



BSM (one-loop) Triple Higgs Couplings at Future e^+e^- Colliders

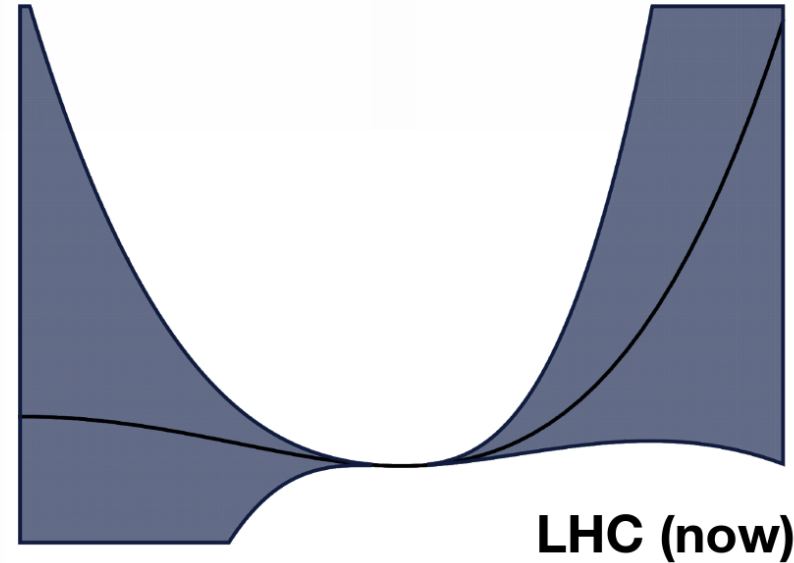
Francisco Arco (*he/him*)

Virtual Overflow Session of 3rd ECFA Workshop
October 16, 2024

Ongoing work with S. Heinemeyer and M. Mühlleitner

Motivation: BSM in the Higgs Sector

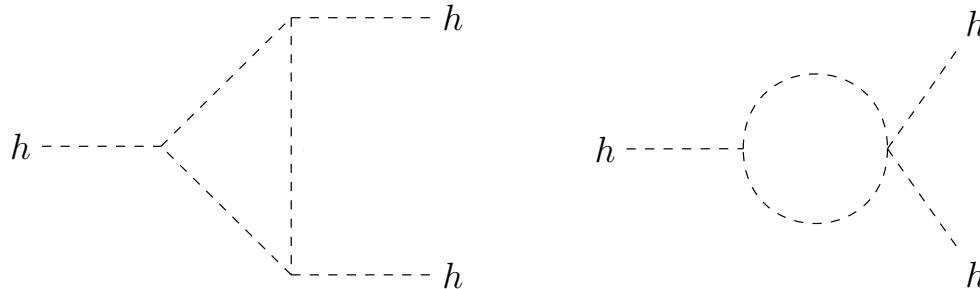
- The Higgs boson potential is essentially *untested*
- Extended Higgs sectors can solve (at least some) of the SM problems
 - Baryon asymmetry
 - Dark matter
 - Hierarchy problem
 - ...
- *Many, many room for BSM physics!*



Sketch of the current uncertainty in the (SM) Higgs potential, by Nathaniel Craig

Large 1L corrections @BSM models!

- Framework: **Two Higgs doublet model (2HDM)**
 - 5 Higgs bosons h, H, A, H^\pm + new scalar interactions
 - **Large scalar couplings** are still allowed! [FA, Heinemeyer, Herrero, 21, 22]
- Large scalar couplings can lead to **large 1L corrections** to $\lambda_{hhh}^{(1)}$ (well above 100% w.r.t. the tree level) [Kanemura, Kiyoura, Okada, Senaha, Yuan, 02]

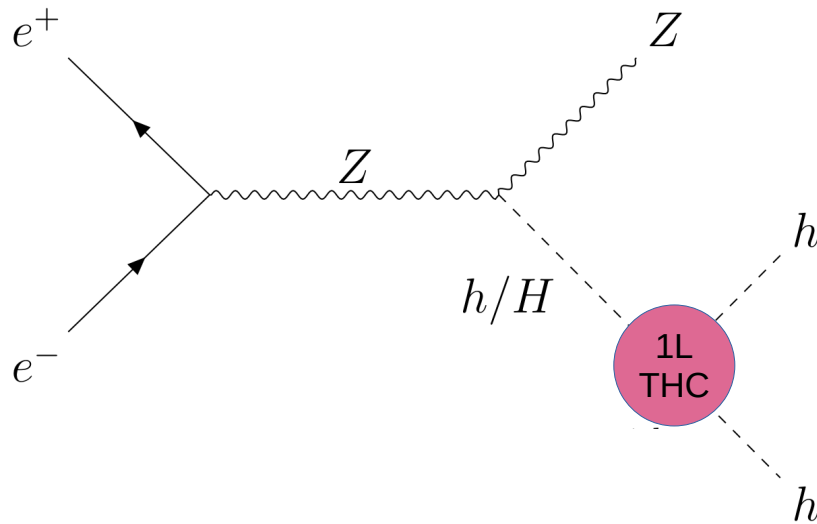


- Other triple Higgs couplings (THCs) could receive similar contributions

Where to look? At e^+e^- colliders!

- Our computation of the xs (@ ILC):

Tree level $e^+e^- \rightarrow hhZ$
 +
 1L corrected $\lambda_{hhh}^{(1)}$ and $\lambda_{hhH}^{(1)}$



- Computation of 1L THC:

1. Effective potential for λ_{hhh} and λ_{hhH}
2. Diagrammatic calculation for λ_{hhh}

- Includes the expected main EW corrections

Two Higgs Doublet Model (2HDM)

- SM + **second Higgs doublet** (CP-conserving)

5 physical Higgs bosons: h, H : (CP-even) A : (CP-odd) and H^\pm

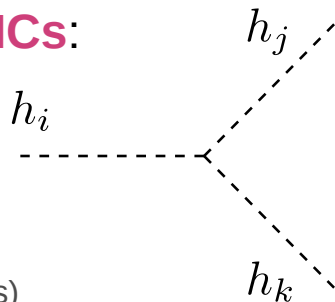
- Z_2 symmetry to avoid FCNC (softly broken by m_{12}^2) \Rightarrow Four 2HDM types

- Input parameters:

$$m_h (= 125 \text{ GeV}), m_H, m_A, m_{H^\pm}, \tan \beta, \cos(\beta - \alpha) \equiv c_{\beta - \alpha}, m_{12}^2 \equiv \bar{m}^2 s_\beta c_\beta$$

- **Alignment limit:** for $c_{\beta - \alpha} = 0$ the SM interactions for h are recovered !!

- Notation for **THCs**:



$$= -i v n! \lambda_{h_i h_j h_k}^{(0)}$$

$$\kappa_\lambda^{(0,1)} \equiv \frac{\lambda_{hhh}^{(0,1)}}{\lambda_{\text{SM}}^{(0)}}$$

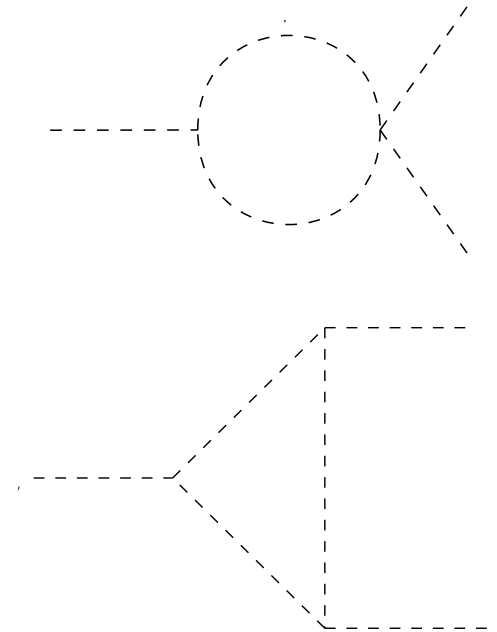
with

$$\lambda_{\text{SM}}^{(0)} = \frac{m_h^2}{2v^2} \simeq 0.13$$

($n = \#$ identical bosons)

Triple Higgs Couplings at 1 Loop

- **Effective potential** for $\lambda_{hhh}^{(1)}$ and $\lambda_{hhH}^{(1)}$
 - ‘On-shell’ renormalization: 1L parameters are set equal to their tree-level values
 - BSMPTv3 [*Basler, Biermann, Mühlleitner, Müller, Santos, Viana, 24*]
- **Full diagrammatic** approach, only for $\lambda_{hhh}^{(1)}$
 - On-shell conditions for masses, angles, and WFRs
MS-bar for m_{12}^2
 - The *finite momentum* effects are included!
 - anyH3/anyBSM [*Bahl, Braathen, Gabelmann, Weiglein, 23*]
- They will capture the pure scalar 1L corrections to $e^+e^- \rightarrow hhZ$
(*expected to be the main EW ones*)



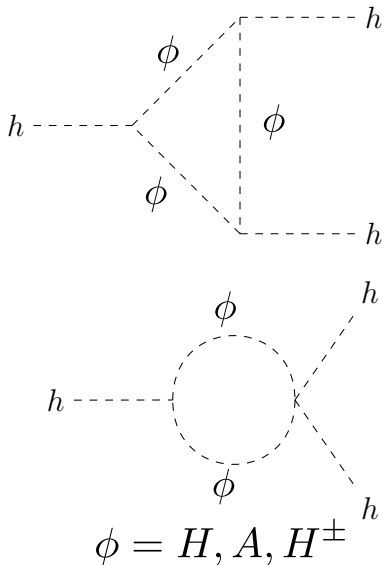
THCs: tree vs 1loop with constraints

Type	$\kappa_\lambda^{(0)}$	$\kappa_\lambda^{(1)}$	$\lambda_{hhH}^{(0)}$	$\lambda_{hhH}^{(1)}$
I	[-0.2, 1.2]	[0.2, 6.8]	[-1.6, 1.5]	[-2.1, 1.9]
II	[0.6, 1.0]	[0.7, 5.7]	[-1.5, 1.6]	[-1.7, 2.0]
LS	[0.5, 1.0]	[0.6, 6.3]	[-1.7, 1.7]	[-2.2, 2.1]
FL	[0.7, 1.0]	[0.8, 5.8]	[-1.6, 1.3]	[-1.9, 1.5]

*(results from
the effective
potential)*

- Scan of the parameter space (550,000 points) *[ScannerS + HiggsTools + HDECAY]*
- Applied **constraints** to the 2HDM
 - EWPO
 - Tree-level unitarity + potential stability
 - BSM Higgs boson searches
 - Properties of the SM-like Higgs boson
 - **Close to the alignment!**
 - Flavor Observables

