Vacuum Polarization Effects During the Reheating Epoch

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ICNFP 2025

17-31 July 2025, OAC, Kolymbari, Crete, Greece

based on A.A., A.Nikitenko, arXiv:2505.03453 [gr-qc]

23th July 2025

Motivation (I)

Why should we consider quantum effects in Cosmology and GR which are classical theories?

- We believe that gravity will be quantized after all
- Here we are going to consider quantum effects due to ordinary (non-GR) fields
- We are looking for perturbations of classical solutions
- High-precision astrophysical observations provide sensitivity to such effects
- It is also interesting to study effects of backreaction in dynamical systems

Motivation (II)

Why to consider the Starobinsky model?

$$S_{\text{Starobinsky}} = -\frac{M_{Pl}^2}{16\pi}\int d^4x\sqrt{-g}\left(R - \frac{R^2}{6M_R^2}\right) + S_{\text{matter}}$$

- There are many modified gravity theories ...
- The Starobinsky model is simple but also yields reach effects
- It is one of the first models providing cosmological inflation
- It is also one of the most successful ones
- It is motivated (induced) by quantum effects

[A.A. Starobinsky, Phys. Lett. B '1980]

The action (I)

QFT in curved spacetime ⇒ GR action with counter terms

$$S = -\frac{M_{Pl}^2}{16\pi}\int d^4x\sqrt{-g}\left[R - 2\Lambda + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu}\right]$$

Terms proportional to $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ and $\Box R$ also appear but in 4D they can be transformed into surface terms and omitted

Variation of the counter terms yields new tensors in the Einstein eqs.

$$\begin{aligned}
{}^{(1)}H_{\mu\nu} &\equiv \frac{1}{\sqrt{-g}}\frac{\delta\left[\sqrt{-g}R^2\right]}{\delta g^{\mu\nu}} = 2\nabla_\nu\nabla_\mu(-R) - 2g_{\mu\nu}\nabla_\rho\nabla^\rho(-R) - \frac{1}{2}g_{\mu\nu}R^2 + 2RR_{\mu\nu} \\
{}^{(2)}H_{\mu\nu} &\equiv \frac{1}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}\left[\sqrt{-g}R_{\alpha\beta}R^{\alpha\beta}\right] = 2\nabla_\alpha\nabla_\nu(-R^\alpha_\mu) - \nabla_\rho\nabla^\rho(-R_{\mu\nu}) \\
&\quad -\frac{1}{2}g_{\mu\nu}\nabla_\rho\nabla^\rho(-R) - \frac{1}{2}g_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} + 2R^\rho_\mu R_{\rho\nu}
\end{aligned}$$

Constants α and β are divergent and should be renormalized ⇒ α_0, β_0

The action (II)

In general, tensors ${}^{(1)}H_{\mu\nu}$ and ${}^{(2)}H_{\mu\nu}$ are not proportional to each other. But in a cosmological conformally flat metric

$$ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j$$

we have

$$\frac{\delta}{\delta g^{\mu\nu}} \int d^4x\sqrt{-g}\left[R_{\alpha\beta}R^{\alpha\beta} - \frac{1}{3}R^2\right] = 0$$

and can absorb the term with ${}^{(2)}H_{\mu\nu}$ into redefinition of α. So, the Starobinsky model emerges naturally.

N.B. The value of the scalaron mass $M_R \sim 1/\sqrt{\alpha_0}$ is not predicted and can be taken from observations (CMB) $\Rightarrow M_R \sim 3 \cdot 10^{13}$ GeV

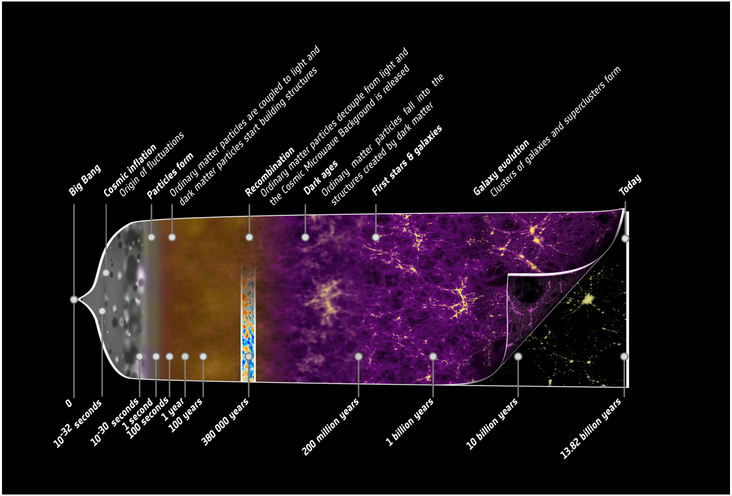
Cosmology in the Starobinsky model

The universe history in the model:

- some initial conditions (fine tuning is not required, no singularity!)
- epoch of cosmological exponential inflation (enough many e-folds)
- scalaron dominance epoch: it is produced by curvature oscillations and decays into dark matter and/or SM particles $\Rightarrow$ (hot) Big Bang
- Baryogenesis (needs an extra mechanism), nucleosynthesis, CMB and so on till the present day

N.B. Scalaron $\approx$ inflaton, there is a duality between $f(R)$ and scalar-tensor gravity models

History of the Universe



* Picture from the European Space Agency

Einstein equations

Einstein equations in the Starobinsky model read

$$R_{\mu\nu} + \frac{R}{2} g_{\mu\nu} - \frac{1}{6M^2} {}^{(1)}H_{\mu\nu} = \frac{8\pi}{M_{Pl}^2} \left[\mathring{T}_{\mu\nu} + \langle T_{\mu\nu} \rangle \right]$$

where $\mathring{T}_{\mu\nu}$ is the stress-energy tensor of particles produced from the scalaron decay and provide backreaction.

So-called vacuum polarization stress-energy tensor

$$\langle T_{\mu\nu} \rangle = k_1 {}^{(1)}H_{\mu\nu} + k_3 {}^{(3)}H_{\mu\nu}$$

k_1 and k_3 are constants. Constant k_3 depends on the number of scalar N_S, fermionic N_F, and gauge N_G fields

$$k_3 = \frac{1}{2880\pi^2} \left(N_S + \frac{11}{2} N_F + 62 N_G \right)$$

[Starobinsky '1980, Grib '1980, Vilenkin '1985]

${}^{(3)}H$ tensor

Our goal is to study the effect of the ${}^{(3)}H_{\mu\nu}$ tensor

$$ {}^{(3)}H_{\mu\nu} = R^{\sigma}_{\mu} R_{\nu\sigma} - \frac{2}{3} R R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\sigma\tau} R_{\sigma\tau} + \frac{1}{4} g_{\mu\nu} R^2 $$

It arises in the regularization and renormalization procedure, but does not correspond to any counterterm, since it cannot be obtained by varying a polynomial metric action.

In general, the tensor is not divergence-free: $\nabla_{\nu} {}^{(3)}H_{\mu}{}^{\nu} \neq 0$. The identity $\nabla_{\nu} {}^{(3)}H_{\mu}{}^{\nu} = 0$ holds only for the conformally flat metric considered here.

Therefore, the presence of ${}^{(3)}H_{\mu\nu}$ in the equations cannot be reduced to any scalar-tensor modified gravity theory.

Without ${}^{(3)}H_{\mu\nu}$

The scalaron dominance epoch without taking into account contributions due to vacuum polarization was considered earlier in [E.Arbuzova, A.Dolgov, L.Reverberi '2012; E.Arbuzova, A.Dolgov, R.Singh '2021; E.Arbuzova, A.Dolgov, A.Rudenko '2023] and other papers. In particular, it is shown that heavy DM particles can be produced by scalaron decays

For cosmological application we can consider the trace of the equation

$$R_{\mu\nu} + \frac{R}{2} g_{\mu\nu} - \frac{1}{6M^2} {}^{(1)}H_{\mu\nu} = \frac{8\pi}{M_{Pl}^2} \mathring{T}_{\mu\nu}$$

Let's consider scalaron decay into a massless scalar field minimally coupled to gravity

$$S_\phi = \frac{1}{2} \int d^4x \sqrt{-g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

For the FLRW metric

$$\ddot{\phi} + 3H(t)\dot{\phi} - \frac{1}{a^2(t)} \Delta\phi = 0, \qquad \mathring{T}^\mu_\mu = -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

Equations (I)

The dymamics in conformal time $\eta = t/a(t)$ is described by

$$R'' + 2\frac{a'}{a}R + M_R^2 a^2 R = \frac{8\pi M_R^2}{M_{Pl}^2}\frac{1}{a^2}\left[\chi'^2 - (\vec{\nabla}\chi)^2 + \frac{a'^2}{a^2}\chi^2 - \frac{a'}{a}(\chi\chi' + \chi'\chi)\right]$$

where $\chi(\eta) \equiv a(t)\phi$, $R' \equiv dR/d\eta$, and we have $R = -6a''/a^3$

