

Jitter amplification in booster linac

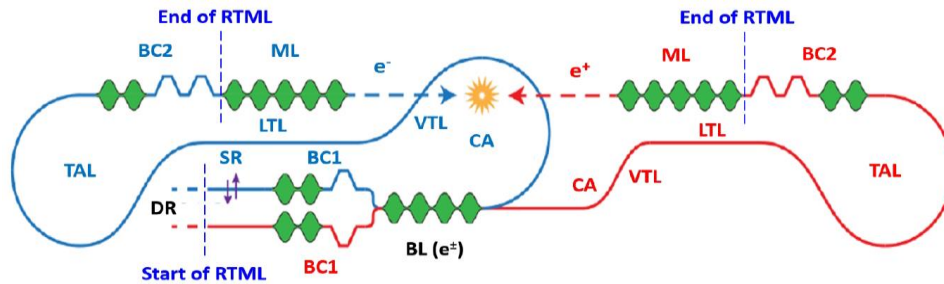
Yongke Zhao

For discussion only

25/09/2024

Introduction

- CLIC RTML layout



- Baseline Booster Linac (BL) lattice

- CLIC “L-band” structure
- 8 structures per FODO cell
- Distance between quadrupoles: 7.5 m
- 272 structures. $G = 15.089$ MV/m

- Beam parameters

Parameter	Unit	Entrance	Exit
Number of bunches per train		352	
Number of particles per bunch		5.2×10^9	
Beam energy	GeV	2.86	9
Bunch length (σ_z)	μm	1800	~ 70
Energy spread (σ_E/E)	%	0.12	< 1.7
Horizontal emittance ($\epsilon_{n,x}$)	nm·rad	700	< 800
Vertical emittance ($\epsilon_{n,y}$)	nm·rad	5	< 6

- Emittance budgets (full RTML)

Emittance budgets	$\epsilon_{n,x}$	$\epsilon_{n,y}$
Perfect machine	< 800	< 6
Static imperfections	< 820	< 8
Static and dynamic imperfections	< 850	< 10

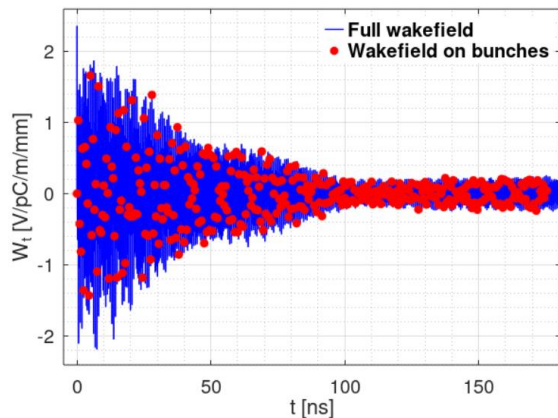
- CLIC “L-band” structure parameters

Parameter	Unit	BC1
Structure name		CLIC L-band
RF frequency	GHz	1.999
Structure length	m	1.5
Number of cells		30
Phase advance per cell	$^\circ$	120
Working RF phase	$^\circ$	90
First iris radius	mm	20
Last iris radius	mm	14
First iris thickness	mm	8
Last iris thickness	mm	8

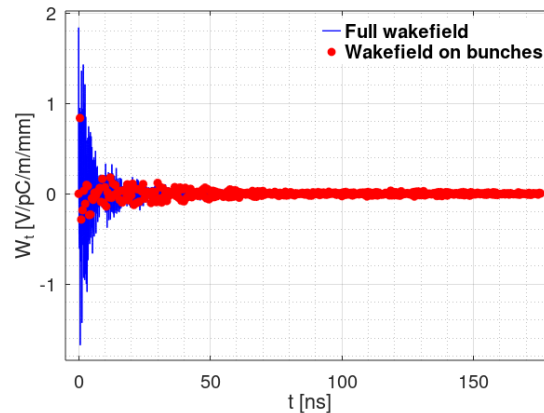
Long-range wakefield

- Using wakefields directly from Adnan:
 - Version 0: Wakefield not used
 - ~~Version 1: $a_0 = 13$ mm. Wakefield with $\text{sum}(|W_t|) = 93.38$ V/pC/m/mm~~
 - Version 2: $a_0 = 15$ mm. Wakefield with $\text{sum}(|W_t|) = 9.86$ V/pC/m/mm
 - Version 3: $a_0 = 16$ mm. Wakefield with $\text{sum}(|W_t|) = 9.36$ V/pC/m/mm

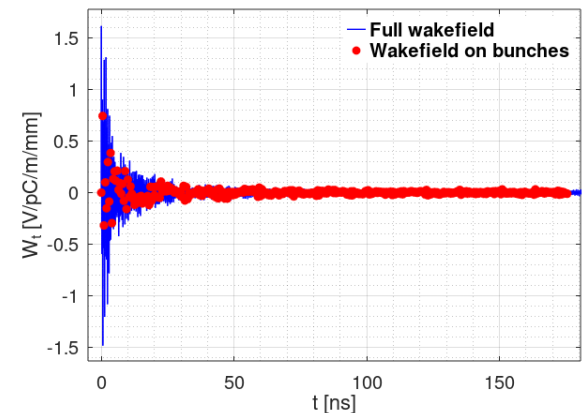
Version 1



Version 2



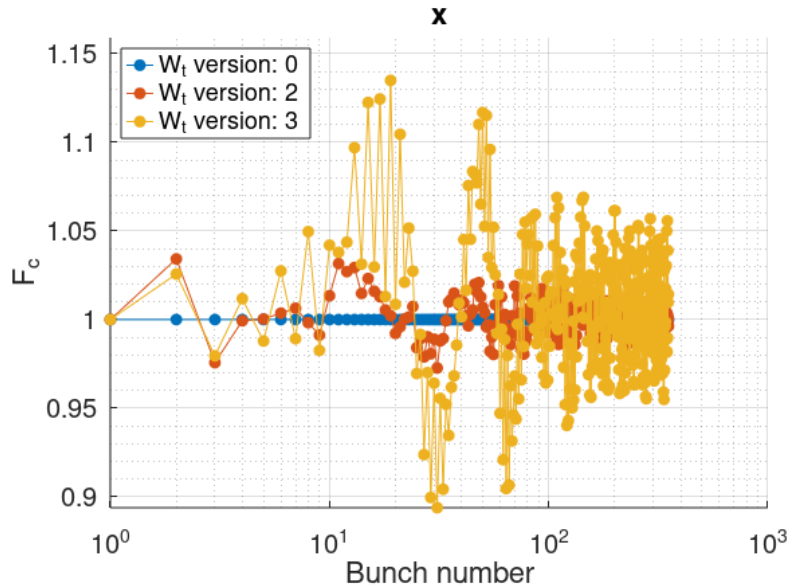
Version 3



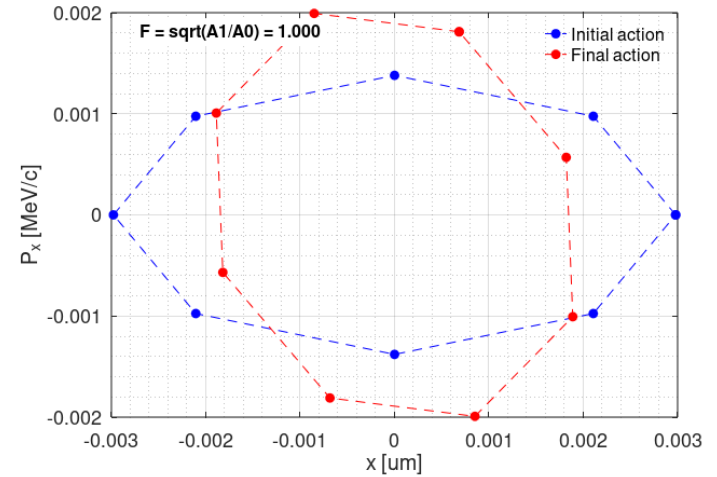
Coherent jitter

- **Single particle tracking** simulation
- Jitter considered: $J = 10\% * \sigma(x, px)$
- 8 trains evenly spaced on ellipse
- 352 bunches per train
- Jitter amplification factor (F) definition in this case:

$$F_c = \frac{F_{W \neq 0}}{F_{W = 0}}, \text{ where } F = \max_{\text{bunches}} \left(\frac{J_{\text{final}}}{J_{\text{initial}}} \right) = \max_{\text{bunches}} \left(\sqrt{\frac{A_{\text{final}}}{A_{\text{initial}}}} \right)$$



Action of last bunch, wakefield not used



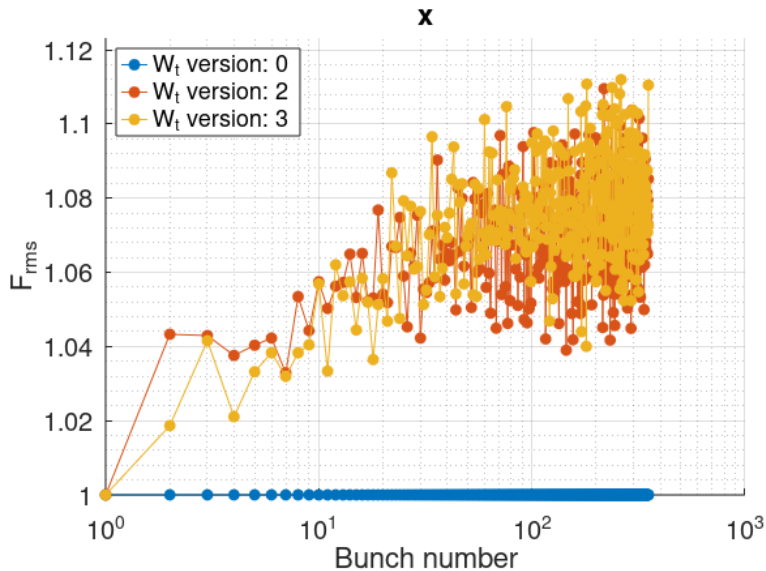
A: action (area of ellipse)

Version	Average F	Maximum F
2	1.001	1.034
3	1.007	1.135

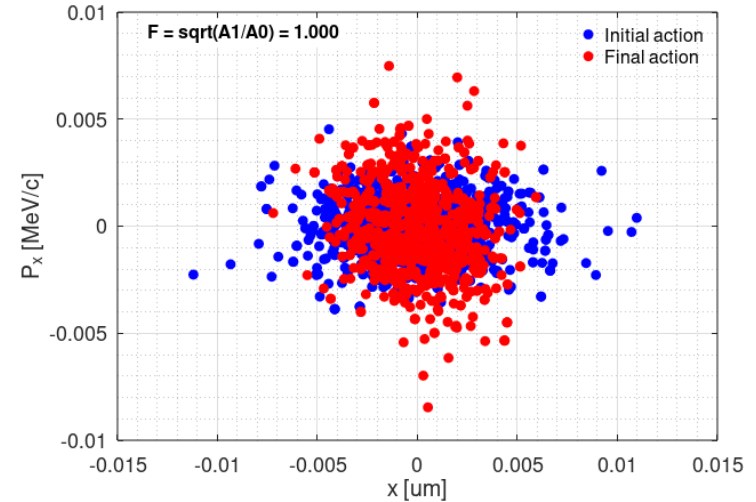
Incoherent jitter

- **Single particle tracking** simulation
- Jitter considered: $J = 10\% * \sigma(x, p_x)$
- 1000 trains randomly spaced in phase space
- 352 bunches per train
- Jitter amplification factor (F) definition in this case:

$$F_{\text{rms}} = \frac{F_{W \neq 0}}{F_{W=0}}, \text{ where } F = \frac{1}{N_{\text{bunches}}} \sum \frac{J_{\text{final}}}{J_{\text{initial}}} = \frac{1}{N_{\text{bunches}}} \sum \sqrt{\frac{A_{\text{final}}}{A_{\text{initial}}}}$$



Action of last bunch, wakefield not used



A: action (area of ellipse)

