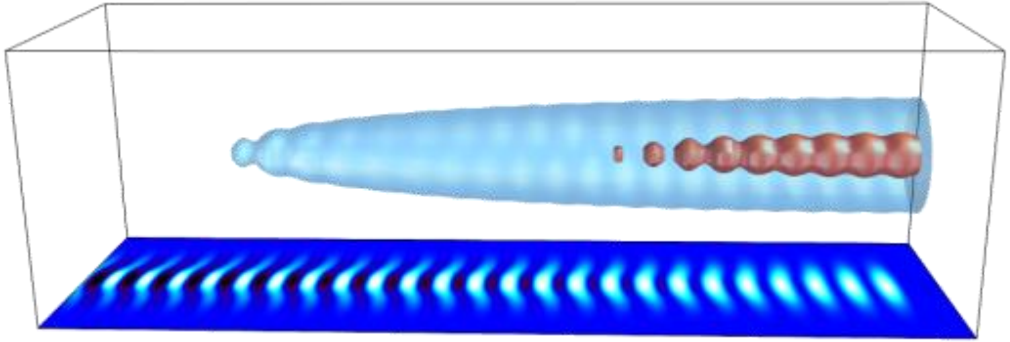
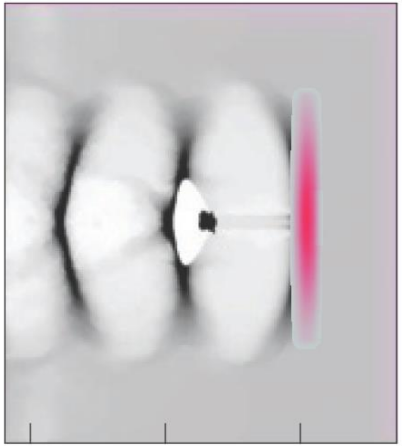


Options for proton-driven particle acceleration and simulation tools



Alexander Pukhov

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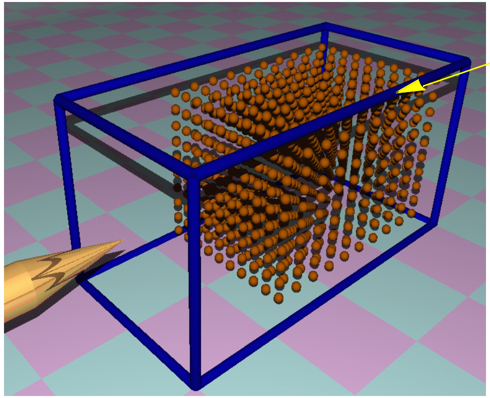
01.10.2024

Simulation tools for plasmas

- **Explicit electromagnetic PIC codes**
- **Lorentz boost**
- **Quasi-static approximation**
- **Hybrid methods**

Virtual Laser Plasma Lab

A. Pukhov, J. Plasma Phys. 61, 425 (1999)



Plasma or neutral gas

Gas of an arbitrary element can be used.

The code VLPL is written in C++, object oriented, parallelized using MPI for **Massively Parallel** performance
10^9 particles and 10^8 cells can be treated

Advanced physics & numerics

Fields

$$\nabla\times\mathbf{B}=\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}+\frac{4\pi}{c}\mathbf{j}$$

$$\nabla\times\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}$$

Particles

$$\frac{d\mathbf{p}}{dt}=q\mathbf{E}+\frac{q}{c\gamma}\mathbf{p}\times\mathbf{B}$$

$$\gamma=\sqrt{1+p^2/(mc)^2}$$

- Inelastic processes
- Radiation damping
- QED effects
- hybrid hydro model
- quasi-static approximation

Basic equations: full Maxwell

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Ampere's law

$$\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{B} - \frac{4\pi}{c}\mathbf{j}$$

Faraday's law

$$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

Dynamic equations

Poisson's eq.

