

BMS particles for an infrared-finite S-matrix

[discussion session]

Based on 2412.06002 with
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Laura DONNAY

Infrared Surprises of Scattering Amplitudes

CERN

15 May 2025

SISSA



Definition:

BMS particle = unitary irreducible representation of the BMS_4 group



asymptotic symmetry group of 4d
asymptotically flat spacetimes

* the name **BMS particle** was introduced in [Barnich, Oblak '14 '15] for BMS_3

Motivation

Why BMS representation theory?

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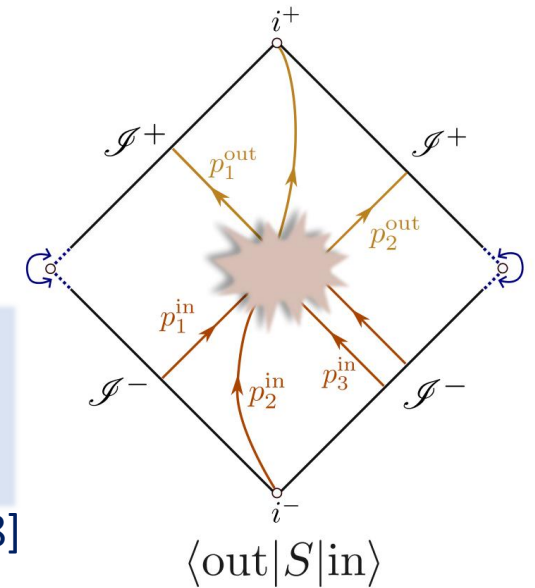
Why BMS representation theory?

- **The gravitational S-matrix is BMS invariant!** [Strominger '14]

The equivalence between BMS and **soft theorems** implies that

IR divergences appear precisely because the usual S-matrix elements fail to conserve BMS charges

[Kapec, Perry, Raclariu, Strominger '17][Akhoury, Choi '18]



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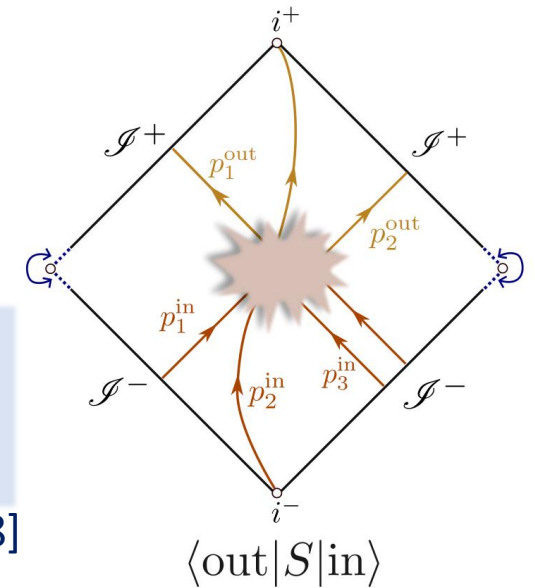
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Pushing this idea further:
states of the **S-matrix** should be **BMS states!**



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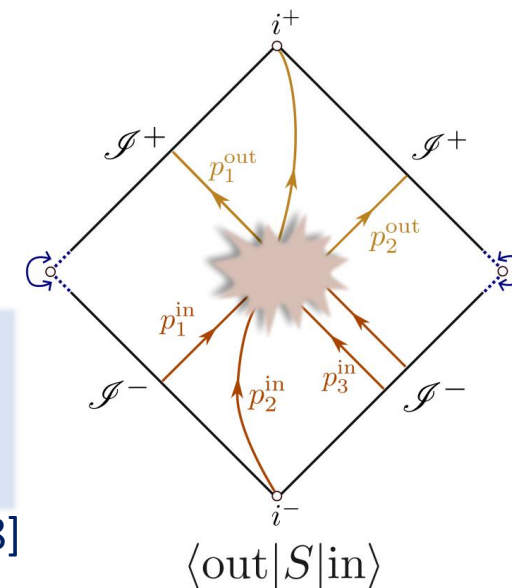
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- **Flat space holography?**

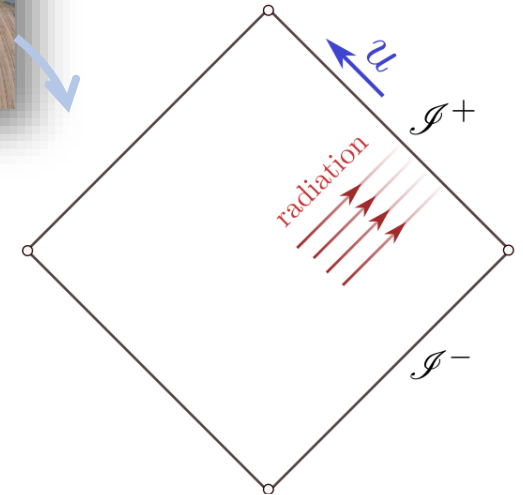
AdS/CFT duality : bulk states and boundary operators organize into the same UIRs.

