Fluctuation-Dissipation Breakdown in a Phase Oscillator Model on the Fruit Fly Connectome

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ICNFP 2025, Kolymbari, Crete
July 28, 2025

Outline

Neural Systems and Non-Equilibrium Physics

- **Neural systems** are inherently out of equilibrium
- Exhibit complex fluctuations across spatial and temporal scales
- **Fluctuation-Dissipation Theorem (FDT)** characterizes relationship between:
  - Spontaneous fluctuations
  - System response to external perturbations
- Deviations from FDT highlight departure from equilibrium

Key Question

How does **network anisotropy** affect fluctuation-dissipation violation in neural oscillator models?

Fluctuation-Dissipation Framework

Auto-response function:

$$\mathcal{R}(t,s) = \frac{\delta E_h(t,s)}{\delta h}\Big|_{h\to 0} \tag{1}$$

Auto-correlation function:

$$A(t,s) = \langle E(t)E(s)\rangle \tag{2}$$

In equilibrium:

$$\mathcal{R}(t,s) = \frac{1}{T}\frac{\partial A(t,s)}{\partial s} \tag{3}$$

In non-equilibrium (FDT violation):

$$\mathcal{R}(t,s) = \frac{1}{T}X(t,s)\frac{\partial A(t,s)}{\partial s} \tag{4}$$

where $X(t,s) \neq 1$ signals departure from equilibrium

Full Fruit Fly (FF) Connectome

Network Statistics:

- $N = 124,891$ nodes
- $L = 3,794,615$ edges
- Average degree: $\langle k \rangle = 64.3$
- Directed network (dendrites/axons)

Dimensions:

- Topological dimension: $d > d_c = 4$
- Spectral dimension: $d_s \simeq 2.1$, reflects the return probability of a random walker:

$$P(t) \sim t^{-d_s/2} \quad (5)$$

- Since $d_s < d_c = 4$: Non-mean-field synch. behavior

Anisotropy Measure

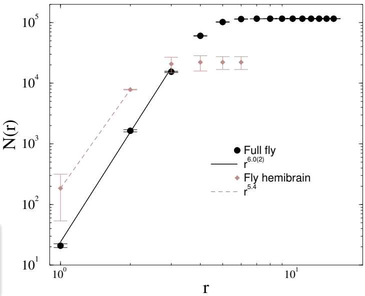
$\alpha = \frac{1}{N} \sum_{i,j}^{N} (W_{ij} - W_{ji})$

Three scenarios:

(o) Original: $\alpha = 0.163$

(i) 20% inhibitory: $\alpha = 0.1$

(w) Renormalized: $\alpha = 2 \times 10^{-5}$



Shinomoto-Kuramoto equation:

$$\dot{\theta}_j(t) = \omega_j^0 + K \sum_{k=1}^{N} W_{jk} \sin[\theta_k(t) - \theta_j(t)] + F \sin(\theta_j(t)) \qquad (6)$$

where:

- ω_j^0: self-frequency (Gaussian distributed)
- K: global coupling strength (control parameter)
- W_{jk}: weighted adjacency matrix
- F: external periodic force

Kuramoto order parameter:

$$R(t) = \left\langle \left| \frac{1}{N} \sum_j e^{i\theta_j(t)} \right| \right\rangle \qquad (7)$$

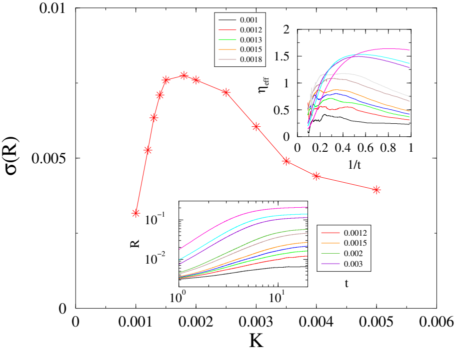
Critical Points and Synchronization

Critical coupling strengths:

- Scenario (o): $K_c = 0.0015$
- Scenario (i): $K_c = 0.01$
- Scenario (w): $K_c = 1.9$

Growth exponent:

$$R(t,N) = N^{-1/2} t^{\eta} f(t/N^Z) \qquad (8)$$

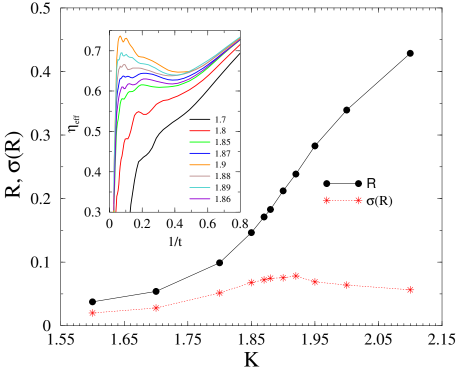


Key Finding:

Higher anisotropy requires **weaker coupling** for synchronization

Non-mean-field scaling:

$\eta \sim 0.65$ vs. $\eta_{MF} = 0.75$



Critical Scaling Behavior

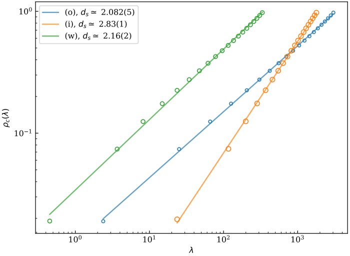
At criticality, observables decay with aging exponents:

$$\mathcal{R}(t/s) \sim s^{-1-a}(t/s)^{-\lambda_{\mathcal{R}}/Z} \quad (9)$$

$$A(t/s) \sim s^{-b}(t/s)^{-\lambda_A/Z} \quad (10)$$

Key observations:

