Differential Study of Local Λ-hyperon Polarization in Heavy-Ion Collisions: Transport Model Approach

Oleksandr Vitiuk, Elizaveta Zherebtsova

University of Wroclaw, Poland

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Uniwersytet Wrocławski

National Science Centre Poland
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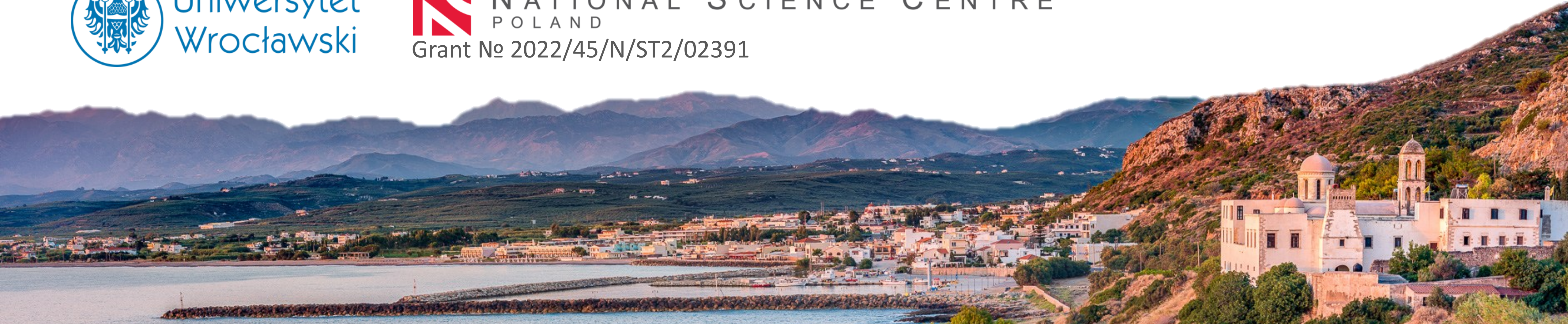

Outline

- Introduction
- Λ polarization in spin-thermal approach
- Central collisions
- UrQMD
- Λ polarization in UrQMD
- Summary

Introduction

- O. W. Richardson
  [Physical Review. Series I. 26 (3): 248–253 (1908)]

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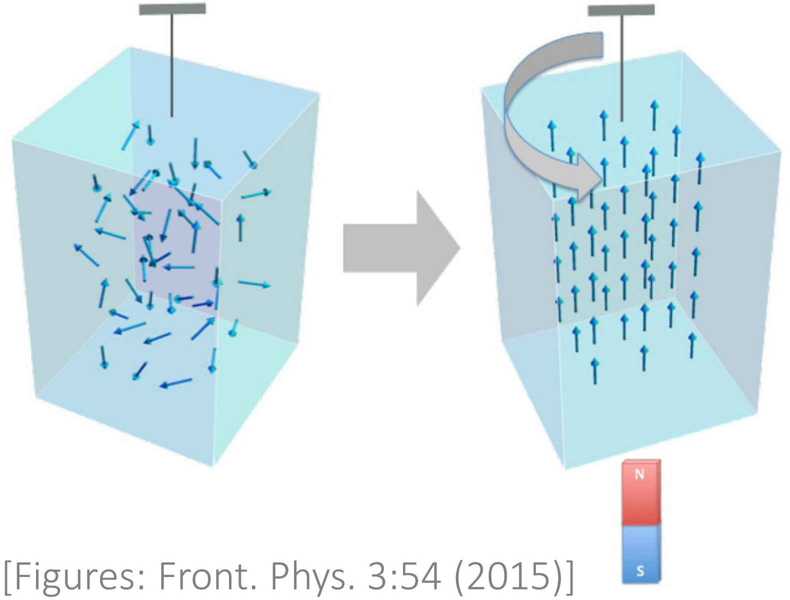

[Figures: Front. Phys. 3:54 (2015)]

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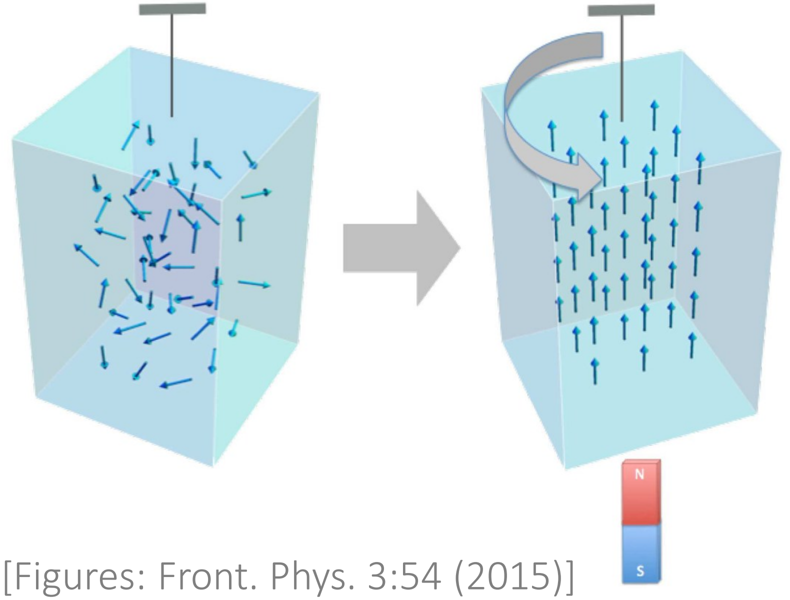

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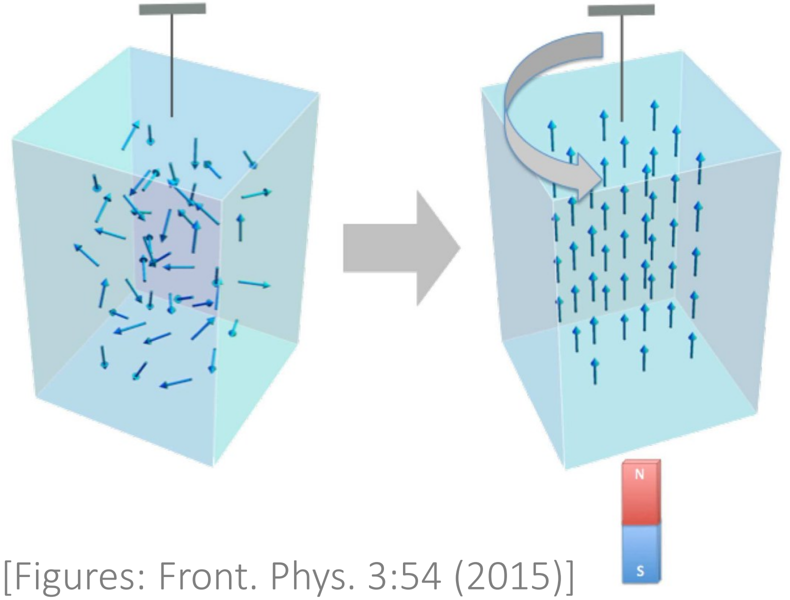

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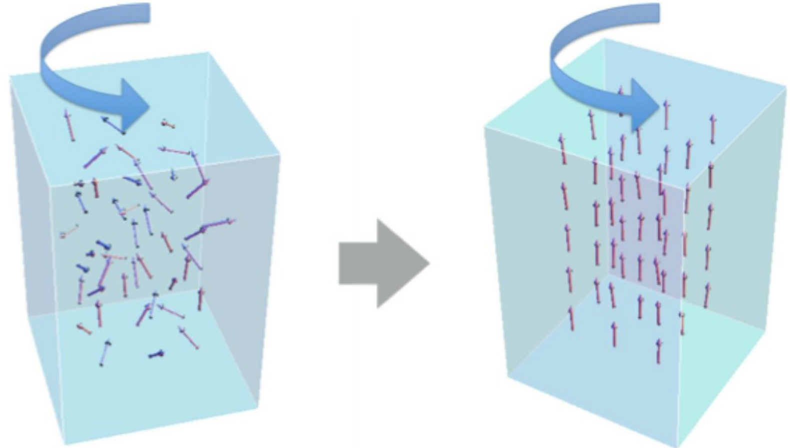
Rotation



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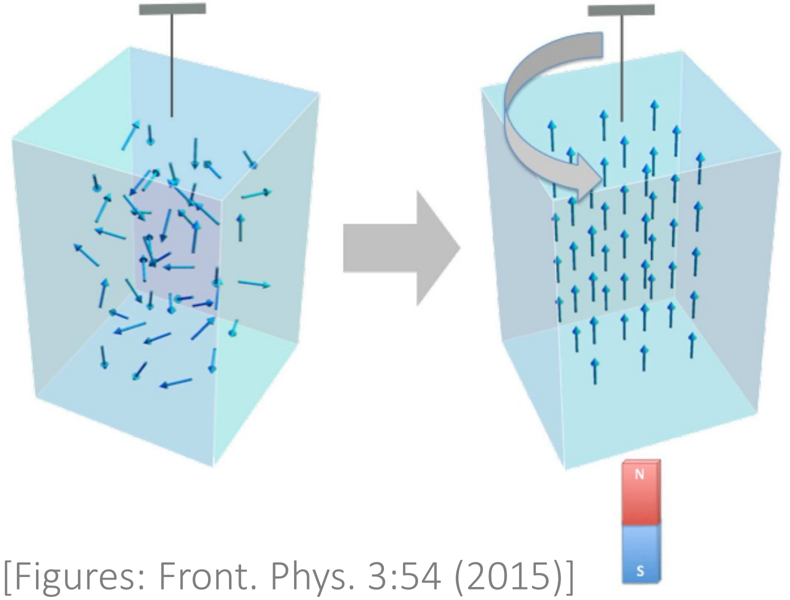

