Center vortices and the properties of the Yang-Mills vacuum

XIV International Conference on New Frontiers in Physics
17-31 July 2025, Kolymbari, Crete, Greece

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Outline

- The necessity of non-perturbative methods and the lattice.
- Lattice evidence for the importance of center vortices.
- The center-vortex approach to infrared Yang-Mills and QCD.
- Wavefunctional for the Yang-Mills vacuum peaked at center vortices.

The necessity of non-perturbative methods

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- The lattice is a very successful non-perturbative approach to QFT.

- Can be thought of as a lab to probe non-perturbative phenomena which are not easily accessible by experiments.

High-Precision Lattice QCD Confronts Experiment

C. T. H. Davies *et al.* (HPQCD and UKQCD Collaborations, MILC Collaboration, HPQCD and Fermilab Lattice Collaborations)
Phys. Rev. Lett. **92**, 022001 – Published 15 January 2004

ABSTRACT

The recently developed Symanzik-improved staggered-quark discretization allows unquenched lattice-QCD simulations with much smaller (and more realistic) quark masses than previously possible. To test this formalism, we compare experiment with a variety of nonperturbative calculations in QCD drawn from a restricted set of "gold-plated" quantities. We find agreement to within statistical and systematic errors of 3% or less. We discuss the implications for phenomenology and, in particular, for heavy-quark physics.

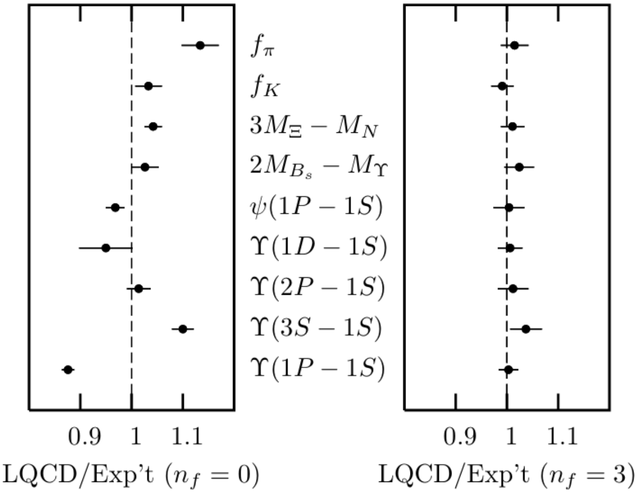

Figure: The vertical dashed lines represent the experimental results, and the points with error bars are the corresponding lattice results. Taken from PRL 92 (2004) 022001.

Model building in the continuum

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- Center vortices are particular field configurations which capture the main properties of QCD in the infrared regime.

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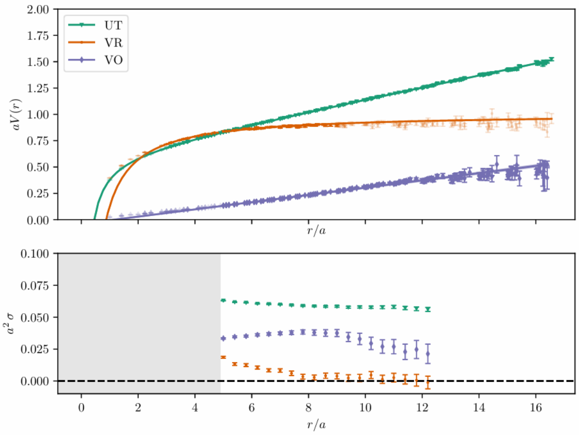

Figure: The static quark potential calculated on vortex-modified ensembles in pure Yang-Mills. From Leinweber et. al (2022).

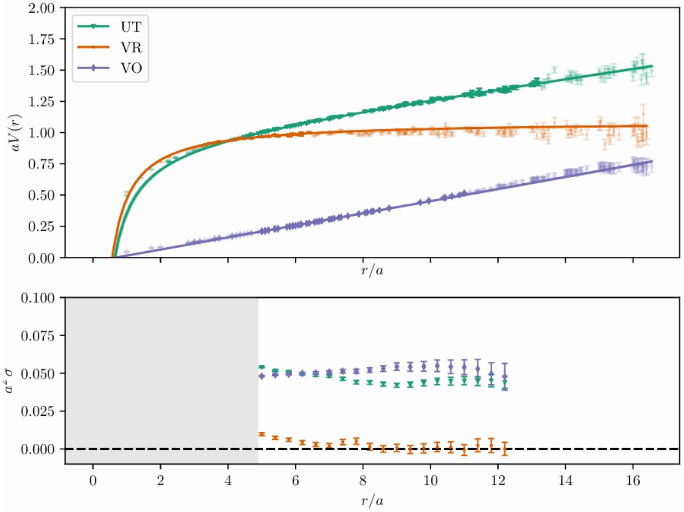

Figure: The static quark potential calculated on vortex-modified ensembles with $m_\pi = 156$ MeV. From Leinweber et. al (2022).