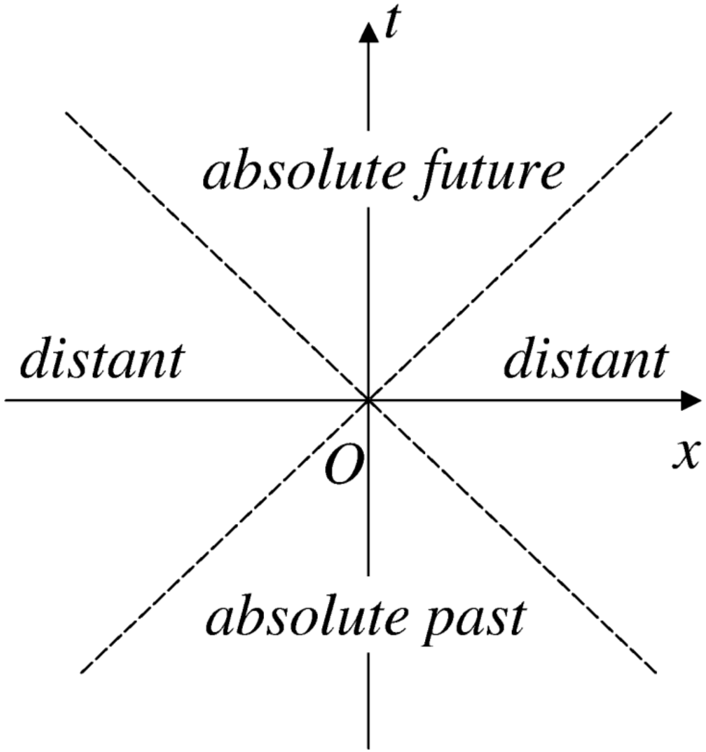
$$\nabla \cdot \mathbf{E} = 4\pi\rho$$

No magnetic dipoles

$$\nabla \cdot \mathbf{B} = 0$$

Static equations

Locality of the equations

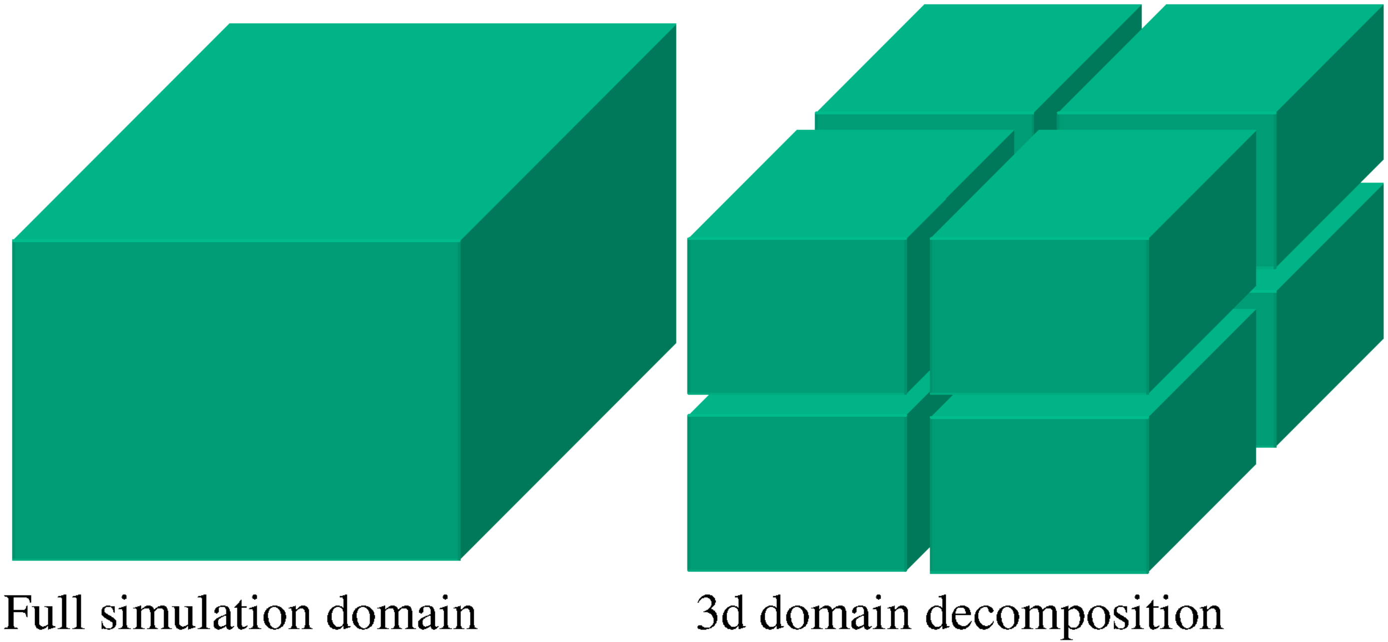


Only particles within the radius of cτ communicate to each other at every time step τ.

This allows for efficient parallelization using domain decomposition

No global communications

Implementation of a PIC code
Parallelization: domain decomposition



Distribution function and kinetic equation

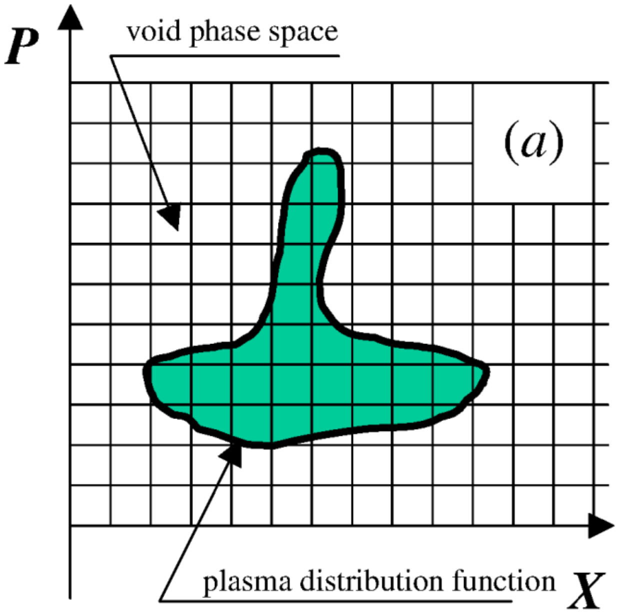
N-particles distribution function: $F_N(t, \mathbf{r}_1, \mathbf{p}_1, \ldots, \mathbf{r}_N, \mathbf{p}_N)$

Nearly ideal plasma:
single particle distribution function: $f(t, \mathbf{r}, \mathbf{p})$

$$\frac{\partial f(t,\mathbf{r},\mathbf{p})}{\partial t} + \frac{\mathbf{p}}{m\gamma}\nabla_{\mathbf{r}} \cdot f(t,\mathbf{r},\mathbf{p}) + q\left[\mathbf{E} + \frac{\mathbf{p}}{mc\gamma} \times \mathbf{B}\right]\nabla_{\mathbf{p}} \cdot f(t,\mathbf{r},\mathbf{p}) = St$$

How to solve this equation?

Eulerian approach, FDTD “Vlasov codes”



Very inefficient:
a lot of empty phase space
has to be processed.

However, temperature effects
may be described more carefully.
Low noise.
May be subject
to numerical diffusion.

“Finite elements” approach, integrating along characteristics

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Vlasov is a transport Eq in 6D space

Characteristics of the Vlasov Eq.
coincide with the equations of motion

$$\frac{d\mathbf{p}}{dt} = q\left[\mathbf{E} + \frac{\mathbf{p}}{mc\gamma} \times \mathbf{B}\right] + \mathbf{F}_{st}$$

$$\frac{d\mathbf{r}}{dt} = \frac{\mathbf{p}}{m\gamma}$$

Thus, we just push
the numerical macroparticles
in self-consistent
electromagnetic fields

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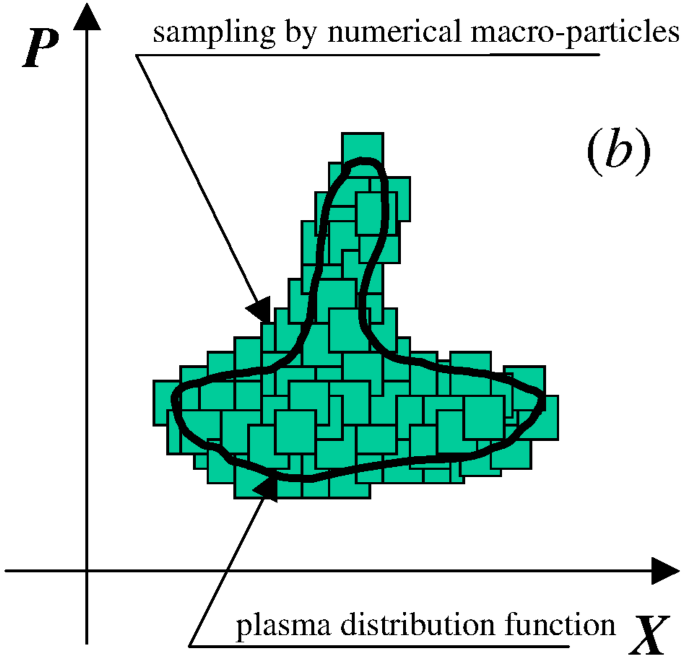
Sampling phase space: numerical macroparticles



Computationally efficient:
only filled parts of phase space
have to be processed.

However, temperature effects
are poorly described,
or require very many particles.
Noisy as $N^{1/2}$.

