Testing Quantum Mechanics at polarised e+e- colliders

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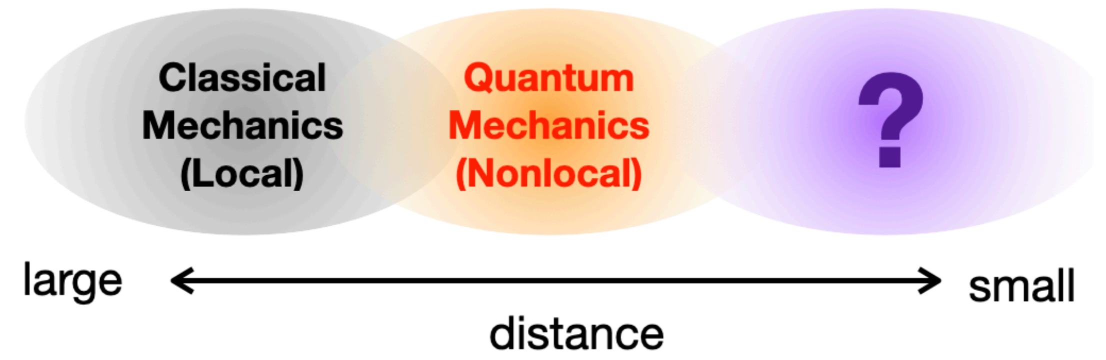
High Energy Test of QM

- At very short-distances (high-energies), QM might be modified.
  - sense of locality may change (again)
  - QM might be modified to be married with gravity



- How can we falsify/test QM?

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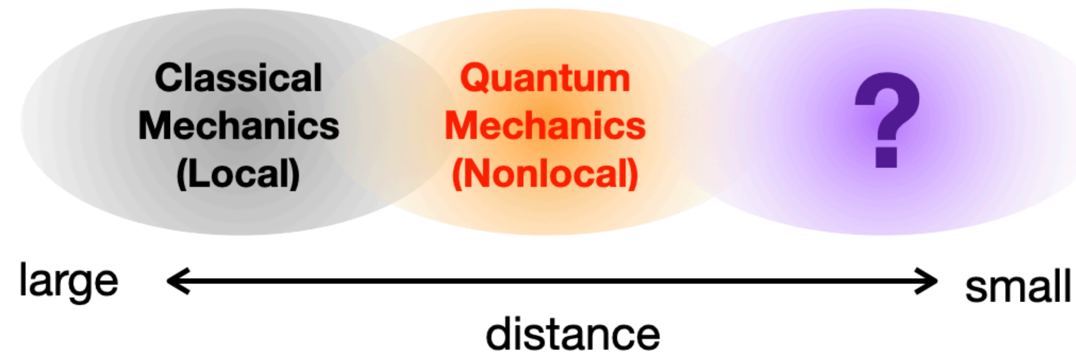

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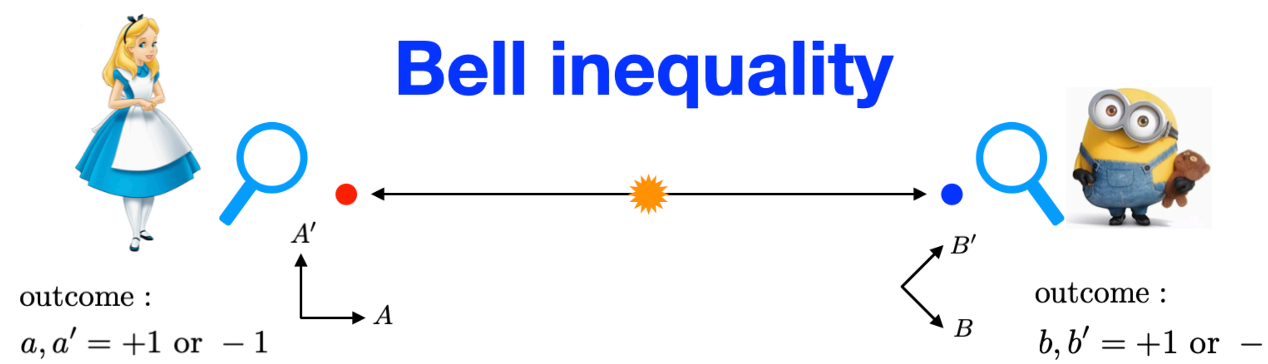
$$\langle \mathcal{B} \rangle = (\langle AB \rangle + \langle A'B \rangle) + (\langle AB' \rangle - \langle A'B' \rangle)$$

$$\Rightarrow \begin{cases} \langle \mathcal{B} \rangle_{\rm LR} \leq 2 & \text{Local-Real [CHSH 1969]} \\ \langle \mathcal{B} \rangle_{\rm QM} \leq 2\sqrt{2} & \text{QM [Tsirelson 1987]} \end{cases}$$



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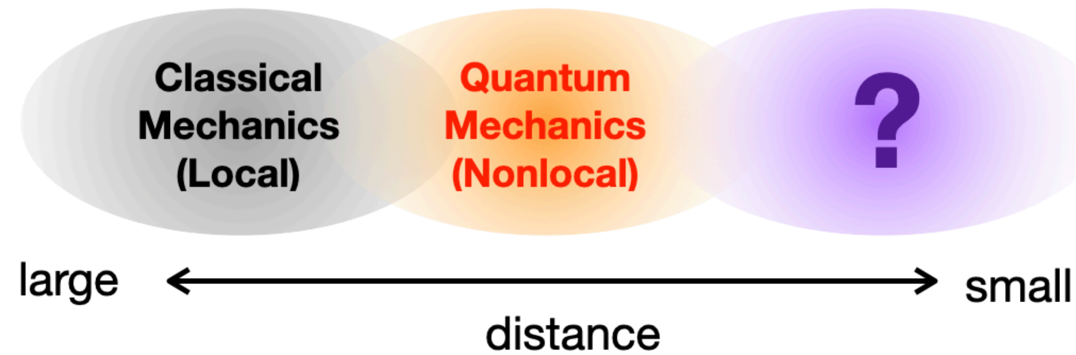

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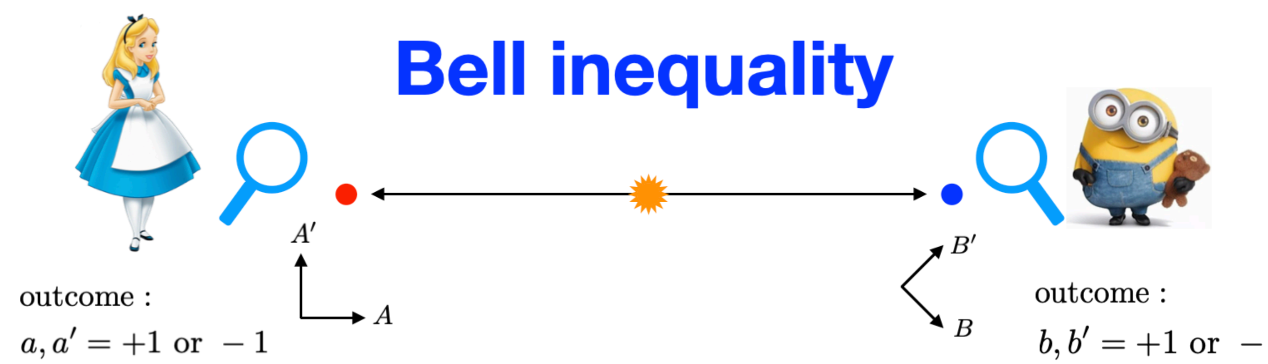

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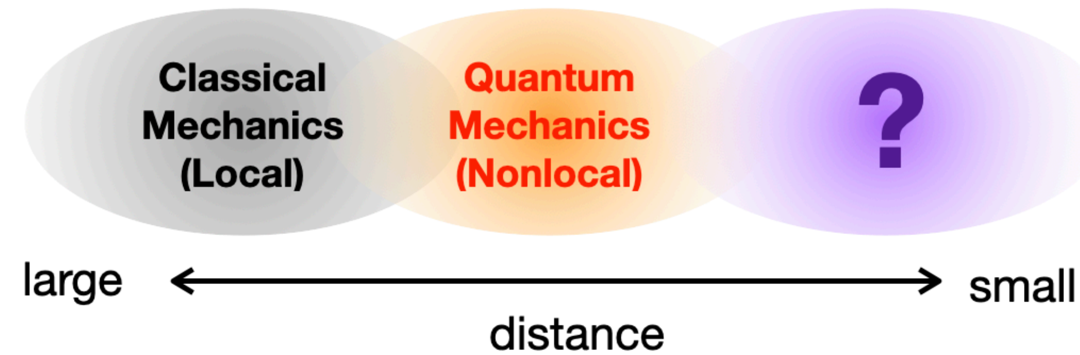

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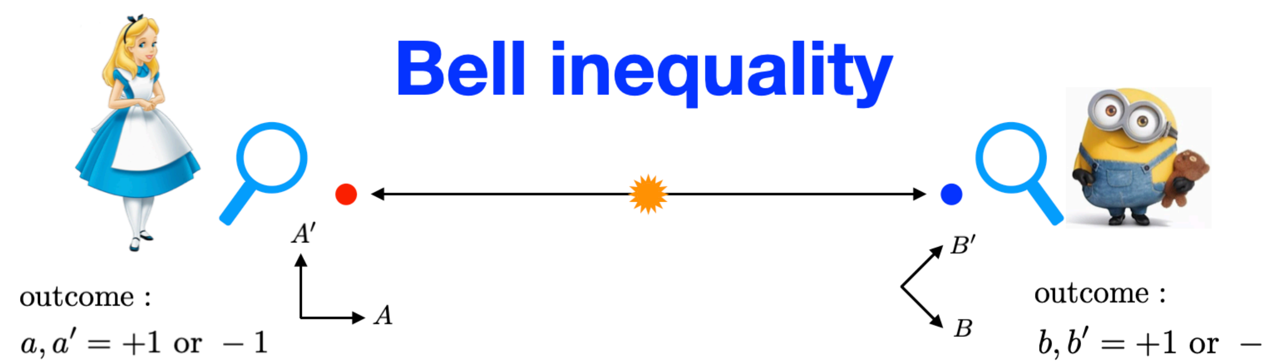

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- Possible modification of QM?
  - No-signalling theories: $\langle \mathcal{B} \rangle_{\rm NS} \le 4$ [Cirel'son (1980), Popescu, Rohrlich (1994)]
  - Non-linear extensions of QM: [Weinberg (1989), Polchinski (1991), D.E.Kaplan, S.Rajendran, (2021)]

$$i\partial_t |\chi\rangle = \int d^3x \left[\hat{\mathcal{H}}(x) + \langle \chi | \hat{\mathcal{O}}_1(x) | \chi \rangle \hat{\mathcal{O}}_2(x) \right] |\chi\rangle$$

non-linear state-dependent term

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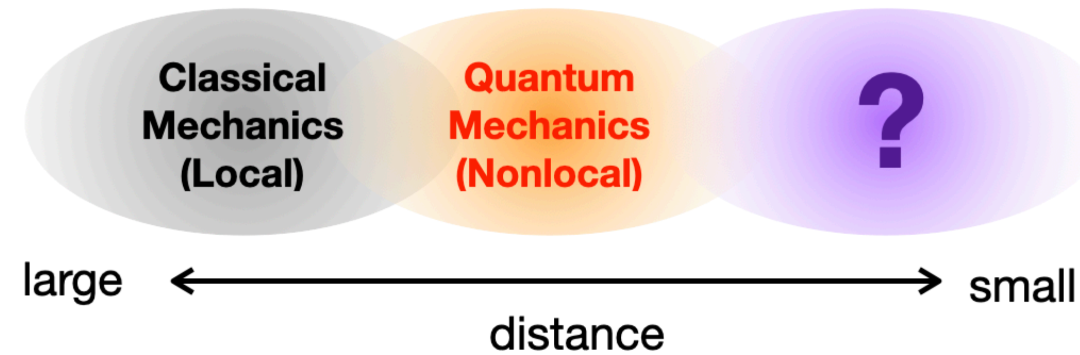

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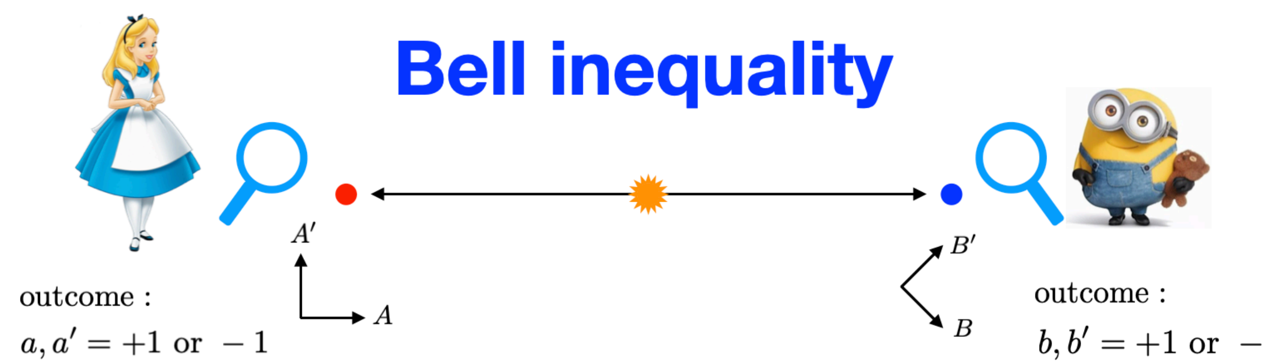

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A new test of QM with Quantum Channels / Instruments

Quantum Channels/Instruments

$e^- e^+$ — input state: $\rho_0 \in \mathcal{S}(\mathcal{H}_0)$ (set of density operators)

$\longrightarrow$

$t\bar{t}$ — output: $\varrho_x \in \mathcal{B}(\mathcal{H}_1)$ (set of positive linear operators (unnormalised density operators))

Quantum Instrument (QI) is a **map**:

$$\mathcal{I}_x : \mathcal{S}(\mathcal{H}_0) \to \mathcal{B}(\mathcal{H}_1)$$

$$\mathcal{I}_x(\rho_0) = \varrho_x$$

generated by

1) unitary evolution with *environment*
2) *measurement* (with measurement outcome x)

Quantum Channels/Instruments

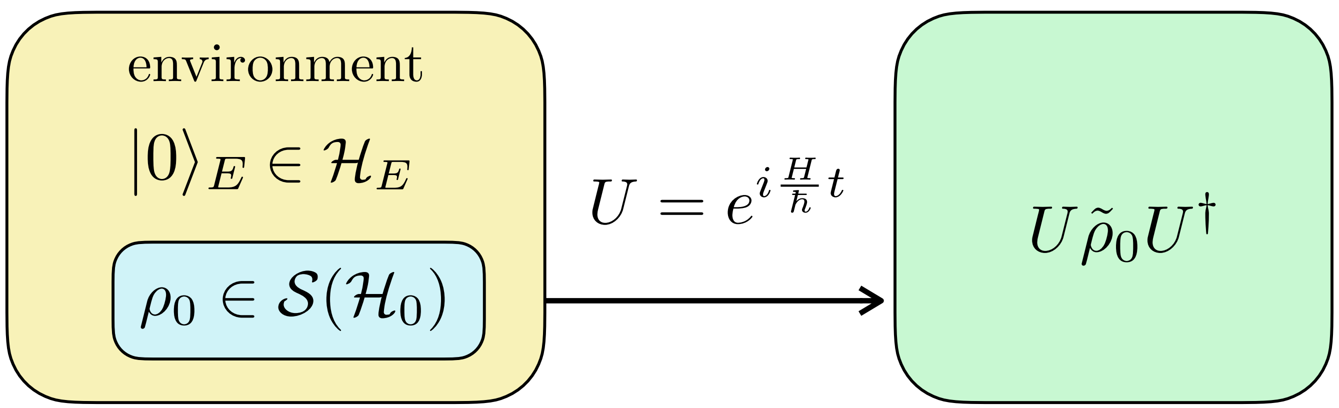
environment

$|0\rangle_E \in \mathcal{H}_E$

$\rho_0 \in \mathcal{S}(\mathcal{H}_0)$

$$\tilde{\rho}_0 = |0\rangle\langle 0|_E \otimes \rho_0$$

Quantum Channels/Instruments



$$\tilde{\rho}_0 = |0\rangle\langle 0|_E \otimes \rho_0 \quad \xrightarrow{\;U\;} \quad U\Big[|0\rangle\langle 0|_E \otimes \rho_0\Big]U^\dagger$$