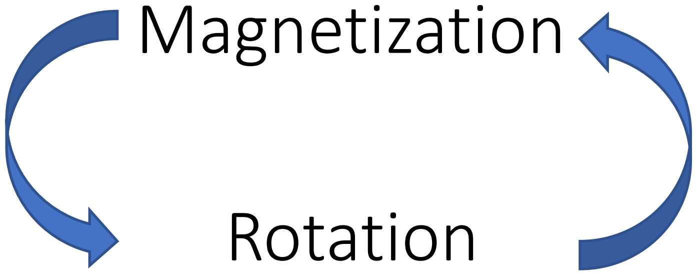

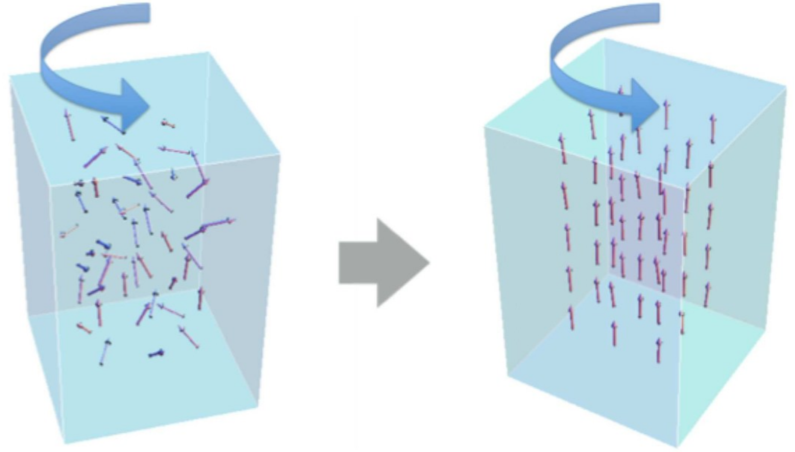

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PRL 94, 102301 (2005) PHYSICAL REVIEW LETTERS week ending 18 MARCH 2005

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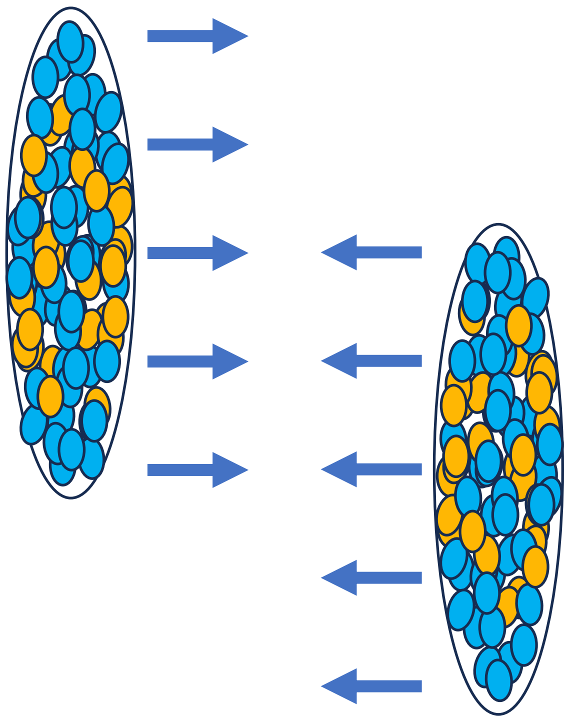
Zuo-Tang Liang[1] and Xin-Nian Wang[2,1]



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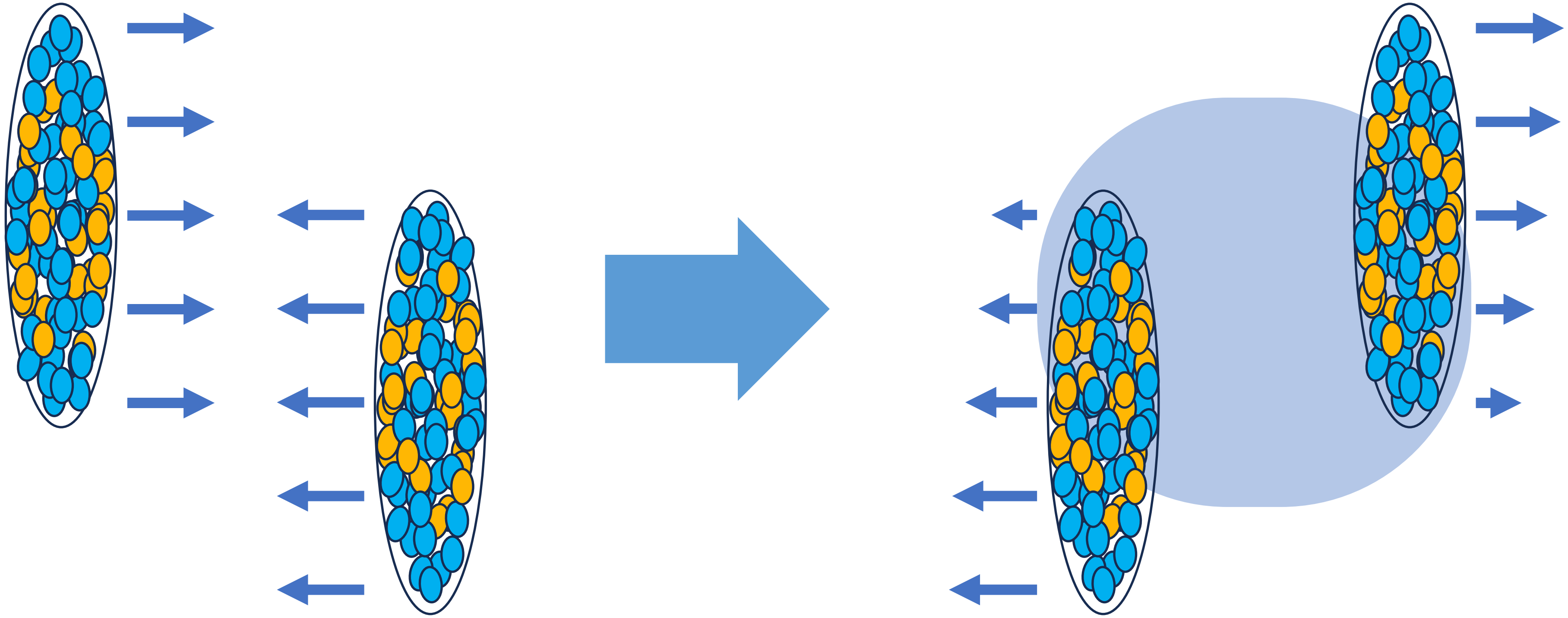

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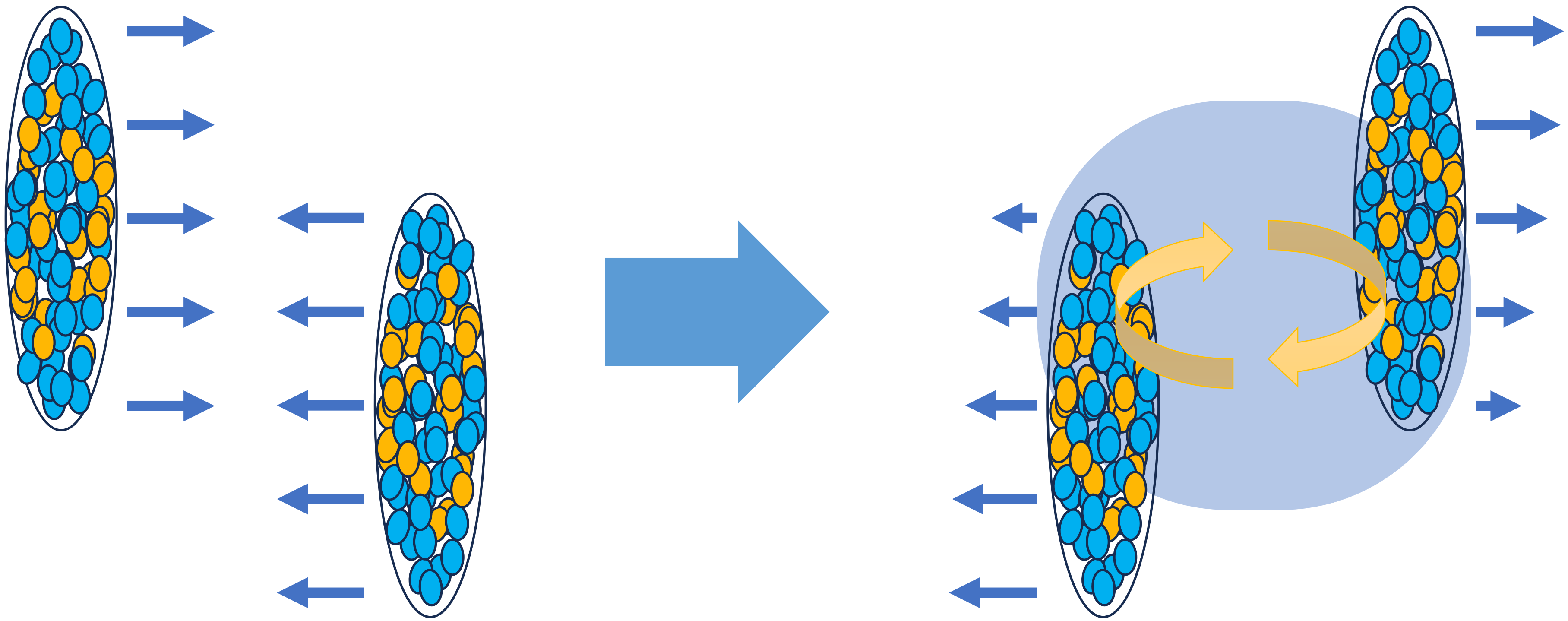