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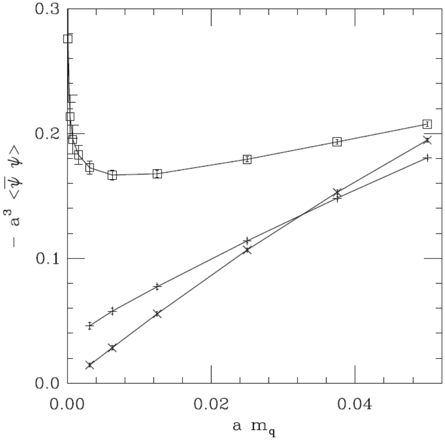

Figure: χSB order parameters $\langle\bar{\psi}\psi\rangle$ vs. lattice bare quark mass, for the unmodified (plus sign), center-projected (open square), and vortex-removed (multiplication sign) configurations in $SU(2)$ lattice gauge theory. From Alexandrou, de Forcrand and D'Elia (2000).

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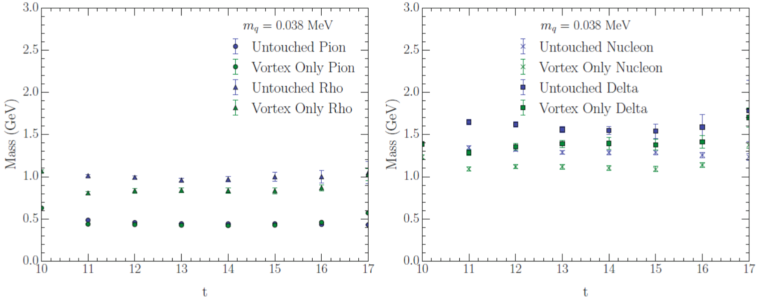


Figure: Effective masses of the low-lying mesons (left) and baryons (right) at bare quark mass $m_q = 38$ MeV, with untouched and vortex-only ensembles.

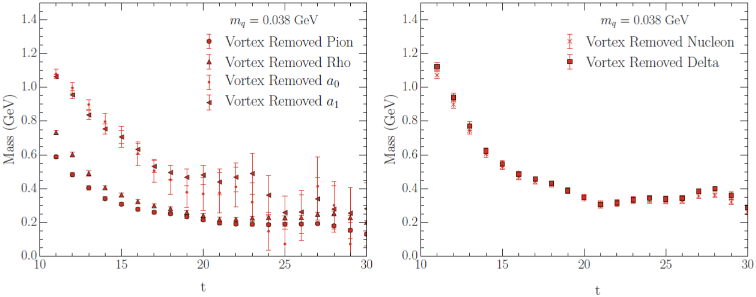


Figure: Effective masses of the low-lying mesons (left) and baryons (right) at bare quark mass $m_q = 38$ MeV, with vortex-removed ensembles.

- Center-vortex approach to probe infrared properties of Yang-Mills in the continuum: evaluate observables using ensembles of these field configurations

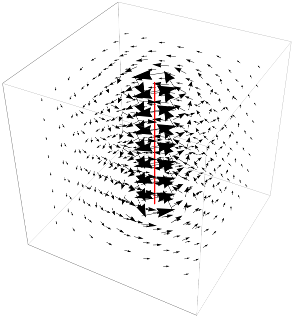
$$\langle O \rangle = \int DA\, O(A) e^{-S_{YM}} \approx \int [DA]_{\text{vortices}}\, O(A) e^{-S_{YM}} \, .$$

- Non-perturbative approach to Yang-Mills and QCD.

Vortices in the continuum limit

- The following gauge field represents a center-vortex in $SU(2)$ YM theory, in 3 Euclidean dimensions (Reinhardt, Engelhardt (1999))

$$A_\mu = \partial_\mu \varphi \frac{\sigma_3}{2} \, .$$



- Why is it a center-vortex? Because $W_C[A_\mu] = (-1)^{L(C,l)}$, where l is the vortex guiding-center.

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- In any dimension,

$$W_C[A_\mu^{\mathrm{vortex}}] = z^{L(C,\Sigma)} \; ,$$

where Σ is the vortex core (or guiding center), which is a point, closed loop, closed surface in $2d, 3d, 4d$ respectively. $L(C,\Sigma)$ is the linking number between C and Σ.