The matter on the right hand side leads to

$$\langle\chi^2\rangle \sim -\frac{1}{48\pi^2}\int_{\eta_0}^{\eta} d\eta_1 \frac{a^2(\eta_1)R(\eta_1)}{\eta - \eta_1}$$

$$\left\langle \langle\partial_\eta\chi\rangle^2 - \langle\vec{\nabla}\chi\rangle^2 \right\rangle \sim -\frac{1}{96\pi^2}\int_{\eta_0}^{\eta} d\eta_1 \frac{\partial_{\eta_1}^2\left(a^2(\eta_1)R(\eta_1)\right)}{\eta - \eta_1}$$

$$\langle\chi\partial_{\eta_1}\chi + (\partial_{\eta_1}\chi)\chi\rangle \sim -\frac{1}{48\pi^2}\int_{\eta_0}^{\eta} d\eta_1 \frac{\partial_{\eta_1}\left(a^2(\eta_1)R(\eta_1)\right)}{\eta - \eta_1}$$

The contribution of $\langle\chi^2\rangle$ is dominant

Equations (II)

So, we get integro-differential equation

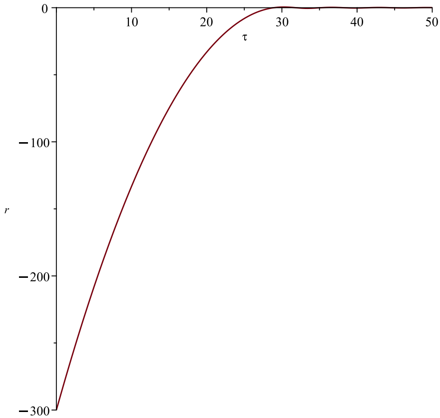
$$\ddot{R} + 3H\dot{R} + M_R^2 R = -\frac{M_R^2}{12\pi M_{Pl}^2} \int_{t_0}^{t} dt_1 \frac{\ddot{R}(t_1)}{t - t_1}$$

In dimensionless variables $\tau = tM_R$, $r = R/M_R^2$, $h = H/M_R$, $y = \rho/M_R^4$ we get the system of equations

$$\begin{cases} r'' + 3hr' + r = -8\pi\mu^2(1 - 3w)y \\ r' = -r/6 - 2h^2 \\ y' + 3(1 + w)hy = 0 \end{cases}$$

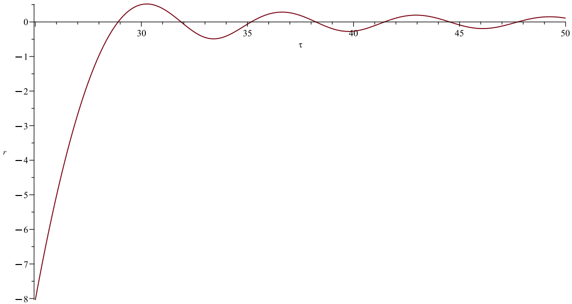
for the matter equation of state $P = w\rho$

Numerical solutions (I)



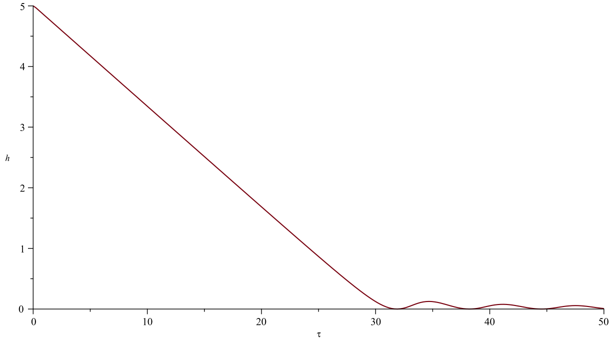
Evolution of the dimensionless scalar curvature r as a function of dimensionless time τ for initial values $r(0) = -300$ and $h(0) = 5$

Numerical solutions (II)



Evolution of the dimensionless scalar curvature $r(\tau)$ as a function of dimensionless time τ for initial values $r(0) = -300$ and $h(0) = 5$. The plot shows the range $\tau = 25..30$, $r = -8...$, where the damped oscillations of the scalar curvature around zero are seen

Numerical solutions (III)



