

On the semileptonic decays of the $B\bar{B}$ pairs and the
branching ratio of the $\Upsilon(5S) \rightarrow B_s^{(*)}\bar{B}_s^{(*)}$ decay

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arXiv:2412.06535

International Conference on New Frontiers in Physics
July 17-31 2025

$$B_s \rightarrow \mu^+ \mu^-$$

This is a rare decay - so good place to look for New Physics.

$$\text{Theory: } \mathcal{B}(B_s \rightarrow \mu^+ \mu^-) = (3.63_{-0.10}^{+0.15}) \times 10^{-9}.$$

This branching ratio was measured at LHCb:

$$\mathcal{B}(B_s \rightarrow \mu^+ \mu^-) = (3.34 \pm 0.27) \times 10^{-9}$$

Uncertainty in the number of B_s -mesons produced at LHC do not allow to measure $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$ with the desired small uncertainty.

$B_s \rightarrow D_s^- \pi^+$ as a normalization channel

We can find the value of $\mathcal{B}(B_s \rightarrow D_s^- \pi^+)$ at $\Upsilon(5S)$ (at Belle II) by the following relation:

$$\frac{N(B_s \rightarrow D_s^- \pi^+) N(B^0)}{N(B^0 \rightarrow D^- \pi^+) N(B_s)} \mathcal{B}(B^0 \rightarrow D^- \pi^+) = \mathcal{B}(B_s \rightarrow D_s^- \pi^+).$$

Consideration of the time dependence of semileptonic decays of both B -mesons produced at $\Upsilon(5S)$ resonance at Belle II allows to determine the fraction of the $\Upsilon(5S) \rightarrow B_s^{(*)} \bar{B}_s^{(*)}$ decays with the percentage accuracy.

Thus we suggest the way to determine the probability of the rare decay $B_s \rightarrow \mu^+ \mu^-$ with the percentage accuracy at the LHC using $B_s \rightarrow D_s^- \pi^+$ as normalisation channel.

$\Upsilon(4S)$ and $\Upsilon(5S)$ decays

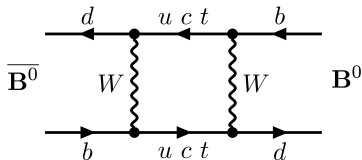
Pairs of B -mesons are produced in Υ decays; in asymmetric e^+e^- colliders time dependencies of semileptonic B decays are studied.

In $\Upsilon(4S)$ decays $B\bar{B}$ pairs are produced.
They are produced in C -odd state.

In $\Upsilon(5S)$ decays $B\bar{B}$ as well as $B_s\bar{B}_s$ pairs are produced.
They can be produced in C -even state as well:

$$\Upsilon(5S) \rightarrow B_{(s)}^* \bar{B}_{(s)}, \quad B_{(s)}^* \rightarrow B_{(s)} \gamma.$$

$B^0 - \bar{B}^0$ mixing and oscillations



$$B_L = pB^0 + q\bar{B}^0, B_H = pB^0 - q\bar{B}^0,$$

$$|B^0(t)\rangle = e^{-iMt} e^{-(\Gamma/2)t} \left[\cos\left(\frac{\Delta m}{2}t\right) |B^0\rangle + i\frac{q}{p} \sin\left(\frac{\Delta m}{2}t\right) |\bar{B}^0\rangle \right],$$

$$|\bar{B}^0(t)\rangle = e^{-iMt} e^{-(\Gamma/2)t} \left[\cos\left(\frac{\Delta m}{2}t\right) |\bar{B}^0\rangle + i\frac{p}{q} \sin\left(\frac{\Delta m}{2}t\right) |B^0\rangle \right],$$

where $M = (m_L + m_H)/2$ and $\Delta m = m_H - m_L$. Since the modulus of the ratio $|p/q| = 1 + \mathcal{O}(10^{-3})$, then in what follows we will assume $|p/q| = 1$.