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Quantum electrodynamics (QED)

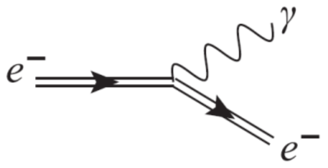
- Quantify importance of QED with parameter

$$\chi = \frac{\sqrt{-\left(F_{\mu\nu}p^{\nu}\right)^2}}{m_e c E_{\mathrm{cr}}}$$

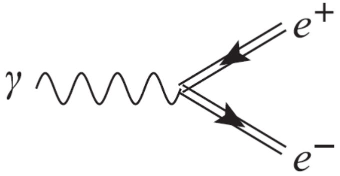
  - $E_{\mathrm{cr}} \approx 1.3 \times 10^{18}\ \mathrm{V/m}$ ("Schwinger field")

- $\chi \to 1$: hard-photon emission, electron-positron pair creation, … become important
- $\chi \approx 1600$: fully non-perturbative QED
  - no experiments
  - only preliminary analytical studies

Photon emission



Pair production



V. Ritus, *Ann. Phys*. **69**, 555 (1972)
V. Yakimenko et al., *PRL* **122**, 190404 (2019)

Modeling QED effects

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Monte-Carlo QED loop (Photon emission)

1. Randomized photon energy

$$\epsilon_\gamma = r_1 \epsilon_e \,, \; r_1 \in [0,1]$$

2. Calculate probability rate

$$\frac{\mathrm{d}W_{\mathrm{rad}}}{\mathrm{d}\epsilon_\gamma} = -\frac{\alpha m^2 c^4}{\hbar \epsilon_e^2}\left[\int_x^\infty \mathrm{Ai}(\xi)\,\mathrm{d}\xi + \left(\frac{2}{x} + \chi_\gamma \sqrt{x}\right)\mathrm{Ai}'(x)\right]$$

3. Probability of decay process

$$W_{\mathrm{decay}} = \frac{dW_{\mathrm{rad}}}{d\epsilon_\gamma}\epsilon_e \Delta t$$

4. Second random number for photon emission

$$r_2 \gtrless W_{\mathrm{decay}} \,, r_2 \in [0,1]$$

(Similar procedure for electron-positron pair creation)

N. Elkina, *PRAB* **14**, 054401 (2011)

Modeling QED effects

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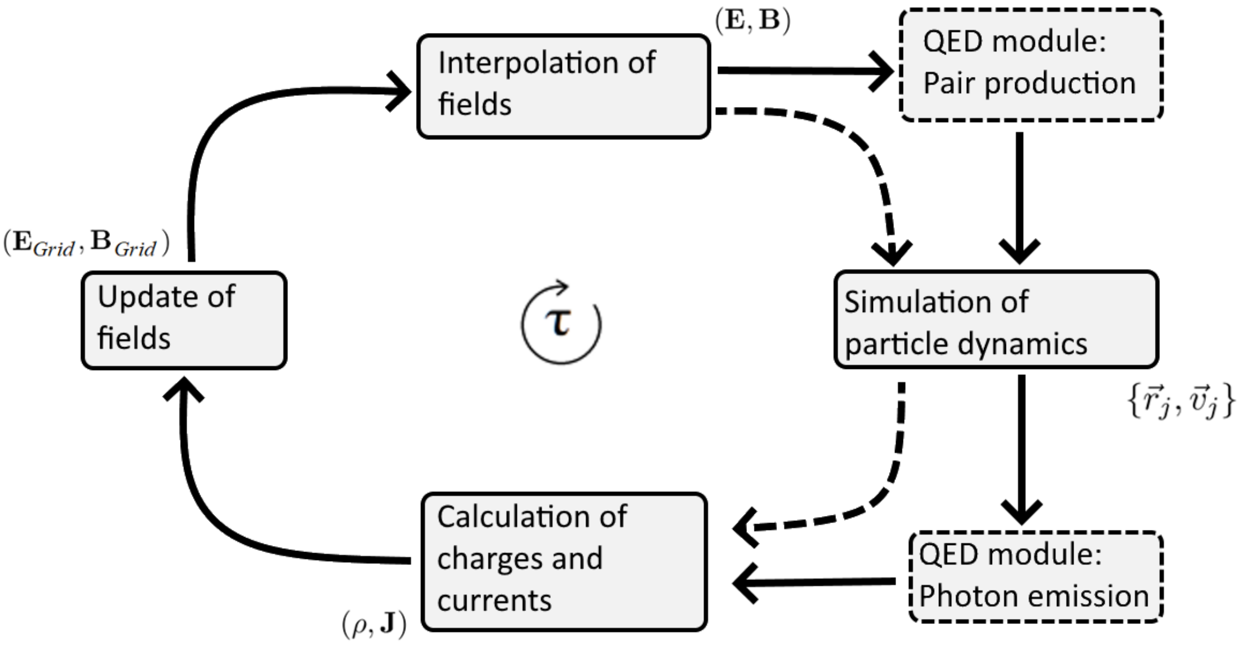
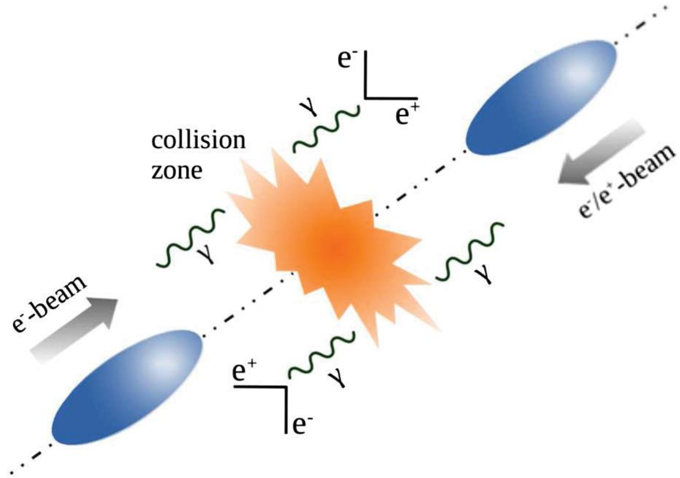
Modified PIC cycle



Beam-beam collisions

- Utilize particle beams instead of intense laser fields
- Particles of one beam experience the strong fields of the other
- Reach fields $> E_{\mathrm{cr}}$ **in reference frame**
  - Creation of secondary particles
  - Study of non-perturbative QED possible



V. Yakimenko et al., *PRL* **122**, 190404 (2019)

Beamstrahlung-enhanced disruption

- Particles are subject to betatron oscillations during beam collision
- Disruption time: characteristic time for particle to reach central axis
  - Disruption parameter D quantifies beam distortion

$$D_0 = \frac{\omega_b^2 \sigma_z^2}{2\gamma c^2}$$

- Generally: small disruption parameter wanted
  - Reduce interference in precision measurements

- Beamstrahlung: radiation reaction losses due to bent particle trajectory
  - Usually unwanted for colliders