κ_λ : tree level vs 1 loop



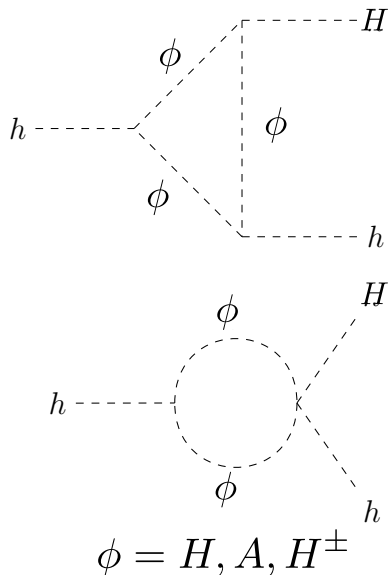
Type	$\kappa_\lambda^{(0)}$	$\kappa_\lambda^{(1)}$	$\lambda_{hhH}^{(0)}$	$\lambda_{hhH}^{(1)}$
I	[-0.2, 1.2]	[0.2, 6.8]	[-1.6, 1.5]	[-2.1, 1.9]
II	[0.6, 1.0]	[0.7, 5.7]	[-1.5, 1.6]	[-1.7, 2.0]
LS	[0.5, 1.0]	[0.6, 6.3]	[-1.7, 1.7]	[-2.2, 2.1]
FL	[0.7, 1.0]	[0.8, 5.8]	[-1.6, 1.3]	[-1.9, 1.5]

(results from the effective potential)

- Very large corrections are possible! $\lambda_{hhh}^{(1)} \gg \lambda_{hhh}^{(0)}$
- Reason: h couplings to heavy Higgs bosons can be large
 - Even at the *alignment limit* !!! $m_H \sim \bar{m} < m_A \sim m_{H^\pm}$

(In the SM, top-loops are ~ -8%)

λ_{hhH} : tree level vs 1 loop



Type	$\kappa_\lambda^{(0)}$	$\kappa_\lambda^{(1)}$	$\lambda_{hhH}^{(0)}$	$\lambda_{hhH}^{(1)}$
I	[-0.2, 1.2]	[0.2, 6.8]	[-1.6, 1.5]	[-2.1, 1.9]
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(results from the effective potential)

- 1L corrections for λ_{hhH} can also yield interesting results!
- Example: $\lambda_{hhH}^{(1)} \gtrsim \lambda_{hhH}^{(0)} \sim 0$ or change of sign in λ_{hhH}

At e^+e^- colliders

Effects from THCs at $e^+e^- \rightarrow hhZ$

A) Non-resonant diagram

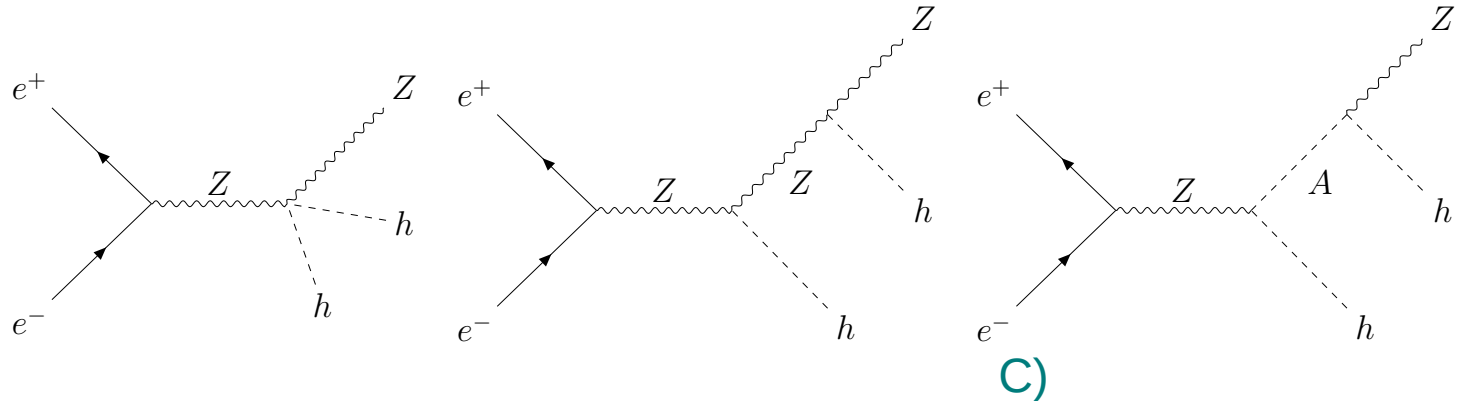
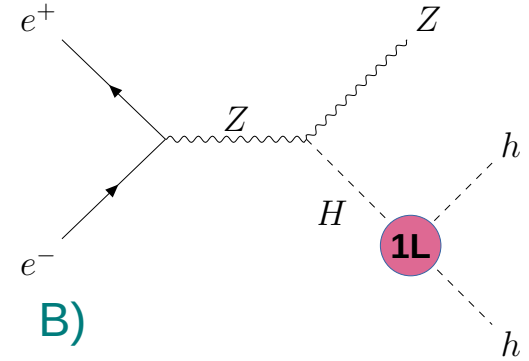
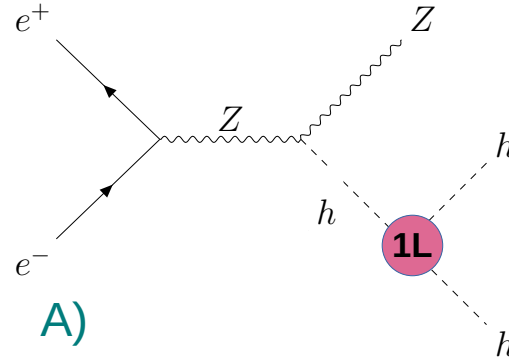
with $\kappa_\lambda \Rightarrow$ at low m_{hh}

B) Resonant H diagram

with $\lambda_{hhH} \Rightarrow$ at $m_{hh} \simeq m_H$

C) Resonant A diagram

(no THC)



In the alignment limit ($c_{\beta-\alpha}=0$)

A) Non-resonant diagram

with $\kappa_\lambda^{(1)} \neq 0$

B) Resonant diagram

with $\kappa_\lambda^{(1)} \approx m_H$

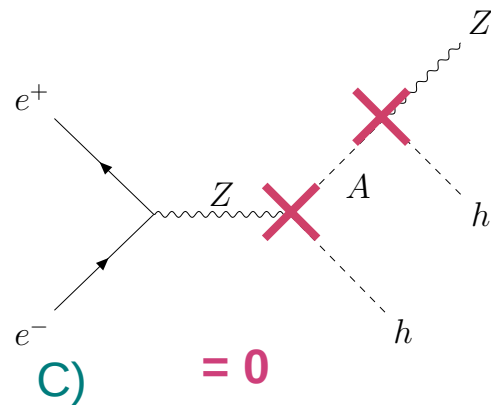
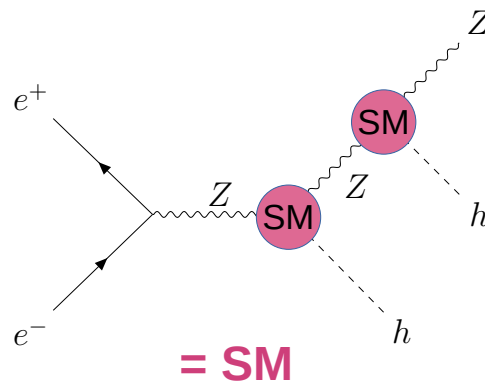
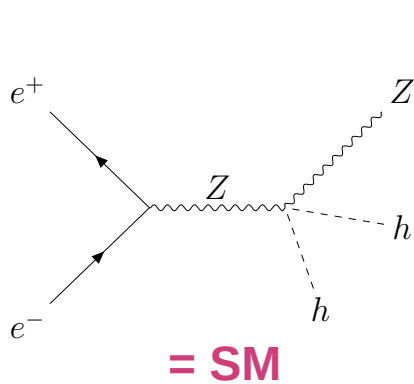
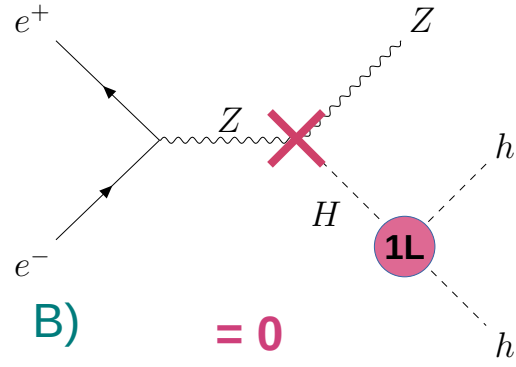
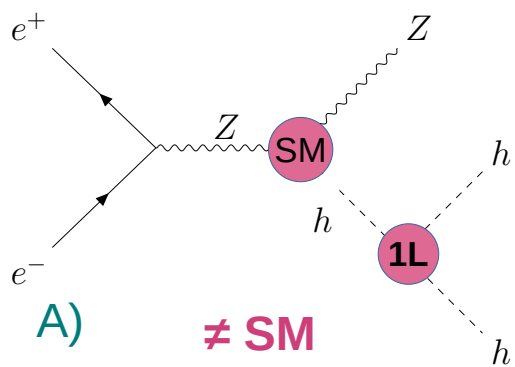
C) Resonant diagram

(no THC)

$$\kappa_\lambda^{(0)} = 1,$$

$$\lambda_{hhH}^{(0)} = 0$$

Only BSM effects in $\kappa_\lambda^{(1)}$



Access to $\kappa_\lambda^{(1)}$ (at ILC)

