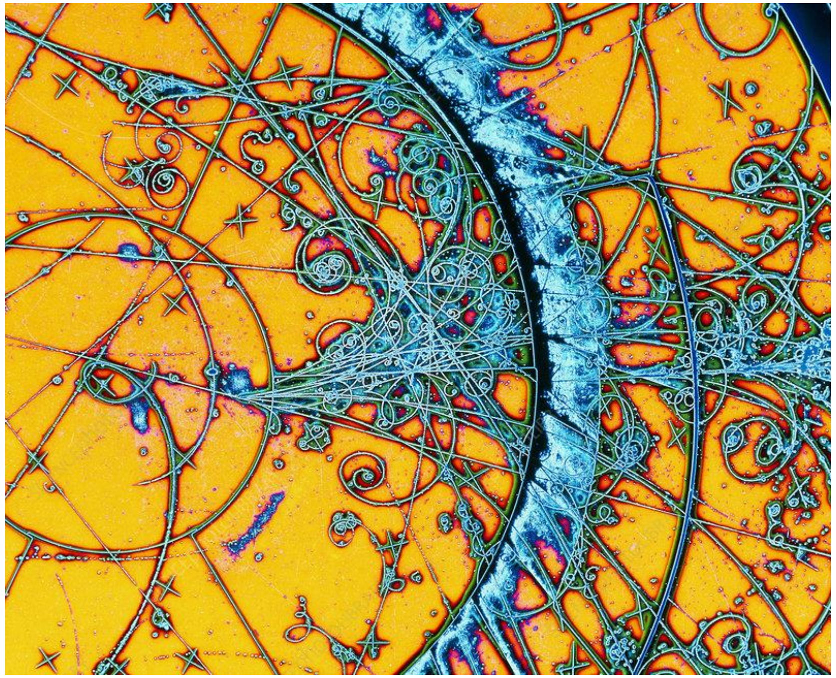

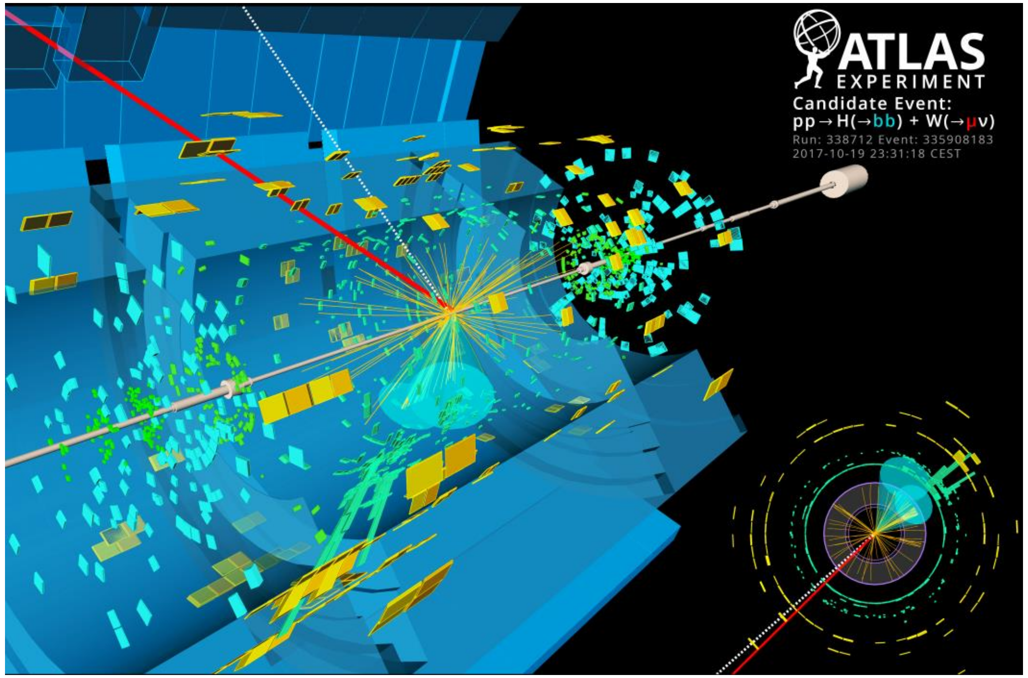
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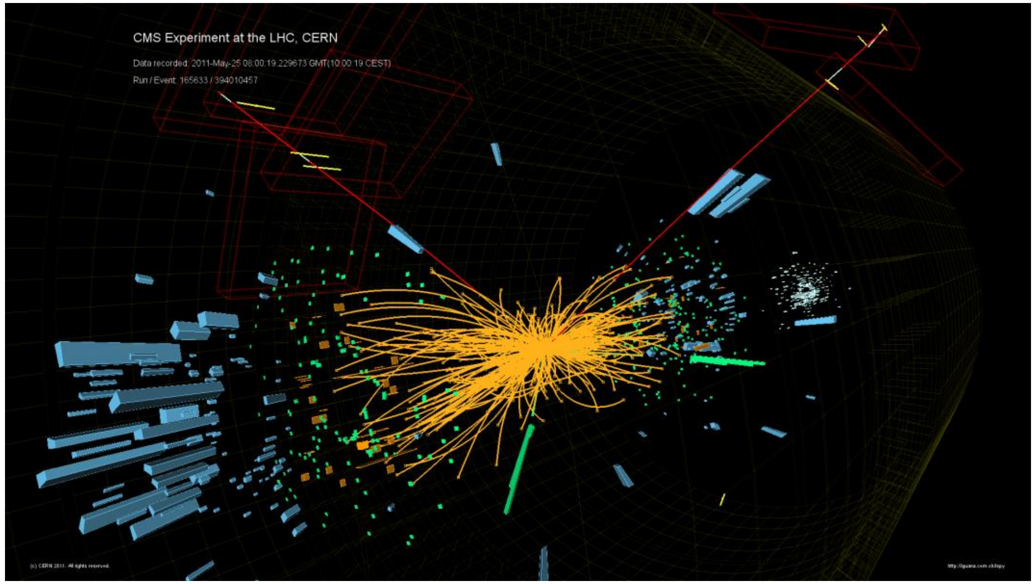
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Signals, Noise and Signal processing in Particle Detectors 3/5

Academic Training Lectures

Werner Riegler, CERN, werner.riegler@cern.ch

Oct. 14-18 2024, Council Chamber

Signals, Noise and Signal processing in Particle Detectors

Mon. 14 Oct.

Lecture 1:

- Recap
- Simulation of resistive elements
- Theorem extensions
- Radio signals for particle detection

Tue. 15 Oct.

Lecture 2:

- Electron avalanches
- Electron-Hole avalanches
- APDs (Avalanche Photo Diodes)
- LGADs (Low Gain Avalanche Diodes)
- SiPMs (Silicon Photo Multipliers)

Wed. 16 Oct.

Lecture 3:

- Linear Signal processing
- Noise

Thu. 17 Oct.

Lecture 4:

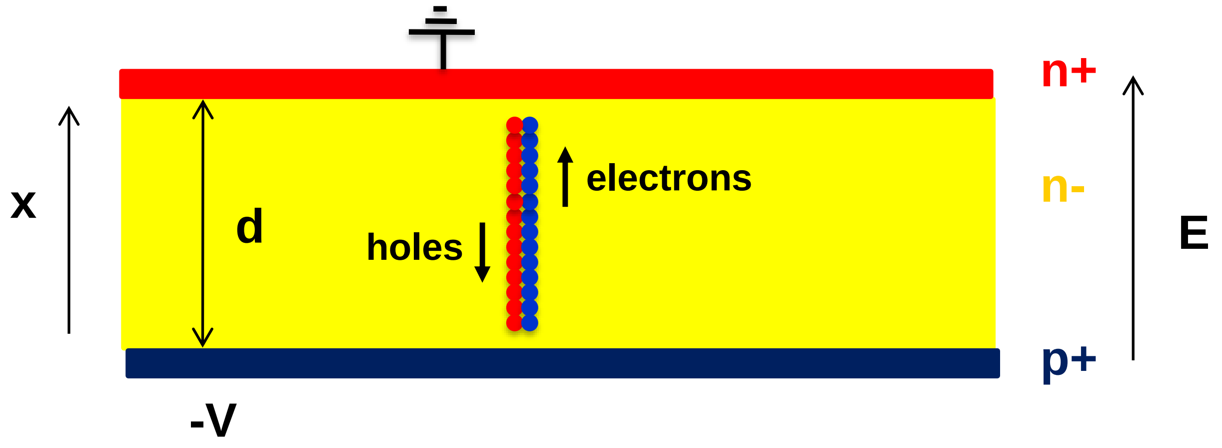
- Optimum Filters
- Sampling Theorem
- Applications

Fri. 18 Oct.

Lecture 5:

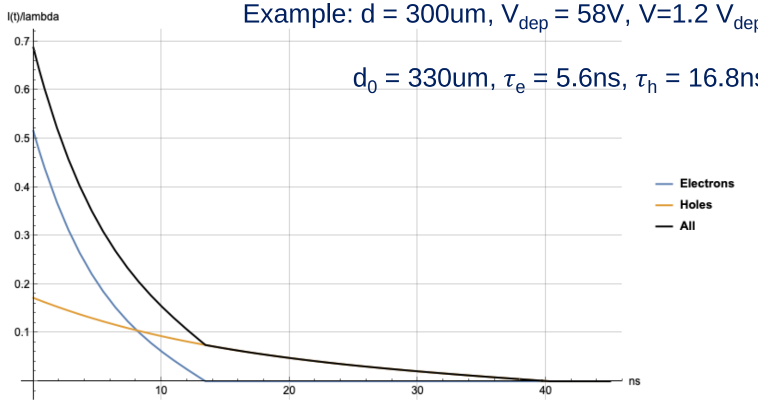
- Overflow, wrap-up and Q&A session

Intrinsic time resolution of silicon sensors



Example: d = 300um, V_{dep} = 58V, V=1.2 V_{dep} = 68V

d_0 = 330um, τ_e = 5.6ns, τ_h = 16.8ns



In silicon sensors the signal edge is instantaneous (i.e. sub ps level)

- acceleration of electrons to 10^7cm/s in vacuum is 0.14ps
- passage of the particle through a 50um sensor takes 0.16ps

In Wire Chambers the electrons first have to move to the wires before an avalanche at the wire leads to an appreciable signal
→ intrinsic resolution limit.

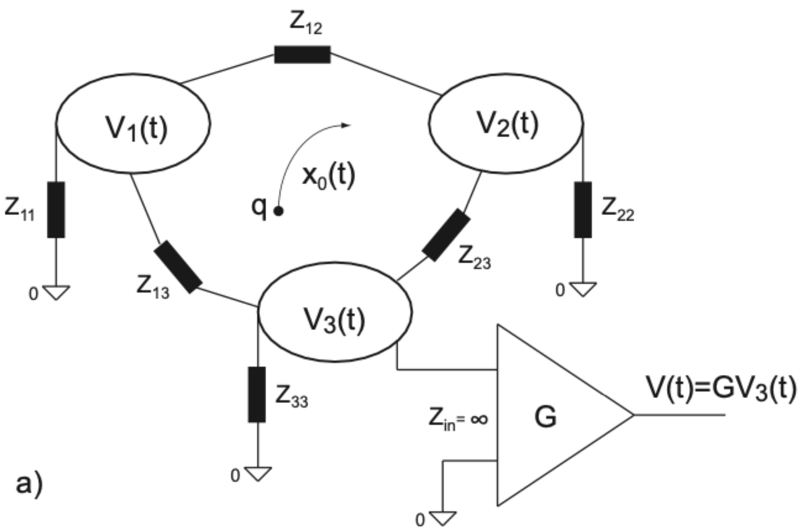
In RPCs and SiPMs the avalanche starts instantly, but it still takes some time until the signal reaches the threshold
→ intrinsic resolution limit.

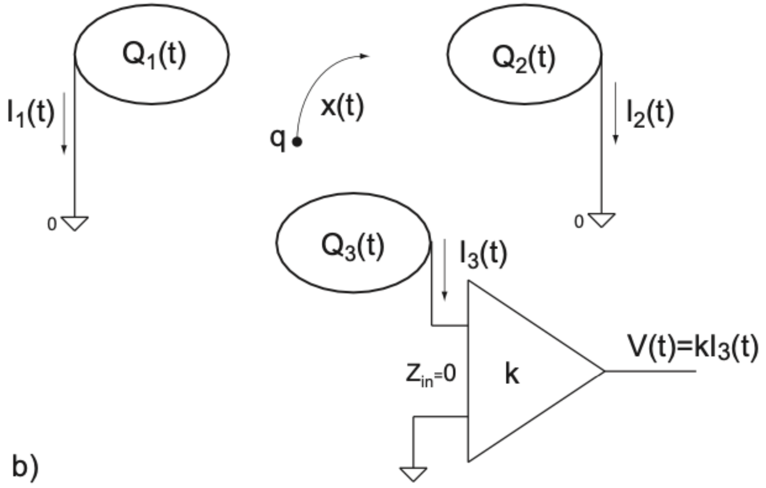
→ The intrinsic time resolution of a silicon sensor is infinite (sub ps).

→ The time resolution in a planar silicon sensors without gain/LGADs is a question of signal/noise/electronics and specifically the Landau fluctuations within the electronics integration time !

Processing of the detector signal

An amplifier is traditionally considered a device that produces an amplified copy of the input signal.

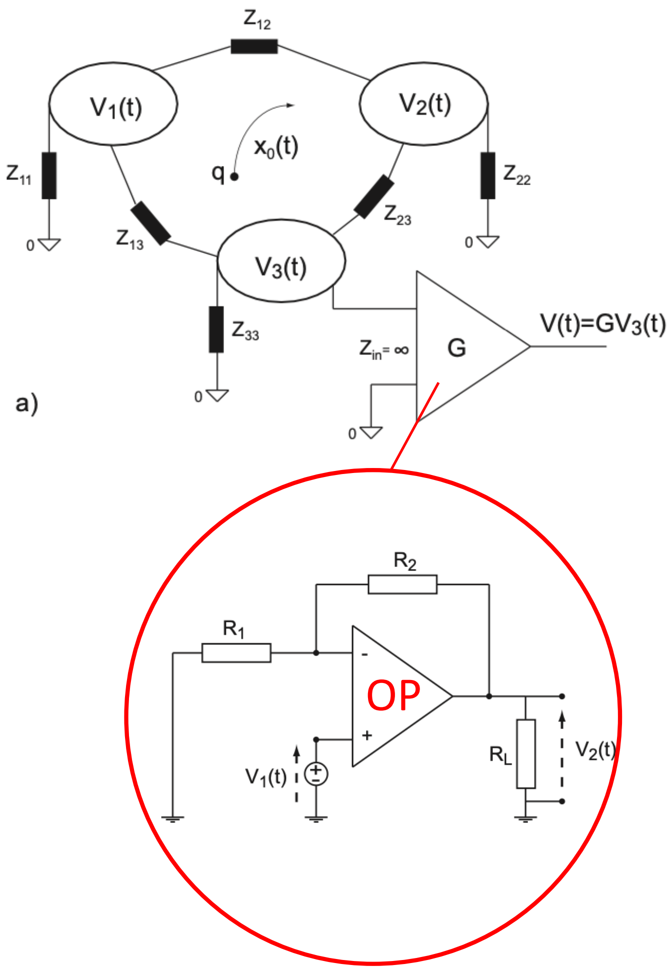




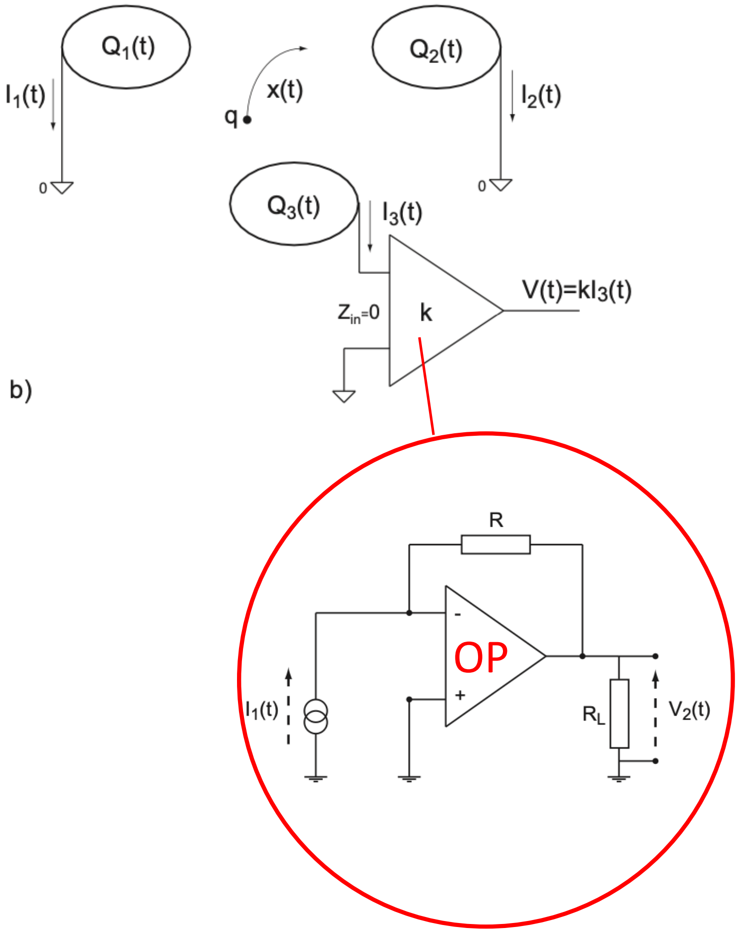
An ideal voltage amplifier, that produces an output voltage $v_{out}(t) = G \times v_{in}(t)$, where G is the (dimensionless) voltage-gain of the amplifier. The input impedance of the amplifier is infinite.