Version	Average F	Maximum F
2	1.070	1.110
3	1.076	1.112

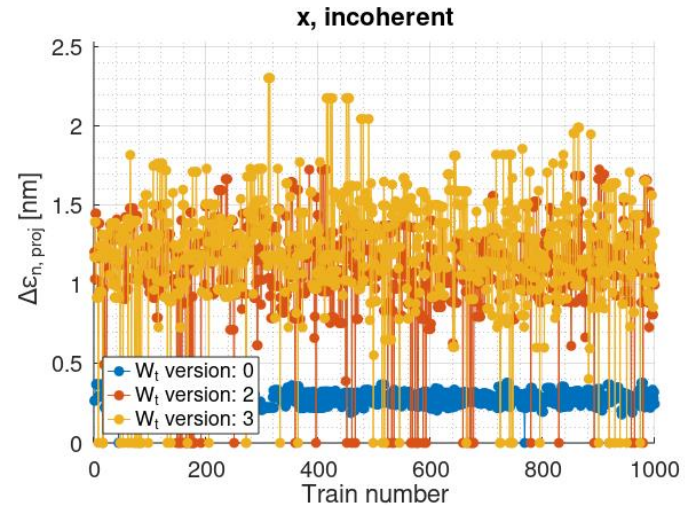
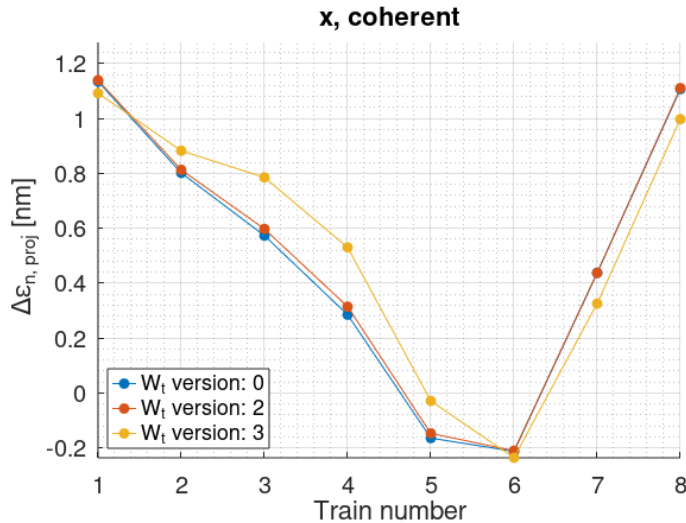
Full bunch tracking

- Full bunch tracking also performed
- Consistent with single particle tracking

Jitter amplification	V2 ($a_0 = 15$ mm)	V3 ($a_0 = 16$ mm)
Single particle tracking, $\max(F_c)$	1.034	1.135
Full bunch tracking, $\max(F_c)$	1.035	1.140
Single particle tracking, $\text{mean}(F_{\text{rms}})$	1.070	1.076
Full bunch tracking, $\text{mean}(F_{\text{rms}})$	1.073	1.081

- Conclusions:
 - $a_0 = 15$ mm has smaller jitter amplifications than $a_0 = 16$ mm
 - $F_c \sim 1.03$, $F_{\text{rms}} \sim 1.07$

Projected emittance growth



Projected emittance growth [nm]	V0 (no wake)	V2 (a0 = 15 mm)	V3 (a0 = 16 mm)
Coherent, average	0.5	0.5	0.5
Coherent, maximum	1.1	1.1	1.1
Incoherent, average	0.3	1.1	1.2
Incoherent, maximum	0.4	1.7	2.3
Incoherent, average (calculated)	0	1.0	1.2

- Calculated incoherent emittance growth:

Consistent with calculation!

$$\Delta \epsilon_{\text{projected}} = \epsilon_{\text{projected}}^{\text{final}} - \epsilon_{\text{projected}}^{\text{inital}} = \epsilon_0 \cdot (1 + J_1^2) - \epsilon_0 \cdot (1 + J_0^2) = \epsilon_0 \cdot J_0^2 \cdot (F^2 - 1)$$

Conclusions

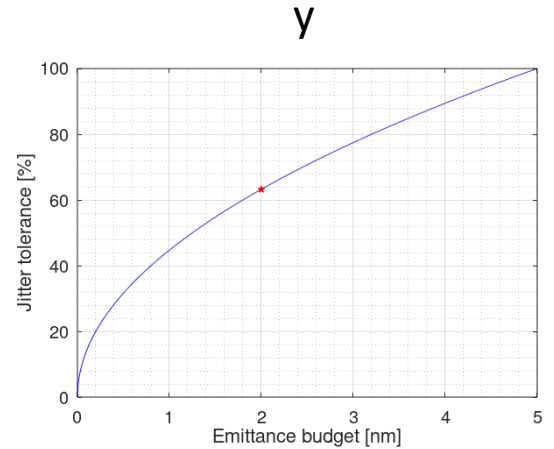
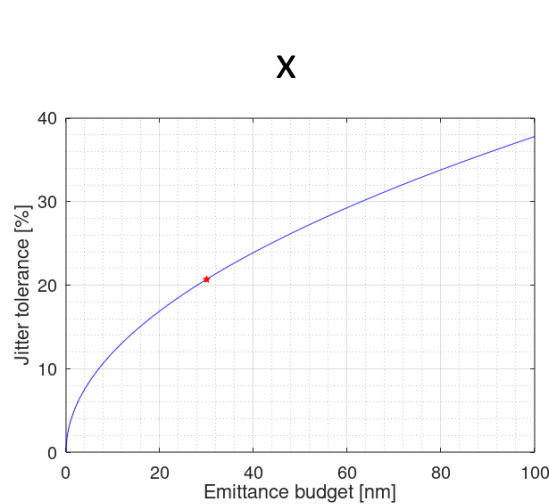
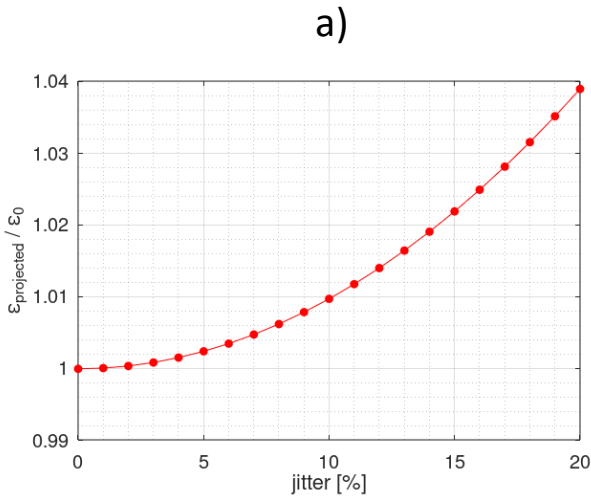
- Latest wakefield from Adnan looks much better now. $a_0 = 15$ mm is better than $a_0 = 16$ mm
- Jitter amplifications: $F_c \sim 1.03$, $F_{rms} \sim 1.07$, which is small
- Projected emittance growth (full train): ~ 1.1 nm, which is small and consistent with the calculation

BACKUP

Jitter tolerance

Test 0

- No tracking. Average of 5 randomly jittered trains (352 random bunches per train)
- Plotting
 - a) projected emittance growth as a function of jitter
 - b) jitter tolerance for x & y



Jitter tolerance definition in this case:

$$\Rightarrow \frac{\epsilon_{\text{projected}}}{\epsilon_0} \approx 1 + J^2$$

$$\Rightarrow J \approx \sqrt{\frac{\epsilon_{\text{projected}}}{\epsilon_0} - 1}$$

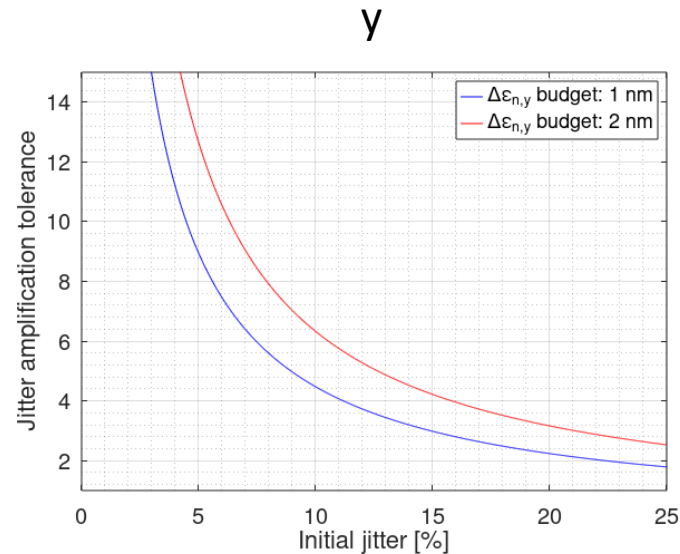
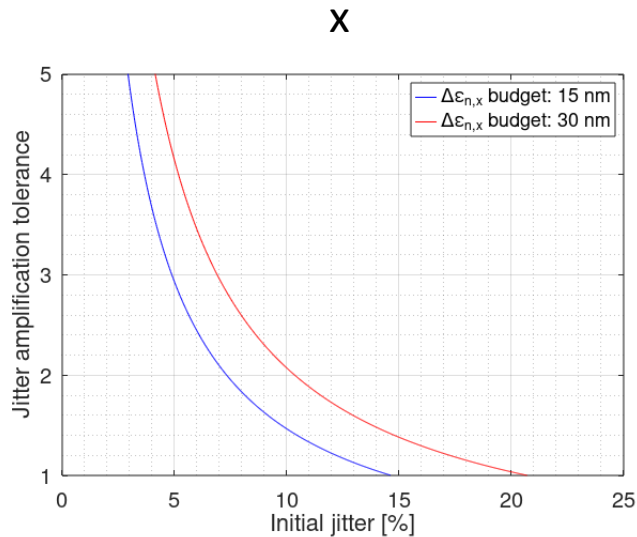
$$\Rightarrow J_{\text{max}} = \sqrt{\frac{\epsilon_{\text{projected}}^{\text{max}}}{\epsilon_0} - 1} = \sqrt{\frac{\Delta\epsilon_{\text{projected}}^{\text{max}}}{\epsilon_0}} = \sqrt{\frac{\text{Budget}}{\epsilon_0}}$$

$$\Rightarrow J_{x,\text{max}} \sim 20\%, J_{y,\text{max}} \sim 60\% \text{ (for given budgets)}$$

Test 0

- No tracking. Average of 5 randomly jittered trains (352 random bunches per train)
- Plotting
 - c) jitter amplification tolerance for x & y

c)



Jitter amplification tolerance definition in this case:

$$\rightarrow F = \frac{J_{\max}}{J_{\text{initial}}} = \sqrt{\frac{\text{Budget}}{\epsilon_0 \cdot J_{\text{initial}}^2}} \rightarrow \begin{aligned} &F_{x,\max} \sim 4, F_{y,\max} \sim 12 @ J_{\text{initial}} = 0.05 \text{ (budgets: 30 nm, 2 nm)} \\ &F_{x,\max} \sim 2, F_{y,\max} \sim 5 @ J_{\text{initial}} = 0.1 \text{ (budgets: 30 nm, 2 nm)} \end{aligned}$$

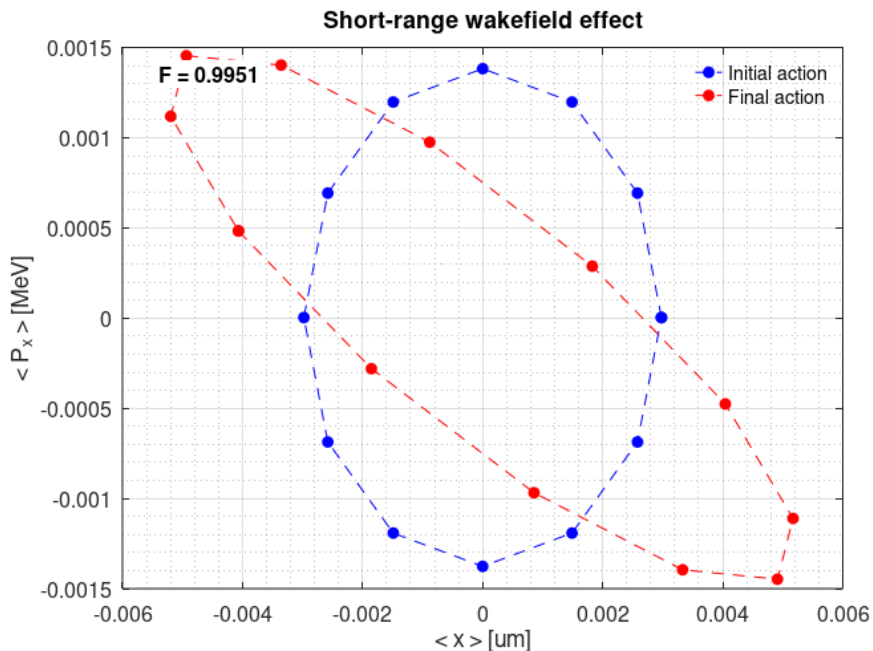
Short-range wakefield effect

Test 1

- **Short-range** wakefield effect in BL. Full **single bunch tracking simulation**
- Jitter considered: $J = 10\% * \sigma(x, p_x)$
- Jitter amplification factor definition in this case:

$$F_s = \frac{J_{\text{final}}}{J_{\text{initial}}} = \sqrt{\frac{A_{\text{final}}}{A_{\text{initial}}}}$$