Quantum Channels/Instruments

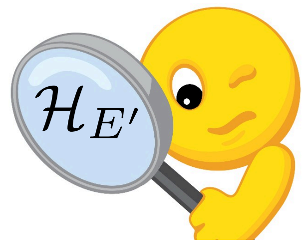
environment

$|0\rangle_E \in \mathcal{H}_E$

$\rho_0 \in \mathcal{S}(\mathcal{H}_0)$

$U = e^{i\frac{H}{\hbar}t}$

$U\tilde{\rho}_0 U^\dagger$



$$\mathcal{P}_x = \sum_{k\in x} |e_k\rangle\langle e_k|_{E'}$$

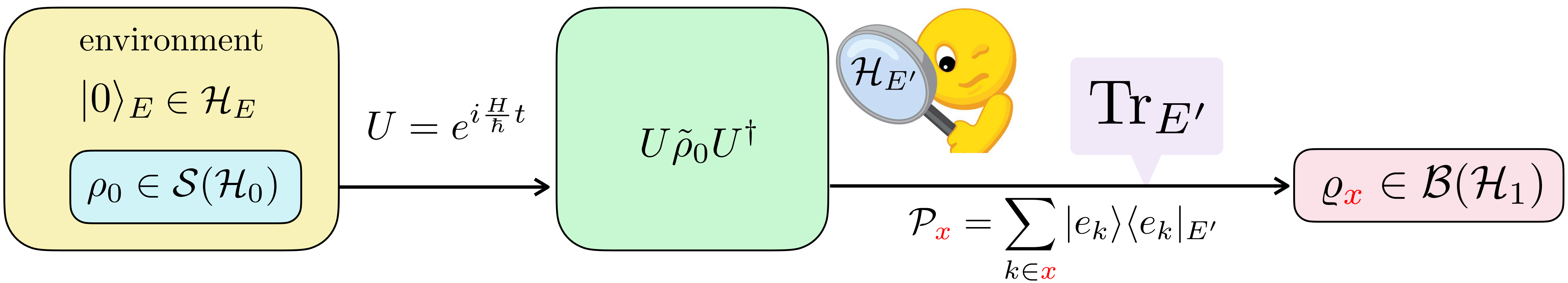
measurement outcome lies in x

$$\tilde{\rho}_0 = |0\rangle\langle 0|_E \otimes \rho_0 \xrightarrow{U} U\Big[|0\rangle\langle 0|_E \otimes \rho_0\Big]U^\dagger$$

measuring E'

$$\mathcal{P}_x U\Big[|0\rangle\langle 0|_E \otimes \rho_0\Big]U^\dagger \mathcal{P}_x$$

Quantum Channels/Instruments



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$$\tilde{\rho}_0 = |0\rangle\langle 0|_E \otimes \rho_0 \xrightarrow{U} U\Big[|0\rangle\langle 0|_E \otimes \rho_0\Big]U^\dagger$$

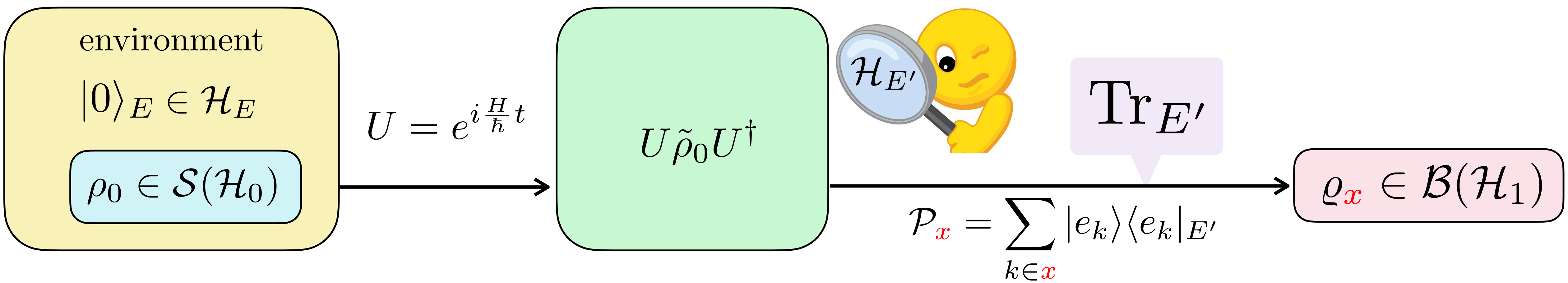
$\downarrow$ measuring E'

$$\mathcal{P}_x U\Big[|0\rangle\langle 0|_E \otimes \rho_0\Big]U^\dagger \mathcal{P}_x$$

$\downarrow$ tracing out E'

$$\varrho_x = \mathrm{Tr}_{E'}\Big[\mathcal{P}_x U\Big[|0\rangle\langle 0|_E \otimes \rho_0\Big]U^\dagger \mathcal{P}_x\Big]$$

Quantum Channels/Instruments



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$$= \sum_{k\in x} \langle e'_k|U|0\rangle\langle 0|_E \otimes \rho_0 U^\dagger |e'_k\rangle$$

$$= \sum_{k\in x} E_k \rho_0 E_k^\dagger$$

Kraus operators:

$$E_k = \langle e'_k|U|0\rangle_E$$

$$E_k : \mathcal{H}_0 \to \mathcal{H}_1$$

Quantum Channels/Instruments

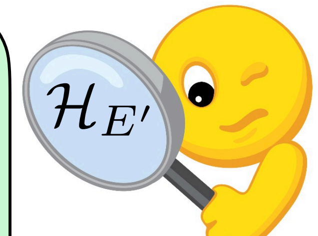
environment

$|0\rangle_E \in \mathcal{H}_E$

$\rho_0 \in \mathcal{S}(\mathcal{H}_0)$

$U = e^{i\frac{H}{\hbar}t}$ →

$U\tilde{\rho}_0 U^\dagger$



$\mathrm{Tr}_{E'}$

→ $\varrho_x \in \mathcal{B}(\mathcal{H}_1)$

$\mathcal{P}_x = \sum_{k\in x} |e_k\rangle\langle e_k|_{E'}$

↑ measurement outcome lies in x

$$\tilde{\rho}_0 = |0\rangle\langle 0|_E \otimes \rho_0 \xrightarrow{U} U\Big[|0\rangle\langle 0|_E \otimes \rho_0\Big]U^\dagger$$

↓ measuring E'

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Quantum Instrument I_x

Kraus operators:

$E_k = \langle e'_k|U|0\rangle_E$

$E_k : \mathcal{H}_0 \to \mathcal{H}_1$

$\mathcal{I}_x : \mathcal{S}(\mathcal{H}_0) \to \mathcal{B}(\mathcal{H}_1)$

$\mathcal{I}_x(\rho_0) = \varrho_x$

Q. What is the general property of the QI map?

Kraus operators:

$$E_k = \langle e'_k | U | 0 \rangle_E$$

$$E_k : \mathcal{H}_0 \to \mathcal{H}_1$$

$$\mathcal{I}_x(\rho_0) = \varrho_x = \sum_{k \in x} E_k \rho_0 E_k^\dagger$$

A.

- Linear
- Trace decreasing $0 \leq \mathrm{Tr}\varrho_x \leq \mathrm{Tr}\rho_0 = 1$
- *Completely* Positive

Completely positive map

$$\mathbf{1}_R \otimes E_k$$

$$R \otimes \mathcal{S}(\mathcal{H}_0) \longrightarrow R \otimes \mathcal{B}(\mathcal{H}_1)$$

is also positive

post-QI (normalised) state

$$\rho_0 \to \rho_x = \frac{\varrho_x}{\mathrm{Tr}\varrho_x} = \frac{\mathcal{I}_x(\rho_0)}{\mathrm{Tr}\mathcal{I}_x(\rho_0)}$$

← state-to-(normalised)state map is ***NOT*** linear

Any map satisfying these properties correspond to **some physical quantum instrument**

Choi Matrix

$$d_0 = \dim\mathcal{H}_0,\ d_1 = \dim\mathcal{H}_1, \quad \rho_0 = \sum_{i,j} \rho_{ij} |i\rangle\langle j|$$

$$\varrho_x = \mathcal{I}_x(\rho_0) = \sum_{i,j} \rho_{ij} \mathcal{I}_x\left(|i\rangle\langle j|\right)$$

If all d_0^2 operators $I_x(|i\rangle\langle j|) \in L(\mathscr{H}_1)$ is fixed, the **QI map is completely determined**.