Goal: find expression for disruption parameter with RR

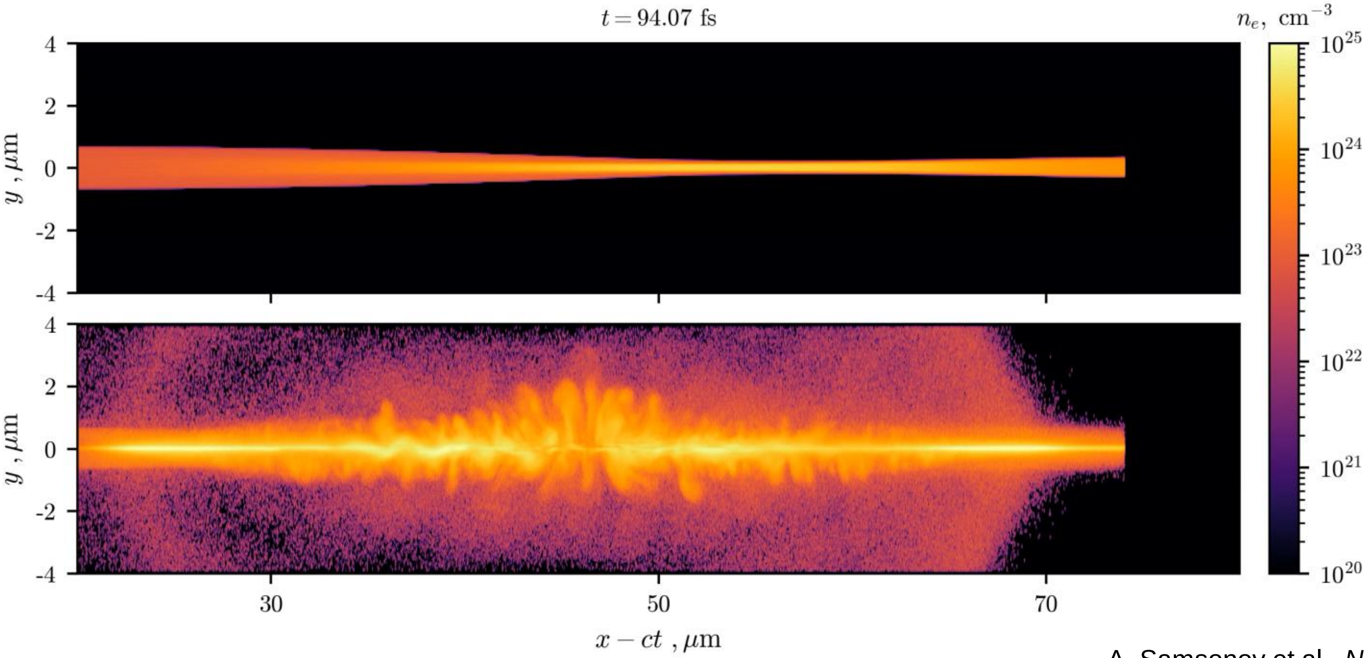
A. Samsonov et al., *NJP* **23**, 103040 (2021)

Beamstrahlung-enhanced disruption

Collision of two cylindrical beams (Movie)

QED off

QED on



A. Samsonov et al., *NJP* **23**, 103040 (2021)

Beamstrahlung-enhanced disruption

Results

- For $\chi_0 = 10$: high yield of electron-positron pairs
  - Formation of a QED cascade
- Simulations with QED exhibit kink instability
  - QED effects are probabilistic → changes in symmetric particle distribution

- Requirement „$D \to 0$“ poses restrictions on beam length and diameter
  - Considering RR strengthens restrictions
- Estimate for disruption parameter in QED regime ($\chi_0 \gg 1$):

$$\frac{D}{D_0} \approx 30.8(\epsilon_b[100\ \mathrm{GeV}]\, n_e[10^{21}\ \mathrm{cm}^{-3}]\, r_b[\mu\mathrm{m}]^4)^{-1/3}$$

A. Samsonov et al., *NJP* **23**, 103040 (2021)

PIC codes for plasma-based acceleration

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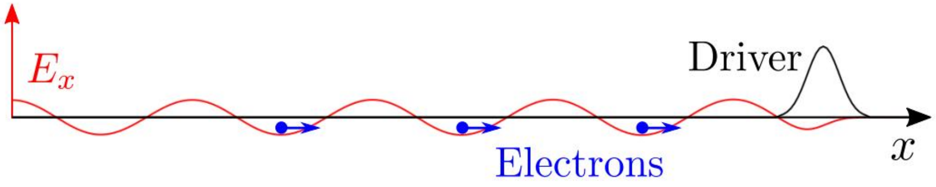
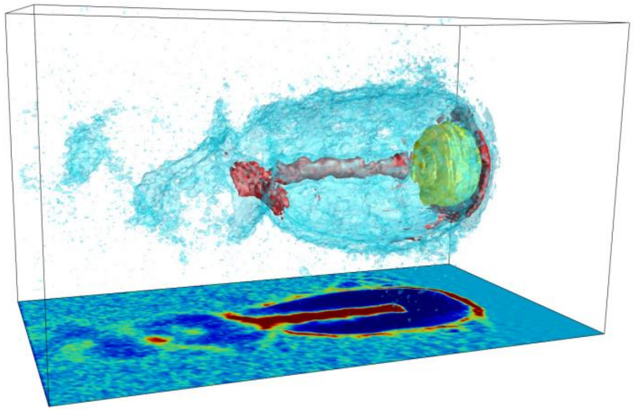


Bubble regime

A.Pukhov & J.Meyer-ter-Vehn, *Appl. Phys. B*, **74,** *p*.355 (2002)

- Plasma wake fields offer a promising alternative to conventional particle accelerators



- Ultra-short laser pulse or particle bunch creates driven plasma oscillation
- Secondary particles can become accelerated in the wake
- Accelerating field reaches hundreds of GV/m
- Simulations carried out with Virtual Laser Plasma Lab

The multi-scale problem

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The plasma acceleration has several very disparate scales

1. Small scale: laser wavelength λ or plasma wavelength λ_p

2. Medium scale: driver length

3. Large scale: acceleration length $L_A=10^4 \ldots 10^7\ \lambda, \lambda_p$

Analytical methods to bridge the scales gap

First-principles PIC codes are universal but computationally expensive. Efficient analytical methods exist to handle the multi-scale problems.

1. Envelope approximation for the laser removes the laser wavelength λ scale

2. Quasi-static approximation explicitly separates the fast coordinate $\zeta = z - ct$ and the slow evolution time $\tau = t$

Any approximation means some physics is neglected though…

Quasi-static approximation

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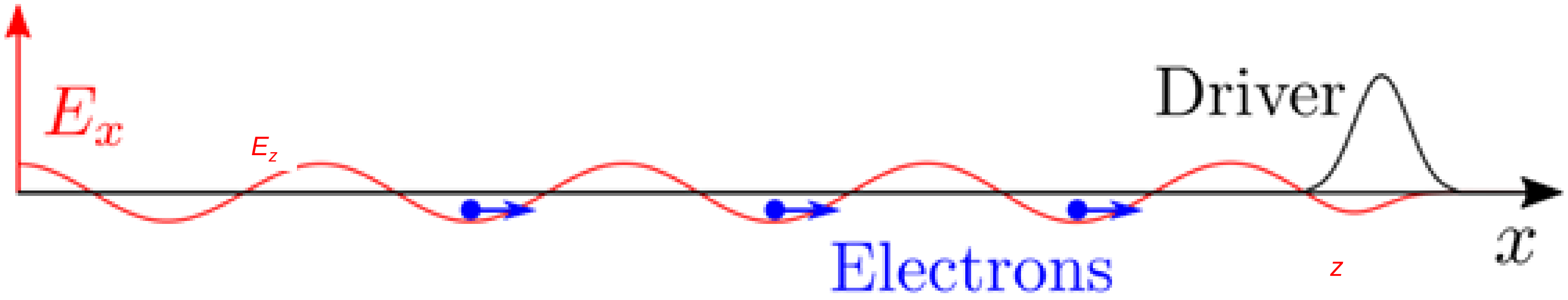

We separate the fast coordinate ζ=*z-ct*
and the slow evolution time τ=*t.*

$$\frac{\partial}{\partial t} << \frac{\partial}{\partial \zeta}$$

It is assumed that the driver does not change during the time it passes its own length

Quasi-static approximation Field equations

We change variables to the fast coordinate ζ=*z-ct*
and the slow evolution time τ=*t* in Maxweel Eqs.:

and neglect
the time derivatives
in (1) and (2), so that

$$\partial_t = -\partial_\xi \qquad \partial_\tau = 0$$

$$\nabla \times B = j + \partial_t E \qquad (1)$$

$$\nabla \times E = -\partial_t B \qquad (2)$$

$$\nabla \cdot B = 0 \qquad (3)$$

$$\nabla \cdot E = \rho \qquad (4)$$

Thus, the quasi-static approximation cannot treat radiation anymore!