J: jitter, A: action (area)



- W/o SRWF, zero E spread: $F_{x,s} = 1.0000$
- W/o SRWF, 1.2% E spread: $F_{x,s} = 0.9862$
 - E spread helps to damp the effect (BNS damping)
- W/ SRWF, 1.2% E spread: $F_{x,s} = 0.9951$ (Nominal)

Long-range wakefield effect – kick on
next bunch only

Test 2.0

- Long-range wakefield effect in BL. **Single particle calculation** using Daniel's formulae
- Transverse **kick on next bunch only** ($a_k = 0$ when $k \neq 1$)
- Jitter amplification factor definition in this case:

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MULTI-BUNCH CALCULATIONS IN THE CLIC MAIN LINAC

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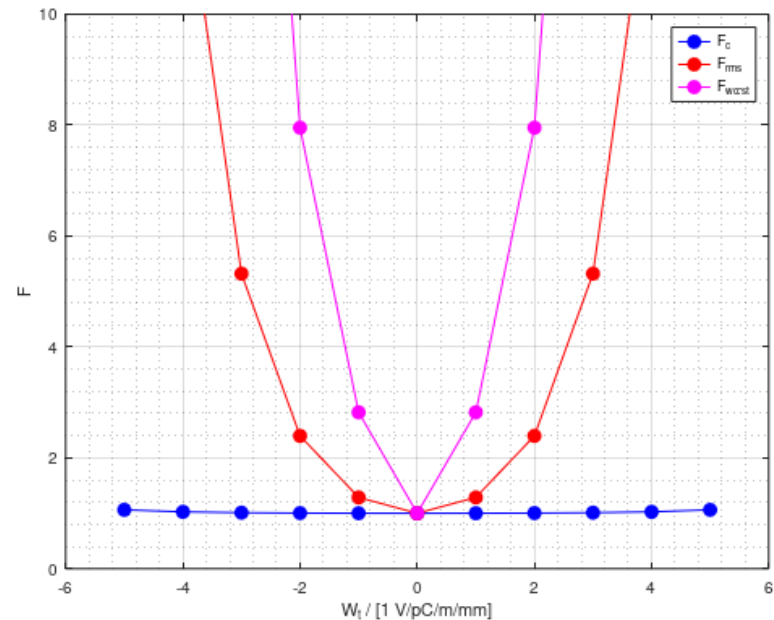
$$A = \lim_{m \rightarrow \infty} \left(1 + \frac{a}{m}\right)^m = \exp(a) = \sum_{k=0}^{\infty} \frac{a^k}{k!} = \sum_{k=0}^{n-1} \frac{a^k}{k!}$$

$$a_k = \sum_i \frac{L_i \beta_i}{2E_i} W(z_k) N e^2$$

$$F_c = \frac{1}{n} \sum_k \left| \sum_j A_{kj} \right|^2$$

$$F_{rms} = \frac{\sum_{k=0}^{n-1} \sum_{j=1}^k A_{k,j} A_{k,j}^*}{n}$$

$$F_{worst} = \text{svd}(A)(1)^2$$



$$F_{x,c} = 1.062 @ \pm 5 \text{ V/pC/m/mm}$$

$$F_{x,rms} = 31.9 @ \pm 5 \text{ V/pC/m/mm}$$

$$F_{x,worst} = 178.4 @ \pm 5 \text{ V/pC/m/mm}$$

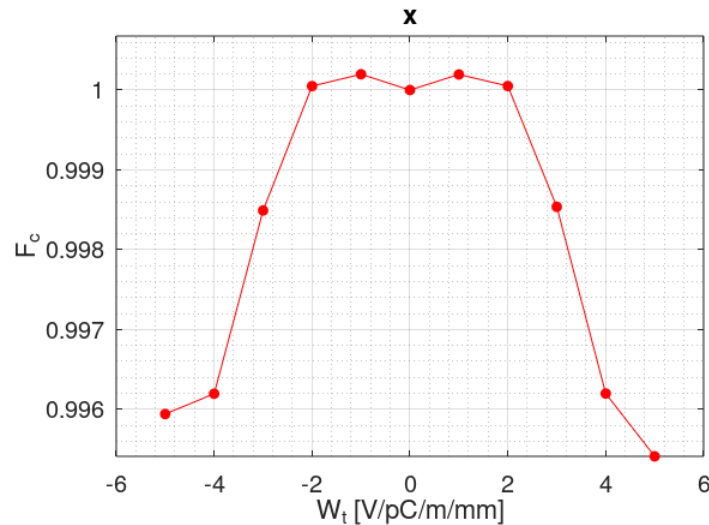
Test 2.1.1a

- Long-range wakefield effect in BL. **Single particle tracking simulation**
- Jitter considered: $J = 0.10$, that is, $\langle x, px \rangle = 0.10 \cdot \sigma(x, px)$
- Transverse **kick on next bunch only**. **Coherent** jitter with 8 specific trains (like Test 1)
- Jitter amplification factor definition in this case:

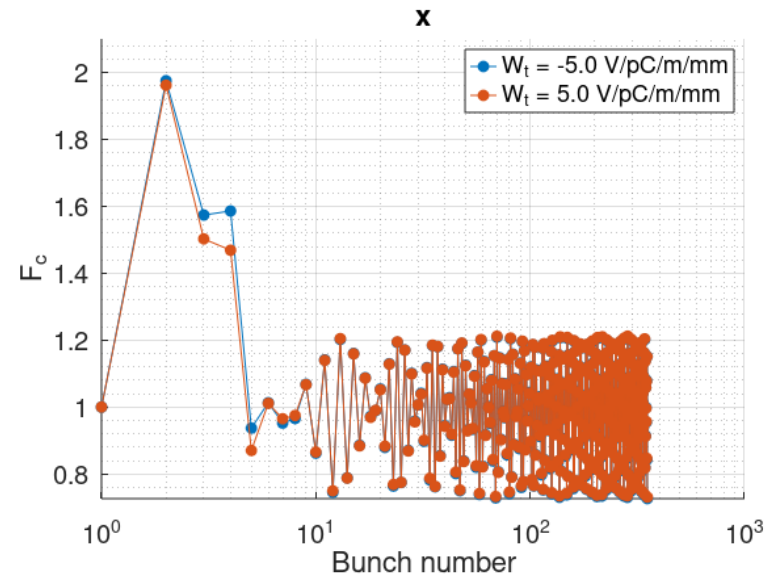
Using average F of all bunches

$$F_c = \frac{F_{W \neq 0}}{F_{W = 0}}, \text{ where } F = \frac{1}{N_{\text{bunches}}} \sum \frac{J_{\text{final}}}{J_{\text{initial}}} = \frac{1}{N_{\text{bunches}}} \sum \sqrt{\frac{A_{\text{final}}}{A_{\text{initial}}}}$$

A: action (area)



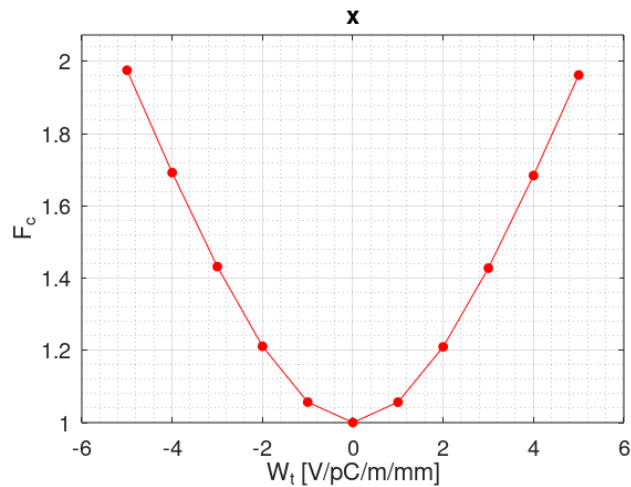
$F_{x,c} \sim 0.996 @ \pm 5 \text{ V/pC/m/mm}$



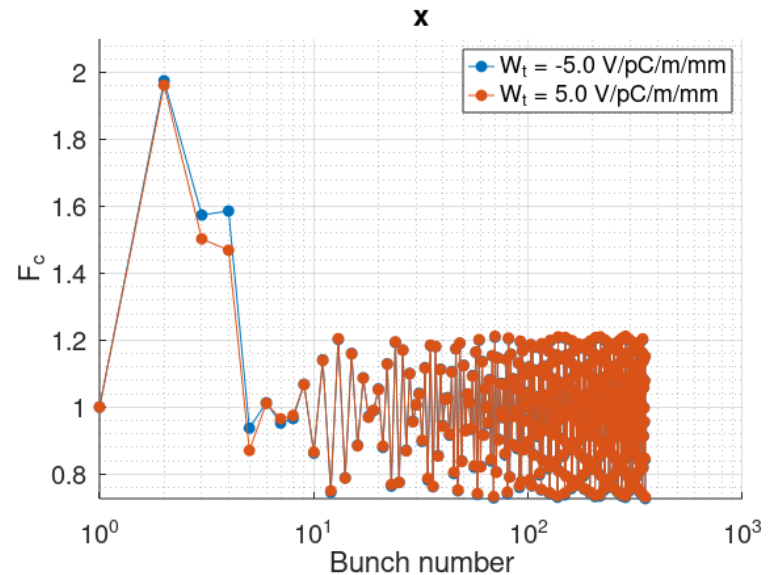
Test 2.1.1b

- Long-range wakefield effect in BL. **Single particle tracking simulation**
- Jitter considered: $J = 0.10$, that is, $\langle x, px \rangle = 0.10 \cdot \sigma(x, px)$
- Transverse **kick on next bunch only**. **Coherent** jitter with 8 specific trains (like Test 1)
- Jitter amplification factor definition in this case: Using maximum F of all bunches

$$F_c = \frac{F_{W \neq 0}}{F_{W=0}}, \text{ where } F = \max_{\text{bunches}} \left(\frac{J_{\text{final}}}{J_{\text{initial}}} \right) = \max_{\text{bunches}} \left(\sqrt{\frac{A_{\text{final}}}{A_{\text{initial}}}} \right) \quad \text{A: action (area)}$$



$F_{x,c} \sim 2.0 @ \pm 5 \text{ V/pC/m/mm}$



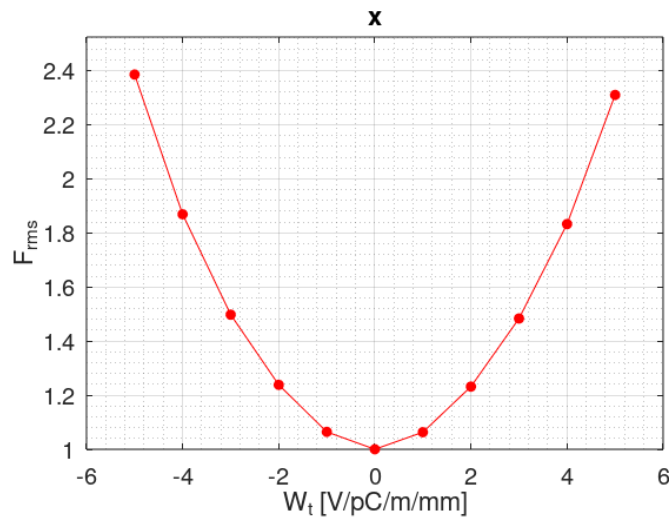
Test 2.1.2a

- Long-range wakefield effect in BL. **Single particle tracking simulation**
- Jitter considered: $J = 0.10$, that is, $\langle x, px \rangle = 0.10 \cdot \sigma(x, px)$
- Transverse **kick on next bunch only**. **Incoherent** jitter with 1000 random trains
- Jitter amplification factor definition in this case:

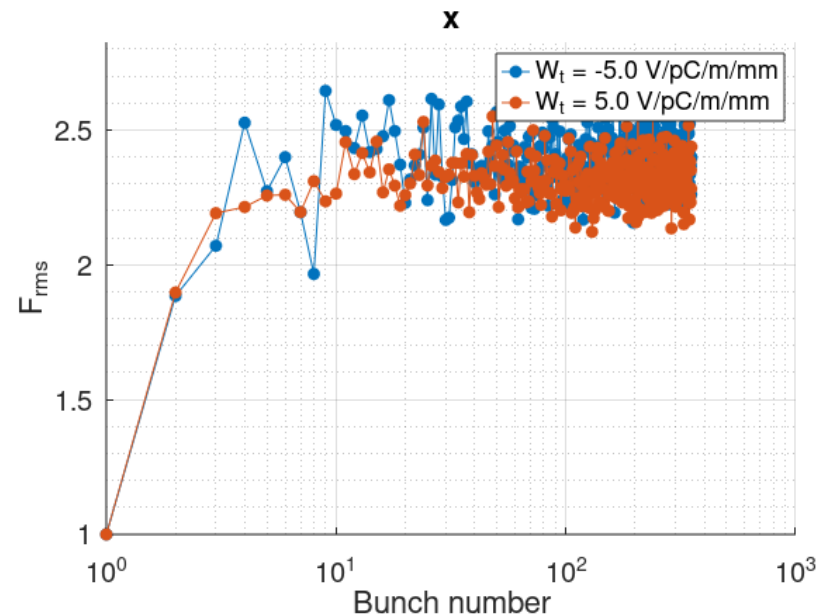
Using average F of all bunches

$$F_{\text{rms}} = \frac{F_{W \neq 0}}{F_{W=0}}, \text{ where } F = \frac{1}{N_{\text{bunches}}} \sum \frac{J_{\text{final}}}{J_{\text{initial}}} = \frac{1}{N_{\text{bunches}}} \sum \sqrt{\frac{A_{\text{final}}}{A_{\text{initial}}}}$$

A: action (area)



$F_{x,c} \sim 2.4 @ \pm 5 \text{ V/pC/m/mm}$

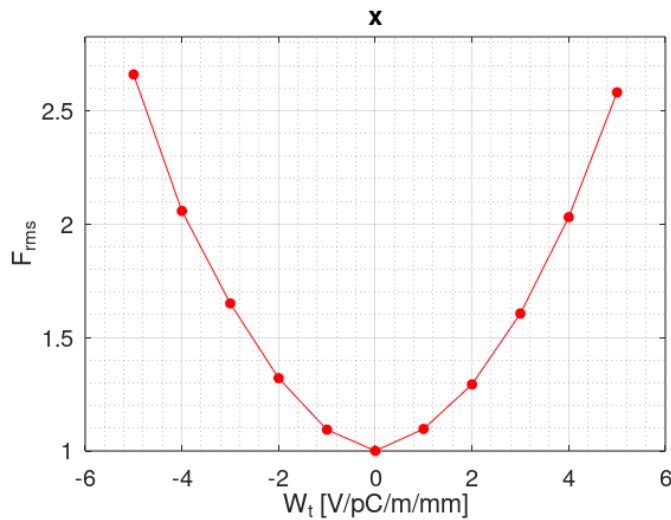


Test 2.1.2b

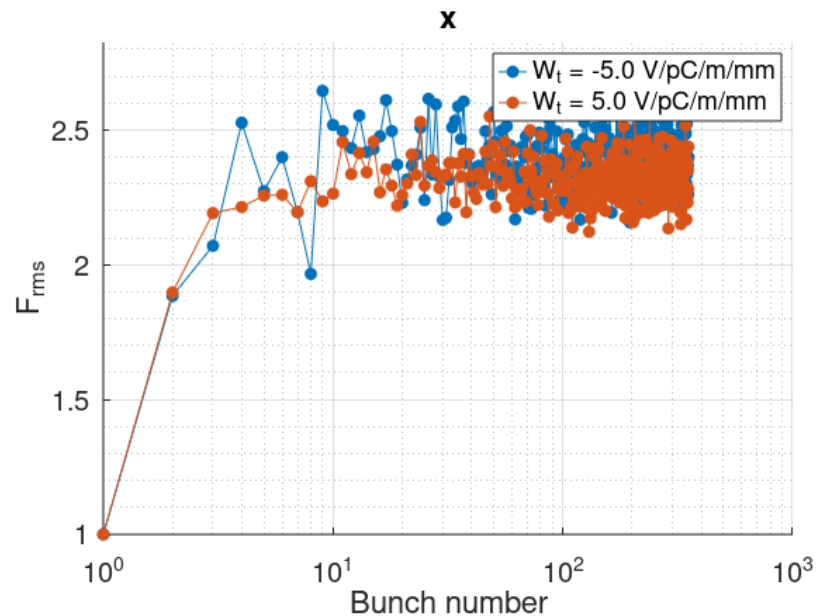
- Long-range wakefield effect in BL. **Single particle tracking simulation**
- Jitter considered: $J = 0.10$, that is, $\langle x, px \rangle = 0.10 \cdot \sigma(x, px)$
- Transverse **kick on next bunch only**. **Incoherent** jitter with 1000 random trains
- Jitter amplification factor definition in this case:

Using maximum F of all bunches

$$F_{\text{rms}} = \frac{F_{W \neq 0}}{F_{W=0}}, \text{ where } F = \max_{\text{bunches}} \left(\frac{J_{\text{final}}}{J_{\text{initial}}} \right) = \max_{\text{bunches}} \left(\sqrt{\frac{A_{\text{final}}}{A_{\text{initial}}}} \right) \quad \text{A: action (area)}$$



$F_{x,c} \sim 2.7 @ \pm 5 \text{ V/pC/m/mm}$



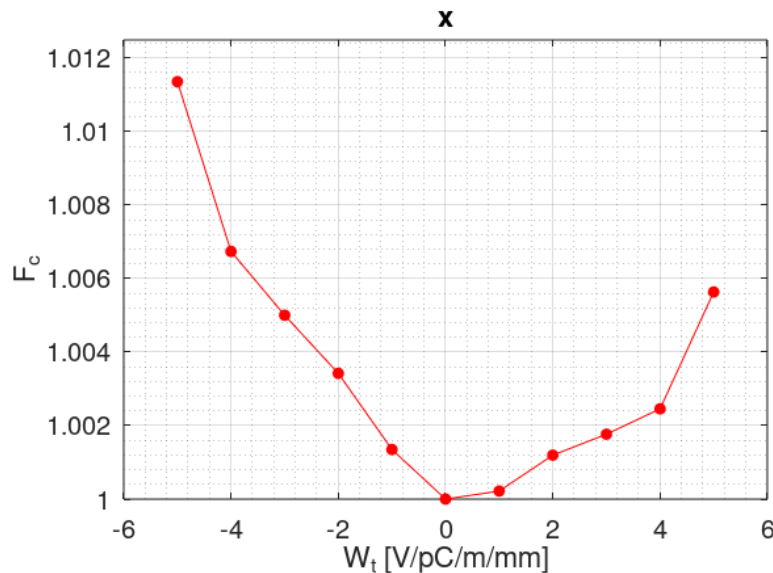
Test 2.2.1a

- Long-range wakefield effect in BL. **Full bunch tracking simulation**. Short-range wake considered
- Jitter considered: $J = 0.10$, that is, $\langle x, px \rangle = 0.10 \cdot \sigma(x, px)$
- Transverse **kick on next bunch only**. **Coherent** jitter with 8 specific trains
- Jitter amplification factor definition in this case:

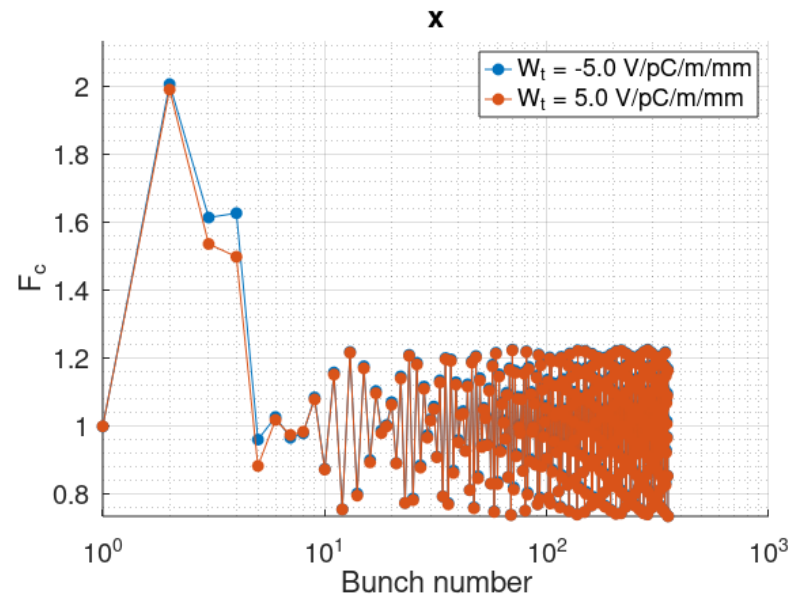
Using average F of all bunches

$$F_c = \frac{F_{W \neq 0}}{F_{W=0}}, \text{ where } F = \frac{1}{N_{\text{bunches}}} \sum \frac{J_{\text{final}}}{J_{\text{initial}}} = \frac{1}{N_{\text{bunches}}} \sum \sqrt{\frac{A_{\text{final}}}{A_{\text{initial}}}}$$

A: action (area)



$F_{x,c} \sim 1.01$ @ ± 5 V/pC/m/mm



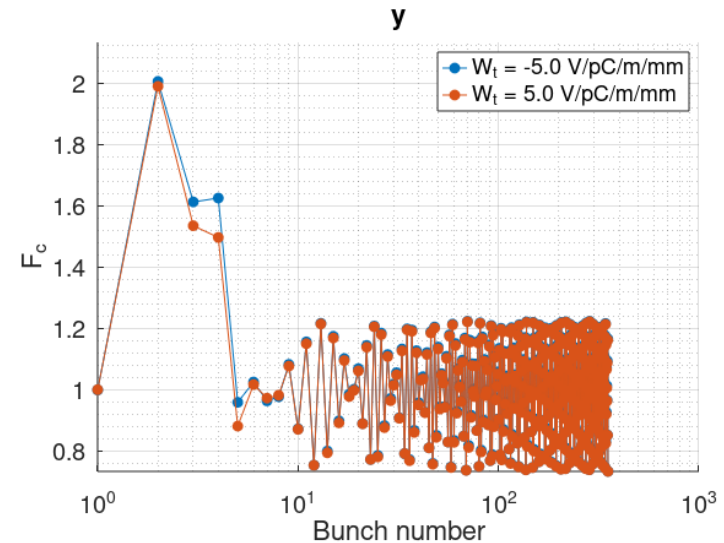
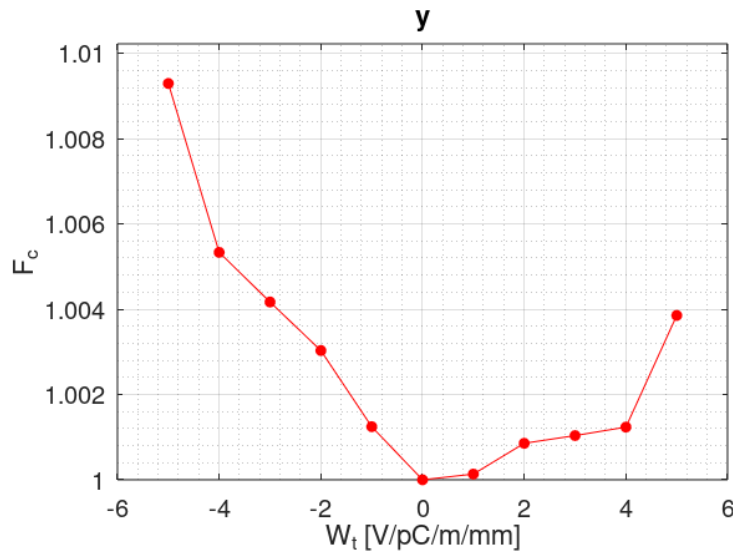
Test 2.2.1a (checking vertical plane)

- Long-range wakefield effect in BL. **Full bunch tracking simulation**. Short-range wake considered
- Jitter considered: $J = 0.10$, that is, $\langle x, px \rangle = 0.10 \cdot \sigma(x, px)$
- Transverse **kick on next bunch only**. **Coherent** jitter with 8 specific trains
- Jitter amplification factor definition in this case:

Using average F of all bunches

$$F_c = \frac{F_{W \neq 0}}{F_{W=0}}, \text{ where } F = \frac{1}{N_{\text{bunches}}} \sum \frac{J_{\text{final}}}{J_{\text{initial}}} = \frac{1}{N_{\text{bunches}}} \sum \sqrt{\frac{A_{\text{final}}}{A_{\text{initial}}}}$$

A: action (area)