Each operator $I_x(|i\rangle\langle j|)$ can be represented by a $d_1 \times d_1$ matrix: $[\mathcal{I}_x(|i\rangle\langle j|)]_{\alpha\beta} = \langle\alpha|\mathcal{I}_x(|i\rangle\langle j|)|\beta\rangle$

Choi matrix: $(d_0 \times d_1)$ dim square matrix

$(d_1 \times d_1)$ matrix

$$\widetilde{\mathcal{I}}_x \equiv \frac{1}{d_0}\begin{pmatrix} [\mathcal{I}_x(|1\rangle\langle 1|)] & [\mathcal{I}_x(|1\rangle\langle 2|)] & \cdots & [\mathcal{I}_x(|1\rangle\langle d_0|)] \\ [\mathcal{I}_x(|2\rangle\langle 1|)] & [\mathcal{I}_x(|2\rangle\langle 2|)] & \cdots & [\mathcal{I}_x(|2\rangle\langle d_0|)] \\ \vdots & \vdots & \ddots & \vdots \\ [\mathcal{I}_x(|d_0\rangle\langle 1|)] & [\mathcal{I}_x(|d_0\rangle\langle 2|)] & \cdots & [\mathcal{I}_x(|d_0\rangle\langle d_0|)] \end{pmatrix}$$

- Linear
- Completely Positive → $\widetilde{\mathcal{I}}_x$ is positive
- Trace decreasing → $\mathrm{Tr}\left[\widetilde{\mathcal{I}}_x\right] \leq 1$

Spin measurement at Colliders

$$e^- e^+ \to t\bar{t}$$

$$\rho_0 \in \mathcal{S}(\mathcal{H}_0) = \mathbb{C}^2_{e^-} \otimes \mathbb{C}^2_{e^+} \longrightarrow \varrho_x \in \mathcal{B}(\mathcal{H}_1) = \mathbb{C}^2_t \otimes \mathbb{C}^2_{\bar{t}}$$

$$\rho_0 = \sum_{s_e} q_{s_e} |s_e\rangle\langle s_e| \qquad s_e = \{++, +-, -+, --\}$$

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$t\bar{t}$ phase-space selection

$$\mathcal{P}_x = \sum_{s_t} \int_x d\Pi_{t\bar{t}} |p_t, s_t\rangle\langle p_t, s_t|$$

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$$\propto \frac{\delta(p)|_{p=0}}{[\delta(p)|_{p=0}]^3} = \frac{T}{V} \sim \frac{1}{[\infty]^2}$$

$$\varrho_x = \frac{V}{T}\frac{1}{2\sigma_{\mathcal{N}}}\varrho'_x$$

$$\sigma_{\mathcal{N}} = \sigma[e^- e^+(\rho_0^{\text{mix}}) \to t\bar{t}]$$

$$\rho_0^{\text{mix}} = \frac{1}{4}\sum_{s_e} |s_e\rangle\langle s_e|$$

$$= \frac{1}{\sigma_{\mathcal{N}}}\sum_{s_e, s_t, s'_t} q_{s_e}\left[\frac{1}{2s}\int_x d\Pi^{t\bar{t}}_{\text{LIPS}}\mathcal{M}^{p_{\text{in}}, s_e}_{p_t, s_t}[\mathcal{M}^{p_{\text{in}}, s_e}_{p_t, s'_t}]^*\right]|s_t\rangle\langle s'_t|$$

The map $\rho_0 \to \varrho_x$ is

- Linear
- Completely Positive
- ~~Trace decreasing~~

$$\text{Tr}\,\varrho_x = \frac{\sigma_x[e^- e^+(\rho_0) \to t\bar{t}]}{\sigma[e^- e^+(\rho_0^{\text{mix}}) \to t\bar{t}]}$$ can be larger than 1

The positive (16 x 16) Choi matrix $\widetilde{I}_x$ exists:

$$\widetilde{\mathcal{I}}_x = \frac{1}{4}\begin{pmatrix} \mathcal{I}_x(|++\rangle\langle++|) & \mathcal{I}_x(|++\rangle\langle+-|) & \mathcal{I}_x(|++\rangle\langle-+|) & \mathcal{I}_x(|++\rangle\langle--|) \\ \mathcal{I}_x(|+-\rangle\langle++|) & \mathcal{I}_x(|+-\rangle\langle+-|) & \mathcal{I}_x(|+-\rangle\langle-+|) & \mathcal{I}_x(|+-\rangle\langle--|) \\ \mathcal{I}_x(|-+\rangle\langle++|) & \mathcal{I}_x(|-+\rangle\langle+-|) & \mathcal{I}_x(|-+\rangle\langle-+|) & \mathcal{I}_x(|-+\rangle\langle--|) \\ \mathcal{I}_x(|--\rangle\langle++|) & \mathcal{I}_x(|--\rangle\langle+-|) & \mathcal{I}_x(|--\rangle\langle-+|) & \mathcal{I}_x(|--\rangle\langle--|) \end{pmatrix}$$

Quantum ***Process*** Tomography

- **QPT** ↔ Experimental reconstruction of QI map, i.e. Choi matrix, $\widetilde{I}_x(|i\rangle\langle j|)$

Why is it useful?

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Once $\widetilde{I}_x$ reconstructed/computed:

- one can compute the QI outcome ϱ_x immediately:

$$\rho_0 = \sum_{i,j} \rho_{ij} |i\rangle\langle j| \quad \longrightarrow \quad \varrho_x = \sum_{i,j} \rho_{ij} \widetilde{\mathcal{I}}_x(|i\rangle\langle j|)$$

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- one can test the property of the process:

Linearity:

- reconstruct $\widetilde{I}_x$ from N input states: $\{\rho_0^{\text{in}}\} = \{\rho_0^{\text{in},1}, \rho_0^{\text{in},2}, \cdots, \rho_0^{\text{in},N}\}$
- check whether the prediction from linearity $\sum_{i,j} \rho_{ij} \widetilde{I}_x(|i\rangle\langle j|)$ agrees with the direct measurement $\widetilde{I}_x(\rho_0)$, for $\rho_0 = \sum_{i,j} \rho_{ij} |i\rangle\langle j|$ not in the input list.

Complete positivity:

- check whether all eigenvalues of the Choi matrix $I_x(|i\rangle\langle j|)$ are non-negative.

Quantum ***Process*** Tomography

- **QPT** $\leftrightarrow$ Experimental reconstruction of QI map, i.e. Choi matrix, $\widetilde{I}_x(|i\rangle\langle j|)$

Why is it useful?

Once $\widetilde{I}_x$ reconstructed/computed:

- one can compute the QI outcome ϱ_x immediately:

$$\rho_0 = \sum_{i,j} \rho_{ij} |i\rangle\langle j| \longrightarrow \varrho_x = \sum_{i,j} \rho_{ij} \widetilde{\mathcal{I}}_x(|i\rangle\langle j|)$$

- one can test the property of the process:

Linearity:

- reconstruct $\widetilde{I}_x$ from N input states: $\{\rho_0^{\text{in}}\} = \{\rho_0^{\text{in},1}, \rho_0^{\text{in},2}, \cdots, \rho_0^{\text{in},N}\}$
- check whether the prediction from linearity $\sum_{i,j} \rho_{ij} \widetilde{I}_x(|i\rangle\langle j|)$ agrees with the direct measurement $\widetilde{I}_x(\rho_0)$, for $\rho_0 = \sum_{i,j} \rho_{ij} |i\rangle\langle j|$ not in the input list.

Complete positivity:

- check whether all eigenvalues of the Choi matrix $I_x(|i\rangle\langle j|)$ are non-negative.

➡ QM would be ***falsified*** if one of them failed! → **New test of QM !**

QPT with *polarised* lepton colliders

- Reconstruction of the diagonal part:

$$\widetilde{\mathcal{I}}_x = \frac{1}{4}\begin{pmatrix} \mathcal{I}_x(|++\rangle\langle++|) & \mathcal{I}_x(|++\rangle\langle+-|) & \mathcal{I}_x(|++\rangle\langle-+|) & \mathcal{I}_x(|++\rangle\langle--|) \\ \mathcal{I}_x(|+-\rangle\langle++|) & \mathcal{I}_x(|+-\rangle\langle+-|) & \mathcal{I}_x(|+-\rangle\langle-+|) & \mathcal{I}_x(|+-\rangle\langle--|) \\ \mathcal{I}_x(|-+\rangle\langle++|) & \mathcal{I}_x(|-+\rangle\langle+-|) & \mathcal{I}_x(|-+\rangle\langle-+|) & \mathcal{I}_x(|-+\rangle\langle--|) \\ \mathcal{I}_x(|--\rangle\langle++|) & \mathcal{I}_x(|--\rangle\langle+-|) & \mathcal{I}_x(|--\rangle\langle-+|) & \mathcal{I}_x(|--\rangle\langle--|) \end{pmatrix}$$

- Consider 4 purely polarised beam settings:

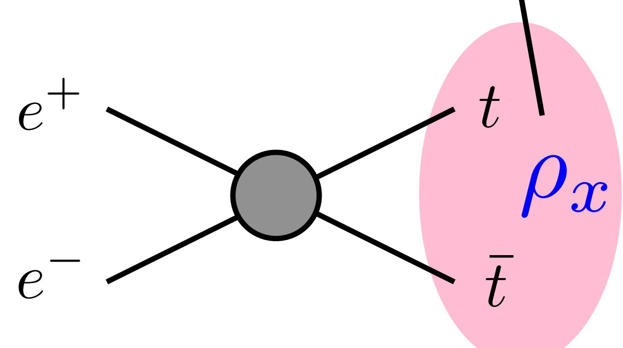
$$\{|i\rangle\} = \{|{++}\rangle, |{+-}\rangle, |{-+}\rangle, |{--}\rangle\} \qquad \rho_0^i = |i\rangle\langle i|$$

$$\mathcal{I}_x(|i\rangle\langle i|) = \varrho_x^i = \mathrm{Tr}\,\varrho_x^i \cdot \frac{\varrho_x^i}{\mathrm{Tr}\,\varrho_x^i}$$

$$e^- e^+ \to t\bar{t}$$

measurable

$$\frac{\sigma_x[e^- e^+(\rho_0^i) \to t\bar{t}]}{\sigma[e^- e^+(\rho_0^{\mathrm{mix}}) \to t\bar{t}]}$$



Quantum **State** Tomography

Y.Afik, J.Nova [2003.02280],
Ashby-Pickering, Barr, Wierzchucka [2209.13990]

- For not perfectly polarised beams,

$$\rho_{e^-}^{(\omega^-)} = \frac{1}{2}(1+\omega^- q)|+\rangle\langle+| + \frac{1}{2}(1-\omega^- q)|-\rangle\langle-|$$

$$\rho_{e^+}^{(\omega^+)} = \frac{1}{2}(1+\omega^+ \bar{q})|+\rangle\langle+| + \frac{1}{2}(1-\omega^+ \bar{q})|-\rangle\langle-|$$

$$0 < q, \bar{q} < 1$$

$$(\omega^-, \omega^+) = \underbrace{\{(+,+), (+,-), (-,+), (-,-)\}}_{\text{4 beam settings}}$$

$$\rho_0^{(\omega^-,\omega^+)} = \rho_{e^-}^{(\omega^-)} \otimes \rho_{e^+}^{(\omega^+)} = \frac{1}{4}\Big[(1+\omega^- q)(1+\omega^+\bar{q})|++\rangle\langle++| + (1+\omega^- q)(1-\omega^+\bar{q})|+-\rangle\langle+-|$$

$$+(1-q)(1+\bar{q})|-+\rangle\langle-+| + (1-q)(1-\bar{q})|--\rangle\langle--|\Big]$$

input beam state

convex linear sum of four pure states

- For not perfectly polarised beams,

$$\rho_{e^-}^{(\omega^-)} = \frac{1}{2}(1+\omega^- q)|+\rangle\langle+| + \frac{1}{2}(1-\omega^- q)|-\rangle\langle-|$$

$$\rho_{e^+}^{(\omega^+)} = \frac{1}{2}(1+\omega^+ \bar{q})|+\rangle\langle+| + \frac{1}{2}(1-\omega^+ \bar{q})|-\rangle\langle-|$$

$$0 < q, \bar{q} < 1$$

$$(\omega^-, \omega^+) = \{(+,+), (+,-), (-,+), (-,-)\}$$

4 beam settings

$$\rho_0^{(\omega^-,\omega^+)} = \rho_{e^-}^{(\omega^-)} \otimes \rho_{e^+}^{(\omega^+)} = \frac{1}{4}\Big[(1+\omega^- q)(1+\omega^+\bar{q})|++\rangle\langle++| + (1+\omega^- q)(1-\omega^+\bar{q})|+-\rangle\langle+-|$$
$$+(1-q)(1+\bar{q})|-+\rangle\langle-+| + (1-q)(1-\bar{q})|--\rangle\langle--|\Big]$$

input beam state

convex linear sum of four pure states

$$\begin{pmatrix} \rho_0^{(+,+)} \\ \rho_0^{(+,-)} \\ \rho_0^{(-,+)} \\ \rho_0^{(-,-)} \end{pmatrix} = \frac{1}{4}\begin{pmatrix} (1+q)(1+\bar{q}) & (1+q)(1-\bar{q}) & (1-q)(1+\bar{q}) & (1-q)(1-\bar{q}) \\ (1+q)(1-\bar{q}) & (1+q)(1+\bar{q}) & (1-q)(1-\bar{q}) & (1-q)(1+\bar{q}) \\ (1-q)(1+\bar{q}) & (1-q)(1-\bar{q}) & (1+q)(1+\bar{q}) & (1+q)(1-\bar{q}) \\ (1-q)(1-\bar{q}) & (1-q)(1+\bar{q}) & (1+q)(1-\bar{q}) & (1+q)(1+\bar{q}) \end{pmatrix} \begin{pmatrix} |++\rangle\langle++| \\ |+-\rangle\langle+-| \\ |-+\rangle\langle-+| \\ |--\rangle\langle--| \end{pmatrix}$$

- For not perfectly polarised beams,

$$\rho_{e^-}^{(\omega^-)} = \frac{1}{2}(1+\omega^- q)|+\rangle\langle+| + \frac{1}{2}(1-\omega^- q)|-\rangle\langle-|$$

$$\rho_{e^+}^{(\omega^+)} = \frac{1}{2}(1+\omega^+ \bar{q})|+\rangle\langle+| + \frac{1}{2}(1-\omega^+ \bar{q})|-\rangle\langle-|$$

$$0 < q, \bar{q} < 1$$

$$(\omega^-, \omega^+) = \{(+,+), (+,-), (-,+), (-,-)\}$$

4 beam settings

$$\rho_0^{(\omega^-,\omega^+)} = \rho_{e^-}^{(\omega^-)} \otimes \rho_{e^+}^{(\omega^+)} = \frac{1}{4}\Big[(1+\omega^- q)(1+\omega^+\bar{q})|++\rangle\langle++| + (1+\omega^- q)(1-\omega^+\bar{q})|+-\rangle\langle+-|$$

$$+(1-q)(1+\bar{q})|-+\rangle\langle-+| + (1-q)(1-\bar{q})|--\rangle\langle--|\Big]$$

input beam state

convex linear sum of four pure states

$$\mathcal{I}_x\left[\begin{pmatrix}\rho_0^{(+,+)}\\ \rho_0^{(+,-)}\\ \rho_0^{(-,+)}\\ \rho_0^{(-,-)}\end{pmatrix} = \frac{1}{4}\begin{pmatrix}(1+q)(1+\bar{q}) & (1+q)(1-\bar{q}) & (1-q)(1+\bar{q}) & (1-q)(1-\bar{q})\\ (1+q)(1-\bar{q}) & (1+q)(1+\bar{q}) & (1-q)(1-\bar{q}) & (1-q)(1+\bar{q})\\ (1-q)(1+\bar{q}) & (1-q)(1-\bar{q}) & (1+q)(1+\bar{q}) & (1+q)(1-\bar{q})\\ (1-q)(1-\bar{q}) & (1-q)(1+\bar{q}) & (1+q)(1-\bar{q}) & (1+q)(1+\bar{q})\end{pmatrix}\begin{pmatrix}|++\rangle\langle++|\\ |+-\rangle\langle+-|\\ |-+\rangle\langle-+|\\ |--\rangle\langle--|\end{pmatrix}\right]$$

diag. entries of Choi matrix

$$\begin{pmatrix}\mathcal{I}_x(|++\rangle\langle++|)\\ \mathcal{I}_x(|+-\rangle\langle+-|)\\ \mathcal{I}_x(|-+\rangle\langle-+|)\\ \mathcal{I}_x(|--\rangle\langle--|)\end{pmatrix} = \Big(\quad\Big)^{-1}\begin{pmatrix}\mathcal{I}_x(\rho_0^{(+,+)})\\ \mathcal{I}_x(\rho_0^{(+,-)})\\ \mathcal{I}_x(\rho_0^{(-,+)})\\ \mathcal{I}_x(\rho_0^{(-,-)})\end{pmatrix}$$

measurable

- Reconstruction of **off**-diagonal elements:

$$\widetilde{\mathcal{I}}_x = \frac{1}{4}\begin{pmatrix} \mathcal{I}_x(|++\rangle\langle++|) & \mathcal{I}_x(|++\rangle\langle+-|) & \mathcal{I}_x(|++\rangle\langle-+|) & \mathcal{I}_x(|++\rangle\langle--|) \\ \mathcal{I}_x(|+-\rangle\langle++|) & \mathcal{I}_x(|+-\rangle\langle+-|) & \mathcal{I}_x(|+-\rangle\langle-+|) & \mathcal{I}_x(|+-\rangle\langle--|) \\ \mathcal{I}_x(|-+\rangle\langle++|) & \mathcal{I}_x(|-+\rangle\langle+-|) & \mathcal{I}_x(|-+\rangle\langle-+|) & \mathcal{I}_x(|-+\rangle\langle--|) \\ \mathcal{I}_x(|--\rangle\langle++|) & \mathcal{I}_x(|--\rangle\langle+-|) & \mathcal{I}_x(|--\rangle\langle-+|) & \mathcal{I}_x(|--\rangle\langle--|) \end{pmatrix}$$

