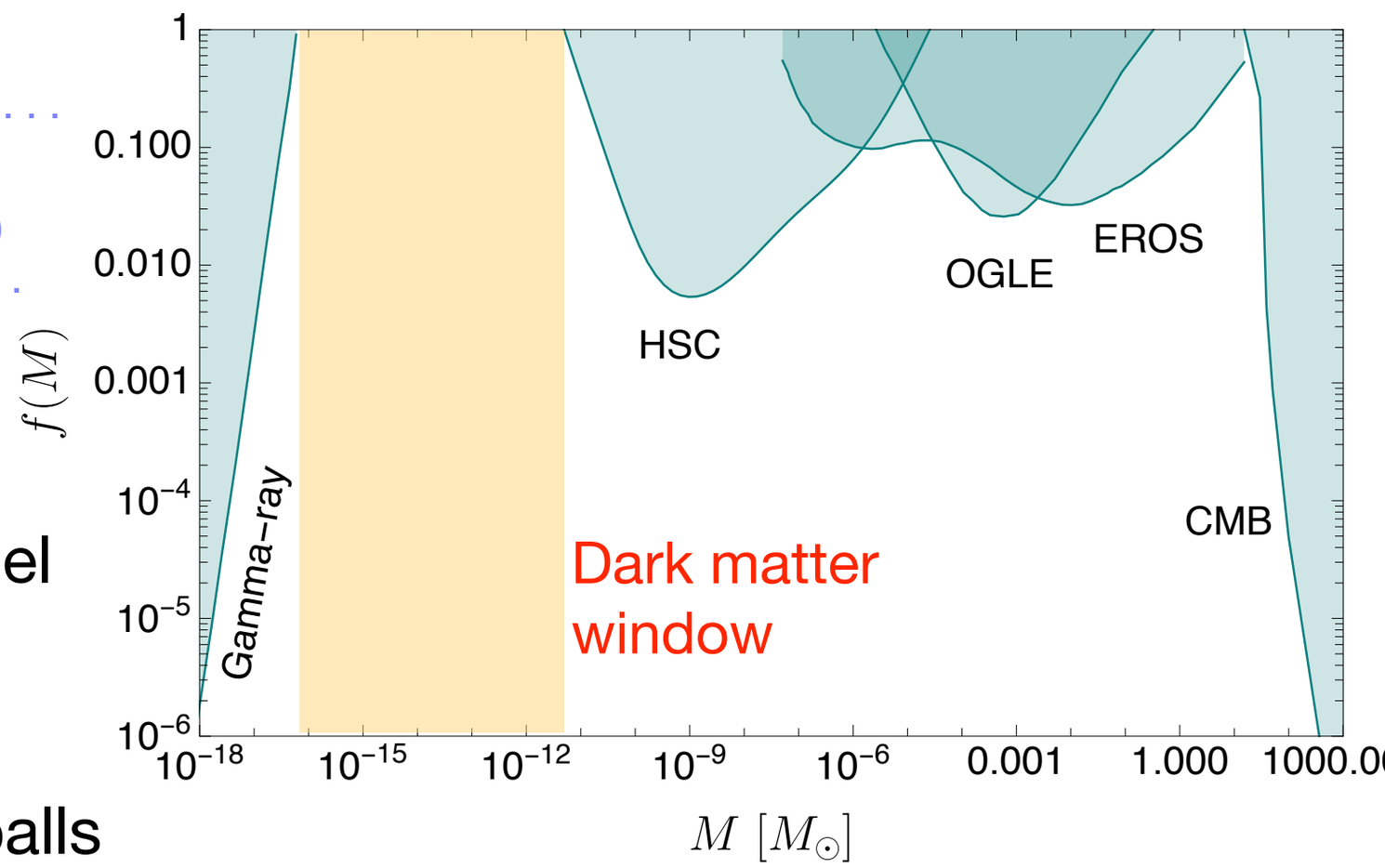
- Primordial Black Holes (PBHs)
  - PBHs are produced by gravitational collapse of over-density regions with Hubble radius in the early universe



- PBHs are of great interest because they could
  - Account for seeds of supermassive BHs $\sim 10^{4-5}\,M_\odot$
  - Account for GW events detected by LIGO-Virgo-KAGRA collaboration $\sim 30 - 50\,M_\odot$
  - Give a significant contribution to dark matter $\sim 10^{17} - 10^{22}\,\mathrm{g}$
- We focus on PBHs that explain dark matter (DM) of the universe

1. Introduction

- Dark matter window is open for PBHs with a mass $M \simeq 10^{17} - 10^{22}\,\mathrm{g}$
- Large density fluctuations with O(0.1) are required for PBH formation but density fluctuations are ~ O(10^{-5}) on CMB scales
- Many sophisticated models have been proposed
  - Ultra-slow-roll
    Ivanov Naselsky Novikov (1994) . . . .
  - Multi-field inflation
    Garcia-Belliido Linde Wands (1996)
    MK, Sugiyama, Yanagida (1998) . . .
  - ……
- We consider two models
  - Axion-like curvaton model
    Kawasaki, Kitajima, Yanagida (2012)
    Ando, Inomata, Kawasaki et al (2018)
    Ando, Kawasaki, Nakatsuka (2018)
  - PBH formation from Q-balls
    Cotner, Kusenko (2016),
    Cotner, Kusenko, Takhistov (2018) . . . .



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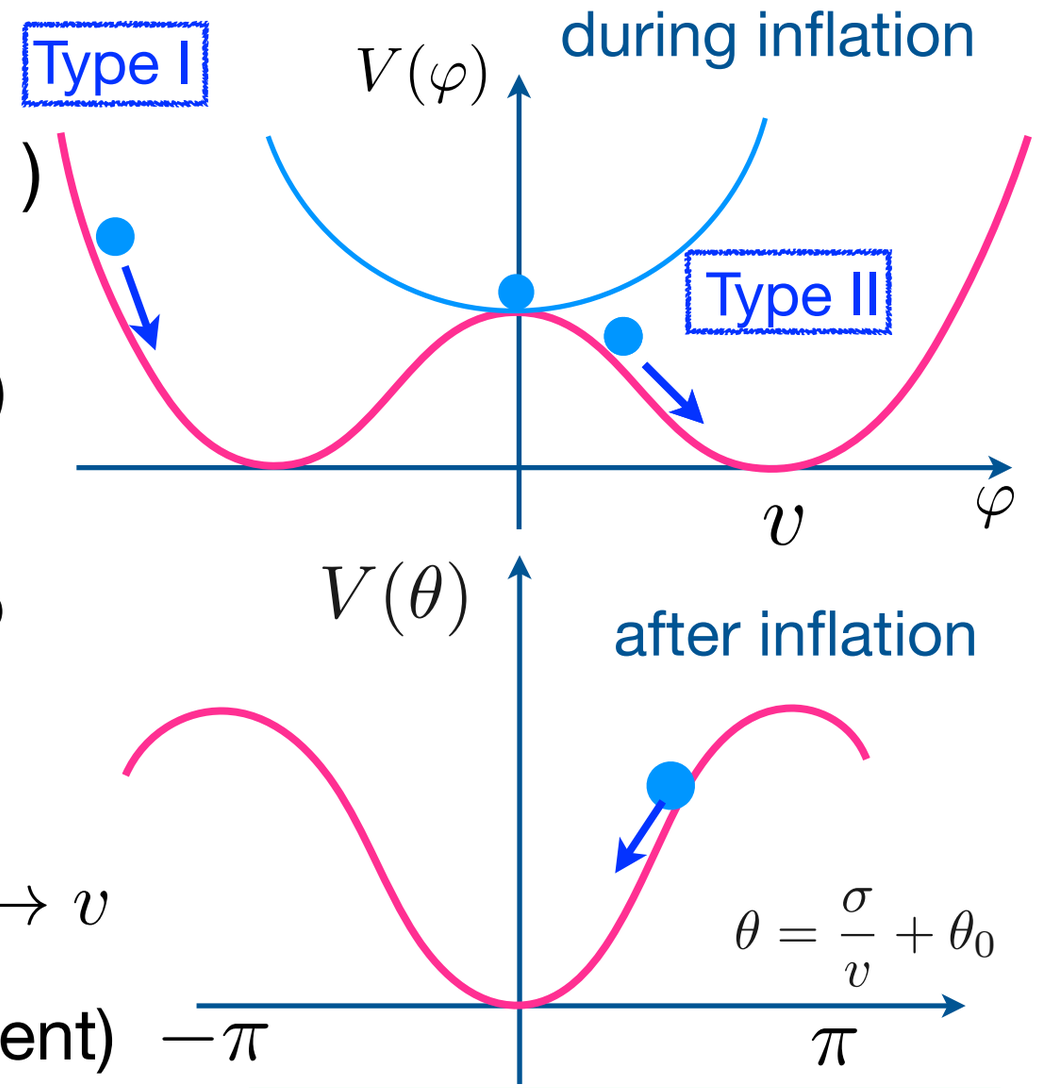
2. Axion-like curvaton models