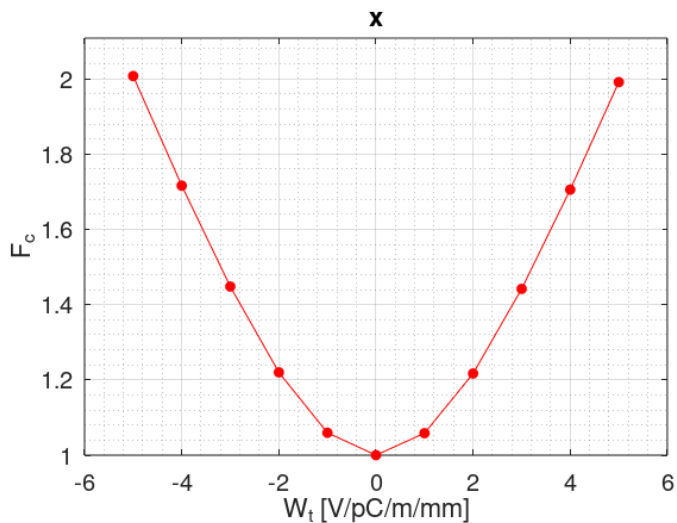
$F_{x,c} \sim 1.01$ @ ± 5 V/pC/m/mm

Test 2.2.1b

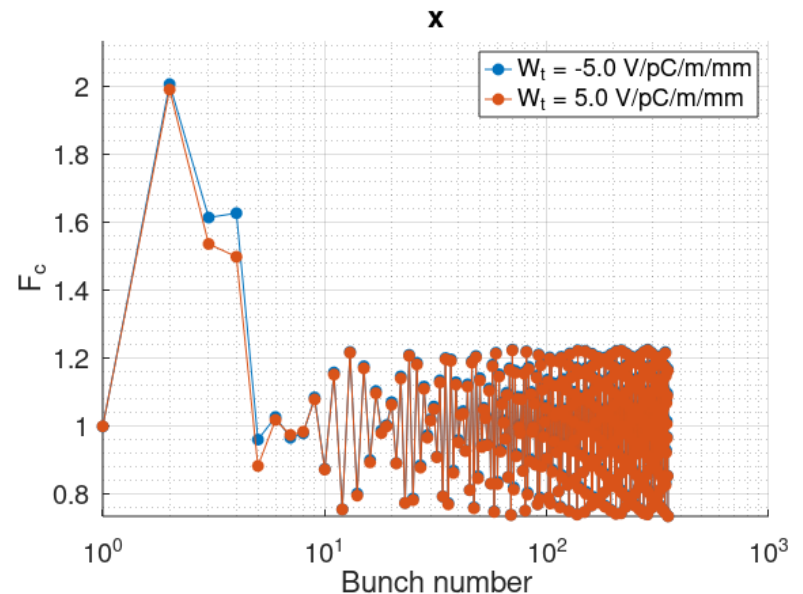
- Long-range wakefield effect in BL. **Full bunch tracking simulation**. Short-range wake considered
- Jitter considered: $J = 0.10$, that is, $\langle x, px \rangle = 0.10 \cdot \sigma(x, px)$
- Transverse **kick on next bunch only**. **Coherent** jitter with 8 specific trains
- Jitter amplification factor definition in this case:

Using maximum F of all bunches

$$F_c = \frac{F_{W \neq 0}}{F_{W=0}}, \text{ where } F = \max_{\text{bunches}} \left(\frac{J_{\text{final}}}{J_{\text{initial}}} \right) = \max_{\text{bunches}} \left(\sqrt{\frac{A_{\text{final}}}{A_{\text{initial}}}} \right) \quad \text{A: action (area)}$$



$F_{x,c} \sim 2.0 @ \pm 5 \text{ V/pC/m/mm}$



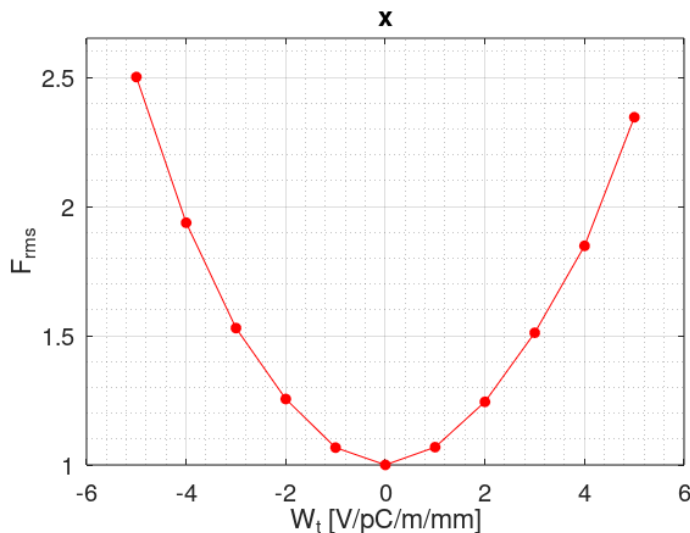
Test 2.2.2.1a

- Long-range wakefield effect in BL. **Full bunch tracking simulation**. Short-range wake considered
- Jitter considered: $J = 0.10$, that is, $\langle x, px \rangle = 0.10 \cdot \sigma(x, px)$
- Transverse **kick on next bunch only**. **Incoherent** jitter with 100 random trains
- Jitter amplification factor definition in this case:

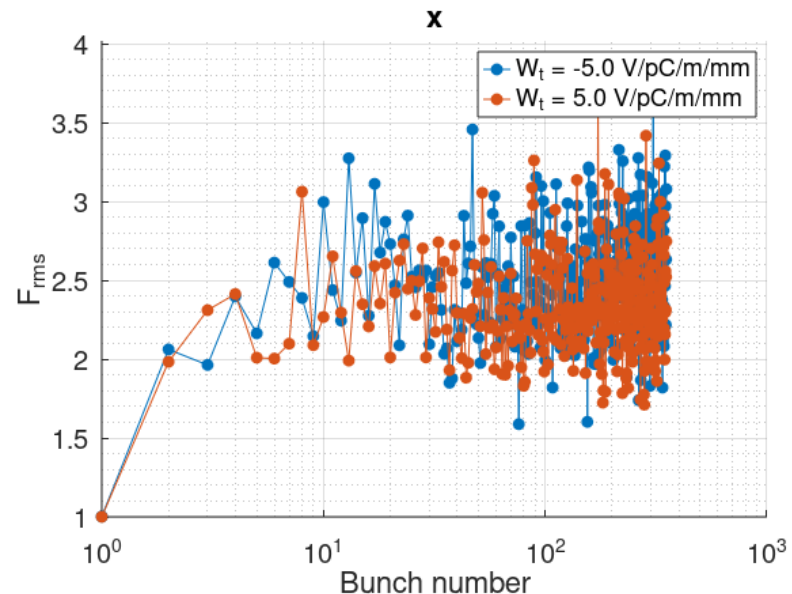
Using average F of all bunches

$$F_{rms} = \frac{F_{W \neq 0}}{F_{W=0}}, \text{ where } F = \frac{1}{N_{bunches}} \sum \frac{J_{final}}{J_{initial}} = \frac{1}{N_{bunches}} \sum \sqrt{\frac{A_{final}}{A_{initial}}}$$

A: action (area) of bunch centers



$F_{x,rms} \sim 2.5$ @ ± 5 V/pC/m/mm

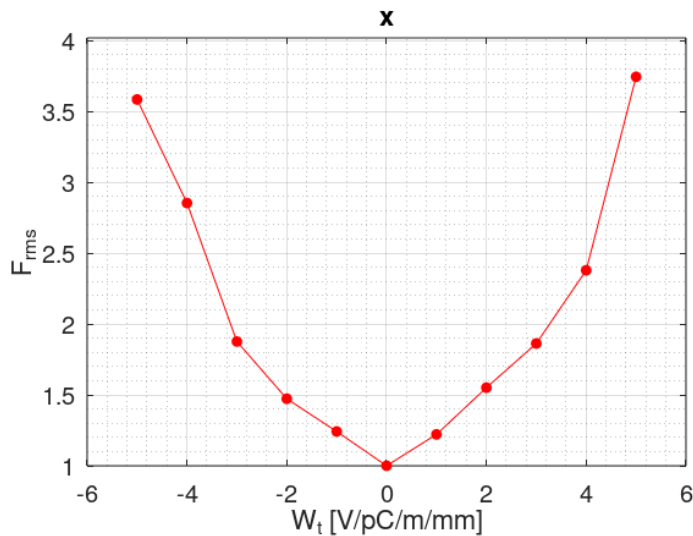


Test 2.2.2.1b

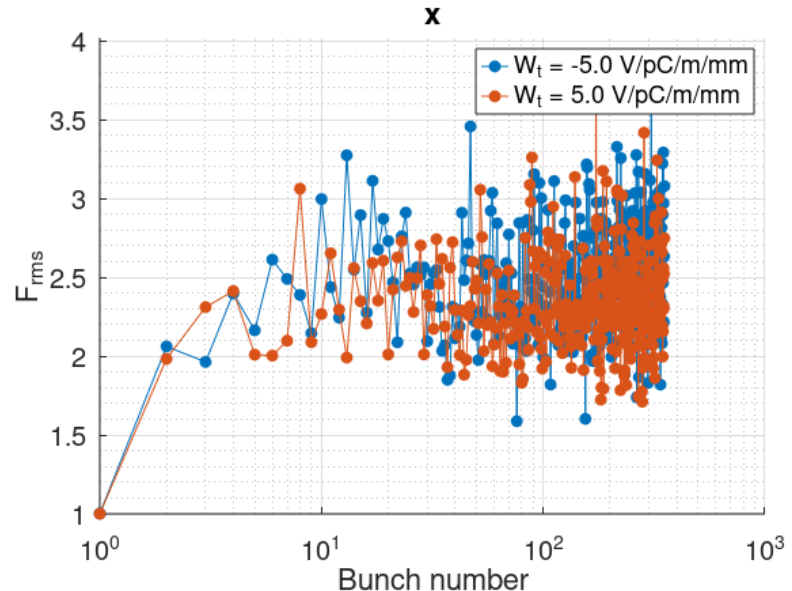
- Long-range wakefield effect in BL. **Full bunch tracking simulation**. Short-range wake considered
- Jitter considered: $J = 0.10$, that is, $\langle x, px \rangle = 0.10 \cdot \sigma(x, px)$
- Transverse **kick on next bunch only**. **Incoherent** jitter with 100 random trains
- Jitter amplification factor definition in this case: Using maximum F of all bunches

$$F_{rms} = \frac{F_{W \neq 0}}{F_{W=0}}, \text{ where } F = \max_{\text{bunches}} \left(\frac{J_{final}}{J_{initial}} \right) = \max_{\text{bunches}} \left(\sqrt{\frac{A_{final}}{A_{initial}}} \right)$$

A: action (area) of bunch centers



$F_{x,rms} \sim 3.7 @ \pm 5 \text{ V/pC/m/mm}$



Test 2.2.2.2a

- **Long-range** wakefield effect in BL. **Full bunch tracking simulation**. Short-range wake considered
- Jitter considered: $J = 0.10$, that is, $\langle x, px \rangle = 0.10 * \sigma(x, px)$
- Transverse **kick on next bunch only**. **Incoherent** jitter with 100 random trains
- Jitter amplification factor definition in this case:

Using average F of all bunches

$$F_{\text{rms}} = \frac{F_{W \neq 0}}{F_{W=0}}, \text{ where } F = \frac{1}{N_{\text{bunches}}} \sum \frac{J_{\text{final}}}{J_{\text{initial}}} = \frac{1}{N_{\text{bunches}}} \sum \sqrt{\frac{\frac{\epsilon_{\text{total}}^{\text{final}}}{\epsilon_{\text{single}}^{\text{initial}} - 1}}{\frac{\epsilon_{\text{total}}^{\text{initial}}}{\epsilon_{\text{single}}^{\text{initial}} - 1}}} = \frac{1}{N_{\text{bunches}}} \sum \sqrt{\frac{\epsilon_{\text{total}}^{\text{final}} - \epsilon_{\text{single}}^{\text{initial}}}{\epsilon_{\text{total}}^{\text{initial}} - \epsilon_{\text{single}}^{\text{initial}}}}$$

ϵ_{total} : projected emittance of all trains for same bunch

ϵ_{single} : single bunch emittance

In progress ...

Seems quite difficult. Need to store very huge data on disk and much longer time for each train or Condor job. I will see if it's possible.

Instead of using projected emittance of all trains, it's much easier to use projected emittance of all bunches in a train, where I just need to store a number instead of all bunches. See Test 2.2.2.3.

Test 2.2.2.2b

- **Long-range** wakefield effect in BL. **Full bunch tracking simulation**. Short-range wake considered
- Jitter considered: $J = 0.10$, that is, $\langle x, px \rangle = 0.10 * \sigma(x, px)$
- Transverse **kick on next bunch only**. **Incoherent** jitter with 100 random trains
- Jitter amplification factor definition in this case:

Using maximum F of all bunches

$$F_{\text{rms}} = \frac{F_{W \neq 0}}{F_{W=0}}, \text{ where } F = \max_{\text{bunches}} \left(\frac{J_{\text{final}}}{J_{\text{initial}}} \right) = \max_{\text{bunches}} \left(\frac{\sqrt{\frac{\epsilon_{\text{total}}^{\text{final}}}{\epsilon_{\text{single}}^{\text{initial}} - 1}}}{\sqrt{\frac{\epsilon_{\text{total}}^{\text{initial}}}{\epsilon_{\text{single}}^{\text{initial}} - 1}}} \right) = \max_{\text{bunches}} \left(\sqrt{\frac{\epsilon_{\text{total}}^{\text{final}} - \epsilon_{\text{single}}^{\text{initial}}}{\epsilon_{\text{total}}^{\text{initial}} - \epsilon_{\text{single}}^{\text{initial}}}} \right)$$

ϵ_{total} : projected emittance of all trains for same bunch
 ϵ_{single} : single bunch emittance

In progress ...

Seems quite difficult. Need to store very huge data on disk and much longer time for each train or Condor job. I will see if it's possible.

Instead of using projected emittance of all trains, it's much easier to use projected emittance of all bunches in a train, where I just need to store a number instead of all bunches. See Test 2.2.2.3.

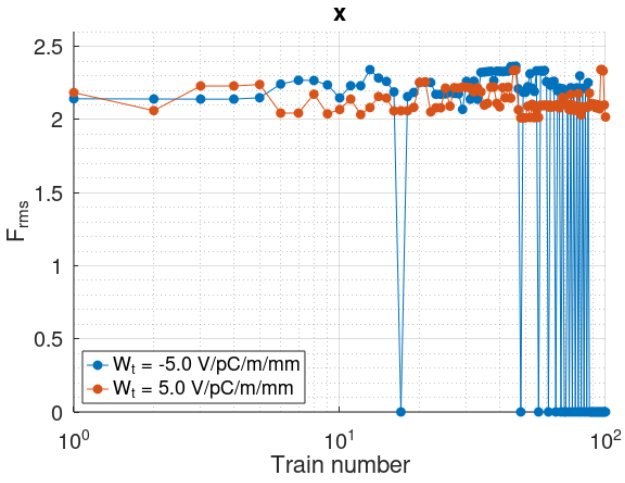
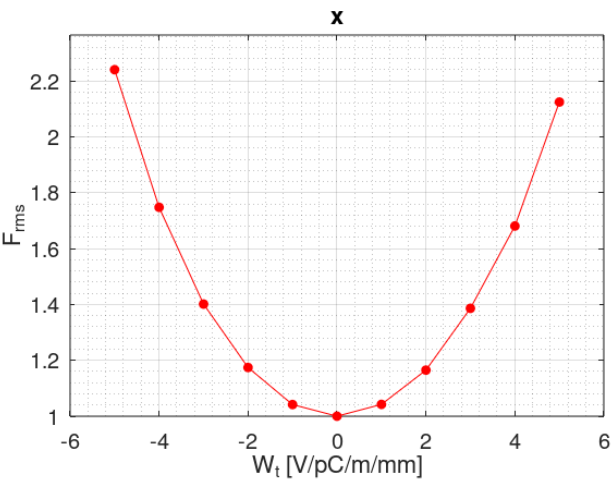
Test 2.2.2.3a

- Long-range wakefield effect in BL. **Full bunch tracking simulation**. Short-range wake considered
- Jitter considered: $J = 0.10$, that is, $\langle x, px \rangle = 0.10 \cdot \sigma(x, px)$
- Transverse **kick on next bunch only**. **Incoherent** jitter with 100 random trains
- Jitter amplification factor definition in this case:

Using average F of all trains

$$F_{rms} = \frac{F_{W \neq 0}}{F_{W=0}}, \text{ where } F = \frac{1}{N_{trains}} \sum \frac{J_{final}}{J_{initial}} = \frac{1}{N_{trains}} \sum \sqrt{\frac{\frac{\epsilon_{total}^{final}}{\epsilon_{single}^{initial}} - 1}{\frac{\epsilon_{total}^{initial}}{\epsilon_{single}^{initial}} - 1}} = \frac{1}{N_{trains}} \sum \sqrt{\frac{\epsilon_{total}^{final} - \epsilon_{single}^{initial}}{\epsilon_{total}^{initial} - \epsilon_{single}^{initial}}}$$

ϵ_{total} : projected emittance of all bunches for same train
 ϵ_{single} : single bunch emittance



$F_{x,rms} \sim 2.2 @ \pm 5 \text{ V/pC/m/mm}$

$F = 0$ means job is killed probably due to long simulation time. Not considered in calculation

Test 2.2.2.3b

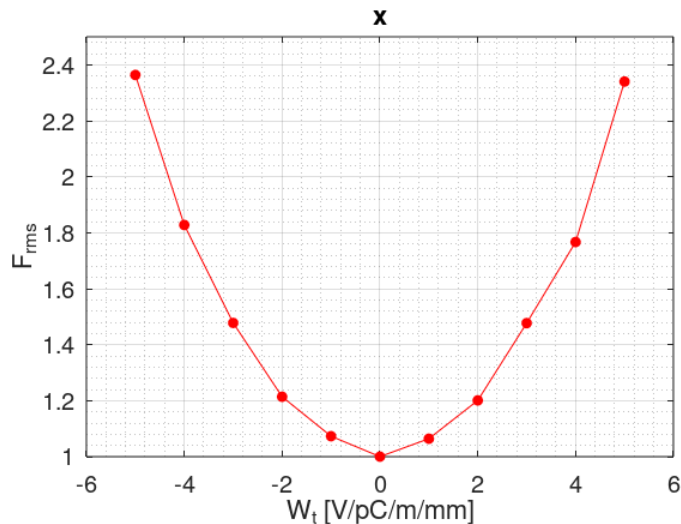
- Long-range wakefield effect in BL. **Full bunch tracking simulation**. Short-range wake considered
- Jitter considered: $J = 0.10$, that is, $\langle x, px \rangle = 0.10 \cdot \sigma(x, px)$
- Transverse **kick on next bunch only**. **Incoherent** jitter with 100 random trains
- Jitter amplification factor definition in this case:

Using maximum F of all trains

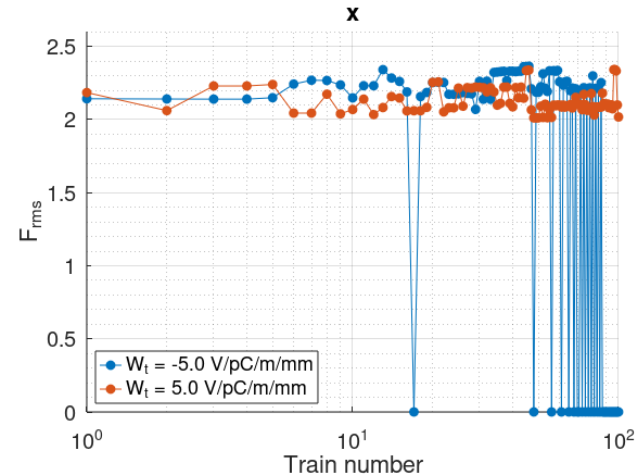
$$F_{rms} = \frac{F_{W \neq 0}}{F_{W=0}}, \text{ where } F = \max_{\text{trains}} \left(\frac{J_{\text{final}}}{J_{\text{initial}}} \right) = \max_{\text{trains}} \left(\frac{\sqrt{\frac{\epsilon_{\text{total}}^{\text{final}}}{\epsilon_{\text{single}}^{\text{initial}} - 1}}}{\sqrt{\frac{\epsilon_{\text{total}}^{\text{initial}}}{\epsilon_{\text{single}}^{\text{initial}} - 1}}} \right) = \max_{\text{trains}} \left(\sqrt{\frac{\epsilon_{\text{total}}^{\text{final}} - \epsilon_{\text{single}}^{\text{initial}}}{\epsilon_{\text{total}}^{\text{initial}} - \epsilon_{\text{single}}^{\text{initial}}}} \right)$$

ϵ_{total} : projected emittance of all bunches for same train

ϵ_{single} : single bunch emittance



F_{x,rms} ~ 2.4 @ ±5 V/pC/m/mm

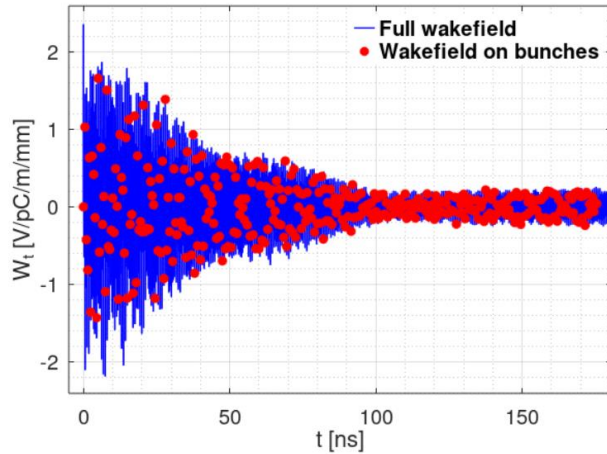


F = 0 means job is killed probably due to long simulation time. Not considered in calculation

Long-range wakefield effect – kick on
all bunches

Test 3.0

- Using wakefield (V1) directly from Adnan:



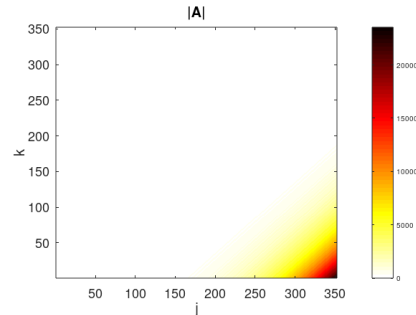
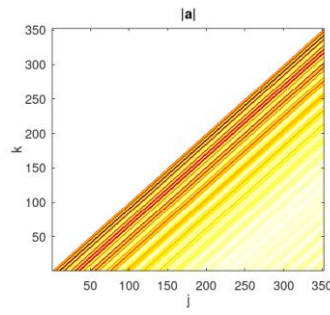
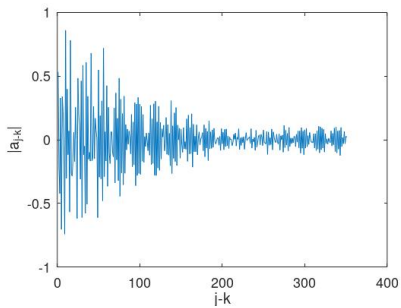
Single particle calculation using Daniel's formulae. Same definitions and configurations as Test 2.0

$$\text{Sum}(\text{Abs}(W)) = 93.4 \text{ V/pC/m/mm}$$

$$a_{j-k} = i \int_0^s \frac{W(z_j - z_k, s) N e^2 \beta(s)}{2E(s)} ds$$

$$a_{jk} = a_{j-k} \text{ for } j > k$$

$$A = \lim_{m \rightarrow \infty} \left(1 + \frac{a}{m}\right)^m = \exp(a) = \sum_{k=0}^{\infty} \frac{a^k}{k!} = \sum_{k=0}^{n-1} \frac{a^k}{k!}$$



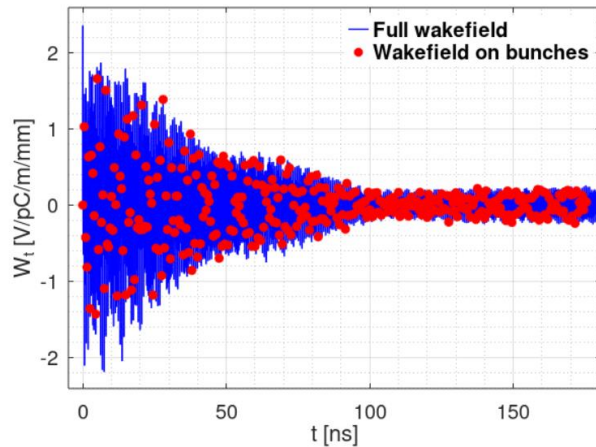
- $F_{x,c} = 1.0E+07$
- $F_{x,rms} = 1.0E+09$
- $F_{x,worst} = 3.6E+11$

$$a1 = 0.53$$

Very very large jitter amplifications!