An ideal current amplifier, that produces an output voltage signal $v(t) = ki_3(t)$. The 'gain' k has dimension of Ω and it would be more precise to call this device a current to voltage converter or a transimpedance amplifier. The input impedance of the device is zero.

Processing of the detector signal



$G = V_2/V_1 = 1 + R_2/R_1$



$k = V_2/I_1 = -R$

An OPerational amplifier is a device with
- very high voltage gain,
- very high input impedance
- very high bandwidth,

ideally all being infinite.

Processing of the detector signal

In most applications we are not interested in an exact copy of the input signal.

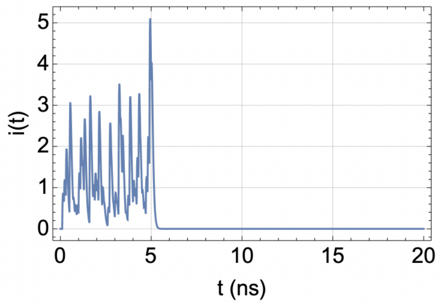
For applications where charge measurement is required one prefers long integration times (slower amplifiers) in order to integrate a large fraction of the detector signal.

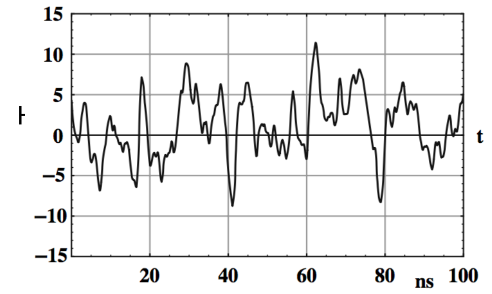
For timing purposes one typically wants fast amplifiers to reduce time walk and jitter effects.

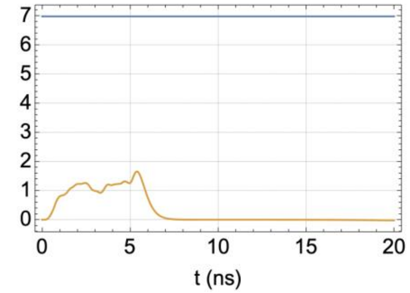
Many detector noise sources, like thermal noise, are ‘white’, meaning that a bandwidth limit will reduce the noise proportionally.
For a given detector signal there will be an optimum bandwidth limit that maximizes the signal to noise ratio.

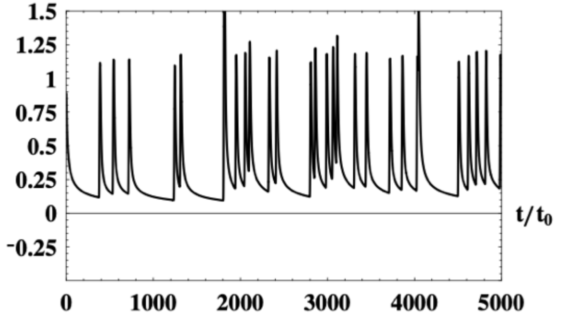
For high rate applications, signal tail cancelation and baseline restoration are important issues in order to avoid signal pileup and baseline fluctuations.

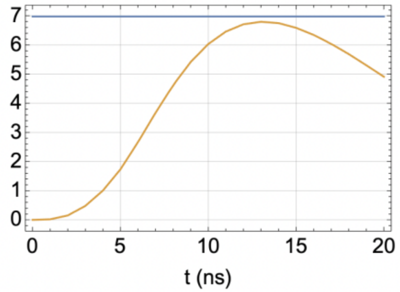
By limiting the bandwidth to the essential frequencies, the detector will also be less sensitive to external noise sources (pick-up …).

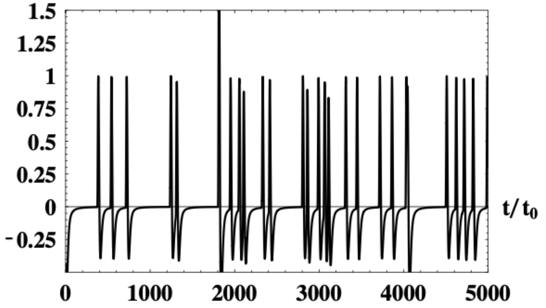










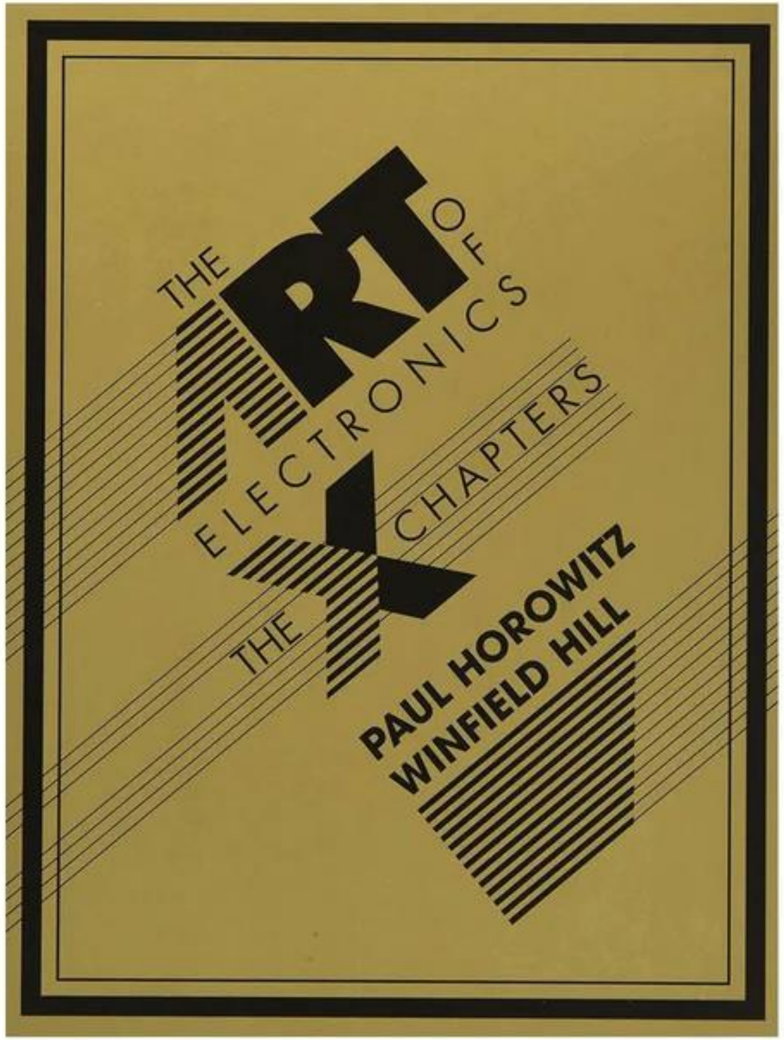


Processing of the detector signal

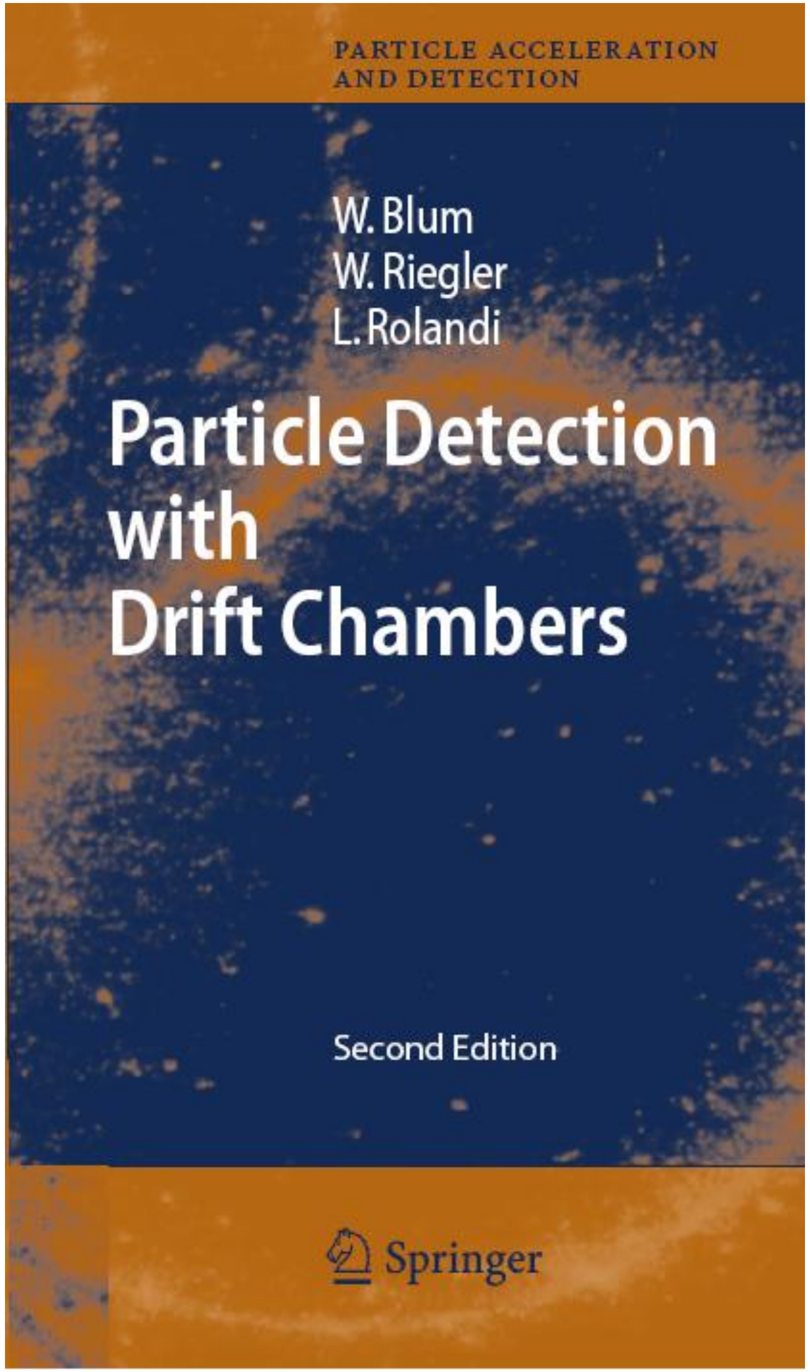
The art and science of electronics is a universe of its own.

The idea of the next two lectures is to collect the concepts that allow us to arrive at the electronics specification for a particular detector application.

The numerous ways of realizing the actual circuits are not discussed.



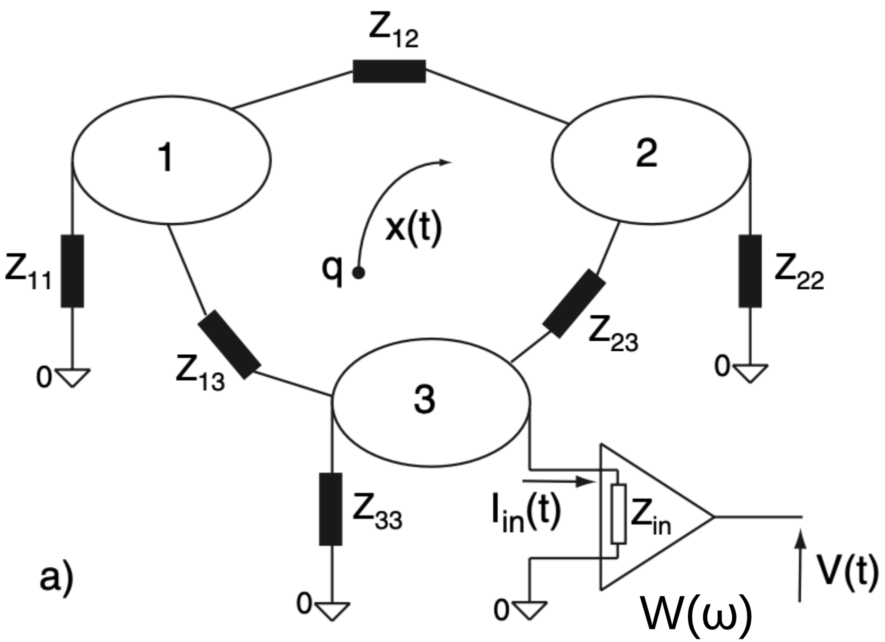
More on signal theorems, readout electronics etc. can be found in this book →

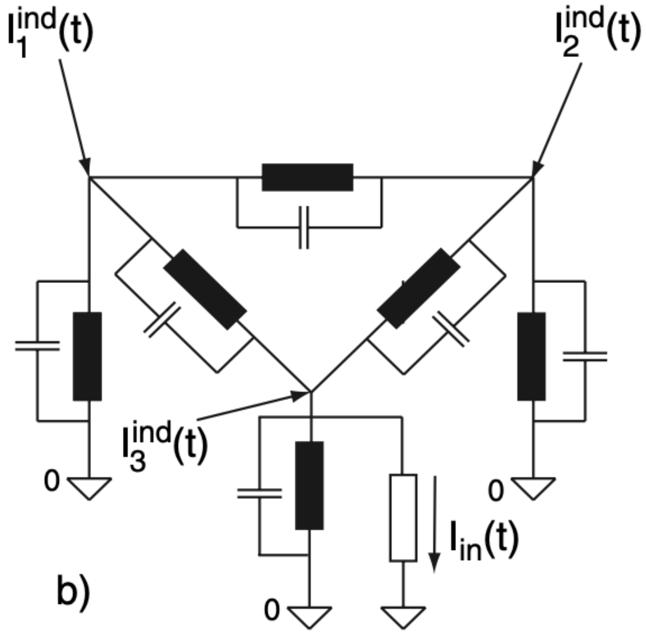


Processing of the detector Signal

We apply a given transfer function to the signal that maximizes the quality of the output that we are looking for (time, charge etc.).

We therefore want to specify the transfer function W(ω) and input impedance $Z_{in}(\omega)$ to optimize our measurement.





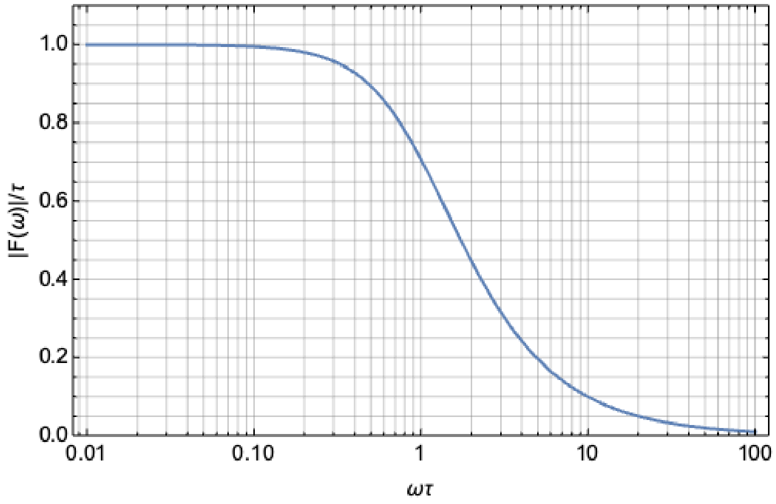
Linear Signal Processing

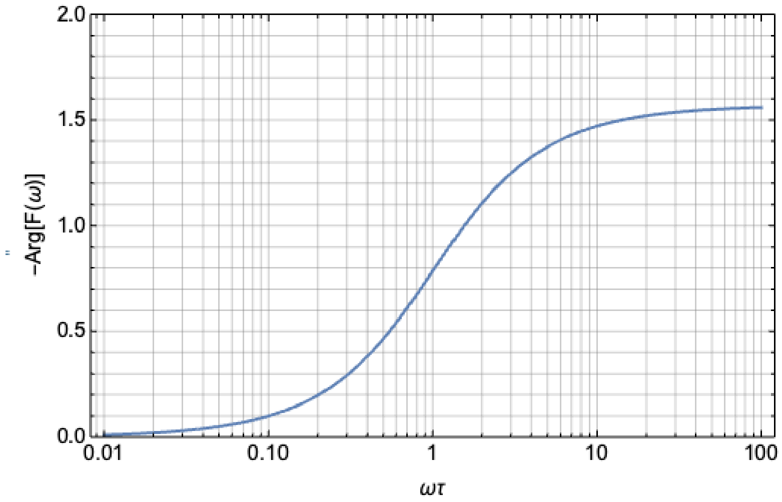
The Fourier Transform of a time dependent signal f(t) is defined as

$$\mathcal{F}[f(t)] = F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t} dt \qquad \omega = 2\pi f \qquad \mathcal{F}^{-1}[F(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega$$

It expresses the signal as a superposition of sinusoidal waves of frequency f with amplitude |F(2π f)| and relative phases Arg[F(2π f)]. Example:

$$f(t) = e^{-t/\tau} \quad t > 0 \qquad F(\omega) = \frac{\tau}{1 + i\omega\tau}$$





Linear Signal Processing

a) Addition

$$\mathcal{F}\left[a f(t) + b g(t)\right] = a F(\omega) + b G(\omega)$$

b) Convolution

$$\mathcal{F}\left[\int_{-\infty}^{\infty} f(t-t')g(t')dt'\right] = F(\omega)\, G(\omega)$$

c) Time differentiation

$$\mathcal{F}\left[f^{(n)}(t)\right] = (i\omega)^n F(\omega)$$

d) Time integration

$$\mathcal{F}\left[\int_{-\infty}^{t} f(t')dt'\right] = \frac{1}{i\omega} F(\omega)$$

e) Time shift

$$\mathcal{F}\left[f(t-t_0)\right] = F(\omega)e^{-i\omega t_0}$$

f) Time scaling

$$\mathcal{F}\left[f(at)\right] = \frac{1}{a} F\left(\frac{\omega}{a}\right)$$

g) Damping

$$\mathcal{F}\left[e^{-i\omega_0 t} f(t)\right] = F(\omega + \omega_0)$$

h) Multiplication

$$\mathcal{F}\left[t^n f(t)\right] = (-1)^n F^{(n)}(\omega)$$

i) Initial-value if $f(t) = 0$ for $t < 0$

$$f(0^+) = \lim_{\omega \to \infty} i\omega\, F(\omega)$$

j) Final-value

$$f(\infty) = \lim_{\omega \to 0} i\omega\, F(\omega)$$

k) Parseval's theorem

$$\int_{-\infty}^{\infty} f(t)^2 dt = \int_{-\infty}^{\infty} |F(2\pi f)|^2 df \qquad = 2\int_{0}^{\infty} |F(2\pi f)|^2 df$$

Linear Signal Processing

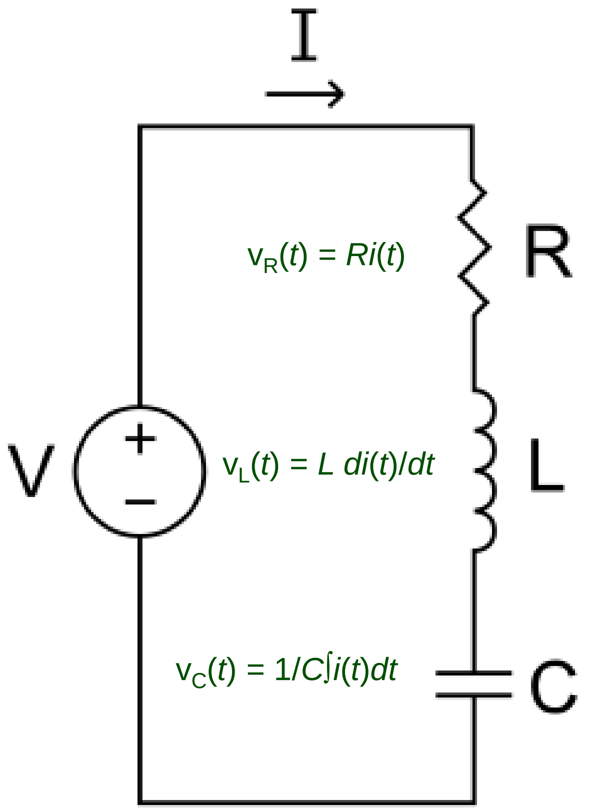
The usefulness of these transformations for electrical circuit analysis is illustrated by this example.