Large 1L κ_λ @ILC500GeV

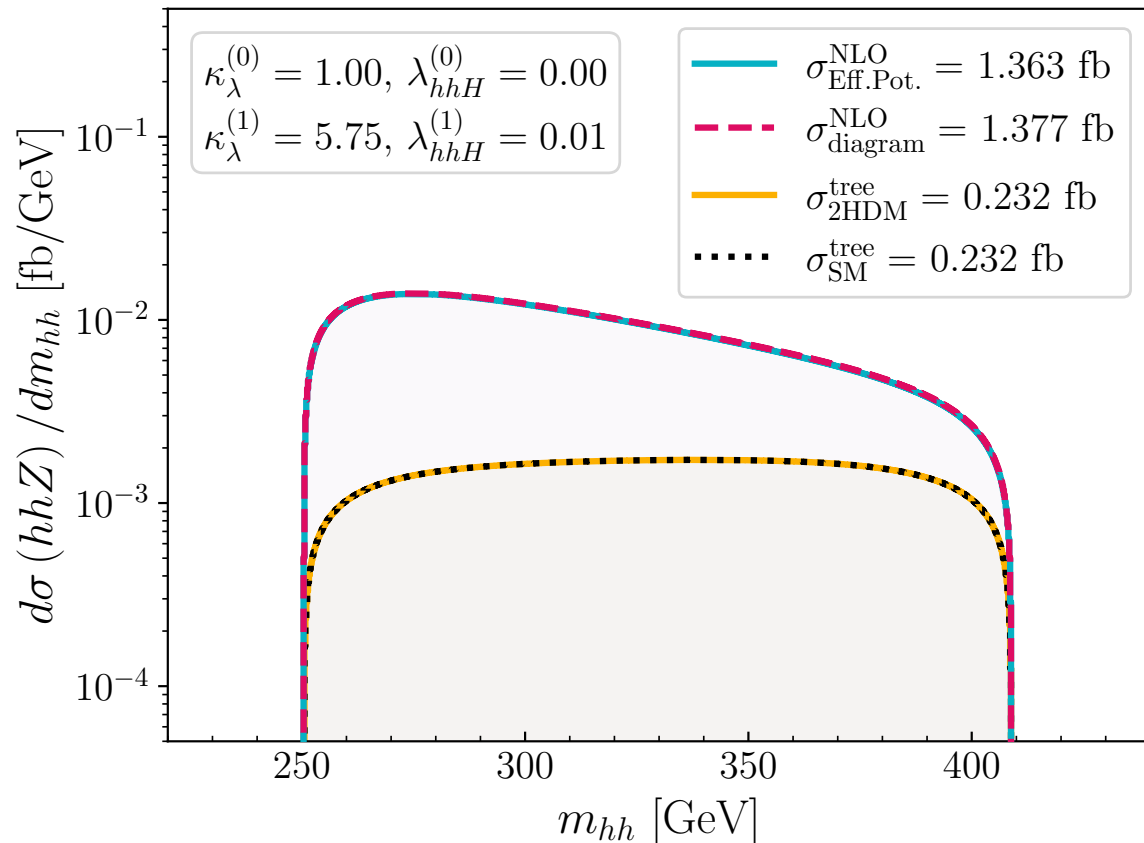
BPal, all types!

$$m_H = \bar{m} = 400 \text{ GeV},$$

$$m_A = m_{H^\pm} = 800 \text{ GeV},$$

$$\tan \beta = 3, \cos(\beta - \alpha) = 0$$

- XS 6 times larger than the tree-level !!!
- Momentum effects on $\kappa_\lambda^{(1)}(m_{hh})$ around 1-2%
- Better access, and sensitivity, to $\kappa_\lambda^{(1)}$ than in the SM!



Large 1L κ_λ @ILC1TeV

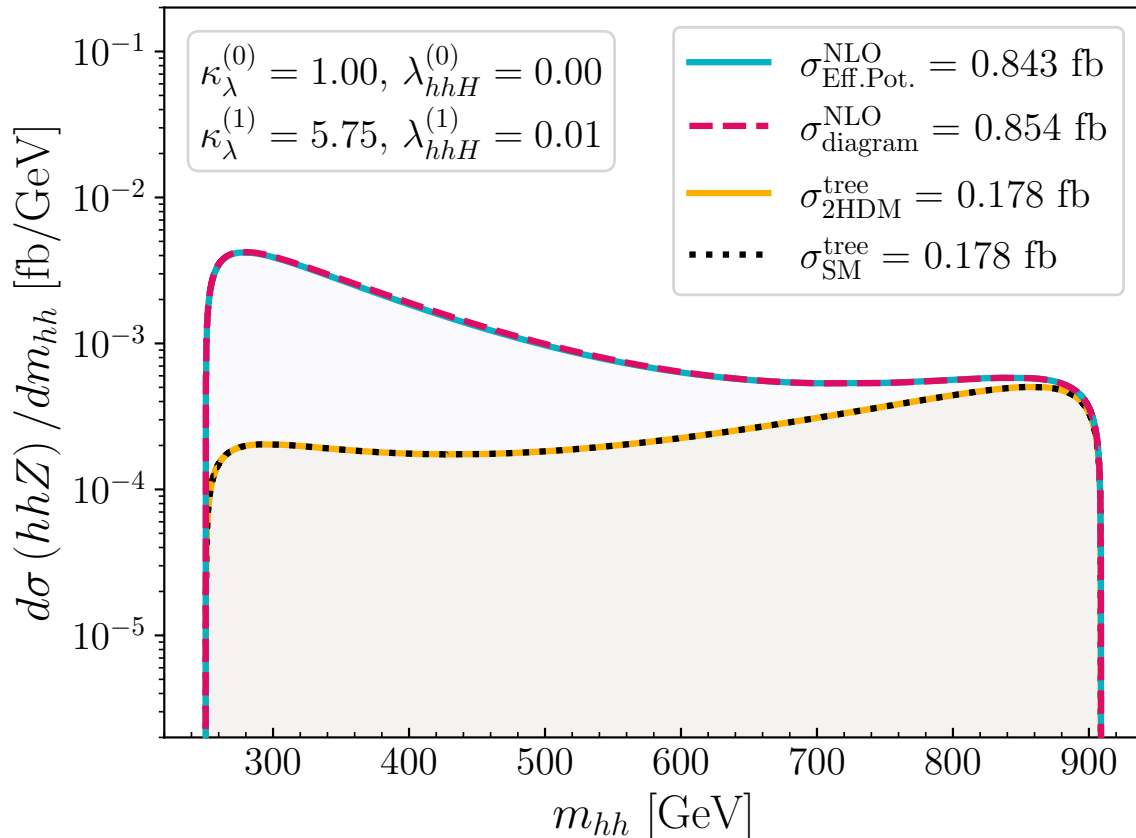
BPal, all types!

$$m_H = \bar{m} = 400 \text{ GeV},$$

$$m_A = m_{H^\pm} = 800 \text{ GeV},$$

$$\tan \beta = 3, \cos(\beta - \alpha) = 0$$

- Similar to the 500 GeV case
- All points with large scalar couplings can potentially lead to large κ_λ at 1L !!!



Access to $\lambda_{hhH}^{(1)}$ (at ILC 500 GeV)

'Sensitivity' to the H resonance

- **Theoretical 'sensitivity'**: significance Z from a likelihood profile ratio testing the H resonance peak vs. the no resonance (i.e. $\lambda_{hhH} = 0$)

- Expected **final 4b-jet events**:

$$\bar{N}_{4bZ} = N_{4bZ} \times \mathcal{A} \times \epsilon_b$$

- 4b-jet tagging efficiency: $\epsilon_b = 85\%$ [Dürig PhD Thesis, 16]

- Acceptance \mathcal{A} after the preselection cuts (around 75%):

$$|\eta_{b,Z}| < 2.5, E_b > 20 \text{ GeV}, y_{b_1 b_2} = \frac{2 \min(E_1^2, E_2^2)(1 - \cos \theta_{12})}{s} > 0.0025$$

- **Smearing** of the m_{hh} distributions due to finite detector resolution

- We consider 2% and 5% Gaussian smearing

- **Size of the bin**: bins with at least 2 events inside the kinematically allowed region

Large 1L λ_{hhH} (no smear)

BP1, type I

$$m_H = \bar{m} = 300 \text{ GeV},$$

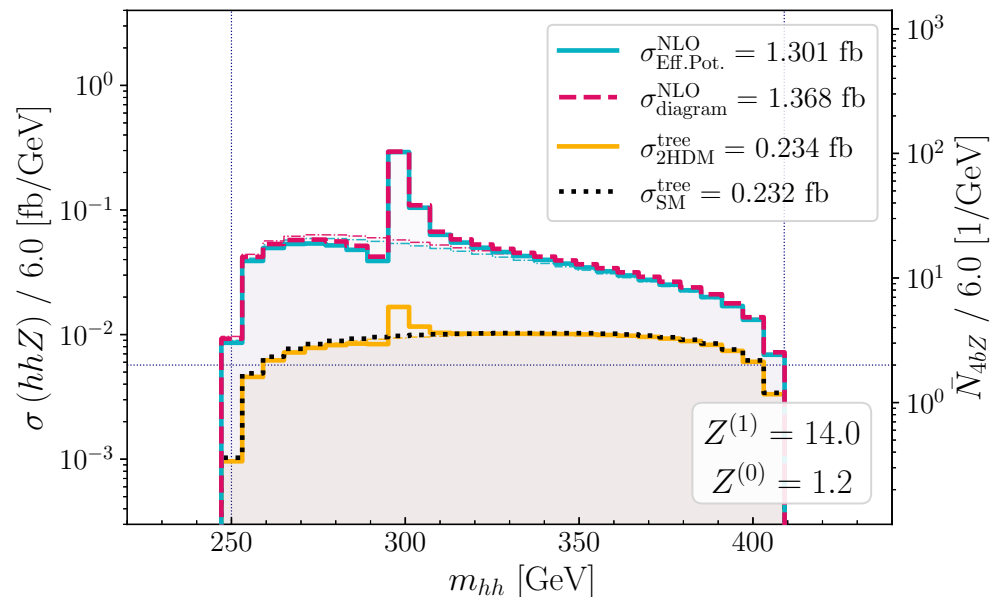
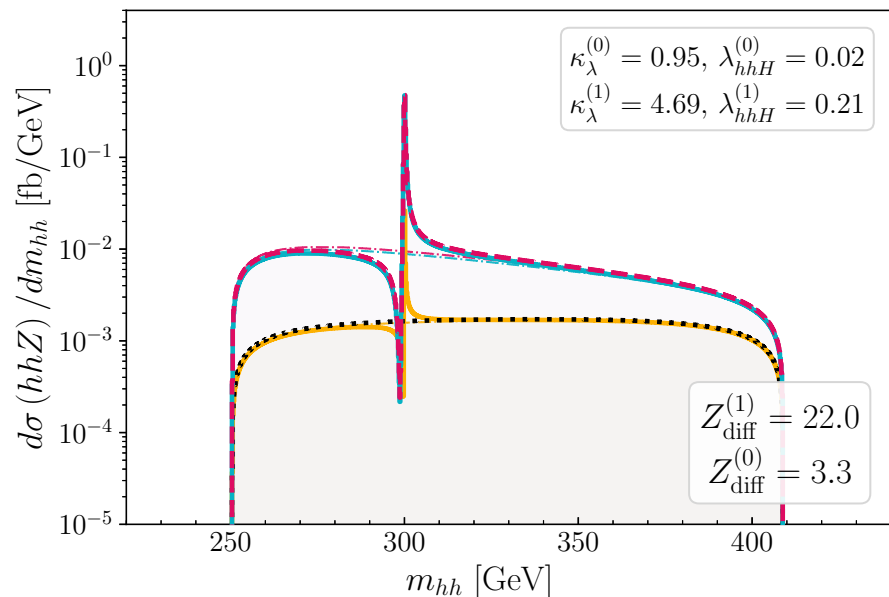
$$m_A = m_{H^\pm} = 650 \text{ GeV},$$

$$\tan \beta = 12, \cos(\beta - \alpha) = 0.12$$

■ Large effect from $\kappa_\lambda^{(1)}$

■ For this point $\lambda_{hhH}^{(0)} \sim 0 \ll \lambda_{hhH}^{(1)}$

⇒ More prominent H resonance (greater Z)



Large 1L λ_{hhH} + 2% smear

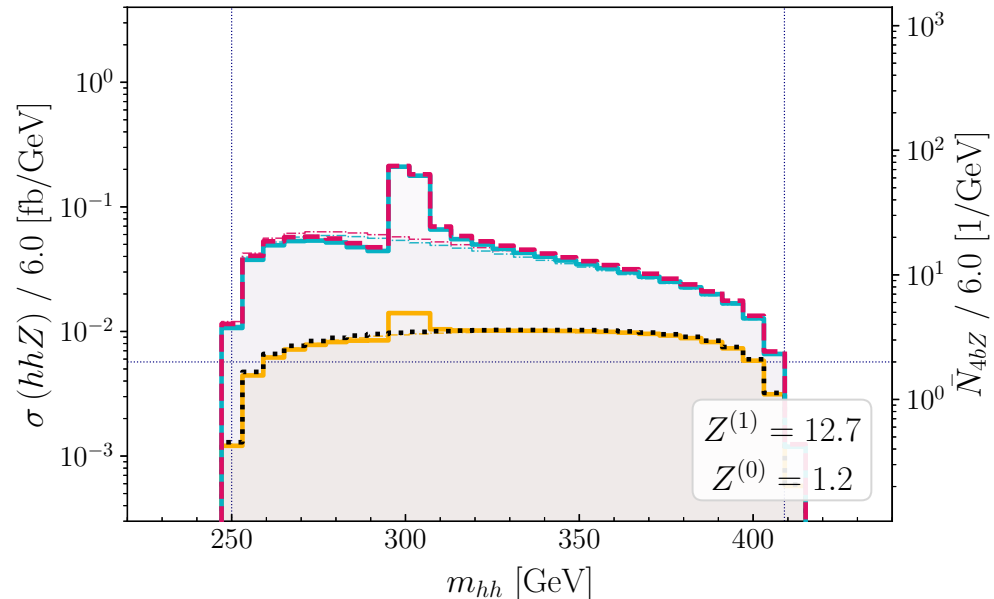
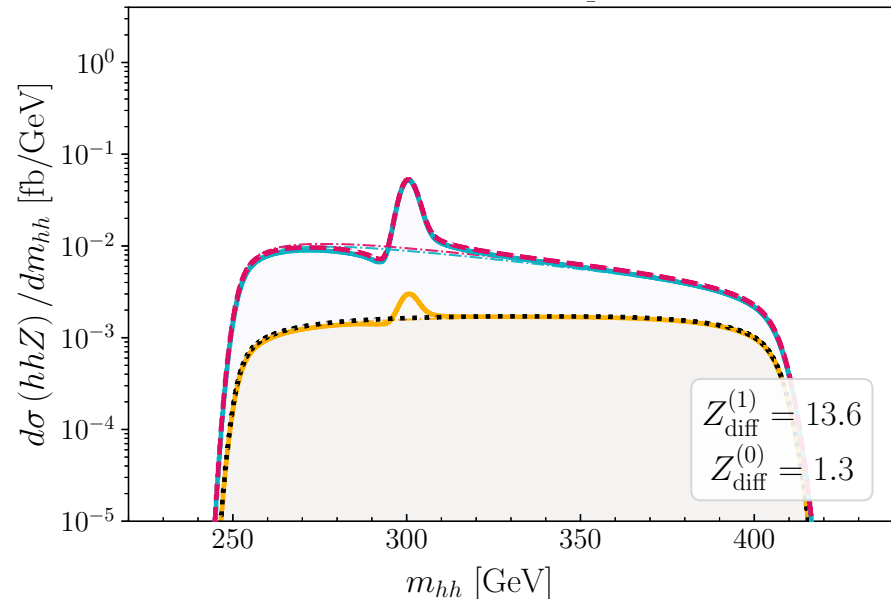
BP1, type I

$$m_H = \bar{m} = 300 \text{ GeV},$$

$$m_A = m_{H^\pm} = 650 \text{ GeV},$$

$$\tan \beta = 12, \cos(\beta - \alpha) = 0.12$$

- The resonance gets ‘smeared’
- Values for Z get worst



Large 1L λ_{hhH} + 5% smear

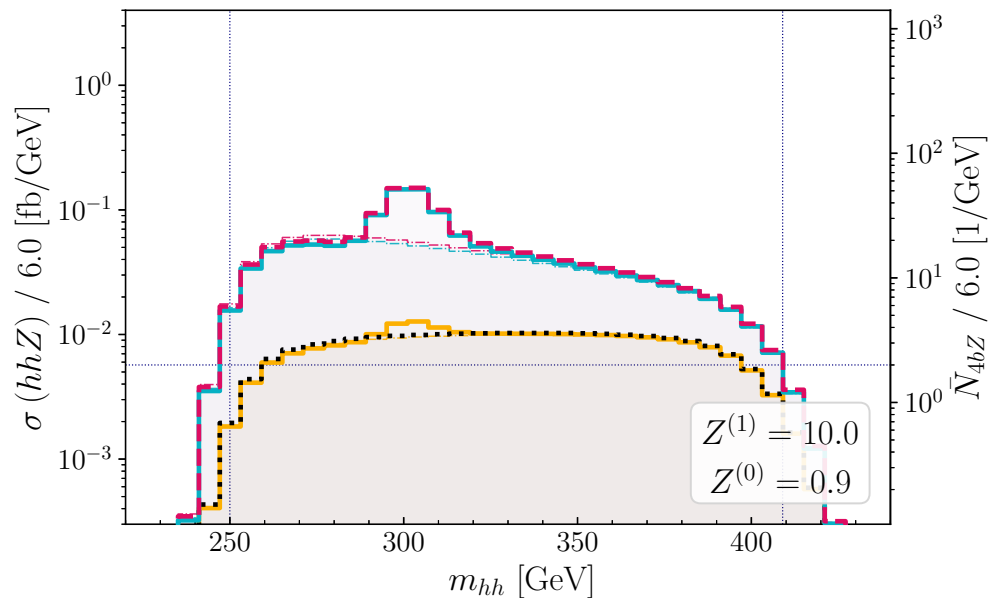
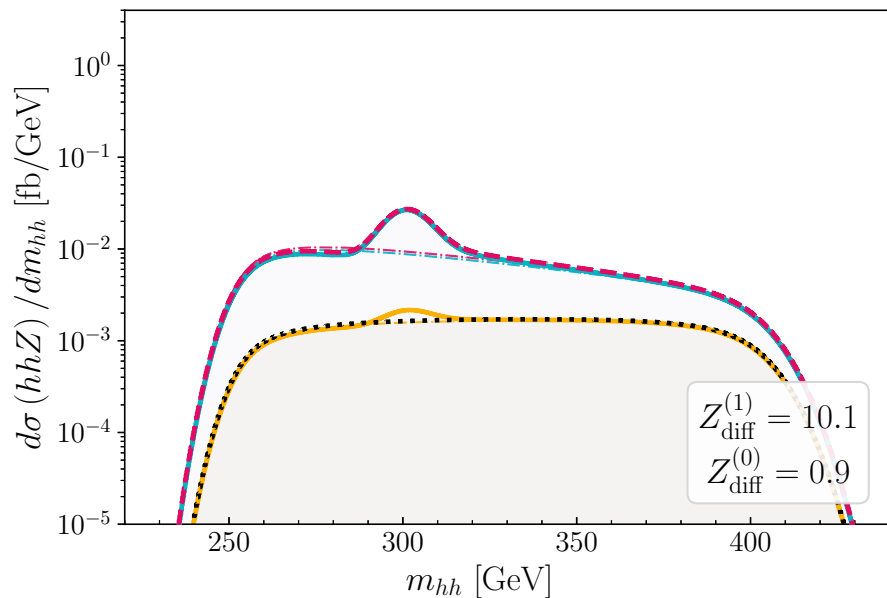
BP1, type I

$$m_H = \bar{m} = 300 \text{ GeV},$$

$$m_A = m_{H^\pm} = 650 \text{ GeV},$$

$$\tan \beta = 12, \cos(\beta - \alpha) = 0.12$$

- The resonance gets even more ‘smeared’
- Values for Z get even worse



1L λ_{hhH} with different sign (no smear)

BPsign, type I

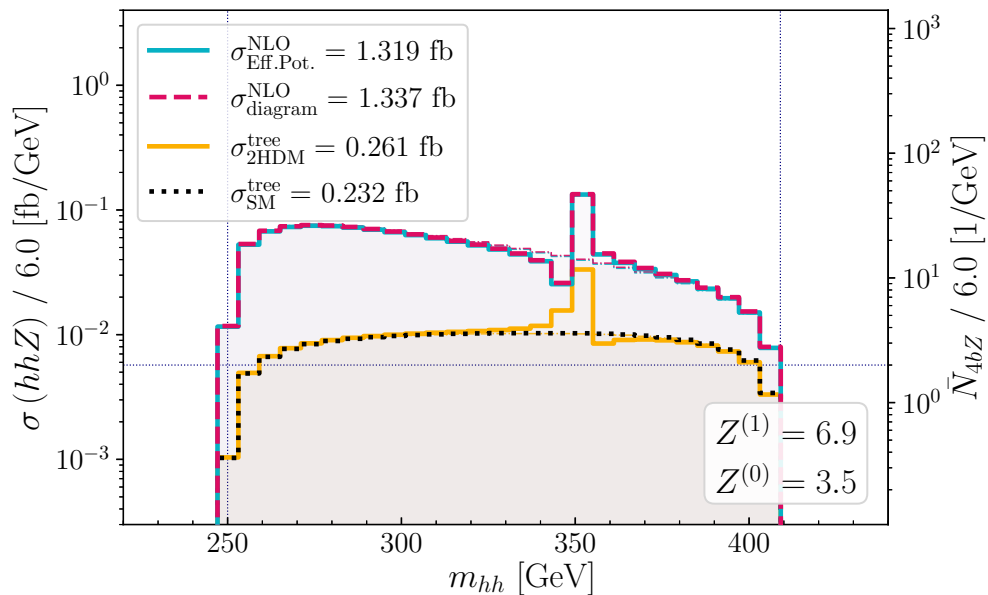
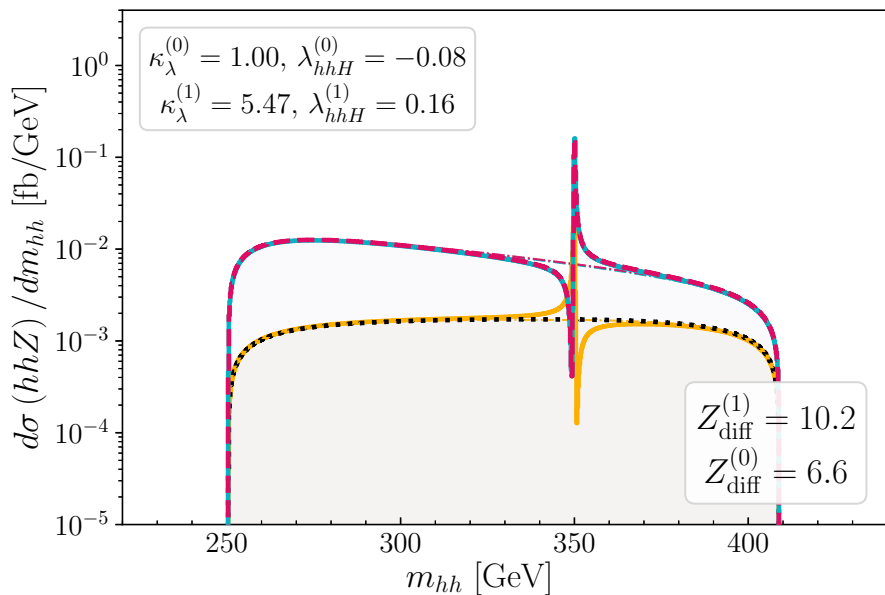
$$m_H = \bar{m} = 350 \text{ GeV},$$

$$m_A = m_{H^\pm} = 650 \text{ GeV},$$

$$\tan \beta = 20, \cos(\beta - \alpha) = 0.1$$

- In this point: $\text{sign} \left(\lambda_{hhH}^{(1)} \right) \neq \text{sign} \left(\lambda_{hhH}^{(0)} \right)$
 \Rightarrow changes the dip-peak structure of the peak!

- Large effect from $\kappa_\lambda^{(1)}$



1L λ_{hhH} with different sign + 2% smear

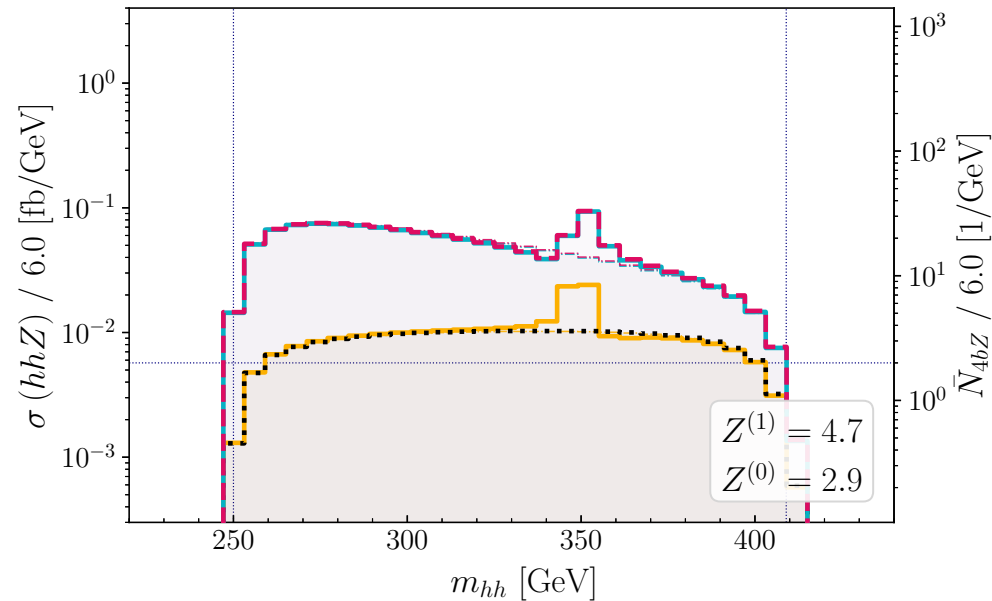
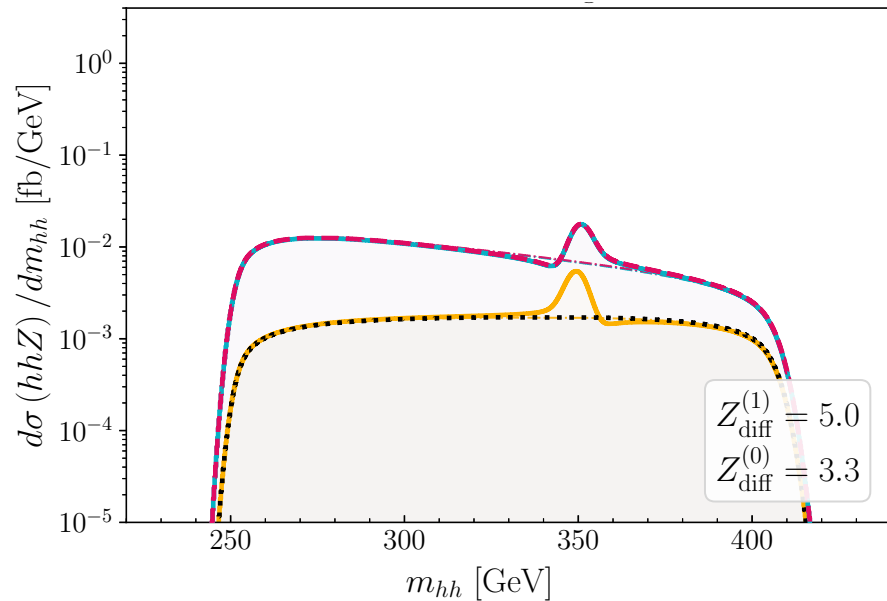
BPsign, type I

$$m_H = \bar{m} = 350 \text{ GeV},$$

$$m_A = m_{H^\pm} = 650 \text{ GeV},$$

$$\tan \beta = 20, \cos(\beta - \alpha) = 0.1$$

- The dip-peak structure gets washed out
- No apparent sensitivity to the coupling sign



1L λ_{hhH} with different sign + 5% smear

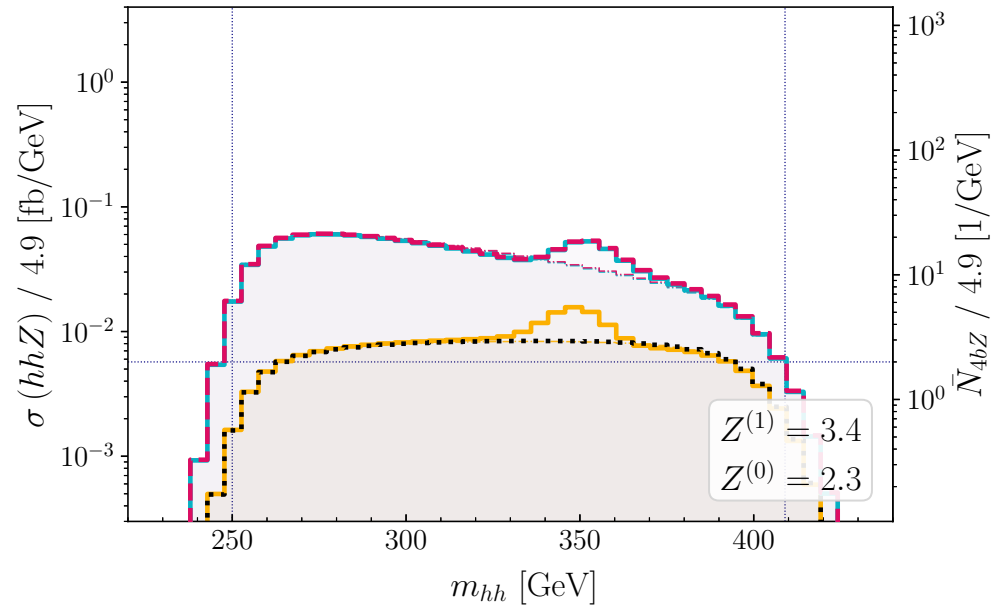
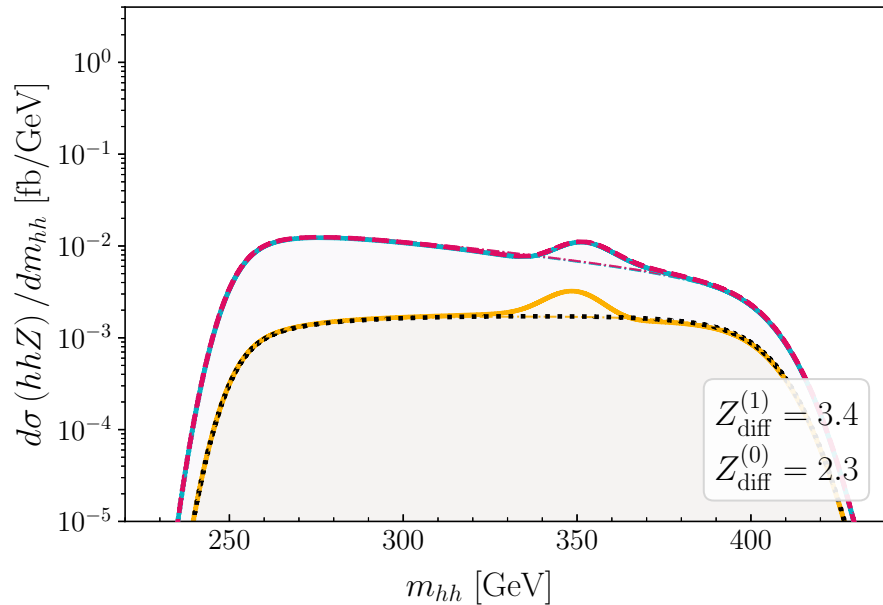
BPsign, type I

$$m_H = \bar{m} = 350 \text{ GeV},$$

$$m_A = m_{H^\pm} = 650 \text{ GeV},$$

$$\tan \beta = 20, \cos(\beta - \alpha) = 0.1$$

- The resonance peak gets very difficult to detect :(



Results for Z :

- Larger values of Z at 1L vs. tree-level
- Smearing decreases the value of Z
 - Still optimistic results: the κ_λ enhancement helps
- Challenging access to resonance H peaks and dip-peak/peak-dip structures
- Still a full experimental analysis is needed!

BP	Smearing	Bin (loop)	$Z_{\text{diff}}^{(1)}$	$Z^{(1)}$	Bin (tree)	$Z_{\text{diff}}^{(0)}$	$Z^{(0)}$
BP1	0%	6.0	22.0	14.0	11.4	3.3	1.2
BP1	2%	6.0	13.6	12.7	11.4	1.3	1.2
BP1	5%	6.0	10.1	10.0	11.4	0.9	0.9
BP1	10%	4.8	7.8	7.8	10.8	0.6	0.6
BP2	0%	8.4	18.0	12.2	11.4	14.4	8.8
BP2	2%	8.4	13.4	12.2	11.4	9.5	8.0
BP2	5%	8.4	10.6	10.4	11.4	7.3	7.1
BP2	10%	8.1	8.5	8.4	10.9	5.7	5.7
BP3	0%	10.1	4.6	4.4	10.8	2.3	2.2
BP3	2%	10.1	4.5	4.3	10.8	2.2	2.2
BP3	5%	10.1	4.2	4.1	10.8	2.0	2.0
BP3	10%	10.1	3.6	3.6	10.8	1.7	1.7
BPsign	0%	6.0	10.2	6.9	11.4	6.6	3.5
BPsign	2%	6.0	5.0	4.7	11.4	3.3	2.9
BPsign	5%	4.9	3.4	3.4	10.8	2.3	2.3
BPsign	10%	3.9	2.5	2.5	10.8	1.8	1.7
BPext	0%	6.0	24.9	17.2	11.4	17.1	11.0
BPext	2%	6.0	15.0	13.8	9.5	11.1	10.3
BPext	5%	4.9	11.9	11.9	9.5	9.3	9.1
BPext	10%	3.9	10.4	10.4	9.3	8.2	8.1

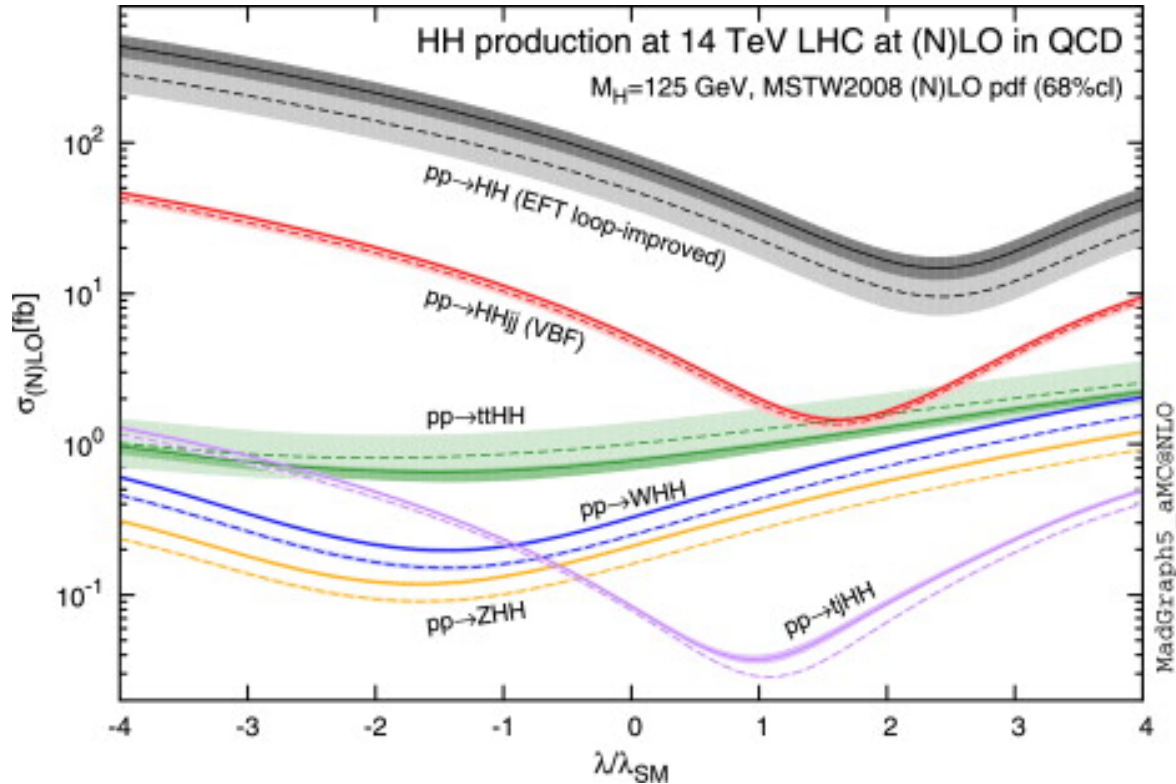
Summary & Conclusions

- Analysis of the **1L corrected triple Higgs couplings** κ_λ and λ_{hhH} , and their impact in **double Higgs production** at e^+e^- colliders in the 2HDM, specifically $e^+e^- \rightarrow hhZ$ at ILC
- **1L corrections to κ_λ can be very large**, *even in the alignment limit !!!*
 - Very distinct prediction even for a very SM-like Higgs boson!
 - No relevant effects from finite momentum
- **1L corrected λ_{hhH} can lead to interesting pheno!** Access via the H resonance peak
 - Analysis of the **final 4b-jet events + smearing + bin size**: access to the resonance peak may be challenging (*but an experimental analysis is needed*)
 - Resolution in the m_{hh} distributions will be crucial

Thanks for your attention! :)

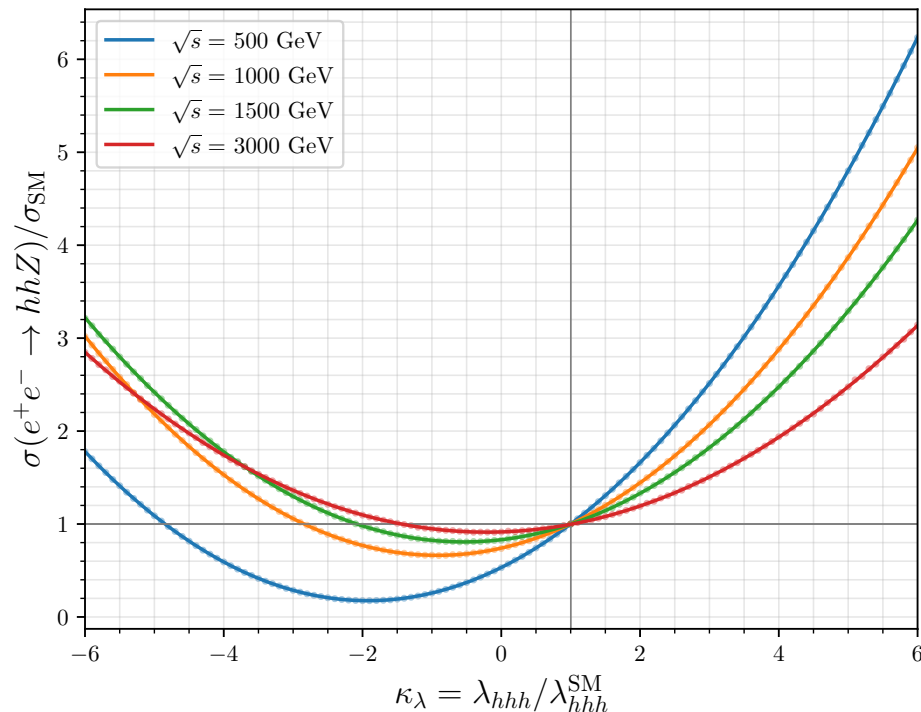
Back up

XS vs κ_λ in the SM at LHC



Di-Higgs production @ e^+e^- colliders

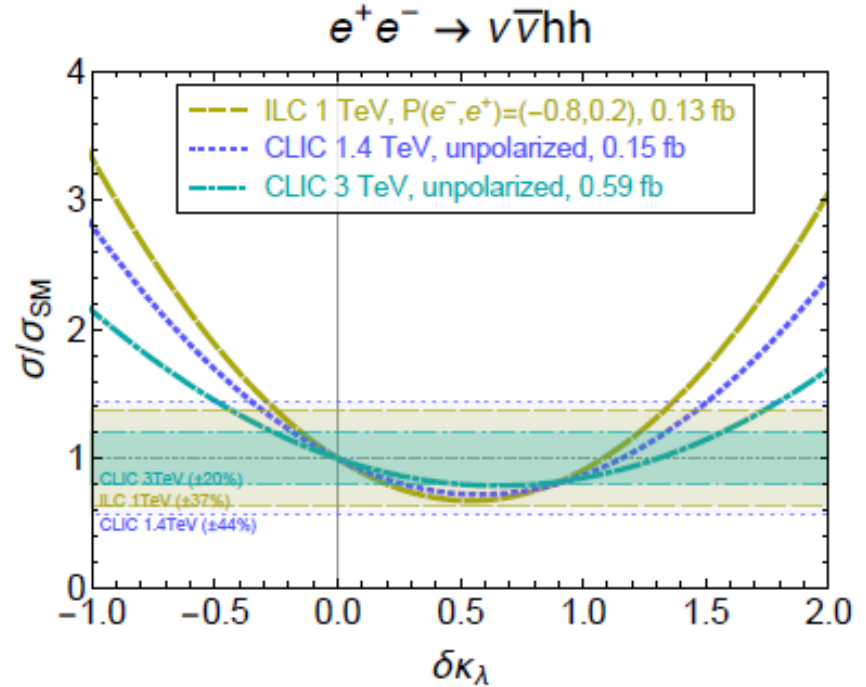
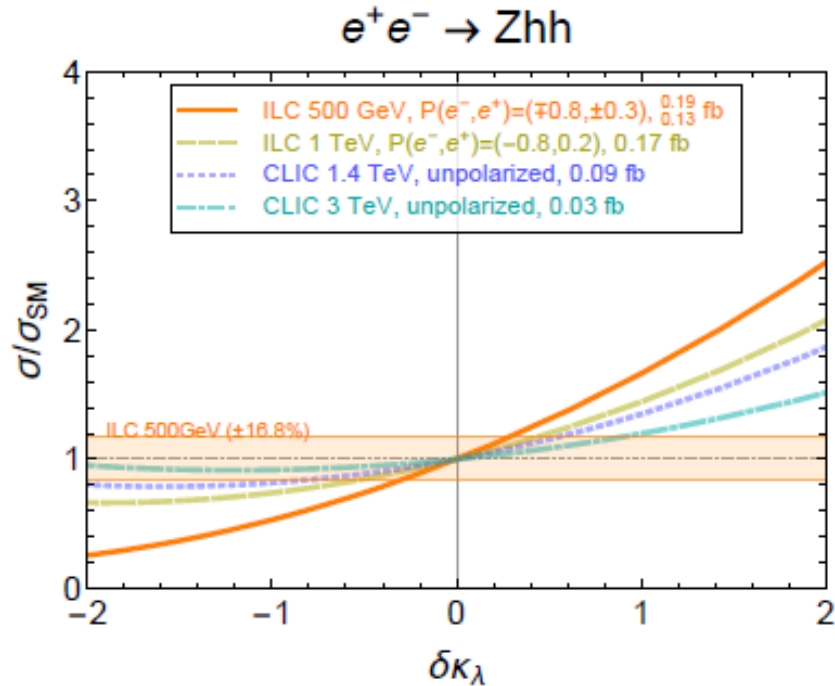
- Main production channel is the double Higgs-strahlung $e^+e^- \rightarrow hhZ$