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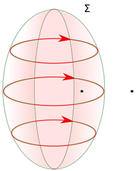

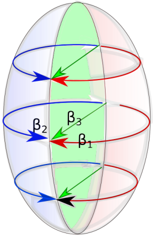

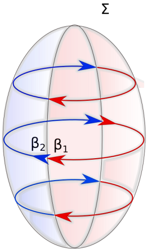

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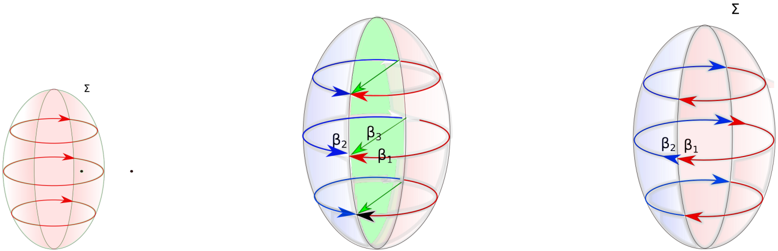

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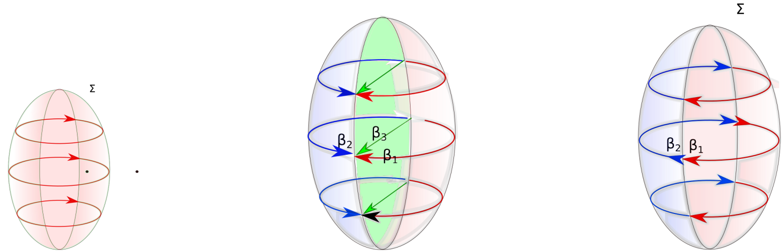

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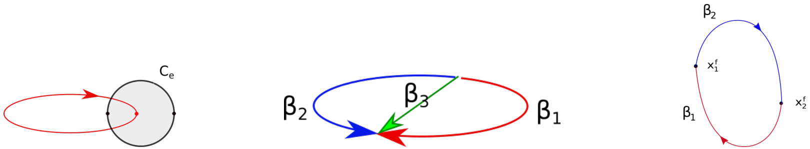

4d Mixed ensembles in the wavefunctional formalism

- Elementary center-vortex loops carrying fundamental magnetic weights $\beta_1, \ldots, \beta_N$, with N-matching: $a = 2\pi\beta \cdot T\, \partial_i \chi + \ldots$

$$\Psi(A) = \sum_{\{\gamma\}} \psi_{\{\gamma\}}\, \delta(A - a(\{\gamma\})) \quad , \quad A_i(\mathbf{x}) \quad , \quad \mathbf{x} \in \mathbb{R}^3 \, .$$

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- Ensemble integration $\to$ effective field representation ($E = \nabla \times \Lambda$)

$$\tilde{\Psi}(E) = \int D\Phi\, e^{-S[\Phi,\Lambda]} \, , |D(\Lambda)\Phi|^2 + m^2 \mathrm{Tr}\, \Phi^\dagger \Phi + \lambda \mathrm{Tr}\, (\Phi^\dagger \Phi)^2 + \det \Phi + \mathrm{c.c.}$$

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- An area law compatible with asymptotic Casimir law was obtained, i.e.

$$\sigma_k = \sigma k (N - k) .$$

Center vortices and topological charge

- The Pontryagin index of a gauge field is

$$Q = \frac{1}{16\pi^2} \int d^4x \mathrm{Tr}(F_{\mu\nu} \tilde{F}^{\mu\nu}) \ .$$

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- Topological susceptibility of the vacuum:

$$\chi = \frac{\langle Q^2 \rangle}{V} \, .$$

- Related to the mass of the η' boson by means of the Witten-Veneziano mass formula

$$m_{\eta'}^2 = \frac{2N_f}{f_\pi^2}\chi \, .$$

- Center-vortex configurations generate topological charge when they contain intersections or twisting

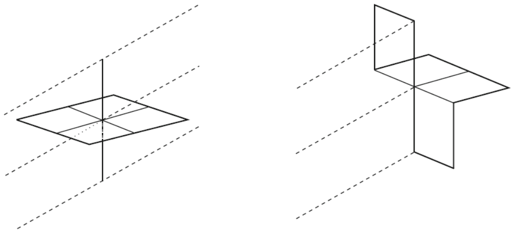

Figure: Intersection points (left) and writhing points (right). From Reinhardt (2002).

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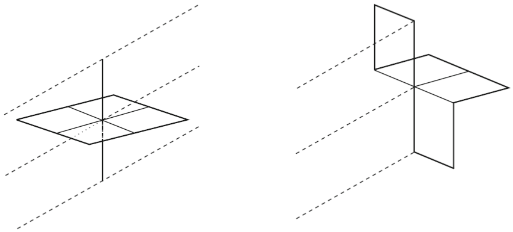

Figure: Intersection points (left) and writhing points (right). From Reinhardt (2002).

- The topological charge calculated with vortex-removed ensembles is zero (Forcrand and D'Elia (1999)).
- Ongoing project: evaluate χ with the wavefunctional peaked at center vortices.

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- It is compatible with an area law for the Wilson loop.
- Work in progress: evaluation of the topological susceptibility and of the 't Hooft loop.

Thank you!