Maybe worth understanding the analogue statement for UIRs in flat space !



BMS symmetries

BMS group = symmetry group of 4D asymptotically flat spacetimes



4 Poincaré translations
 ↓ Symmetry enhancement
 ∞ BMS supertranslations

$$\begin{aligned}
 ds^2 = & -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}} dzd\bar{z} \\
 & + \frac{2M}{r} du^2 + rC_{zz} dz^2 + D^z C_{zz} dudz \\
 & + \frac{1}{r} \left(\frac{4}{3}(N_z + u\partial_z m_B) - \frac{1}{4}\partial_z(C_{zz}C^{zz}) \right) dudz + c.c. + \dots
 \end{aligned}$$

what was expected

what was found



Poincaré



Bondi-Metzner-Sachs (BMS) ('62)
 + van der Burg

BMS group

$$BMS_4 \simeq \mathcal{E}[1](S^2) \rtimes SO(3, 1)$$

supertranslations = weight-1 densities on the celestial sphere
 $u \rightarrow u + \mathcal{T}(z, \bar{z})$

Important remarks on the BMS group

- The Poincaré group sits inside BMS:

$$ISO(3, 1) \simeq \mathbb{R}^4 \rtimes SO(3, 1) \subset BMS_4 \simeq \mathcal{E}[1](S^2) \rtimes SO(3, 1)$$

However, the inclusion is not unique

→ **Infinitely many** non-equivalent Poincaré groups inside BMS

[Ashtekar, 1984]

choice of $ISO(3, 1) \subset BMS_4$ = choice of “gravity vacuum”

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- But, there is a canonical inclusion of translations into **supertranslations**

$$q_\mu \left| \begin{array}{ll} \mathbb{R}^{3,1} & \hookrightarrow \mathcal{E}[1] \\ T^\mu & \mapsto \mathcal{T}(z, \bar{z}) = T^\mu q_\mu(z, \bar{z}) \end{array} \right.$$

$$q^\mu(z, \bar{z}) = (1 + |z|^2, z + \bar{z}, -i(z - \bar{z}), 1 - |z|^2)$$

IR divergences: a way out?

The equivalence between BMS and **soft theorems** implies that

Infrared divergences arise due to the impossibility for the usual S-matrix elements to conserve BMS charges



The notion of asymptotic states needs to be improved to obtain infrared finite amplitudes

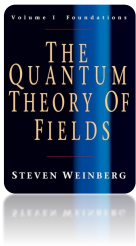
Faddeev — Kulish (1970), Kapec — Perry — Raclariu — Strominger (2017)

Choi—Akhoury (2017), Prabhu—Satishchandran—Wald (2022).

“soft gravitons need to be added to ensure conservation of charges”

Taking this very seriously → BMS particles

BMS UIRs

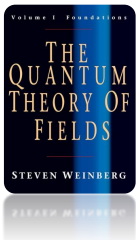


Poincaré

$$(T^\mu, M^{\mu\nu}) \in \mathbb{R}^4 \rtimes SO(3, 1)$$

1. Define momenta as dual to translations

$$p_\mu \in (\mathbb{R}^4)^* \quad \langle p, T \rangle := p_\mu T^\mu$$



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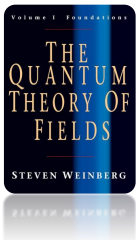
$$p_\mu \in (\mathbb{R}^4)^* \quad \langle p, T \rangle := p_\mu T^\mu$$

2. Fix an **orbit** of the Lorentz group $\mathcal{O}_k \subset (\mathbb{R}^4)^*$

In practice fix $k_\mu \in (\mathbb{R}^4)^*$ and consider

$$p_\mu := k_\nu M^\nu{}_\mu \in \mathcal{O}_k \simeq \frac{SO(3, 1)}{\ell_k}$$

ℓ_k	$SU(2)$	$ISO(2)$	$SL(2, \mathbb{R})$	$SO(3, 1)$
\mathcal{O}_k	H^3	$\mathbb{R} \times S^2$	dS_3	$\{0_\mu\}$