Voltages and currents in a circuit containing *R*, *L*, *C* elements are determined by applying Kirchhoff's Laws, stating that the sum of voltages in every loop must be zero and the sum of the currents on every node must be zero. This analysis will therefore lead to a linear differential equation for the currents in the circuit.

Writing the above relations in the Fourier domain, the circuit relations become algebraic equations.

Instead of having to solve a set of coupled differential equations in the time domain we just have to solve a set of linear algebraic equations in the Fourier domain!

In addition, due to the theorems stated above, many signal manipulations are strongly simplified when working in the Fourier domain.



$v_R(t) = Ri(t)$ $\quad V_R(\omega) = RI(\omega)$

$v_L(t) = L\, di(t)/dt$ $\quad V_L(\omega) = i\omega L I(\omega)$

$v_C(t) = 1/C \int i(t)dt$ $\quad V_C(\omega) = I(\omega)/i\omega C$

Linear Signal Processing

A linear device is defined by the following property:

If the input signals $i_1(t)$ and $i_2(t)$ result in output signals $v_1(t)$ and $v_2(t)$,
the input signal $i(t) = c_1 i_1(t) + c_2 i_2(t)$ will result in the output signal $v(t) = c_1 v_1(t) + c_2 v_2(t)$.

The input signal i(t) and output signal v(t) of a linear, time invariant and causal device are related by a differential equation of the following form

$$a_0\, v(t) + a_1 \frac{dv(t)}{dt} + \ldots + a_m \frac{d^m v(t)}{dt^m} = b_0\, i(t) + b_1 \frac{di(t)}{dt} + \ldots + b_n \frac{d^n i(t)}{dt^n}$$

where the coefficients *a* and *b* are independent of time. The term ’time-invariant’ describes the fact that the relation between input and output signal is independent of time.

For a ’causal’ system the output signal $v(t)$ is zero as long as the input signal $i(t)$ is zero.

Performing the Fourier transform this equation is transformed into an algebraic equation with the solution

$$a_0 V(\omega) + a_1 i\omega V(\omega) + \ldots + a_m (i\omega)^m V(\omega) = b_0 I(\omega) = b_1 i\omega I(\omega) + \ldots + b_n (i\omega)^n I(\omega)$$

and we have

$$V(\omega) = \frac{b_0 + b_1 i\omega + \ldots + b_n (i\omega)^n}{a_0 + a_1 i\omega + \ldots + a_m (i\omega)^m} I(\omega) = W(\omega) I(\omega)$$

Linear Signal Processing

$$V(\omega) = \frac{b_0 + b_1 i\omega + \ldots + b_n (i\omega)^n}{a_0 + a_1 i\omega + \ldots + a_m (i\omega)^m} I(\omega) = W(\omega) I(\omega)$$

The function $W(\omega)$ is called the *transfer function* of the system and we see that the transfer function of a linear time invariant system can be expressed by the ratio of two polynomials in the Fourier domain.

Separating the signal $I(\omega)$ and the transfer function $W(\omega)$ into the absolute value and the phase, indicates how the frequencies contained in the input signal are separately transformed in order to yield the output signal

$$V(\omega) = |I(\omega)||W(\omega)| \exp\left[i \arg[I(\omega)] + i \arg[W(\omega)]\right]$$

The sinusodial components of the input signal I are scaled by $|W|$ and phase shifted by $\arg[W]$.

For very high frequencies we have

$$|W(\omega)| =\propto \omega^{n-m}$$

A system where $n > m$ is un-physical because it would result in infinite amplification at infinite frequency. We can therefore state that for the transfer function of a realistic linear device we have $n \leq m$.

Linear Signal Processing

A n^{th} order polynomial has n (real and complex) roots, so the transfer function $W(\omega)$ can be expressed as

$$W(\omega) = A\,\frac{(i\omega - z_1)(i\omega - z_2)\ldots(i\omega - z_n)}{(i\omega - p_1)(i\omega - p_2)\ldots(i\omega - p_m)}$$

The roots $i\omega = z_1, z_2 \ldots$ are the zeros of $W(\omega)$ and $i\omega = p_1, p_2 \ldots$ are the poles of $W(\omega)$. In general, z_i and p_i are complex numbers, some of the roots may be repeated.

→ In the Fourier domain, the transfer function of a linear signal processing device can therefore be fully described by its poles and zeros.

A unit input signal $I(\omega) = 1$ will result in the output signal $V(\omega) = W(\omega)$. Returning to the time domain we have

$$\mathcal{F}^{-1}[1] = \delta(t) \text{ and } \mathcal{F}^{-1}[W(\omega)] = w(t).$$

The inverse Fourier transform of the transfer function is therefore the output signal for a unit delta input signal which we call the *delta response* of the system. In the time domain we therefore have

$$v(t) = \int_{-\infty}^{\infty} w(t-t')\,i(t')\,dt' \qquad\qquad V(\omega) = W(\omega)I(\omega)$$

The transfer function $W(\omega)$ and the delta response $w(t)$ are two equivalent ways of describing a linear system.

Linear Signal Processing

$$W(\omega) = A\,\frac{(i\omega - z_1)(i\omega - z_2)\ldots(i\omega - z_n)}{(i\omega - p_1)(i\omega - p_2)\ldots(i\omega - p_m)}$$

Using partial fraction expansion, $W(\omega)$ with $m \geq n$ can always be written as a sum of terms with the form $1/(i\omega - p_i)^{k_i}$, where $k_i \geq 0$ are integers and the poles are complex numbers ($p_i = a_i + ib_i$).

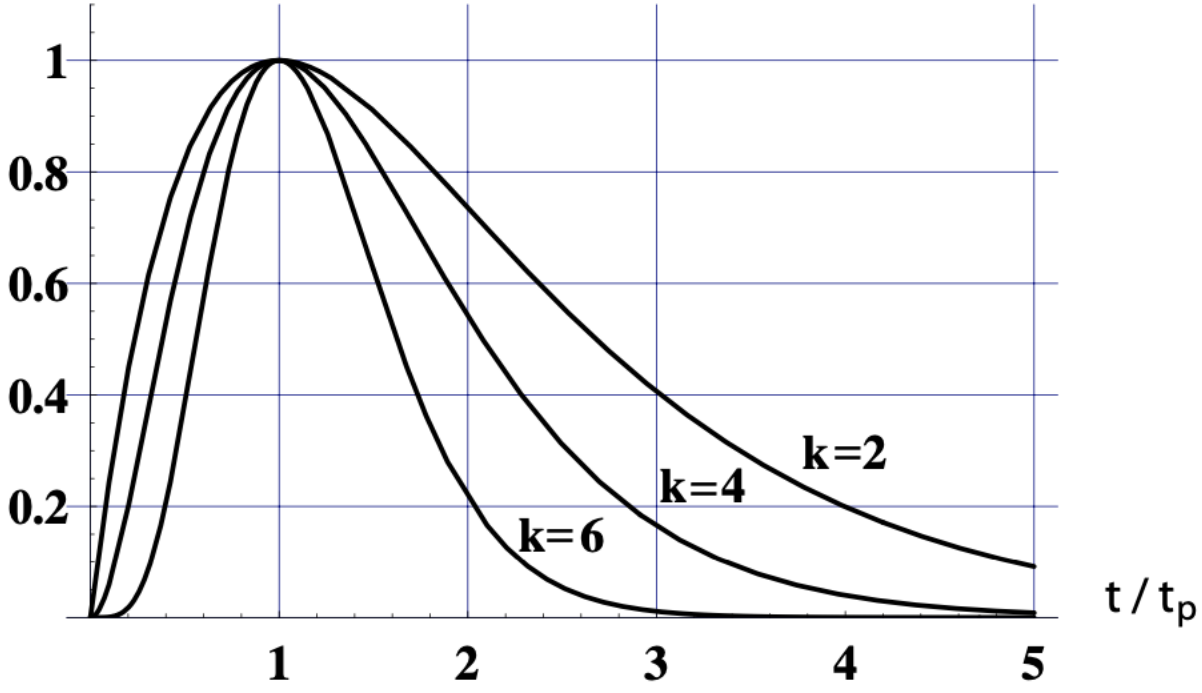
$$\mathcal{F}^{-1}\left[\frac{1}{(i\omega - p_i)^{k_i}}\right] = \frac{t^{k_i-1}}{(k_i-1)!}\, e^{a_i t}\ [\cos(b_i t) + i\sin(b_i t)]\ \Theta(t)$$

In case $a_i > 0$, the delta response tends to infinity for $t \to \infty$, so the criterion for stability of a linear system is given by the requirement that the real parts of *all* poles of $W(\omega)$ must be negative.

In case $a_i + ib_i$ is a root of a polynomial, the complex conjugate $a_i - ib_i$ is also a root of the polynomial. The terms appear therefore always in complex conjugate pairs, the imaginary part is cancelled and the delta response is therefore always real.

Linear Signal Processing

Example of the k = 2, 4, 6 and b_i = 0. We will mainly discuss transfer functions with real poles.

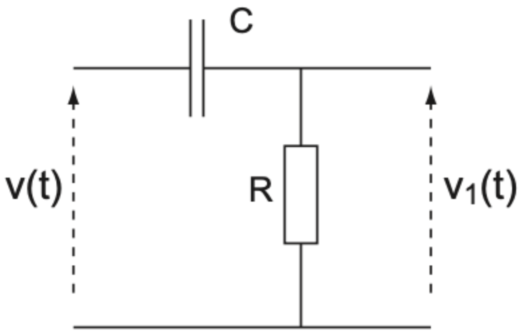


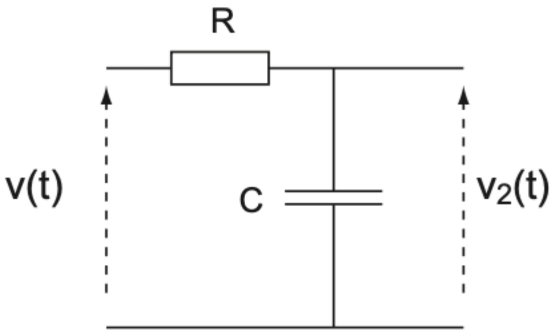
$$\frac{t^{k_i - 1}}{(k_i - 1)!} e^{a_i t}$$

CR, RC, Pole-Zero, Zero-Pole Filters

These filters are elementary in the sense that by cascading of these four filter types we can construct any desired transfer function $W(\omega)$ with real poles.

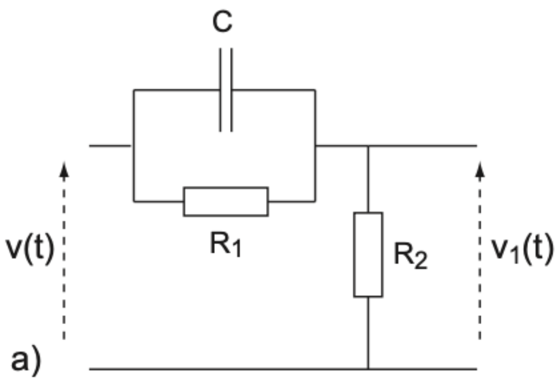
CR

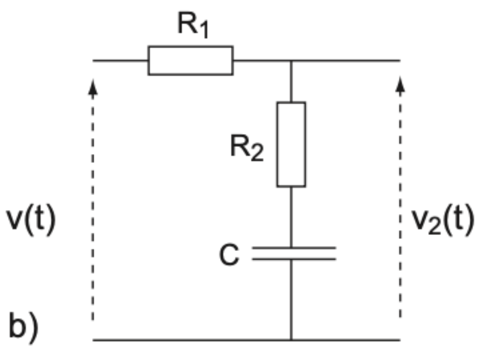




RC

Pole-Zero

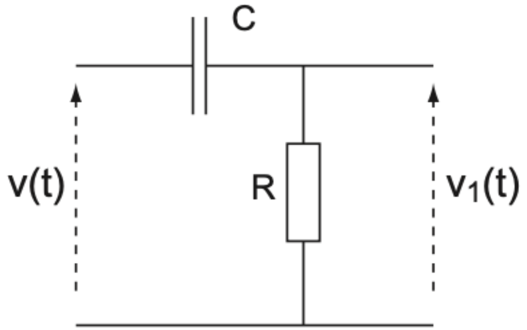


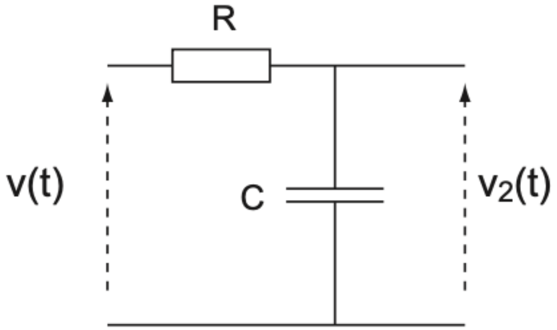


Zero-Pole

CR, RC

CR





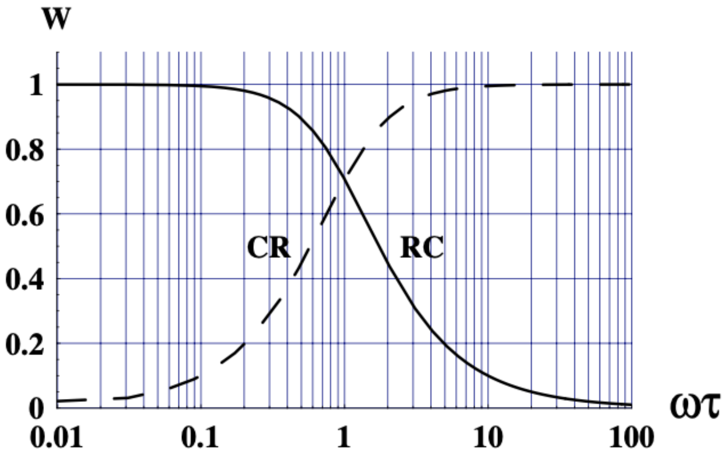
RC

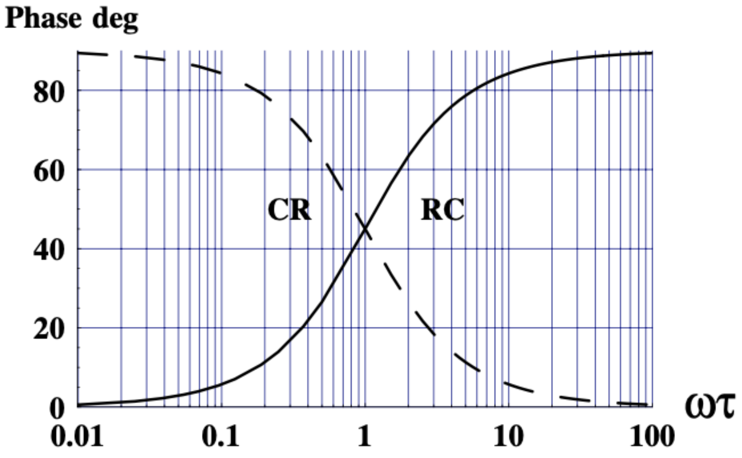
$$V_1(\omega) = \frac{i\omega RC}{1+i\omega RC} V(\omega) = \frac{i\omega\tau}{1+i\omega\tau} V(\omega) = W_{CR}(\omega)\,V(\omega)$$

$$V_2(\omega) = \frac{1}{1+i\omega RC} V(\omega) = \frac{1}{1+i\omega\tau} V(\omega) = W_{RC}(\omega)\,V(\omega).$$

$$W_{CR}(i\omega) = \frac{i\omega\tau}{1+i\omega\tau} = \frac{\omega\tau}{\sqrt{1+\omega^2\tau^2}} \exp\left[i \ \arctan 1/\omega\tau\right]$$

$$W_{RC}(i\omega) = \frac{1}{1+i\omega\tau} = \frac{1}{\sqrt{1+\omega^2\tau^2}} \exp\left[i \ \arctan \omega\tau\right].$$





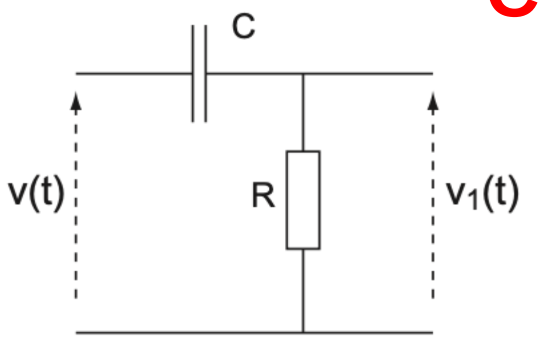
At the frequency $\omega_0 = 1/\tau$ the input voltage is attenuated by $1/\sqrt{2} \approx 0.707$ and the phase shift between input and output signal is $\pi/4 = 45°$.