- Curvaton is a non-inflaton scalar field that generates curvature fluctuations
- Axion-like curvaton (Type I and Type II)
  - Phase of a complex scalar field Φ with spontaneously broken global U(1) plays a role of the curvaton
  - Evolution of Φ during inflation leads to distinctive scale dependence of fluctuations
  - Type I large $\Phi \to v$ Type II $\Phi \simeq 0 \to v$
  - After inflation, axion (= phase component) starts oscillation



$$V_\sigma = \Lambda^4 \left[1 - \cos\left(\frac{\sigma}{v} + \theta_0\right)\right]$$

- Axion fluctuations → curvature perturbations

2.1 Type II axion-like curvaton model

- Model $\quad I$: inflaton $\quad V_I$: inflaton potential

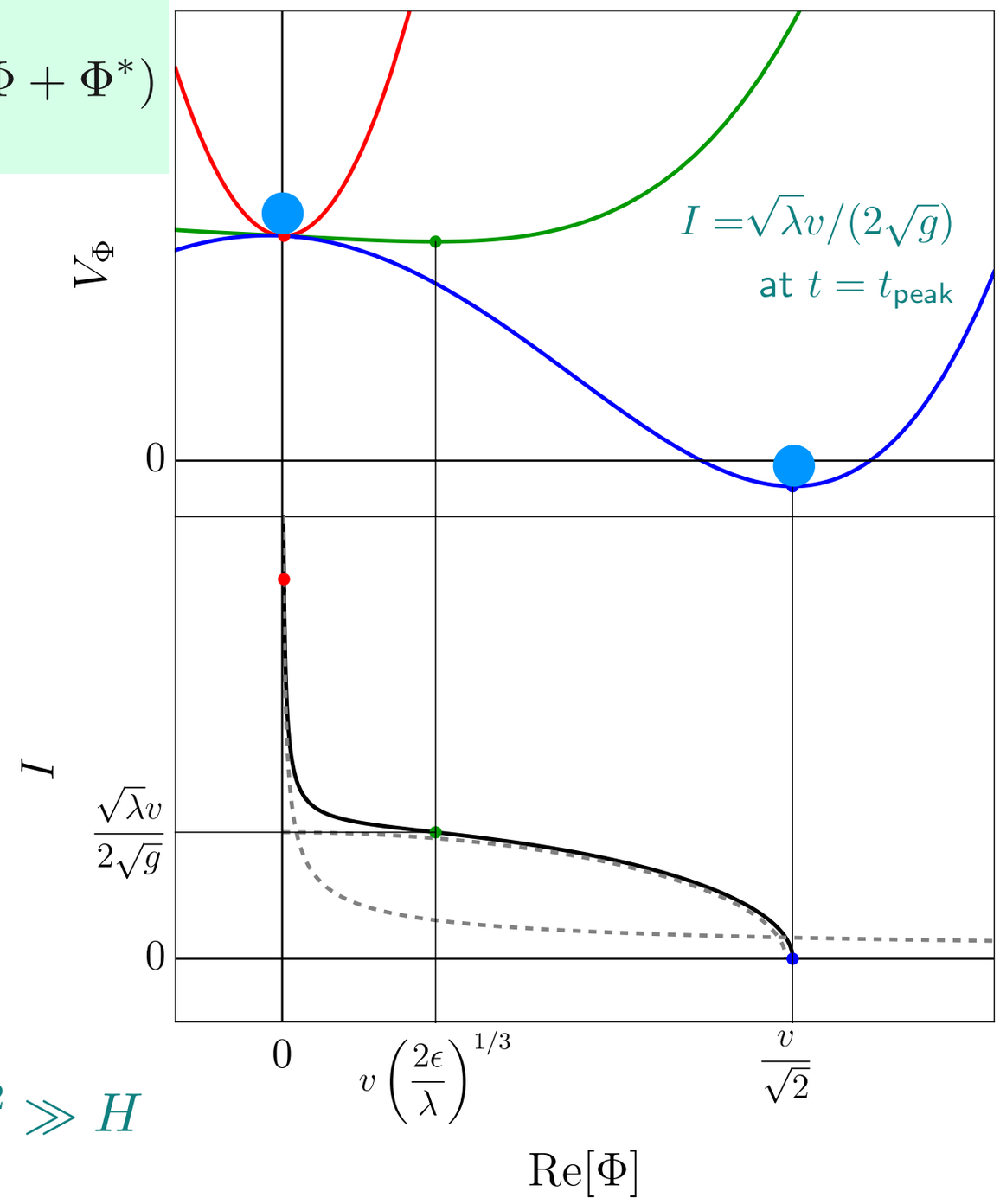
$$\mathcal{L} = \frac{1}{2}\partial_\mu I \partial^\mu I + \partial_\mu \Phi \partial^\mu \Phi^* - V_I(I) - V_\Phi(I,\Phi)$$

$$V_\Phi(I,\Phi) = \frac{\lambda}{4}\left(|\Phi|^2 - \frac{v^2}{2}\right)^2 + gI^2|\Phi|^2 - v^3\epsilon\left(\Phi + \Phi^*\right)$$

  - Effective mass of Φ depends on I

$$I \gtrsim \sqrt{\lambda}v/(2\sqrt{g}) \Rightarrow \Phi \sim 0$$

$$I \lesssim \sqrt{\lambda}v/(2\sqrt{g}) \Rightarrow \Phi \to v/\sqrt{2}$$



- Radial component = saxion
- Axion and saxion fluctuations

$$\Phi = \frac{1}{\sqrt{2}}(\varphi_0 + \delta\varphi)e^{i\delta\sigma/\varphi_0}$$

  - Saxion finally settles at $\varphi \sim v$
  - We can neglect $\delta\varphi$ ⟵ $m_\varphi^2 \sim \lambda v^2 \gg H$

2.1 Type II axion-like curvaton model

- Axion fluctuations
  - Phase component (axion)

$$m_{\delta\sigma}^2 \begin{cases} \gtrsim \lambda v^2/4 \gg H^2 & (t \lesssim t_{\text{peak}}) \\ \simeq \sqrt{2}\epsilon v^2 \ll H^2 & (t \gtrsim t_{\text{peak}}) \end{cases}$$