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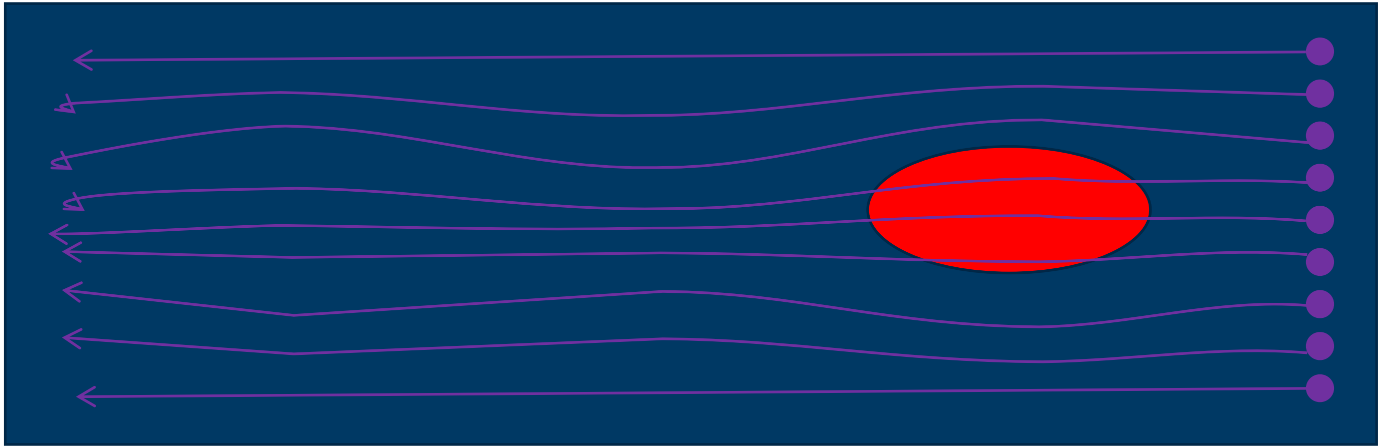
Pushing particles

In quasi-static codes, one seeds a single layer of particles at the leading edge of the simulation box and pushes them through the driver



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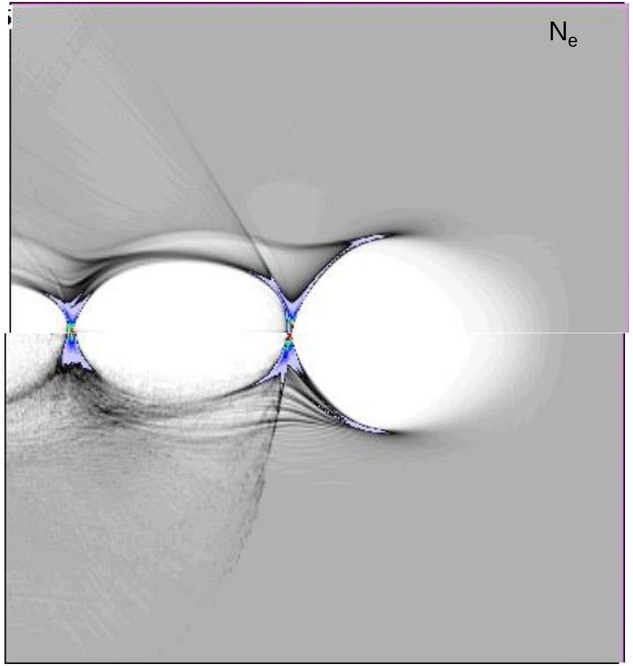
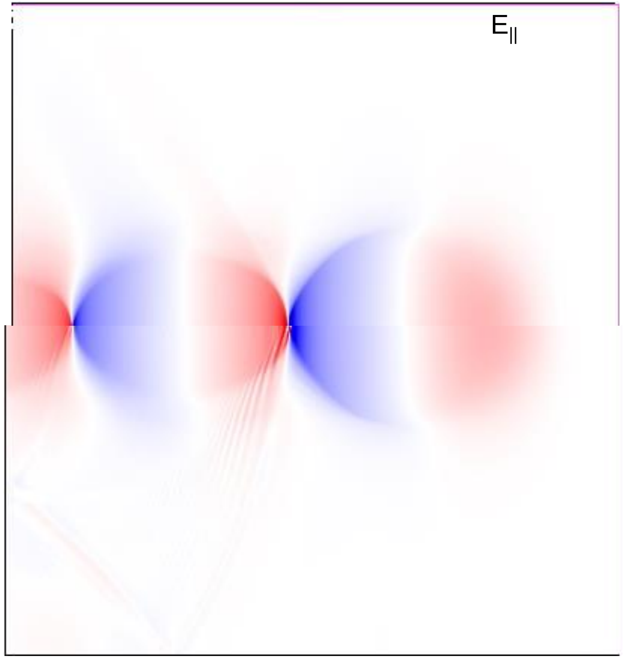
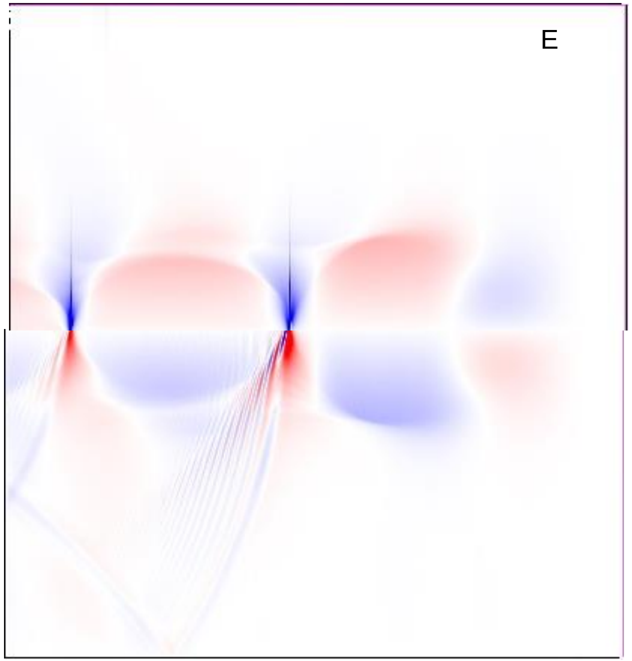
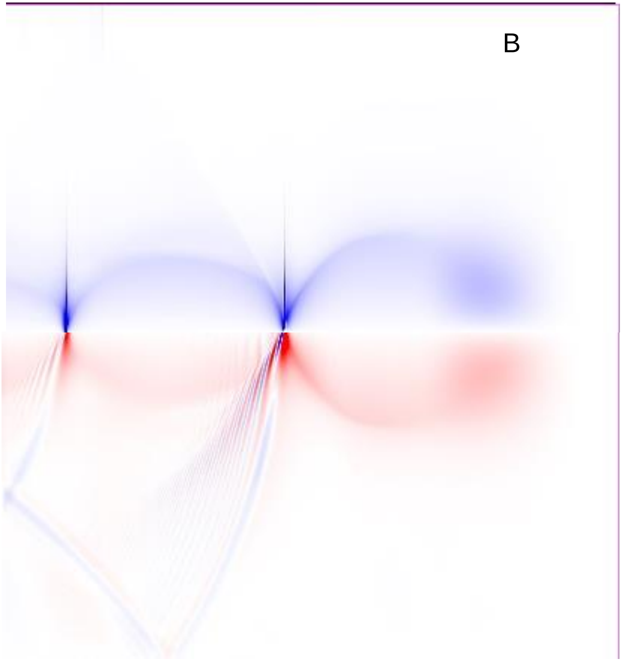
Compare
quasi-static qs-VLPL3D vs explicit VLPL3D





