A Solution of the Scalar Nonet Mass Puzzle

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Astract

We propose an explanation of the inverse mass hierarchy of the low-lying nonet of the scalar mesons in the framework of the massless Nambu – Jona-Lasinio $U_R(3)$ x $U_L(3)$ quark model. The collective meson states are described via quark–antiquark pairs, whose condensates lead simultaneously to spontaneous breaking of chiral and flavour symmetries.
It is shown that, due to flavour symmetry breaking, two iso-doublets of K_0*(700) (or κ) mesons play the role of Goldstone bosons.
It is also proven that there exists a solution with degenerate masses of the a_0(980) and f_0(980) mesons and a zero mass of the f_0(500) (or σ) meson.

The Model

Let us consider scalar sector of the massless Nambu – Jona-Lasinio $U_R(3)$ x $U_L(3)$ quark model:

$$\mathcal{L} = i\bar{\Psi}\partial\!\!\!/\Psi + \frac{G_0}{2}\left(\bar{\Psi}\Psi\right)^2 + \frac{\bar{G}_0}{2}\sum_{a=1}^{8}\left(\bar{\Psi}\lambda_a\Psi\right)^2, \text{ where } \lambda_a\,(a=1,\dots,8) \text{ are the Gell-Mann matrices, } \Psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}.$$

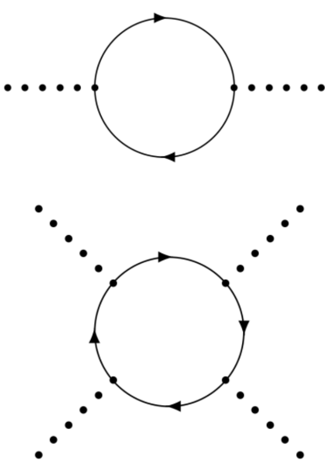
The coupling constants G_0 and $\bar{G}_0$ have dimension $[\text{mass}]^{-2}$ and lead to non-renormalizable theory. Therefore, we linearize the Lagrangian introducing dimensionless coupling constants g_0 and g_a

$$\mathcal{L} = i\bar{\Psi}\partial\!\!\!/\Psi + g_0\bar{\Psi}S_0\Psi - \frac{g_0^2}{2G_0}S_0^2 + \sum_{a=1}^{8}\left(g_a\bar{\Psi}\lambda_a S_a\Psi - \frac{g_a^2}{2\bar{G}_0}S_a^2\right)$$

and the auxiliary fields (without kinetic terms) $S_0 = \frac{G_0}{g_0}\bar{\Psi}\Psi$ and $S_a = \frac{\bar{G}_0}{g_a}\bar{\Psi}\lambda_a\Psi$

The kinetic terms and self-interactions arise from the Feynman diagrams:

$$\begin{aligned} V_{\text{eff}} = {} & \frac{\mu^2}{2}S_0^2 + \frac{\tilde{\mu}^2}{2}\sum_{a=1}^{8}S_a^2 + \frac{g^2}{2}S_0^4 + 3g^2S_0^2\sum_{a=1}^{8}S_a^2 + \frac{3g^2}{4}\left(\sum_{a=1}^{8}S_a^2\right)^2 \\ & + \frac{3\sqrt{3}}{\sqrt{2}}g^2S_0S_3\left(S_4^2 + S_5^2 - S_6^2 - S_7^2\right) - \sqrt{2}g^2S_0S_8^3 \\ & + 3\sqrt{2}g^2S_0S_8\left(S_1^2 + S_2^2 + S_3^2 - \frac{S_4^2 + S_5^2 + S_6^2 + S_7^2}{2}\right) \\ & + 3\sqrt{6}g^2S_0\left(S_1S_4S_6 + S_1S_5S_7 - S_2S_4S_7 + S_2S_5S_6\right) \end{aligned}$$



Results

Dimensionless variables of expectation values

$x = 3\sqrt{2}g\langle S_0\rangle / \sqrt{-\tilde{\mu}^2}: \quad 1 \le x^2 \le 1+\sqrt{3}$

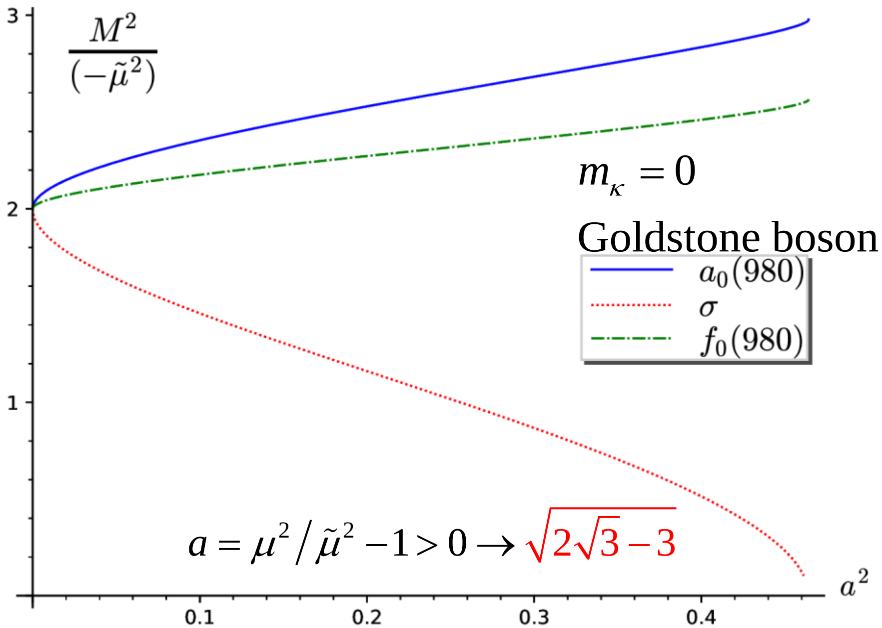
$x \neq 0:$ breaking of chiral symmetry

$y = 3g\langle S_3\rangle / \sqrt{-\tilde{\mu}^2} = 0:$ no $SU_f(2)$ breaking

$z = 3g\langle S_8\rangle / \sqrt{-\tilde{\mu}^2}: \quad z = \frac{x+\sqrt{3(4-x^2)}}{2}$

$z \neq 0:$ breaking of $U_f(3)$ symmetry

Physical point: $x_0 = \sqrt{1+\sqrt{3}}$



- Breaking of $U_R(3)$ x $U_L(3)$ symmetry leads to the appearance of Goldstone bosons, which results into zero masses for the κ mesons.
- Our model suggests also a zero mass for the σ meson. We have also determined that the σ meson state consists almost completely of ($s\bar{s}$) quark combination. In previous studies, it was assumed that the σ meson for massless quarks consists predominantly of light quarks.
- We have also shown that $f_0(980)$ consists predominantly of light quarks, contrary to the previous assumptions that it consists mainly of strange quarks.
- There exists also a solution with degenerate masses of the $a_0(980)$ and $f_0(980)$ mesons.

Update from yesterday's paper of the ALICE Collaboration

Yesterday's paper of the ALICE Collaboration **arXiv:2507.19347** confirms one of our main statement and breaks down about 50 years old dogma about tetraquark origin of low lying scalar nonet mesons:
"The thermal model calculations provide a better description of the decreasing trend of particle ratios when no strange or antistrange quark composition for $f_0(980)$ is assumed, which suggests that the tetraquark interpretation of the $f_0(980)$ is disfavored."

Please, compare with our statement:

- We have also shown that $f_0(980)$ consists predominantly of light quarks, contrary to the previous assumptions that it consists mainly of strange quarks.