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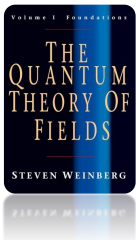
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3. Poincaré particle = wavefunction with support on a given orbit of the Lorentz group
+ spin (UIR of little group)

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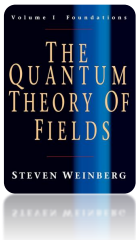
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BMS

$$(\mathcal{T}(z, \bar{z}), M^{\mu\nu}) \in \mathcal{E}[1](S^2) \rtimes SO(3,1)$$

Define **supermomenta** as dual to **supertranslations**

$$\mathcal{P}(z, \bar{z}) \in (\mathcal{E}[1])^* \sim \mathcal{E}[-3] \quad \langle \mathcal{P}, \mathcal{T} \rangle := \int d^2z \mathcal{P} \mathcal{T}$$



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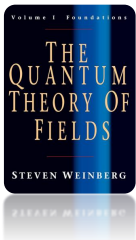
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[McCarthy] : there are **10** possible **little groups**

$\ell_\mathcal{K} = \text{stab}(\mathcal{K}(z, \bar{z}))$ with mass-shell of dimensions
 $\text{Dim}(\mathcal{O}_\mathcal{K}) \in \{0, 3, 4, 5, 6\}$



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Seminal papers

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But these works also raise more physics questions than they answer:
What do BMS states look like? Where is the (IR) physics?

hard BMS UIR

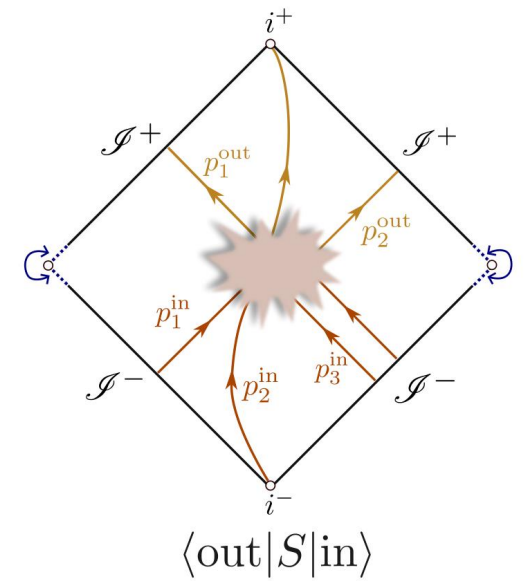
Hard BMS UIRs

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$$\Phi(X) = \int \frac{d^3 p}{(2\pi)^3 2p^0} \left[a(p) e^{ip \cdot X} + a(p)^\dagger e^{-ip \cdot X} \right]$$



Hard BMS UIRs

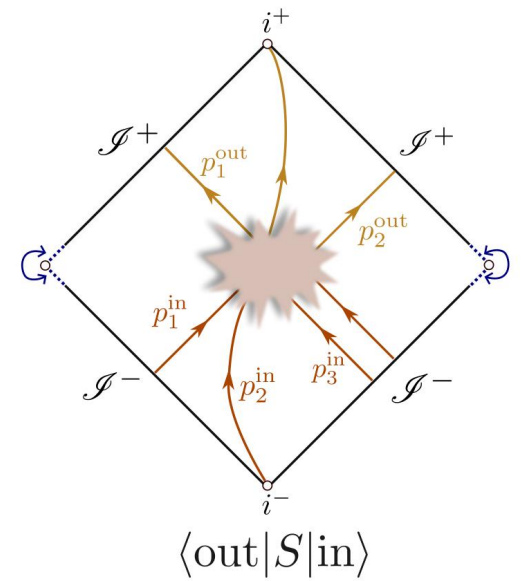
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- massless scalar near \mathcal{I}

$$\phi(u, \zeta) \propto \int_0^{+\infty} d\omega [a(\omega, \zeta, \bar{\zeta})e^{-i\omega u} - a(\omega, \zeta, \bar{\zeta})^\dagger e^{i\omega u}]$$