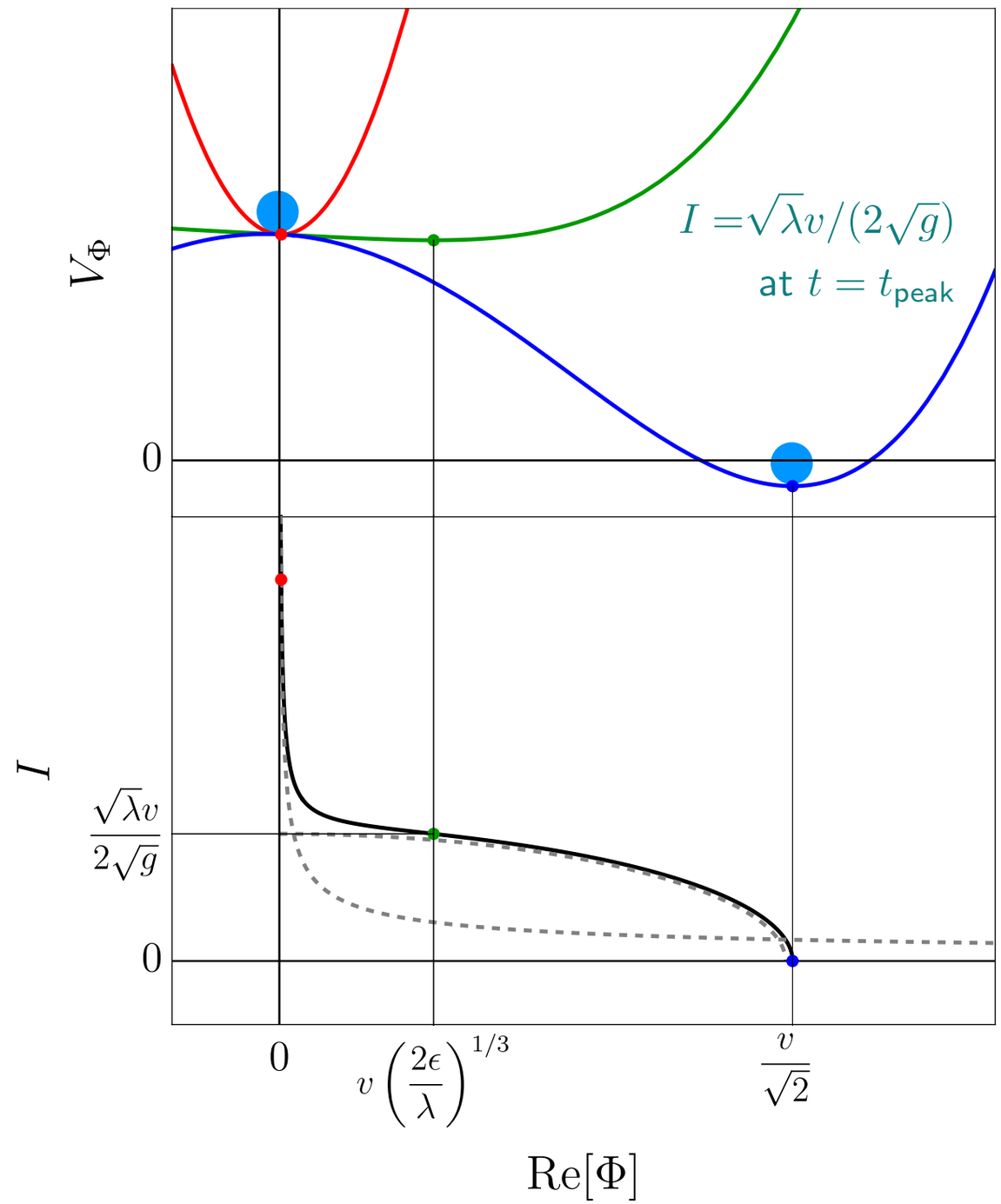
  - At horizon exit $\delta\sigma_k \simeq \dfrac{H}{2\pi}$
  - After horizon exit

$$\delta\sigma_k \propto \begin{cases} a^{-3/2} & (t_k < t < t_{\text{peak}}) \\ \text{constant} & (t_{\text{peak}} < t) \end{cases}$$

- Final axion fluctuation is given by misalignment angle fluctuation

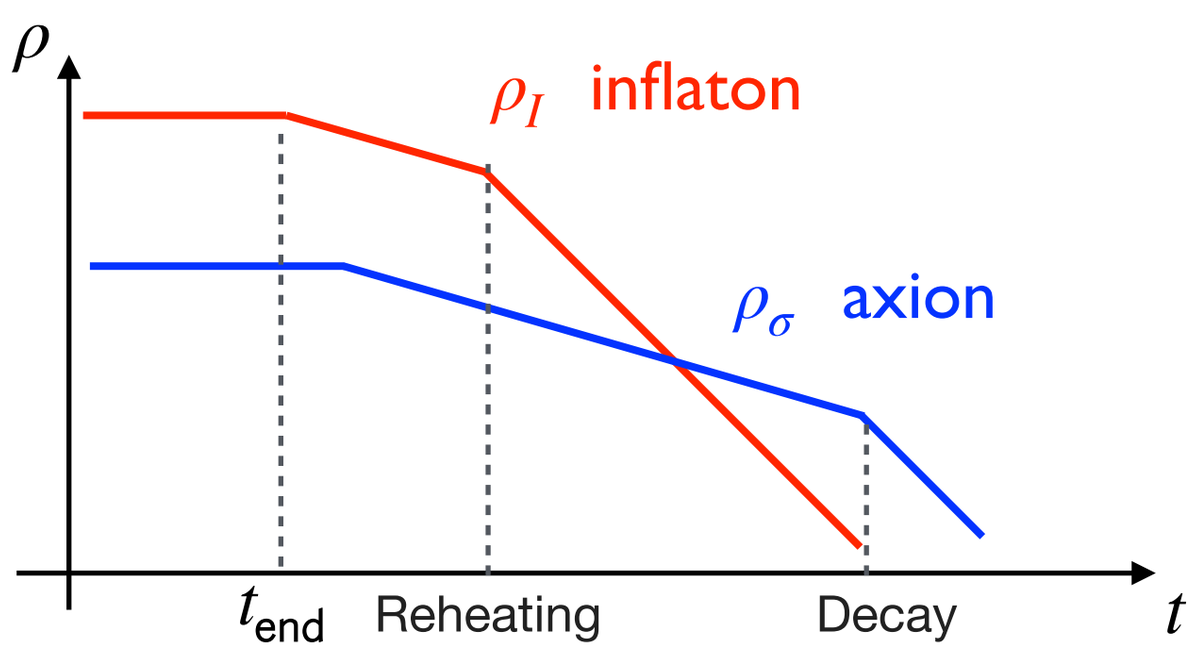
$$\delta\theta_k \simeq \frac{\delta\sigma_k}{\varphi_0} \quad \Rightarrow \quad \frac{\delta\rho_\sigma}{\rho_\sigma} \simeq \frac{2\delta\theta_k}{\theta_i}$$

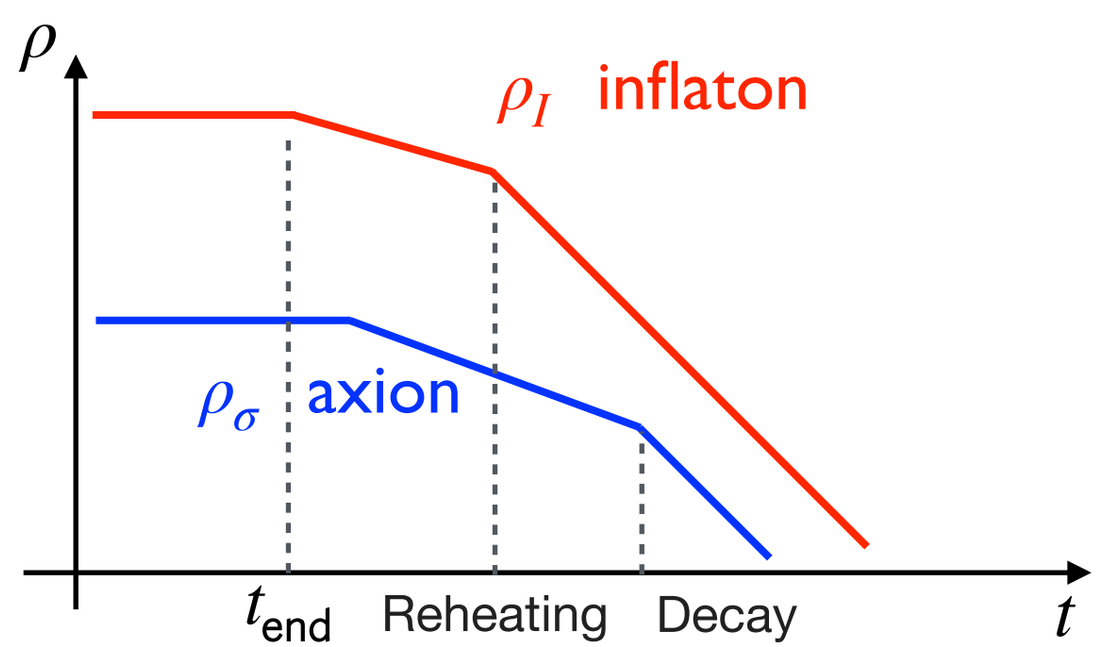
  - Axion fluctuations decreases as saxion moves $0 \to v$ at $t > t_{\text{peak}}$