Evolution of the Hubble parameter $h(\tau)$ as a function of dimensionless time τ for initial values $r(0) = -300$ and $h(0) = 5$. The plot shows oscillations of $h(\tau)$ near zero for $\tau > 30$

Damping of oscillations

In the equation

$$\ddot{R} + (3H + \Gamma)\dot{R} + M_R^2 R = 0$$

term $(3H(t) + \Gamma)\dot{R}$ describes the damping of oscillations. Γ here is the scalaron decay width. Earlier the contribution of $3H\dot{R}$ was omitted in calculations of Γ without sufficient justification. We have shown that indeed that can be done, since for the first approximation the two damping effects are factorised in

$$R(t) \sim \cos(M_R t + \theta) e^{-(\Gamma_0 + \Gamma)t/2}$$

where $\Gamma_0 \approx 2M_R/\tau \sim 10^{12}$ GeV for the characteristic values $\tau \sim 40$ provides damping not related to scalaron decays

Approximate solution

At sufficiently large t one can see the following asymptotical solutions

$$H(t) = \frac{2}{3t}\left[1 + \sin(M_R t + \theta)\right], \qquad R(t) = -\frac{4M_R}{t}\cos(M_R t + \theta)$$

Substituting these asymptotic solutions into the integro-differential equation for $\Gamma \ll M_R$ we get

$$\left[\left(\omega^2 - M_R^2\right)\cos(\omega t + \theta) + \Gamma\omega\sin(\omega t + \theta)\right]e^{-\Gamma t/2}$$
$$= \frac{\omega^2 M_R^2}{12\pi M_{Pl}^2}e^{-\Gamma t/2}\int_0^{t-t_0}\frac{d\xi}{\xi}\left[\cos(\omega t + \theta)\cos\omega\xi + \sin(\omega t + \theta)\sin\omega\xi\right]$$

Looking at particular oscillation modes we get renormalization of M_R and the value of the scalaron width [E.Arbuzova, A.Dolgov et al.]

$$\Gamma = \frac{M_R^3}{24\pi M_{Pl}^2}$$

Let's include vacuum polarization

$$R_{\mu\nu} + \frac{R}{2} g_{\mu\nu} - \frac{1}{6M_R^2} {}^{(1)}H_{\mu\nu} = \frac{8\pi}{M_{Pl}^2} \left[\mathring{T}_{\mu\nu} + k_1 {}^{(1)}H_{\mu\nu} + k_3 {}^{(3)}H_{\mu\nu} \right]$$

For the trace we get

$$R + \frac{1}{6M_R^2} {}^{(1)}H^\mu_\mu = -\frac{8\pi}{M_{Pl}^2} \mathring{T}^\mu_\mu - \frac{8\pi}{M_{Pl}^2} k_3 {}^{(3)}H^\mu_\mu, \qquad \frac{8\pi}{M_{Pl}^2} \mathring{T}^\mu_\mu = \frac{M_R^2}{12\pi M_{Pl}^2} \int_{t_0}^{t} dt_1 \frac{\ddot{R}(t_1)}{t - t_1}$$

$$R + \frac{1}{6M_R^2} {}^{(1)}H^\mu_\mu = \frac{1}{M_R^2} \left(\ddot{R} + 3H\dot{R} \right) + R, \qquad {}^{(3)}H^\mu_\mu = 12 \left(H^2 \dot{H} + H^4 \right)$$

What is the ${}^{(3)}H$ impact?

Correction to the scalaron decay width

By using the same approximate solution (ansatz) we get

$$\Gamma = \frac{M_R^3}{24 M_{Pl}^2} + \frac{256\pi M_R^3}{27 M_{Pl}^2 \tau^3} k_3 e \approx 7.56 + 0.01 \text{ GeV}$$

for

$$k_3 = \frac{1}{2880\pi^2}\left(N_S + \frac{11}{2}N_F + 62 N_G\right) \sim 0.036 \qquad \text{and} \qquad \tau \sim 40$$

The correction is small but not extremely

More accurate numerical solution of the equations is possible, but after adopting renormalization of divergent contributions

Approximate analytic procedure allows to understand the nature of the explored effects

Conclusions

- We confirmed earlier results on scalaron decays
- Additional contribution to the scalaron decay width due to vacuum polarization is computed
- It is suppressed not by M_{Pl} but by a characteristic dimensionless time
- It is not the full story, quantization of gravity is required
- But the considered effects will (might?) be still around