$$a(p) \xrightarrow[BMS_4]{} e^{i\omega \mathcal{T}(\zeta, \bar{\zeta})} a(p)$$



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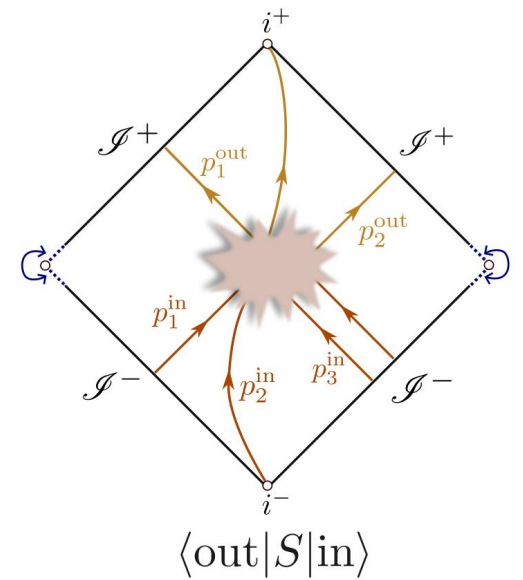
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$$a(p) \xrightarrow{BMS_4} e^{i\omega \mathcal{T}(\zeta, \bar{\zeta})} a(p)$$

- massive scalar near i^\pm

$$\phi(u, \mathbf{k}) \propto a(\mathbf{k})e^{-im\omega}$$

$$a(\mathbf{k}) \xrightarrow{} e^{-\int d^2 z \frac{\mathcal{T}(z, \bar{z})m^4}{(q(z, \bar{z}) \cdot p(\mathbf{k}))^3}} a(\mathbf{k})$$



Hard rep

$$a(p) \xrightarrow{BMS_4} e^{i\langle P\mathcal{T} \rangle} a(p)$$

Hard massless

$$P(z, \bar{z}) = \omega \delta^{(2)}(z - \zeta)$$

Hard massive

$$P(z, \bar{z}) = -\frac{m^4}{\pi} (q(z, \bar{z}) \cdot p)^{-3}$$

great but

... what about **soft stuff** ?

soft vs hard

Supermomenta decomposition

Key result

$$\mathcal{P}(z, \bar{z}) = \boxed{\partial_z^2 \partial_{\bar{z}}^2 \mathcal{N}} \quad \text{Soft part} \quad + \quad \boxed{P(z, \bar{z})} \quad \text{Hard part}$$

[Bekaert, LD, Herfray '24][Bekaert, Herfray '25]

Supermomenta decomposition

Key result

Soft part

Hard part

$$\mathcal{P}(z, \bar{z}) = \partial_z^2 \partial_{\bar{z}}^2 \mathcal{N} + P(z, \bar{z})$$

\simeq 'memory'

\simeq extra IR degrees of freedom

$$\mathcal{N}(z, \bar{z}) \in \mathcal{E}[1]/\mathbb{R}^{3,1}$$

\rightarrow massive case ($p^2 < 0$)

$$P(z, \bar{z}) = -\frac{m^4}{\pi} (q(z, \bar{z}) \cdot p)^{-3}$$

\rightarrow massless case ($p^2 = 0$) $p^\mu = \omega q^\mu(\zeta, \bar{\zeta})$

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Supermomenta decomposition

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Soft part Hard part

Important property:

Supermomenta can be projected to momenta via

$$\mathcal{P}(z, \bar{z}) \mapsto p_\mu = \int d^2 z \mathcal{P}(z, \bar{z}) q_\mu(z, \bar{z})$$

-> massive case ($p^2 < 0$)

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Supermomenta decomposition

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$$\mathcal{P}(z, \bar{z}) = \underbrace{\partial_z^2 \partial_{\bar{z}}^2 \mathcal{N}}_{\text{Soft part}} + \underbrace{P(z, \bar{z})}_{\text{Hard part}}$$

This decomposition is **unique**. It is **Lorentz invariant** and **nonlinear**.

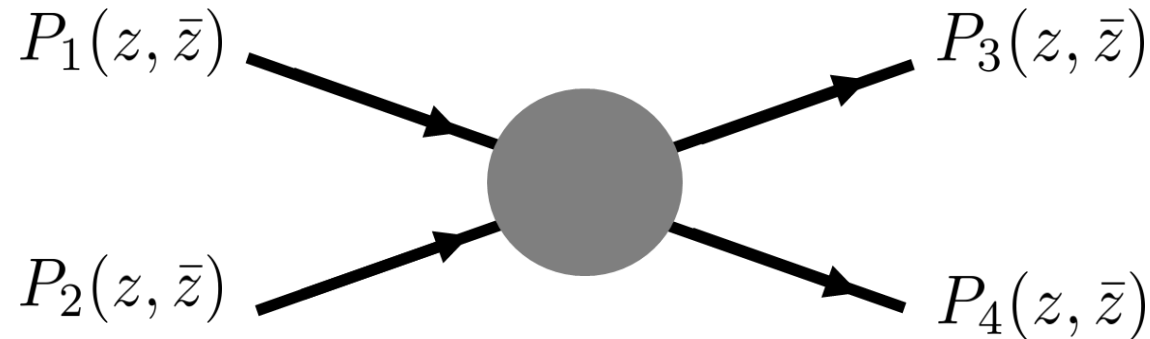
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Consequence:



Momentum conservation does not imply conservation of hard supermomentum:

$$p_1^\mu + p_2^\mu = p_3^\mu + p_4^\mu \quad P_1(z, \bar{z}) + P_2(z, \bar{z}) \neq P_3(z, \bar{z}) + P_4(z, \bar{z})$$

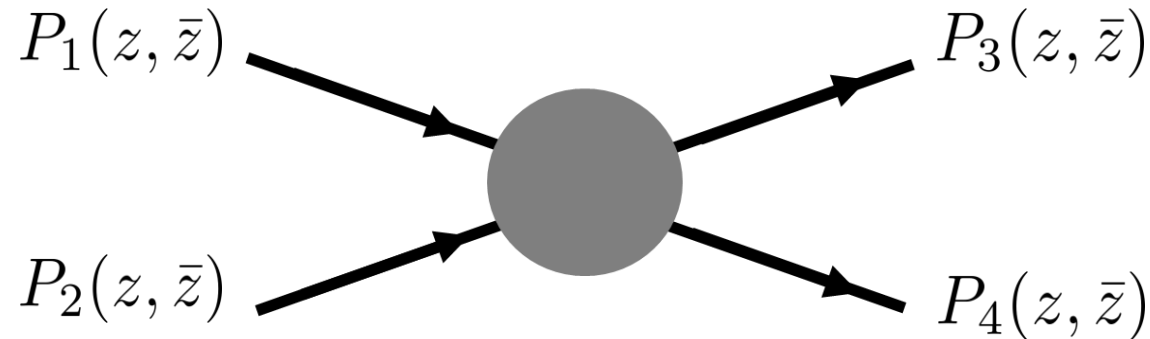
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[Bekaert, LD, Herfray '24][Bekaert, Herfray '25]

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This decomposition is **unique**. It is **Lorentz invariant** and **nonlinear**.

Consequence:



One is forced to consider non hard representations

Momentum conservation does not imply conservation of hard supermomentum:

$$p_1^\mu + p_2^\mu = p_3^\mu + p_4^\mu$$

$$P_1(z, \bar{z}) + P_2(z, \bar{z}) \neq P_3(z, \bar{z}) + P_4(z, \bar{z})$$

BMS wavefunctions

$$|\Psi\rangle = \int \mathcal{D}\mathcal{P} \Psi(\mathcal{P}) |\mathcal{P}\rangle$$

eigenstate of the supertranslations generator

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seems formal but the integrals*
are **finite-dimensional**

wavefunctions with support only on a given
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* see [Bekaert, Herfray '25] for explicit examples

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- All BMS particles can be given a physical interpretation:
They are **quantum superposition** of usual particles
over the space of gravity vacua.

* see [Bekaert, Herfray '25] for explicit examples

Conclusion

- Usual QFT asymptotic states are UIR BMS* representations: the *hard* ones

$$\mathcal{P}(z, \bar{z}) = P(z, \bar{z})$$

*here we focused on gravity but of course there is also a similar story for the gauge theory analogues

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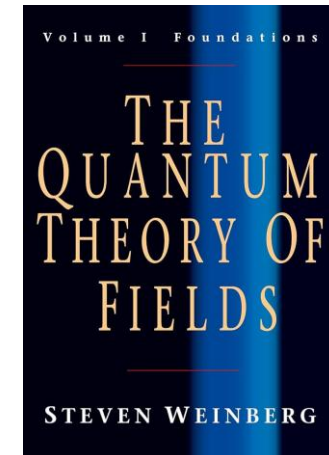
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This will necessarily require the notion of BMS particles.