- Consider polarisations ***NOT*** in the direction of the beam:

$$|\mathbf{m}\rangle = \alpha\,|+\rangle + \beta\,|-\rangle\,, \qquad |-\mathbf{m}\rangle = \bar{\alpha}\,|+\rangle + \bar{\beta}\,|-\rangle\,, \qquad |\alpha|^2 + |\beta|^2 = |\gamma|^2 + |\delta|^2 = 1$$

$$|\mathbf{n}\rangle = \gamma\,|+\rangle + \delta\,|-\rangle\,, \qquad |-\mathbf{n}\rangle = \bar{\gamma}\,|+\rangle + \bar{\delta}\,|-\rangle\,. \qquad \alpha\bar{\alpha}^* + \beta\bar{\beta}^* = \gamma\bar{\gamma} + \delta\bar{\delta}^* = 0$$

- Consider the beam setting $(e^-, e^+) = (+, \mathbf{m})$

$$\begin{aligned} \rho_0^{(+,\mathbf{m})} &= |+\rangle\langle+| \otimes |\mathbf{m}\rangle\langle\mathbf{m}| \\ &= |\alpha|^2|++\rangle\langle++| + \alpha\beta^*|++\rangle\langle+-| + \alpha^*\beta|+-\rangle\langle++| + |\beta|^2|--\rangle\langle--| \end{aligned}$$

- Reconstruction of **off**-diagonal elements:

$$\widetilde{\mathcal{I}}_x = \frac{1}{4}\begin{pmatrix} \mathcal{I}_x(|++\rangle\langle++|) & \mathcal{I}_x(|++\rangle\langle+-|) & \mathcal{I}_x(|++\rangle\langle-+|) & \mathcal{I}_x(|++\rangle\langle--|) \\ \mathcal{I}_x(|+-\rangle\langle++|) & \mathcal{I}_x(|+-\rangle\langle+-|) & \mathcal{I}_x(|+-\rangle\langle-+|) & \mathcal{I}_x(|+-\rangle\langle--|) \\ \mathcal{I}_x(|-+\rangle\langle++|) & \mathcal{I}_x(|-+\rangle\langle+-|) & \mathcal{I}_x(|-+\rangle\langle-+|) & \mathcal{I}_x(|-+\rangle\langle--|) \\ \mathcal{I}_x(|--\rangle\langle++|) & \mathcal{I}_x(|--\rangle\langle+-|) & \mathcal{I}_x(|--\rangle\langle-+|) & \mathcal{I}_x(|--\rangle\langle--|) \end{pmatrix}$$

- Consider polarisations ***NOT*** in the direction of the beam:

$$|\mathbf{m}\rangle = \alpha\,|+\rangle + \beta\,|-\rangle\,, \quad |-\mathbf{m}\rangle = \bar{\alpha}\,|+\rangle + \bar{\beta}\,|-\rangle\,, \qquad |\alpha|^2 + |\beta|^2 = |\gamma|^2 + |\delta|^2 = 1$$

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- Consider the beam setting $(e^-, e^+) = (+, \mathbf{m})$

$$\rho_0^{(+,\mathbf{m})} = |+\rangle\langle+| \otimes |\mathbf{m}\rangle\langle\mathbf{m}|$$

$$= |\alpha|^2|++\rangle\langle++| + \alpha\beta^*|++\rangle\langle+-| + \alpha^*\beta|+-\rangle\langle++| + |\beta|^2|--\rangle\langle--|$$

$\mathcal{I}_x$ ↓

$\varrho_x^{(+,\mathbf{m})}$ — measure this time

$\varrho_x^{(+,+)}$ — already reconstructed

$\mathcal{I}_x(|++\rangle\langle+-|)$ — target

$\mathcal{I}_x(|+-\rangle\langle++|)$ — target

$\varrho_x^{(-,-)}$ — already reconstructed

- Reconstruction of **off**-diagonal elements:

$$\widetilde{\mathcal{I}}_x = \frac{1}{4}\begin{pmatrix} \mathcal{I}_x(|++\rangle\langle++|) & \mathcal{I}_x(|++\rangle\langle+-|) & \mathcal{I}_x(|++\rangle\langle-+|) & \mathcal{I}_x(|++\rangle\langle--|) \\ \mathcal{I}_x(|+-\rangle\langle++|) & \mathcal{I}_x(|+-\rangle\langle+-|) & \mathcal{I}_x(|+-\rangle\langle-+|) & \mathcal{I}_x(|+-\rangle\langle--|) \\ \mathcal{I}_x(|-+\rangle\langle++|) & \mathcal{I}_x(|-+\rangle\langle+-|) & \mathcal{I}_x(|-+\rangle\langle-+|) & \mathcal{I}_x(|-+\rangle\langle--|) \\ \mathcal{I}_x(|--\rangle\langle++|) & \mathcal{I}_x(|--\rangle\langle+-|) & \mathcal{I}_x(|--\rangle\langle-+|) & \mathcal{I}_x(|--\rangle\langle--|) \end{pmatrix}$$

- Consider polarisations ***NOT*** in the direction of the beam:

$$|\mathbf{m}\rangle = \alpha\,|+\rangle + \beta\,|-\rangle\,, \quad |-\mathbf{m}\rangle = \bar{\alpha}\,|+\rangle + \bar{\beta}\,|-\rangle\,, \qquad |\alpha|^2 + |\beta|^2 = |\gamma|^2 + |\delta|^2 = 1$$

$$|\mathbf{n}\rangle = \gamma\,|+\rangle + \delta\,|-\rangle\,, \quad |-\mathbf{n}\rangle = \bar{\gamma}\,|+\rangle + \bar{\delta}\,|-\rangle\,. \qquad \alpha\bar{\alpha}^* + \beta\bar{\beta}^* = \gamma\bar{\gamma} + \delta\bar{\delta}^* = 0$$

- Consider the beam setting $(e^-, e^+) = (+, \mathbf{m})$

$$\begin{aligned} \rho_0^{(+,\mathbf{m})} &= |+\rangle\langle+| \otimes |\mathbf{m}\rangle\langle\mathbf{m}| \\ &= |\alpha|^2|++\rangle\langle++| + \alpha\beta^*|++\rangle\langle+-| + \alpha^*\beta|+-\rangle\langle++| + |\beta|^2|--\rangle\langle--| \end{aligned}$$

$\mathcal{I}_x$ ↓

$\varrho_x^{(+,\mathbf{m})}$	$\varrho_x^{(+,+)}$	$\mathcal{I}_x(\vert++\rangle\langle+-\vert)$	$\mathcal{I}_x(\vert+-\rangle\langle++\vert)$	$\varrho_x^{(-,-)}$
measure this time	already reconstructed	target	target	already reconstructed

- With another beam setting $(e^-, e^+) = (+, \mathbf{n})$

$$\begin{pmatrix} \varrho_x^{(+,\mathbf{m})} - |\alpha|^2\varrho_x^{(+,+)} - |\beta|^2\varrho_x^{(+,-)} \\ \varrho_x^{(+,\mathbf{n})} - |\gamma|^2\varrho_x^{(+,+)} - |\delta|^2\varrho_x^{(+,-)} \end{pmatrix} = \begin{pmatrix} \alpha\beta^* & \alpha^*\beta \\ \gamma\delta^* & \gamma^*\delta \end{pmatrix} \begin{pmatrix} \mathcal{I}_x(|++\rangle\langle+-|) \\ \mathcal{I}_x(|+-\rangle\langle++|) \end{pmatrix}$$

12 polarisation settings

$$(+,\mathbf{m}),(+,\mathbf{n}),(-,\mathbf{m}),(-,\mathbf{n})$$
$$(\mathbf{m},+),(\mathbf{n},+),(\mathbf{m},-),(\mathbf{n},-)$$
$$(\mathbf{m},-\mathbf{m}),(\mathbf{m},\mathbf{n}),(\mathbf{n},-\mathbf{m}),(\mathbf{n},-\mathbf{n})$$

