Tau-pair invariant mass estimation using MLE and collinear approximation

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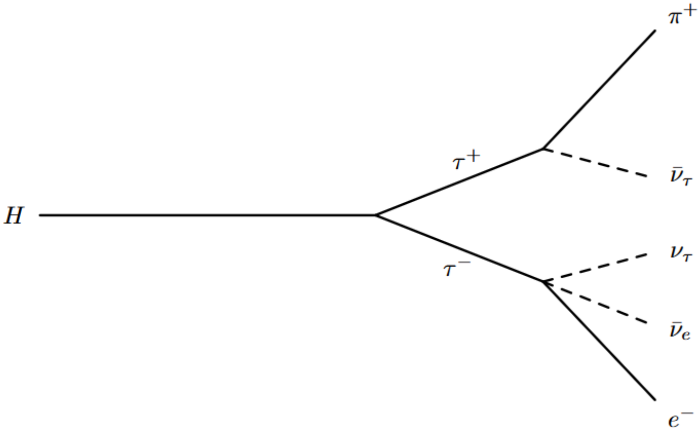
Higgs mass reconstruction



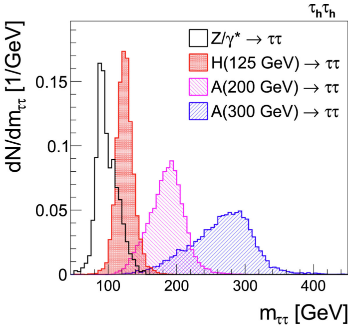
Information we have:

- $p_{1,2}^{\text{vis}}$ – four-momentum of visible products
- $p_x^{\text{rec}}, p_y^{\text{rec}}$ – reconstructed Missing Transverse Energy
- V_{cov} – covariance matrix of reconstructed MET

Existing solutions

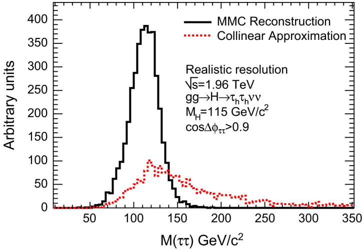
- Matrix element techniques (CMS)

  arXiv:1603.05910 [hep-ex]



- Missing Mass Calculator (ATLAS)

  arXiv:1012.4686 [hep-ex]



fastMTT

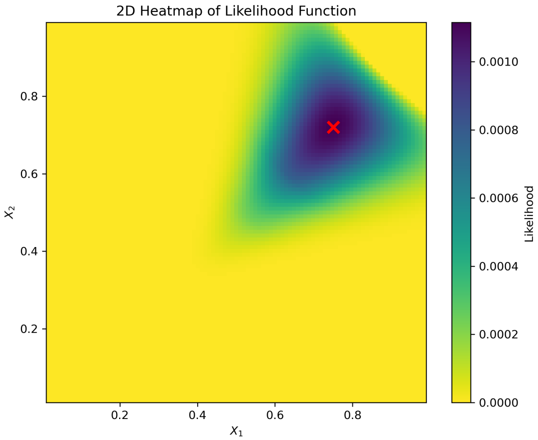
fastMTT algorithm reconstruct di-tau invariant mass with good mass resolution and high computing performance (e.g. $\sim$ 100 times faster then CSVfit).

- It estimates likelihood of invariant mass of taons by Matrix Element Method (in the same fashion as CSVfit).
- Then it uses Collinear Approximation to simplify most of the terms, that is needed to calculate.
- Using that, the result is calculated analytically.
- Finally, a scan over the space of possible masses is performed (grid search) and the most probable one is chosen.