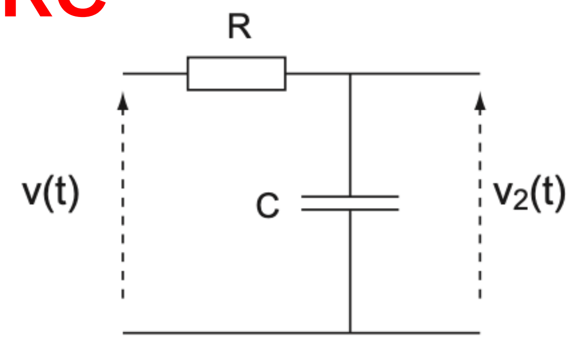
CR = high-pass = differentiator

RC = low-pass = integrator

CR, RC

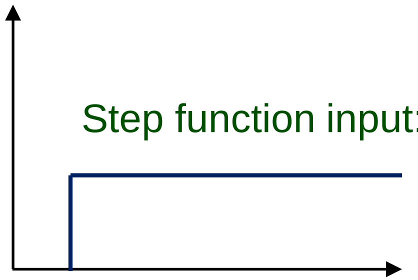
CR





RC

Step function input:

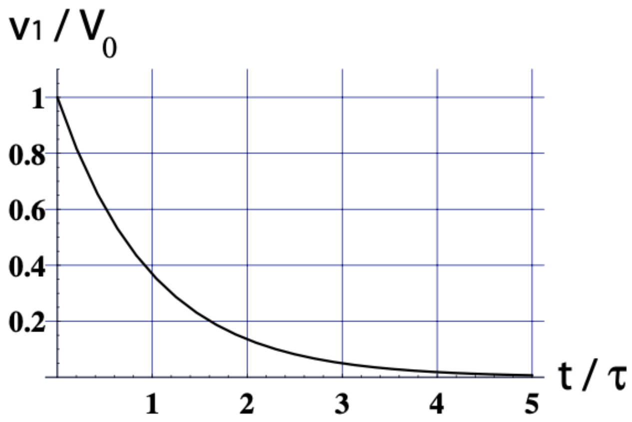


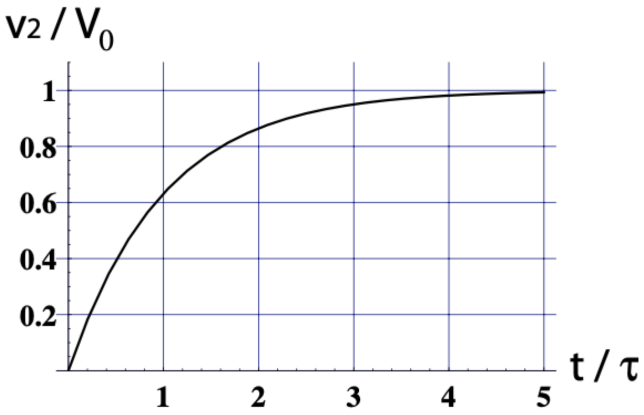
$$v(t) = \Theta(t) = \begin{cases} 0 & t \leq 0 \\ 1 & t > 0 \end{cases} \qquad V(\omega) = \mathcal{F}[v(t)] = \frac{1}{i\omega}$$

$$V_1(\omega) = \frac{V_0\,\tau}{1 + i\omega\tau} \qquad V_2(\omega) = \frac{V_0}{i\omega(1 + i\omega\tau)}$$

$$v_1(t) = V_0\, e^{-\frac{t}{\tau}}\, \Theta(t) \qquad v_2(t) = V_0\,(1 - e^{-\frac{t}{\tau}})\, \Theta(t)$$

CR = high-pass = differentiator





RC = low-pass = integrator

CR, RC

Same calculation in the Time domain. First calculate the delta response, i.e. the inverse Fourier transform of the transfer function:

$$W_{CR}(i\omega) = \frac{i\omega\tau}{1+i\omega\tau} \qquad W_{RC}(i\omega) = \frac{1}{1+i\omega\tau}$$

$$w_{CR}(t) = \delta(t) + \frac{1}{\tau}e^{-\frac{t}{\tau}}\,\Theta(t) \qquad w_{RC}(t) = \frac{1}{\tau}e^{-\frac{t}{\tau}}\,\Theta(t)$$

and then convolute with the input signal:

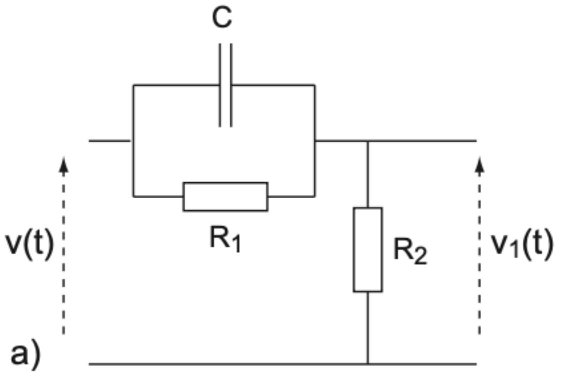
$$v_1(t) = \int_0^t \left(\delta(t-t') - \frac{1}{\tau}e^{-\frac{t-t'}{\tau}}\Theta(t-t')\right) V_0\,\Theta(t')dt' = V_0\,e^{-\frac{t}{\tau}}\,\Theta(t)$$

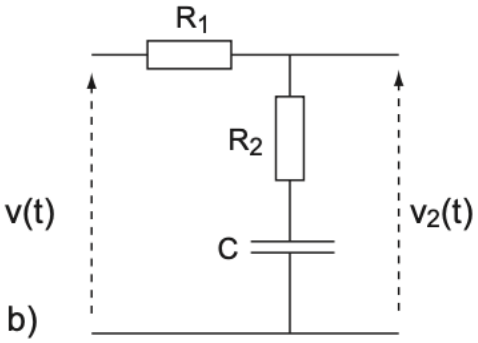
$$v_2(t) = \int_0^t \left(\frac{1}{\tau}e^{-\frac{t-t'}{\tau}}\Theta(t-t')\right) V_0\,\Theta(t')dt' = V_0\,(1-e^{-\frac{t}{\tau}})\,\Theta(t)$$

→ same result (of course)

Pole-Zero, Zero-Pole Filters

Pole-Zero





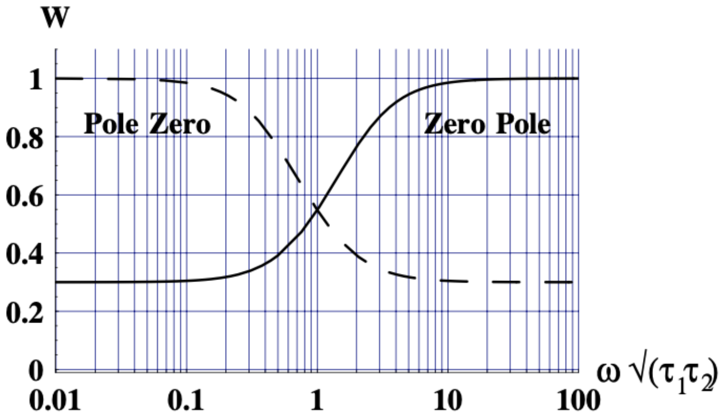
Zero-Pole

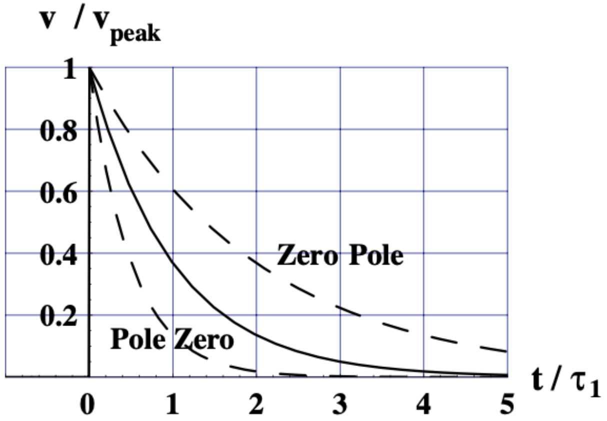
$$W_{PZ}(\omega) = \frac{i\omega + 1/\tau_1}{i\omega + 1/\tau_2} \qquad \tau_1 = R_1 C \quad \tau_2 = \frac{R_1 R_2}{R_1 + R_2} C \qquad \tau_1 > \tau_2$$

$$W_{ZP}(\omega) = \frac{\tau_1}{\tau_2} \frac{i\omega + 1/\tau_1}{i\omega + 1/\tau_2} \qquad \tau_1 = R_2 C \quad \tau_2 = (R_1 + R_2) C \qquad \tau_1 < \tau_2$$

Let's send an exponential signal with time constant $\boldsymbol{\tau}_1$ through the filter.

$$V(\omega) = \mathcal{L}[e^{-\frac{t}{\tau_1}} \Theta(t)] = \frac{1}{i\omega + 1/\tau_1} \qquad V_1(\omega) = W_{PZ}(\omega) V(\omega) = \frac{1}{i\omega + 1/\tau_2}$$





These filters turn an exponential signal with decay constant $\boldsymbol{\tau}_1$ into an exponential signal with decay constant $\boldsymbol{\tau}_2$

→ tail cancellation filter

Tail cancellation

In case we want to eliminate the long tail of a signal in order to reduce signal pileup, we can proceed the following way:
In general signal tails do not have exponential form, like e.g. in wire chambers where the shape is $1/(t+t_0)$.

$$i(t) = \frac{q}{2t_0 \ln \frac{b}{a}} \frac{1}{1+t/t_0} = I_0 \frac{1}{1+t/t_0}$$

We can approximate this signal to arbitrary precision by a sum of exponentials

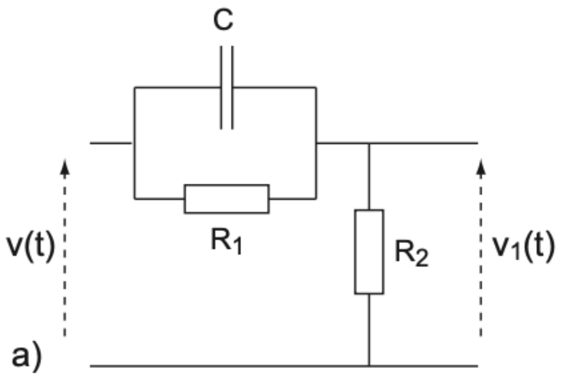
$$i(t) \simeq I_0 \sum_{n=1}^{N} A_n e^{-\alpha_n t/t_0} = I_0 \sum_{n=1}^{N} A_n e^{-t/\tau_n} \qquad \tau_n < \tau_{n+1} \qquad \Rightarrow \qquad I(\omega) = \mathcal{F}[i(t)] = I_0 \sum_{n=1}^{N} \frac{A_n}{i\omega + 1/\tau_n}$$

Example of two exponentials:

$$i(t) = I_0 \left(A_1 e^{-t/\tau_1} + A_2 e^{-t/\tau_2} \right) \qquad I(\omega) = I_0 \left(\frac{A_1}{i\omega + 1/\tau_1} + \frac{A_2}{i\omega + 1/\tau_2} \right)$$

Rewrite:

$$I(\omega) = I_0 \frac{(A_1 + A_2)}{(i\omega + 1/\tau_1)} \frac{(i\omega + 1/\tau)}{(i\omega + 1/\tau_2)} \qquad \tau = \tau_1 \tau_2 \frac{A_1 + A_2}{A_1 \tau_1 + A_2 \tau_2} < \tau_2.$$



Applying a Pole-Zero Filter of form $W_{PZ}(\omega) = (i\omega + 1/\tau_2)/(i\omega + 1/\tau)$, the output is

$$I_2(\omega) = I(\omega) \frac{i\omega + 1/\tau_2}{i\omega + 1/\tau} = I_0 \frac{A_1 + A_2}{i\omega + 1/\tau_1} \qquad i_2(t) = I_0 (A_1 + A_2) e^{-t/\tau_1}$$

→ We have removed the second exponential and are just left with the shorter exponential term.

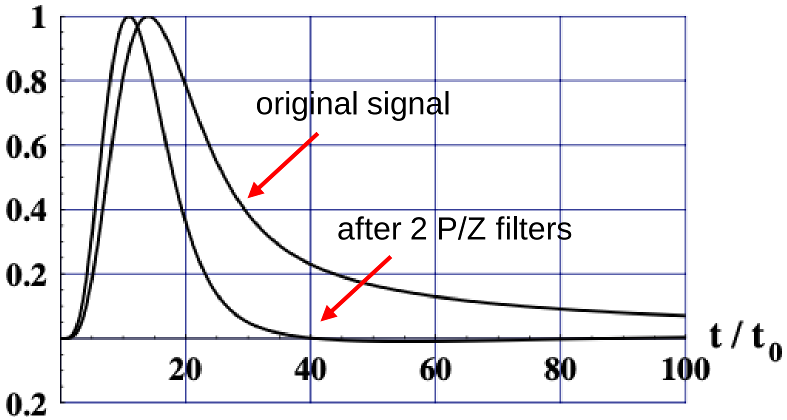
Tail cancellation

Using 3 exponentials for a wire chamber signal:

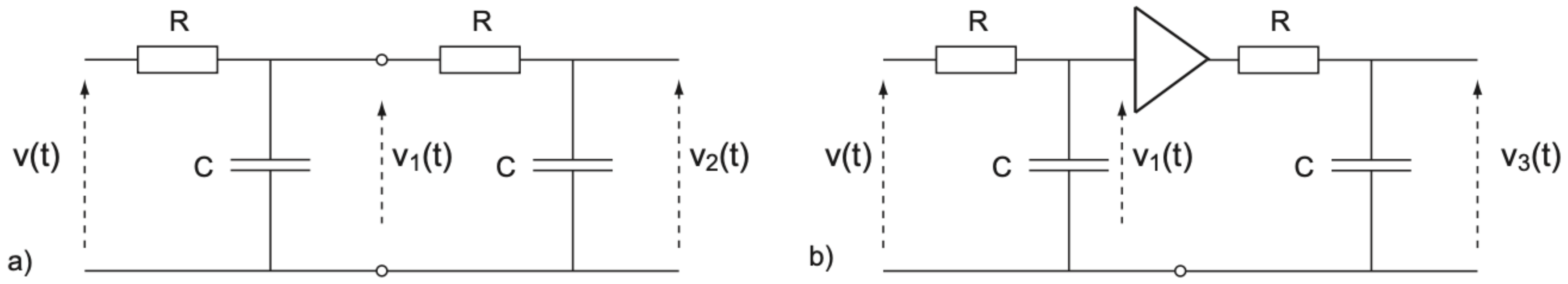
$$I_0 \frac{1}{1+t/t_0} \simeq I_0 \left(A_1 e^{-\alpha_1 t/t_0} + A_2 e^{-\alpha_2 t/t_0} + A_3 e^{-\alpha_3 t/t_0} \right) \quad \Longrightarrow \quad I(\omega) \simeq I_0 \frac{a}{(i\omega + 1/\tau_1)} \frac{(i\omega + 1/\tau_a)}{(i\omega + 1/\tau_2)} \frac{(i\omega + 1/\tau_b)}{(i\omega + 1/\tau_3)}.$$

Using two Pole/Zero filters with the respective time constants we have

$$I_2(\omega) = I(\omega) \frac{(i\omega + 1/\tau_2)}{(i\omega + 1/\tau_a)} \frac{(i\omega + 1/\tau_3)}{(i\omega + 1/\tau_b)} = I_0 \frac{a}{i\omega + 1/\tau_3} \quad \Longrightarrow \quad i(t) = I_0 (A_1 + A_2 + A_3) e^{-t/\tau_1}$$



Cascading of circuit Elements



$$V_2(\omega) = \frac{V(\omega)}{(k + i\omega RC)(1/k + i\omega RC)} \neq \frac{V(\omega)}{(1 + i\omega RC)^2}$$

The transfer function of two cascaded circuit elements is therefore not simply equal to the product of the two individual transfer functions.

In order to decouple the two circuits we must introduce a so called voltage buffer between them, which is an active device of infinite input impedance, infinite bandwidth and voltage gain G. Such a buffer produces an output signal which is an exact copy of the input signal scaled by G. Due to the infinite input impedance, no current is taken out of the first RC circuit and the transfer function indeed becomes the product of the individual transfer functions.