Test 3.1.1

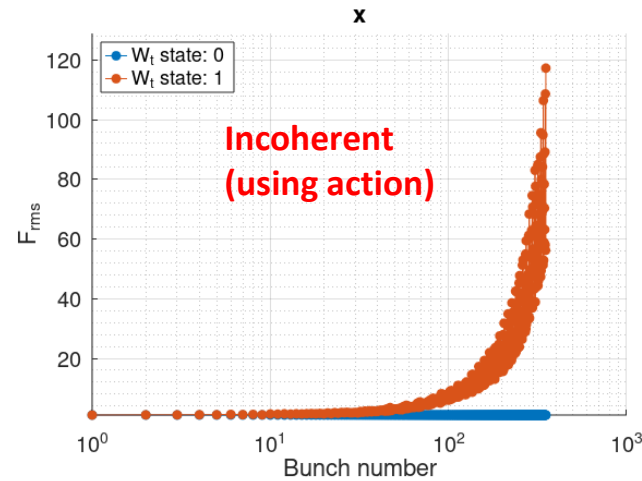
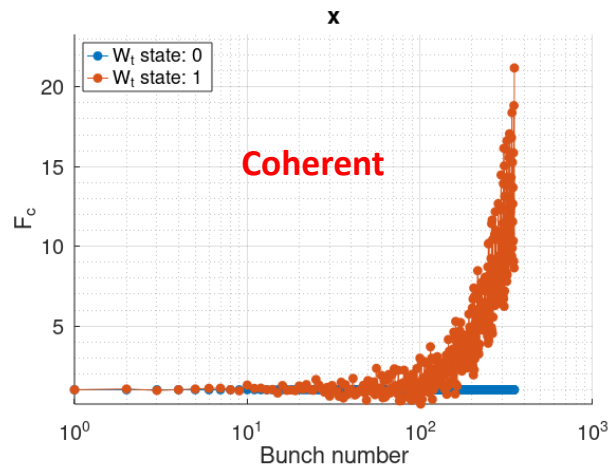
- Using wakefield (V1) directly from Adnan:



$$\text{Sum}(\text{Abs}(W)) = 93.4 \text{ V/pC/m/mm}$$

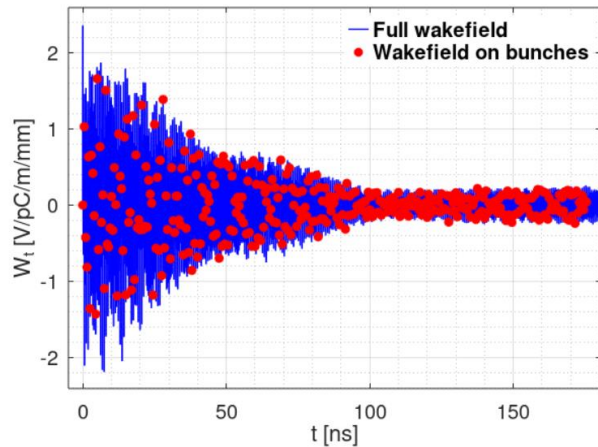
Single particle tracking simulation

- $F_{x,c} = 4.9$ (average) or 21.2 (maximum)
- $F_{x,rms} = 24.4$ (average) or 117.3 (maximum)



Test 3.1.2

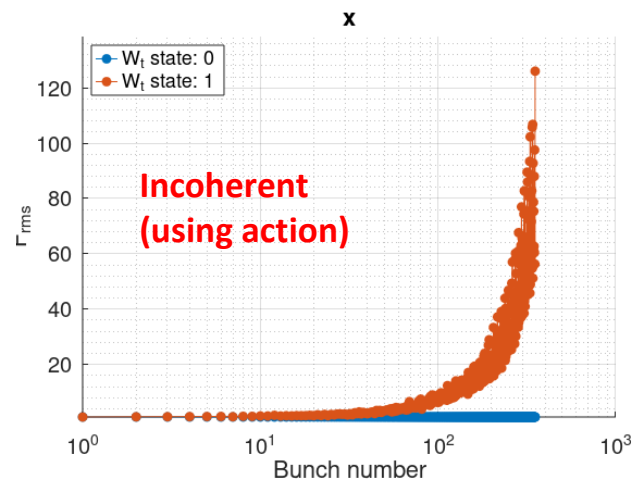
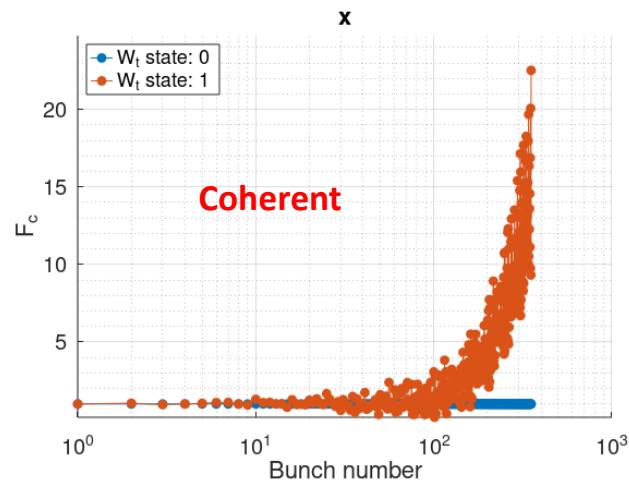
- Using wakefield (V1) directly from Adnan:



$$\text{Sum}(\text{Abs}(W)) = 93.4 \text{ V/pC/m/mm}$$

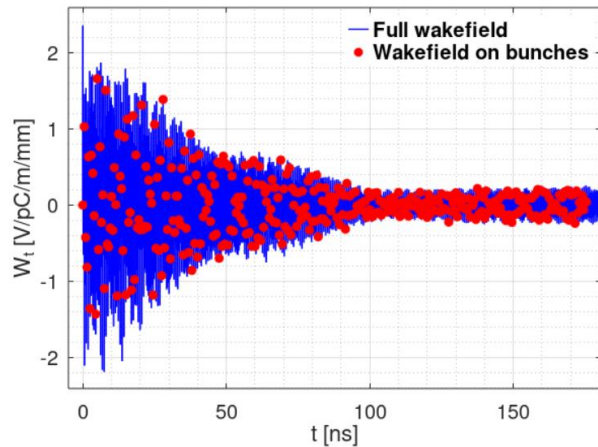
Full bunch tracking simulation

- $F_{x,c} = 5.1$ (average) or 22.5 (maximum)
- $F_{x,rms} = 24.9$ (average) or 126.0 (maximum)



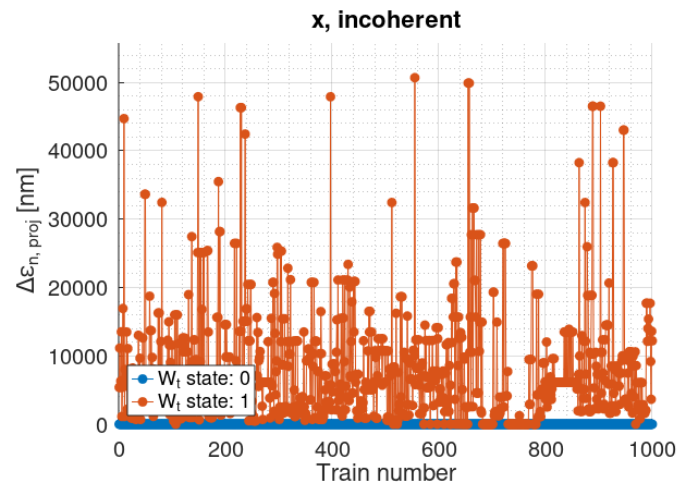
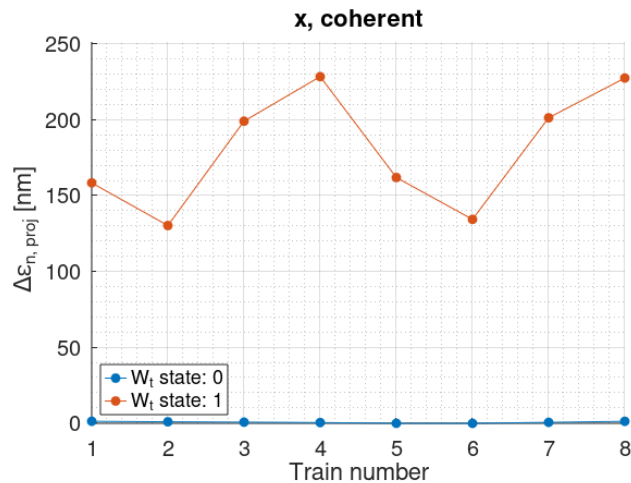
Test 3.1.2

- Using wakefield (V1) directly from Adnan:



$\text{Sum}(\text{Abs}(W)) = 93.4 \text{ V/pC/m/mm}$

Full bunch tracking simulation



Test 3.1.1 & 3.1.2

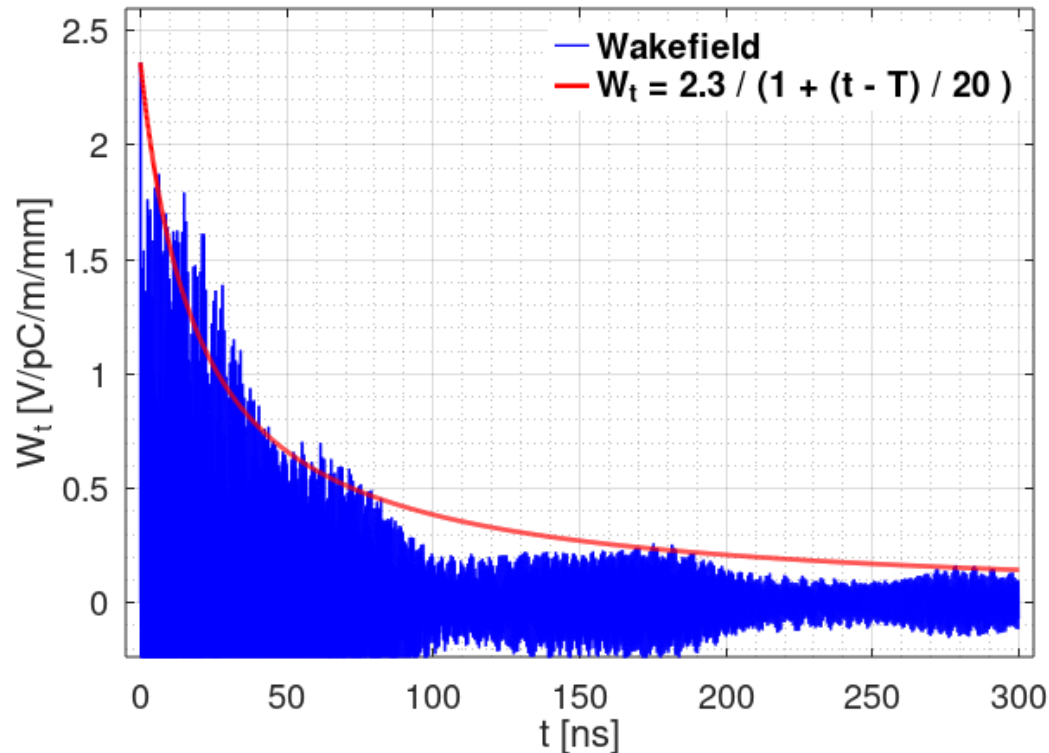
- Comparison between single particle & full bunch
 - Consistent with very small difference

x	Average Fc	Maximum Fc	Average Frms	Maximum Frms
Single particle	4.9	21.2	24.4	117.3
Full bunch	5.1	22.5	24.9	126.0

Test 3.2 – Wake scan

- Wake formula assumption:

$$W_{\perp}(t) = \frac{k}{1 + \frac{t - T}{\alpha}}, \quad t \geq T = 0.5 \text{ ns}$$