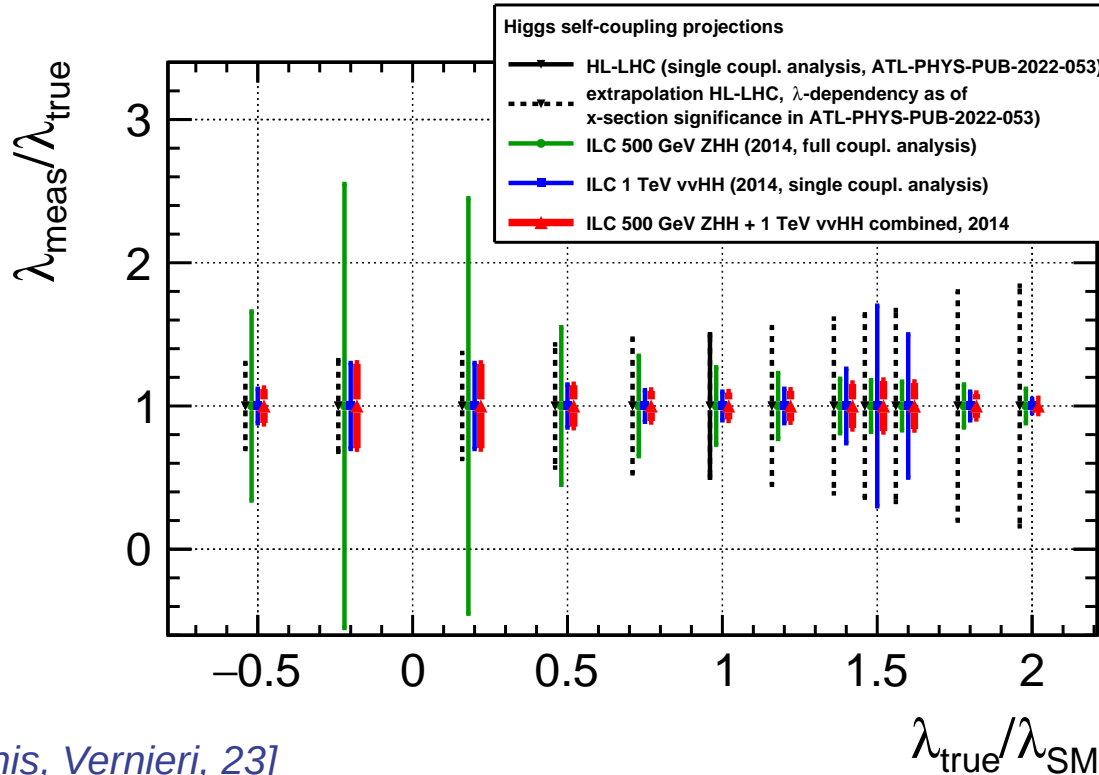
- We expect *larger cross sections* for larger values of κ_λ !

X_S vs κ_λ in the SM at e^+e^- colliders

[Di Vita, Durieux, Grojean, Gu, Liu, Panico, Riembau, Vantalon, 18]



$\kappa_\lambda \neq 1$ at HL-LHC and e^+e^- colliders



[Torndal, List, Ntounis, Vernieri, 23]

Feynman Rules @tree level @alignment

$$\begin{aligned}
 hhh &: \frac{-3im_h^2}{v} = -6iv\lambda_{\text{SM}}^{(0)}, \\
 hhH = hhhH &: 0, \\
 hHH/v = hHHH &: \frac{-i(m_h^2 + 2m_H^2 - 2\bar{m}^2)}{v^2}, \\
 h\phi\phi/v = hh\phi\phi &: \frac{-i(m_h^2 + 2m_\phi^2 - 2\bar{m}^2)}{v^2}, \\
 HHH/(3v) = hHHH = H\phi\phi/v = hH\phi\phi &: \frac{2i(m_H^2 - \bar{m}^2) \cot 2\beta}{v^2}, \\
 HHHH/3 = HH\phi\phi &: \frac{-i(m_h^2 + 4(m_H^2 - \bar{m}^2) \cot^2 2\beta)}{v^2},
 \end{aligned}$$

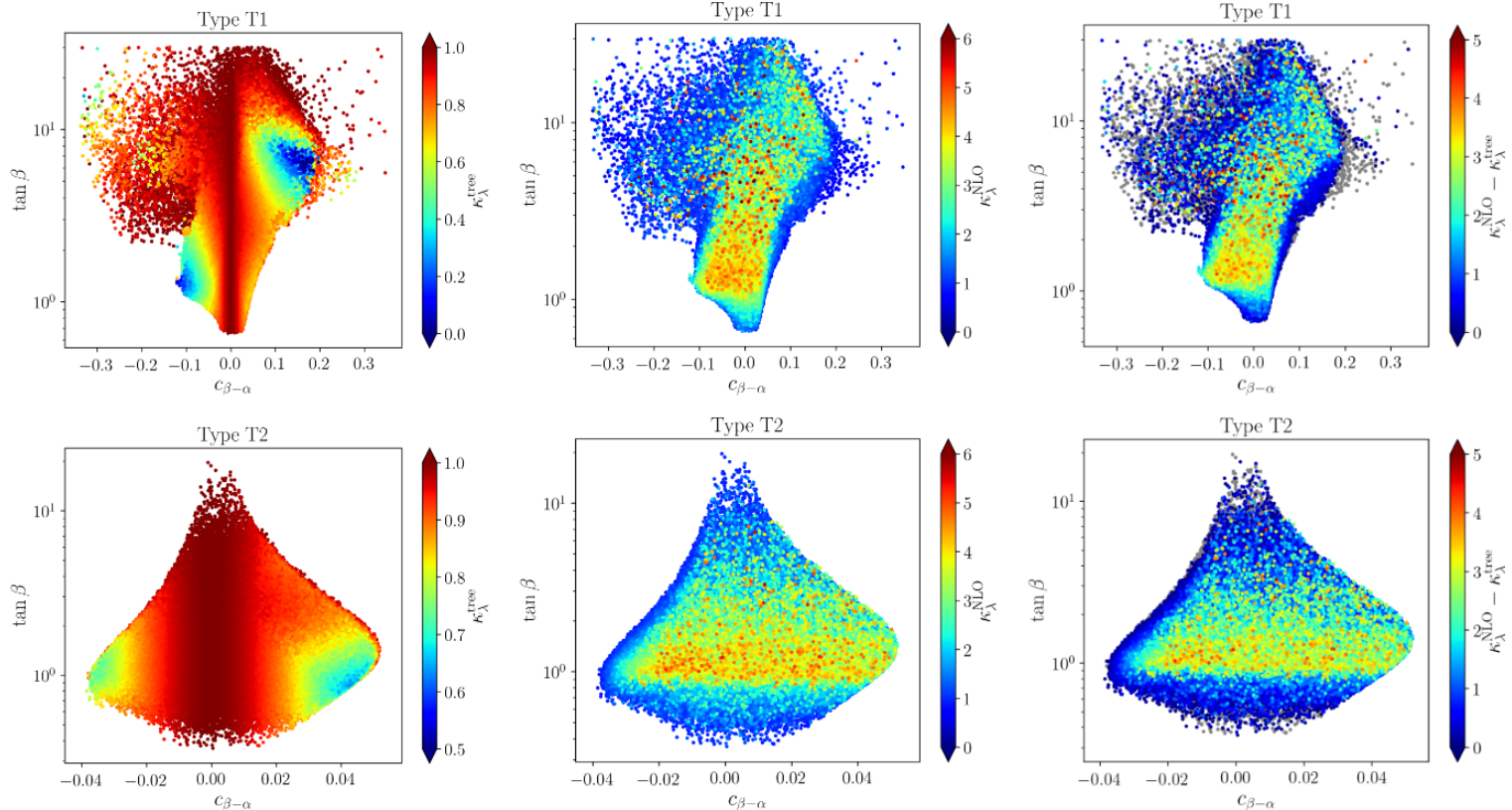
Main corrections to κ_λ

[Kanemura, Kiyoura, Okada, Senaha, Yuan, 02]