All 12 off-diagonal elements can be reconstructed

$$\widetilde{\mathcal{I}}_x = \frac{1}{4}\begin{pmatrix} \mathcal{I}_x(|++\rangle\langle++|) & \mathcal{I}_x(|++\rangle\langle+-|) & \mathcal{I}_x(|++\rangle\langle-+|) & \mathcal{I}_x(|++\rangle\langle--|) \\ \mathcal{I}_x(|+-\rangle\langle++|) & \mathcal{I}_x(|+-\rangle\langle+-|) & \mathcal{I}_x(|+-\rangle\langle-+|) & \mathcal{I}_x(|+-\rangle\langle--|) \\ \mathcal{I}_x(|-+\rangle\langle++|) & \mathcal{I}_x(|-+\rangle\langle+-|) & \mathcal{I}_x(|-+\rangle\langle-+|) & \mathcal{I}_x(|-+\rangle\langle--|) \\ \mathcal{I}_x(|--\rangle\langle++|) & \mathcal{I}_x(|--\rangle\langle+-|) & \mathcal{I}_x(|--\rangle\langle-+|) & \mathcal{I}_x(|--\rangle\langle--|) \end{pmatrix}$$

12 polarisation settings

$(+, \mathbf{m}), (+, \mathbf{n}), (-, \mathbf{m}), (-, \mathbf{n})$

$(\mathbf{m}, +), (\mathbf{n}, +), (\mathbf{m}, -), (\mathbf{n}, -)$

$(\mathbf{m}, -\mathbf{m}), (\mathbf{m}, \mathbf{n}), (\mathbf{n}, -\mathbf{m}), (\mathbf{n}, -\mathbf{n})$

All 12 off-diagonal elements can be reconstructed

$$\widetilde{\mathcal{I}}_x = \frac{1}{4}\begin{pmatrix} \mathcal{I}_x(|++\rangle\langle ++|) & \mathcal{I}_x(|++\rangle\langle +-|) & \mathcal{I}_x(|++\rangle\langle -+|) & \mathcal{I}_x(|++\rangle\langle --|) \\ \mathcal{I}_x(|+-\rangle\langle ++|) & \mathcal{I}_x(|+-\rangle\langle +-|) & \mathcal{I}_x(|+-\rangle\langle -+|) & \mathcal{I}_x(|+-\rangle\langle --|) \\ \mathcal{I}_x(|-+\rangle\langle ++|) & \mathcal{I}_x(|-+\rangle\langle +-|) & \mathcal{I}_x(|-+\rangle\langle -+|) & \mathcal{I}_x(|-+\rangle\langle --|) \\ \mathcal{I}_x(|--\rangle\langle ++|) & \mathcal{I}_x(|--\rangle\langle +-|) & \mathcal{I}_x(|--\rangle\langle -+|) & \mathcal{I}_x(|--\rangle\langle --|) \end{pmatrix}$$

$$\rho_0 = \sum_{i,j} \rho_{ij} |i\rangle\langle j| \longrightarrow \varrho_x = \sum_{i,j} \rho_{ij} \widetilde{\mathcal{I}}_x(|i\rangle\langle j|)$$

12 polarisation settings

$(+, \mathbf{m}), (+, \mathbf{n}), (-, \mathbf{m}), (-, \mathbf{n})$
$(\mathbf{m}, +), (\mathbf{n}, +), (\mathbf{m}, -), (\mathbf{n}, -)$
$(\mathbf{m}, -\mathbf{m}), (\mathbf{m}, \mathbf{n}), (\mathbf{n}, -\mathbf{m}), (\mathbf{n}, -\mathbf{n})$

All 12 off-diagonal elements can be reconstructed

$$\widetilde{\mathcal{I}}_x = \frac{1}{4}\begin{pmatrix} \mathcal{I}_x(|++\rangle\langle++|) & \mathcal{I}_x(|++\rangle\langle+-|) & \mathcal{I}_x(|++\rangle\langle-+|) & \mathcal{I}_x(|++\rangle\langle--|) \\ \mathcal{I}_x(|+-\rangle\langle++|) & \mathcal{I}_x(|+-\rangle\langle+-|) & \mathcal{I}_x(|+-\rangle\langle-+|) & \mathcal{I}_x(|+-\rangle\langle--|) \\ \mathcal{I}_x(|-+\rangle\langle++|) & \mathcal{I}_x(|-+\rangle\langle+-|) & \mathcal{I}_x(|-+\rangle\langle-+|) & \mathcal{I}_x(|-+\rangle\langle--|) \\ \mathcal{I}_x(|--\rangle\langle++|) & \mathcal{I}_x(|--\rangle\langle+-|) & \mathcal{I}_x(|--\rangle\langle-+|) & \mathcal{I}_x(|--\rangle\langle--|) \end{pmatrix}$$

$$\rho_0 = \sum_{i,j} \rho_{ij} |i\rangle\langle j| \longrightarrow \varrho_x = \sum_{i,j} \rho_{ij} \widetilde{\mathcal{I}}_x(|i\rangle\langle j|)$$

- Is this prediction agrees with the measurement?

$\Longrightarrow$ Test of **Linearity**

12 polarisation settings

$$(+,\mathbf{m}),(+,\mathbf{n}),(-,\mathbf{m}),(-,\mathbf{n})$$
$$(\mathbf{m},+),(\mathbf{n},+),(\mathbf{m},-),(\mathbf{n},-)$$
$$(\mathbf{m},-\mathbf{m}),(\mathbf{m},\mathbf{n}),(\mathbf{n},-\mathbf{m}),(\mathbf{n},-\mathbf{n})$$

All 12 off-diagonal elements can be reconstructed

$$\widetilde{\mathcal{I}}_x = \frac{1}{4}\begin{pmatrix} \mathcal{I}_x(|++\rangle\langle++|) & \mathcal{I}_x(|++\rangle\langle+-|) & \mathcal{I}_x(|++\rangle\langle-+|) & \mathcal{I}_x(|++\rangle\langle--|) \\ \mathcal{I}_x(|+-\rangle\langle++|) & \mathcal{I}_x(|+-\rangle\langle+-|) & \mathcal{I}_x(|+-\rangle\langle-+|) & \mathcal{I}_x(|+-\rangle\langle--|) \\ \mathcal{I}_x(|-+\rangle\langle++|) & \mathcal{I}_x(|-+\rangle\langle+-|) & \mathcal{I}_x(|-+\rangle\langle-+|) & \mathcal{I}_x(|-+\rangle\langle--|) \\ \mathcal{I}_x(|--\rangle\langle++|) & \mathcal{I}_x(|--\rangle\langle+-|) & \mathcal{I}_x(|--\rangle\langle-+|) & \mathcal{I}_x(|--\rangle\langle--|) \end{pmatrix}$$

$$\rho_0 = \sum_{i,j} \rho_{ij} |i\rangle\langle j| \longrightarrow \varrho_x = \sum_{i,j} \rho_{ij} \widetilde{\mathcal{I}}_x(|i\rangle\langle j|)$$

- Is this prediction agrees with the measurement?

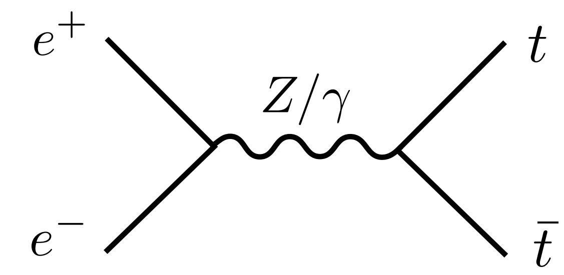
$\Longrightarrow$ Test of **Linearity**

- Are all eigenvalues non-negative?

$\Longrightarrow$ Test of **Complete-Positivity**

Theoretical Prediction

$$\mathcal{L} \ni \sum_i \frac{1}{\Lambda_i^2}[\bar{\psi}_e\gamma_\mu(c_L^i P_L + c_R^i P_R)\psi_e][\bar{\psi}_t\gamma^\mu(d_L^i P_L + d_R^i P_R)\psi_t]$$



i	Λ_i^2	c_L^i	c_R^i	d_L^i	d_R^i
A	s	$-e$	$-e$	$\frac{2}{3}e$	$\frac{2}{3}e$
Z	$s - m_Z^2$	$g_Z\left(-\frac{1}{2} + \sin^2\theta_w\right)$	$g_Z \sin^2\theta_w$	$g_Z\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_w\right)$	$g_Z\left(-\frac{2}{3}\sin^2\theta_w\right)$

e^-e^+ (upper indices), $t\bar{t}$ (lower indices)