2.2 Power spectrum of curvature perturbations

- Power spectrum of axion density fluctuation has a peak at $k = k_{\text{peak}}$
  (k_{peak} is the scale that exits horizon at $t = t_{\text{peak}}$)
  - $k < k_{\text{peak}}$: spectrum is blue-tilted due to a decrease of $\delta\sigma_k$ $(t < t_{\text{peak}})$
  - $k > k_{\text{peak}}$: spectrum is red-tilted due to an increase of φ_0 $(t > t_{\text{peak}})$
- Thermal histrory
  - Axion density relatively increases after reheating of inflaton





2.2 Power spectrum of curvature perturbations

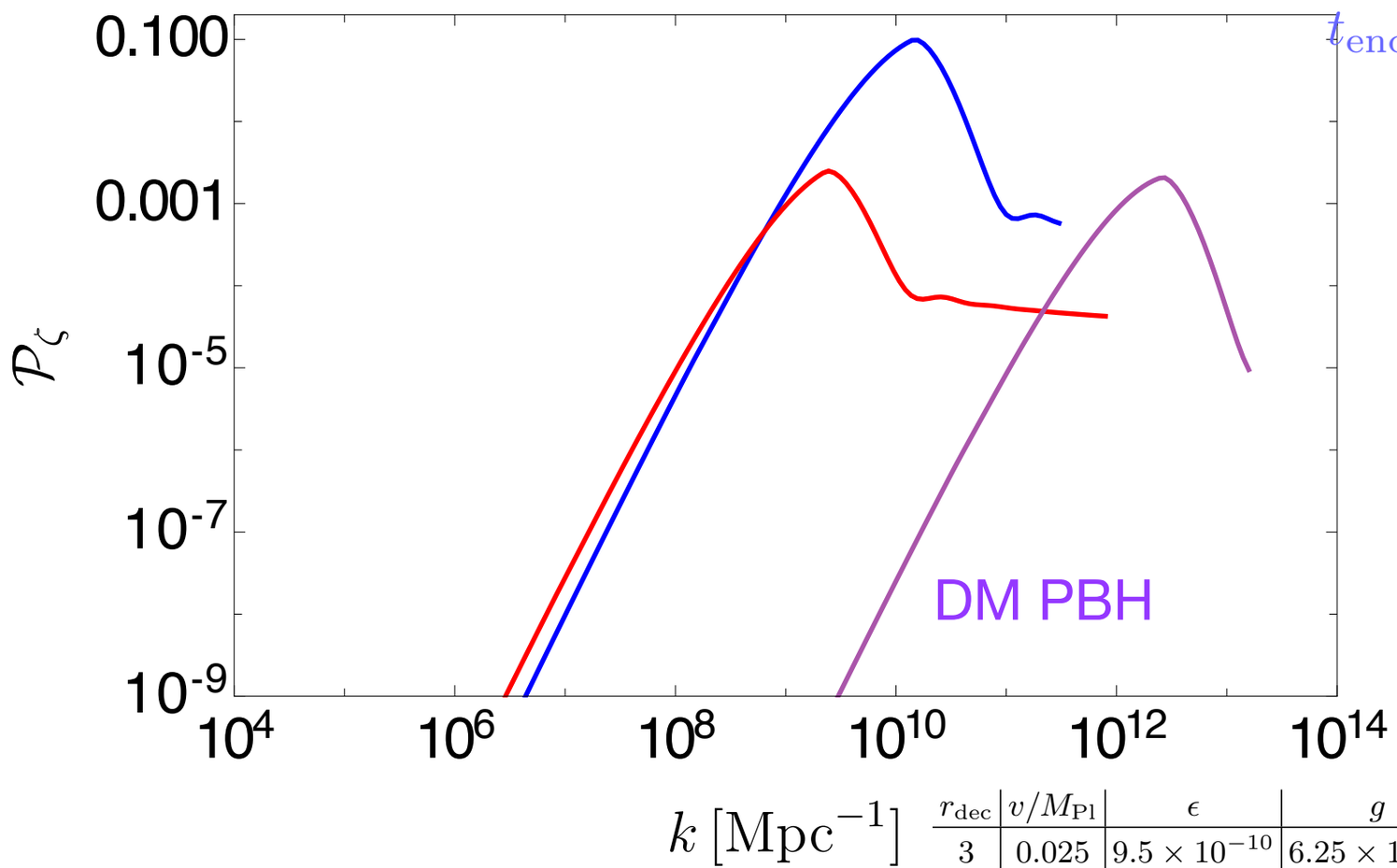
- Axion decays into radiation and produce curvature perturbations

$$\zeta = \frac{r_{\rm dec}}{4+3r_{\rm dec}}\frac{\delta\rho_\sigma}{\rho_\sigma} \qquad r_{\rm dec} = \left.\frac{\rho_\sigma}{\rho_I}\right|_{\rm dec}$$

- Power spectrum

$$\boxed{\mathcal{P}_\zeta(k,t_{\rm dec}) = \left(\frac{2r_{\rm dec}}{4+3r_{\rm dec}}\right)^2 \frac{\mathcal{P}_{\delta\sigma}(k,t_{\rm end})}{v^2\theta_i^2}}$$

$t_{\rm end}$: time at the end of inflation



$r_{\rm dec}$	$v/M_{\rm Pl}$	ϵ	g	θ_i	λ	$T_{\rm R}$ [GeV]	$M/M_{\rm Pl}$
3	0.025	9.5×10^{-10}	6.25×10^{-10}	0.00194	8.4×10^{-5}	10^{14}	1.23×10^{-5}
0.3	0.025	1.7×10^{-9}	6.03×10^{-10}	0.00115	8.4×10^{-5}	10^{14}	1.23×10^{-5}
0.3	0.0235	9.0×10^{-10}	6.10×10^{-10}	0.00532	8.4×10^{-5}	10^{14}	1.23×10^{-5}

2.3 PBH formation

- Curvaton models induce Non-Gaussianity
  - Local-type non-Gassuanity $\zeta(x) = \zeta_g(x) + \frac{3}{5} f_{\rm NL}(\zeta_g^2(x) - \langle \zeta_g^2(x) \rangle)$
  - Curavton model

ζ_g : Gaussian curvature perturbation

$$f_{\rm NL} = \frac{5}{12}\left(-3 + \frac{4}{r_{\rm dec}} + \frac{8}{4 + 3r_{\rm dec}}\right)$$

- PBH formation is determined by density perturbations $\delta(R)$ and their variance $\sigma^2(R)$ coarse-grained over horizon scale $R = (aH)^{-1}$
  - They are calculated from $\mathcal{P}_\zeta(k)$ and $r_{\rm dec}$
- Using Press-Schechter formalism, PBH formation rate is given by

$$\beta(R) = \int_{-\infty}^{\infty} {\rm d}\delta_g \, P_g(\delta_g; R)\Theta(\delta(\delta_g; R) - \delta_{\rm th})$$

δ_g : Gaussian density perturbation

$P_g(\delta_g; R)$: Gauusian distribution

$$P_g(\delta_g; R) = \frac{1}{\sqrt{2\pi}\sigma(R)} \exp\left[-\frac{\delta_g^2}{2\sigma^2(R)}\right]$$

  - Threshold for PBH formation $\delta_{\rm th} = 0.53$

2.3 PBH formation

- PBH mass

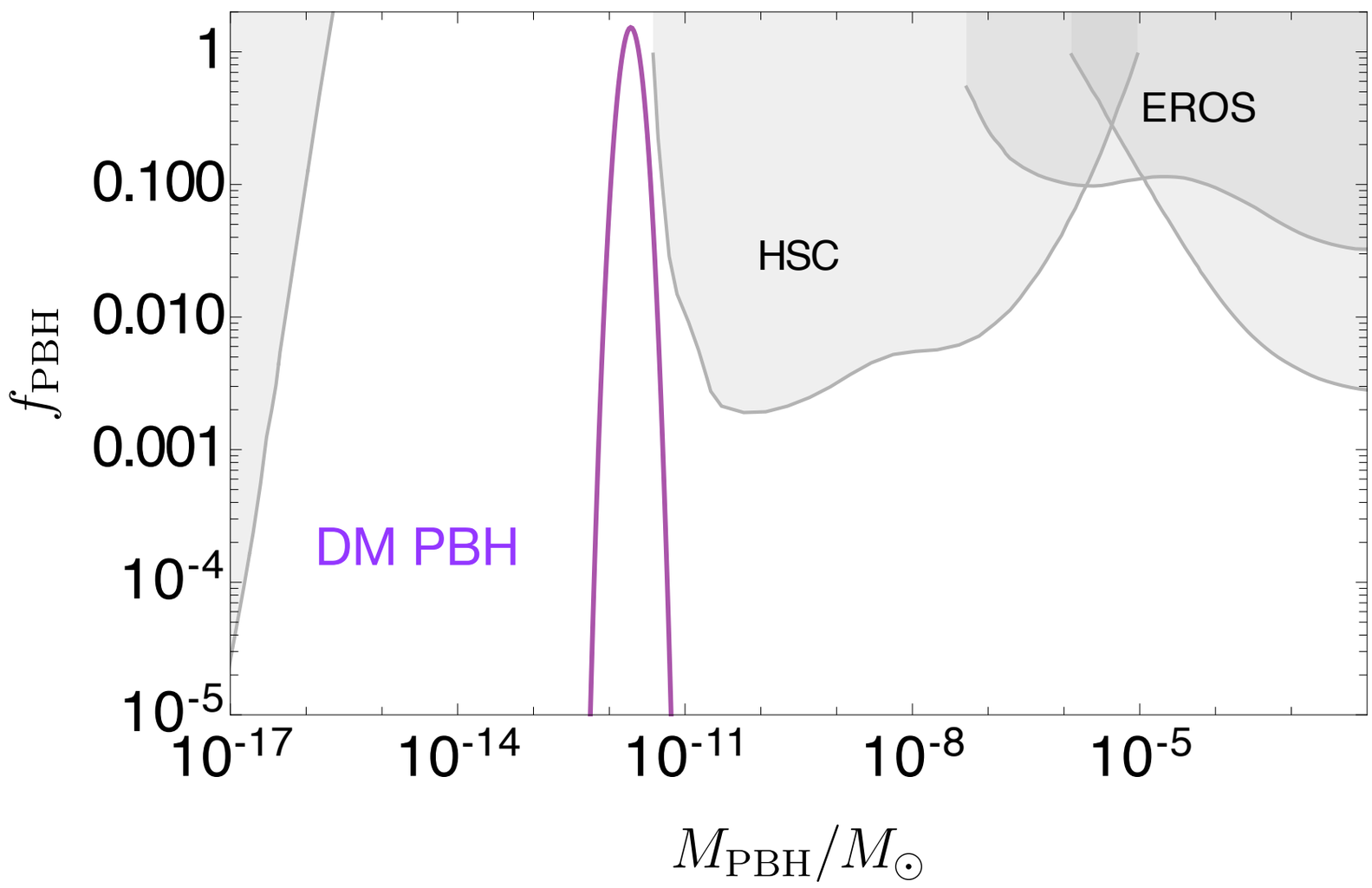
$$M_{\rm PBH} \simeq \gamma M_H$$

$$M_{\rm PBH} \simeq M_\odot \left(\frac{\gamma}{0.2}\right)\left(\frac{k_{\rm PBH}}{2.0\times 10^6\,{\rm Mpc}^{-1}}\right)^{-2}$$

- PBH fraction of DM

$$f_{\rm PBH}(M_{\rm PBH}) \simeq \frac{\beta(M_{\rm PBH})}{1.7\times 10^{-8}}\left(\frac{\gamma}{0.2}\right)^{3/2}\left(\frac{M_{\rm PBH}}{M_\odot}\right)^{-1/2}$$

- Axion-like curvaton model can produce PBHs that explain DM



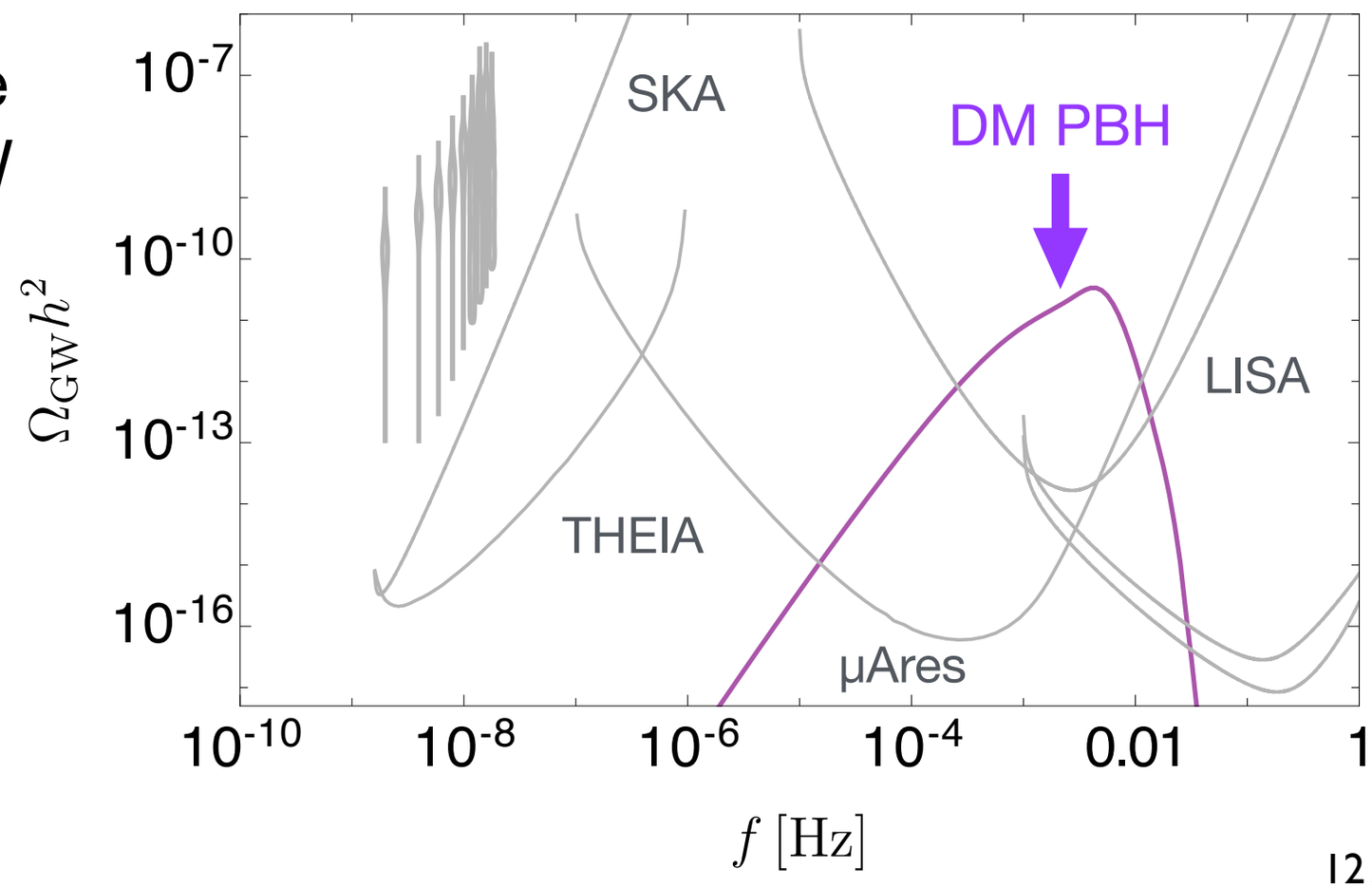
2.4 Scalar-induced gravitational waves

- Scalar and tensor perturbations do not couple in the linear order but they do in the second order

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = \mathcal{O}(\zeta^2)$$

- GWs are produced by the 2nd order effect of scalar perturbations
  = scalar induced GWs