Compare quasi-static VLPL3D vs explicit





Challenges for full scale simulations

What limits the time- and space-steps

- Witness betatron frequency limits the slow time step:
  injecting 1GeV witness limits times step to $c\tau \leq 30\mathrm{mm}$
  for 300m acceleration distance we need $\sim 10^4$ time steps

- The simulation box must be ~5 plasma wavelengths because of dephasing

- Step in ξ must be small enough, $\Delta\xi k_p \sim 0.1$

- Because of the low normalized emittance, $\varepsilon_n \sim 100\mathrm{nm}$, the witness bunch pinches to very small transverse sizes. The radial resolution must be $\sim 10\mathrm{nm}$

- Reliable results require full 3D simulations, or at least quasi-3D.
  2D radially symmetric should be considered with a grain of salt

Available codes

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- Full electromagnetic codes: VLPL3D, OSIRIS, WARP-X…
- Quasi-3D full electromagnetic codes: FBPIC
- Full 3D quasi-static codes: QV3D, HIPACE++, LCODE3D
- 2D radially-symmetric quasi-static codes: LCODE, WAKE-T
- quasi-3D quasi-static PIC code: QUICKPIC (?)

All codes have their advantages and caveats…

What are we simulating?

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Alternative
configurations

Proton-driven acceleration. Why?

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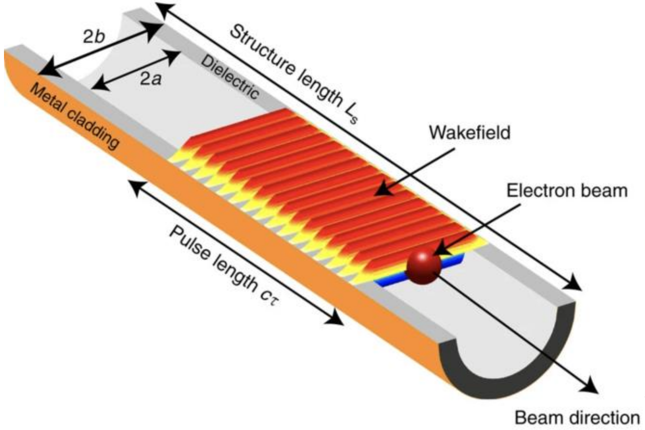
Universität Düsseldorf

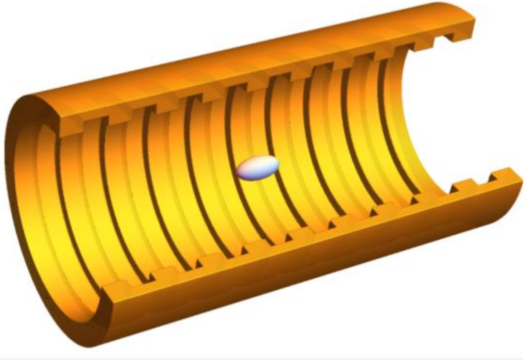
- We know how to accelerate protons to multi-TeV energies
- This energy must be converted into energy of leptons
- Single stage acceleration
- Wake field acceleration

