Influence of the momentum compaction factor on the beam dynamics in muon RCSs

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The momentum compaction factor α_c

Slide with input from from L. Soubirou, 25th HEMAC

- **Definition:** $\alpha_c = \frac{dL/L}{dp/p} = \frac{1}{L}\oint \frac{D_x(s)}{\rho(s)} ds$
- Slip factor: $\eta = \alpha_c - \frac{1}{\gamma^2}$ with $\gamma \gg 1$ for muons

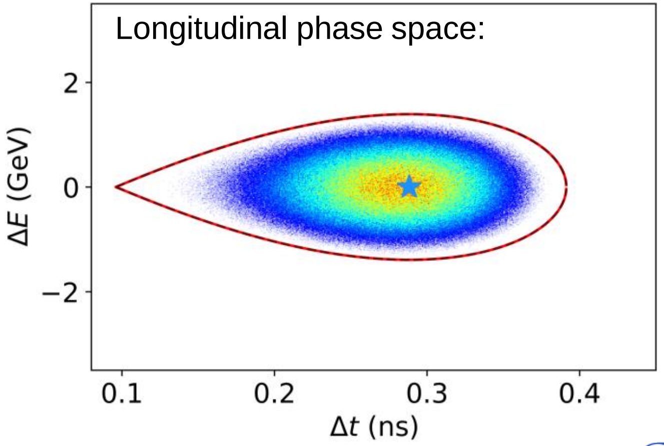
- **Effects on the longitudinal beam dynamics:**

Synchrotron tune: $Q_s = \sqrt{-\frac{h\boldsymbol{\eta}}{2\pi E_s} eV cos(\phi_s)}$

RF bucket area: $A_0 = \frac{8\sqrt{2}}{2\pi h f_{rev}} \sqrt{\frac{E_s\, V}{\pi\, h\, \boldsymbol{\eta}}}$

Energy acceptance : $\Delta E \propto \sqrt{\frac{E_s\, V}{h\, \boldsymbol{\eta}}}$

for Φ_s=0 or Φ_s=π

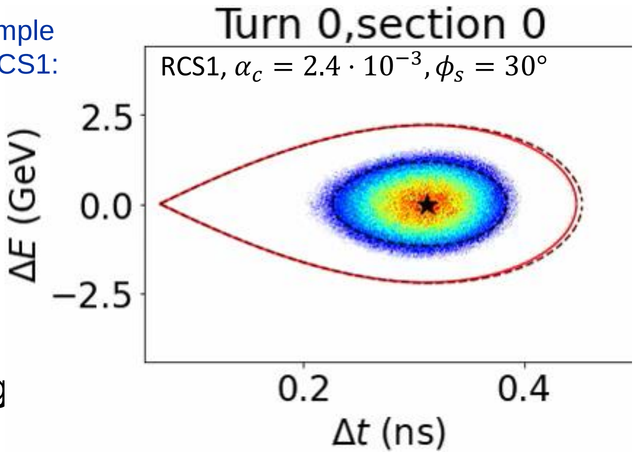


First-guess for momentum compaction factor

→ The transition gamma γ_t ($\alpha_c = \frac{1}{\gamma_t^2}$) was assumed to be similar to that of the SPS

→ $\boldsymbol{\gamma_t = 20.41, \alpha_c = 2.4 \cdot 10^{-3}}$

Example of RCS1:



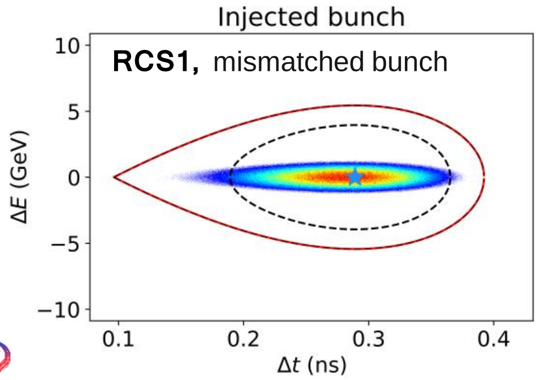
Consequences on beam dynamics:

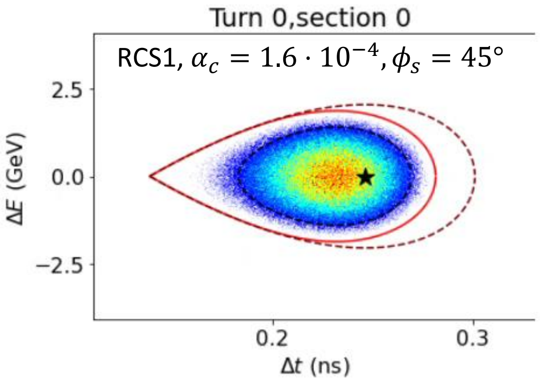
- **Extremely high synchrotron tune Q_s of up to 1.5**
- **Required that $n_{\mathrm{RF}} \neq 1$**
- **Effect of induced voltages mitigated due to phase mixing**

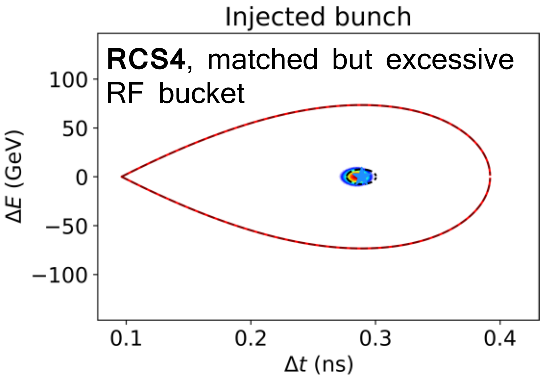
→ **RF voltage has to be sufficiently high to compensate that**

Calculated momentum compaction factor

- **Scales as $1/n_c^2$ with the number of cells per arc and is calulated as:** $\alpha_{c,1f} = \frac{\theta_c \langle D \rangle}{L_c} = \left(\frac{\pi}{n_c}\right)^2 \left(\frac{1}{sin^2\left(\frac{\mu}{2}\right)} - \frac{1}{12}\right)$
- Extreme changes with respect to initial guess:
  - RCS1: from $\gamma_t = 20$ up to 79 → **15 times smaller momentum compaction factor α_c**
  - RCS4: up to 8 times larger transition gamma, **62.5 times smaller α_c**
- Large effects on RF, RF bucket, beam dynamics, bunch length, etc.
- Probably strong mismatch in RCS1 due to bunch length reduction, extreme changes of the RF bucket in RCS4







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