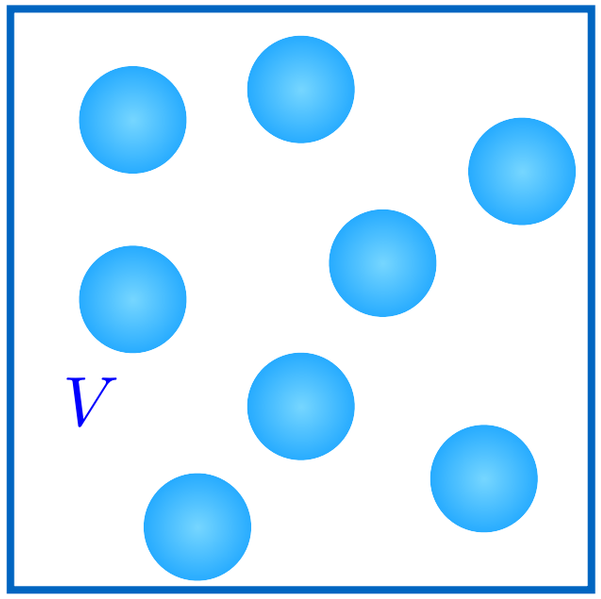
- Axion-like curvaton model for DM PBHs predicts GWs
  - Predicted GWs can be detected by future GW experiments



3. PBHs from non-topological solitons

Cotner, Kusenko (2016), Cotner Kusenko, Takhistov (2018)
Cotner, Kusenko, Sasaki, Takhistov (2019)

- Q-balls and Osillons : non-topological solitons in scalar field theory
  - Q-balls have global U(1) charges Q
  - Formed during oscillation of a scalar field in the early universe
  - Suppose N solitons in a volume V ⇒ N follows Poisson statistics



$$\frac{\delta N}{N} \simeq \frac{1}{\sqrt{N}}$$

$$\frac{\delta \rho}{\rho} \simeq \frac{1}{\sqrt{N}}$$

large for small N

- If solitons dominate the universe and its number in the Horizon volume is small ($\sim 10^4$)

  → Density fluctuations are large enough to produce PBHs

3.1 Q-ball with charge distribution

- Produced Q-balls have a charge distribution (in a volume V)

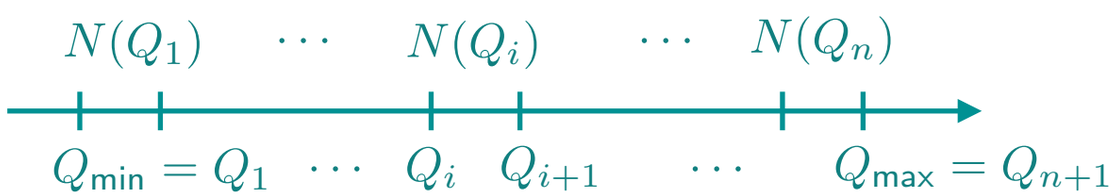
$$N(Q) \text{ between } Q \text{ and } Q + \Delta Q$$

- Q-ball mass

$$M_Q(Q) = mQ^p \ (p = 3/4 \text{ Gauge-mediation type})$$

- Q-ball energy

$$E = \sum_{i=1}^{n} M_Q(Q_i) N(Q_i) \Delta Q_i$$



- Average Q-ball energy

$$\langle E \rangle = \sum_{i=1}^{n} M_Q(Q_i) \langle N(Q_i) \rangle \Delta Q_i = \sum_{i=1}^{n} M_Q(Q_i) \bar{N}(Q_i) \Delta Q_i$$

- Variance of energy fluctuation

Poisson statistics

$$\langle (\delta E)^2 \rangle = \sum_{i=1}^{n} M_Q^2(Q_i) \left\langle (\delta N(Q_i) \Delta Q_i)^2 \right\rangle = \sum_{i=1}^{n} M_Q^2(Q_i) \bar{N}(Q_i) \Delta Q_i$$

$$\delta N(Q) = N(Q) - \bar{N}(Q)$$

3.1 Q-ball with charge distribution

- Average Q-ball distribution

$$\bar{N}(Q) = \begin{cases} \mathcal{N}_Q V \left(\frac{Q}{Q_*}\right)^{\alpha_Q} \frac{1}{Q} & (Q < Q_*) \\ 0 & (Q > Q_*) \end{cases}$$

- Average Q-ball density

$$\bar{\rho}_Q = \frac{\langle E \rangle}{V} = \mathcal{N}_Q \int^{Q_*} dQ\, M_Q(Q) \left(\frac{Q}{Q_*}\right)^{\alpha_Q} \frac{1}{Q} = \mathcal{N}_Q \frac{M_Q(Q_*)}{\alpha_Q + p}$$

- Variance of density fluctuations

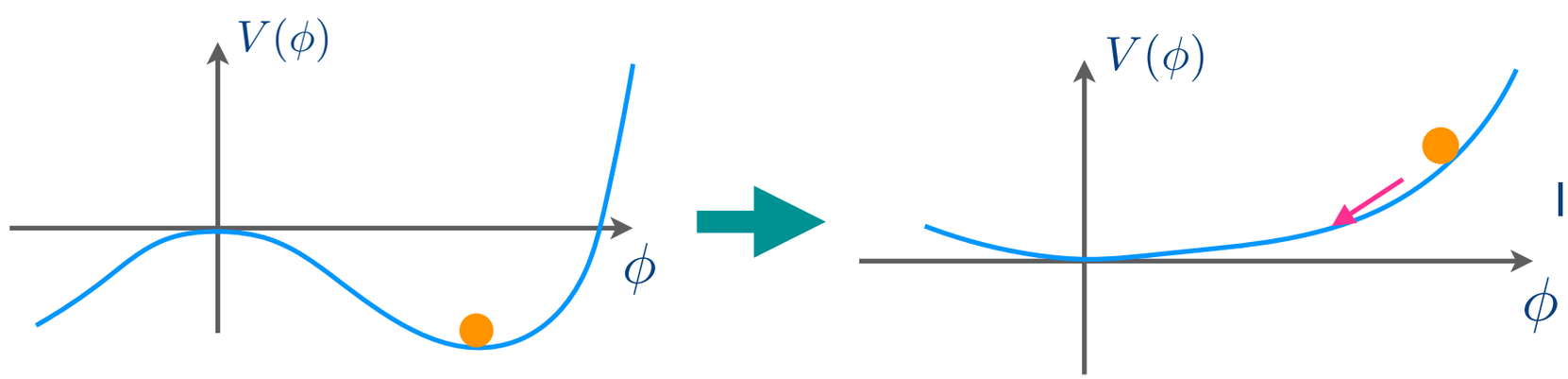
$$\langle(\delta\rho_Q\rangle)^2 \equiv \frac{\langle(\delta E)^2\rangle}{V^2} = \frac{\mathcal{N}_Q}{V} \int^{Q_*} dQ\, M_Q^2(Q) \left(\frac{Q}{Q_*}\right)^{\alpha_Q} \frac{1}{Q} = \frac{\mathcal{N}_Q}{V} \frac{M_Q^2(Q_*)}{\alpha_Q + 2p}$$

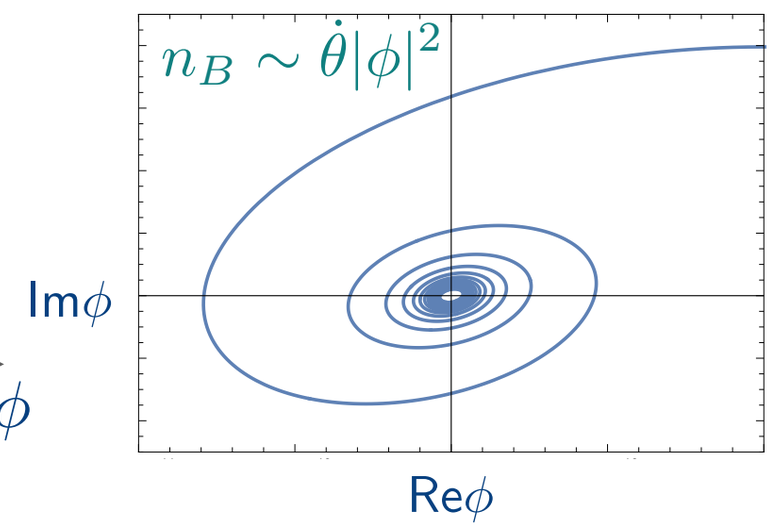
- Variance of Q-ball density perturbations in a volume V

$$\boxed{\sigma_Q = \sqrt{\frac{\langle(\delta\rho_Q)^2\rangle}{\bar{\rho}_Q^2}} = \frac{1}{\sqrt{\mathcal{N}_Q V}} \frac{\alpha_Q + p}{\sqrt{\alpha_Q + 2p}}}$$