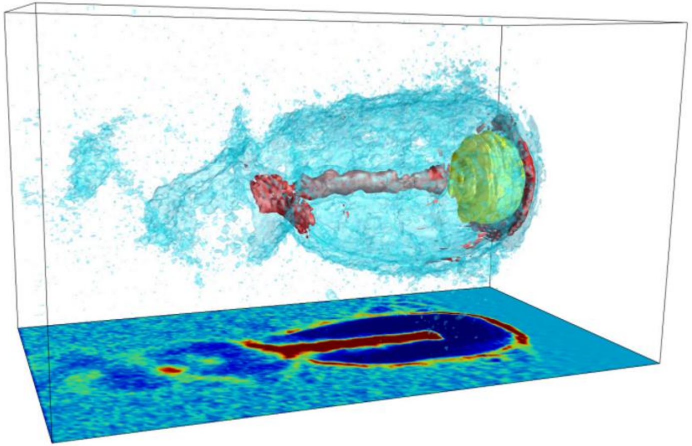
Options for acceleration

Dielectric structures | Corrugated metallic pipe | Plasmas

~1 GV/m | ~100 MV/m | 1GV…10TV/m







BBU limits wakefield in structures

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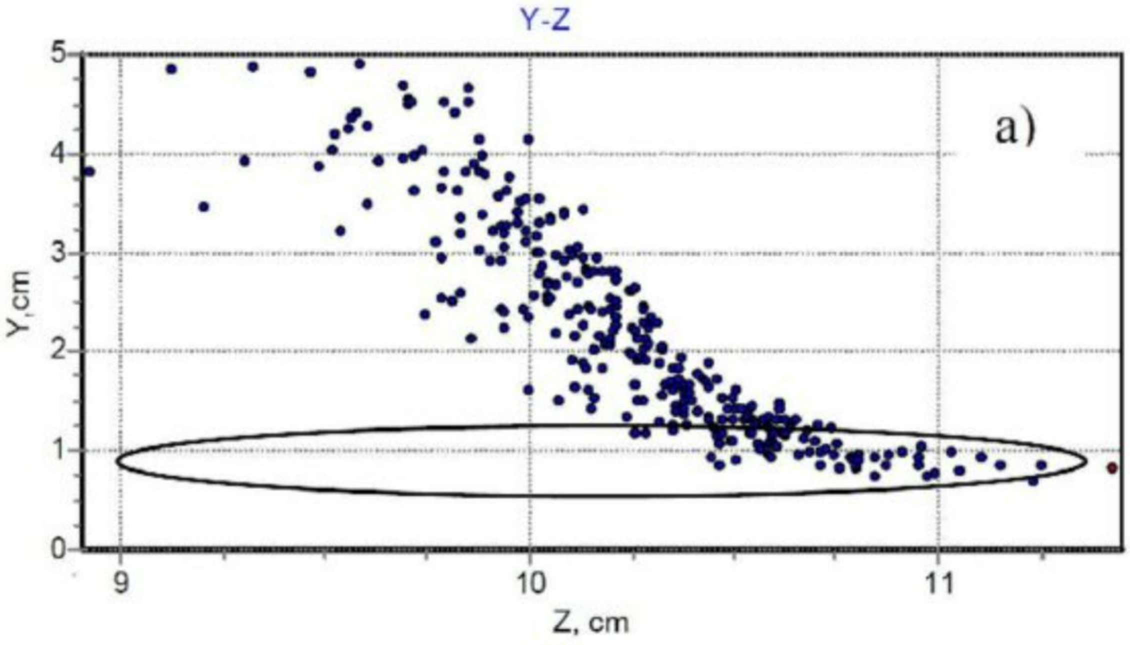
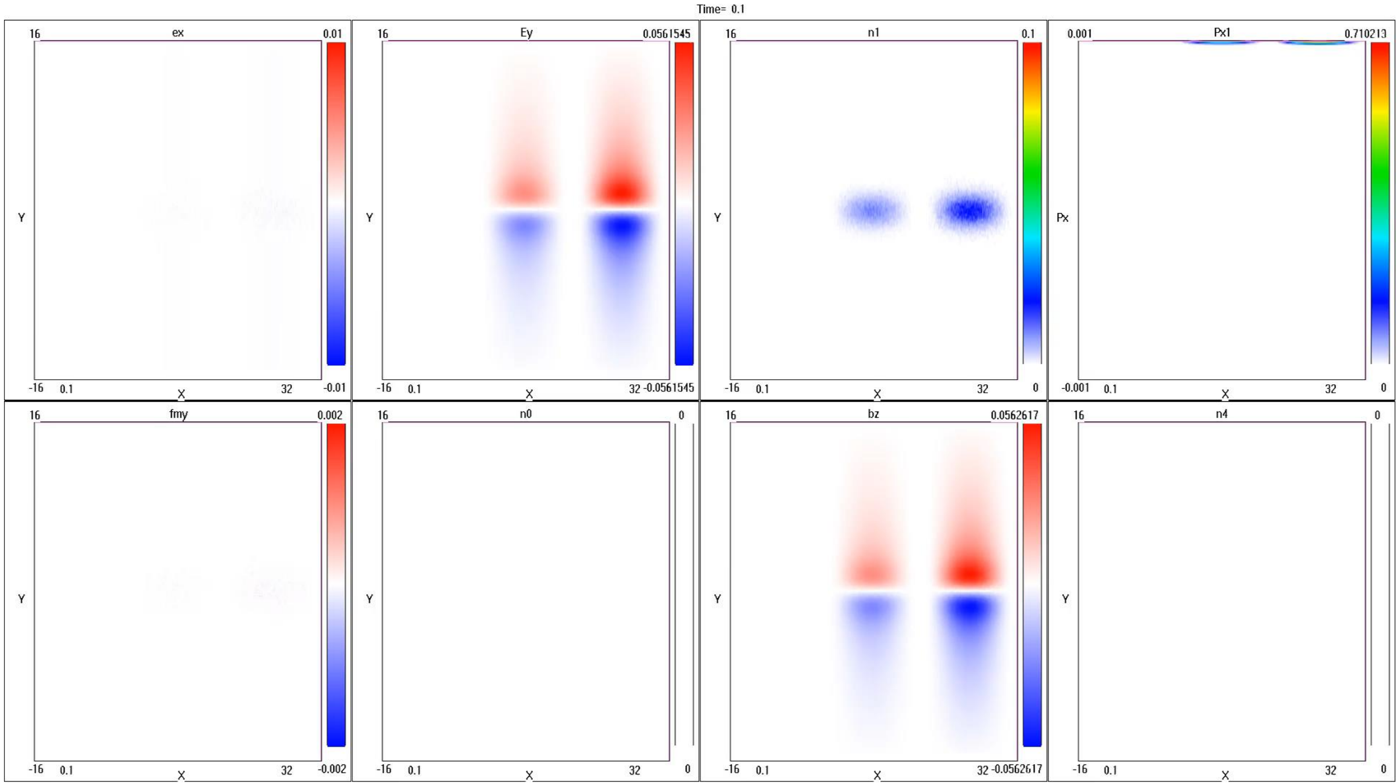


Tail of the bunch is attracted to the wall

Fig. by Alexandr Moiseevich Altmark

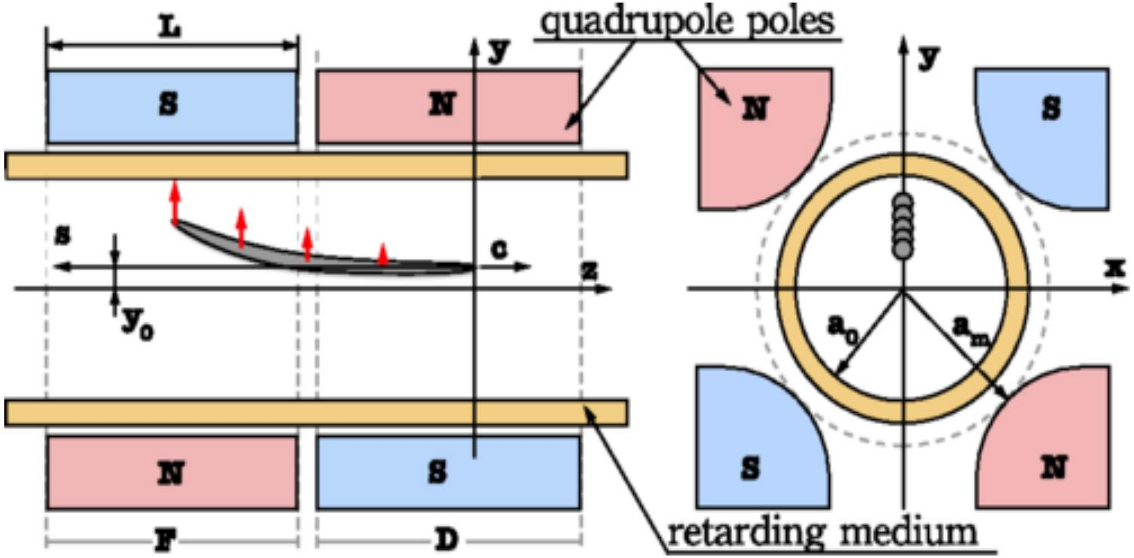
The main limit for SWFA: Beam Break Up (BBU)



Driver and witness in a corrugated waveguide

VLPL3D simulation

Strong focusing (and chirp) to stabilize the BBU



1.5 T quadrupoles can provide (may be) up to 100 MV/m stable accelerating fields

S. S. Baturin and A. Zholents Stability condition for the drive bunch in a collinear wakefield accelerator Phys. Rev. Accel. Beams **21**, 031301 (2018)