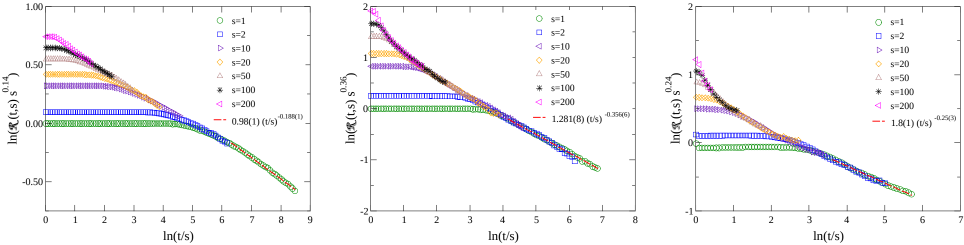
- Scaling collapse achieved for auto-response functions
- Power-law tails for $\ln(t/s) > 4$
- Different decay exponents for different scenarios
- Consistent with $d_s < 4$ expectation



Universal Relationship

Scaling collapses imply: $1 + a = \lambda_{\mathcal{R}}/Z$

Auto-response Scaling Collapse

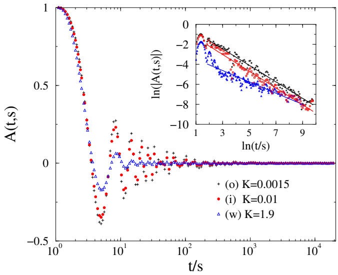


Collapse of auto-response functions for **scenarios** (o), (i), and (w) at their estimated critical points respectively $K_{c_{(o)}} = 0.0015$, $K_{c_{(i)}} = 0.01$, $K_{c_{(w)}} = 1.9$. The dashed line shows PL fitting for the tail: $\ln(t/s) > 4$.

Auto-correlation Results

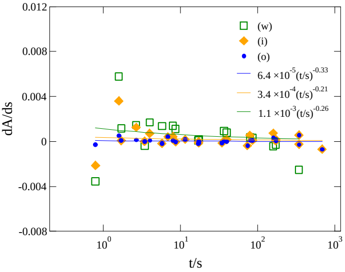
Power-law tail fits for $A(t, s = 1)$:

- Scenario (o): $A_T(t) = 1.02 \times t^{-0.82}$
- Scenario (i): $A_T(t) = 0.6 \times t^{-0.83}$
- Scenario (w): $A_T(t) = 0.05 \times t^{-0.66}$



Key Features:

- Damped oscillatory behavior after ensemble averaging
- Asymptotic power-law decay consistent with criticality
- Scenario-dependent exponents reflect different network properties



FDT Violation Results

Fluctuation-dissipation ratios: $X(y) = \mathcal{R}_T(y)/\dot{A}_T(y)$

$$X(y)_o = 1.5 \times 10^4 \times y^{0.14} \quad (11)$$

$$X(y)_i = 3.7 \times 10^3 \times y^{-0.14} \quad (12)$$

$$X(y)_w = 1.6 \times 10^3 \times y^{0.01} \quad (13)$$

$$X(y)_{ER} = 0.95 \times 10^4 \times y^{-0.02} \quad (14)$$

Key Finding

FDR follows anisotropy level: Higher anisotropy ⇒ Larger FDR ⇒ Further from equilibrium

Comparison: Erdős-Rényi (symmetric) graph shows intermediate behavior

Anisotropy-FDT Relationship

Clear correlation observed:

- Scenario (o): Highest α, largest FDR
- Scenario (i): Medium α, medium FDR
- Scenario (w): Lowest α, smallest FDR

Physical interpretation:

- More anisotropic $\Rightarrow$ Further from equilibrium
- Symmetric interactions $\Rightarrow$ Closer to equilibrium
- Network asymmetry drives non-equilibrium behavior

Anisotropy Values:

- (o): $\alpha = 0.163$
- (i): $\alpha = 0.1$
- (w): $\alpha = 2 \times 10^{-5}$

FDR Amplitudes:

- (o): 1.5×10^4
- (i): 3.7×10^3
- (w): 1.6×10^3

Main Conclusions

1. **FDT violation correlates with network anisotropy**
   - Higher asymmetry $\Rightarrow$ larger deviation from equilibrium
   - Quantitative relationship established
2. **Non-mean-field critical behavior**
   - Spectral dimension $d_s < 4$ drives anomalous scaling
   - Universal aging relationships confirmed
   - Anisotropy as key control parameter

Broader Impact

Framework for characterizing brain states in health and disease through equilibrium distance measures

Future Directions

Immediate extensions:

- Module-dependent analysis of FDT violation
- Effects of external forcing on aging behavior
- Comparison with other neural oscillator models

Speculative Applications:

- Precision neuroscience: Pathology-specific whole-brain models
- Therapeutic interventions: Optimizing brain stimulation protocols
- Disease biomarkers: FDT-based classification of brain states

Thank You!

Questions?

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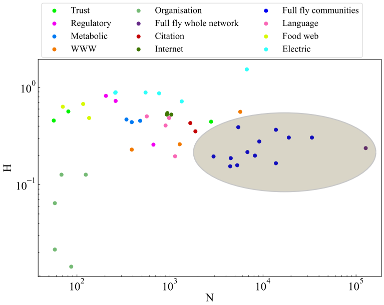
arXiv: 2503.20708

Appendix: FF Structure & Response Scaling for ER-graph

Random Walk Hierarchy:



ER Graph Auto-response Scaling: