



Probing Lorentz Invariance Violation in Z Boson Mass Measurements at High- Energy Colliders

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Outline

- Motivation & Background
- Z Boson LIV Model & Phenomenology
- Experimental Strategy & Outlook

Motivation: Importance of Lorentz Invariance Violation (LIV)

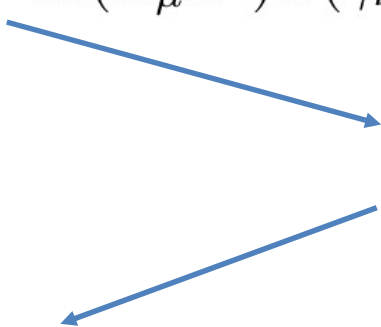
- Lorentz invariance as cornerstone of modern physics
 - Theoretical interest in possible LIV
 - a) Spontaneous LIV is a source for emergent vector (tensor) fields, where they appear as Goldstones of spacetime symmetry violation.
 - b) LIV may be used as a form of UV completion for the theory to make it finite. (Hořava's model for example).
 - Experimental significance at high energies
- LIV while insignificant at low energies, can heavily influence ultra high energy physics.

How to approach LIV on the experiment?

While on fundamental level we may not understand full picture, on practical level we can always introduce some probable LIV operator, calculate its effect and think how to make it visible in the experiments/observations.

- We can introduce some LIV interaction. For example if we introduce preferred LIV direction in space time fixed by the unit vector n_μ , we can have the simplest interaction form

$$\Delta L_{int} = e\delta(A_\mu n^\mu)\bar{\Psi}(\gamma_\nu n^\nu)\Psi$$


$$\sigma_{new} = \sigma_{LI}(1 + \delta_{int} \cdot f(\Omega))$$

$\delta_{int} > 10^{-5}$ to be visible at LHC, which might be unrealistically high for LIV

CMS Collaboration (arXiv.2405.14757).

E. Lunghi, N. Sherrill, A. Szczepaniak, A. Vieira (arXiv.2011.02632).

- Modify dispersion relation of the particle in Non Lorentz invariant way. In the most general form we can write

$$P^\mu P_\mu = m_{eff}^2 \longrightarrow m_{eff}^2 = m^2 + \delta(n, p) P_0^2 \longrightarrow \frac{\Delta c}{c} \sim \delta(n, p)$$

Restriction from cosmic rays are very tight for $\delta(n, p)$ for stable the particles. There many different restrictions from different processes, but shortly we can summarize that $|\delta_{\gamma, e, p}| < 10^{-16}$.

But neutrino sector:

The upper limit defined for the atmospheric neutrinos with energies 1GeV~1TeV, by Super-Kamiokande is 10^{-8} . The same limit for 20-100 TeV neutrinos defined by IceCube collaboration is $10^{-10} \sim 10^{-11}$

And there is not understood neutrino events from SN1987a with lightspeed variation $\delta < 10^{-9}$

Whether this is a feature or experimental inability remains to be seen.

- Electro-magnetic sector is very restricted
- Weak sector is less explored

There are weak limits only on neutrinos.

Unstable weak bosons is totally unexplored. We can study them basically only on accelerators.

The question is: how can this be probed, given that the LHC's sensitivity to such a small parameter is extremely low — several orders of magnitude lower than the existing limits for neutrinos, especially if we attempt to generalize these limits to the entire weak sector?

Key is the study of the resonance energy region of the intermediate boson

- If we are to modify the dispersion relation of the intermediate boson

$$p_\alpha^2 = M_{B,eff}^2 = M_B^2 - \delta(n^\lambda p_\lambda)^2$$

- We affect the decay rate and propagator

$$\Gamma_{LIV} = \frac{M_{B,eff}^2}{M_B^2} \Gamma_{LI}$$

$$D_B = \frac{i}{p_\alpha^2 - M_B^2} \rightarrow \frac{i}{p_\alpha^2 - M_{B,eff}^2 (1 - i\Gamma_{LI}/2M_B)^2}$$

Cross-section is

$$\sigma_{LIV} \sim |D_B|^2$$

And then resonance mass

$$M_{resonance}^2 = M_{B,eff}^2 (1 - \Gamma_{LI}^2 / 4M_B^2)$$

If we are to apply this to neutral Z-boson, since accuracy of its mass measurement is order better than of W-bosons and it can notice smaller effects, we roughly estimate

$$\Delta M_Z = |M_Z - M_{Zresonance}| \approx 2 \text{ MeV}$$

$$\delta < 10^{-8} \text{ to } 10^{-9}$$

Transition to Z-Boson Focus

- The Z-boson's narrow width, clean Leptonic final states, more accurate measurements, make it better candidate.
- We modify kinetic sector of Z-boson, and study its impact on Drell–Yan at the LHC, specifically near the resonance energies.
- Also we discuss historical tension between LHC and Tevatron data regarding W -boson, which aligns with LIV concept.

Z-Boson LIV Operators

- Quadratic LIV terms in the broken phase (no higher derivatives):

$$\Delta L_{LIV} = \frac{\delta_{LIV}}{2} (\partial_n Z^\mu)(\partial_n Z_\mu) + \frac{\delta_{1LIV}}{2} (\partial_\mu Z_n)(\partial^\mu Z_n) + \delta_{2LIV} (\partial_\mu Z^\mu)(\partial_n Z_n)$$

within specific scenario we can only connect δ parameters to each other, but only δ_{LIV} term alters the on-shell dispersion relation;

Note: This might appear contentious, but LIV and gauge invariance does not sit well together. One can not have gauge invariance and physical Lorentz violation same time.

Dispersion Relation, effective Mass and propagator

- Modified dispersion relation

$$k_\mu k^\mu = M_{eff}^2 = M_Z^2 + \delta_{LIV} (k_n)^2$$

- Modified propagator

$$D_{corrected} = -i \frac{g_{\mu\nu} - k_\mu k_\nu / M_{eff}^2}{k_\mu^2 - (M_{eff} - ik_0 \Gamma_{eff}(k) / 2M_{eff})^2}$$

where any farther correction to the propagator due to the loop effects may only be proportional to $\alpha \cdot \delta_{LIV}$ (α is fine structure constant).

Modified Decay Rate

Exact Lepontic Decay rate:

$$\Gamma_{eff} = \frac{\alpha(g_l^2 + 1)}{12 \sin^2 2\theta_w} \frac{1}{k_0} (M_{eff}^2 + 2 \frac{g_l^2 - 2}{g_l^2 + 1} m^2) \sqrt{1 - 4 \frac{m^2}{M_{eff}^2}}$$

which in the leading order allows to write

$$\Gamma_{eff} \approx \Gamma_{SM} \left[1 + \frac{\delta k_n^2}{M_Z^2} \right] = \Gamma_{SM} \frac{M_{eff}^2}{M_Z^2}$$

Kinematics of Drell-Yan processes

Two protons collide with following momenta

$$P_1 = (E, P \vec{r}), \quad P_2 = (E, -P \vec{r})$$

\vec{r} indicates orientation of the collision axe (or equivalently detector), which rotates in the space with an Earth.

Transferred momentum of the intermediate boson is often parametrized by the invariant mass M (which defines Mandelstam variable $S = M^2$) and rapidity Y

$$Q_\mu = ((x_1 + x_2)E, (x_1 - x_2)P \vec{r}) = M(\cosh Y, \vec{r} \sinh Y)$$

$$Q_\mu^2 = M^2$$

Cross section

DY-cross section

$$\frac{d^2\sigma_p}{dM^2 dY} = \sum_f \frac{f_{qf}(x_1) f_{\bar{q}f}(x_2)}{4E^2} \sigma_{EM} \left[1 + \frac{g_q g_l \left(M^2 - M_{eff}^2 (1 - \Gamma_{0SM}^2 / 4M_Z^2) \right)}{2 |e_f| M^2 \sin^2 2\theta_w} R \right. \\ \left. + \frac{(1 + g_q^2)(1 + g_l^2)}{16e_f^2 \sin^4 2\theta_w} R \right]$$
$$R = \frac{M^4}{\left(M^2 - M_{eff}^2 (1 - \Gamma_{0SM}^2 / 4M_Z^2) \right)^2 + M_{eff}^4 \Gamma_{0SM}^2 / M_Z^2}$$

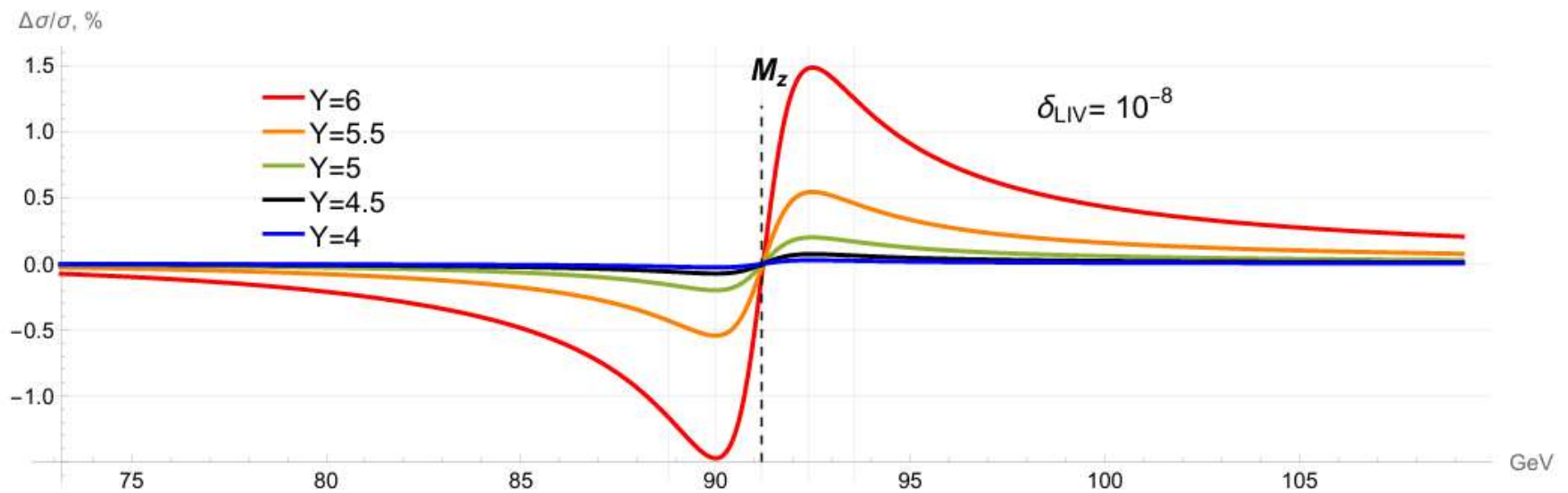
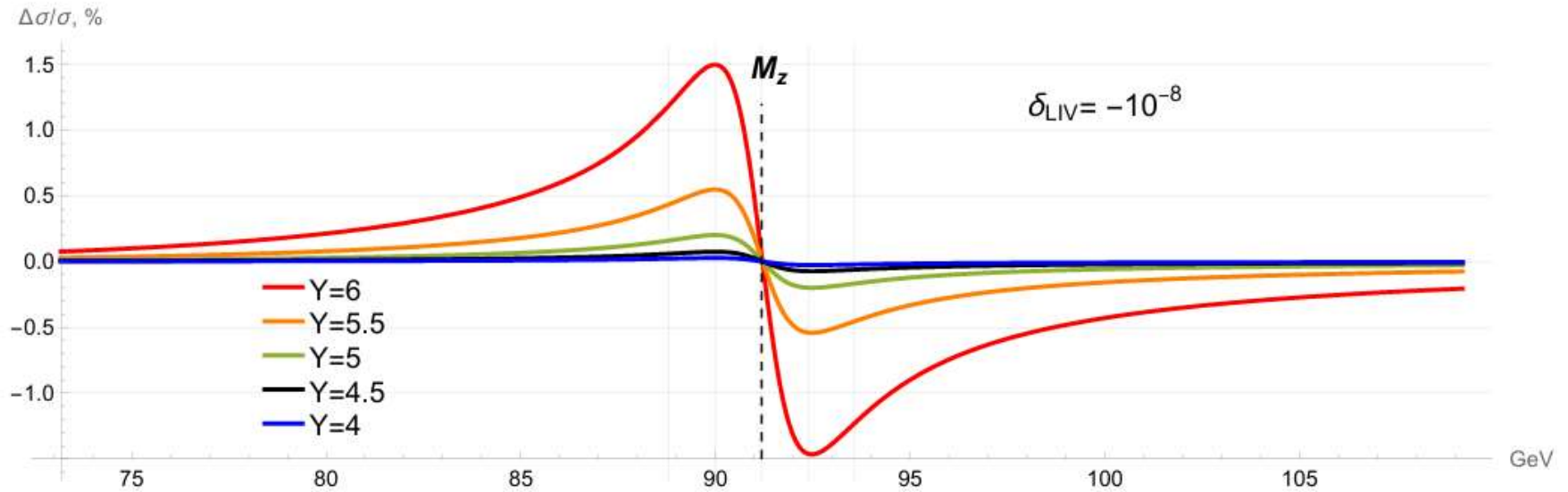
Resonance value of the invariant mass now is energy and orientation dependent. It is exponentially amplified by the rapidity Y , therefore it is very sensitive to rapidity and experimentally more promising

$$\begin{array}{ccc}
 & M_r^2 \approx M_{eff}^2 - \Gamma_{0SM}^2/4 & \\
 \text{timelike violation} \nearrow & & \searrow \text{lightlike violation} \\
 M_r^2 \approx M_Z^2(1 + \delta_{LIV} \cosh^2 Y) - \Gamma_{0SM}^2/4 & \text{spacelike violation} & \\
 & \downarrow & \\
 M_r^2 \approx M_Z^2(1 + \delta_{LIV} \sinh^2 Y \cos^2 \beta) - \Gamma_{0SM}^2/4 & & \\
 & & \\
 & & M_r^2 \approx M_Z^2(1 + \delta_{LIV} (\cosh Y - \sinh Y \cos \beta)^2) - \Gamma_{0SM}^2/4
 \end{array}$$

where β is an angle between \vec{r} and \vec{n} (special direction of the Lorentz violation)

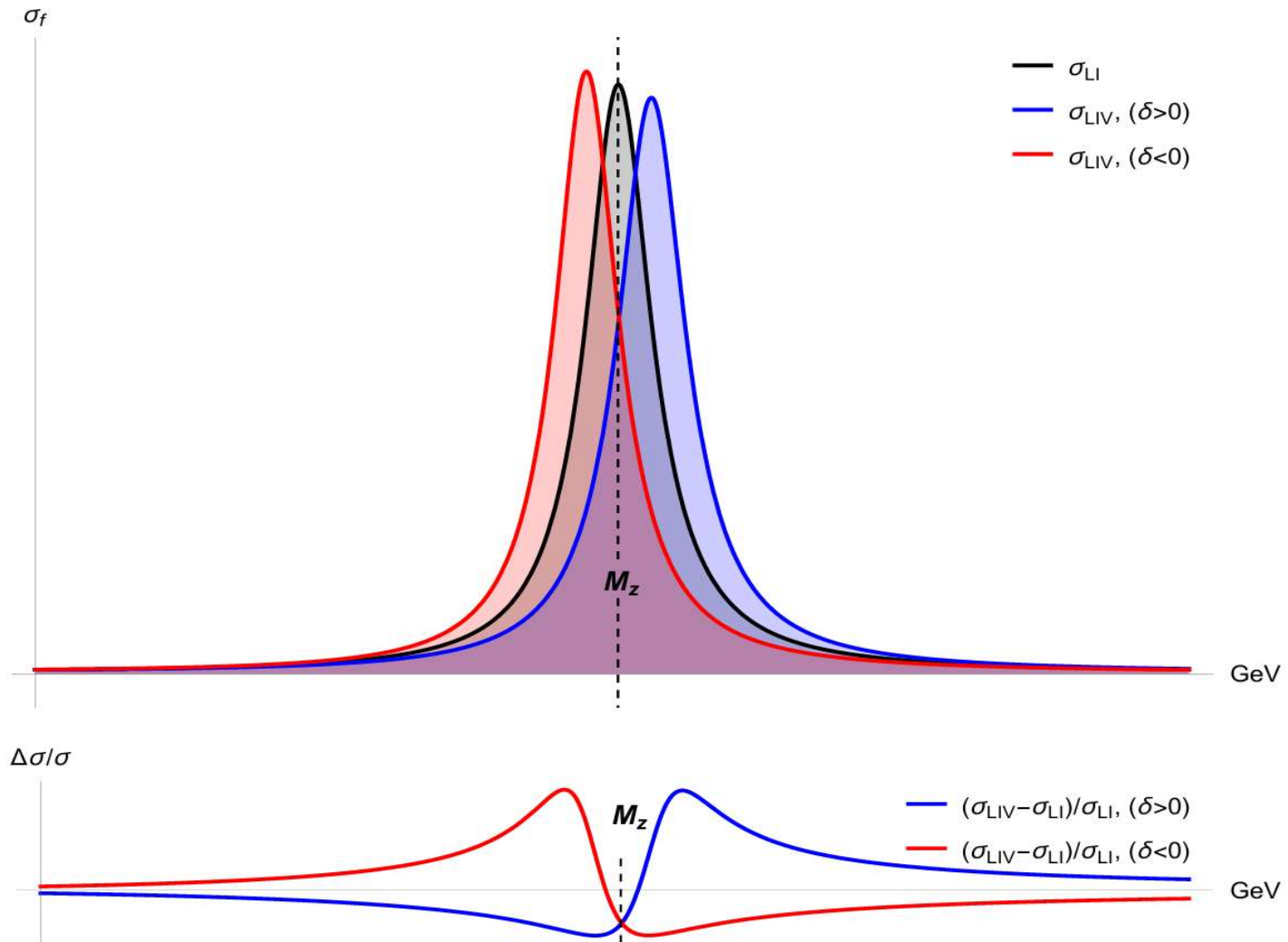
The case of timelike violation

$$(\sigma_{LIV} - \sigma_{LI})/\sigma_{LI}$$



This $Y = 8$ plot is for demonstrative purposes only. For the LHC data, the expected difference is limited to $\sim 0.1\%$ for the rapidity $Y = 4.5$, rendering it practically indistinguishable by visual inspection alone.

Figure 2: Highly exaggerated comparison of LIV and LI cross-sections.



Experimental strategy

- What we have?

LIV distorted distribution \longrightarrow we try to fit LIV cross-section in LI framework \longrightarrow diluted effect, we can not identify even if it is there, because high rapidity events are small in numbers.

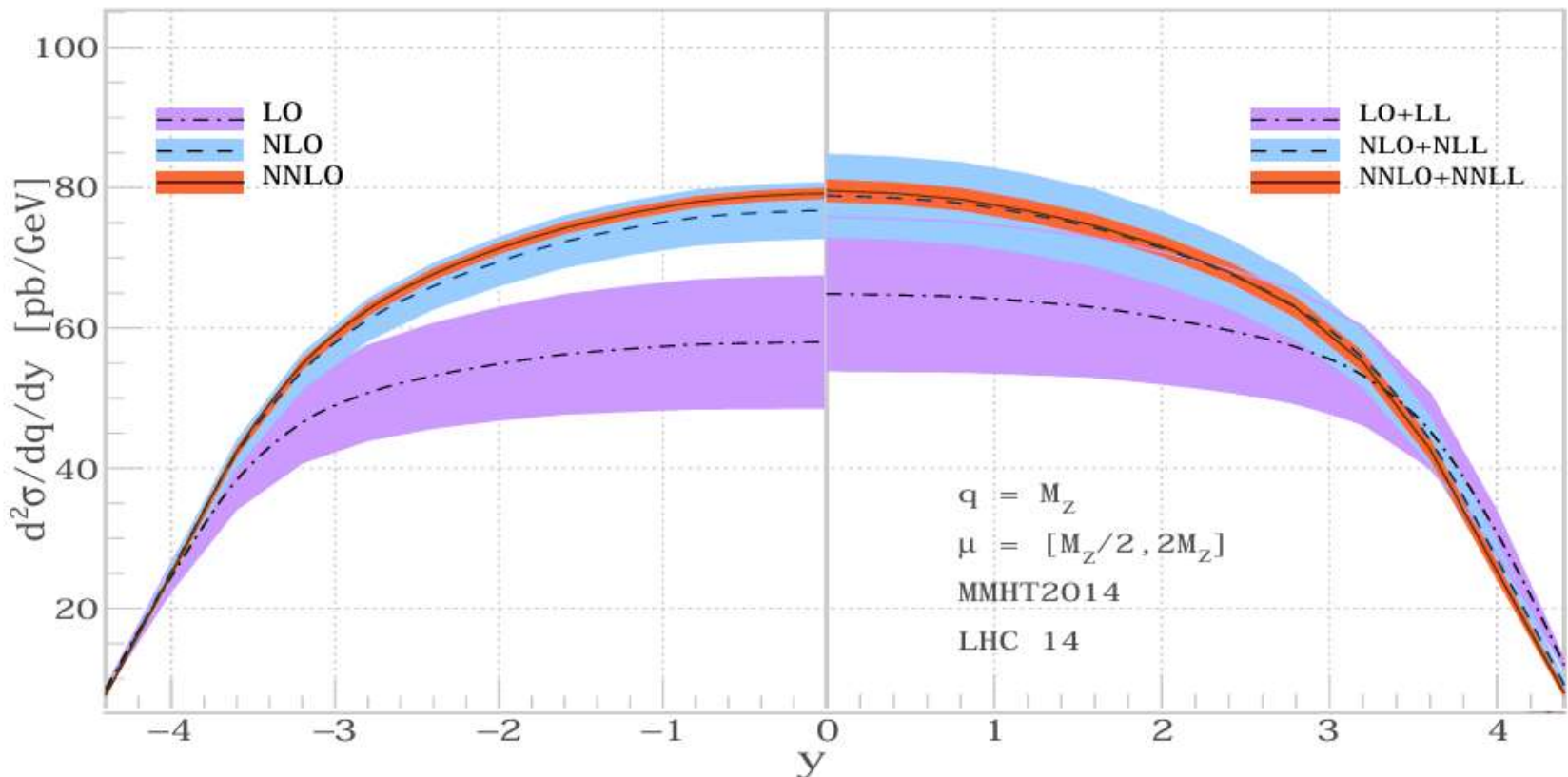


Table: Absolute mass shift $|\Delta M_Z|$ as a function of rapidity Y and the fractional composition of LI and LIV contributions of cross sections in the data.

$LI :$	90%	70%	50%	30%	10%	0%
$LIV :$	10%	30%	50%	70%	90%	100%
Y=0.5	0.1keV	0.2keV	0.3keV	0.4keV	0.5keV	0.6keV
Y=1.0	0.1keV	0.3keV	0.5keV	0.8keV	1.0keV	1.1keV
Y=1.5	0.3keV	0.8keV	1.3keV	1.8keV	2.3keV	2.5keV
Y=2.0	0.6keV	1.9keV	3.2keV	4.5keV	5.8keV	6.5keV
Y=2.5	1.7keV	5.1keV	8.6keV	12.keV	15.4keV	17.1keV
Y=3.0	4.6keV	13.9keV	23.1keV	32.4keV	41.6keV	46.2keV
Y=3.5	12.5keV	37.6keV	62.6keV	87.7keV	112.7keV	125.3keV
Y=4.0	34keV	102keV	170keV	238keV	306keV	340keV
Y=4.5	92keV	277keV	462keV	0.6MeV	0.8MeV	0.9MeV
Y=5.0	251keV	0.8MeV	1.3MeV	1.8MeV	2.3MeV	2.5MeV
Y=5.5	0.7MeV	2.0MeV	3.4MeV	4.8MeV	6.1MeV	6.8MeV
Y=6.0	1.9MeV	5.6MeV	9.3MeV	13.0MeV	16.7MeV	18.6MeV
Y=6.5	5MeV	15MeV	25MeV	35MeV	45MeV	51MeV
Y=7.0	14MeV	41MeV	69MeV	96MeV	124MeV	137MeV

What we could do to enhance detectability?

- For the timelike violation we should segregate events by the rapidity. When data is mixed 90% LI and 10% LIV (at $Y=5$) perceived $|\Delta M_z| \approx 0.25 \text{ MeV}$, but 100% LIV (at $Y=5$) gives $|\Delta M_z| \approx 2.5 \text{ MeV}$ (for $|\delta_{LIV}| = 10^{-8}$)
- Separation for $Y=4$ to 5 , should give a reasonable statistic and fairly big shift in mass.
- For space and light like violations additional anisotropy appears. This farther may dilute the LIV signal since variation respect to sidereal time appears. This depends on the orientation of the experiment with respect to Lorentz violation direction. Farther separation of events by sidereal time should LIV effect, but how small bin size by the rapidity and sidereal can get depends on the statistics of the experiment. There may be practical bin size limit we can practically afford.

Conclusion and Outlook

- Looking into resonance region of the intermediate bosons on the accelerators, should give better understanding of LIV on the level comparable to cosmic rays.
- Without accounting for LIV, higher-energy colliders would experience greater LIV data contamination, leading to a more pronounced systematic underestimation for negative δ_{LIV} (or overestimation for positive δ_{LIV}) of resonance mass. This aligns with historical data, where, measurements showed discrepancies between LHC and Tevatron results for the W mass measurements.
- While we analyzed in details only time like violation case, in principal, for general case for $|\delta_{LIV}| = 10^{-8}$ (may be even for 10^{-9}), LIV should detectable for the current LHC energy and accuracy.

Thanks for attention