3.2 Q-ball formation

- Q-balls are produced in the Affleck-Dine mechanism that uses one of flat directions (= AD field ϕ) in MSSM $\ni$ (squark, slepton, Higgs)
- AD field has a baryon or lepton number
  - AD field has a large field value during inflation
  - After inflation AD field starts oscillation at $H \simeq m_{\text{eff}}$
  - AD field is kicked into phase direction and generates baryon number (= AD baryogenesis)



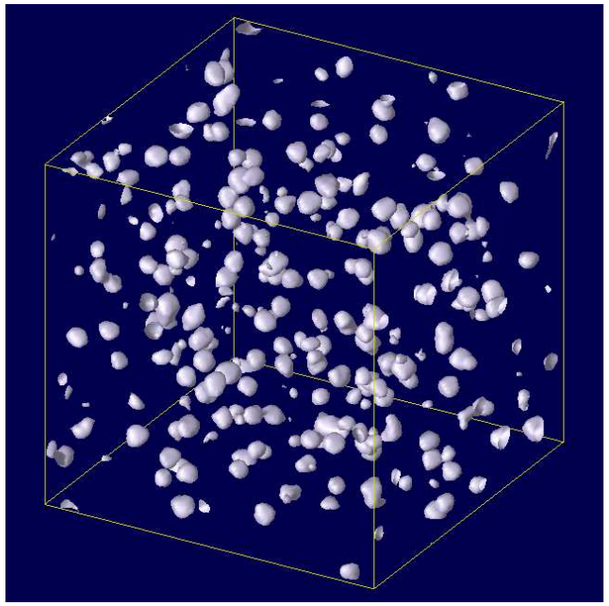


- AD field oscillation has spatial instabilities if the potential is flatter than quadratic one
- AD field fragments into spherical lumps (Q-balls)

3.2 Q-ball formation

- Q-ball properties depend on SUSY breaking scenario
- We consider gauge-mediated SUSY breaking models
  - Potential $V(\Phi) = M_F^4 \left[\log \left(1 + \frac{|\Phi|^2}{M_{\text{mess}}^2} \right) \right]^2$
  - Q-ball mass

$$M_Q = \frac{2\sqrt{2}\pi}{3} \zeta M_F Q^{3/4} \equiv m Q^{3/4} \quad (\zeta \sim 3.6)$$



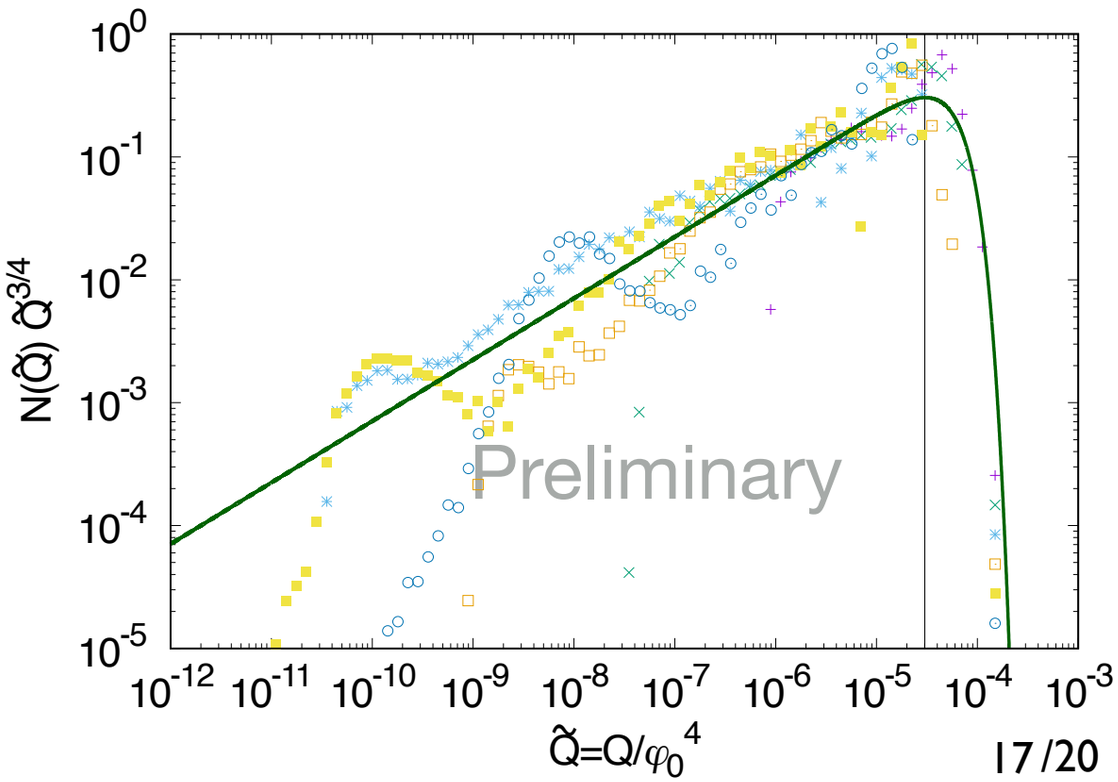
Hiramatsu MK Takahashi (2010)

- Lattice simulation
- Q-ball charge distribution Kasuya MK (2025)

$$\bar{N}(Q) = \begin{cases} \mathcal{N}_Q V \left(\frac{Q}{Q_*} \right)^{\alpha_Q} \frac{1}{Q} & (Q < Q_*) \\ 0 & (Q > Q_*) \end{cases}$$

$$\alpha_Q \simeq -0.25 \quad \mathcal{N}_Q V_H \simeq 9 \times 10^4$$

(V_H : horizon volume at formation)



- Q-ball density perturbation

$$\sigma_{Q,H} = \sigma_Q(V_H) \simeq 1.5 \times 10^{-3}$$

$$\sigma_Q(V) = \sigma_{Q,H} \left(\frac{V}{V_H} \right)^{-1/2}$$

3.3 PBH formation

- We assume Q-balls dominate the universe at formation
- PBH formation rate in Q-ball (=matter) dominated universe

$$\beta_f = 0.056\,(\sigma_Q(M))^5 \left(\frac{M}{M_{f,H}}\right)^{10/3} \qquad (M < M_{f,H})$$

Khlopov Pplnarev (1980)
Harada et al. (2016)
Kasuya Kawasaki Kusenko Neda (2025)

$M_{f,H}$: horizon mass at Q-ball formation$(= t_f)$

$$M_{f,H} = 1.23 \times 10^{-15}\, M_\odot \left(\frac{m_{\phi,\mathrm{eff}}}{\mathrm{MeV}}\right)^{-1}$$
$$= 2.4 \times 10^{18}\,\mathrm{g} \left(\frac{m_{\phi,\mathrm{eff}}}{\mathrm{MeV}}\right)^{-1}$$

- PBH mass $M_{\mathrm{PBH}} \simeq M = \bar{\rho}_Q V$

- Since $\sigma_Q \propto 1/\sqrt{V} \propto M^{-1/2}$

$$\sigma_Q(M) = \sigma_{Q,H} \left(\frac{M}{M_{f,H}}\right)^{-1/2} \Rightarrow \beta_f(M) = 0.056\,\sigma_{Q,H}^5 \left(\frac{M}{M_{f,H}}\right)^{5/6} \quad (M \lesssim M_{f,H})$$

- Perturbations at superhorizon scale grow after horizon crossing

$$\Rightarrow \beta_f(M) = 0.056\,\sigma_{Q,H}^5 \left(\frac{M}{M_{f,H}}\right)^{-5/2} \quad (M_{f,H} \lesssim M)$$

- Q-balls decay and the universe becomes RD with temperature T_{dec}

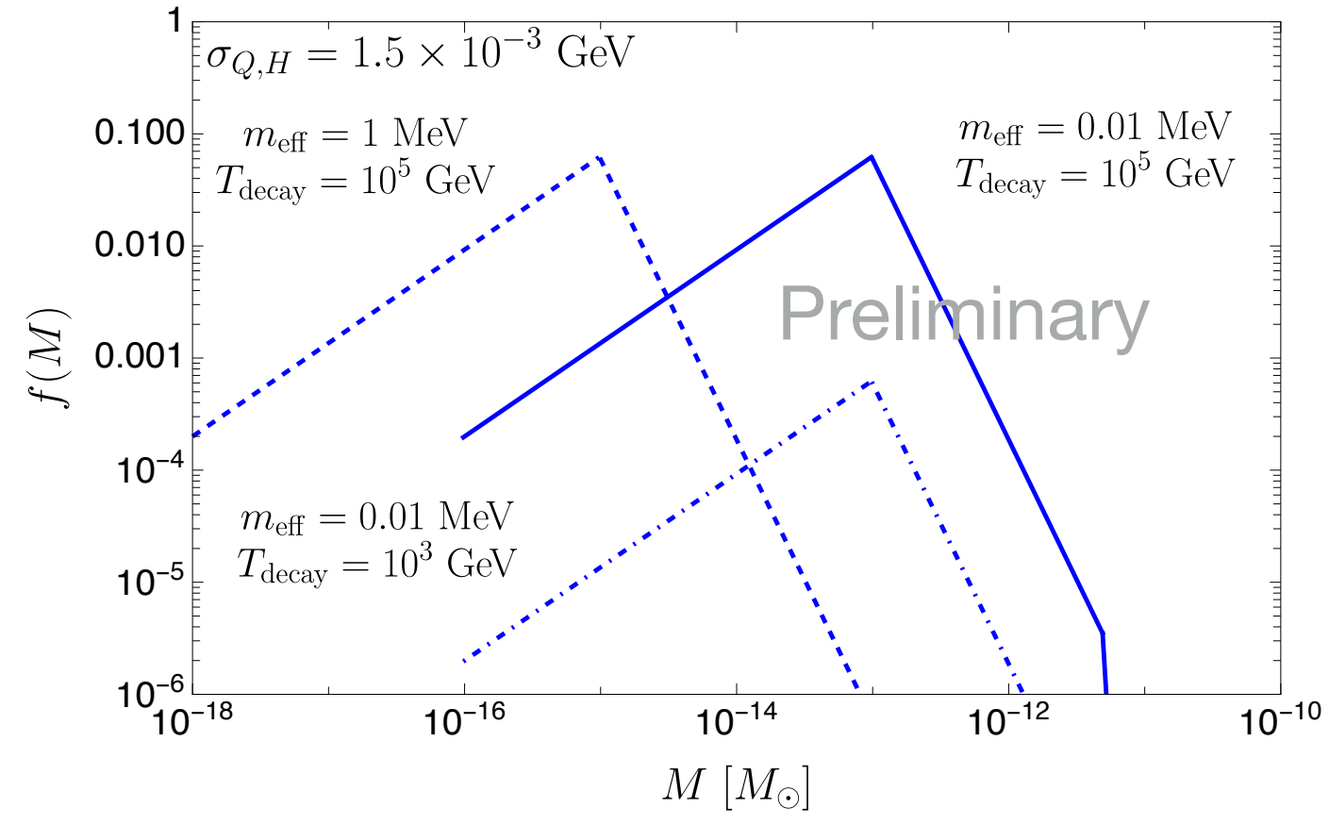
$\Rightarrow$ PBH formation stops

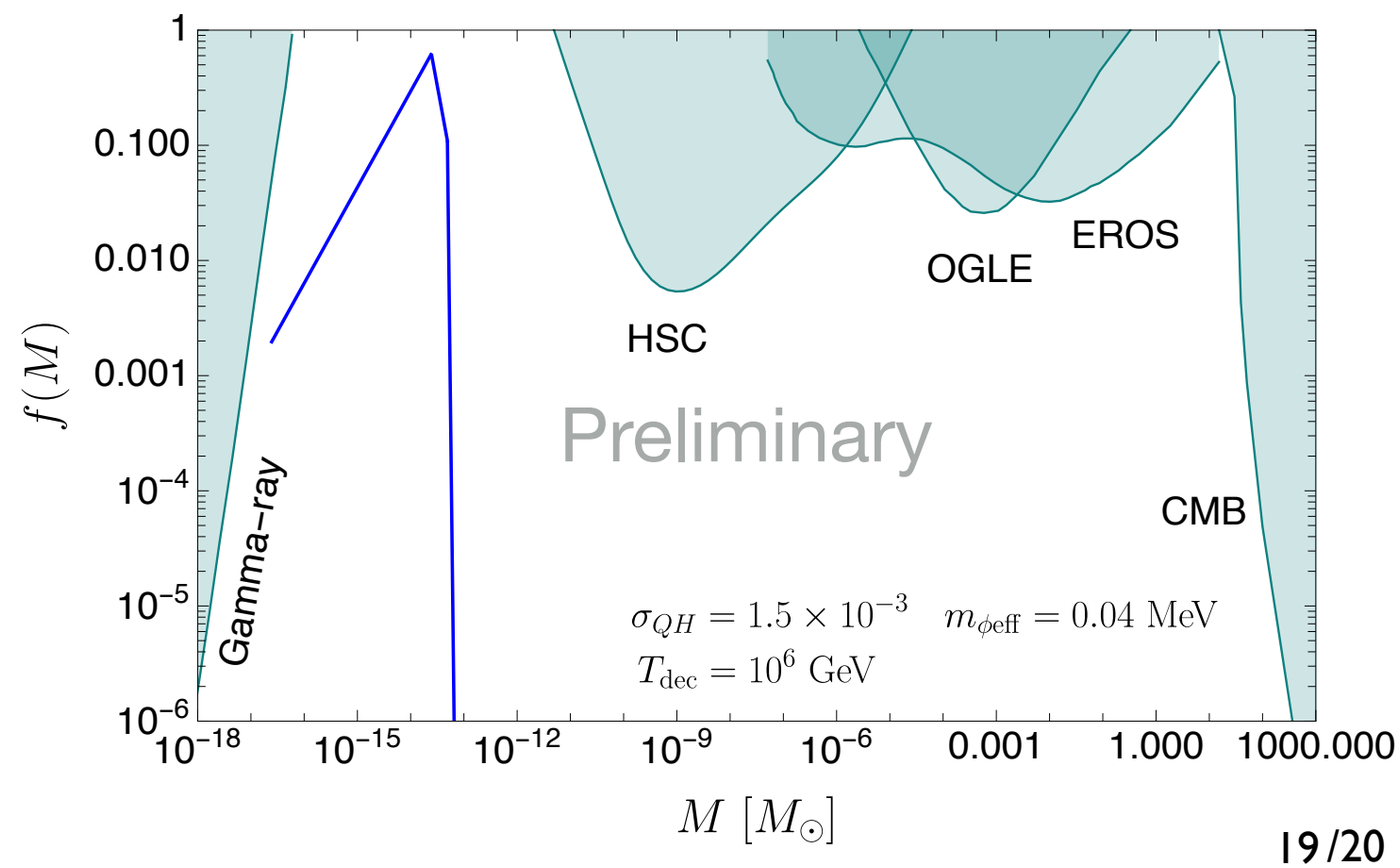
3.3 PBH formation

- The present PBH fraction

$$f(M) = \frac{\Omega_{\rm PBH}(M)}{\Omega_{\rm DM}} = \beta_f(M)\frac{T_{\rm dec}}{T_{\rm eq}}\frac{\Omega_m}{\Omega_{\rm DM}}$$

- PBH fraction has a peak at horizon mass at Q-ball formation determined by $m_{\rm eff}$
- PBH abundance is determined by $\sigma_{Q,H}^5 T_{\rm dec}$
- This scenario can produce PBHs with mass $\sim 10^{-14} M_\odot$ that account for all dark matter





4. Conclusion

- PBHs are a good candidate for dark matter of the universe
- We considered the axion-like curvaton model and Q-ball model for PBH formation
- Axion-like curvaton model can produce PBHs that account for dark matter, and also produce GWs that can be detected by future experiments
- Q-balls induce the density perturbations coming from Poisson fluctuations of Q-ball number density.
- Q-ball density perturbation can be large enough to produce PBHs that account for dark matter