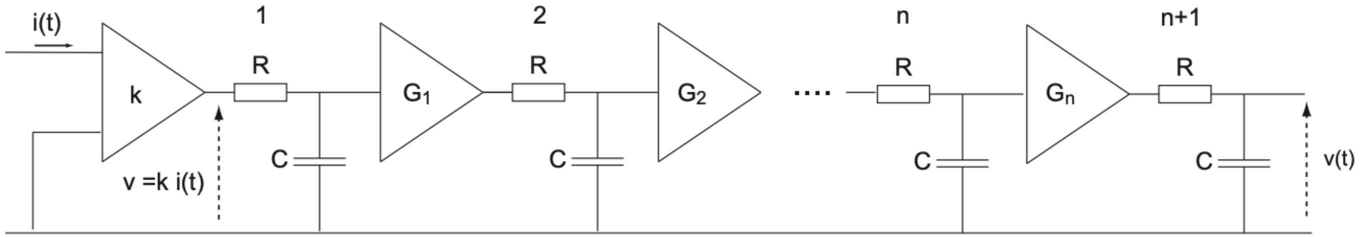
A cascade of circuit elements with individual transfer functions $W_1(\omega)$,$W_2(\omega)$...$W_n(\omega)$, that are decoupled by ideal voltage buffers, has a transfer function equal to the product

$$W(\omega) = W_1(\omega) \times W_2(\omega) \times \ldots \times W_n(\omega)$$

Signal shaping, bandwidth limitation

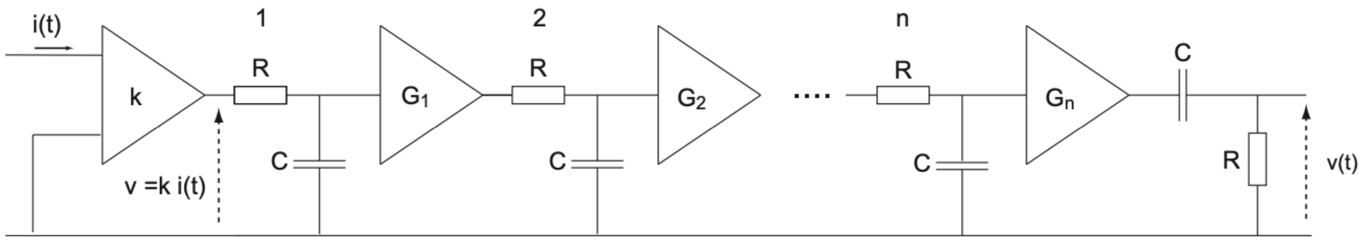
In order to realize a given bandwidth (peaking time, risetime …) of the signal processing chain, we look at two commonly used schemes, called the $(RC)^n$ and $CR\text{-}(RC)^n$ shapers.

$$W_{uni}(\omega) = kA\,\frac{1}{(1+i\omega\tau)^{n+1}}$$



b)

$$W_{bip}(\omega) = kA\,\frac{i\omega\tau}{(1+i\omega\tau)^{n+1}}$$



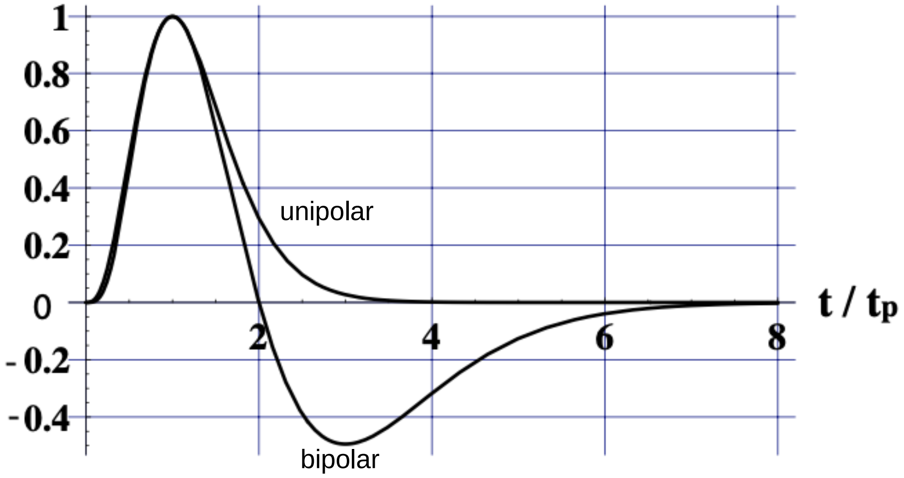
The first one is called a ‘unipolar’ shaper and the second one is called a ‘bipolar’ shaper.

Signal shaping, bandwidth limitation

$$H_{uni}(\omega) = \frac{t_p\ e^n n!}{(n + i\omega t_p)^{n+1}}$$

$$h_{uni}(t) = \left(\frac{t}{t_p}\right)^n e^{n(1-t/t_p)}\,\Theta(t)$$

$$t_p = n\tau$$



$$H_{bip}(\omega) = \frac{1}{\sqrt{n}}\,\frac{i\omega t_p^2\ e^r n!}{(r + i\omega t_p)^{n+1}}$$

$$h_{bip}(t) = \frac{1}{\sqrt{n}}\left(n - \frac{rt}{t_p}\right)\left(\frac{t}{t_p}\right)^{n-1} e^{r(1-t/t_p)}\,\Theta(t)$$

$$t_p = \tau(n - \sqrt{n})$$

$$r = n - \sqrt{n}.$$

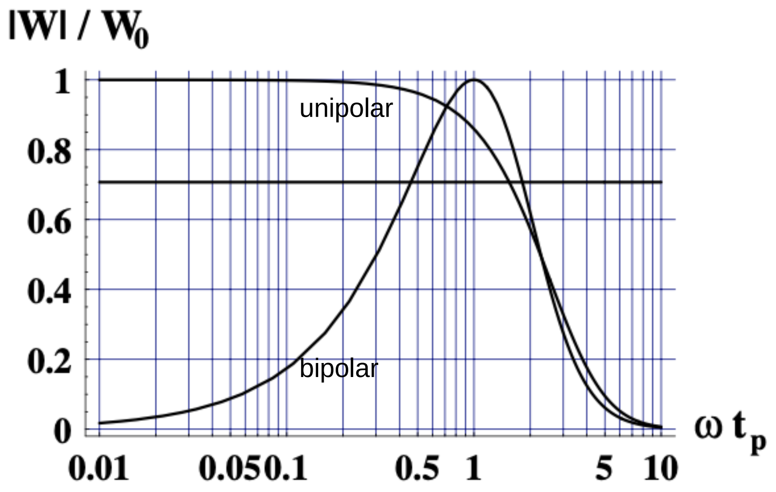
The delta response of the bipolar shaper integrates to zero:

$$\lim_{t\to\infty} g \int_{-\infty}^{t} h_{bip}(t')dt' = \lim_{\omega\to 0} i\omega \frac{1}{i\omega} H_{bip}(\omega) = H_{bip}(0) = 0$$

Any signal processed by the bipolar shaper integrates to zero:

$$\lim_{t\to\infty} \int_{-\infty}^{t} v_1(t')dt' = \lim_{i\omega\to 0} i\omega \frac{1}{i\omega} W(\omega)V(\omega) = W(0)V(0) = 0$$

In physical terms, the zero in the transfer function at $s = i\omega = 0$ implies that DC signals are fully attenuated. Only sinusodial components with frequencies $\omega > 0$ pass the circuit, and because all individual sinusodial components are bipolar the entire signal is bipolar.

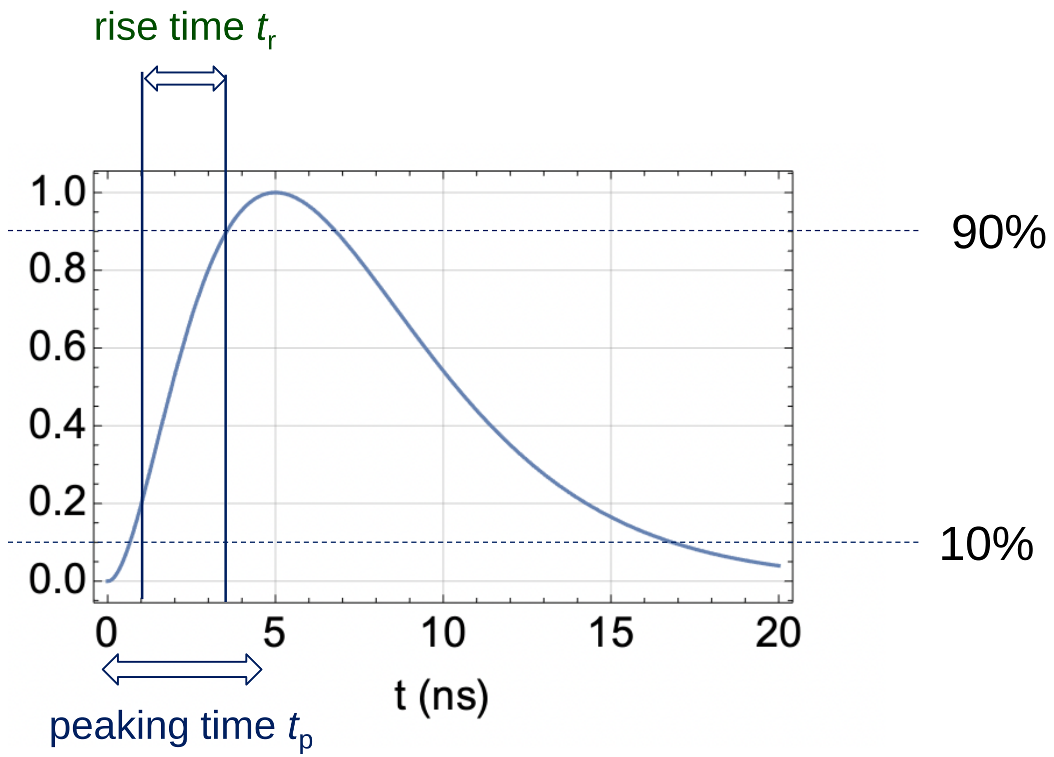


Vocabulary: Rise Time, Peaking Time

The rise time t_r of a pulse is defined as the time taken for its leading edge to rise from 10% to 90% of the peak height.

The peaking time t_p of a pulse is defined as the time taken for its leading edge to rise from zero to the peak height.

When we talk about the peaking time of an amplifier we mean the peaking time of its delta response.



Vocabulary: Bandwidth Limit

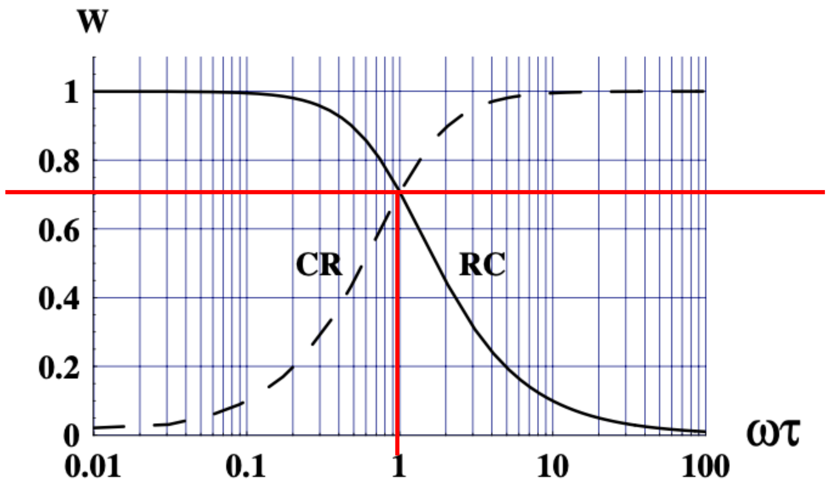
In the frequency domain, an amplifier is characterized by the gain $|W(i\omega)|$ and the phase shift $\arg[W(i\omega)]$ for each frequency.

The bandwidth limit of an amplifier is defined as the frequency where the signal transmission has been reduced by 3 dB from the central or midrange reference value.

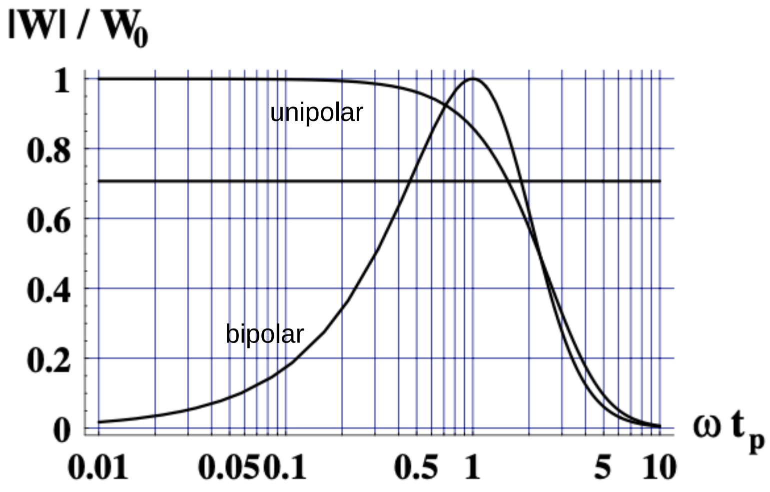
A -3dB reduction corresponds to a power level of ≈ 0.5 and a voltage level equal to ≈ 0.708 ≈ 1/√2 of the value at the center frequency reference.

The bandwidth limit of the RC lowpass filter is therefore given by $f_{bw} = 1/2\pi\,\boldsymbol{\tau} = 1/2\pi\,RC$.

$$W_{RC}(i\omega) = \frac{1}{1+i\omega\tau} = \frac{1}{\sqrt{1+\omega^2\tau^2}}\exp\left[i\ \arctan\omega\tau\right].$$



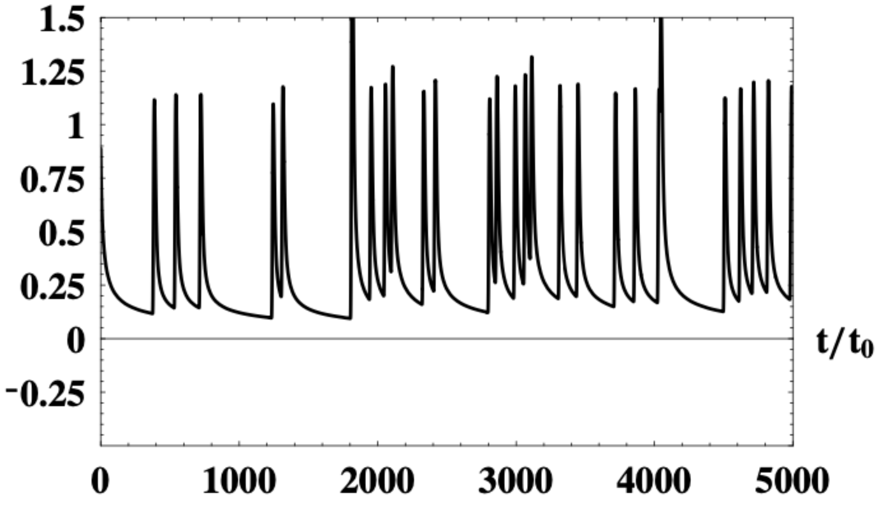
$$\frac{1}{\sqrt{1+\omega_{bw}^2\tau^2}} = \frac{1}{\sqrt{2}} \quad \rightarrow \quad \omega_{bw} = \frac{1}{\tau} \qquad f_{bw} = \frac{1}{2\pi\tau}$$



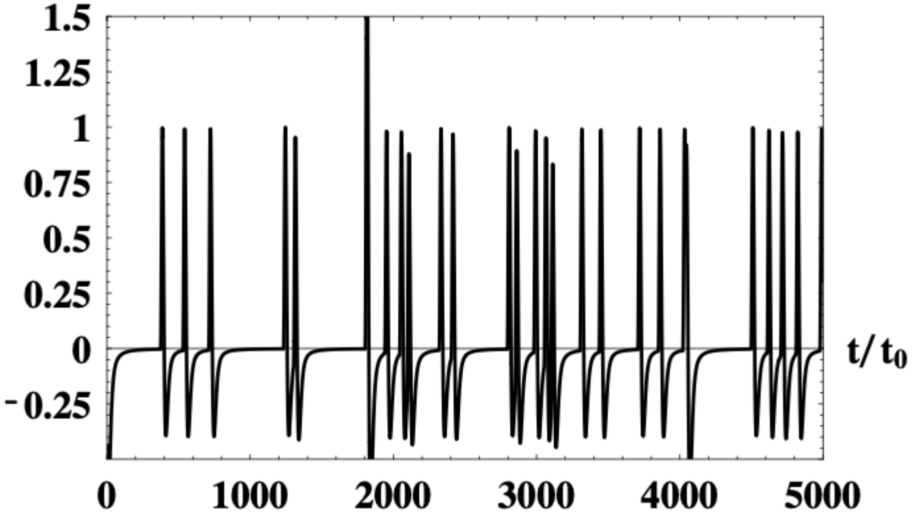
$$\omega_{bw} \approx \frac{2}{t_p} \qquad f_{bw} \approx \frac{1}{\pi t_p}$$