$$\begin{aligned}
\mathcal{M}_{00}^{++} = \mathcal{M}_{11}^{++} &= e^{i\phi}\sum_i \frac{s}{2\Lambda_i^2}\gamma^{-1}c_R^i \sin\theta(d_L^i + d_R^i)\,,\\
\mathcal{M}_{01}^{++} &= -e^{i\phi}\sum_i \frac{s}{2\Lambda_i^2}c_R^i(1+\cos\theta)[d_L^i(1-\beta) + d_R^i(1+\beta)]\,,\\
\mathcal{M}_{10}^{++} &= -e^{i\phi}\sum_i \frac{s}{2\Lambda_i^2}c_R^i(1-\cos\theta)[d_L^i(1+\beta) + d_R^i(1-\beta)]\,,\\
\mathcal{M}_{00}^{--} = \mathcal{M}_{11}^{--} &= e^{-i\phi}\sum_i \frac{s}{2\Lambda_i^2}\gamma^{-1}c_L^i \sin\theta(d_L^i + d_R^i)\,,\\
\mathcal{M}_{01}^{--} &= -e^{-i\phi}\sum_i \frac{s}{2\Lambda_i^2}c_L^i(1-\cos\theta)[d_L^i(1-\beta) + d_R^i(1+\beta)]\,,\\
\mathcal{M}_{10}^{--} &= -e^{-i\phi}\sum_i \frac{s}{2\Lambda_i^2}c_L^i(1+\cos\theta)[d_L^i(1+\beta) + d_R^i(1-\beta)]
\end{aligned}$$

$$\mathcal{M}_{A,B}^{+-} = \mathcal{M}_{A,B}^{-+} = 0$$

$$d\widetilde{\mathcal{I}}(|I,J\rangle\langle K,L|)_{(A,B),(C,D)} = \frac{1}{\sigma_{\mathcal{N}}}\frac{1}{2s}\int_x d\Pi\,\mathcal{M}_{A,B}^{I,J}\left(\mathcal{M}_{C,D}^{K,L}\right)^*$$

$$\sigma_{\mathcal{N}} = \frac{q^3}{16\pi^2 s\sqrt{s}}\int d\cos\theta\, d\phi \sum_{I,J,A,B}\left|\mathcal{M}_{A,B}^{I,J}\right|^2$$

$$\widetilde{\mathcal{I}}_x = \begin{pmatrix} I_x^{(++,++)} & 0 & 0 & I_x^{(++,--)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ I_x^{(--,++)} & 0 & 0 & I_x^{(--,--)} \end{pmatrix} \qquad I_x^{(--,++)} = [I_x^{(++,--)}]^\dagger$$

$$\frac{dI^{(++,++)}}{d\cos\theta d\phi} = \begin{pmatrix} a_{11}^{(+)} s_\theta^2 & a_{12}^{(+)} s_\theta(1+c_\theta) & a_{13}^{(+)} s_\theta(1-c_\theta) & a_{14}^{(+)} s_\theta^2 \\ a_{21}^{(+)} s_\theta(1+c_\theta) & a_{22}^{(+)}(1+c_\theta)^2 & a_{23}^{(+)}(1-c_\theta^2) & a_{24}^{(+)} s_\theta(1+c_\theta) \\ a_{31}^{(+)} s_\theta(1-c_\theta) & a_{32}^{(+)}(1-c_\theta^2) & a_{33}^{(+)}(1-c_\theta)^2 & a_{34}^{(+)} s_\theta(1-c_\theta) \\ a_{41}^{(+)} s_\theta^2 & a_{42}^{(+)} s_\theta(1+c_\theta) & a_{43}^{(+)} s_\theta(1-c_\theta) & a_{44}^{(+)} s_\theta^2 \end{pmatrix},$$

$$\frac{dI^{(++,--)}}{d\cos\theta d\phi} = e^{i2\phi} \begin{pmatrix} a_{11}^{(+-)} s_\theta^2 & a_{12}^{(+-)} s_\theta(1-c_\theta) & a_{13}^{(+-)} s_\theta(1+c_\theta) & a_{14}^{(+-)} s_\theta^2 \\ a_{21}^{(+-)} s_\theta(1+c_\theta) & a_{22}^{(+-)}(1-c_\theta^2) & a_{23}^{(+-)}(1+c_\theta)^2 & a_{24}^{(+-)} s_\theta(1+c_\theta) \\ a_{31}^{(+-)} s_\theta(1-c_\theta) & a_{32}^{(+-)}(1-c_\theta)^2 & a_{33}^{(+-)}(1-c_\theta^2) & a_{34}^{(+-)} s_\theta(1-c_\theta) \\ a_{41}^{(+-)} s_\theta^2 & a_{42}^{(+-)} s_\theta(1-c_\theta) & a_{43}^{(+-)} s_\theta(1+c_\theta) & a_{44}^{(+-)} s_\theta^2 \end{pmatrix}$$

$$\frac{dI^{(--,--)}}{d\cos\theta d\phi} = \begin{pmatrix} a_{11}^{(-)} s_\theta^2 & a_{12}^{(-)} s_\theta(1-c_\theta) & a_{13}^{(-)} s_\theta(1+c_\theta) & a_{14}^{(-)} s_\theta^2 \\ a_{21}^{(-)} s_\theta(1-c_\theta) & a_{22}^{(-)}(1-c_\theta)^2 & a_{23}^{(-)}(1-c_\theta^2) & a_{24}^{(-)} s_\theta(1-c_\theta) \\ a_{31}^{(-)} s_\theta(1+c_\theta) & a_{32}^{(-)}(1-c_\theta^2) & a_{33}^{(-)}(1+c_\theta)^2 & a_{34}^{(-)} s_\theta(1+c_\theta) \\ a_{41}^{(-)} s_\theta^2 & a_{42}^{(-)} s_\theta(1-c_\theta) & a_{43}^{(-)} s_\theta(1+c_\theta) & a_{44}^{(-)} s_\theta^2 \end{pmatrix}.$$

$$a^{(+)}\big|_{\sqrt{s}=355\,\mathrm{GeV}} = \begin{pmatrix} 0.188 & -0.705 & 0.338 & 0.188 \\ -0.705 & 2.643 & -1.268 & -0.705 \\ 0.338 & -1.268 & 0.608 & 0.338 \\ 0.188 & -0.705 & 0.338 & 0.188 \end{pmatrix} \cdot 10^{-2}, \qquad a^{(+)}\big|_{\sqrt{s}=1\,\mathrm{TeV}} = \begin{pmatrix} 3.679 & -1.557 & -0.988 & 3.679 \\ -1.557 & 0.659 & 0.418 & -1.557 \\ -0.988 & 0.418 & 0.265 & -0.988 \\ 3.679 & -1.557 & -0.988 & 3.679 \end{pmatrix} \cdot 10^{-2}$$

$$a^{(+-)}\big|_{\sqrt{s}=355\,\mathrm{GeV}} = \begin{pmatrix} 0.300 & 0.356 & -0.940 & 0.300 \\ -1.124 & -1.334 & 3.525 & -1.124 \\ 0.539 & 0.640 & -1.692 & 0.539 \\ 0.300 & 0.356 & -0.940 & 0.300 \end{pmatrix} \cdot 10^{-2}, \qquad a^{(+-)}\big|_{\sqrt{s}=1\,\mathrm{TeV}} = \begin{pmatrix} 5.706 & -1.621 & -2.328 & 5.706 \\ -2.415 & 0.686 & 0.985 & -2.415 \\ -1.533 & 0.435 & 0.625 & -1.533 \\ 5.706 & -1.621 & -2.328 & 5.706 \end{pmatrix} \cdot 10^{-2}$$

$$a^{(-)}\big|_{\sqrt{s}=355\,\mathrm{GeV}} = \begin{pmatrix} 0.478 & 0.567 & -1.499 & 0.478 \\ 0.567 & 0.673 & -1.779 & 0.567 \\ -1.499 & -1.779 & 4.700 & -1.499 \\ 0.478 & 0.567 & -1.499 & 0.478 \end{pmatrix} \cdot 10^{-2}, \qquad a^{(-)}\big|_{\sqrt{s}=1\,\mathrm{TeV}} = \begin{pmatrix} 8.849 & -2.514 & -3.610 & 8.849 \\ -2.514 & 0.714 & 1.025 & -2.514 \\ -3.610 & 1.025 & 1.473 & -3.610 \\ 8.849 & -2.514 & -3.610 & 8.849 \end{pmatrix} \cdot 10^{-2}$$

Sensitive to BSM extension!

Summary and Discussion

- High-energy tests of QM are important:
  - Locality may be an emerging property.
  - QM may be modified at high-energy to be reconciled with gravity.

- The spin correlation measurement at polarised colliders can be understood as a quantum instrument. ex. $e^+e^- \to t\bar{t}$

- The QI map is completely determined by the Choi matrix, which can be experimentally reconstructed by the quantum state tomography of the final state with various beam polarisation settings. ← Quantum *Process* Tomography

- The reconstructed Choi matrix can be used to test the linearity and complete-positivity of the map, which offers a novel test of QM.

- The proposed method requires unconventional beam polarisations, i.e. polarisations ***not*** in the direction of the beam.

Thank you for listening!