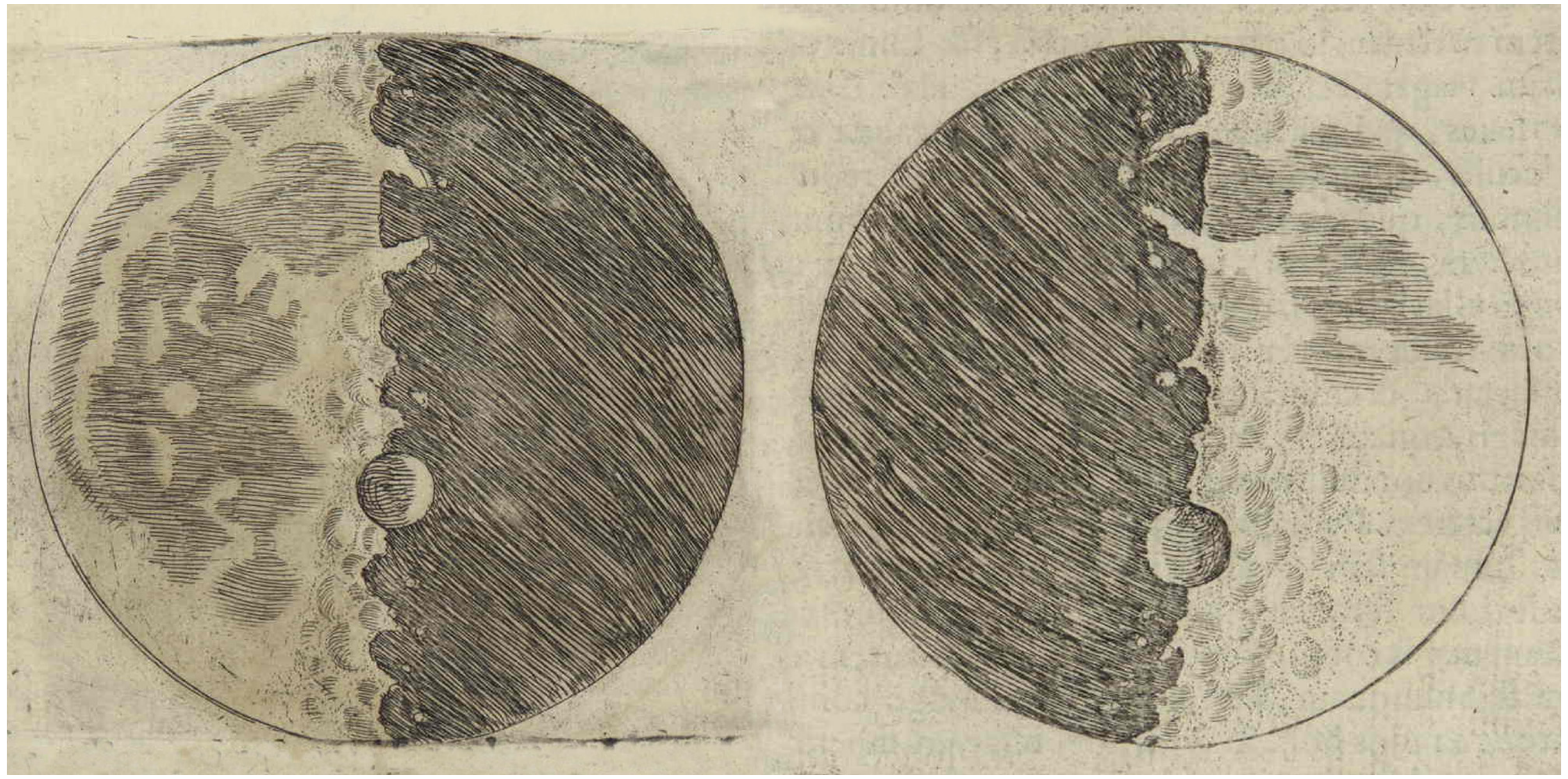
Majorana phases beyond $0\nu\beta\beta$

On the yet unknown parameters of the lepton sector



Galileo Galilei, 1610

AD, Gori, Grossman, Ligeti [arXiv:2406.18647]
Aloni, **AD** [arXiv:2211.09638]

Avital Dery, CERN TH, BSM Forum, Nov 21 2024

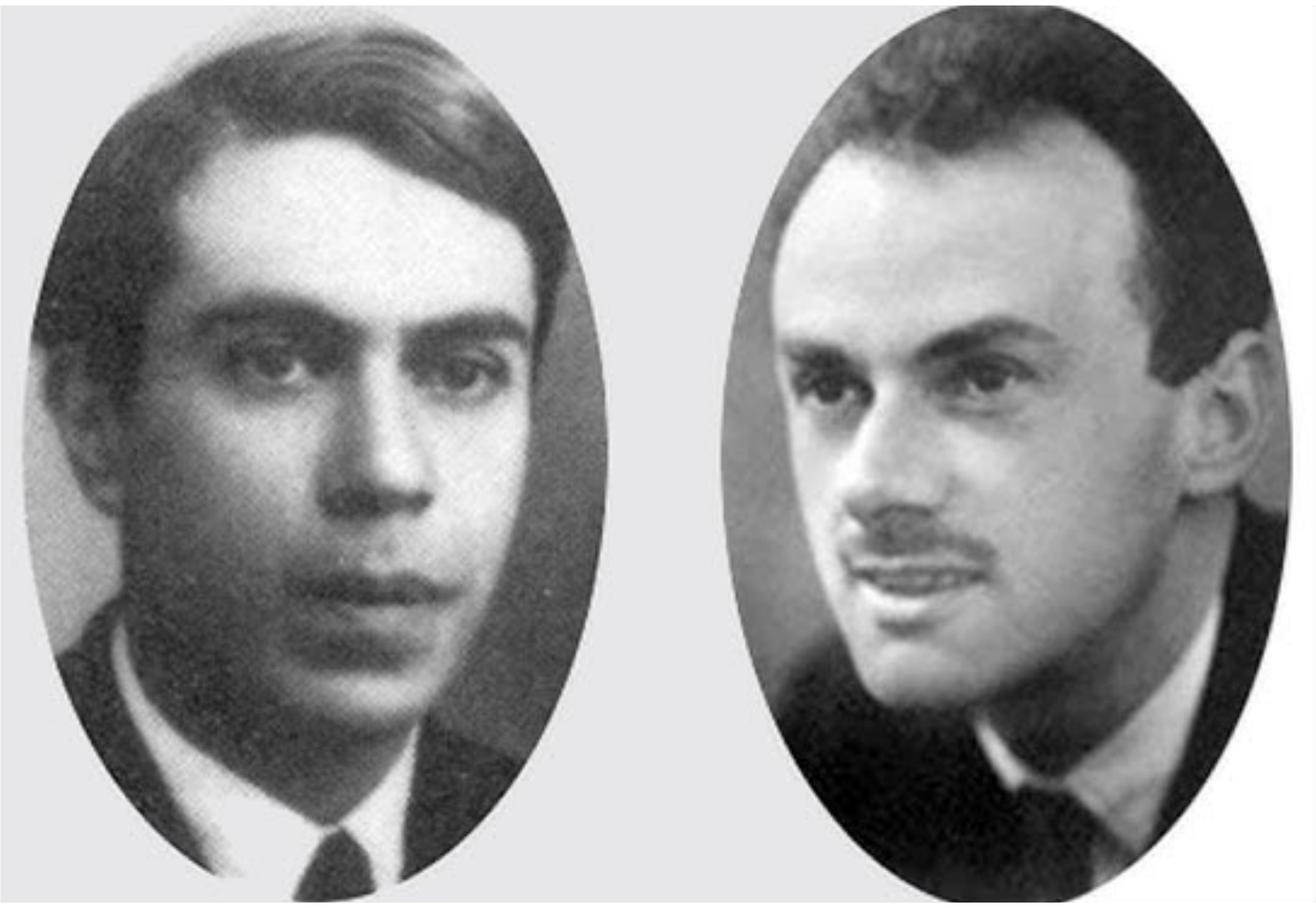
Neutrino masses = BSM

$$Y_M \frac{\phi\,\phi\,L\,L}{\Lambda}$$

$\Delta L = 2$

ν_MSM

$\cancel{L}$



$$Y_D\,\bar{L}_L\,\widetilde{\phi}\;\nu_R$$ + impose L

$\Delta L = 0$

ν_DSM

L

Leptonic mixing parameters

Determined by oscillation experiments

Mixing angles known to $\mathcal{O}(1\%)$

Two mass squared differences known, up to one sign

$$\Delta m^2_{32} > 0 \quad \rightarrow \quad \text{Normal Ordering (NO)}$$

$$\Delta m^2_{32} < 0 \quad \rightarrow \quad \text{Inverted Ordering (IO)}$$

Determining the ordering is one of the primary goals of DUNE

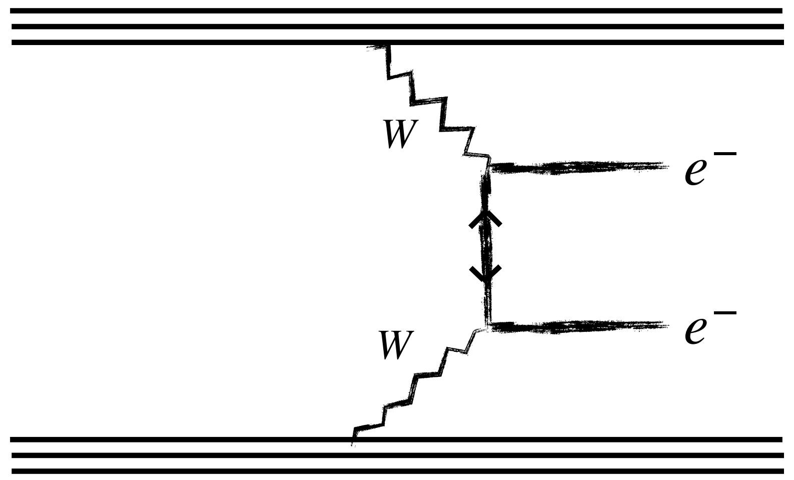
Parameter	Value NO	Value IO
θ_{12}	$(33.41^{+0.75}_{-0.72})^\circ$	same
θ_{23}	$(49.1^{+1.0}_{-1.3})^\circ$	$(49.5^{+0.9}_{-1.2})^\circ$
θ_{13}	$(8.54^{+0.11}_{-0.12})^\circ$	$(8.57^{+0.12}_{-0.11})^\circ$
$\Delta m^2_{21}/(10^{-5}\,\mathrm{eV}^2)$	$7.41^{+0.21}_{-0.20}$	same
$\Delta m^2_{32}/(10^{-3}\,\mathrm{eV}^2)$	$2.437^{+0.028}_{-0.027}$	$-2.498^{+0.032}_{-0.025}$

Prospects for determining the Dirac CP-phase at Hyper-Kamiokande (after ~ 10 years of running):

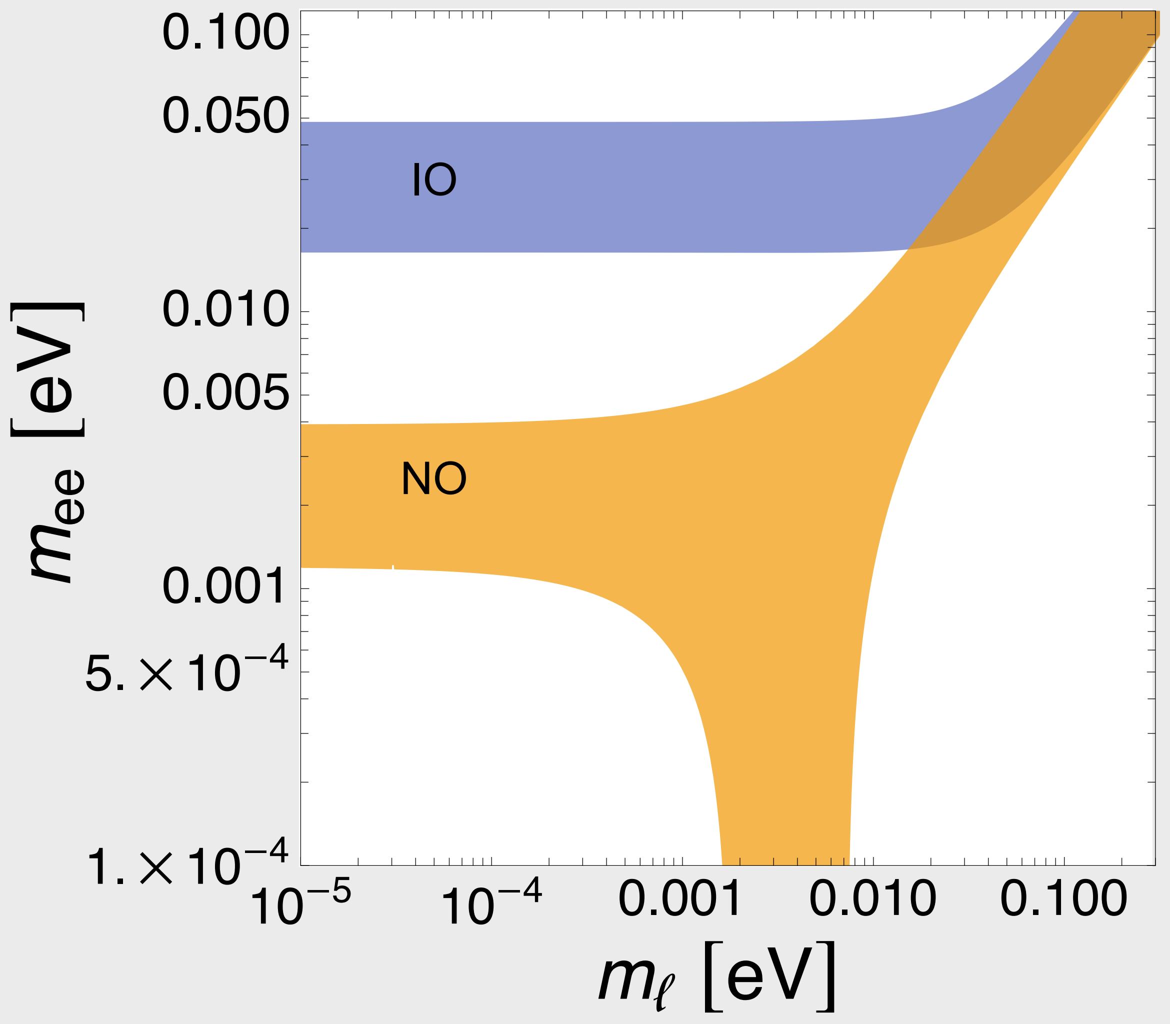
$$\delta = (0 \pm 7)^\circ, \qquad \delta = (90 \pm 22)^\circ$$

Determining the nature of ν's

$0\nu\beta\beta$



$\Delta L = 2$



Current exp. bound:

$m_{ee} < (0.028\ -\ 0.122)\,\mathrm{eV}$

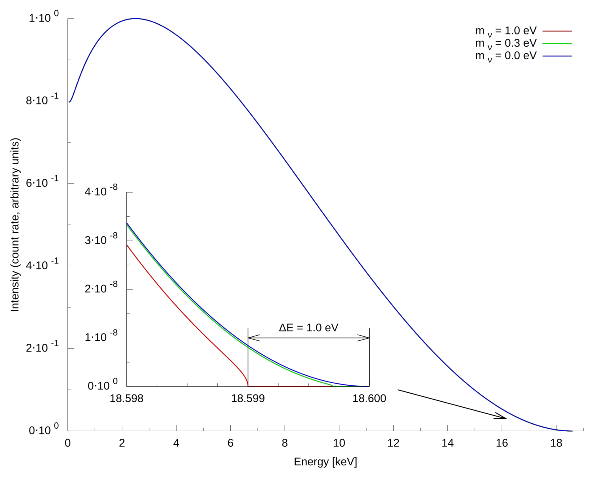
(Decay half life $T^{0\nu}_{1/2} > 3.8\times10^{26}\,\mathrm{yr}$)

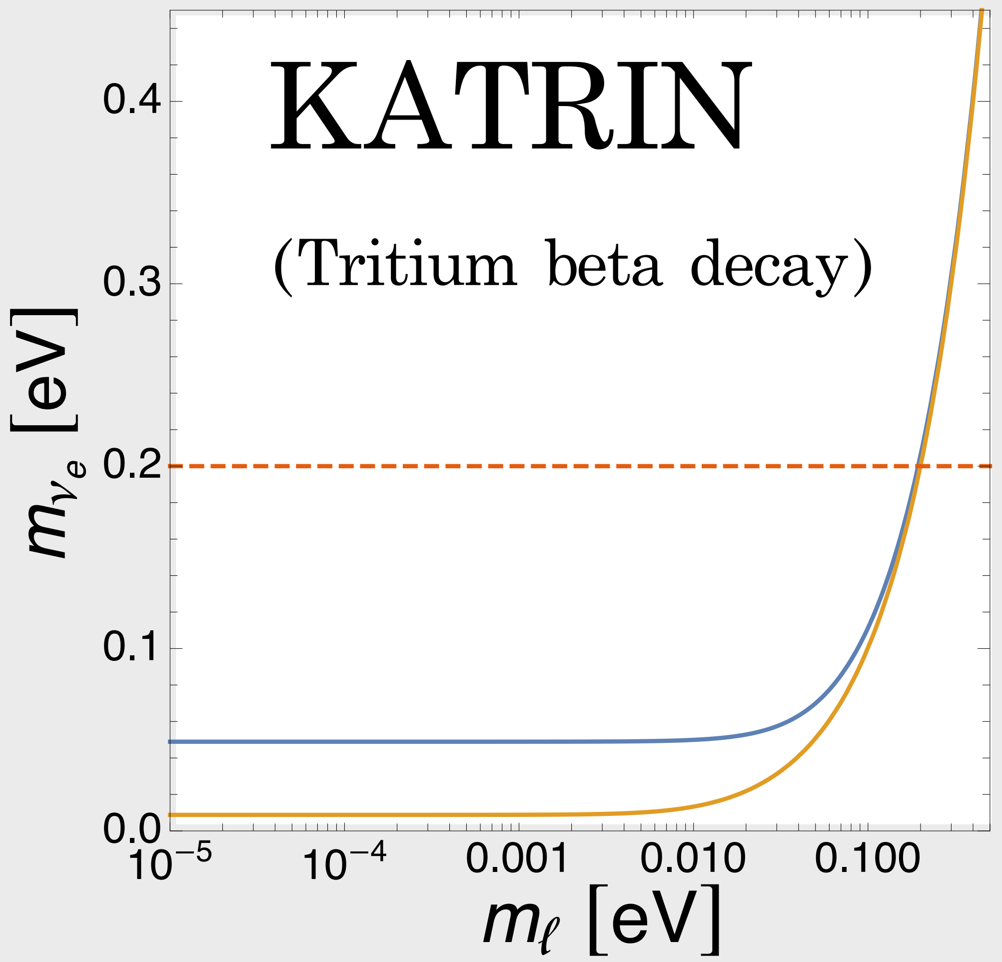
[KamLAND-Zen 2406.11438]

Future exp. sensitivity:

$m_{ee} \sim (0.005\ -\ 0.020)\,\mathrm{eV}$

in ~ 10 years



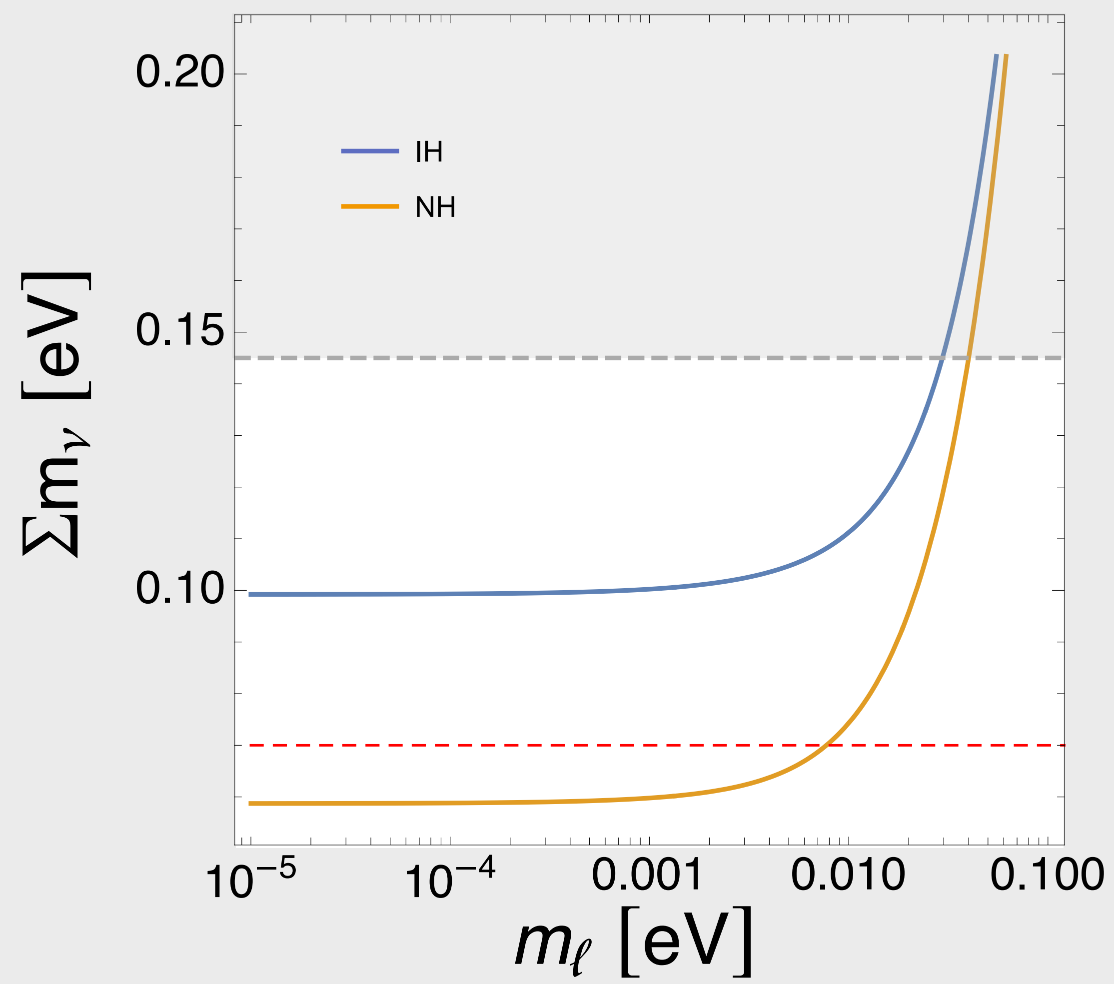


$m_{\nu_e} < 0.45\,\mathrm{eV}$

target sensitivity:

$m_{\nu_e} \sim 0.2\,\mathrm{eV}$

in ~ 5 years



DESI BAO + CMB

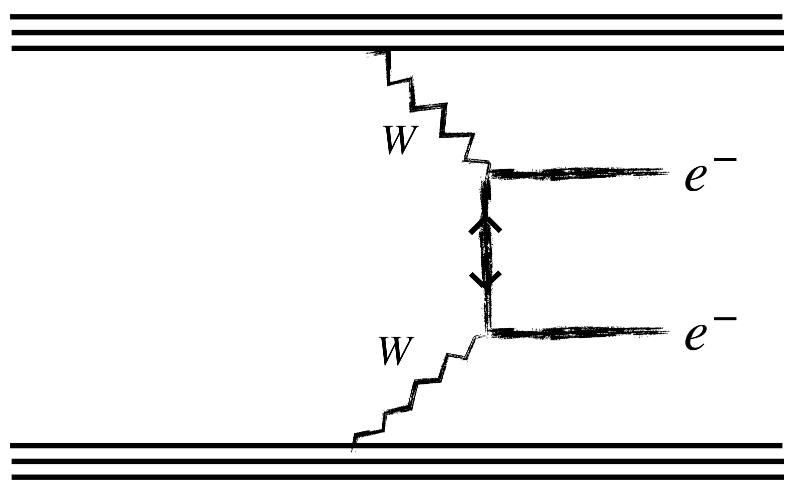
$\sum_i m_i < 0.072\,\mathrm{eV}$

$\sum_i m_i < 0.113\,\mathrm{eV}$ depending on prior

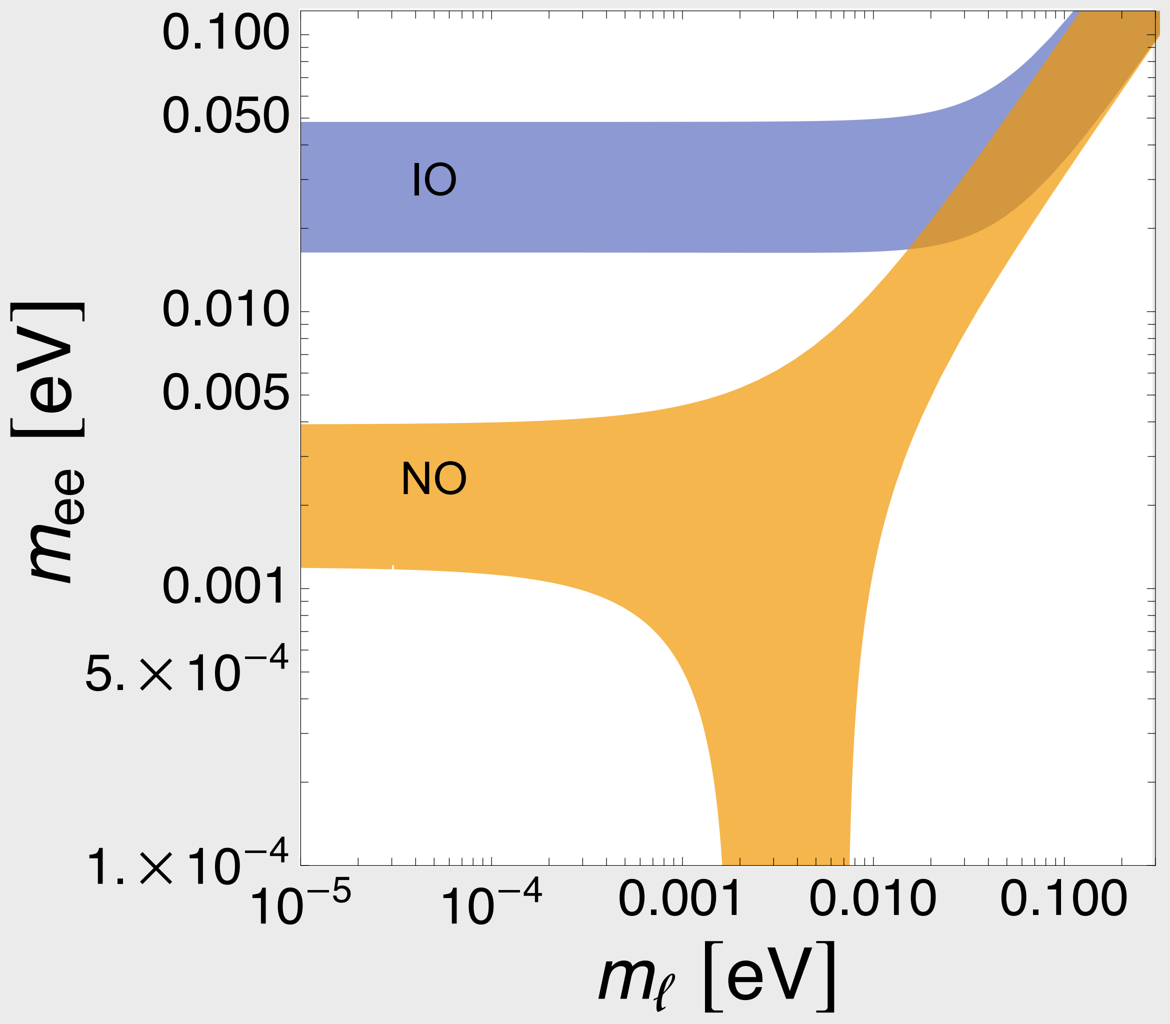
$\sum_i m_i < 0.145\,\mathrm{eV}$

Determining the nature of ν's

$0\nu\beta\beta$



$\Delta L = 2$



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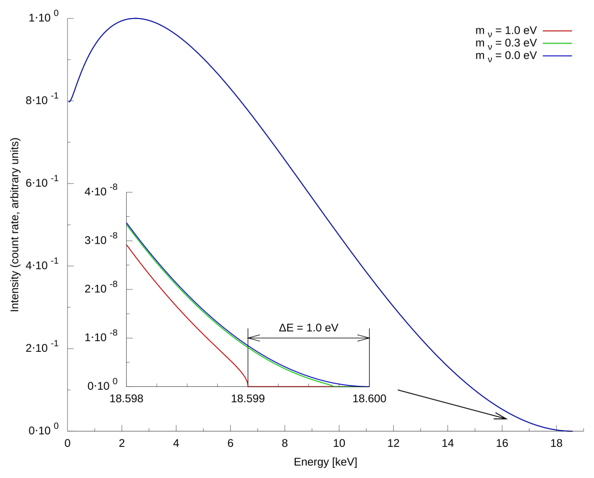
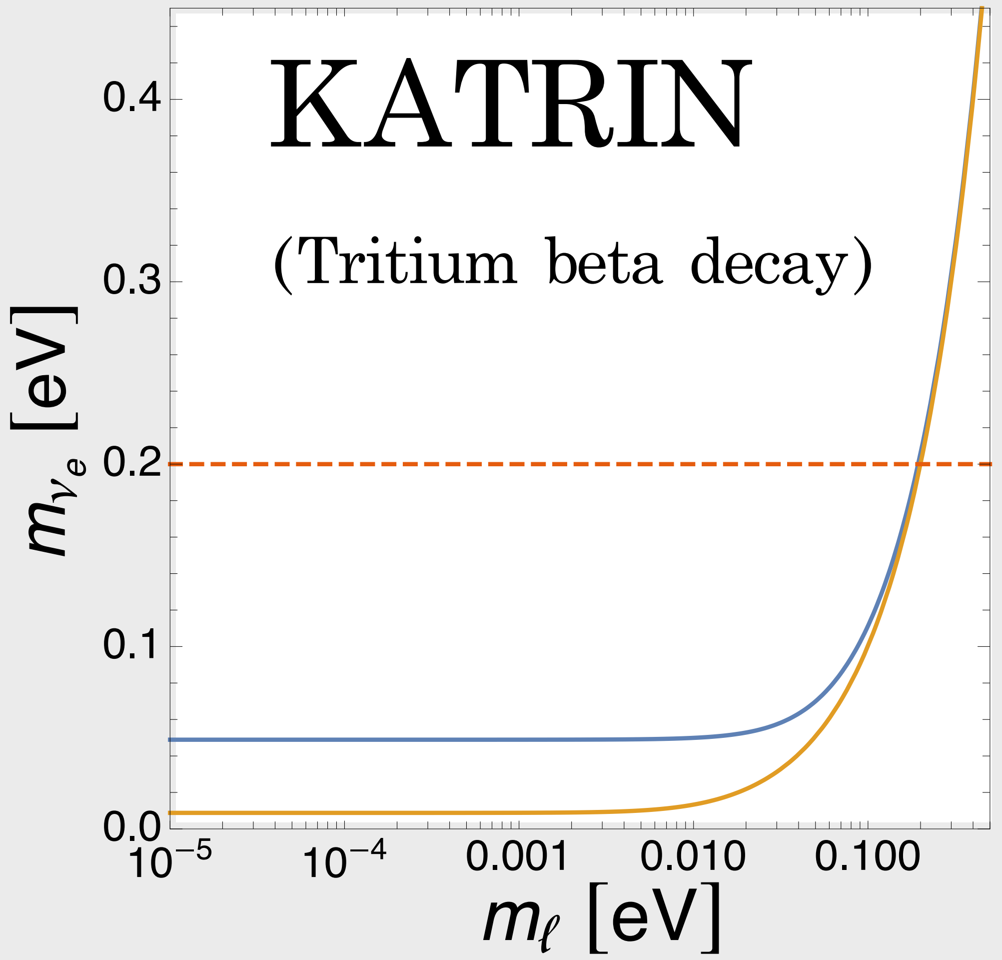
[KamLAND-Zen
2406.11438]

Future exp. sensitivity:
$m_{ee} \sim (0.005 \; - \; 0.020)\,\mathrm{eV}$
in $\sim$ 10 years

If neutrino masses are Normally Ordered (NO), non-observation of $0\nu\beta\beta$ may not be sufficient to exclude the Majorana nature of neutrinos





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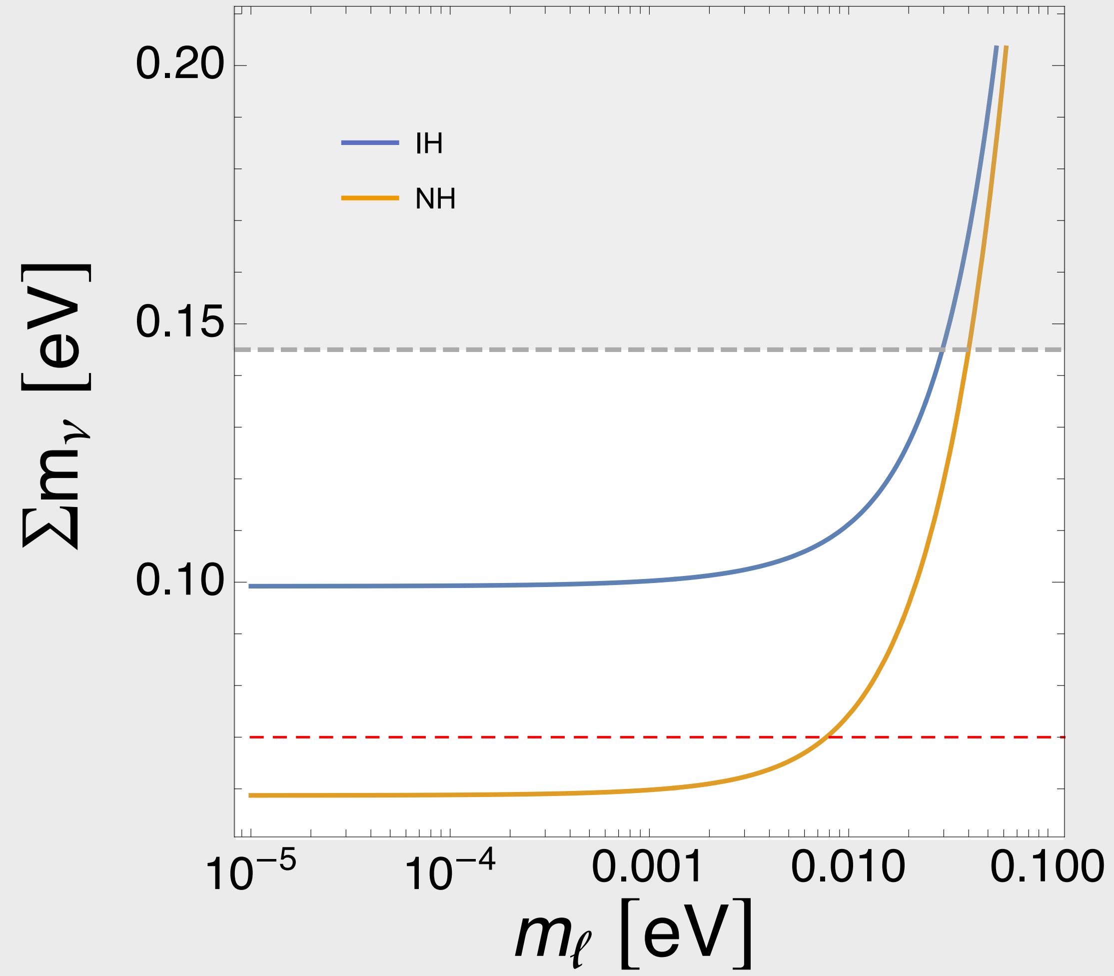

DESI BAO + CMB

$\sum_i m_i < 0.072\,\mathrm{eV}$

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$\sum_i m_i < 0.145\,\mathrm{eV}$

depending on prior

Leptonic CP phases

$$U = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \times \begin{pmatrix} c_{13} & & s_{13}\, e^{-i\delta} \\ & 1 & \\ -s_{13}\, e^{i\delta} & & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \times \begin{pmatrix} e^{i\eta_1} & & \\ & e^{i\eta_2} & \\ & & 1 \end{pmatrix}$$

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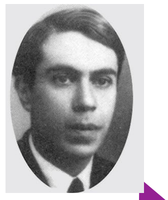
Majorana

$$(\bar{e} \quad \bar{\mu} \quad \bar{\tau})\, \gamma^\rho\, W^-_\rho \begin{pmatrix} 3R + 6I \\ \text{parameters a priori} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

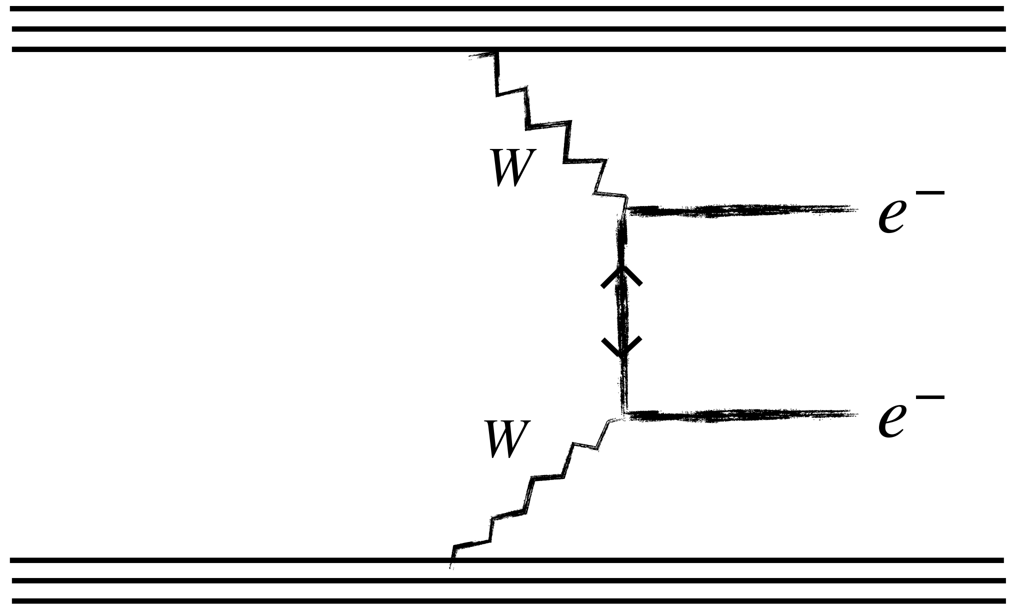
$$\left(e^{i\phi_e}\,\bar{e} \quad e^{i\phi_\mu}\,\bar{\mu} \quad e^{i\phi_\tau}\,\bar{\tau}\right)$$

$$3R + 3I$$

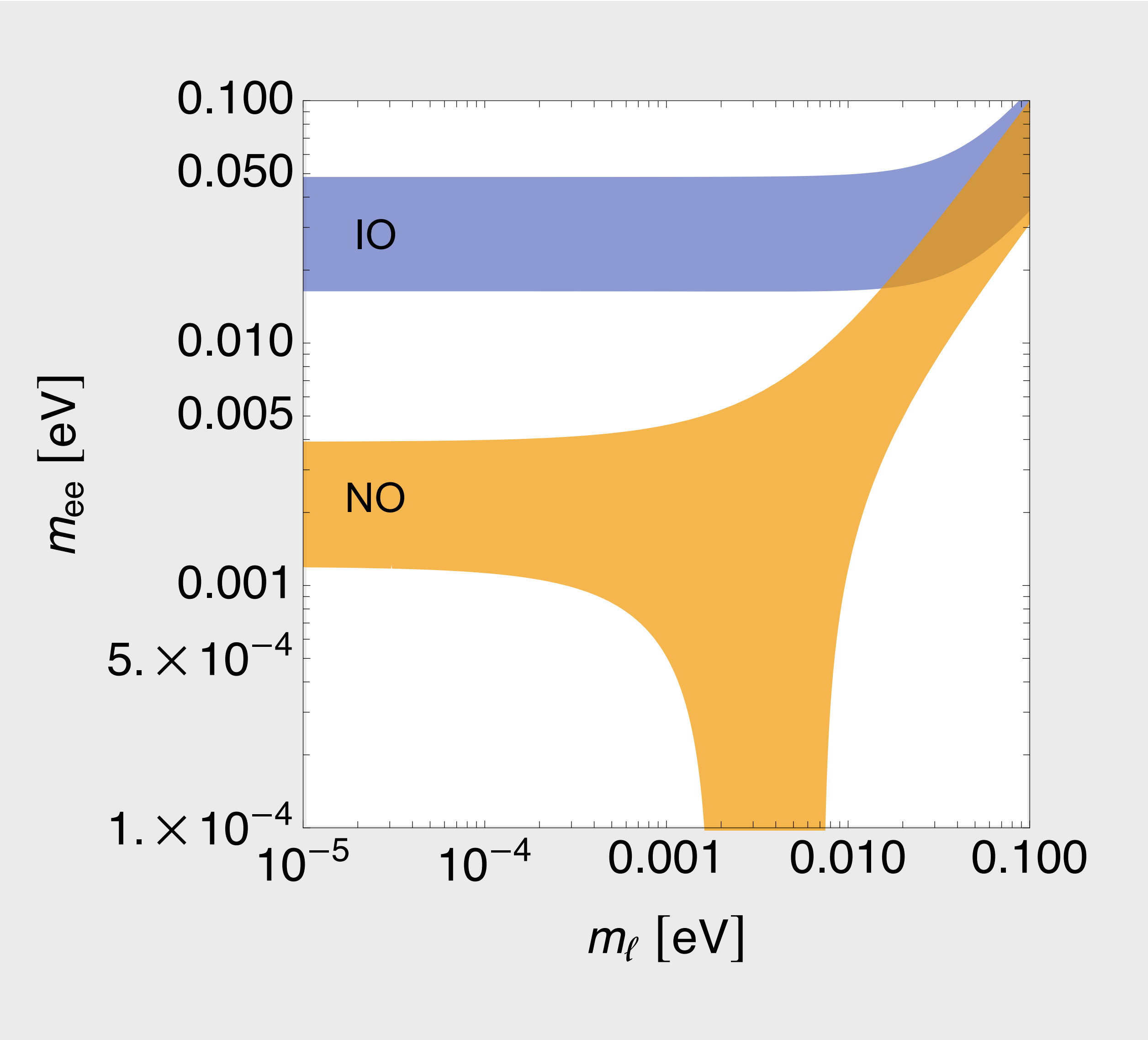


$3R + 3I$ $\qquad$ $3R + 1I$ $\quad$ physical parameters

massless lightest ν $\quad$ $3R + 2I$



$0\nu\beta\beta$



$$\Gamma_{0\nu\beta\beta} \propto m_{ee}^2 = \left| \sum_i m_i U_{ei}^2 \right|^2$$

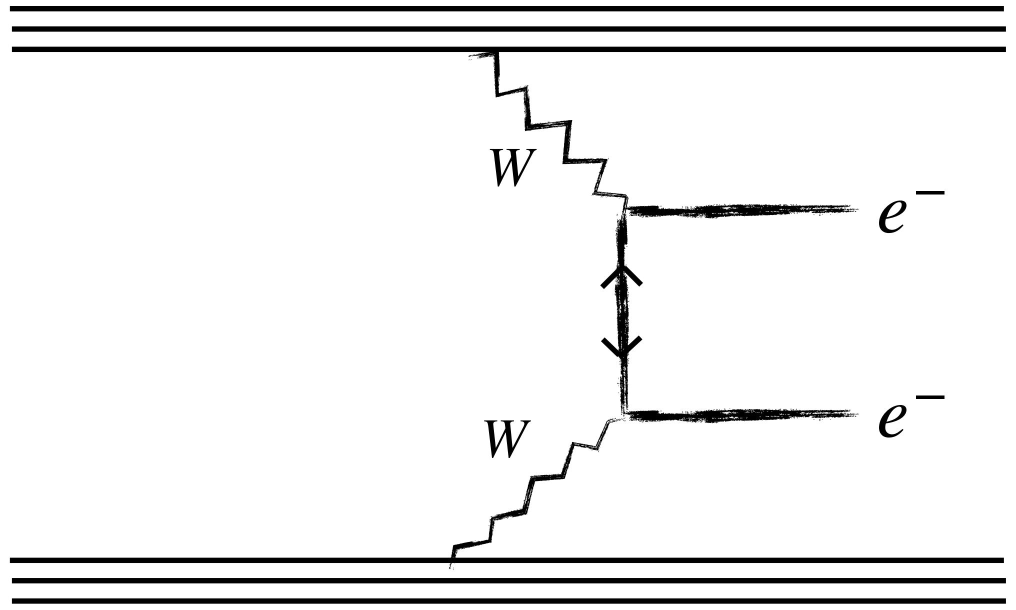
$$m_{ee}^2 = \sum_i m_i^2 |U_{ei}|^4 + 2\, m_1 m_2 |U_{e1}|^2 |U_{e2}|^2 \cos(\eta_1 - \eta_2)$$
$$+ 2\, m_2 m_3 |U_{e2}|^2 |U_{e3}|^2 \cos(\eta_2 + \delta)$$
$$+ 2\, m_1 m_3 |U_{e1}|^2 |U_{e3}|^2 \cos(\eta_1 + \delta)$$

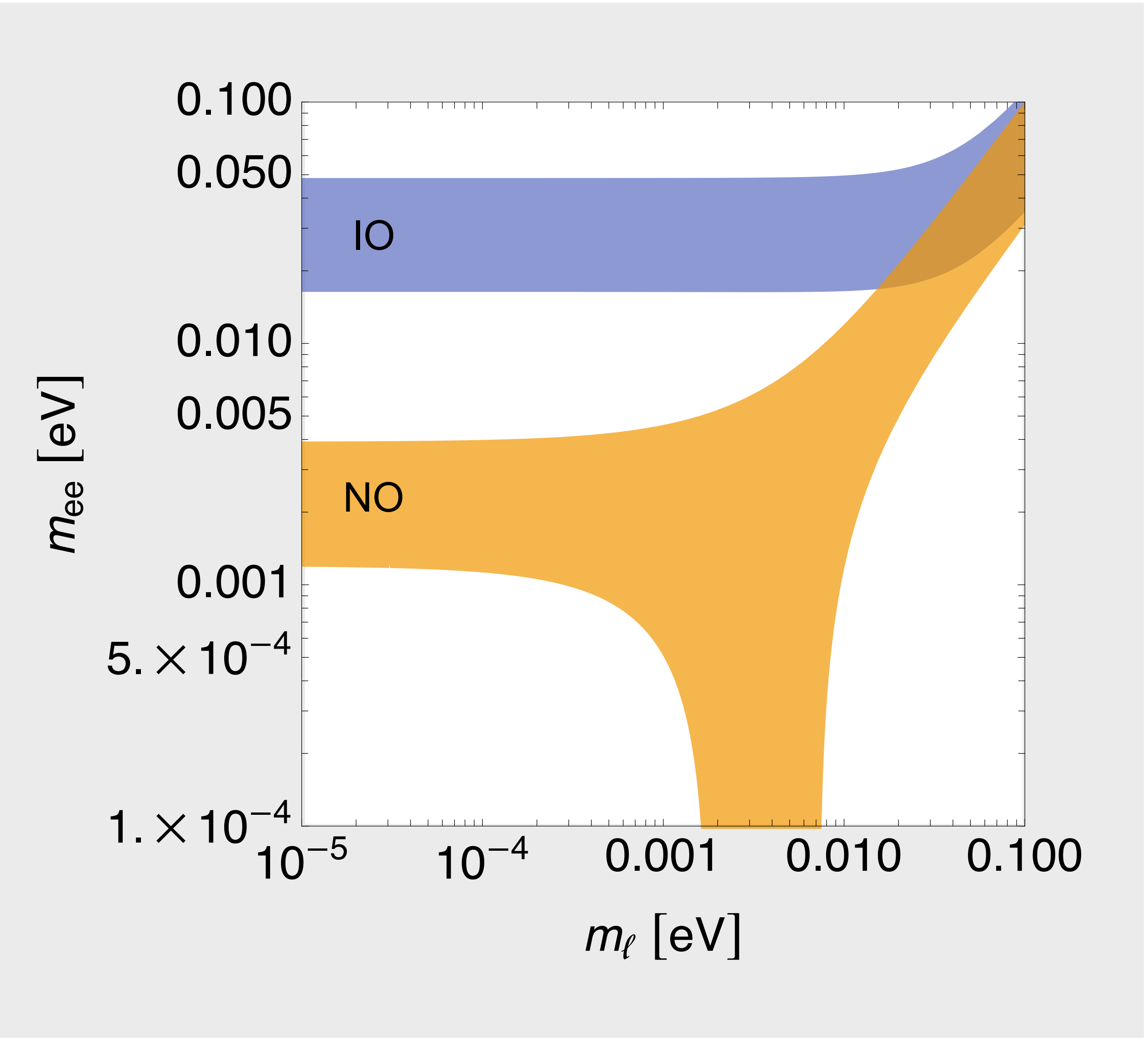
Q:

a) What type of observables would have sensitivity to the orthogonal combination of Majorana phases?

b) How does the sensitivity to Majorana phases vary as the lightest neutrino mass approaches zero (but non-zero)?

$0\nu\beta\beta$





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$$\begin{aligned} m_{ee}^2 = \sum_i m_i^2 |U_{ei}|^4 &+ 2\, m_1 m_2 |U_{e1}|^2 |U_{e2}|^2 \cos(\underline{\eta_1 - \eta_2}) \\ &+ 2\, m_2 m_3 |U_{e2}|^2 |U_{e3}|^2 \cos(\underline{\eta_2 + \delta}) \\ &+ 2\, m_1 m_3 |U_{e1}|^2 |U_{e3}|^2 \cos(\eta_1 + \delta) \end{aligned}$$

Q:

a) What type of observables would have sensitivity to the orthogonal combination of Majorana phases?

b) How does the sensitivity to Majorana phases vary as the lightest neutrino mass approaches zero (but non-zero)?

Notice that:

I. Dependent on two phase combinations

II. Allowed range is <u>independent of any knowledge of δ</u>

$\Big\}$ This is no coincidence.

Phase convention dependence

$$U = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \times \begin{pmatrix} c_{13} & & s_{13}\, e^{-i\delta} \\ & 1 & \\ -s_{13}\, e^{i\delta} & & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \times \begin{pmatrix} e^{i\eta_1} & & \\ & e^{i\eta_2} & \\ & & 1 \end{pmatrix}$$

could have chosen a different convention, e.g., $\begin{pmatrix} 1 & & \\ & e^{i\eta_2} & \\ & & e^{i\eta_3} \end{pmatrix}$

Phase-convention dependent quantities (e.g., η_1) cannot correspond to physical parameters.

Working with phase-convention invariants

Quartic invariants
$$t_{\alpha i\beta j} = U_{\alpha i}U_{\beta j}U^*_{\alpha j}U^*_{\beta i}$$
$$\delta^{\alpha\beta}_{ij} = \arg(t_{\alpha i\beta j})$$

Quadratic invariants
$$s_{\alpha ij} = U_{\alpha i}U^*_{\alpha j}$$
$$\Phi^{\alpha}_{ij} = \arg(s_{\alpha ij})$$

physical if ν's are Majorana

[Nieves and Pal, *Phys. Rev. D* **36** (1987) 315]

A choice of basis:

$$\{\,|t_{e1e2}|,\ |t_{e3e3}|,\ |t_{\mu 2e3}|,\ \Psi_D\} \qquad \{\Phi^e_{12},\ \Phi^e_{23}\}$$

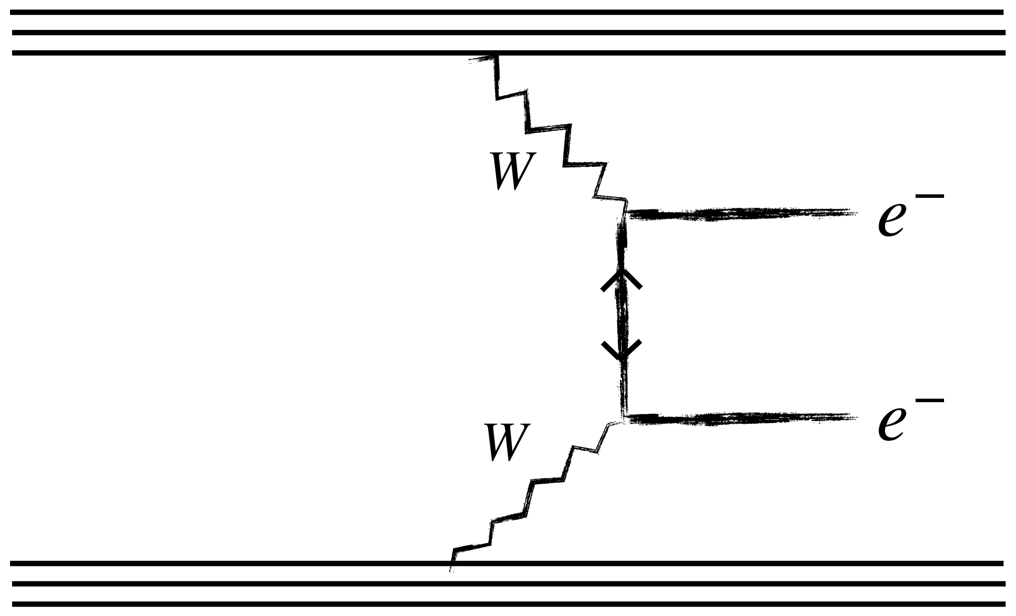
3R+3I ✔

with $\Psi_D \equiv \delta^{\mu e}_{23}$

Phases of other flavors are related by

$$\Phi^{\beta}_{ij} = \Phi^{\alpha}_{ij} + \delta^{\beta\alpha}_{ij}$$

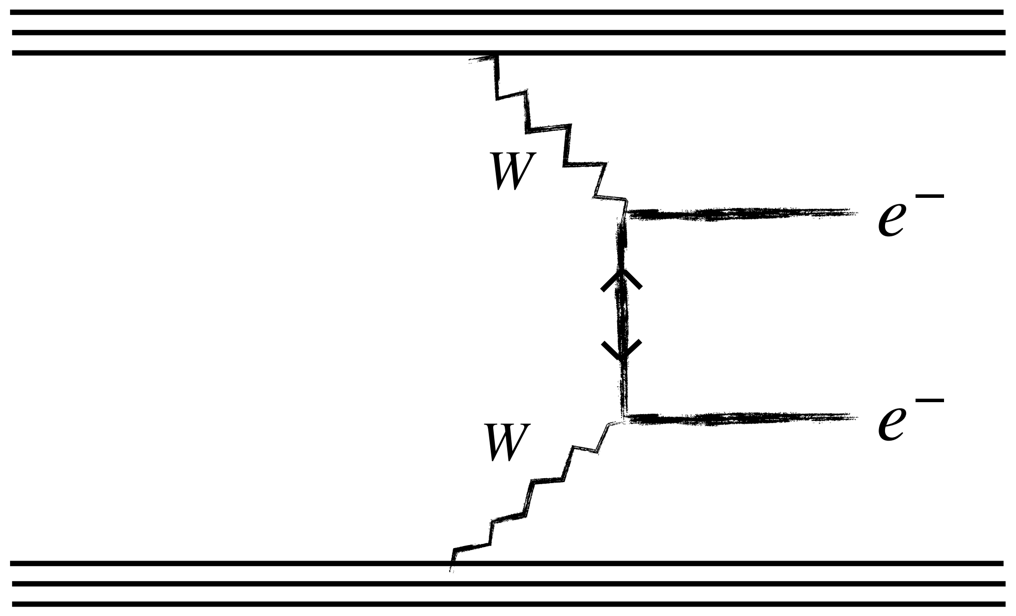
The sum of a Majorana phase and a Dirac phase is a Majorana phase.



$0\nu\beta\beta$

$$m_{ee}^2 = \sum_i m_i^2 |t_{eiei}| + 2\, m_1 m_2 |t_{e1e2}| \cos(2\Phi^e_{12})$$
$$+ 2\, m_2 m_3 |t_{e2e3}| \cos(2\Phi^e_{23})$$
$$+ 2\, m_1 m_3 |t_{e1e3}| \cos(2(\Phi^e_{12} + \Phi^e_{23}))$$

I. Dependent on two Majorana phases
II. Independent of the Dirac phase
III. The sensitivity to phase Φ_{ij} is reduced linearly as $m_i \to 0$ or $m_j \to 0$



$0\nu\beta\beta$

Generalized effective Majorana mass matrix

$$m_{\alpha\beta} = \left| \sum_i m_i U_{\alpha i} U_{\beta i} \right| \qquad \alpha, \beta \in \{e, \mu, \tau\}$$

$$m_{ee}^2 = \sum_i m_i^2 |t_{eiei}| + 2\, m_1 m_2 |t_{e1e2}| \cos(2\Phi_{12}^e) + 2\, m_2 m_3 |t_{e2e3}| \cos(2\Phi_{23}^e) + 2\, m_1 m_3 |t_{e1e3}| \cos(2(\Phi_{12}^e + \Phi_{23}^e))$$

Generalize to any α, β

$$m_{\alpha\beta}^2 = \sum_i m_i^2 |t_{\alpha i\beta i}| + 2\, m_1 m_2 |t_{\alpha 1\beta 2}| \cos(2\Phi_{12}^{\alpha\beta}) + 2\, m_2 m_3 |t_{\alpha 2\beta 3}| \cos(2\Phi_{23}^{\alpha\beta}) + 2\, m_1 m_3 |t_{\alpha 1\beta 3}| \cos(2(\Phi_{12}^{\alpha\beta} + \Phi_{23}^{\alpha\beta})),$$

where $\Phi_{ij}^{\alpha\beta} = \dfrac{\Phi_{ij}^{\alpha} + \Phi_{ij}^{\beta}}{2}$

(relevant observables are, e.g., $\mu^- \to e^+$ conversion, $\tau^- \to \ell^+ M_1^- M_2^-$)

I. Dependent on two Majorana phases
II. Independent of the Dirac phase
III. The sensitivity to phase Φ_{ij} is reduced linearly as $m_i \to 0$ or $m_j \to 0$

When considering more than one entry of $m_{\alpha\beta}$ at once, the Dirac phase does creep in.

$$m_{\alpha\beta}$$

$$m_{\alpha'\beta'}$$

$$\begin{aligned} m_{\alpha\beta}^2 = \sum_i m_i^2 |t_{\alpha i\beta i}| &+ 2\, m_1 m_2 |t_{\alpha 1\beta 2}| \cos(2\Phi_{12}^{\alpha\beta}) \\ &+ 2\, m_2 m_3 |t_{\alpha 2\beta 3}| \cos(2\Phi_{23}^{\alpha\beta}) \\ &+ 2\, m_1 m_3 |t_{\alpha 1\beta 3}| \cos(2(\Phi_{12}^{\alpha\beta} + \Phi_{23}^{\alpha\beta}))\,, \end{aligned}$$

$$\Phi_{ij}^{\alpha'\beta'} = \Phi_{ij}^{\alpha\beta} + (\delta_{ij}^{\alpha'\alpha} + \delta_{ij}^{\beta'\beta})/2$$

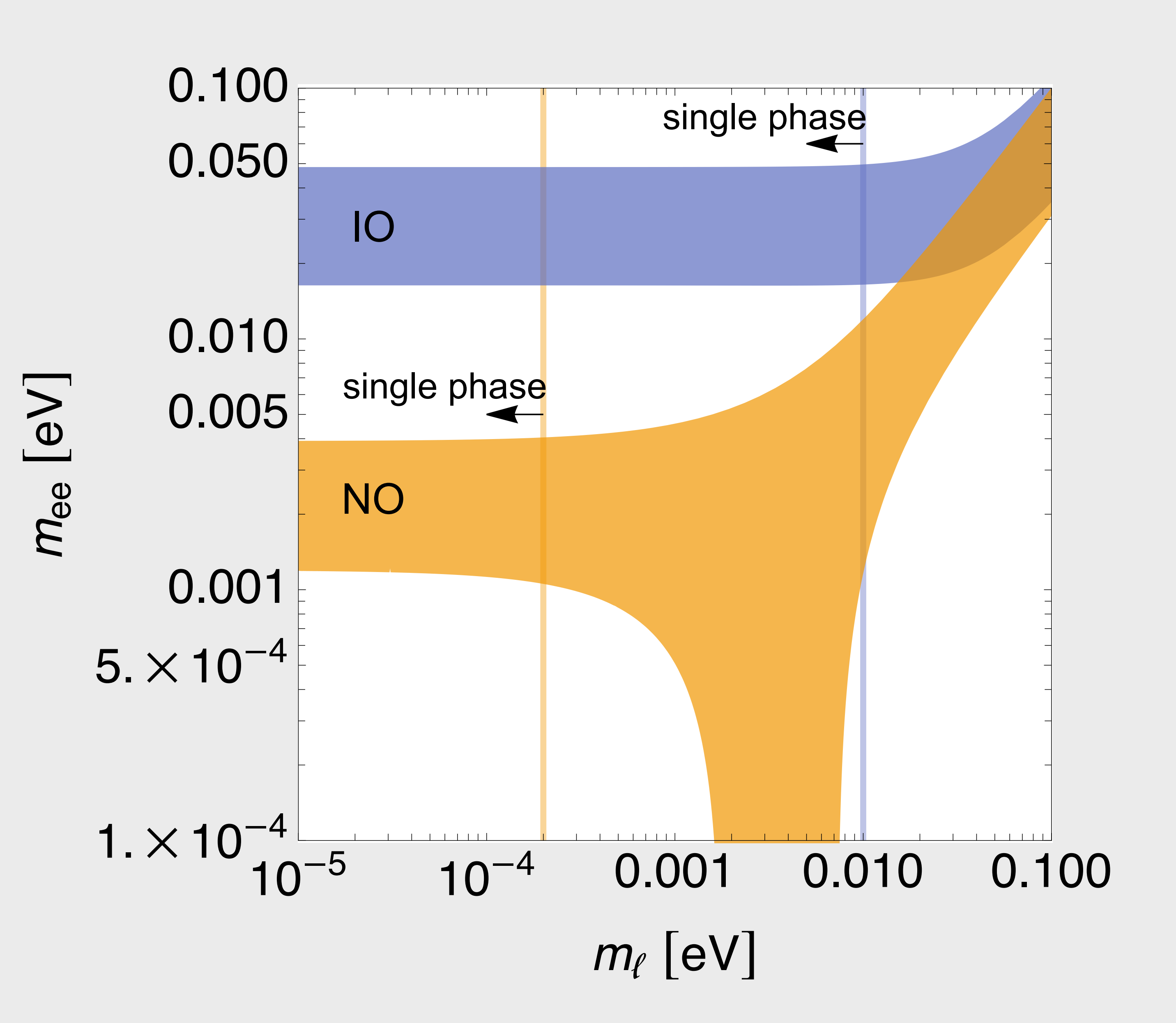
In order to write an expression for $m_{\alpha'\beta'}$ in the same basis, we have to involve Dirac-type phases.

=> In principle, a measurement of three different entries of $m_{\alpha\beta}$ would determine all three phases - two Majorana and one Dirac.

The single phase limit

Unified naming scheme for both orderings -

$$\{\underset{lightest}{\nu_\ell},\ \nu_2,\ \underset{other}{\nu_o}\}$$



$$m^2_{\alpha\beta} = \sum_i m_i^2 |t_{\alpha i\beta i}| + 2\, m_2 m_o |t_{\alpha 2\beta o}| \cos(2\Phi^{\alpha\beta}_{2o}) + 2\, m_\ell m_2 |t_{\alpha\ell\beta 2}| \cos(2\Phi^{\alpha\beta}_{2\ell}) + 2\, m_\ell m_o |t_{\alpha\ell\beta o}| \cos(2(\Phi^{\alpha\beta}_{2\ell} + \Phi^{\alpha\beta}_{2o}))$$

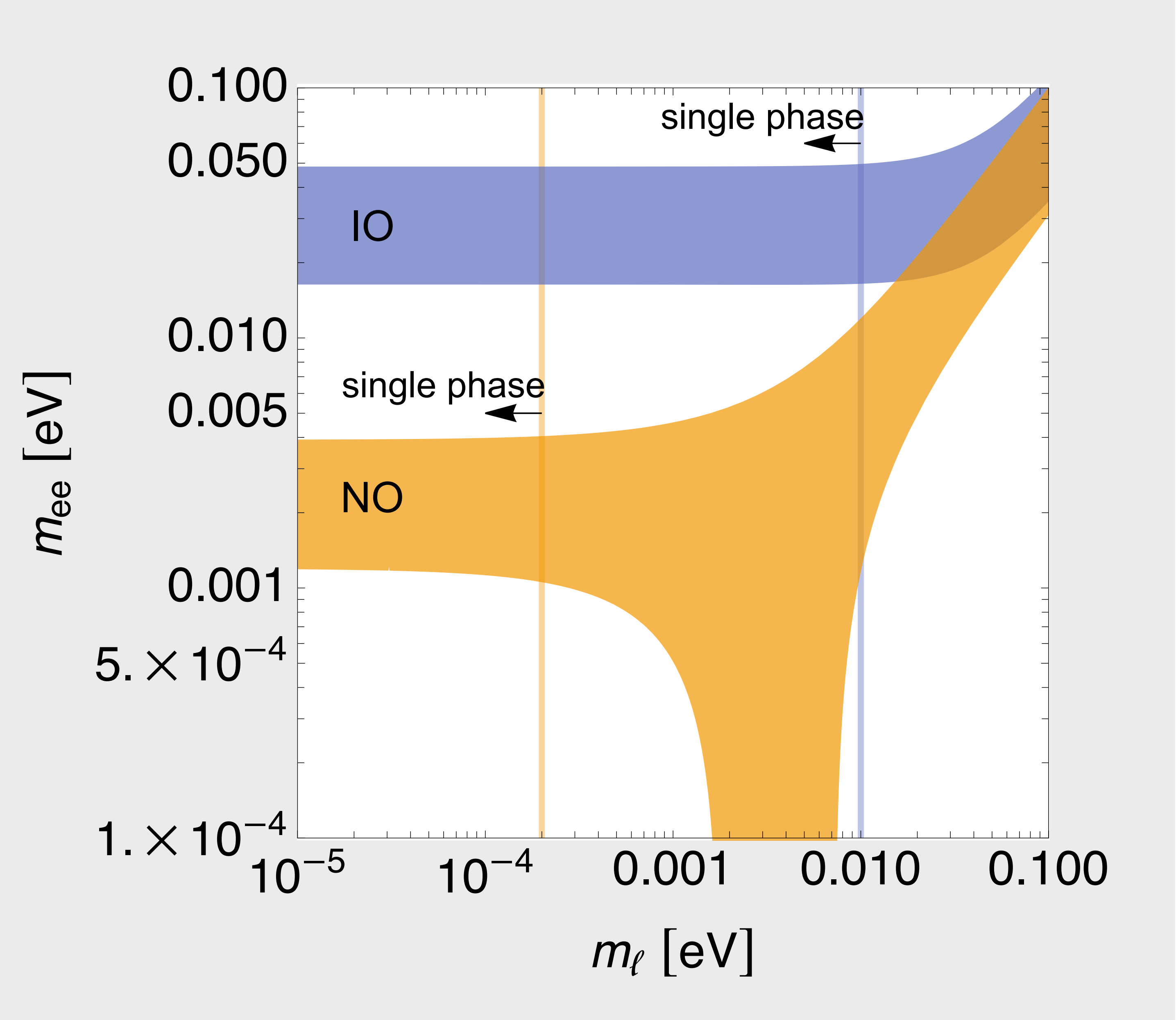
For hierarchical masses, $m_\ell \ll m_o$, one phase term dominates, and we can write the approximate expression

$$m^2_{\alpha\beta} \approx m_2^2 |t_{\alpha 2\beta 2}| \left[1 + \frac{m_o^2}{m_2^2}\frac{|t_{\alpha o\beta o}|}{|t_{\alpha 2\beta 2}|} + 2\frac{m_o}{m_2}\frac{|t_{\alpha o\beta o}|^{1/2}}{|t_{\alpha 2\beta 2}|^{1/2}} \cos(2\Phi^{e}_{2o} - \delta^{\alpha e}_{2o} - \delta^{\beta e}_{2o}) \right]$$

Dependent on the same Majorana phase for all α, β

(assuming we know the Dirac phase from oscillation data)

The single phase limit



The corrections to this expression are of

$$\mathcal{O}\left(\max\left[\frac{m_\ell}{m_2}\frac{|t_{\alpha\ell\beta 2}|}{|t_{\alpha 2\beta 2}|},\frac{m_\ell m_o}{m_2^2}\frac{|t_{\alpha\ell\beta o}|}{|t_{\alpha 2\beta 2}|}\right]\right)$$

For given $\{\alpha,\beta\}$, this gives us the precision needed in order to be sensitive to the sub-leading Majorana phase.

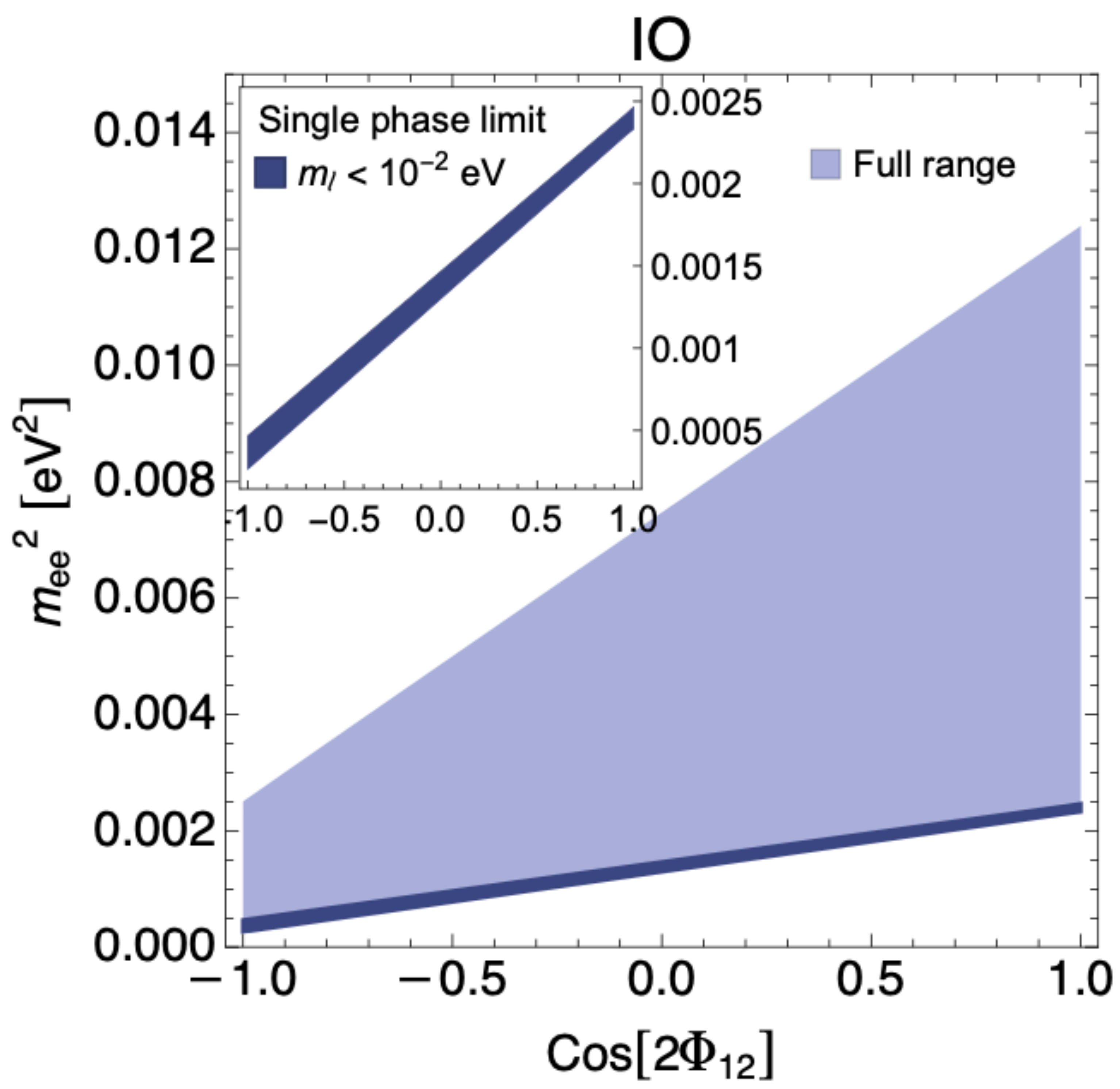
$$m_{\alpha\beta}^2 \approx m_2^2|t_{\alpha 2\beta 2}|\left[1+\frac{m_o^2}{m_2^2}\frac{|t_{\alpha o\beta o}|}{|t_{\alpha 2\beta 2}|}+2\frac{m_o}{m_2}\frac{|t_{\alpha o\beta o}|^{1/2}}{|t_{\alpha 2\beta 2}|^{1/2}}\cos(2\Phi_{2o}^{e}-\delta_{2o}^{\alpha e}-\delta_{2o}^{\beta e})\right]$$

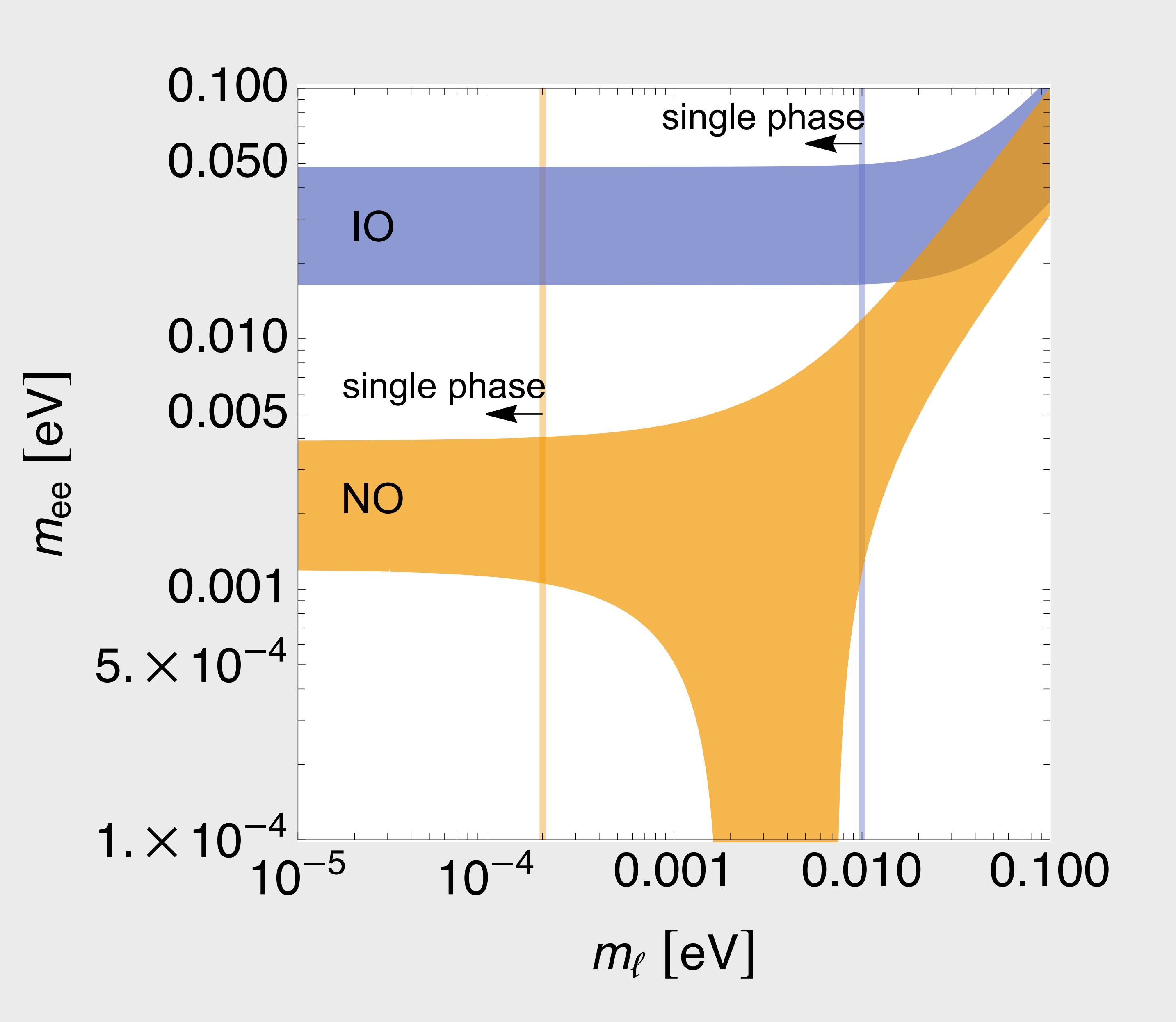
Dependent on the same Majorana phase for all α,β

(assuming we know the Dirac phase from oscillation data)

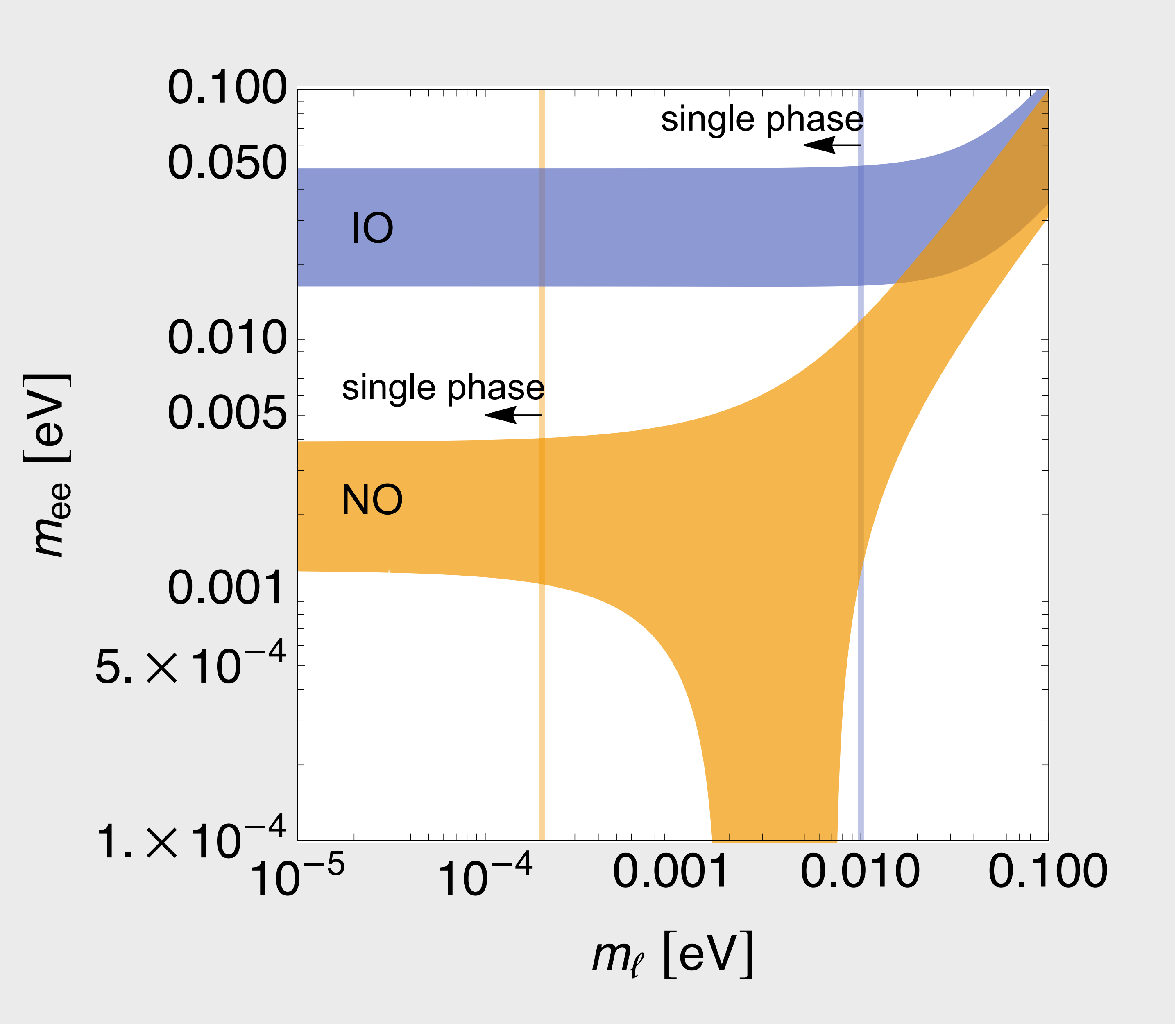
The single phase limit

In this limit ($m_\ell \ll m_o$), a measurement of any entry of $m_{\alpha\beta}$ (e.g., m_{ee}) is a measurement of $\cos(2\Phi^e_{2o})$



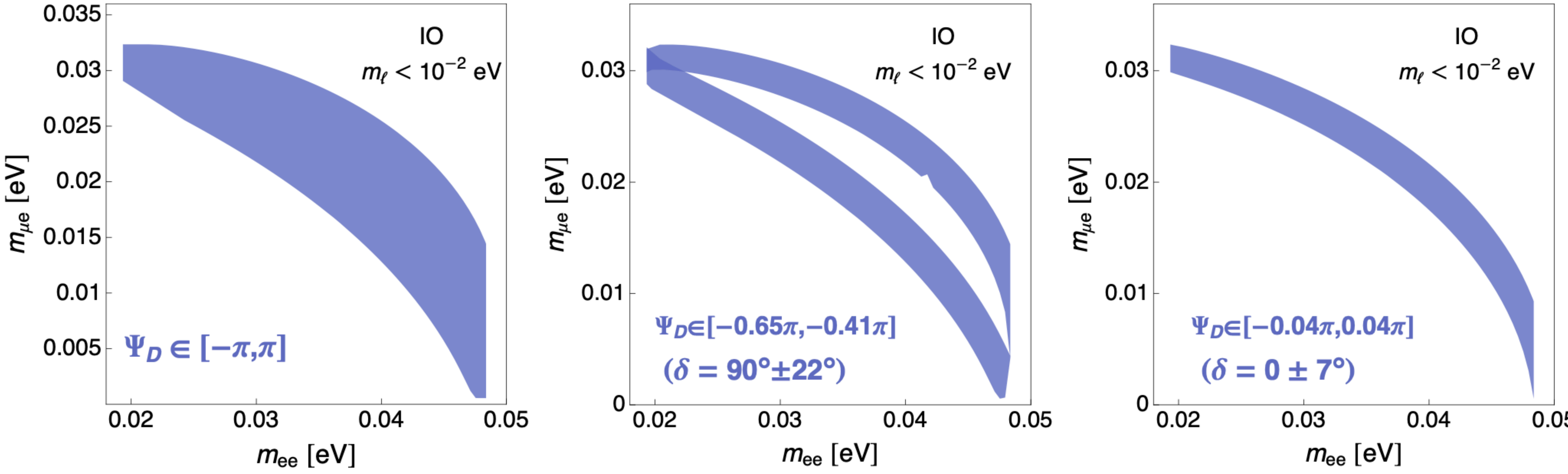


The single phase limit



In this limit, only one of the six entries of $m_{\alpha\beta}$ is independent.

=> e.g. knowledge of m_{ee} fixes $m_{\mu e}$



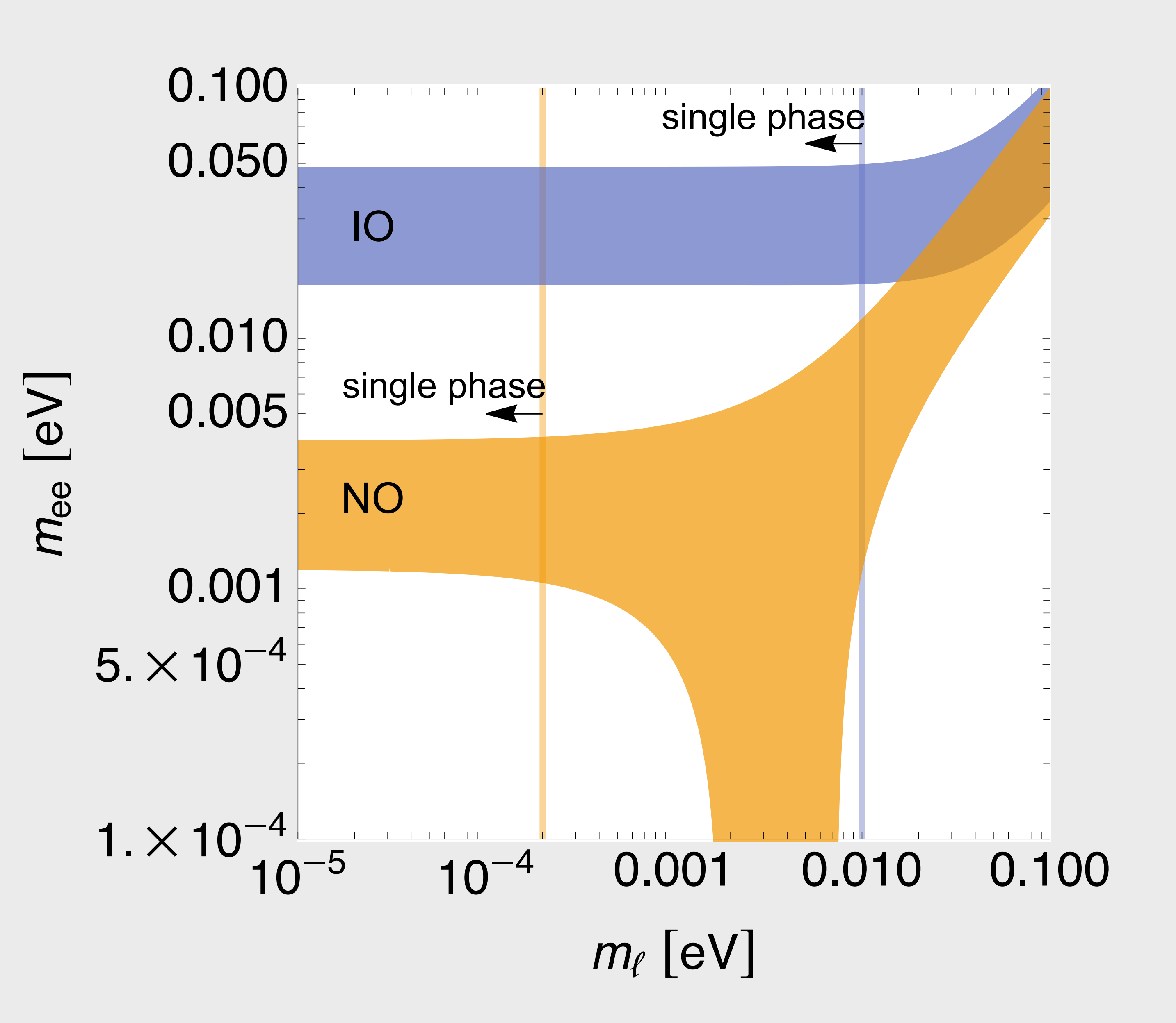
(a) (b) (c)

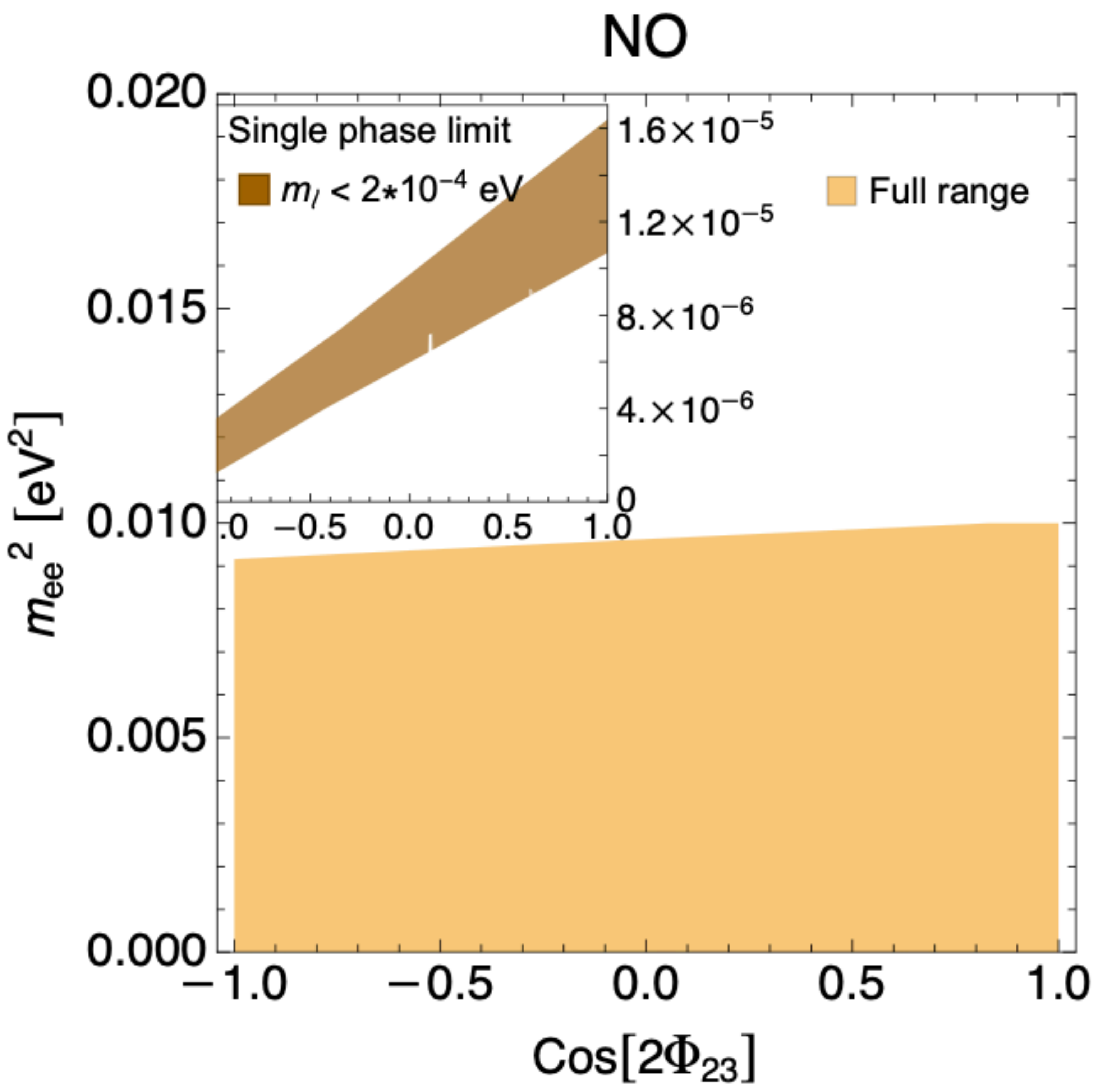
Assuming no knowledge of δ

Assuming we know δ, for two representative values

The single phase limit, NO

In this limit ($m_\ell \ll m_o$), a measurement of any entry of $m_{\alpha\beta}$ (e.g., m_{ee}) is a measurement of $\cos(2\Phi^{e}_{2o})$





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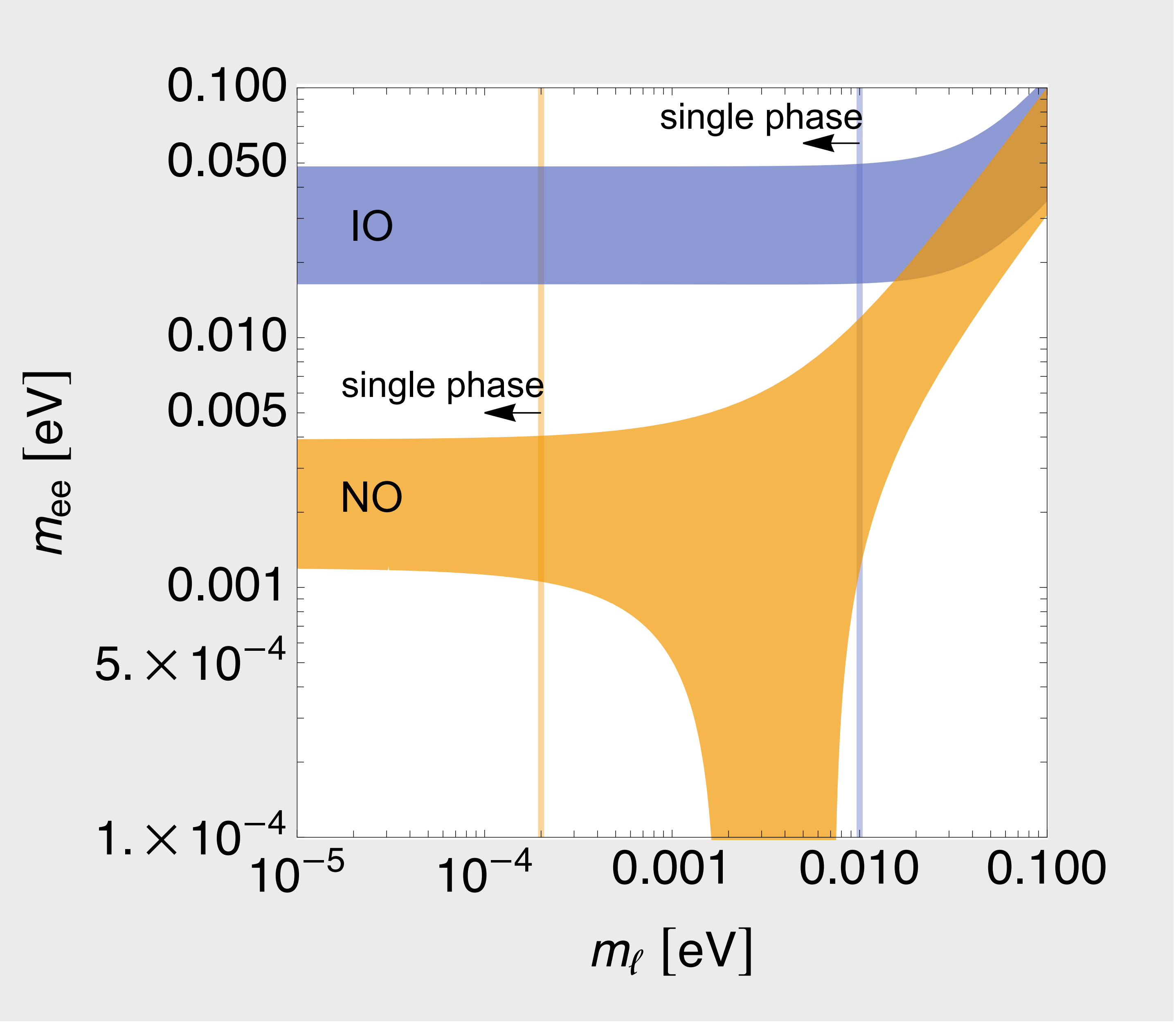
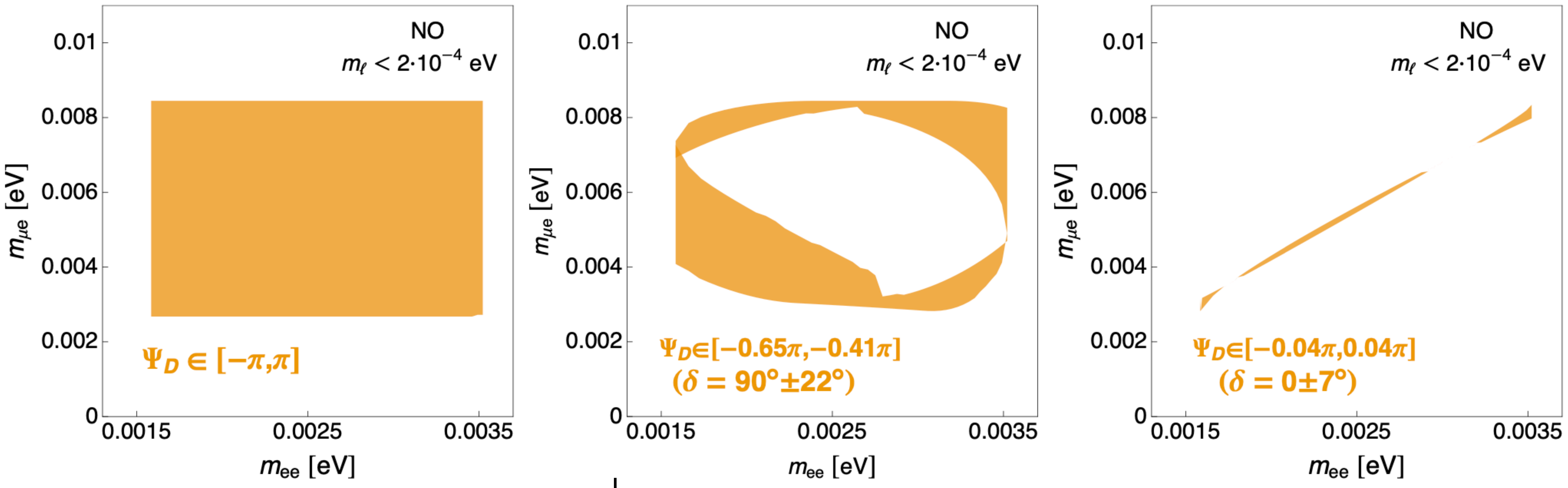


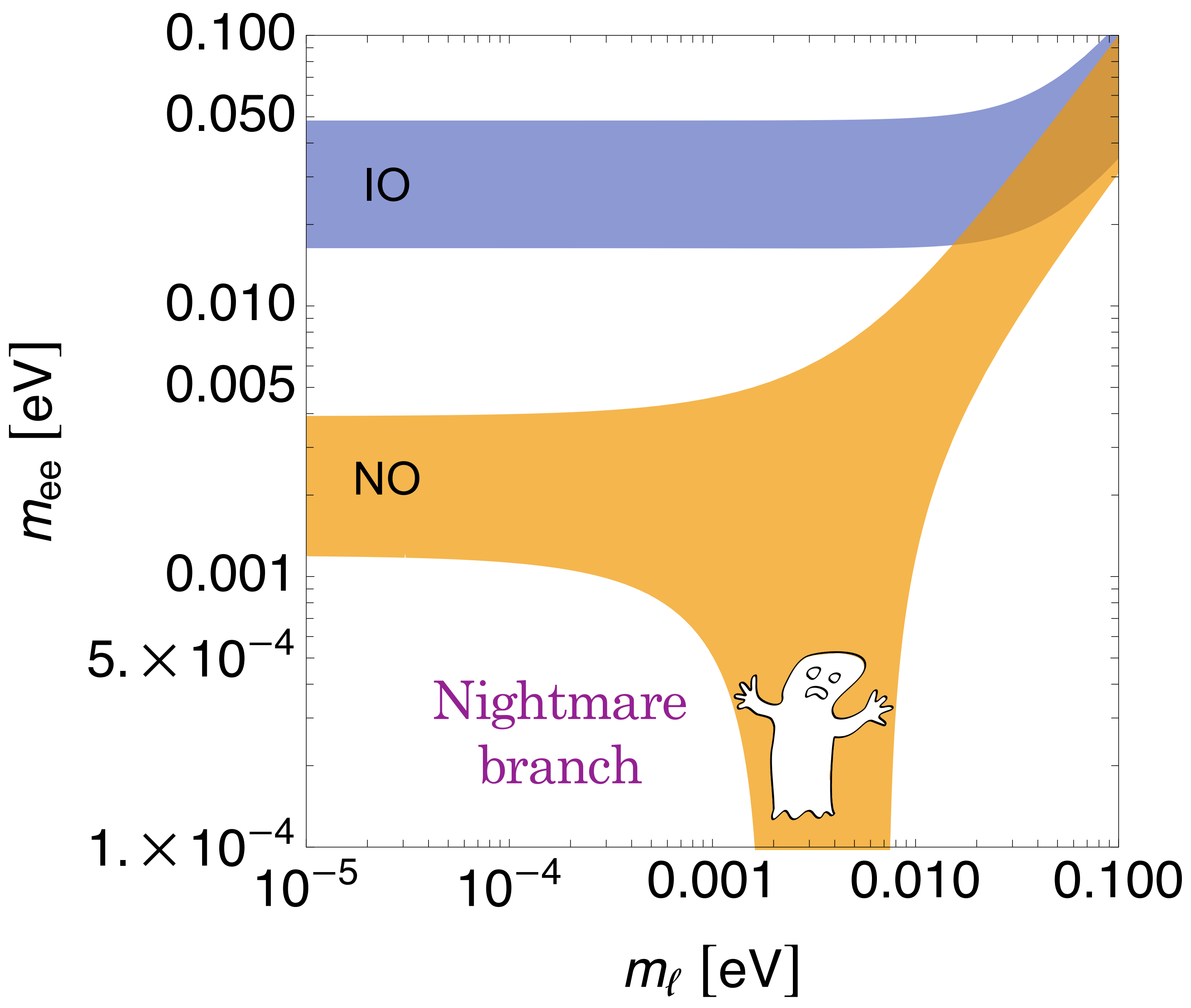


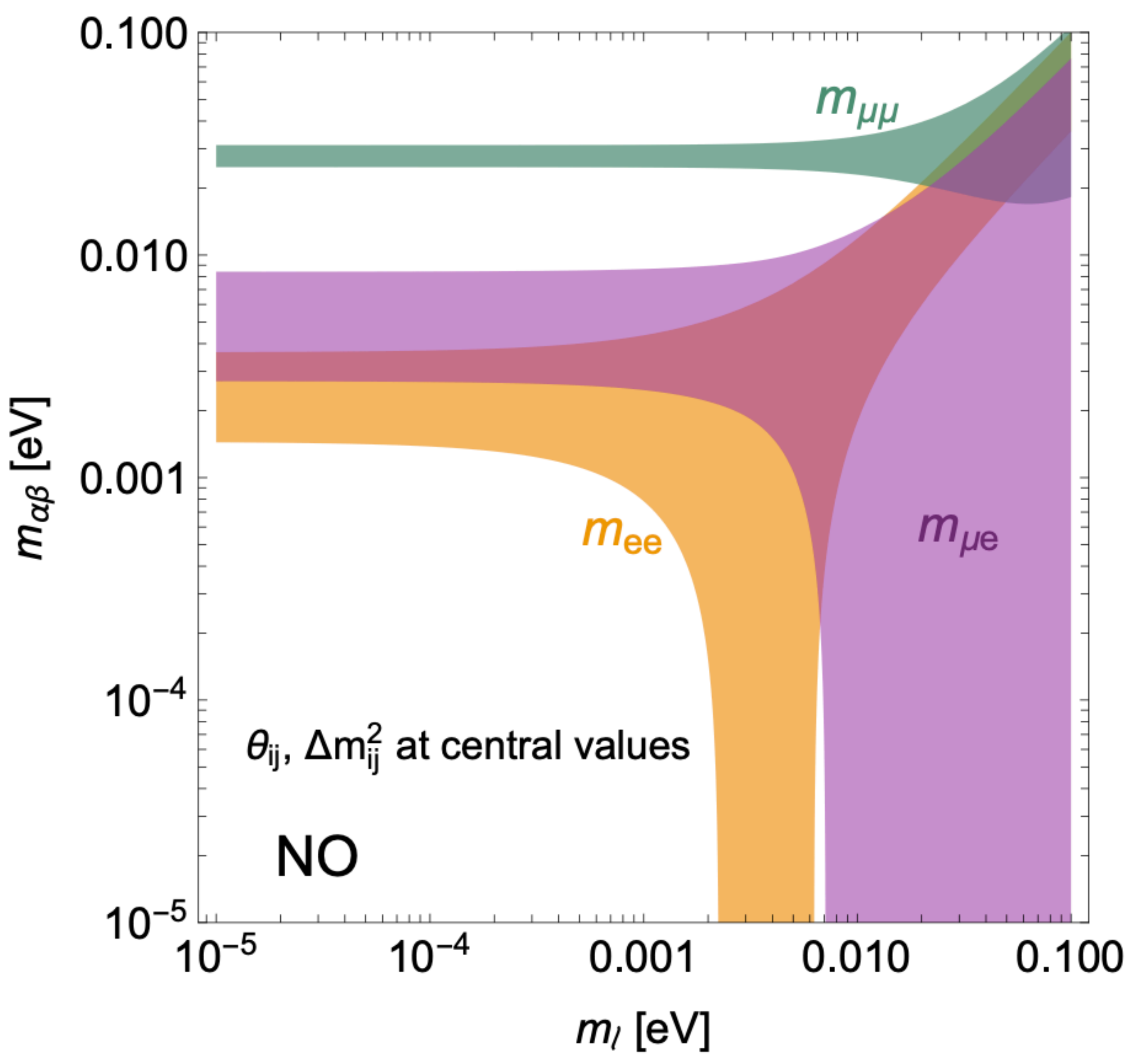
(d) (e) (f)

Assuming no knowledge of δ

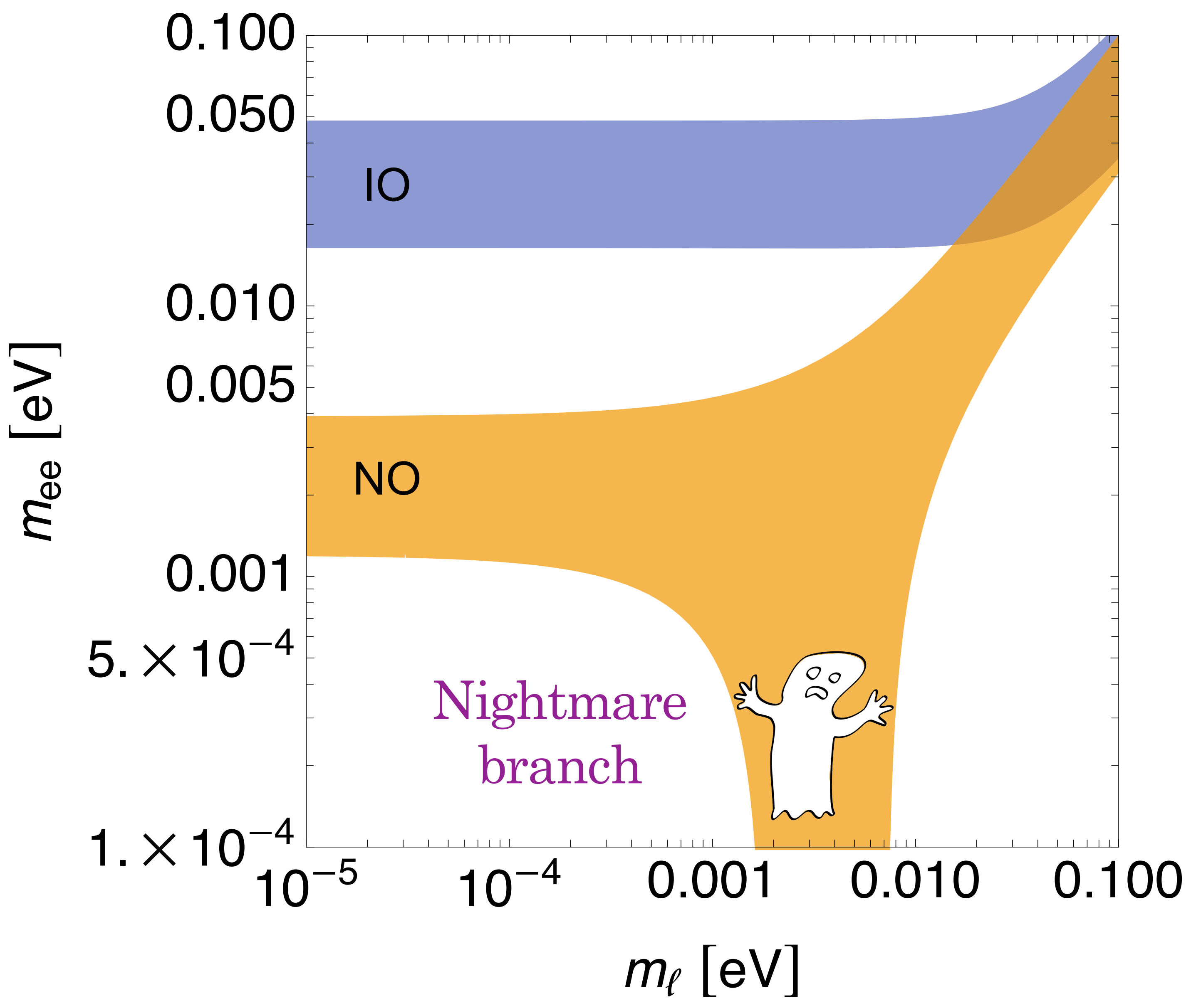
Assuming we know δ, for two representative values

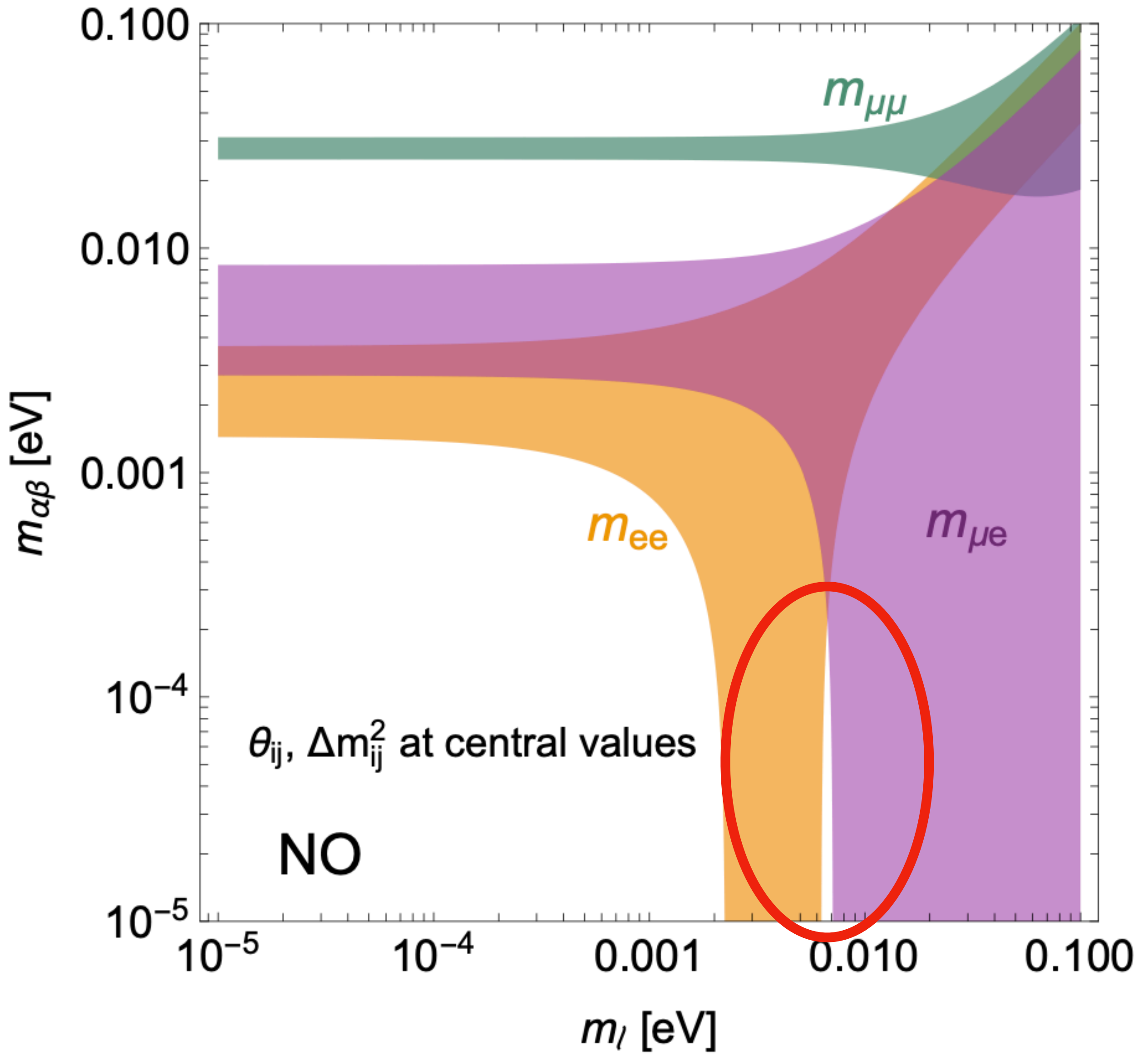
A no-lose proposition





A no-lose proposition

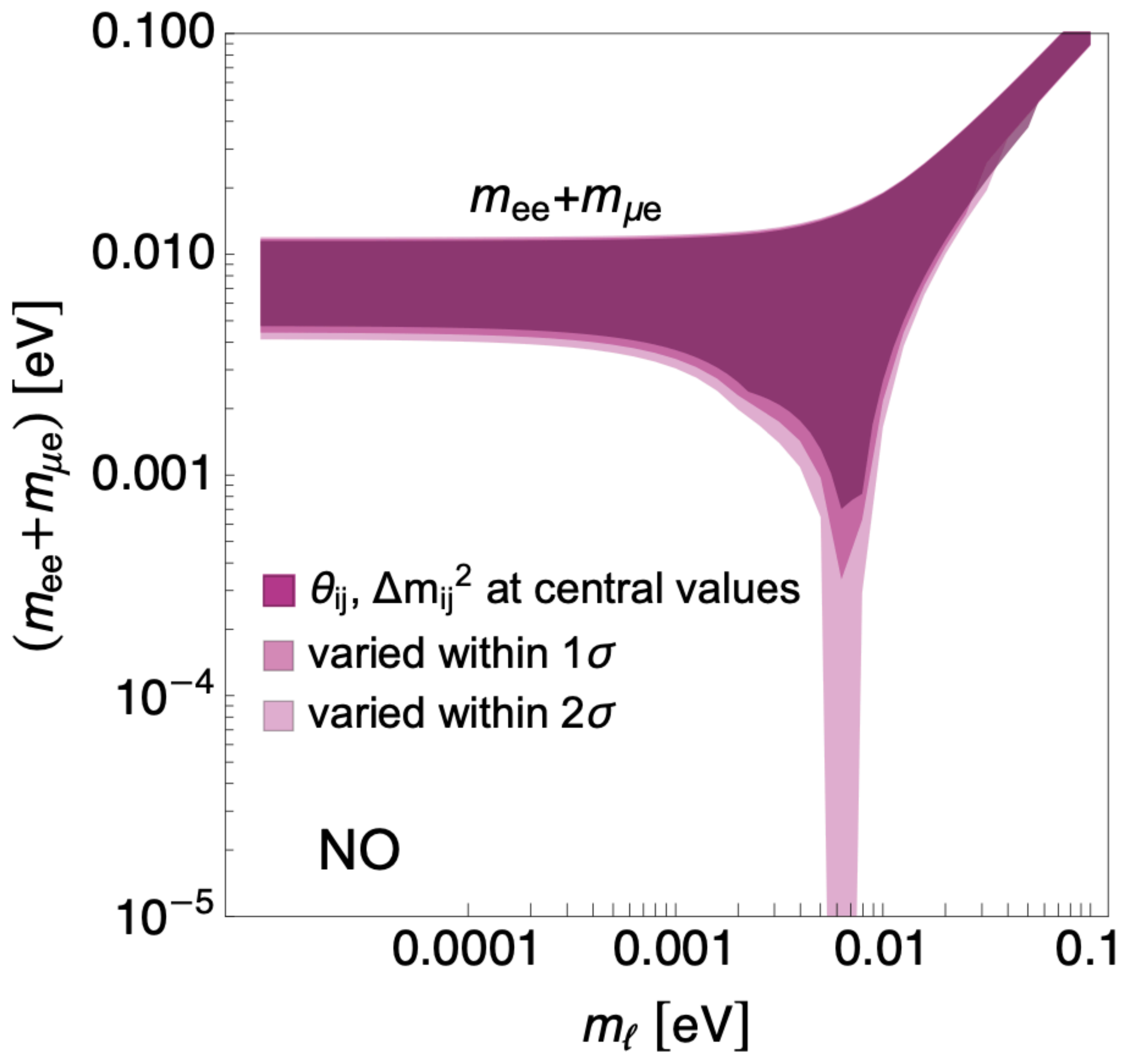




Neither $m_{\mu e}$ nor $m_{\mu\mu}$ vanish together with m_{ee}, for current central values of the oscillation parameters.

In principle, a measurement (or non-measurement) of two of them with the required sensitivity is able to exclude the ν_MSM, or discover LNV.

A no-lose proposition



This picture will be impacted by further precision on oscillation parameters!

Currently, expected sensitivity to $m_{\mu e}^2$ (Mu2e, COMET) is orders of magnitudes away from ν_MSM sensitivity (10^{-16} vs. 10^{-40})

Crazy ideas on how to probe $m_{\mu e}$??

Aloni, **AD** [arXiv:2211.09638]

Revisiting Leptonic non-unitarity

$$\begin{pmatrix} \begin{pmatrix} \text{LMM} \\ \end{pmatrix}_{3\times 3} & \cdots \\ \vdots & \end{pmatrix}$$

Motivation

1. IceCube has measured astrophysical tau neutrinos
2. Collider ν experiments (FASERν, SND@LHC) started running, expected to see $\mathcal{O}(100)$ ν_τ events

What can we learn about τ elements?
Could we have $\mathcal{O}(1)$ deviation from τ-row unitarity?

In analogy - CKM non-unitarity

$$-\frac{g}{\sqrt{2}}\begin{pmatrix}\bar{u}_L & \bar{c}_L & \bar{t}_L\end{pmatrix}\begin{pmatrix} & & \\ & CKM & \\ & & \end{pmatrix} W_\mu^+ \gamma^\mu \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

In analogy - CKM non-unitarity

β decay $\quad K \to \pi \ell \nu \quad B \to \pi \ell \nu$

$$-\frac{g}{\sqrt{2}} \begin{pmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} W^+_\mu \gamma^\mu \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

$D \to \pi \ell \nu$

D_s decays $\quad t \to Wb \quad B \to DX$

$$N_u \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985 \pm 0.0007$$

$$|1 - N_c| < 0.025 \quad @\ 2\sigma$$

$$|1 - N_t| < 0.089 \quad @\ 2\sigma$$

Deviations from unitarity of at most $\mathcal{O}(10\%)$

What about the PMNS?

$$-\frac{g}{\sqrt{2}}\left(\bar{e}_L \quad \bar{\mu}_L \quad \bar{\tau}_L\right)\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} W_\mu^+ \gamma^\mu \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Cannot measure neutrino mass eigenstates

(unlike the quark case - quarks hadronize before they oscillate)

$$\cancel{\tau \to \nu_1 \bar{u} d}$$

What about the PMNS?

$$-\frac{g}{\sqrt{2}}\begin{pmatrix}\bar{e}_L & \bar{\mu}_L & \bar{\tau}_L\end{pmatrix}\begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3}\\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}\end{pmatrix} W_\mu^+ \gamma^\mu \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

Oscillation experiments measure combinations of elements, e.g.

$$P(\nu_\mu \to \nu_\tau) = 4\,|U_{\mu 3}|^2\,|U_{\tau 3}|^2 \sin^2\left(\frac{\Delta m_{32}^2 L}{4E}\right)$$

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$\boxed{U_{\tau 1}\,,\ U_{\tau 2}\,,}$ difficult to directly constrain (no ν_τ beam)

What about the PMNS?

$$-\frac{g}{\sqrt{2}}\begin{pmatrix}\bar{e}_L & \bar{\mu}_L & \bar{\tau}_L\end{pmatrix}\begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3}\\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}\end{pmatrix} W^+_\mu \gamma^\mu \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

Our a-priori naive expectation (backed by the literature):

$\{|U_{ei}|\}$ elements well measured => fix N_e

$\{|U_{\mu i}|\}$ elements somewhat constrained => N_μ constrained

$|U_{\tau 3}|$ constrained by $\nu_\mu \to \nu_\tau$ => N_τ unconstrained

$$N_\alpha \equiv \sum_i |U_{\alpha i}|^2$$

Define scenario: non-unit. from kinematically inaccessible states

Extra ν states exist, kinematically inaccessible, $M \gg m_W$, resulting in effective low-E non-unitarity.

$$\left(\begin{array}{cc} \left(\ \text{LMM} \ \right) & \cdots \\ \vdots & \end{array} \right)$$

=> Low energy processes are affected, e.g.,

$$\pi^+ \to \mu^+ \nu_\mu \qquad |\nu_\mu\rangle^{\text{eff.}} = \frac{1}{\sqrt{N_\mu}} \sum_{i=1}^{3} U^*_{\mu i} |\nu_i\rangle$$

$$\Gamma(\pi \to \mu\nu) = N_\mu \cdot \Gamma(\pi \to \mu\nu)^{\text{SM}}$$

Rates of processes with an external neutrino are modified by a normalization factor

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Rates of processes with an external neutrino are modified by a normalization factor

=> non-orthogonality, Zero distance effect

$$\langle \nu_\beta | \nu_\alpha \rangle = \underbrace{\sum_{i=1}^{3} U^*_{\alpha i} U_{\beta i}}_{t_{\alpha\beta}} \neq 0$$

non-closure factors

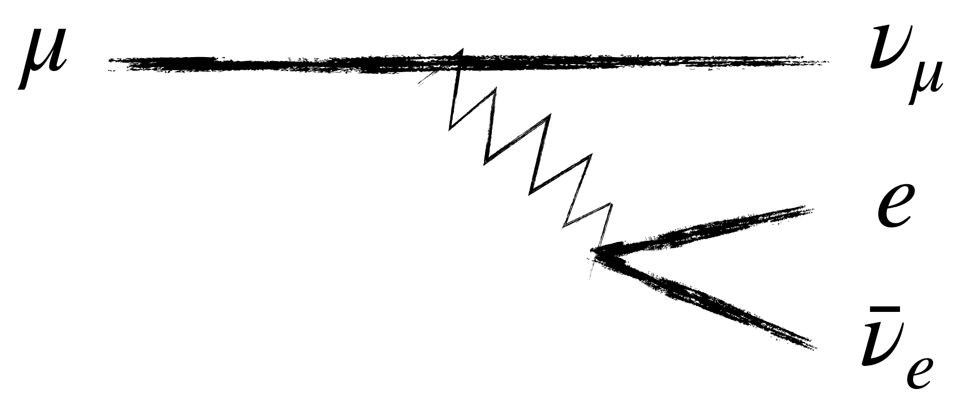
NOMAD experiment $\quad \frac{|t_{\mu e}|^2}{N_\mu N_e} \lesssim 10^{-3} \quad \frac{|t_{\mu\tau}|^2}{N_\mu N_\tau} \lesssim 10^{-4} \quad \frac{|t_{e\tau}|^2}{N_e N_\tau} \lesssim 10^{-2}$

Cauchy-Schwarz $\quad |t_{\alpha\beta}|^2 \leq (1 - N_\alpha)(1 - N_\beta)$

Indirect constraints on N_α

[Antusch and Fischer, *JHEP* **10** (2014) 094]

G_F measured in muon decay



$\propto N_\mu N_e$

$$G_\mu^2 = N_e N_\mu \, G_F^2$$

Compare to LHC measurements of $\sin^2\theta_W$ in FB asymmetries

$$Z \to f\bar{f}$$

(no ν's here)

$$\sin^2\theta_W \cos^2\theta_W = \frac{\alpha(m_Z)\,\pi}{\sqrt{2}\,G_F\,m_Z^2}$$

$$\Rightarrow \quad \sqrt{N_e N_\mu} = 1.0004 \pm 0.0007$$

N_τ ?

Lepton non-universality (LFU) constraints

$$\frac{\Gamma(\tau \to \mu \nu_\tau \bar{\nu}_\mu)}{\Gamma(\tau \to e \nu_\tau \bar{\nu}_e)} \propto \frac{N_\mu}{N_e}$$

$$\frac{\Gamma(\mu \to \mu \nu_\mu \bar{\nu}_\mu)}{\Gamma(\tau \to e \nu_\tau \bar{\nu}_e)} \propto \frac{N_e}{N_\tau}$$

$$\vdots$$

$$\frac{N_\mu}{N_e} = 1.004 \pm 0.003$$

$$\frac{N_\tau}{N_e} = 1.006 \pm 0.003$$

$$\frac{N_\tau}{N_\mu} = 0.999 \pm 0.003$$

=> $N_e \approx N_\mu \approx N_\tau \equiv N_{\text{univ.}}$ Up to $\mathcal{O}(10^{-3})$

LFU + weak angle

Without any input from ν experiments, the combination of these two constraints leads to

$$|1-N_e| < 0.005\,, \qquad |1-N_\mu| < 0.001\,, \qquad |1-N_\tau| < 0.002 \qquad @\ 2\sigma$$

$$|t_{\alpha\beta}|^2 \leq (1-N_\alpha)(1-N_\beta) \lesssim 10^{-6}\,, \qquad \alpha,\beta = e,\mu,\tau$$

$$\sum_i N_i = \sum_\alpha N_\alpha \;\to\; (1-N_i) \lesssim 10^{-3}, \qquad i = 1,2,3$$

$\mathcal{O}(10^{-3})$, better than CKM.

$\downarrow$

$\mathcal{O}(0.1)$

Fairly robust bound, hard to evade.

How well can oscillation exp.'s do?

$$P_{\alpha\beta}: \quad |U_{\alpha i}|^2 \to \frac{|U_{\alpha i}|^2}{N_\alpha}$$

The oscillation probability is only sensitive to the normalized magnitudes of elements, $\{|U_{\alpha i}|^2/N_\alpha\}$, and <u>not to the overall normalization itself</u>.

$$n_{\nu_\beta} = \Phi_{\nu_\alpha} \cdot P_{\alpha\beta} \cdot \sigma_{\nu_\beta}$$

According to the literature, the sensitivity to $\{N_\alpha\}$ comes from the normalization of fluxes and cross sections.

How well can oscillation exp.'s do?

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$$\Gamma(\pi \to \mu\nu) \;=\; N_\mu \cdot \Gamma(\pi \to \mu\nu)^{\text{SM}}$$

Rates of processes with an external neutrino are modified by a normalization factor, with respect to the SM

However, in practice, SM predictions often involve measured inputs.

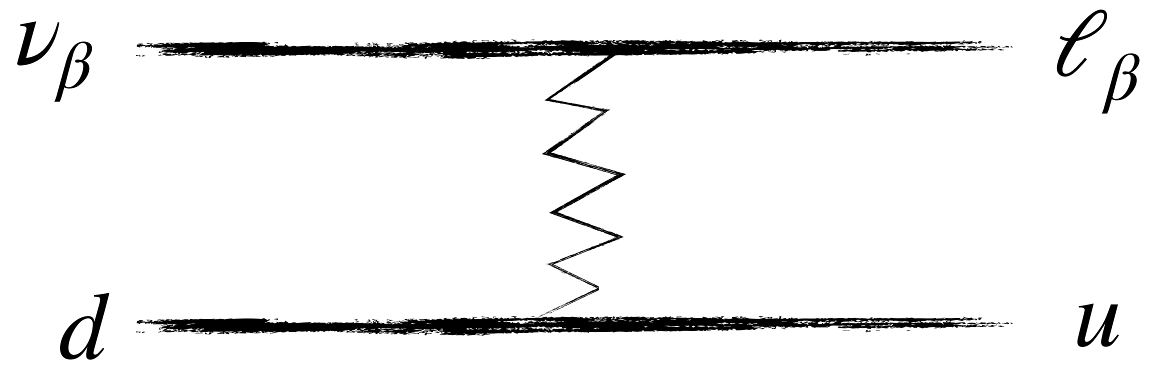
e.g., if one uses as input the pion decay constant, f_π, from a measurement of $\pi \to \ell\nu$, then there is no need to normalize (no sensitivity to N)

(Recurring mistake in the leptonic non-unit. literature)

Deep Inelastic Scattering predictions are also contaminated.

$$n_{\nu_\beta} = \Phi_{\nu_\alpha} \cdot P_{\alpha\beta} \cdot \sigma^{\mathrm{DIS}}_{\nu_\beta}$$

High energy, no need for hadronic input



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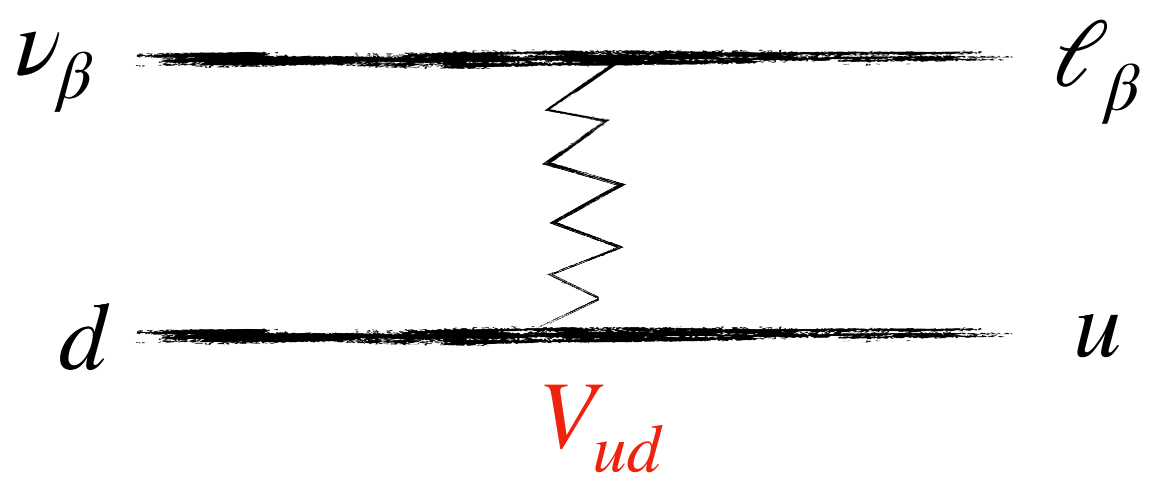
High energy, no need for hadronic input

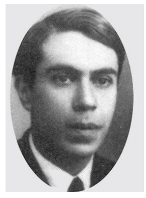
BUT, $|V_{ud}|$ is measured in beta decays, involving a neutrino in the final state (and G_μ).



=> Again, no need to normalize, meaning that, in general, there is <u>no sensitivity of oscillation experiments to $\{N_\alpha\}$.</u> (Matter effects do not change this conclusion)

This is contrary to current literature, including the DUNE physics book, which claims $\mathcal{O}(10\%)$ sensitivity to $(1-\alpha_{ee})^2 = N_e$.

Summary and conclusions

For $\Delta L = 2$ processes within the ν_MSM: 

1. For small lightest neutrino mass, $m_\ell \ll m_o$, there is approximately only one independent element of the symmetric matrix $m_{\alpha\beta}$

2. The sensitivity to the subleading Majorana phase scales as m_ℓ (and requires a second observable in order to disentangle)

3. If future progress allows to reach ν_MSM sensitivity to $m_{\mu e}$, the Majorana nature of neutrinos could be ruled out by non observation of both m_{ee} and $m_{\mu e}$ (no-lose proposition). Precision measurement of oscillation parameters will impact this.

Summary and conclusions

For an extended leptonic mixing matrix (kinematically inaccessible states):

1. The combination of LFU + weak angle measurements places a strong constraint of $\mathcal{O}(10^{-3})$ on non-unit. of the 3×3 LMM, surpassing analogous CKM constraints by two orders of magnitude.

2. Oscillation experiments are insensitive to $\{N_\alpha\}$. The focus (in terms of non-unit.) should be on improving the NOMAD bounds on the non-closure factors, $\{t_{\alpha\beta}\}$.

3. Measured inputs to SM predictions should be treated with care.

Thank you for your attention!