SWFA and Plasma

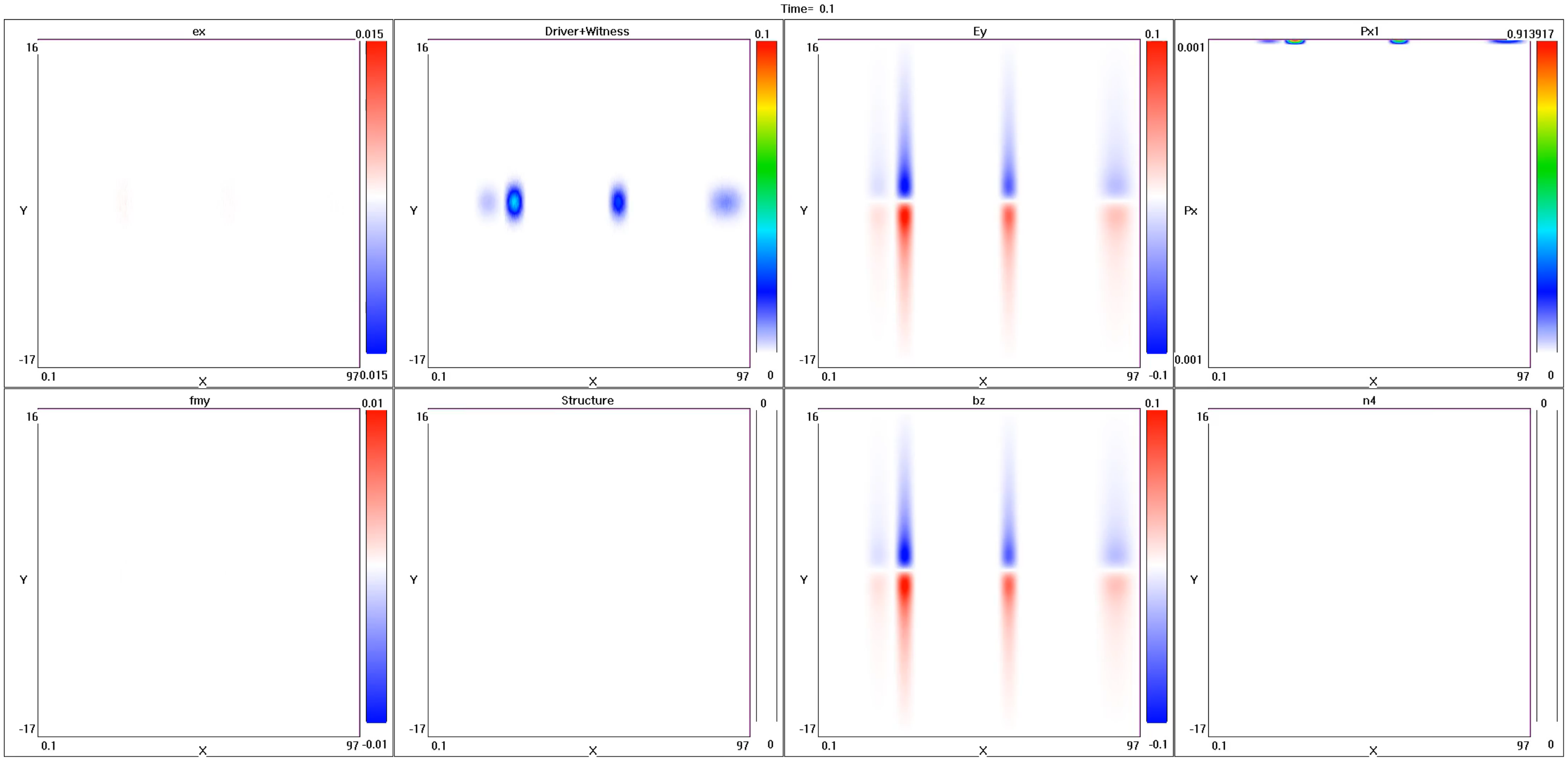
Let us decouple acceleration and focusing
We use the structure to support accelerating wake
while plasma provides focusing only!

Plasma can provide focusing strength as high as

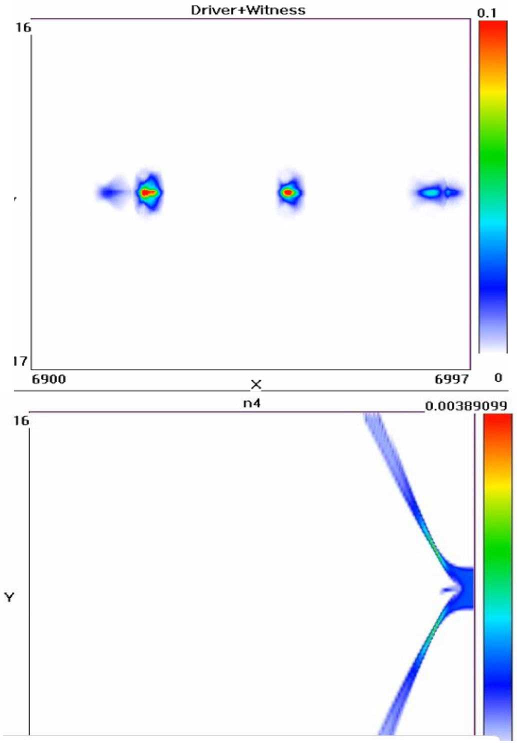
$$\frac{B}{r}\,[\mathrm{T/mm}] \approx 6 \cdot 10^4 \sqrt{\frac{n_e[\mathrm{cm}^{-3}]}{10^{15}}}$$

when all electrons are swept away from the plasma channel

Narrow plasma column: stabilization



SWFA and Plasma



bunches

plasma electrons

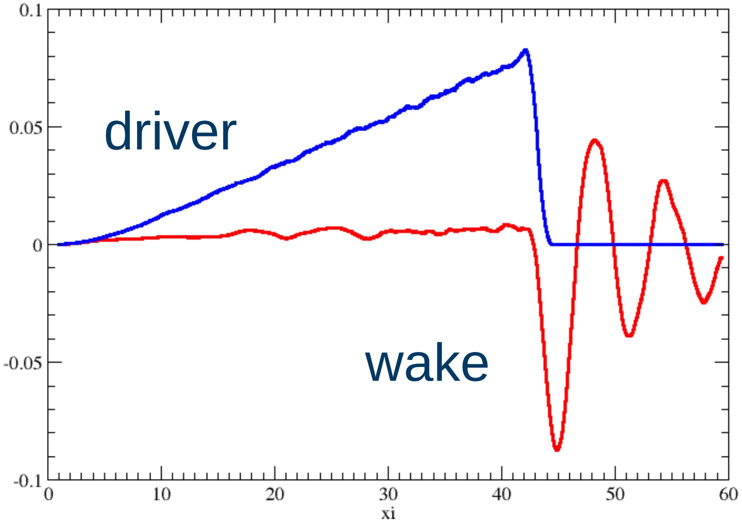
The leading bunch scatters away electrons off the narrow plasma column

The trailing bunches stay stable and tightly focused by the ion column

How to create the plasma column?

Driver with triangular profile

- Wake field: $E(\xi) = \int_0^{\xi} j_0 \frac{\xi'}{\sigma_z} \cos(\xi - \xi')\, d\xi' = j_0 \begin{cases} \frac{1}{\sigma_z}(1 - \cos\xi), & \xi < \sigma_z \\ \sin(\xi - \sigma_z), & \xi > \sigma_z \end{cases}$



Triangular drivers with length $\sigma_z \gg 1/k_p$

can excite wakes
with large transformer ratios

$R = k_p \sigma_z \gg 1$

Positions are open at Dusseldorf Uni

- Positions at Ph.D. and PostDoc level are open at HHU, Dusseldorf
- Focus on Relativistic Plasma Simulations
- If interested, please, contact

  pukhov@tp1.hhu.de