Shaping and baseline stability, e.g wire chamber signals

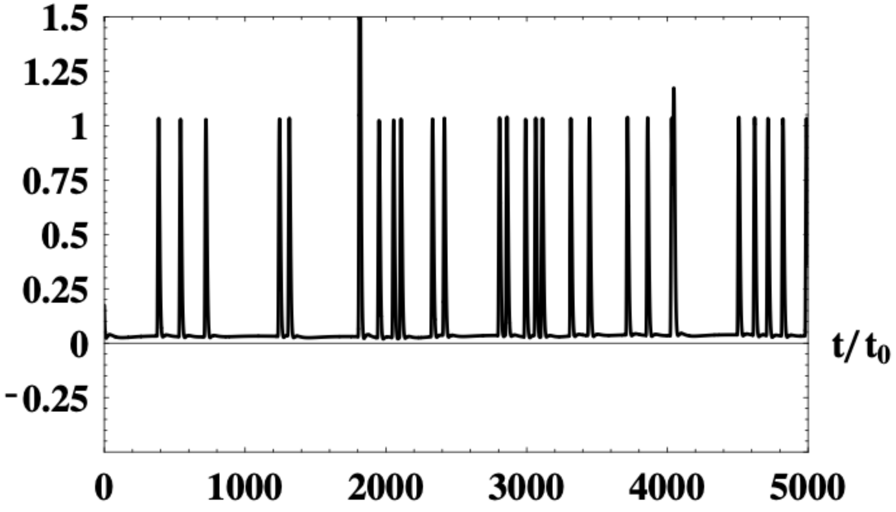
unipolar shaper



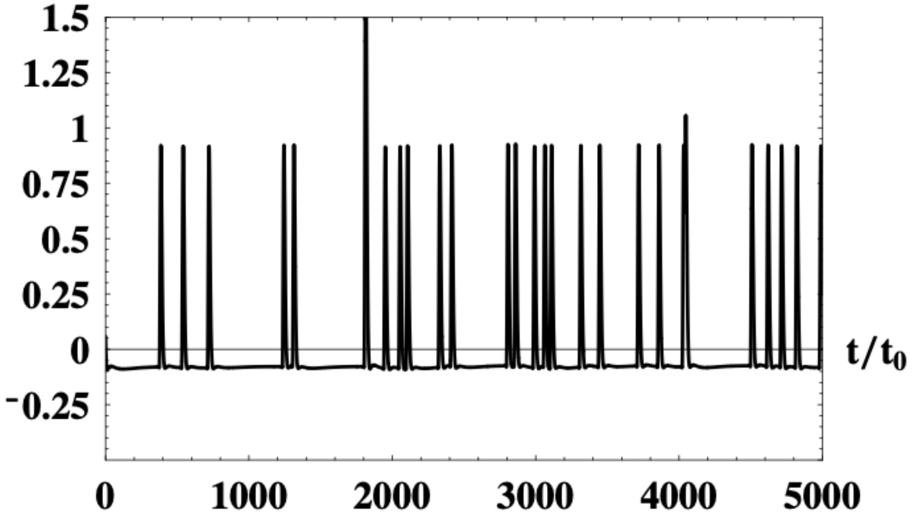
bipolar shaper



unipolar & 2x pole/zero



unipolar & 2x pole/zero + AC coupling



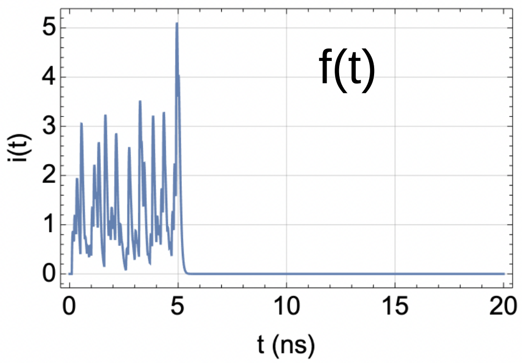
Bipolar shaping guaranties a stable baseline but produces a significant undershoot and can have slightly worse noise characteristics.

An unipolar shaper with a tail cancellation filter that does not remove ALL the long term components will result in an average offset of the baseline.

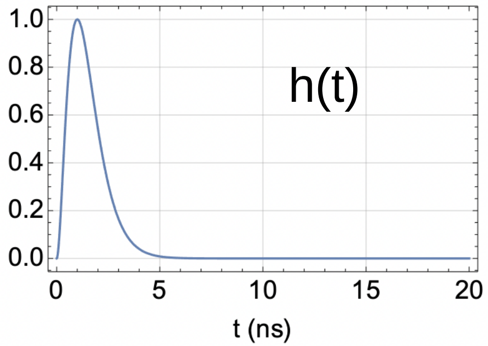
AC coupling cannot eliminate this offset, because in an AC coupled system the entire signal has to integrate to zero, so the baseline will be shifted below zero by an amount that depends on the occupancy.

The only way to stabilize the baseline without undershoot is a non-linear circuit element (‘Baseline –restorer’).

Electronics processing of a detector signal



Signal

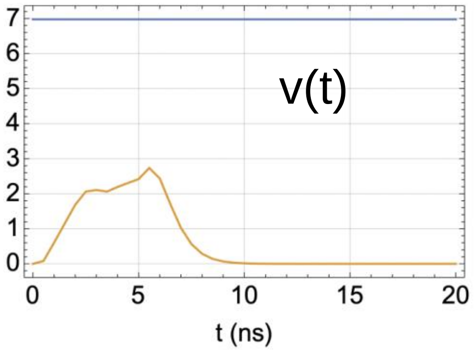


Frontend delta response:

$$h(t) = \left(\frac{t}{t_p}\right)^n e^{n(1-t/t_p)}\,\Theta(t)$$

Corresponding transfer function:

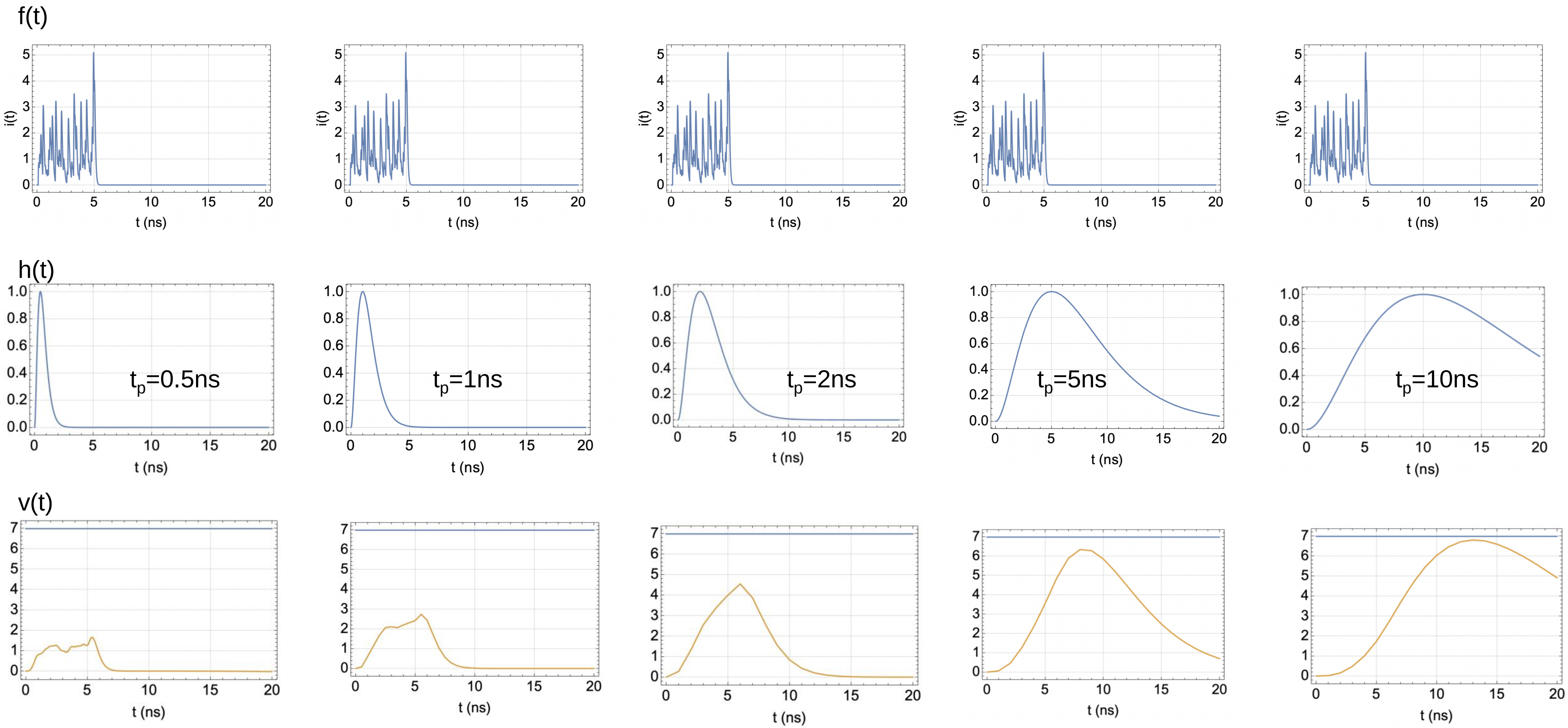
$$H(\omega) = \frac{t_p e^n n!}{(n + i\omega t_p)^{n+1}}$$



Frontend output

$$v(t) = \int_0^t h(t-t')f(t')dt'$$

Electronics processing of a detector signal



If the peaking time t_p becomes larger than the signal length, the peak of the output signal approaches the total charge.

Sometimes the peaking time is also called the ‘integration’ time, because it gives the time over which the input signal is ‘integrated’ or ‘averaged’

Ballistic deficit

Processing a signal i(t) by an amplifier with transfer function h(t) and peaking time t_p that is much longer than the duration of $i(t)$, the convolution integral can be approximated and the output pulse is given by

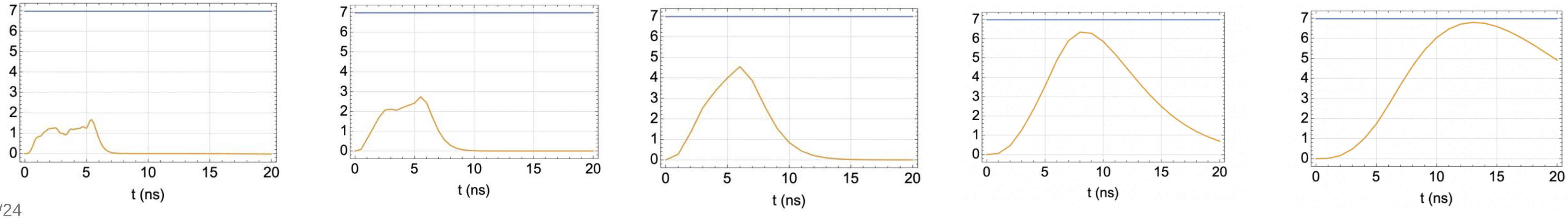
$$v(t) = g\, h(t)\, Q_{tot}$$

The peak of the output signal is gQ_{tot}. The output pulse height is therefore proportional to the total signal charge and such an amplifier is called *charge amplifier*.

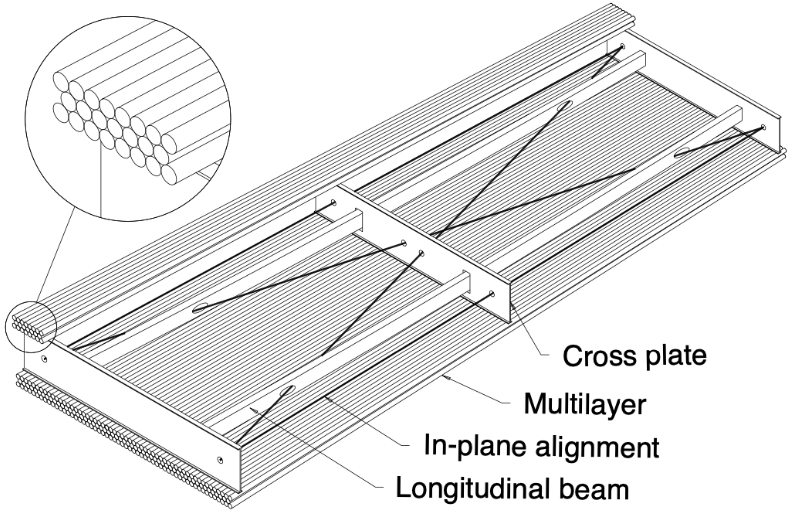
For some applications we want to preserve the speed of the signal and therefore use peaking times that are smaller than the length of the signal i(t).

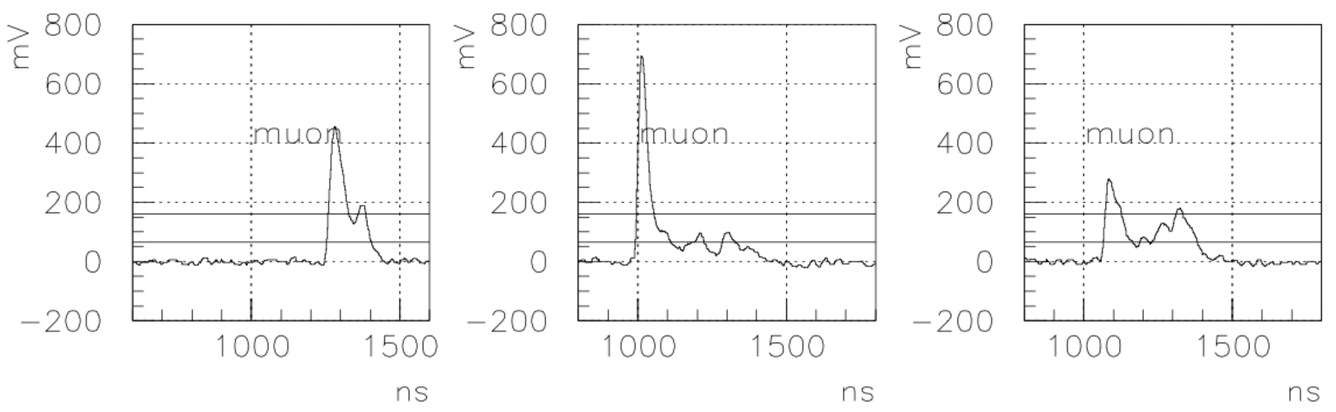
The peak of the signal will then not be proportional to the total charge, and we call the difference the ‘ballistic deficit’.

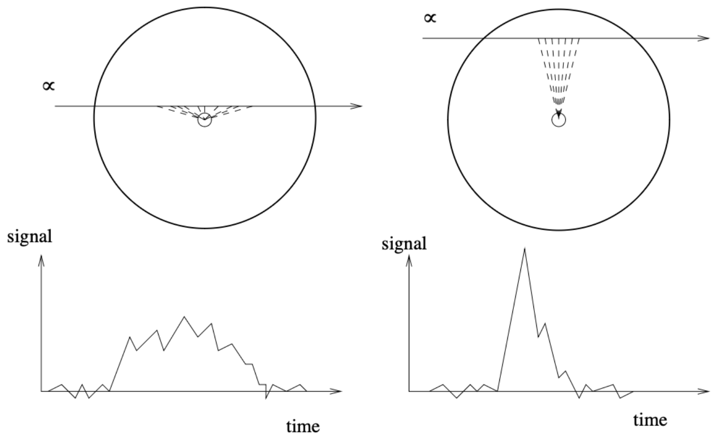
The ballistic deficit is defined as the difference between the amplifier output pulse-height for the input signal $i(t)$ and the output pulse-height in case the entire input signal charge $Q_{tot} = \int i(t)dt$ would be contained in a delta current pulse $Q_{tot}\delta(t)$.



ATLAS muon chambers







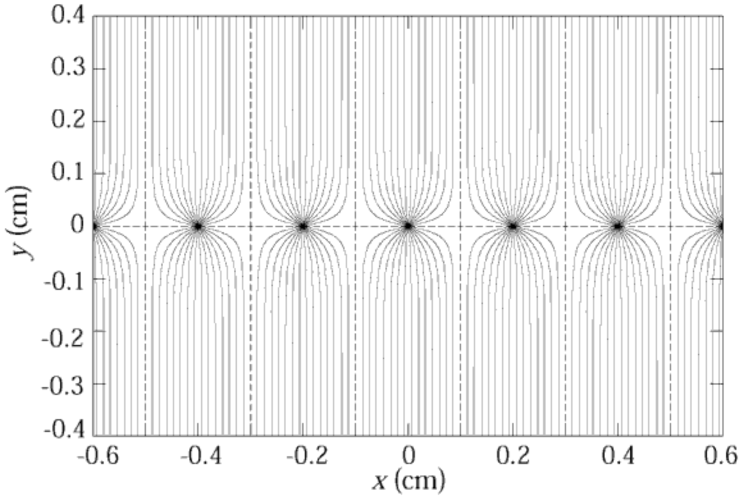
The signals are up to 900ns long.

We are interested in the arrival time of the first electrons
→ track position.

We are not interested in the total signal charge.

We use a fronted with peaking time O(15ns).

Strip readout

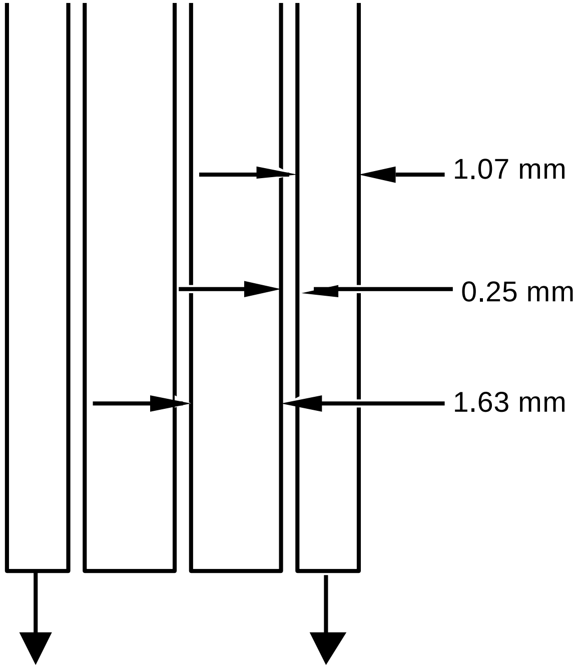


Cathode strip chamber:

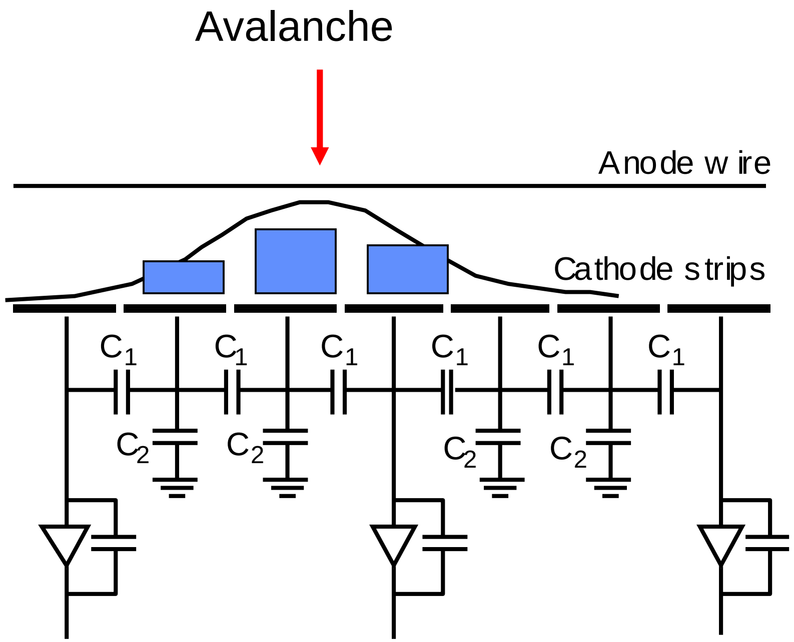
We are interested in the total signal charge for the best possible charge interpolation.