[Bekaert, LD, Herfray '24]

[Bekaert, Herfray '25]

$$|\mathcal{P}\rangle = |p, \partial^2 \mathcal{N}\rangle$$



Asymptotic one-particle states should belong to UIRs of the BMS group

Multi-particle states should be tensor products thereof: $|p_1, \partial^2 \mathcal{N}_1\rangle \otimes \cdots \otimes |p_n, \partial^2 \mathcal{N}_n\rangle$

More material discussion

Contrasting with the literature

- **Dressed states** (“coherent states”, “attaching soft clouds”, etc.) [Chung ‘65][Kibble ‘68]
[Dollard ‘71][Kulish, Fadeev ‘70]
 - ✓ IR divergences cancel in S-matrix elements but
 - the construction has an intrinsic ambiguity
 - the states themselves are now IR divergent [Hannesdottir, Schwartz ‘19]

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- **Dressed states + BMS** [Kapec, Perry, Raclariu, Strominger ‘17][Akhoury, Choi ‘17]
Each Fadeev-Kulish state has ‘large gauge/BMS’ charge = 0
Too restrictive! It is enough to require the **total conservation** of BMS charges.
→ Constructed **eigenstates** of the **soft** charge and showed the equivalence to FK construction.

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From the BMS particle point of view

the dressed state construction \approx **shoehorn BMS states** into the **conventional QFT** language.

→ Dressed states are not **momentum eigenstates**, and thus not supermomentum eigenstates.
They do not belong to a separable, Lorentz invariant, Hilbert space. [Prabhu, Satishchandran, Wald ‘22]

McCarthy (1975):

there are 10 possible connected little groups:

Representatives $\varphi(z, \bar{z}) := \mathcal{K}(z, \bar{z})$ for the corresponding orbits are given by :

TABLE 2. INVARIANT VECTORS OF CONNECTED SUBGROUPS OF G

	$\Phi(G) = 0$			
	$\Phi(D) = 0$			
$\Phi(\Delta)$		$\Phi(SU(2))$	$\Phi(SL(2, R))$	
$\varphi = K$	$\Phi(J) = 0$	$\Phi(K) = 0$	$\varphi = K(1 + z ^2)^{-3}$	$\varphi = K \left(\frac{z - \bar{z}}{i} \right)^{-3} + A\delta^2 \left(\frac{z - \bar{z}}{i} \right)$
$\hat{\varphi} = K z ^{-6} + A\delta^{2,2} + C\delta$			$\hat{\varphi} = K(1 + z ^2)^{-3}$	$\hat{\varphi} = K \left(\frac{\bar{z} - z}{i} \right)^{-3} + A\delta^2 \left(\frac{\bar{z} - z}{i} \right)$
	$\Phi(\Sigma)$	$\Phi(P)$	$\Phi(\Omega)$	
$\varphi = K$		$\varphi = K z ^{-3}$	$\varphi = K \left(\frac{z - \bar{z}}{i} \right)^{-3} + A\delta^2 \left(\frac{z - \bar{z}}{i} \right)$	
$\hat{\varphi} = K z ^{-6} + A\delta^{2,2} + B\delta^{2,0} + \bar{B}\delta^{0,2} + C\delta$		$\hat{\varphi} = K z ^{-3}$	$\hat{\varphi} = K \left(\frac{\bar{z} - z}{i} \right)^{-3} + A\delta^2 \left(\frac{\bar{z} - z}{i} \right)$	
	$\Phi(\Pi)$	$\Phi(\Gamma)$	$\Phi(\Lambda)$	$\Phi(Q)$
$\varphi = r^{-3}\beta(\theta) + A\delta^{1,0} + \bar{A}\delta^{0,1}$	$\varphi = \beta(r)$	$\varphi = \beta(z + \bar{z})$		$\varphi = r^{-3}\beta(\theta)$
$\hat{\varphi} = r^{-3}\beta(-\theta) + B\delta^{1,0} + \bar{B}\delta^{0,1}$	$\hat{\varphi} = r^{-6}\beta(r^{-1})$	$\hat{\varphi} = z ^{-6}\beta(z^{-1} + \bar{z}^{-1}) + A\delta^{2,2} + B\delta^{2,0} + \bar{B}\delta^{0,2} + C\delta$		$\hat{\varphi} = r^{-3}\beta(-\theta)$
$(z = re^{i\theta})$	$(z = re^{i\theta})$			$(z = r^{(1+i\rho)}e^{i\theta})$

- 4 from Wigner

$ISO(2)$ $SU(2)$ $SL(2, \mathbb{R})$ 1

- 6 extra little groups