Test 3.2 – Wake scan

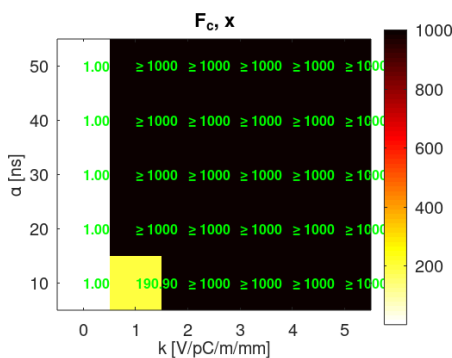
- Range

- k: [0:1:5] V/pC/m/mm
- alpha: [10:10:50] ns

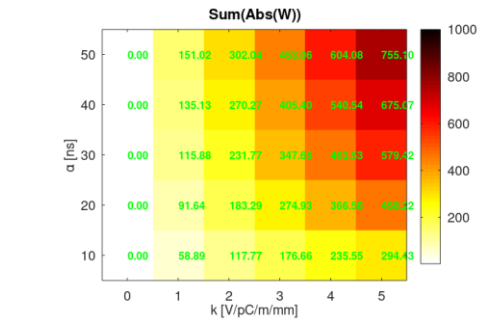
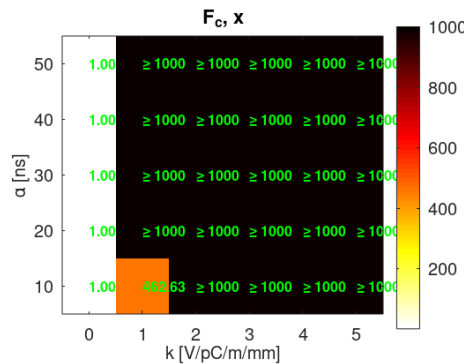
$$W_{\perp}(t) = \frac{k}{1 + \frac{t - T}{\alpha}}, \quad t \geq T = 0.5 \text{ ns}$$

Coherent

Average

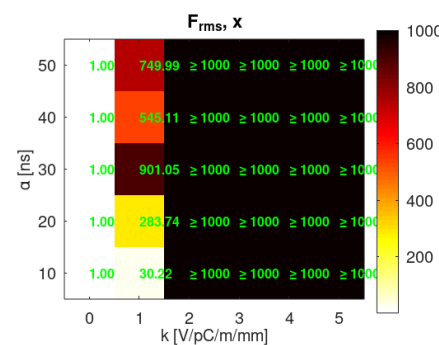


Maximum

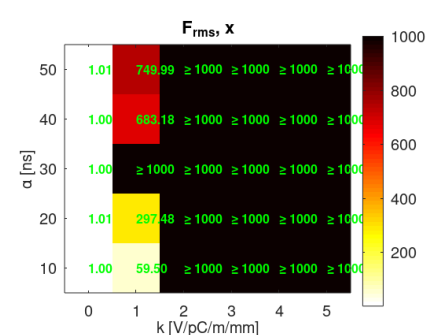


Incoherent

Average



Maximum



Very very large jitter amplifications!

Full bunch tracking simulation. Same definitions and configurations as Test 2.2.1 and Test 2.2.2.3 (but **only 10 trains simulated**)

Summary (table in next slide)

- Test 0: **general study**. **Jitter budgets** are 0.2σ for x and 0.6σ for y (assuming projected emittance budgets are same with budget numbers in PIP report). **Jitter amplification (F) budgets** are plotted as functions of initial jitter, e.g. $F_x < 4$, $F_y < 12$ @ 0.05σ , $F_x < 2$, $F_y < 5$ @ 0.1σ
- Test 1: $F = 0.995$ due to **short-range** wakefield for full bunch tracking (w/ BNS damping)
- Test 2.0: F plotted as **kick on next bunch** only using Daniel's **analytic** formulae for single particle, for x. E.g. $F_c = 1.06$, $Frms = 32$, $F_{worst} = 178$ @ 5 V/pC/m/mm
- Test 2.1.1a: F_c (average of all bunches) plotted as kick on next bunch only for single particle tracking, using action (area), for x. E.g. $F_c = 0.996$ @ 5 V/pC/m/mm
- Test 2.1.1b: F_c (**maximum** of all bunches) plotted as **kick on next bunch** only for **single particle** tracking, using action (area), for x. E.g. $F_c = 2.0$ @ 5 V/pC/m/mm
- Test 2.1.2a: $Frms$ (average of all bunches) plotted as kick on next bunch only for single particle tracking, using action (area), for x. E.g. $Frms = 2.4$ @ 5 V/pC/m/mm
- Test 2.1.2b: $Frms$ (**maximum** of all bunches) plotted as **kick on next bunch** only for **single particle** tracking, using action (area), for x. E.g. $Frms = 2.7$ @ 5 V/pC/m/mm
- Test 2.2.1a: F_c (average of all bunches) plotted as kick on next bunch only for full bunch tracking, using action (area), for x. E.g. $F_c = 1.01$ @ 5 V/pC/m/mm
- Test 2.2.1b: F_c (**maximum** of all bunches) plotted as **kick on next bunch** only for **full bunch** tracking, using action (area), for x. E.g. $F_c = 2.0$ @ 5 V/pC/m/mm
- Test 2.2.2.1a: $Frms$ (average of all bunches) plotted as kick on next bunch only for full bunch tracking, using action (area), for x. E.g. $Frms = 2.5$ @ 5 V/pC/m/mm
- Test 2.2.2.1b: $Frms$ (**maximum** of all bunches) plotted as **kick on next bunch** only for **full bunch** tracking, using action (area), for x. E.g. $Frms = 3.7$ @ 5 V/pC/m/mm
- Test 2.2.2.2a and Test 2.2.2.2b (average and maximum of all bunches) using projected emittance of all trains in progress (seems difficult technically)
- Test 2.2.2.1a: $Frms$ (average of all trains) plotted as kick on next bunch only for full bunch tracking, using projected emittance of all bunches, for x. E.g. $Frms = 2.2$ @ 5 V/pC/m/mm
- Test 2.2.2.1a: $Frms$ (**maximum** of all trains) plotted as **kick on next bunch** only for **full bunch** tracking, using projected emittance of all bunches, for x. E.g. $Frms = 2.4$ @ 5 V/pC/m/mm
- Test 3.0: F calculated using Daniel's **analytic** formulae for single particle calculation, with **full wakefield** map, for x. E.g. $F_c = 1.0E+07$, $Frms = 1.0E+09$, $F_{worst} = 3.6E+11$
- Test 3.1: F estimated for **full bunch** tracking, with **full wakefield** map, and plotted as function of bunch number, for x. E.g. $F_c = 5.1$ (average) or 22.5 (**maximum**), $Frms = 26.2$ (average) or 62.2 (**maximum**)
- Test 3.2: F estimated for full bunch tracking, with wakefield **envelop** assumption, and plotted as function of parameters **2D scan**, for x. **Very very large F** is found

Summary table

F for x @ 5 V/pC/m/mm due to long-range wake (with kick on next bunch only)	Fc	Frms	Fworst
Analytic using Daniel's formulae	1.06	32	178
Single particle tracking	0.996 (average) 2.0 (maximum)	2.4 (average) 2.7 (maximum)	-
Full bunch tracking - Using action for Frms (100 trains, to increase statistics?)	1.01 (average) 2.0 (maximum)	2.5 (average) 3.7 (maximum)	-
Full bunch tracking - Using projected emittance for Frms – Using projection emittance of all trains	-	In progress (difficult)	-
Full bunch tracking - Using projected emittance for Frms – Using projection emittance of all bunches	-	2.2 (average) 2.4 (maximum)	-
F for x due to long-range wake (full fieldmap)	Fc	Frms	Fworst
Analytic using Daniel's formulae	1.0E+07	1.0E+09	3.6E+11
Single particle tracking	-	-	-
Full bunch tracking - Using action for Frms (100 trains, to increase statistics?)	5.1 (average) 22.5 (maximum)	26.2 (average) 62.2 (maximum)	-
F for x due to long-range wake (2D scan)	Fc	Frms	Fworst
Full bunch tracking - Using action for Frms (10 trains, to increase statistics?)	>> 100	>> 100	-

Follow-up study

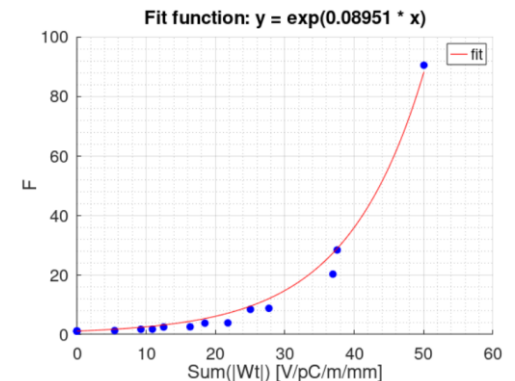
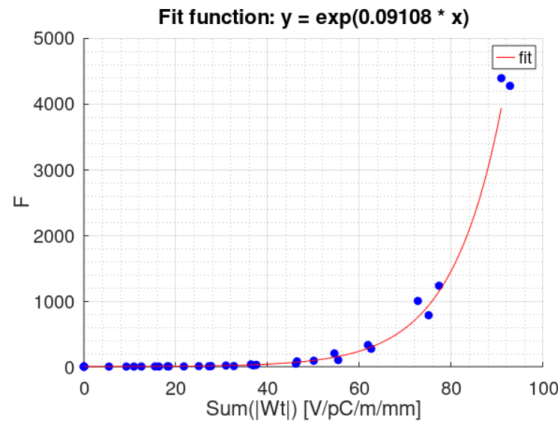
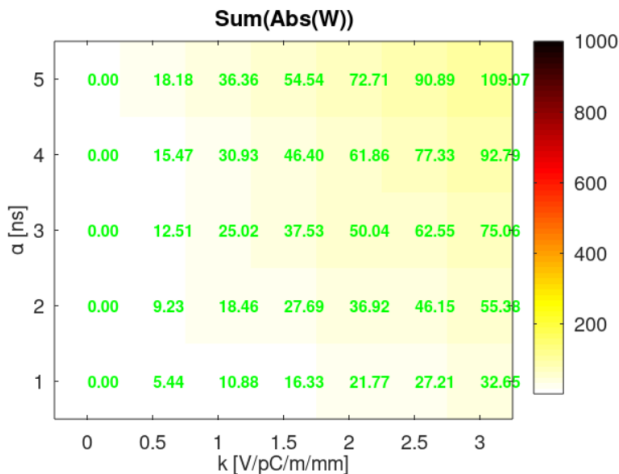
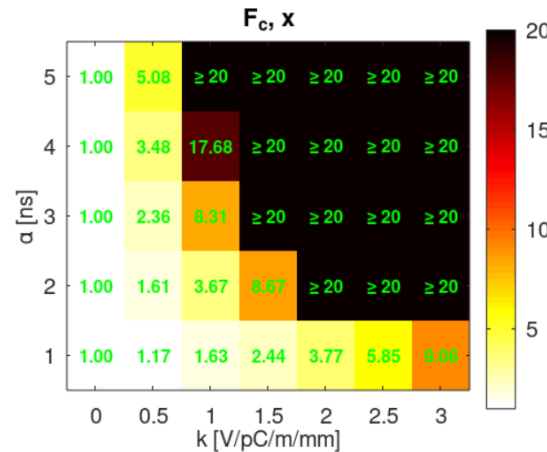
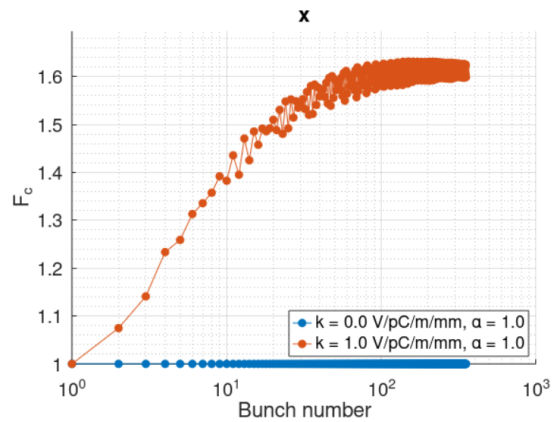
Strategy

- The analytic formulae are being verified
- For the moment, **the most fast, reliable and conservative** way to estimate jitter amplification might be using **single particle** tracking and maybe better to take the **maximum** number if statistics allows
- To be conservative, envelop of wakefield is also used and assumed to be the **$k/[1+(t-T)/\alpha]$** function, with a **2D scan of k and α** , but with small numbers (corresponding to wakefield after damping)

Test 3.3.1

- Following strategy in last slide (single particle tracking)

Coherent case. 8 trains (evenly spaced on ellipse)
Using maximum number



Test 3.3.2

- Following strategy in last slide (single particle tracking)

**Incoherent case. 1000 trains (randomly spaced in phase space)
Using average number (due to large fluctuations)**