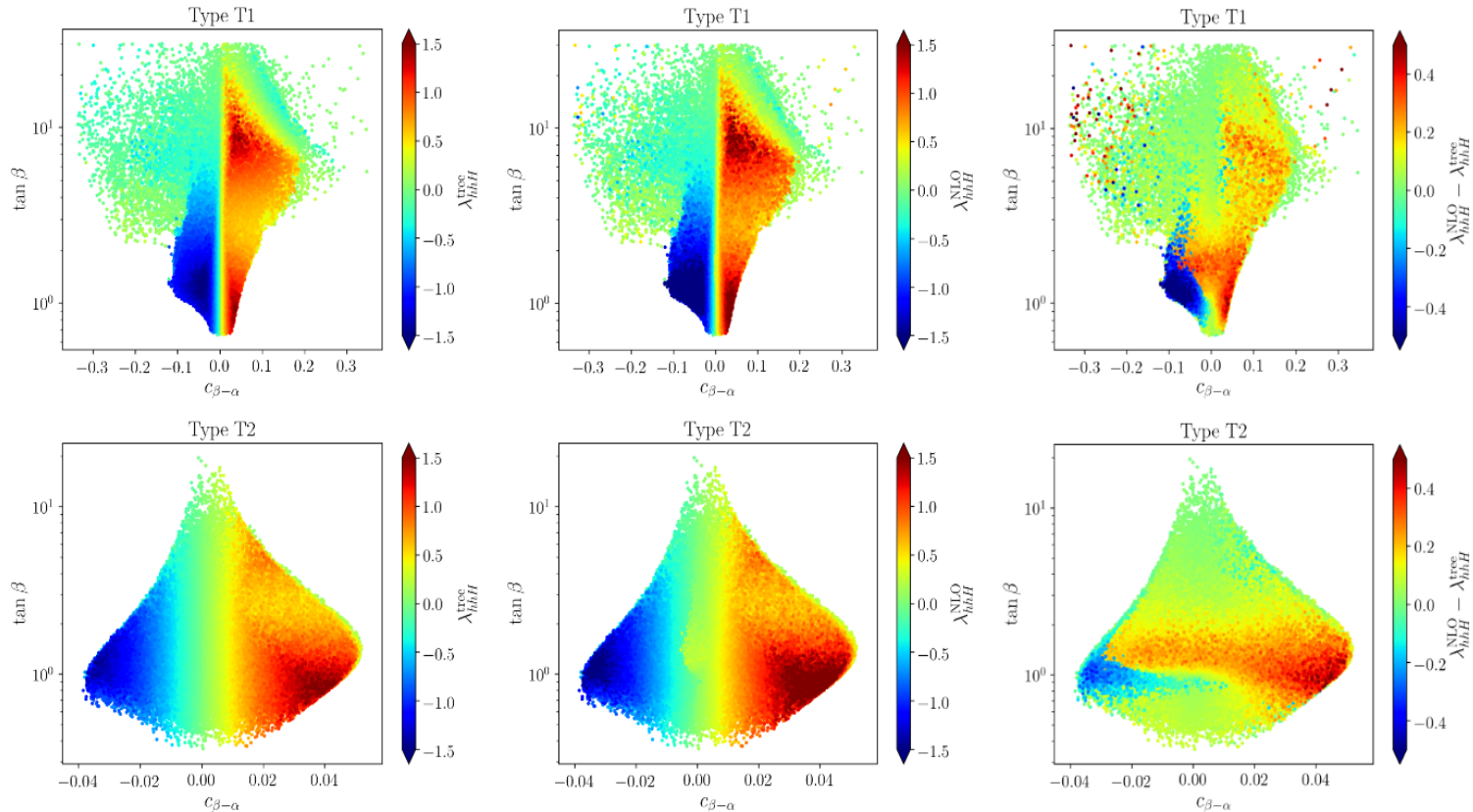
$$\kappa_\lambda^{(1)} \equiv \frac{\lambda_{hhh}^{(1)}}{\lambda_{\text{SM}}^{(0)}} \simeq 1 + \sum_{\phi=H,A,H^\pm} \frac{m_\phi^4}{12\pi^2 m_h^2 v^2} \left(1 - \frac{\bar{m}^2}{m_\phi^2} \right)^3$$

$$\lambda_{\text{SM}}^{(1)} \simeq \lambda_{\text{SM}}^{(0)} \left(1 - \frac{m_t^4}{\pi^2 m_h^2 v^2} \right) \quad \lambda_{\text{SM}}^{(0)} = \frac{2m_h^2}{v^2} \simeq 0.13$$

Results for κ_λ



Results for λ_{hhH}



Example for large κ_λ at 1 loop

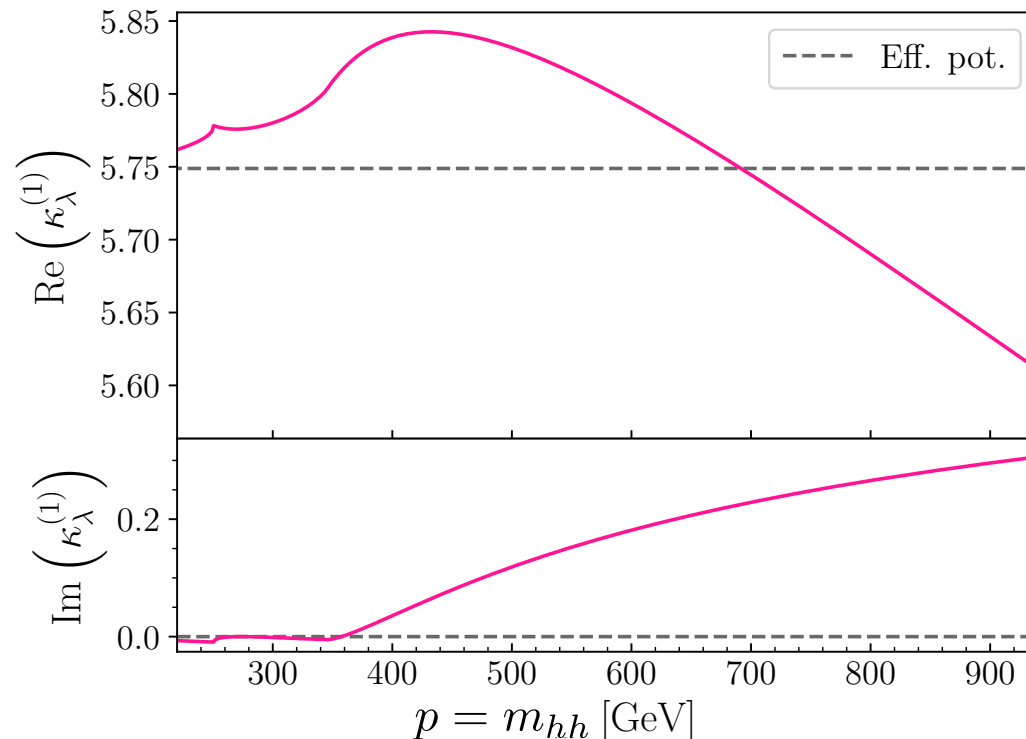
BPal, all types!

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$$m_A = m_{H^\pm} = 800 \text{ GeV},$$

$$\tan \beta = 3, \quad \cos(\beta - \alpha) = 0$$

- Large $\kappa_\lambda^{(1)}$ due to large $\lambda_{hAA}^{(0)}$ and $\lambda_{hH^+H^-}^{(0)}$
- Good agreement between effective potential and diagrammatic computation
 - Momentum dependence more important for large momentum



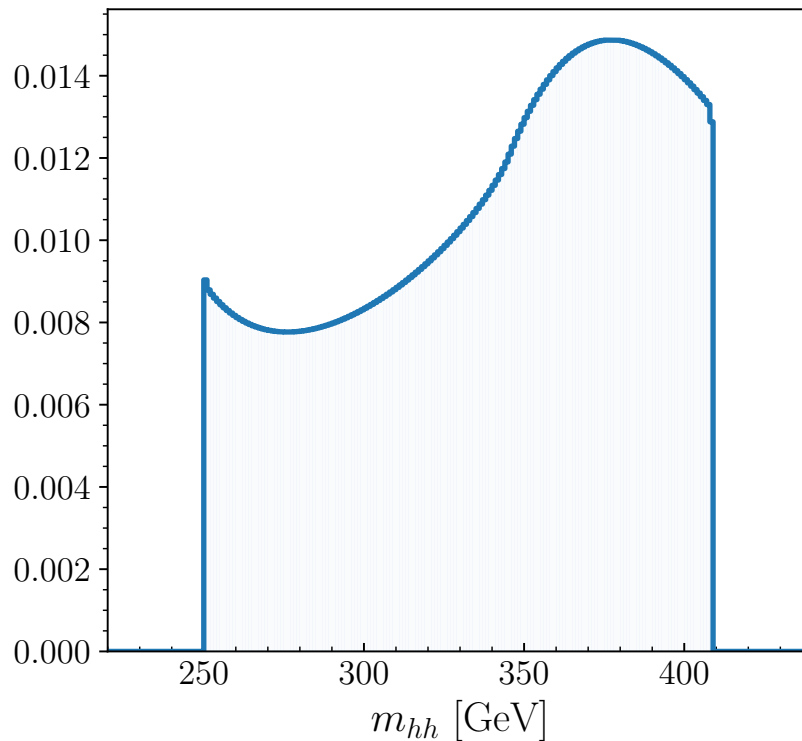
Relative difference w/ and wo/ p

BPal, all types!

$$m_H = \bar{m} = 400 \text{ GeV},$$

$$m_A = m_{H^\pm} = 800 \text{ GeV},$$

$$\tan \beta = 3, \cos(\beta - \alpha) = 0$$



2HDM Yukawa couplings

$$\begin{aligned}
 L_{\text{Yukawa}} \supset & - \sum_{f=u,d,l} \frac{m_f}{v} \left[\xi_f^h \bar{f} f h + \xi_f^H \bar{f} f H + \xi_f^A \bar{f} \gamma_5 f A \right] \\
 & - \frac{\sqrt{2}}{v} \left[\bar{u} (\xi_d V_{\text{CKM}} m_d P_R - \xi_u m_u V_{\text{CKM}} P_L) d H^+ + \xi_l \bar{\nu} m_l P_R l H^+ + \text{h.c.} \right]
 \end{aligned}$$

	Type I	Type II	Type III	Type IV
ξ_u	$\cot \beta$	$\cot \beta$	$\cot \beta$	$\cot \beta$
ξ_d	$\cot \beta$	$-\tan \beta$	$-\tan \beta$	$\cot \beta$
ξ_l	$\cot \beta$	$-\tan \beta$	$\cot \beta$	$-\tan \beta$

with $\xi_f^h = s_{\beta-\alpha} + \xi_f c_{\beta-\alpha}$, $\xi_f^H = c_{\beta-\alpha} - \xi_f s_{\beta-\alpha}$, $\xi_u^A = -i\xi_u$, $\xi_{d,l}^A = i\xi_{d,l}$

'Sensitivity' to the H resonance

[Cowan, Cranmer, Gross, Vitells, 13]

■ Our **theoretical 'estimator'**:

- Significance Z from a likelihood profile ratio statistical test: H resonance vs no resonance (i.e. $\lambda_{hhH} = 0$, the 'continuum')
- Notion of the '**sensitivity**' to the H resonance and hence to λ_{hhH}

$$L(\mu) = \prod_i \frac{(\mu s_i + b_i)^{n_i}}{n_i!} e^{-(\mu s_i + b_i)}$$

$$Z = \sqrt{-2 \log \left(\frac{L(0)}{L(1)} \right)} \equiv \sqrt{\sum_i (Z_i)^2}$$

$$s_i = \bar{N}_{i,4bZ} - \bar{N}_{i,4bZ}^C$$

$$b_i = \bar{N}_{i,4bZ}^C$$

$$n_i = s_i + b_i$$

$$Z_i = \sqrt{2 \left((s_i + b_i) \log \left(1 + \frac{s_i}{b_i} \right) - s_i \right)}$$

Disclaimer! This is *NOT* an experimental significance!