$B\bar{B}$ from $\Upsilon(4S)$ decays

They are produced in C-odd state, so at the moment of semileptonic decay of the first one we know flavor of the second one:

$B \rightarrow l^+ \nu X$ - the second one is \bar{B} (EPR paradox).

So the one-particle wave function is all what we need:

$$N_{++} = N_{--} = \frac{1}{2\Gamma} \int_0^{+\infty} d(\Delta t) e^{-\Gamma\Delta t} \sin^2 \frac{\Delta m \Delta t}{2} = \frac{1}{4\Gamma^2} \frac{x^2}{1+x^2},$$

$$N_{+-} = N_{-+} = \frac{1}{2\Gamma} \int_0^{+\infty} d(\Delta t) e^{-\Gamma\Delta t} \cos^2 \frac{\Delta m \Delta t}{2} = \frac{1}{4\Gamma^2} \frac{2+x^2}{1+x^2}.$$

At $\Delta t = 0$ $N_{++} = N_{--} = 0$.

The ratio of the number of decays into leptons of the same signs to the number of decays into leptons of the opposite signs in the case of a C -odd wave function is equal to

$$R_{\text{odd}} = \frac{N_{++} + N_{--}}{N_{+-} + N_{-+}} = \frac{x^2}{2+x^2},$$

where the dimensionless parameter $x = \Delta m/\Gamma$ was introduced. In case $x = 0$ (no oscillations) $N_{++} = N_{--} = 0$; in case $x \gg 1$ (fast oscillations) $N_{++} = N_{+-}$.

two-paricle wave functions

For semileptonic decays of C-even states this trick does not work and we need two-paricle wave function.

C-odd wave function:

$$|\psi(t_1, t_2)\rangle_{\text{odd}} = |B^0(t_1)\rangle \otimes |\bar{B}^0(t_2)\rangle - |\bar{B}^0(t_1)\rangle \otimes |B^0(t_2)\rangle.$$

C-even wave function:

$$|\psi(t_1, t_2)\rangle_{\text{even}} = |B^0(t_1)\rangle \otimes |\bar{B}^0(t_2)\rangle + |\bar{B}^0(t_1)\rangle \otimes |B^0(t_2)\rangle.$$

$$N_{++} = N_{--} = \frac{1}{4\Gamma(1+x^2)} \int_0^{+\infty} d(\Delta t) e^{-\Gamma\Delta t} \left(1 + x^2 - \cos \Delta m \Delta t + \right. \\ \left. + x \sin \Delta m \Delta t \right) = \frac{1}{4\Gamma^2} \frac{3x^2 + x^4}{(1+x^2)^2},$$

$$N_{+-} = N_{-+} = \frac{1}{4\Gamma(1+x^2)} \int_0^{+\infty} d(\Delta t) e^{-\Gamma\Delta t} \left(1 + x^2 + \cos \Delta m \Delta t - \right. \\ \left. - x \sin \Delta m \Delta t \right) = \frac{1}{4\Gamma^2} \frac{2 + x^2 + x^4}{(1+x^2)^2},$$

$$R_{\text{even}} = \frac{N_{++} + N_{--}}{N_{+-} + N_{-+}} = \frac{3x^2 + x^4}{2 + x^2 + x^4}.$$

Branchings of the $\Upsilon(5S) \rightarrow B_s^{(*)} \bar{B}_s^{(*)}$ decays

C -parity of BB pair	Decay modes				Branching notation
$B^0 \bar{B}^0$ in the final state					
C -odd state	$B^0 \bar{B}^0$	$B^{0*} \bar{B}^{0*}$	$B^0 \bar{B}^0 \pi^0$		$(\epsilon_{00})^{\text{odd}}$
C -even state	$B^{0*} \bar{B}^{0*} \pi^0$	$B^0 \bar{B}^0$	$B^{0*} \bar{B}^{0*} \pi^0$		$(\epsilon_{00})^{\text{even}}$
$B^+ B^-$ in the final state					
C -odd and C -even states	$B^+ B^-$	$B^{+*} B^{-*}$	$B^+ B^- \pi^0$		ϵ_{+-}
	$B^{+*} B^{-*} \pi^0$	$B^+ B^-$	$B^{+*} B^{-*} \pi^0$		
	$B^+ B^{-*}$	$B^{+*} B^-$			
$B_S^+ \bar{B}_S^-$ in the final state					
C -odd state	$B_S^+ \bar{B}_S^-$	$B_S^{+*} \bar{B}_S^{-*}$			$(\epsilon_{SS})^{\text{odd}}$
C -even state	$B_S^{+*} \bar{B}_S^{-*}$	$B_S^+ \bar{B}_S^-$			$(\epsilon_{SS})^{\text{even}}$
$B^\pm B^0$ in the final state					
No definite C -parity	$B^+ \bar{B}^0 \pi^-$	$B^{+*} \bar{B}^0 \pi^-$	$B^+ \bar{B}^{0*} \pi^-$	$B^{+*} \bar{B}^{0*} \pi^-$	ϵ_+
	$B^- B^0 \pi^+$	$B^{-*} B^0 \pi^+$	$B^- B^{0*} \pi^+$	$B^{-*} B^{0*} \pi^+$	ϵ_-

$$\begin{aligned}
 \frac{dN_{++}}{d\Delta t} = & L\sigma_{e^+e^- \rightarrow \Upsilon(5S)} \mathcal{B}^2(B^0 \rightarrow \ell^+ \nu_\ell X) \frac{\Gamma}{2} e^{-\Gamma\Delta t} \left[(\epsilon_{00})^{\text{odd}} \sin^2 \frac{\Delta m \Delta t}{2} + \right. \\
 & + \frac{(\epsilon_{00})^{\text{even}}}{2(1+x^2)} \left(1 + x^2 - \cos \Delta m \Delta t + x \sin \Delta m \Delta t \right) + \frac{(\epsilon_{SS})^{\text{odd}} + (\epsilon_{SS})^{\text{even}}}{2} + \\
 & \left. + \frac{\epsilon_+}{4+x^2} \left(2 + x^2 - 2 \cos \Delta m \Delta t + x \sin \Delta m \Delta t \right) \right] \quad (1)
 \end{aligned}$$

where L is a luminosity of the e^+e^- collider, $\sigma_{e^+e^- \rightarrow \Upsilon(5S)}$ is a cross section of the $\Upsilon(5S)$ production, $\mathcal{B}(B^0 \rightarrow \ell^+ \nu_\ell X)$ is the branching ratio of the semileptonic B^0 -meson decay and Γ is B^0 width.

Analogous expression holds for $dN_{--}/d\Delta t$.

Analogous formulas are obtained for the case of dileptons of the opposite signs:

$$\begin{aligned} \frac{dN_{+-}}{d\Delta t} = & L\sigma_{e^+e^- \rightarrow \Upsilon(5S)} \mathcal{B}^2(B^0 \rightarrow \ell^+ \nu_\ell X) \frac{\Gamma}{2} e^{-\Gamma\Delta t} \left[(\epsilon_{00})^{\text{odd}} \cos^2 \frac{\Delta m \Delta t}{2} + \right. & (2) \\ & + \frac{(\epsilon_{00})^{\text{even}}}{2(1+x^2)} \left(1 + x^2 + \cos \Delta m \Delta t - x \sin \Delta m \Delta t \right) + \epsilon_{+-} + \frac{(\epsilon_{SS})^{\text{odd}} + (\epsilon_{SS})^{\text{even}}}{2} + \\ & \left. + \frac{\epsilon_+ + \epsilon_-}{2(4+x^2)} \left(6 + x^2 + 2 \cos \Delta m \Delta t - x \sin \Delta m \Delta t \right) \right]. \end{aligned}$$

It is convenient to form the following ratios from (1) and (2):

$$\frac{d(N_{+-} + N_{-+} - N_{--} - N_{++})/d\Delta t}{d(N_{+-} + N_{-+} + N_{--} + N_{++})/d\Delta t} = C + A \sin(\Delta m \Delta t + \varphi), \quad (3)$$

$$\frac{d(N_{++} + N_{--})/d\Delta t}{d(N_{+-} + N_{-+} + N_{--} + N_{++})/d\Delta t} = C' - \frac{A}{2} \sin(\Delta m \Delta t + \varphi), \quad (4)$$

where the parameters C, C', A and ϕ are equal to

$$C = \epsilon_{+-} + \left(\epsilon_+ + \epsilon_- \right) \frac{2}{4 + x^2},$$

$$C' = \frac{(\epsilon_{00})^{\text{odd}} + (\epsilon_{00})^{\text{even}} + (\epsilon_{SS})^{\text{odd}} + (\epsilon_{SS})^{\text{even}}}{2} + \left(\epsilon_+ + \epsilon_- \right) \frac{1 + x^2/2}{4 + x^2},$$

$$A = - \left[\left((\epsilon_{00})^{\text{odd}} + \frac{(\epsilon_{00})^{\text{even}}}{1+x^2} + \frac{2}{4+x^2}(\epsilon_+ + \epsilon_-) \right)^2 + \left(\frac{x}{1+x^2}(\epsilon_{00})^{\text{even}} + \frac{x}{4+x^2}(\epsilon_+ + \epsilon_-) \right)^2 \right]^{1/2},$$

$$\phi = -\arcsin \left[\frac{(\epsilon_{00})^{\text{odd}} + \frac{(\epsilon_{00})^{\text{even}}}{1+x^2} + \frac{2}{4+x^2}(\epsilon_+ + \epsilon_-)}{-A} \right].$$

Experimental measurement of the time dependences of the left sides of (3) and (4) allows to obtain the branching ratio $\epsilon_{SS} = (\epsilon_{SS})^{\text{odd}} + (\epsilon_{SS})^{\text{even}}$:

$$2C' - C = \epsilon_{SS} + \left(\epsilon_+ + \epsilon_- \right) \frac{x^2}{4 + x^2},$$

where we used the isotopic invariance: $\epsilon_{+-} = (\epsilon_{00})^{\text{odd}} + (\epsilon_{00})^{\text{even}}$.

Let us estimate the uncertainty in ϵ_{SS} determination which comes from the last term. The value of this term is

$$\frac{2}{3} \times (7 \pm 1)\% \times \frac{(0.77)^2}{4 + (0.77)^2} = (0.62 \pm 0.09)\%,$$

where factor $2/3$ accounts for the decays with charged π mesons and $x=0.77$ is substituted. Comparing it with the approximate ϵ_{SS} value $(22 \pm 2)\%$ we see that the relative uncertainty due to this correction is less than 1%.

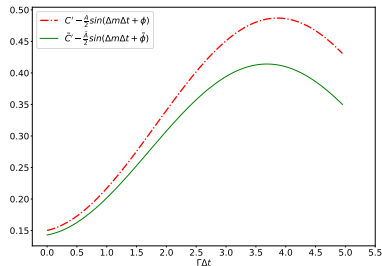
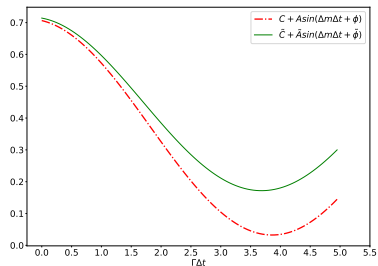
Conclusion

The fraction of B_s -mesons produced in $\Upsilon(5S)$ decays can be determined from the time dependences of semileptonic decays at Belle II.

It can be used to find the branching ratio of normalization channel $B_s \rightarrow D_s^- \pi^+$ at Belle II with the percentage accuracy opening the way to measure $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$ at LHC with the same accuracy.

Corrections due to the longer lifetime of B^+ meson

While lifetimes of B^0 and B_s mesons coincide at a percent accuracy the lifetime of B^+ meson is considerably higher: $\tau_{B^+}/\tau_{B^0} = 1.076 \pm 0.004$.



$\Upsilon(4S) \rightarrow B\bar{B} \rightarrow J/\psi K_S J/\psi K_S$

Two-particle wave function is needed:

$$\begin{aligned} \langle J/\psi K_S, J/\psi K_S | \psi(t_1, t_2) \rangle_{\text{odd}} &= e^{-2iMt - \Gamma t} \left[A \cos\left(\frac{\Delta m}{2} t_1\right) + i \frac{q}{p} \bar{A} \sin\left(\frac{\Delta m}{2} t_1\right) \right] \times \\ &\times \left[\bar{A} \cos\left(\frac{\Delta m}{2} t_2\right) + i \frac{p}{q} A \sin\left(\frac{\Delta m}{2} t_2\right) \right] - (t_1 \leftrightarrow t_2) = e^{-2iMt - \Gamma t} \left[\left(i \frac{p}{q} A^2 - \right. \right. \\ &\left. \left. - i \frac{q}{p} \bar{A}^2 \right) \cos\left(\frac{\Delta m}{2} t_1\right) \sin\left(\frac{\Delta m}{2} t_2\right) + \left(i \frac{q}{p} \bar{A}^2 - i \frac{p}{q} A^2 \right) \sin\left(\frac{\Delta m}{2} t_1\right) \cos\left(\frac{\Delta m}{2} t_2\right) \right] \\ &= -e^{-2iMt - \Gamma t} \left(i \frac{p}{q} A^2 \right) \left[1 - \lambda^2 \right] \sin\left(\frac{\Delta m \Delta t}{2}\right) \end{aligned}$$

where $A(\bar{A})$ is an amplitude of $B^0(\bar{B}^0) \rightarrow J/\psi K_S$ decay and $\lambda = (q/p)(\bar{A}/A)$ was introduced. In the last line the fact that $A = \bar{A}$ was used. The ratio q/p can be expressed via the angle β of the unitarity triangle: $q/p = e^{-2i\beta}$. Then the probability of the decay equals

$$\begin{aligned} \mathbb{P}(J/\psi K_S, J/\psi K_S) &= e^{-2\Gamma t} A^4 [1 - e^{4i\beta}] [1 - e^{-4i\beta}] \sin^2 \left(\frac{\Delta m \Delta t}{2} \right) = \\ &= e^{-2\Gamma t} A^4 \cdot 4 \sin^2(2\beta) \sin^2 \left(\frac{\Delta m \Delta t}{2} \right). \end{aligned}$$

It equals zero for $\Delta t = 0$ due to Bose statistics, for $\Delta m = 0$ because of absence of oscillations and for $\beta = 0$ due to CP conservation. The point is that $\Upsilon(4S)$ is CP even while final state is CP odd.

Let us estimate the amount of $\Upsilon(4S) \rightarrow J/\psi K_S J/\psi K_S$ decays which could be detected at Belle II. The total number of $\Upsilon(4S)$ which should be produced at SuperKEKB with integrated luminosity $L = 50 \text{ ab}^{-1}$ equals $\approx 10^{11}$. For the number of decays $\Upsilon(4S) \rightarrow J/\psi K_S J/\psi K_S$ we get:

$$N(\Upsilon(4S) \rightarrow J/\psi K_S J/\psi K_S) = \epsilon \cdot N(\Upsilon(4S)) \cdot \mathcal{B}(\Upsilon(4S) \rightarrow J/\psi K_S J/\psi K_S) \approx 150,$$

where $\epsilon = 0.04$ is an efficiency of Belle II detector in the $\Upsilon(4S) \rightarrow J/\psi K_S J/\psi K_S$ channel. Taking into account excited charmonium states this number can be doubled.