The same is of course true for silicon sensors.

(a)



(b)



Centroid Time, (center of gravity time)

Signal duration of T i.e. f(t) = 0 for t>T

Electronics peaking time t_p >> T

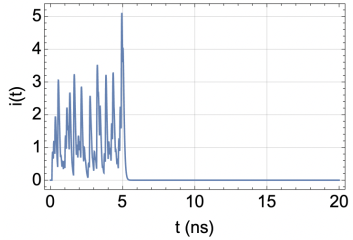
We are interested in times t>T

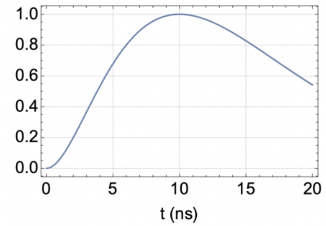
$$
\begin{aligned}
v(t) &= \int_0^t h(t-t')f(t')dt' \\
&\approx \int_0^T \left[h(t) - h'(t)t'\right] f(t')dt' \\
&= h(t)\int_0^T f(t')dt' - h'(t)\int_0^T t' f(t')dt' \\
&= \int_0^T f(t')dt' \left[h(t) - h'(t)\frac{\int_0^T t' f(t')dt'}{\int_0^T f(t')dt'}\right] \\
&= q\left[h(t) - h'(t)t_{cog}\right] \\
&= q\,h(t - t_{cog})
\end{aligned}
$$

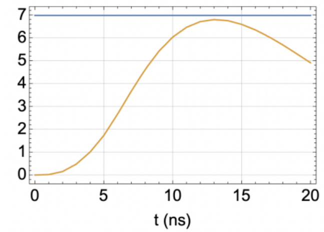
In case the electronics peaking time t_p is longer than the signal duration T, the electronics output signal has

- the same shape as the delta response
- a pulse-height equal to the total charge of the signal
- a 'time displacement' of this delta response by the center of gravity time t_{cog} of the signal.

→ An amplifier that is 'slower' than the signal measures the center of gravity time of the signal

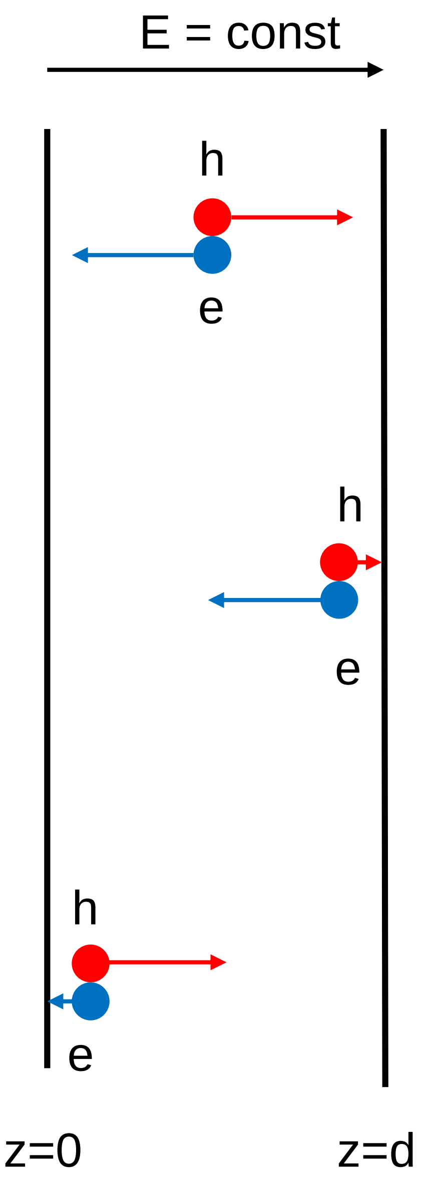




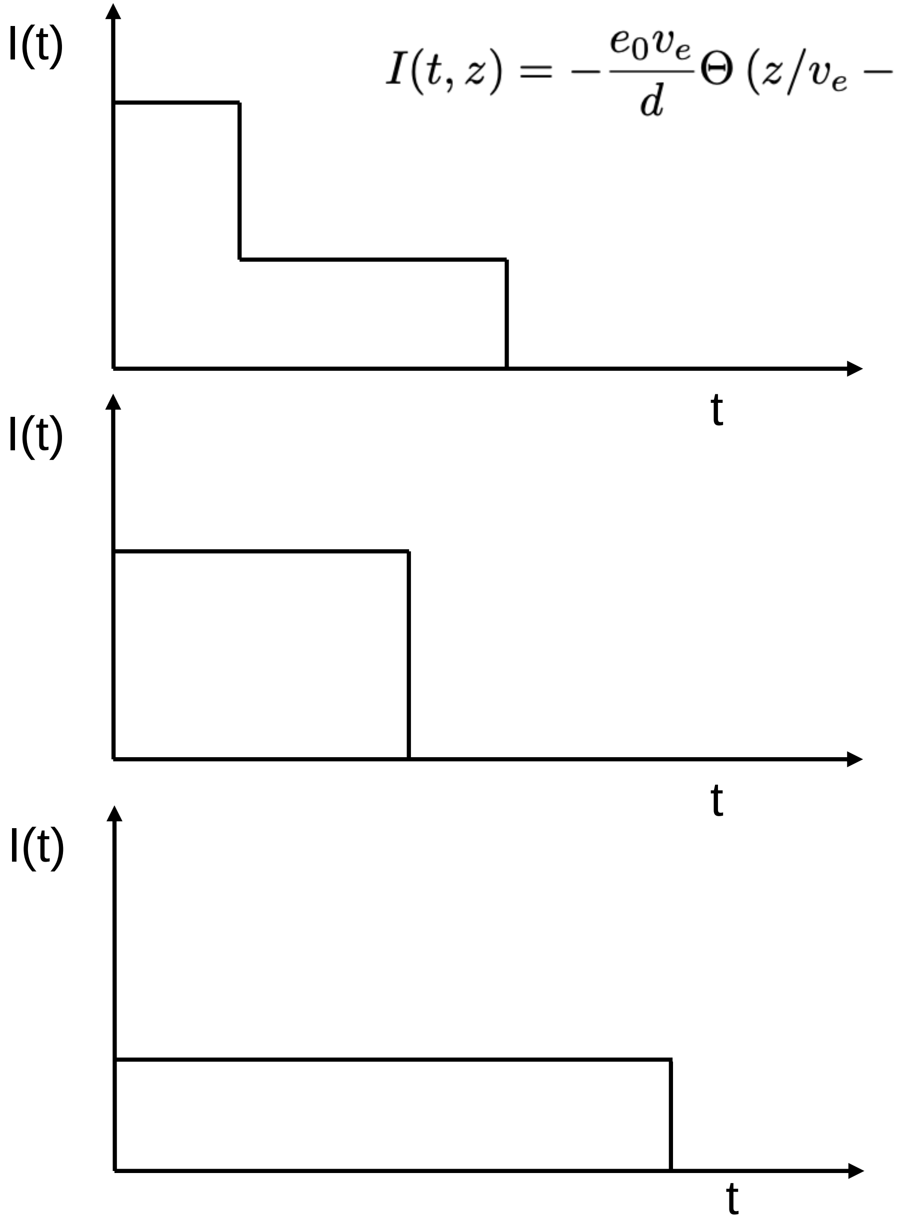


$$
q = \int_0^T f(t')dt' \qquad t_{cog} = \frac{\int_0^T t' f(t')dt'}{\int_0^T f(t')dt'} = \frac{1}{q}\int_0^T t' f(t')dt'
$$

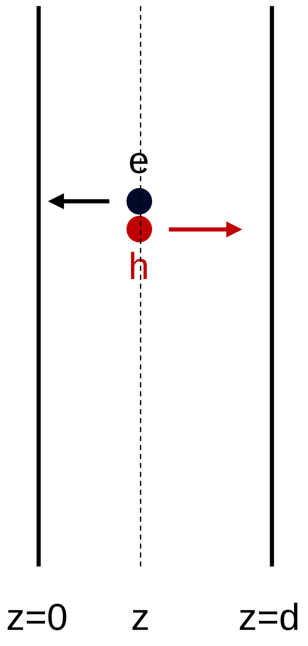
Single e-h pair in silicon



$$I(t,z) = -\frac{e_0 v_e}{d}\Theta\left(z/v_e - t\right) - \frac{e_0 v_h}{d}\Theta\left((d-z)/v_h - t\right)$$



Single e-h pair in silicon, centroid time



Induced signal for different positions of the e-h pair.

$$I(t,z) = -\frac{e_0 v_e}{d}\Theta\left(z/v_e - t\right) - \frac{e_0 v_h}{d}\Theta\left((d-z)/v_h - t\right)$$

giving a centroid time of

$$\tau(z) = \frac{1}{2d}\left(\frac{z^2}{v_e} + \frac{(d-z)^2}{v_h}\right)$$

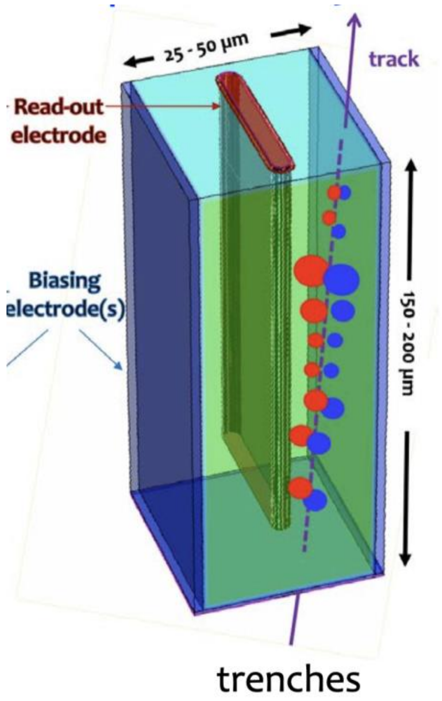
Let's assume the z position has a uniform random distribution:

$$\overline{\tau} = \frac{1}{d}\int_0^d \tau(z)dz \qquad \overline{\tau^2} = \frac{1}{d}\int_0^d \tau(z)^2 dz$$

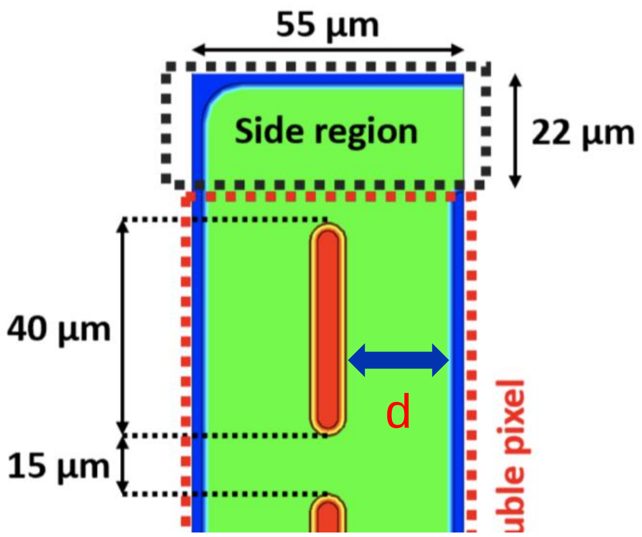
$$\sigma_\tau = \sqrt{\overline{\tau^2} - \overline{\tau}^2} = \sqrt{\frac{4}{180}\frac{d^2}{v_e^2} - \frac{7}{180}\frac{d^2}{v_e v_h} + \frac{4}{180}\frac{d^2}{v_h^2}}$$

The variance of the centroid time represents the time resolution in case we use an amplifier peaking time that is larger than the signal duration.

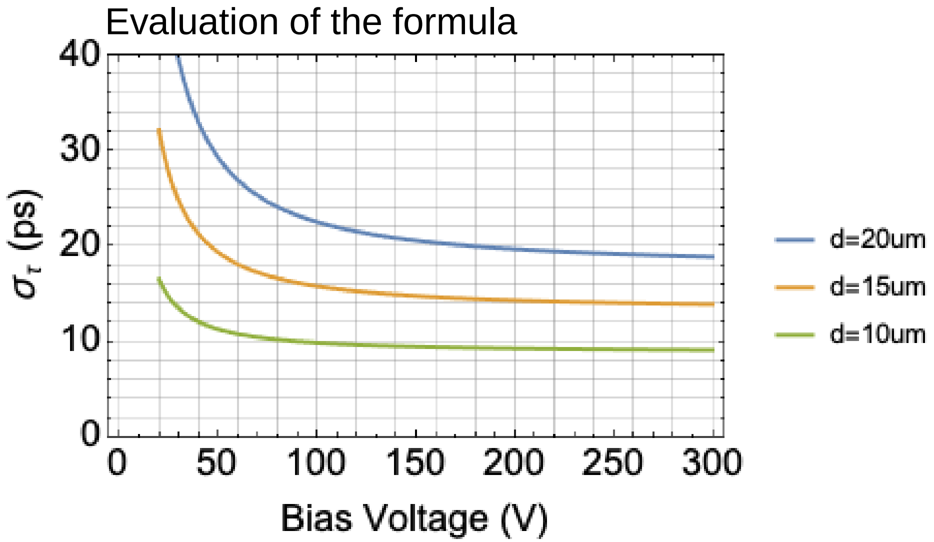
3D sensor realising a parallel plate geometry, TimeSPOT



$$\sigma_\tau = \sqrt{\overline{\tau^2} - \overline{\tau}^2} = \sqrt{\frac{4}{180}\frac{d^2}{v_e^2} - \frac{7}{180}\frac{d^2}{v_e v_h} + \frac{4}{180}\frac{d^2}{v_h^2}}$$



Total charge from the 200um sensor but timing characteristics from a 25um sensor !



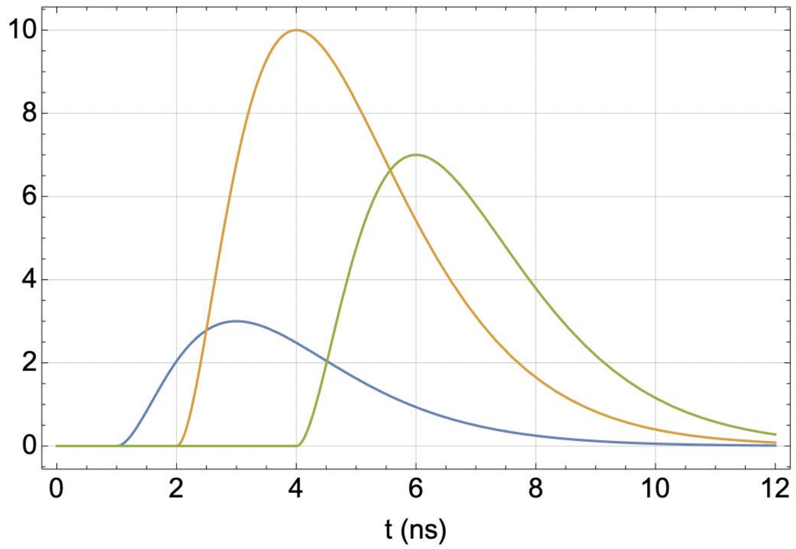
→ 10-20ps achievable and indeed achieved !

L. Anderlini et al., *Intrinsic time resolution of 3D-trench silicon pixels for charged particle detection*. JINST 15, P09029, 2020.

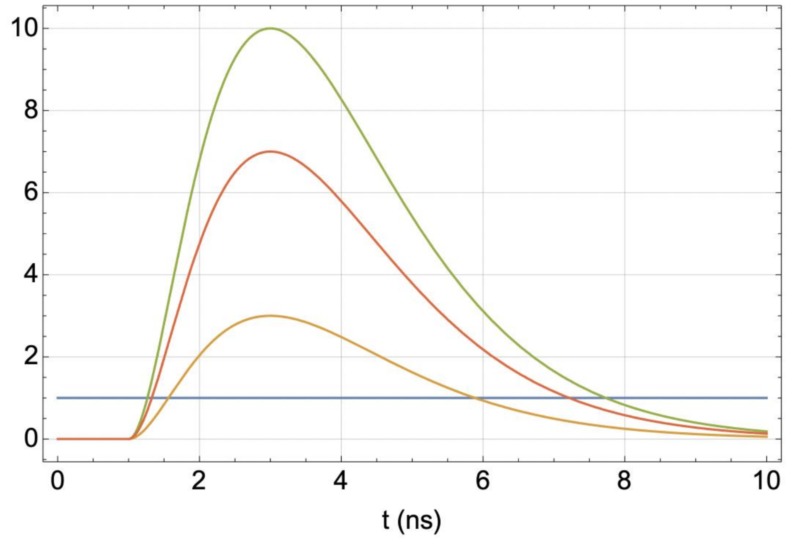
D. Brundu et al., *Accurate modelling of 3D-trench silicon sensor with enhanced timing performance and comparison with test beam measurements*. JINST 16, P09028, 2021.

Electronics 'slower' than the detector signal, time slewing

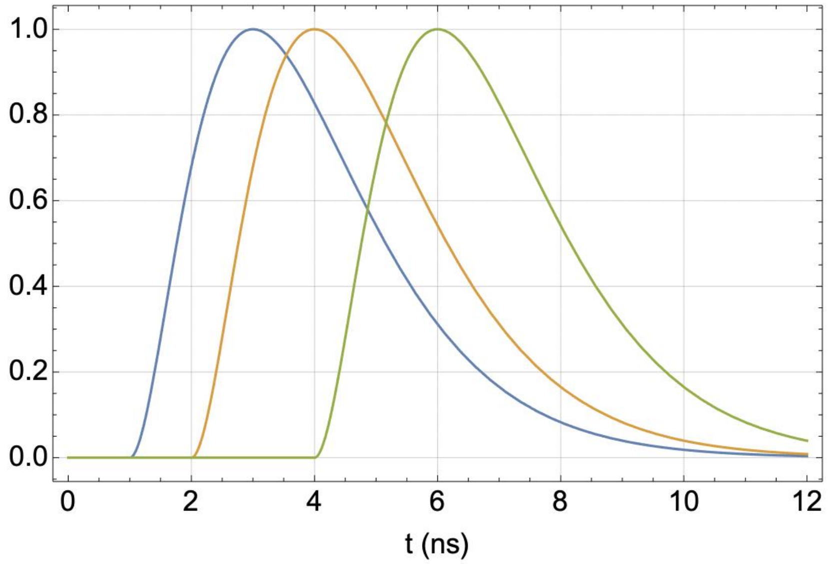
Delta response shifted by t_{cog} and scaled by Q



'time slewing'



Signal normalized to same amplitude → time



There are many different ways to correct for this slewing effect

- Constant Fraction discrimination
- Standard discrimination using time over threshold to correct for pulse-height
- Standard discrimination + pulseheight to correct for pulse-height
- Standard discrimination + total charge to correct for pulse-height
- Multiple sampling and 'fitting' the know signal shape
- ….

Vocabulary: Voltage, Current and Charge amplifier

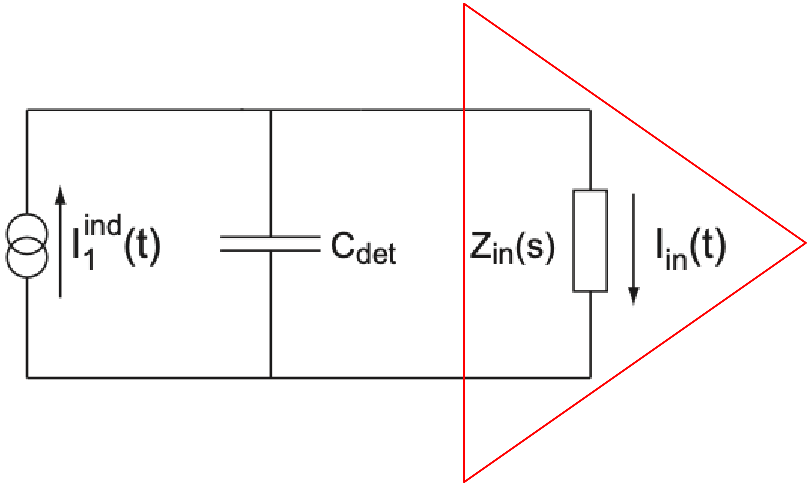
A voltage amplifier processes a voltage signal presented at its input and is characterized by large input impedance.

A current amplifier processes the current signal flowing into the amplifier and is characterized by low input impedance.

In most detectors, the detector capacitance C_{det} together with the input resistance R_{in} forms an ‘integration stage’ with bandwidth limit of $f_{bw} = 1/2\pi R_{in} C_{det}$ which is undesirable in case one wants to preserve the fast signal.

In order to preserve the chamber signal shape, the input impedance of the amplifier must therefore be small compared to all other impedances in the detector or equal to zero in the ideal case, which means that we use current amplifiers for readout of our detectors.

If the bandwidth of the current amplifier is such that it integrates a significant fraction of the chamber signal, it is usually called a charge amplifier or ‘charge sensitive amplifier’.



Sensitivity of a charge amplifier

The filters discussed up to now transform an input voltage signal to an output voltage signal, and therefore the dimension of the transfer function is [W(ω)] = 1.

The transfer function of a current amplifier that transforms a current input signal into a voltage output signal can be written as $V(\omega) = kW(\omega)I(\omega)$ where k has dimensions of V/A=Ω and W(ω) keeps the dimension 1.

In the time domain the connection of input and output signal is

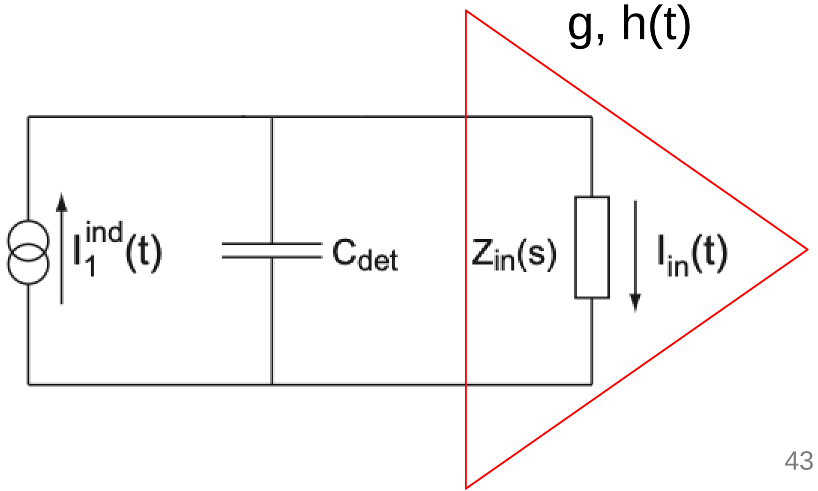
$$v(t) = k\int w(t-t')i(t')dt' = kw(t_p)\int \frac{w(t-t')}{w(t_p)}i(t')dt' = g\int h(t-t')i(t')dt'$$

The value $w(t_p)$ is the peak of the delta response $w(t)$, and the dimensionless function $h(t)$ is the normalized delta response. An input current pulse of $i(t) = Q\delta(t)$ results in an amplifier output peak voltage of $v(t_p) = g\,Q$.

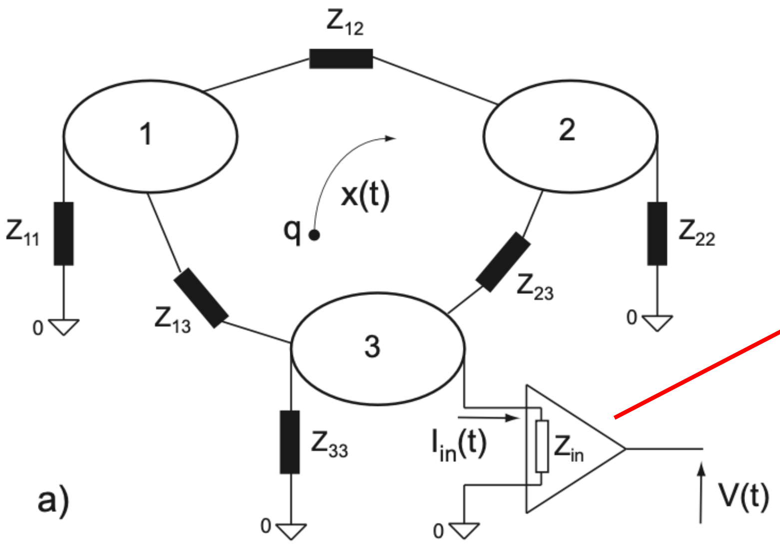
$$v(t) = g\int h(t-t')Q\delta(t')dt' = gQ$$

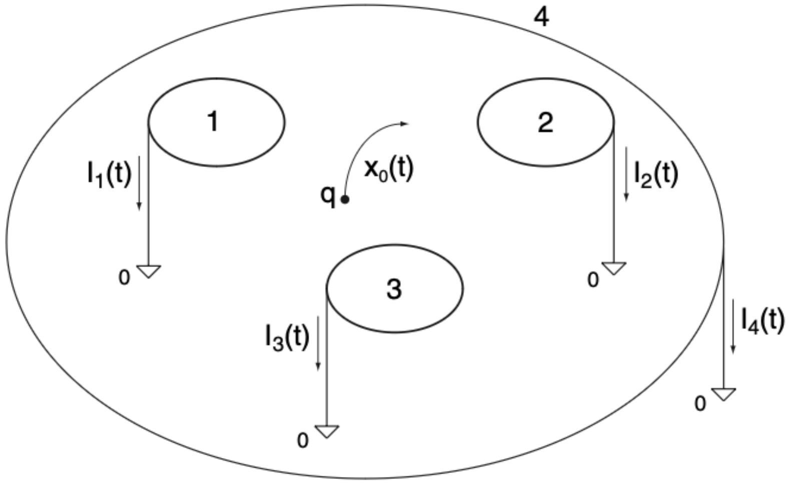
We therefore call g the sensitivity of the amplifier which has the dimension V/C.

Typical amplifiers have sensitivities in the range of 1-50 mV/fC.



Input circuit





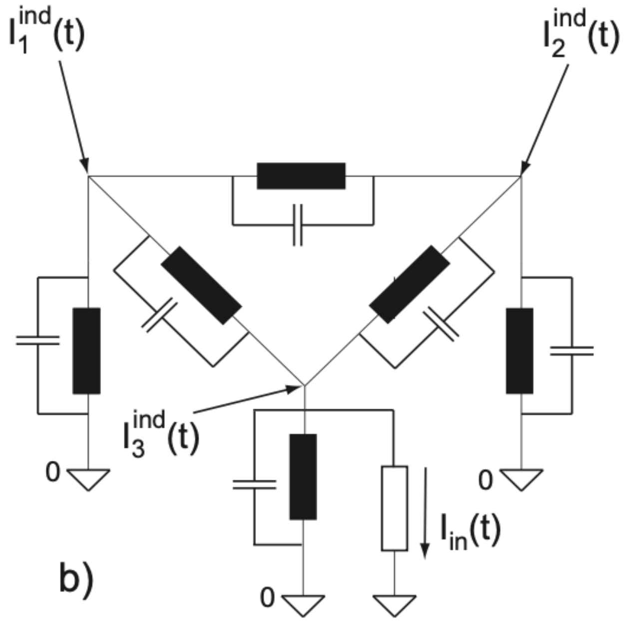
The amplifier is characterized by

- input impedance $Z_{in}(\omega)$
- sensitivity g (V/C)
- normalized delta response h(t) or $H(\omega)$

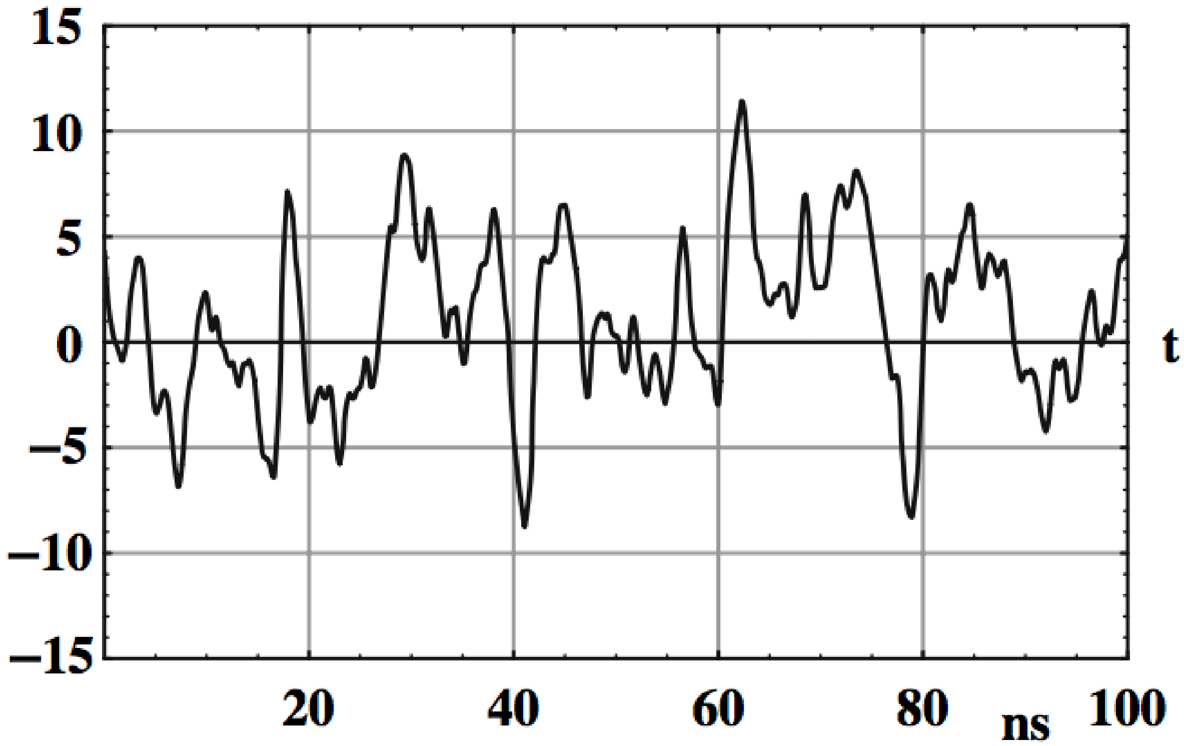
The current flowing into the amplifier has to be calculated by placing the induced currents on the input network that represents the detector.

The amplifier output is then

$$v(t) = g \int h(t-t')I_{in}(t')dt'$$



Tomorrow



Conclusion

In most applications we are not interested in an exact copy of the input signal.

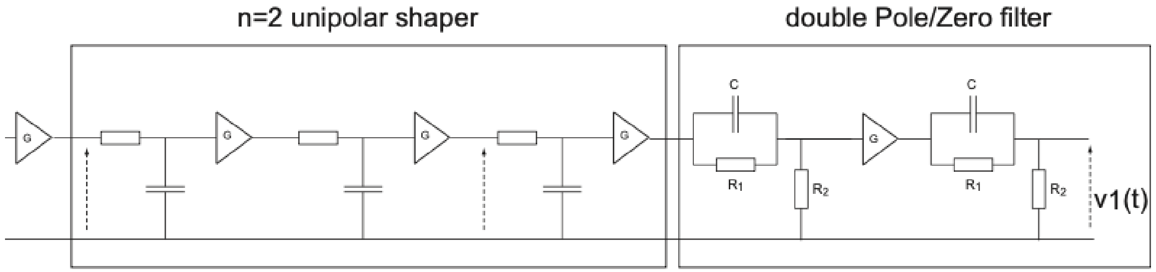
For applications where charge measurement is required one prefers long integration times (slower amplifiers) in order to integrate a large fraction of the detector signal.

For timing purposes one typically wants fast amplifiers to reduce time walk and jitter effects.

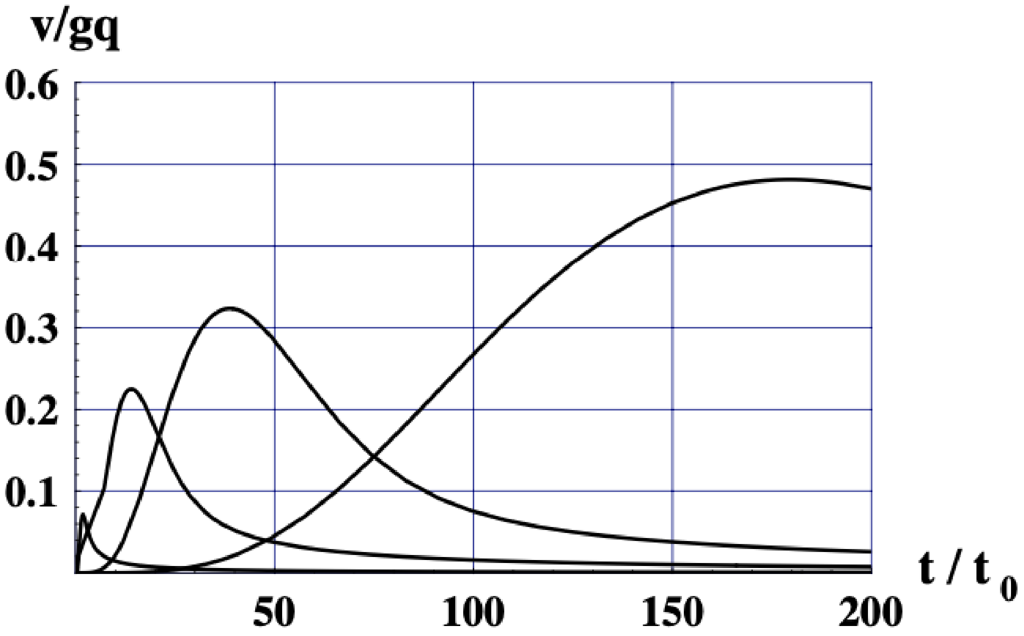
For high rate applications, signal tail cancelation and baseline restoration are important issues in order to avoid signal pileup and baseline fluctuations.

The delta response or the corresponding transfer function in the frequency domain fully characterize a linear signal processing chain.

In general, a linear signal processing system is completely defined by the poles and zeros of the transfer function.



Ballistic deficit



Amplifier output for the unipolar shaper with n = 4 and various values of t_p /t_0 = 1, 10, 30, 150. The longer the peaking time, the larger is the signal, because more signal charge is integrated.

For t_p larger than the total signal length, the peak approaches it's maximum.