$\omega_y = (\nabla \times \vec{v})_y \approx -\frac{1}{2}\frac{\partial v_z}{\partial x}$ ⇒ Guess: $\Delta x \sim 5\ fm, \Delta v \sim 0.2$ ⇒ $\frac{\omega}{T} \sim$ up to few percent

Λ polarization in spin-thermal approach

In the assumption of local thermal equilibrium one can find expression for spin 4-vector: [PRC 95, 054902 (2017)]

$$S^\mu(p,x) \approx -\frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\nu\varpi_{\rho\sigma}(x), \qquad \varpi^{\mu\nu} = \frac{1}{2}\left(\partial^\nu \frac{u^\mu}{T} - \partial^\mu \frac{u^\nu}{T}\right)$$

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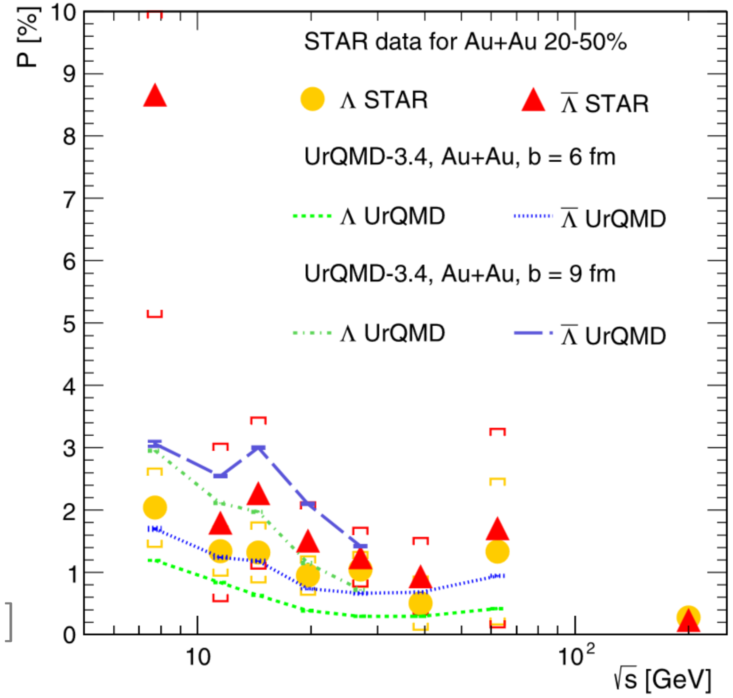
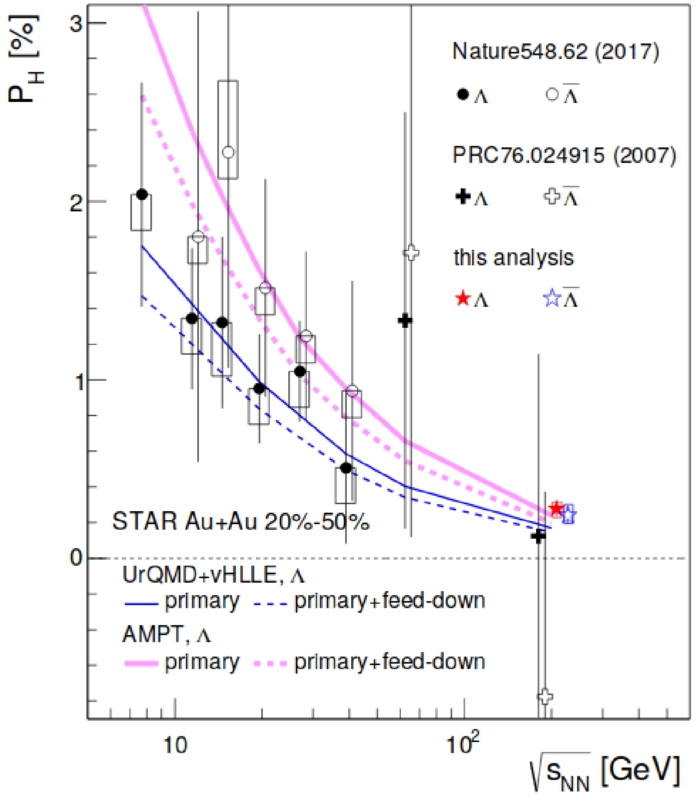
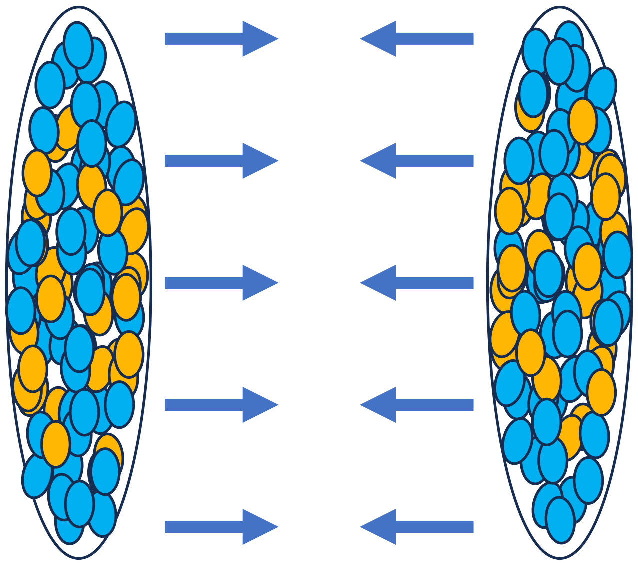
[PRC 98 (2018) 14910; PLB 803, 135298 (2020)]





Central collisions



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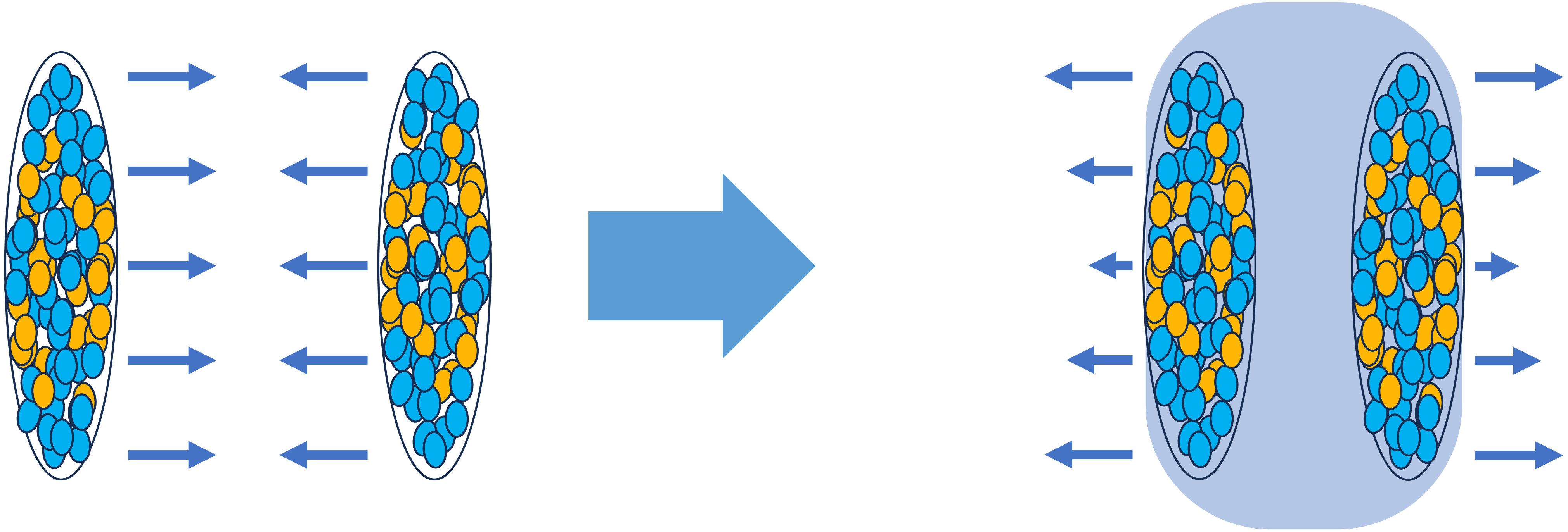

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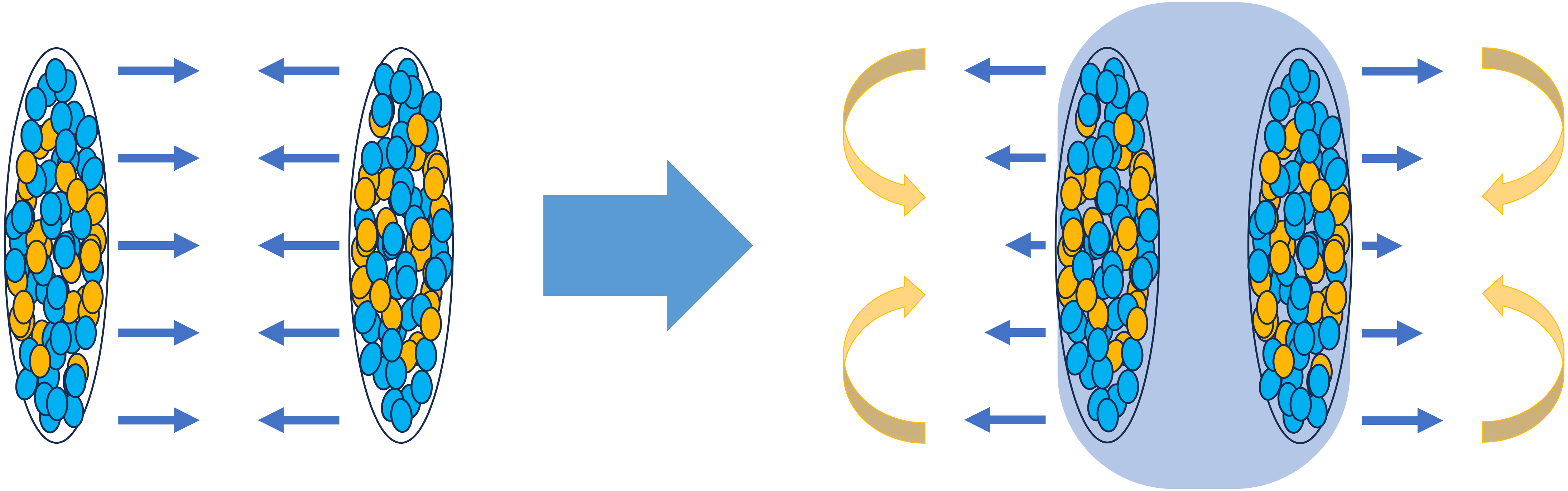

Ultrarelativistic Quantum Molecular Dynamics (UrQMD)

- Represents a Monte Carlo method for the time evolution of the various phase space densities of particle species
- Based on the covariant propagation of all hadrons on classical trajectories, stochastic binary scatterings, resonance and string formation with their subsequent decay
- Ideologically based on the Boltzmann equation
- The collision criterion (black disk approximation): $d < d_0 = \sqrt{\sigma(\sqrt{s}, type)/\pi}$
- 55 baryons and 32 mesons are included. All antiparticles and isospin-projected states are implemented
- Cross sections are taken from PDG
- Resonances are implemented in Breit–Wigner form

[S. A. Bass et al, Prog. Part. Nucl. Phys. 41 (1998) 255-369,
M. Bleicher et al, J. Phys. G: Nucl. Part. Phys. 25 (1999) 1859-1896]

Methodology

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Solution: Coarse-graining approach + HRG Model

[PLB 803, 135298 (2020)]

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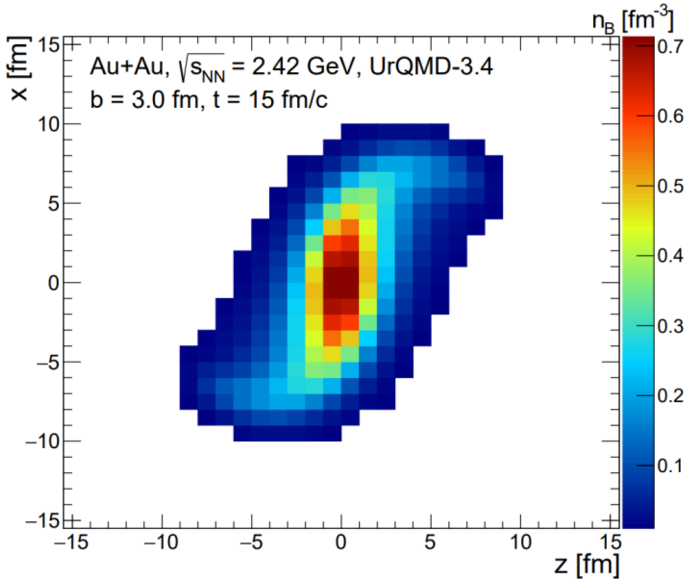

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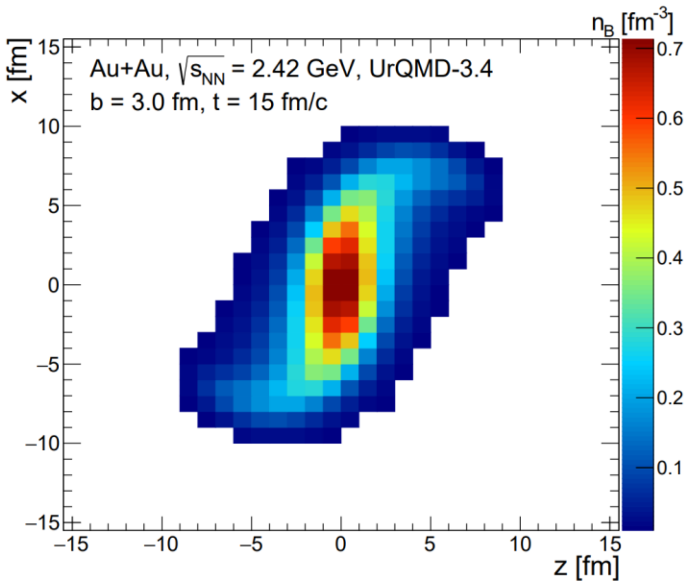

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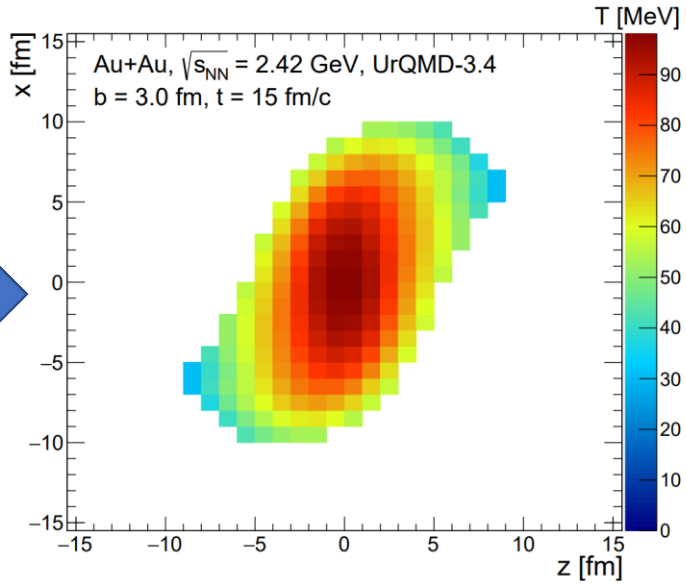

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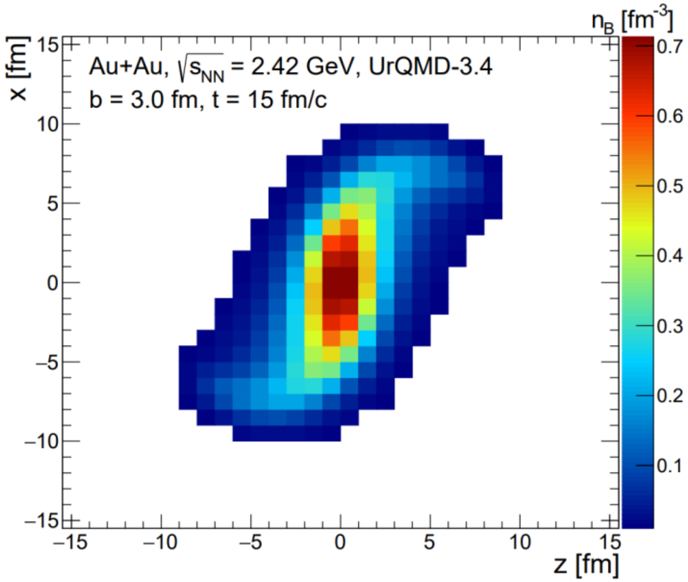

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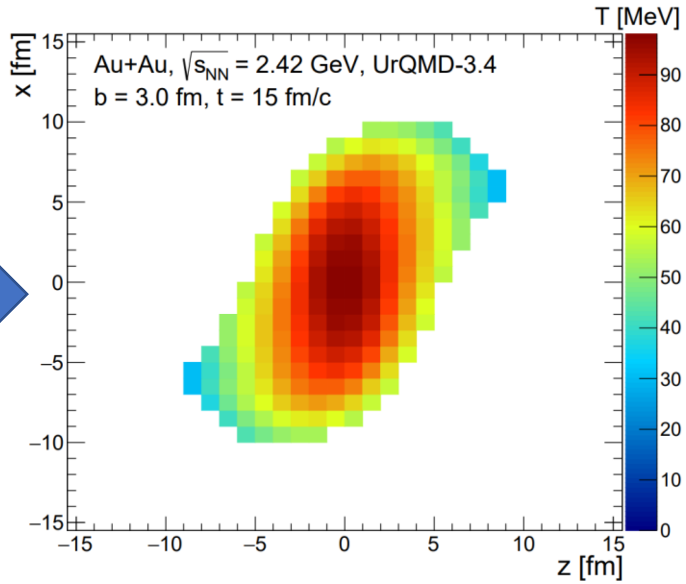

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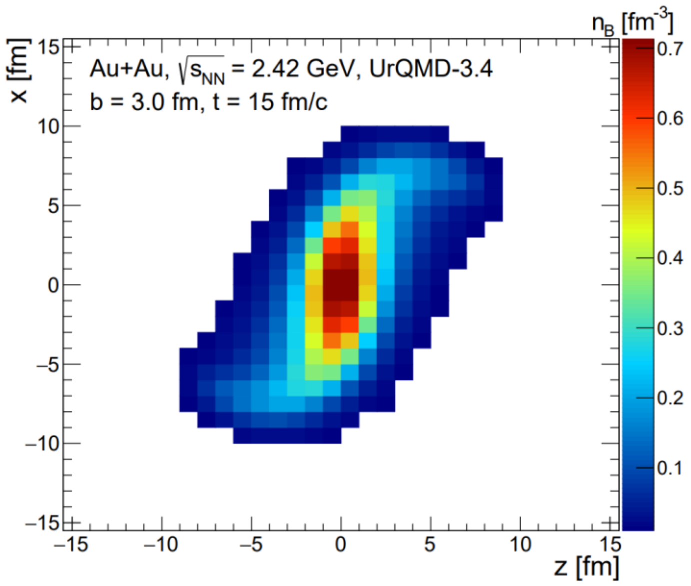

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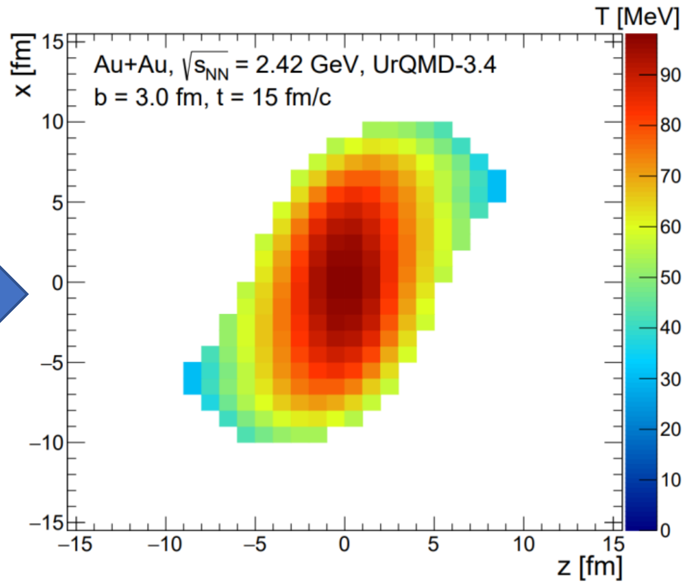

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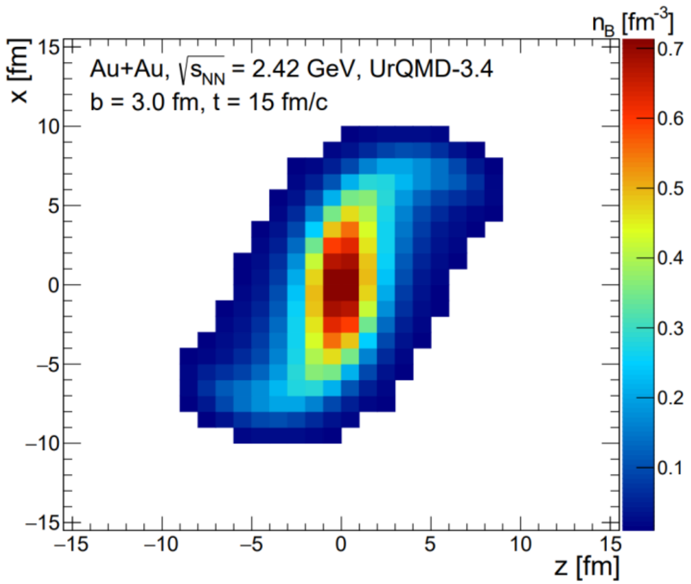
7. Finally, polarization and other observables can be calculated



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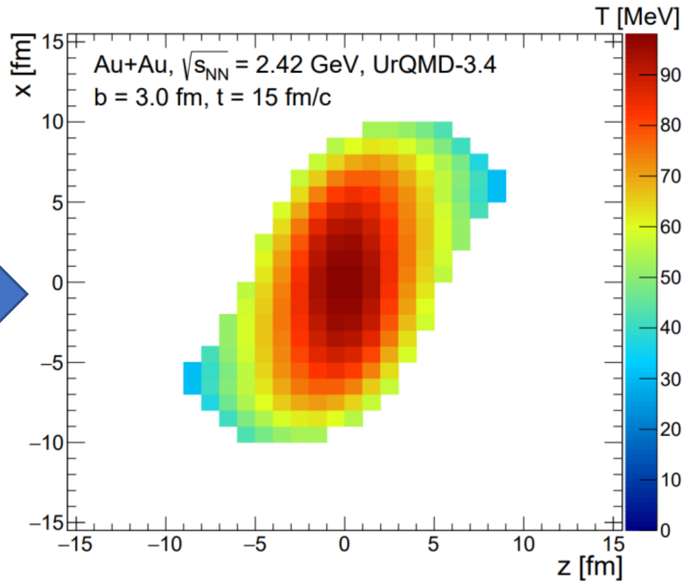

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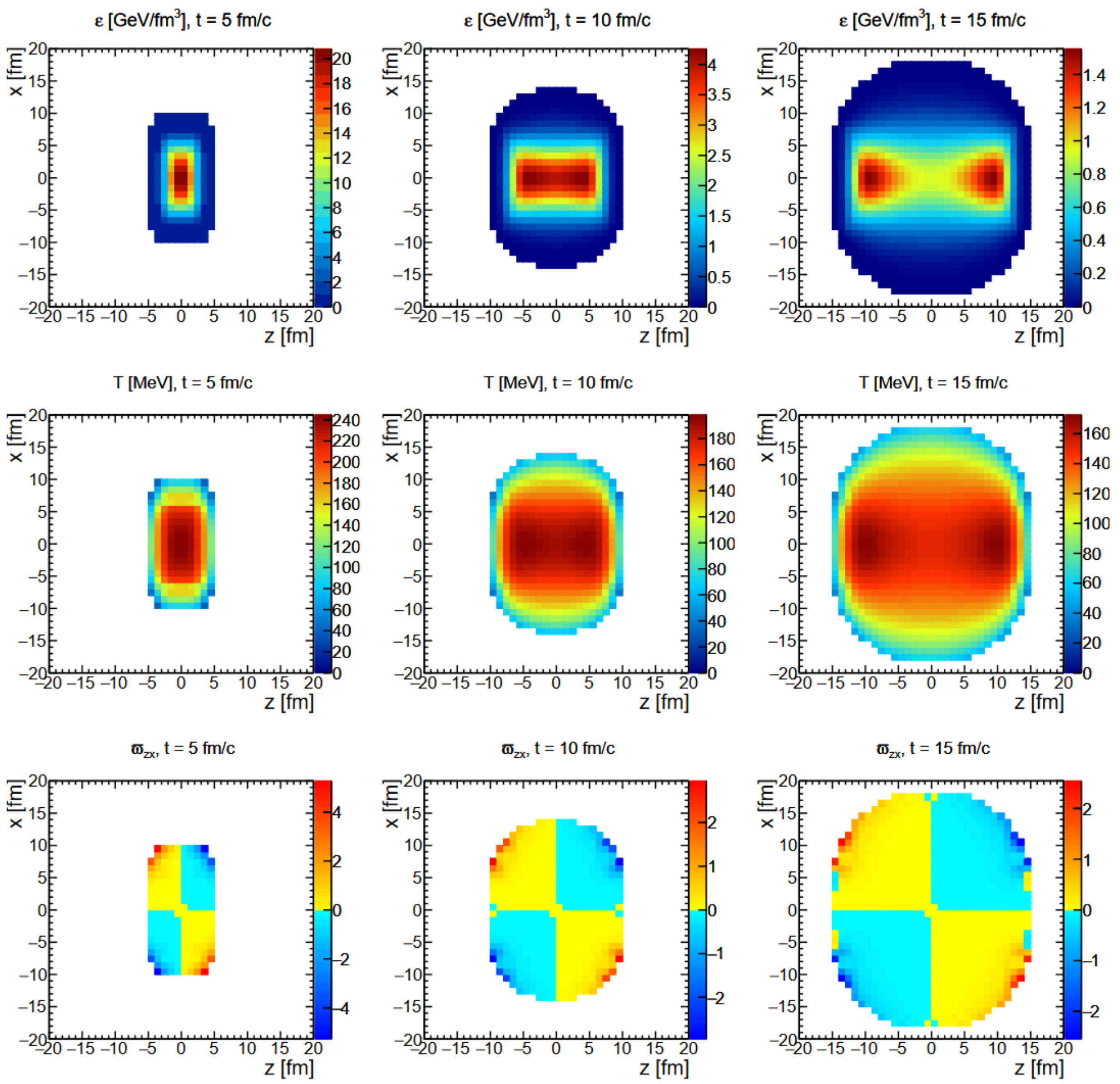

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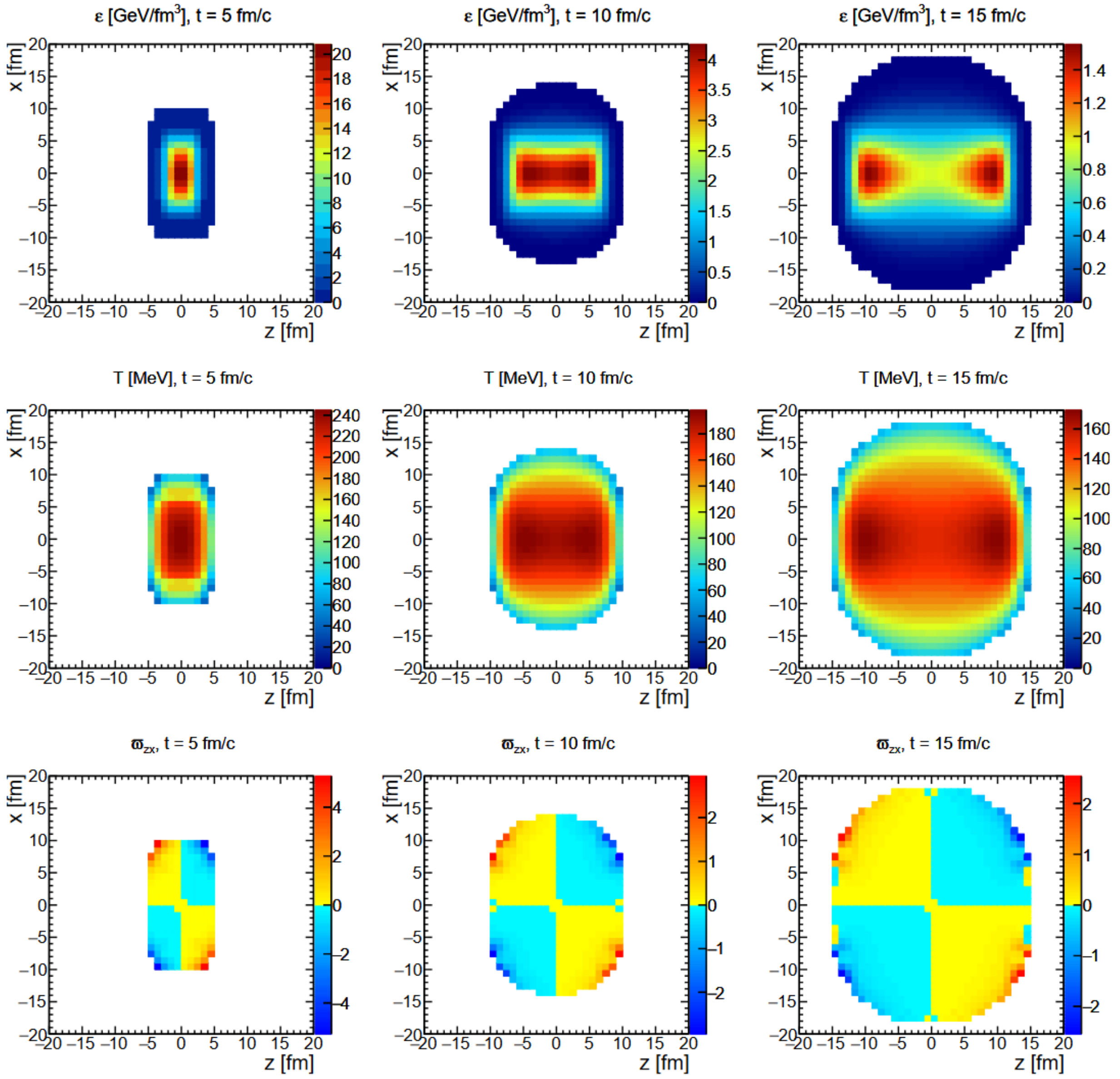

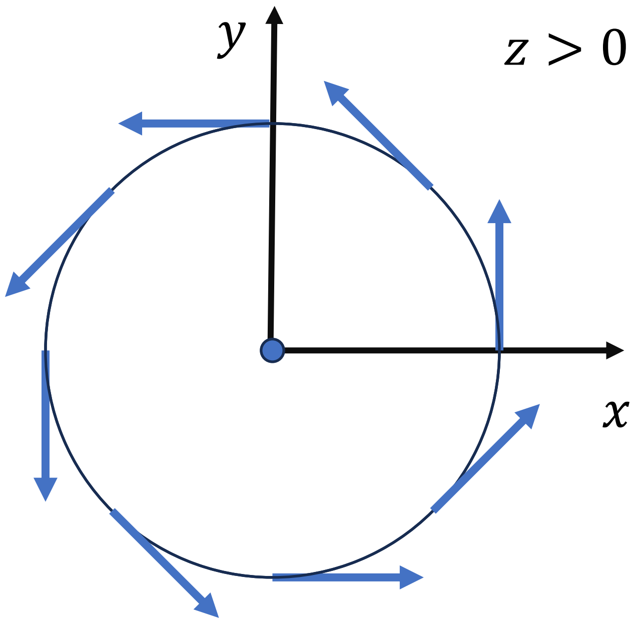

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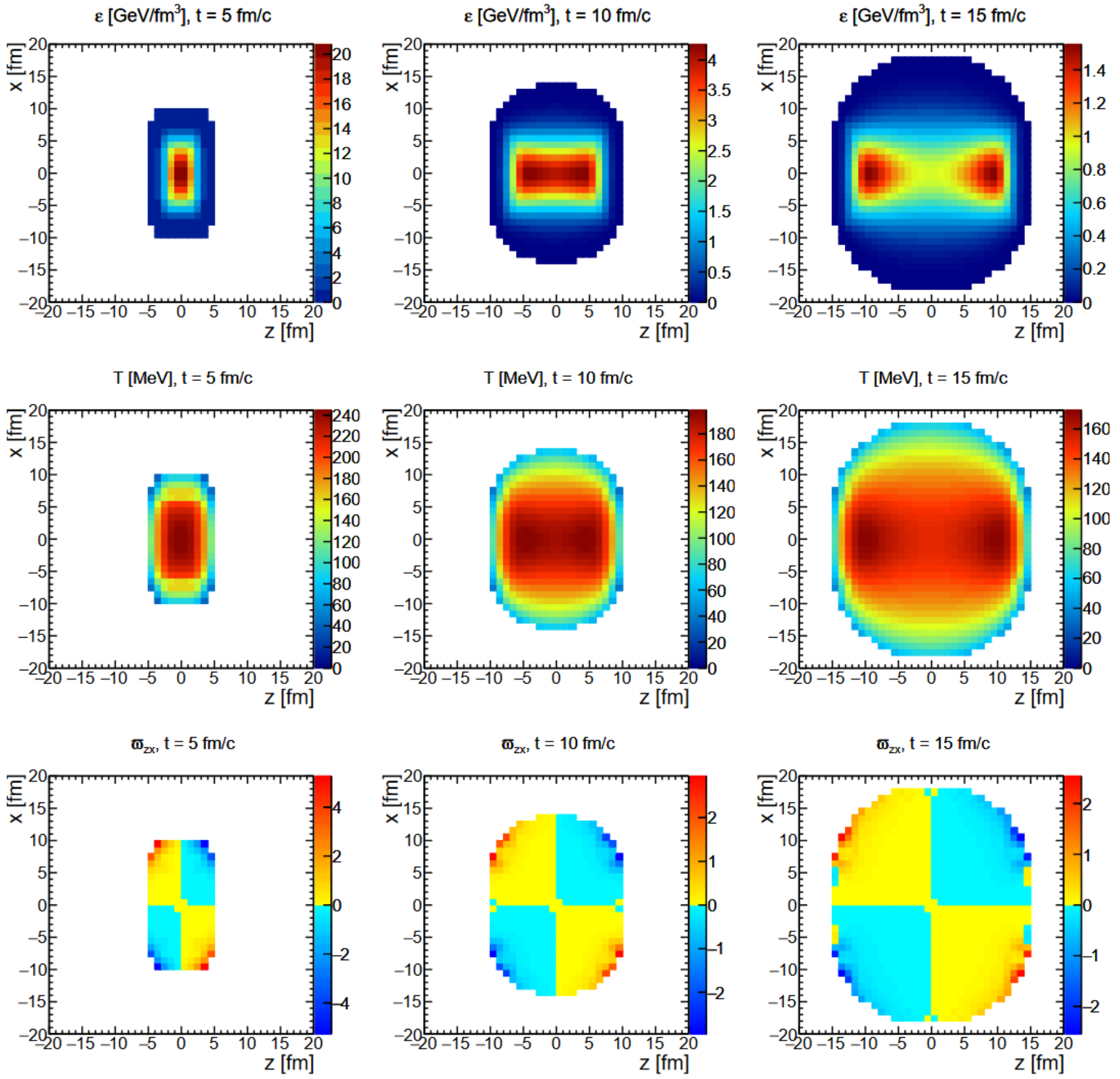

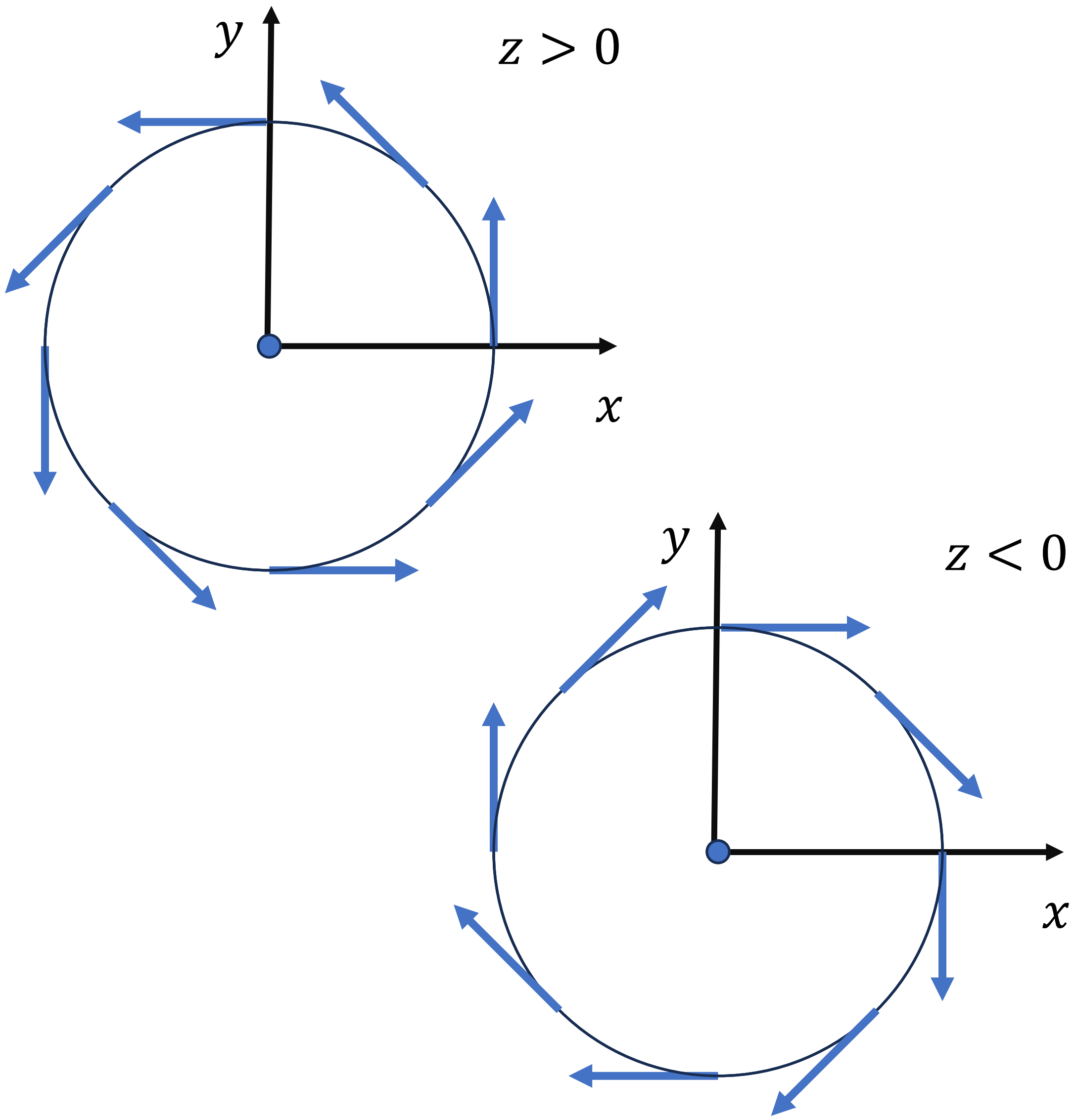

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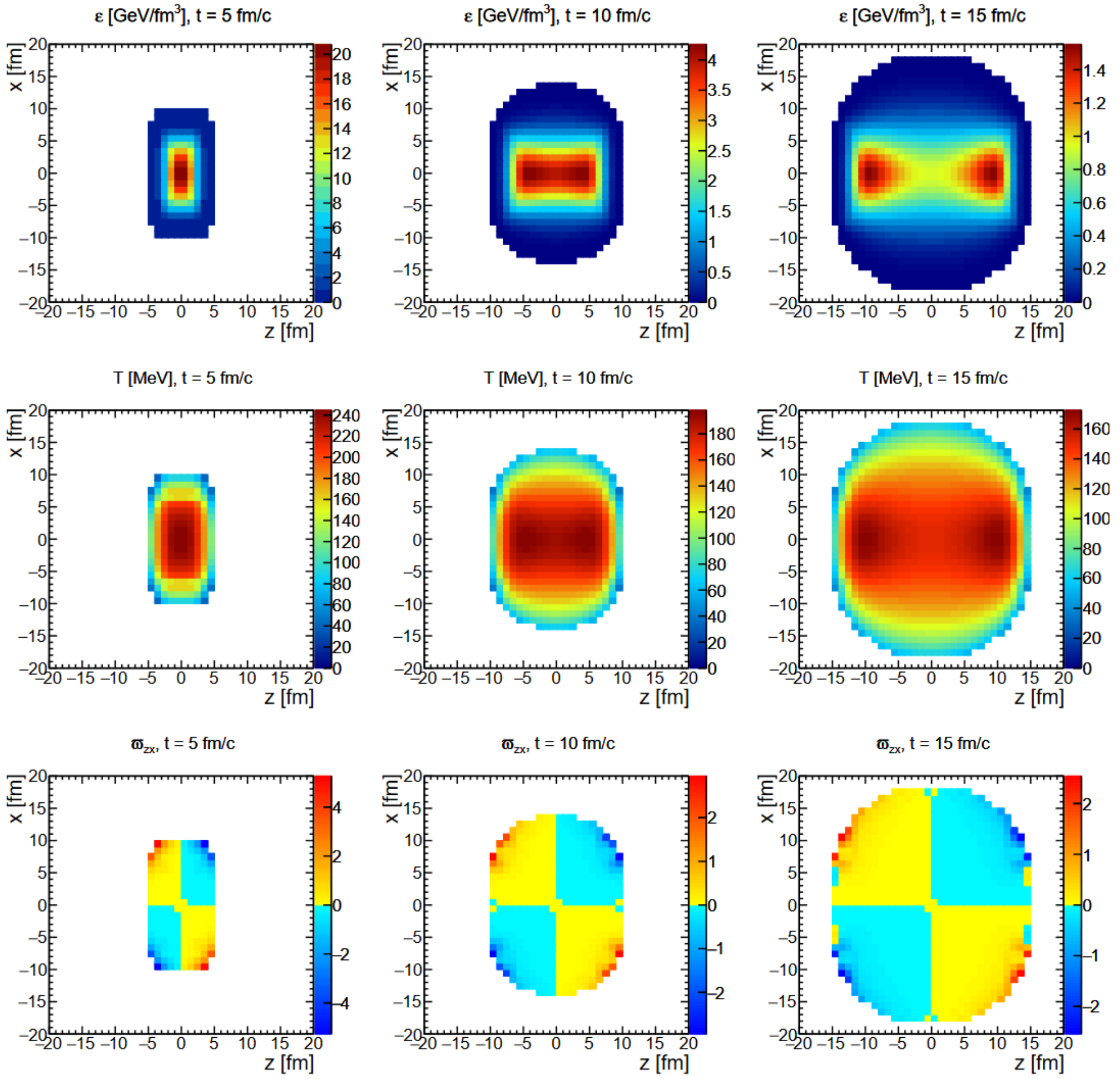
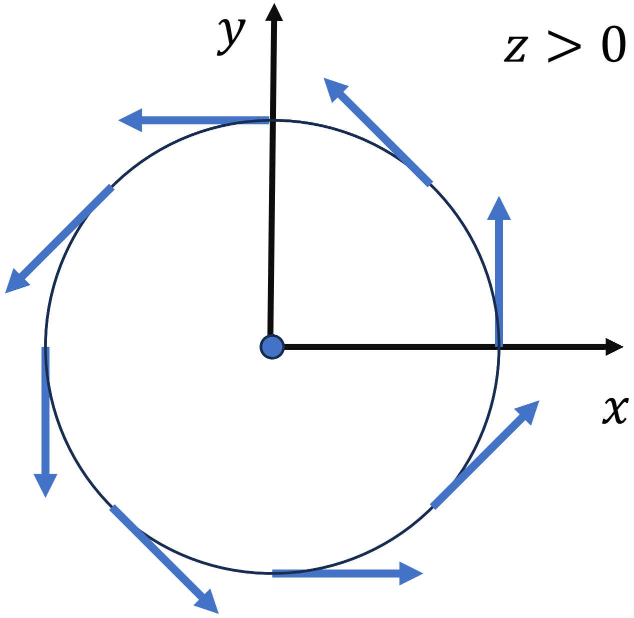
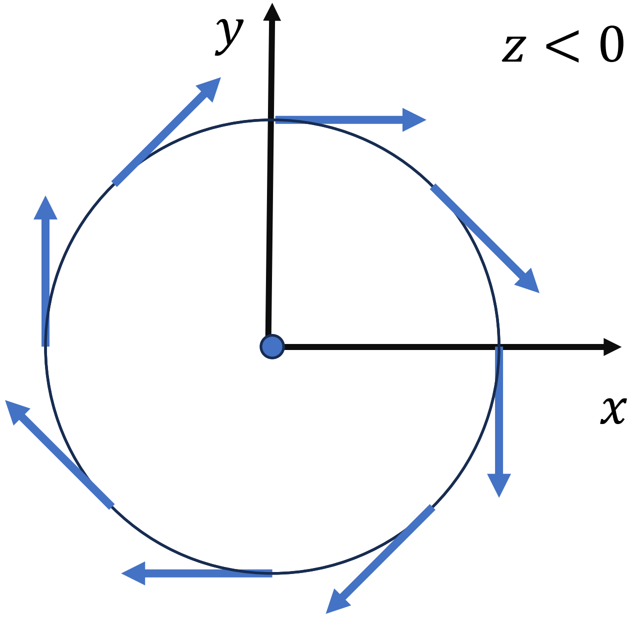


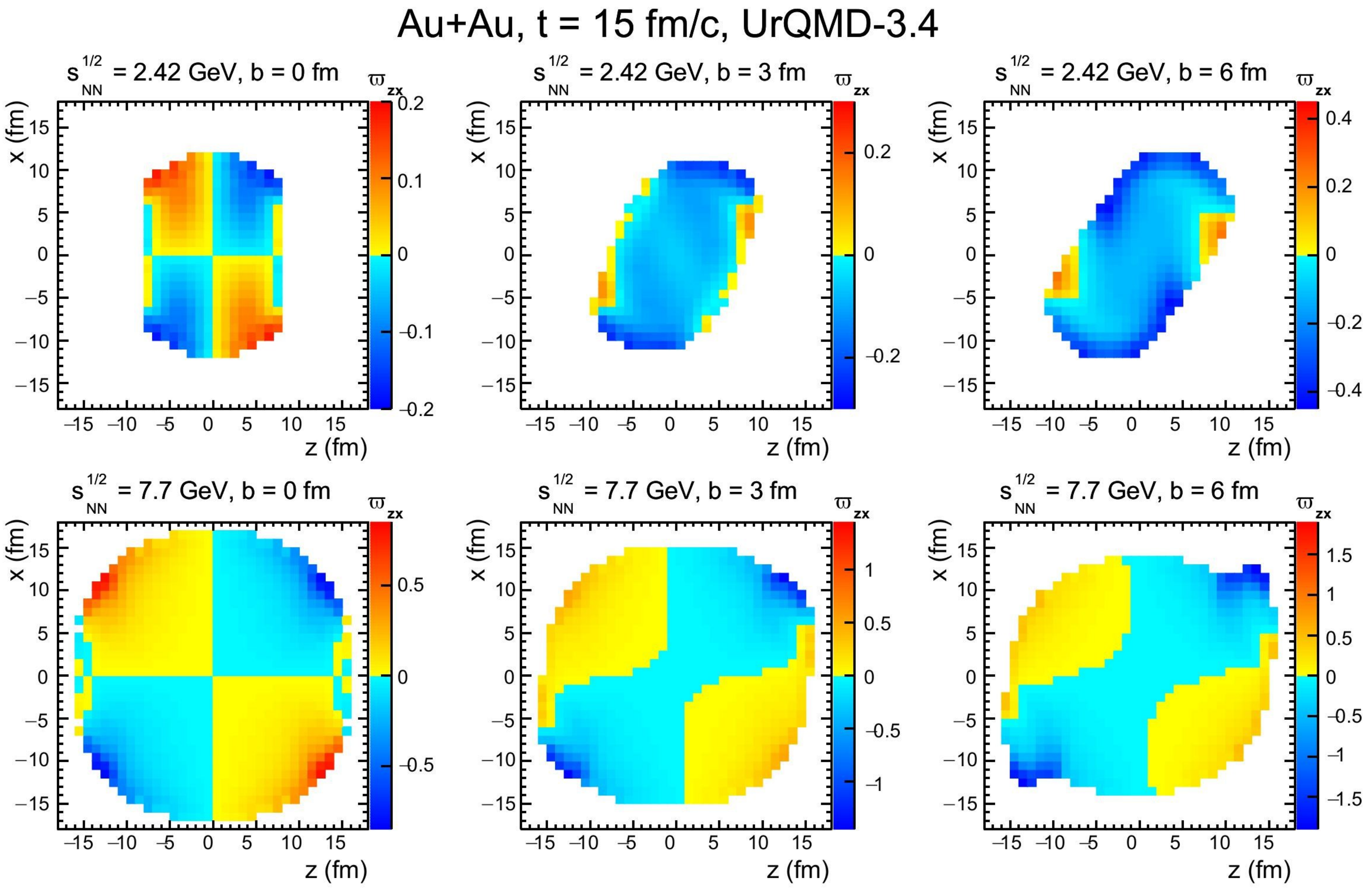


Vorticity magnitude is constant along vortex lines

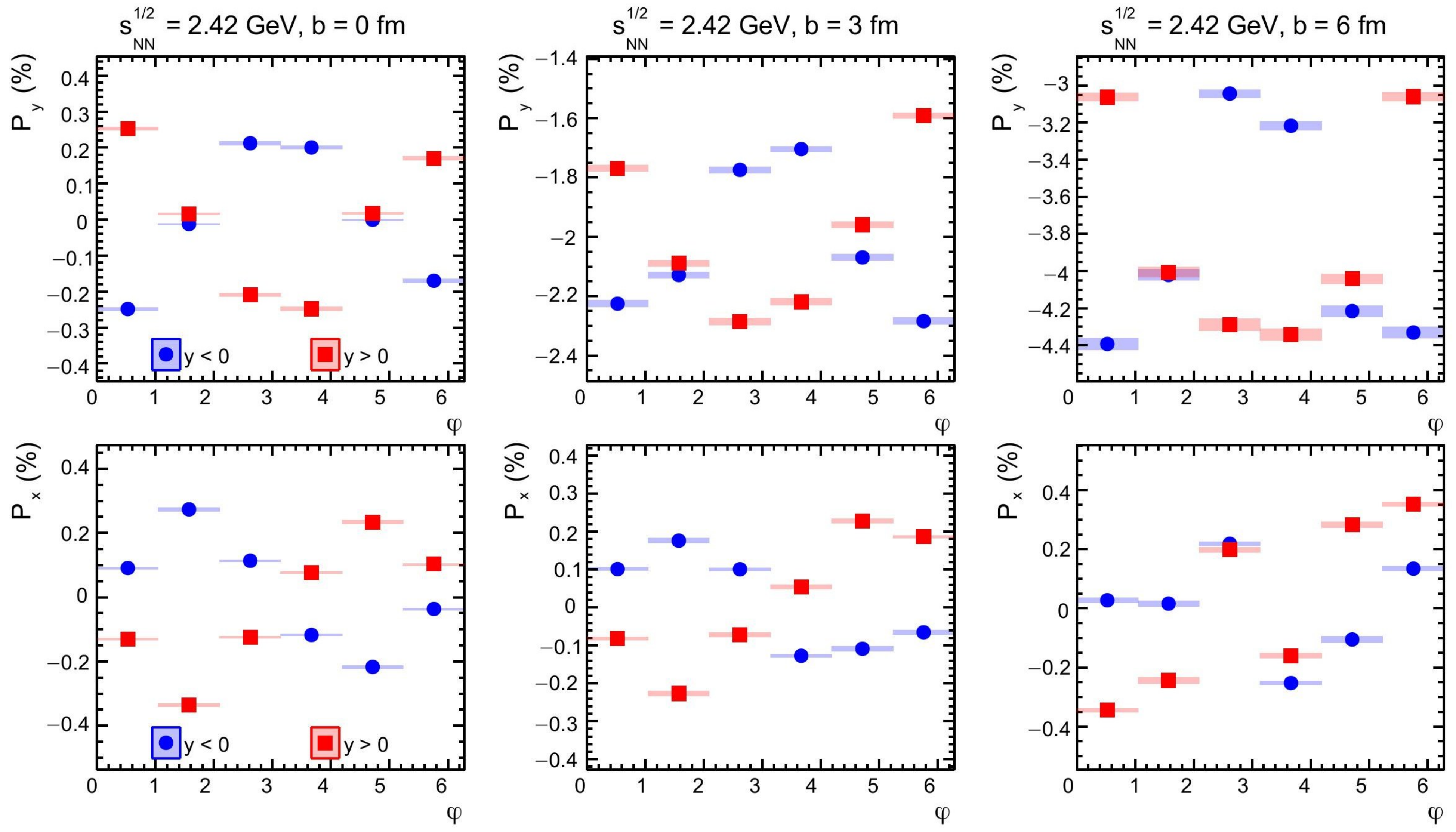
Vortex lines are circles in the transverse plane



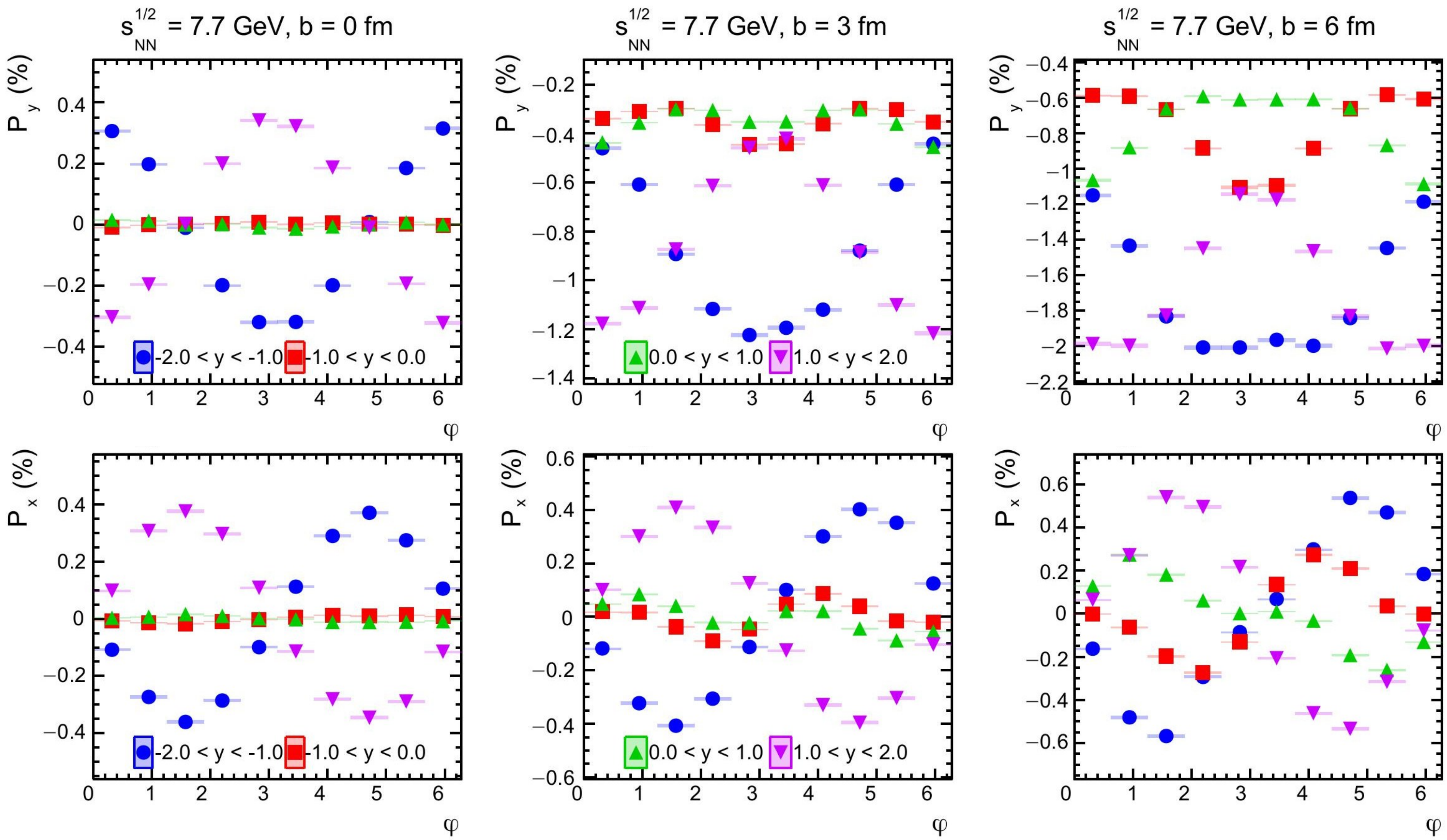
Impact parameter scan



Λ polarization in UrQMD



Λ polarization in UrQMD



Summary

- In central collisions, the thermal vorticity field has a structure which effectively resembles two vortex rings in the forward and backward hemispheres. The structure is stable in time, but the vorticity magnitude decreases due to system expansion.
- Consequently, the local polarization of Λ-hyperons exhibits oscillatory behaviour as a function of the hyperon azimuthal angle in central collisions.
- In semi-central collisions the axial symmetry is broken, resulting in a non-zero global polarization due to appearance of anisotropic flow.
- The magnitude of the local Λ polarization has nontrivial rapidity dependence.
- The measurement of the azimuthal-angle dependence of local polarization can serve as a novel probe to investigate the internal structure and evolution of the fireball in central and semi-central heavy-ion collisions.