Maximum Likelihood Estimation

We maximize likelihood function to obtain the most probable parameter value for given data:

$$\hat{m} = \text{arg max } \mathcal{L}(m|\text{data})$$



Matrix Element Method

We calculate likelihood as:

$$\mathcal{L}(m|\text{data}) = \frac{32\pi^4}{s} \int \prod_{j}^{n} \frac{d^3 p_j}{(2\pi)^3 2E_j} \prod_{i=1}^{2} |\mathrm{BW}_{\tau}^{(i)}|^2$$

$$|\mathrm{M}_{\tau\to\ldots}^{(i)}|^2 \mathrm{TF}(\not{p}_x^{\text{rec}}, \not{p}_y^{\text{rec}} | \not{p}_x^{\text{true}}, \not{p}_y^{\text{true}}) \frac{1}{m_{\tau\tau}^{\kappa}}$$

$\mathrm{BW}_{\tau}^{(i)}$ – taon Breit-Wigner distribution function

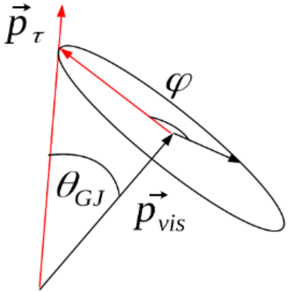
$\mathrm{M}_{\tau\to\ldots}^{(i)}$ – matrix element for taon decay

$\mathrm{TF}(\not{p}_x^{\text{rec}}, \not{p}_y^{\text{rec}} | \not{p}_x^{\text{true}}, \not{p}_y^{\text{true}})$ – transfer function between true and reconstructed MET

$\frac{1}{m_{\tau\tau}^{\kappa}}$ – bayesian regularization term (accelerates algorithm convergence)

Collinear Approximation

- We usually introduce Gotfried-Jackson angle θ_{GJ} to parametrize angle between visible products and taons.
- In the collinear approximation we assume that $\theta_{\mathrm{GJ}} = 0$.
- It is well justified with the energies of Z^0 or H, much larger than m_τ.



fastMTT algorithm

$$\mathcal{L}(m|\text{data}) = \frac{32\pi^4}{s}\text{TF}(\not{p}_x^{\text{rec}}, \not{p}_y^{\text{rec}}|\not{p}_x^{\text{true}}, \not{p}_y^{\text{true}})\frac{1}{m_{\tau\tau}^{\kappa}} \cdot I$$

$$I = \int \prod_j^n \frac{d^3 p_j}{(2\pi)^3 2E_j} \prod_{i=1}^2 |\text{BW}_\tau^{(i)}|^2 |\text{M}_{\tau\to\ldots}^{(i)}|^2$$

Using Collinear Approximation:

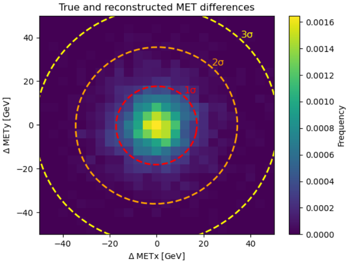
$$I = \int_{x_{1,\min}}^1 dx_1 \int_{x_{2,\min}}^1 dx_2 \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \int_0^{1-x_1 m_\tau^2} dm_{\nu\nu}^2\, \delta\left(m_{\tau\tau} - \frac{m_{\text{vis}}}{\sqrt{x_1 x_2}}\right)$$

$$= 4\pi^2 m_\tau^2 \frac{m_{\text{vis}}^2}{m_{\tau\tau}^3}\left[\log(x_{2,\max}) - \log(x_{2,\min}) + \left(\frac{m_{\text{vis}}}{m_{\tau\tau}}\right)^2\left(\frac{1}{x_{2,\max}} - \frac{1}{x_{2,\min}}\right)\right]$$

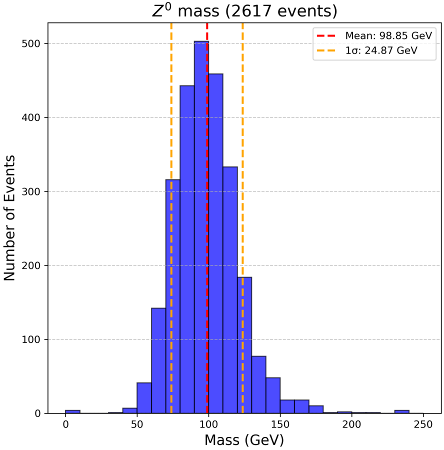
Performance

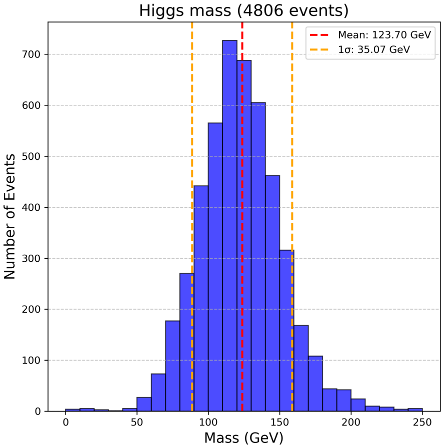
We tested the algorithm using:

- MC samples from Pythia 8.3 with CUEP8M1 tune and different Higgs masses
- Delphes 3.5.0 with standard CMS card (and small adjustments)
- No pile-up
- MET covariance matrix calculated by comparing simulated and reconstructed MET values
- Different Higgs masses

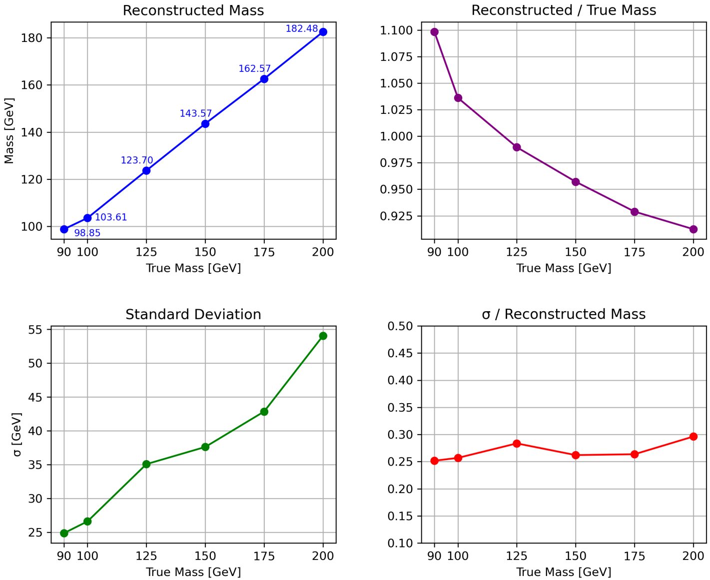


Performance





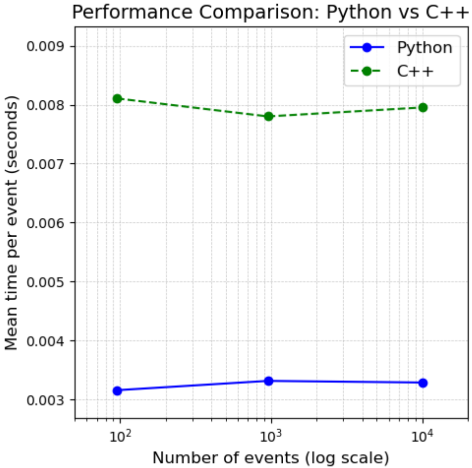
Performance



Computing Performance

```
for(int iX2 = 1; iX2<nGridPoints;++iX2){
  x[1] = iX2*gridFactor;
  for(int iX1 = 1; iX1<nGridPoints;++iX1){
    x[0] = iX1*gridFactor;
    lh = myLikelihood.value(x);
    if(lh<bestLH){
      bestLH = lh;
```

```
X1 = np.arange(1, nGridPoints) * gridFactor
X2 = np.arange(1, nGridPoints) * gridFactor
# Cartesian product
pairs = np.column_stack((np.repeat(X1, len(X2)),
                         np.tile(X2, len(X1))))
lh = self.myLikelihood.value(pairs)
minimum = np.argmin(lh, axis=1)
```



Final remarks

- fastMTT is algorithm that reconstructs invariant mass of system with two taons.
- It is faster then other algorithms used by CMS.
- We plan to develop it further (event by event uncertainty, Z/H mass constraints for better momentum estimation).
- We plan to document and publish the algorithm (paper + code) for easy use outside CMS (ATLAS, FCC, etc.).

Already used by CMS e.g. in: HIG-22-004 or in PLB 857 (2024) 138964